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SMR 1161/3

AUTUMN COLLEGE ON PLASMA PHYSICS

25 October - 19 November 1999

Matter and Light under Extreme Conditions. High Densities Scales and Reactive Astrophysics

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Berkeley, 1999

MATTER and LIGHT UNDER EXTREME CONDITIONS

Collective and Nonlinear Interaction of Lasers, Particle Beams and Plasmas

LECTURE III

"HIGH DENSITIES"

Scales and Reactive Astrophysics.

Friday, Oct. 29
1999

Autumn College in Plasma Physics
Abdus Salam ICTP
Trieste, Italy.

FUNDAMENTALITY, NECESSITY OF SCALES AND DIVERSITY

It is not surprising that systems with different constituents in different geometrical configurations would belong to different scales and as such, would require very different methods and language of discussion.

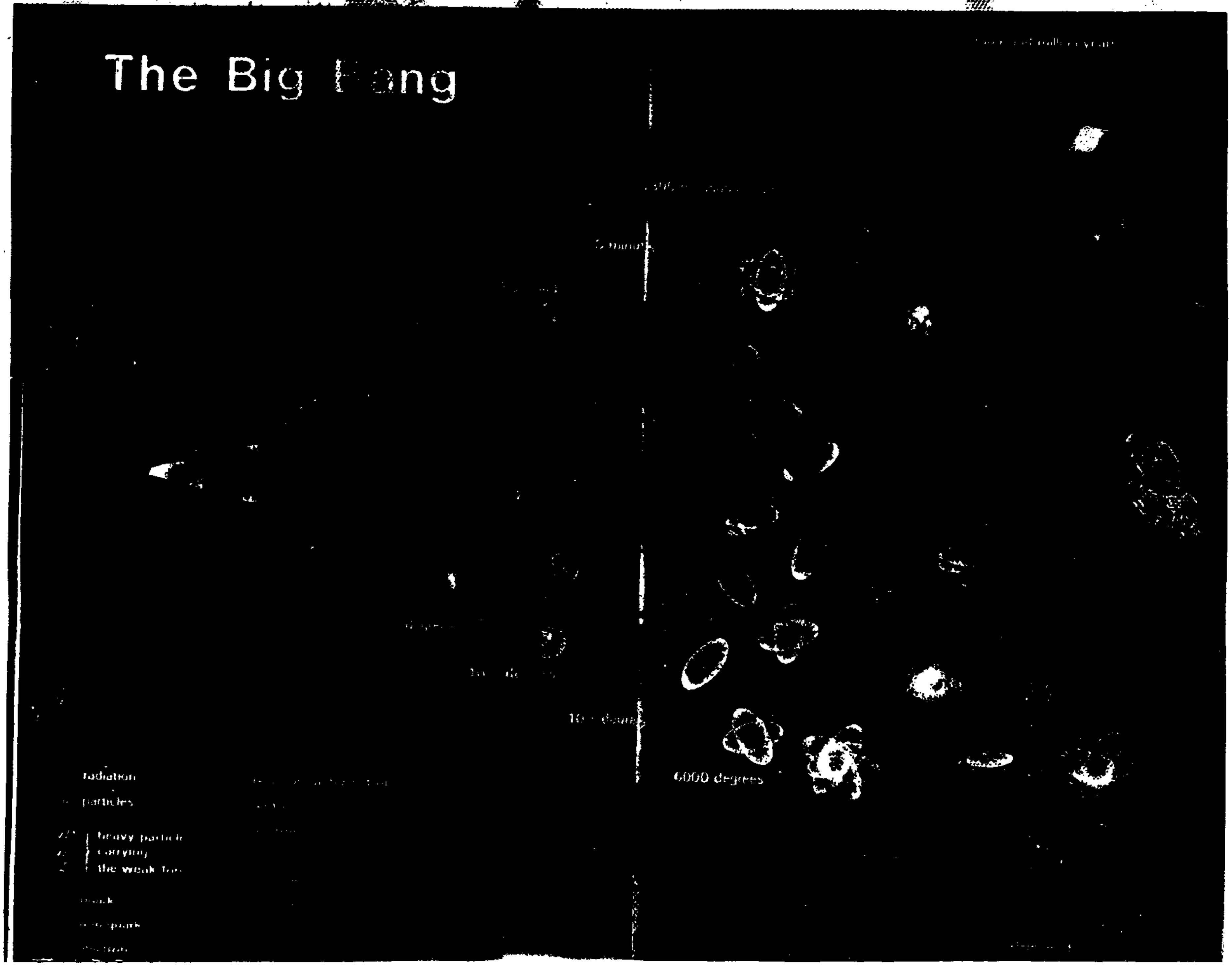
Jupiter $\not\Longleftrightarrow$ Ceramic Superconductor

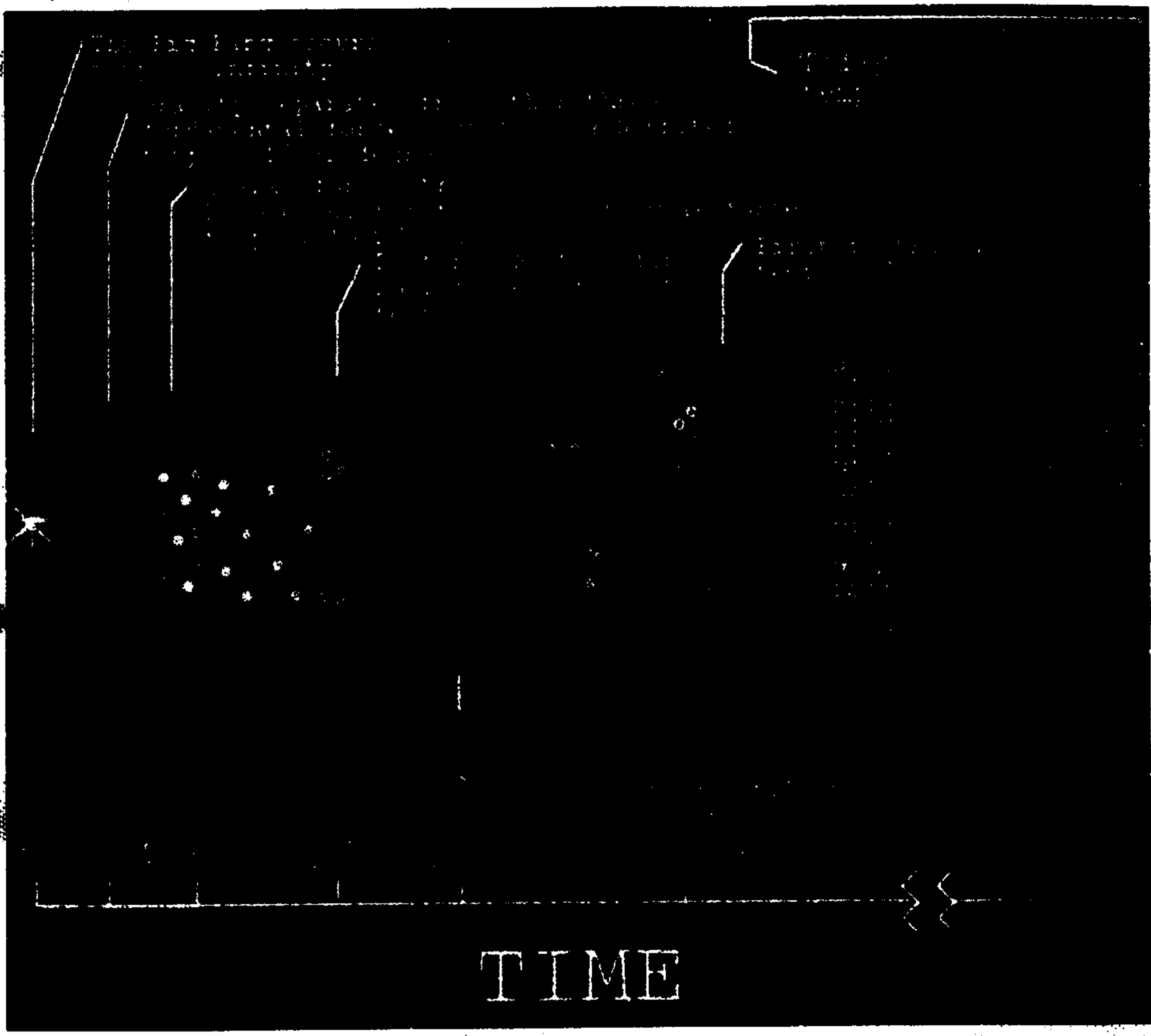
Obvious Incompatibility of scale and quality

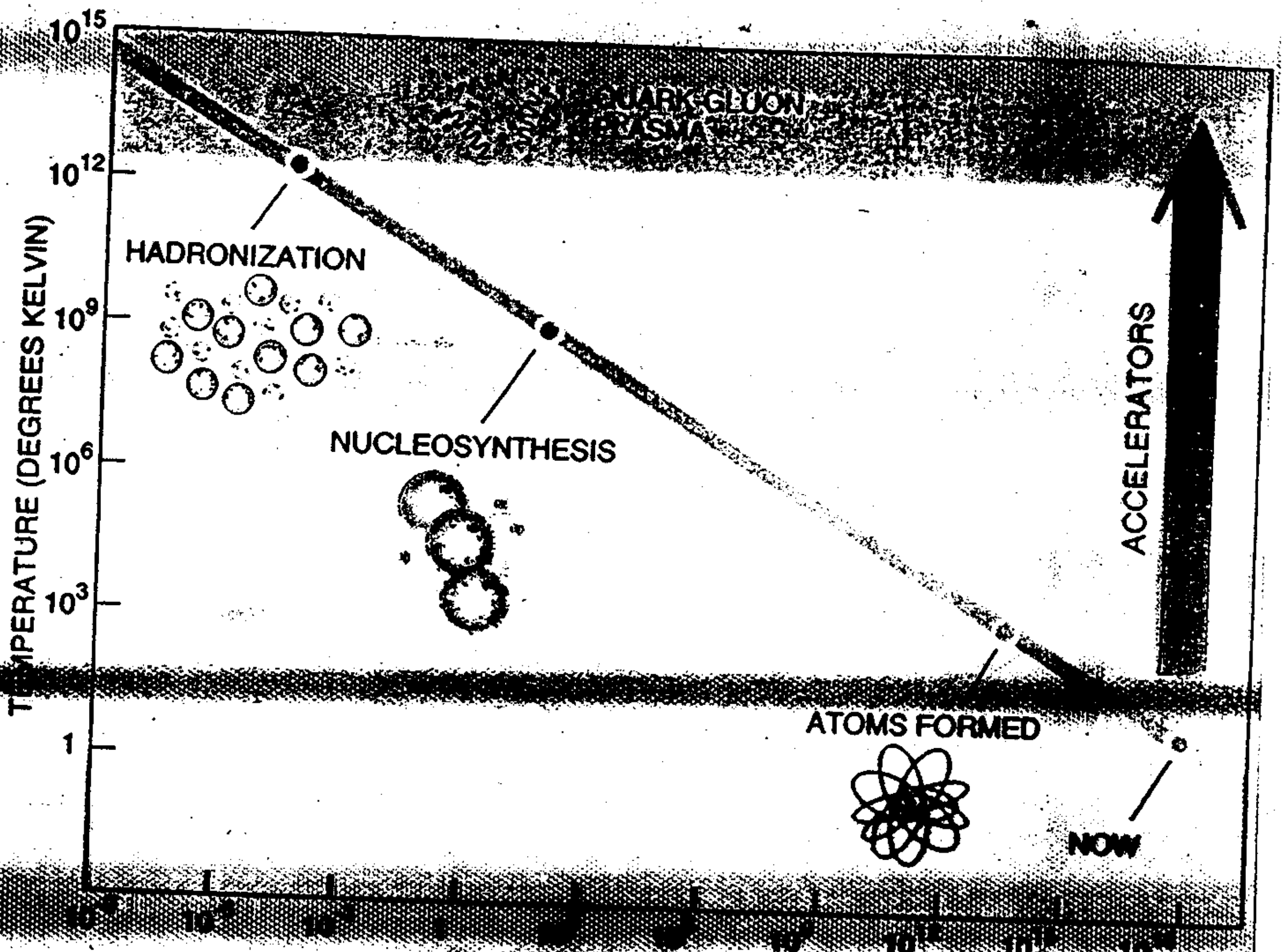
However, that systems with identical constituents in identical geometrical configuration can exhibit such different behaviors that they belong to different scales and require different methods of description is less obvious.

Quantitative Differences of "numerics" $\Longleftrightarrow$ Radically different Qualitative Behavior

The evolution of matter-radiation system in the early universe is a case in


The Big
6000 degrees
radiation
particles
carrying


TIME



TEMPERATURE OF THE UNIVERSE has been falling since the big bang. During the first microsecond, all matter is thought to have existed as quark-gluon plasma. As the universe expanded and cooled, more complex matter condensed out of the plasma, eventually forming the atoms observable today. Accelerators now under construction should be able to heat nuclei to 2×10^{12} kelvins (200 million electron volts [MeV]), perhaps creating the much sought after primordial quark matter.

Consider a physical system consisting of N_1 electrons, N_2 protons and N_3 neutrons. This system is confined in a cubical box of volume Ω, the side of the cube 'l', so that $\Omega = l^3$. Ω could be infinite; if Ω is finite, the particle collissions with the walls are perfectly elastic.

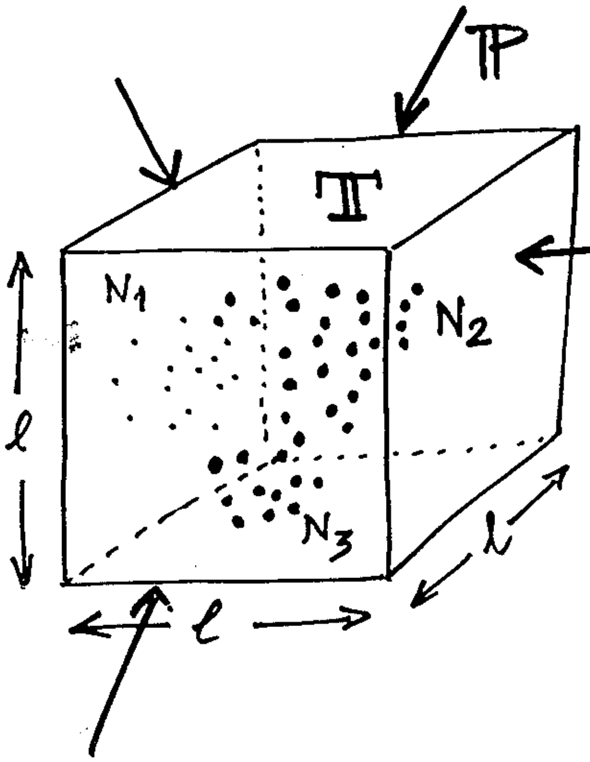
The system is conservative; if it is an isolated system, its energy is E. If the system is coupled to a heat bath, its temperature is T.

The center-of-mass of the system is at rest (the box not moving). The initial total angular momentum is L.

The system is subjected to an outside pressure p.

$$\left\{ \begin{array}{ll} \text{Ordinary electrons}, & (-e, m, s=1/2) \\ \quad\text{"}\quad \text{protons}, & (e, M, s=1/2) \\ \quad\text{"}\quad \text{neutrons} & (0, \cdots) \end{array} \right\}$$

What is the energy spectrum, mechanical and electromagnetic properties, thermodynamic



- e^- $(m, -e, 1/2\uparrow)$
- p $(M, +e, 1/2\uparrow)$
- n

$\Omega = \ell^3$ = Volume

If "Ω" is finite, elastic collisions with walls.

$\vec{P}$ = Net Momentum = 0

- CONSERVATIVE : (a) Energy $\boxed{\text{"E"}}$ is fixed if isolated ; No dissipation
  or (b) Temperature $\boxed{\text{"T"}}$ fixed if coupled to a Heat Bath
- Outside pressure : $\boxed{\mathbb{P}}$

CASE I

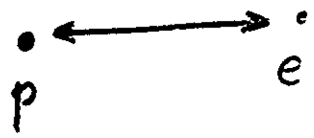
$N_1 = 1$

$N_2 = 1$

$N_3 = 0$

$\mathbb{P} = 0$

$\Omega = \infty$



$E_e = 0.01$ eV

CLASSICAL

1 - 2 Å

slow approach

$\Downarrow$

- Hydrogen-Atom
- Quantum Mechanics
- Non-relativistic

$E_e \sim 10$ MeV

$\Downarrow$

- SCATTERING
- QUANTUM MECHANICS
- Relativistic

$E_e \sim 10$ GeV

$\Downarrow$

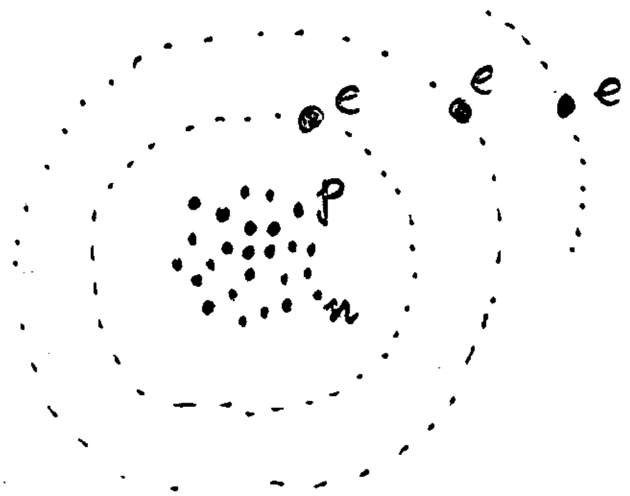
- QED
- QCD

CASE II

$N_1 = 2$ $\qquad$ $\mathbb{P} = 0$

$N_2 = 2$ $\qquad$ $\Omega = \infty$

$N_3 = 2$



$1 - 3\ \text{Å}$, $\quad .01\ \text{eV}$

$\boxed{\text{ATOMS}}$

If $E_A <$ Binding Energy

$\Longrightarrow$ Molecule H_2

$N_1, N_2, N_3 = 3, 4, 5, \ldots$ Molecular Aggregates

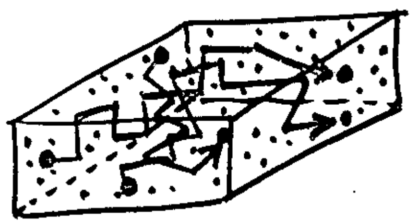
CASE III

$N_1 = N_2 = 10^{12}$

$\Omega = 1$ cubic cm.

$T = 273\ ^0K$

$P = 1$ atmosphere



Mean free path $\sim 10^{-4} - 10^{-5}$ cm.

Typical $\boxed{\text{" GAS "}}$: Kinetic Theory

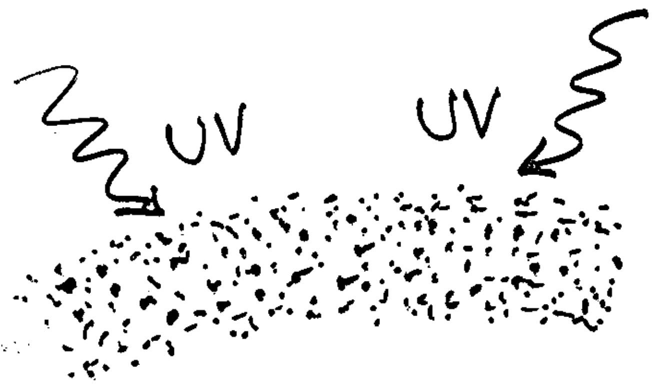
Classical Statistical Mechanics

CASE IV

$N_1 \sim 10^{10} - 10^{15}\ e^-/c.c.$

$N_2 \sim 10^{10} - 10^{15}\ /c.c.$

$T \sim 10^6\ {}^oK - 10^9\ {}^oK$



Tenuous

→ $\boxed{\text{PLASMA PHYSICS}}$ ←

Magneto Hydrodynamics

Occurs 50 km – 100 km up in the atmosphere due to ionizing UV radiation from the SUN.

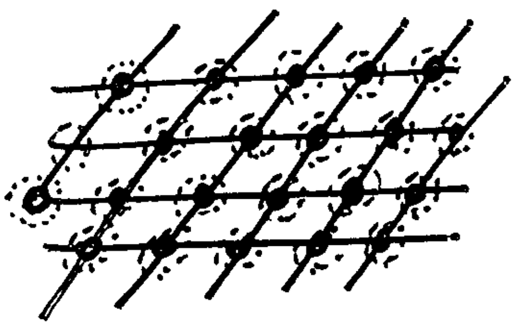
CASE V

$$N_1 = N_2 = 10^{24}$$

$$\Omega \cong 10 \text{ cc.}$$

$$T \sim 4\,^\circ K$$

$$P \sim 10^4 - 10^6 \text{ Atmospheres}$$



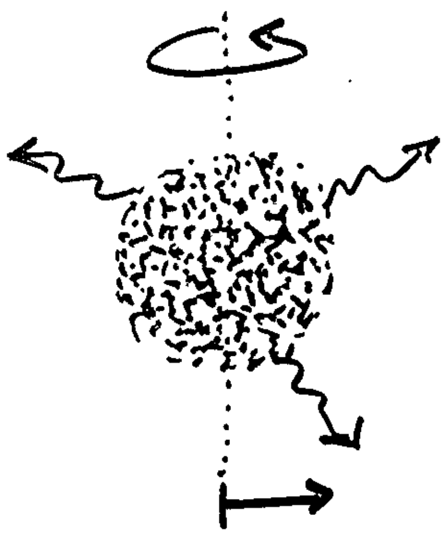
$\boxed{\text{SOLID HYDROGEN}}$

Solid State Physics,
Crystal Structures, ...

CASE VI

$N_1 \cong N_2 \sim 10^{57} \cong$ No. of nuclei in the SUN

$$\rho \sim \frac{N_1}{V} \sim \frac{N_2}{V} \sim 10^{15} - 10^{16} \ \text{g/c.c.}$$



$T \sim 5 \times 10^{6}\ {}^{\circ}K$

30 rps

$B \sim 10^{12}$ Gauss $\Uparrow$

$a \sim 5{,}000\,\text{km}$

$\rightarrow 10^{-13}$ cm.

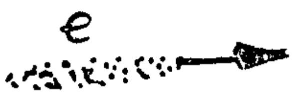
$$\boxed{\text{NEUTRON STAR}}$$

CASE VII

$N_1 \sim 10^{8-11}$

$\vec{p} \neq 0$

$N_\gamma \neq 0$



$\vec{p} \sim$ GeV/c

$T \sim$ 1–10 MeV

$\mathbb{P} \sim \ll 1$ Atmosphere

Photons come into play.

PARTICLE &
LIGHT
BEAM

CASE VIII "CLUSTERS"

10^{-13} cm : Nuclear Domain

10^{-8} cm : Atomic Domain

10^{-7} cm $\downarrow$ $10^{-4}, 10^{-3}$ cm : Mesoscopic Domain : $10^{-7} - 10^{-4}$ cm.

- macromolecules
- Buckyballs $10^{-5} - 10^{-6}$ cm.
- Rydberg Atom

10^{-4} cm

$n = 100$

- Quantum Dots $10^{-6} - 10^{-7}$ cm.

$10^{-3} - 10^{-1}$ cm : Oil Drops

0.3 cm : Macroscopic

Most recently, a new scale has "emerged"

CLUSTERS $\longrightarrow 10^{-7} - 10^{-4} - 10^{-3}$ cm.

Clusters of atoms forming a stable unit : 5 atoms - 20,000 atoms aggregrate

Macromolecules - crystals $\xleftrightarrow[\text{CLUSTERS}]{}$ macroscopic crystals

Ionization $\longrightarrow$ "CLUSTER PLASMAS"

DAWNING OF NUCLEAR PLASMA ASTROPHYSICS IN HOT DENSE STARS — UNRUH RADIATION HAWKING RADIATION and BLACK HOLES

The extremely high energy densities of matter-radiation systems experienced in 'collapsing' or 'exploding' stars lead to collective and reactive, nonlinear and dissipative structures. Under high densities with large gravitational acceleration, even light and radiation gain 'effective mass' and extreme acceleration can manifest in an effective temperature of accelerating bodies, leading to radiation e.g. UNRUH radiation from accelerating particles or the 'HAWKING radiation from a collapsing black hole.

Let us consider a light quanta, a photon, moving against the tremendous acceleration of a gravitational field, for example. In climbing out of the accelerating field, 'g' or 'a', the photon frequency gets shifted!!

Effective Mass $\equiv M_{eff} = h\nu/c^2$

The photon gains a potential energy in climbing a distance 'z' against acceleration, given by:

$$\Delta E = m_{eff} \cdot a \cdot z = \left(\frac{h\nu}{c^2}\right) a z$$

Let us consider the photons to be arising out of the vacuum fluctuations of the electromagnetic field energy, ΔE, with properties:

$$\langle \Delta E \rangle = 0 \quad \text{but} \quad \langle (\Delta E)^2 \rangle \neq 0$$

A photon, manifested out of the zero-point fluctuation of the vacuum, has the zero-point root-mean-square energy:

$$[\langle (\Delta E)^2 \rangle]^{1/2} \sim \frac{1}{2} h\nu$$

How long does this vacuum fluctuation photon hang around? We use the uncertainty principle of quantum mechanics:

$$[\langle (\Delta E)^2 \rangle]^{1/2} \cdot [\langle (\Delta t)^2 \rangle]^{1/2} \sim \frac{\hbar}{2}$$

$$\Longrightarrow \quad \overline{\Delta t} = [\langle (\Delta t)^2 \rangle]^{1/2} = \frac{(\hbar/2)}{(h\nu/2)} = (2\pi\nu)^{-1}$$

The potential energy gain, in travelling a distance $z = c \cdot \overline{\Delta t} = (c/2\pi\nu)$, is:

$$\Delta E = \left(\frac{h\nu}{c^2}\right) \cdot a \cdot \left(\frac{c}{2\pi\nu}\right) = \frac{\hbar a}{c}$$

If this energy is interpreted as a kinetic energy, $(k_B T)$ equivalent to a temperature T, then:

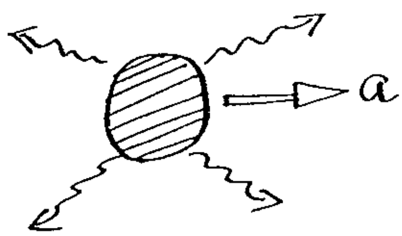
$$k_B T \simeq \frac{\hbar a}{c}$$

$$\Longrightarrow \quad T \simeq \hbar a / k_B c$$

Thus one can consider a non-charged, but conducting object to acquire an effective temperature T which is non-zero for finite non-zero acceleration. Therefore, an accelerating object must radiate as a black-body at temperature $T = \hbar a / k_B c$.

"UNRUH" Radiation

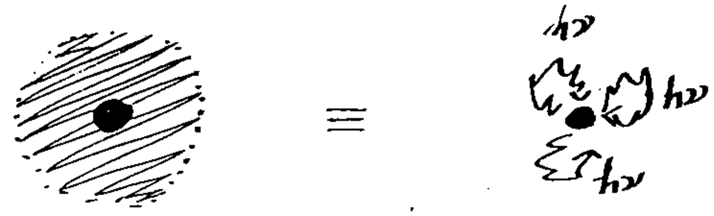
$$T = \frac{\hbar a}{k_B c}$$



Physical Interpretation: The zero-point fluctuation energy of the vacuum implies a finite rms electric field:

$$\langle vac | \tfrac{1}{2} \left(|\vec{E}|^2 + |\vec{B}|^2 \right) | vac \rangle = \langle vac | \, |E|^2 \, | vac \rangle = \tfrac{1}{2} \hbar \omega$$

This average electromagnetic field causes spontaneous emission. Thus a charged particle, like an electron, is continuously interacting with the virtual photons associated with the zero-point fluctuations, ... when at rest.



When the electron is accelerated, it still sees the ensemble of virtual photons, of course, but because of the particular form of motion, some of these 'virtual' states appear as if they were 'real' photons with a thermal distribution:

$$n(\omega)d\omega = [e^{(\hbar\omega/k_BT)} - 1]^{-1} d\omega$$

$$= [\exp(\frac{\omega c}{a}) - 1]^{-1} d\omega$$

associated with the temperature $k_BT = (\frac{\hbar a}{c})$

Consider a proton and an electron separated by an angstrom.

$$\underset{p}{\bullet} \overset{1\,\mathring{A}}{\longleftrightarrow} \underset{e^-}{\cdot} \Rightarrow a = 10^{21}\ cm/s^2.$$

For $a = 2\times10^{22}\ cm/s^2 \Rightarrow T = 1\,^\circ K$

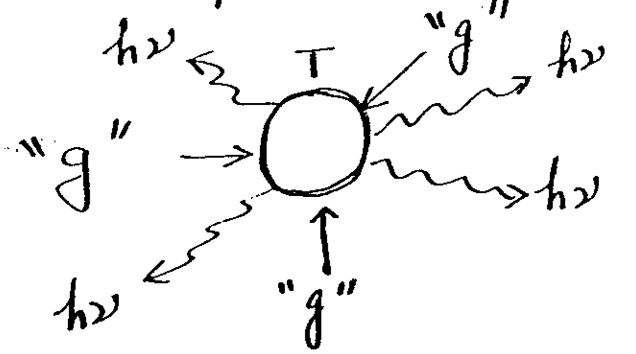
Thus an electron, moving in a laboratory-generated electric field gradient of E = 100 GV/m, can be tested for its temperature rise!!

In a simple-minded way, a star collapsing under a gravitational acceleration 'g', will acquire a temperature:

$$T = \frac{\hbar g}{k_B c}$$

$$= 6 \times 10^{-8} \left(\frac{M_\odot}{M}\right) \; ^{o}K$$

where $M_\odot = 2 \times 10^{33}$ gm. Thus a collapsing star on the way to a black hole will have a temperature and they radiate.



$g \equiv$ acceleration at surface.

"HAWKING" Radiation

The typical surface temperature of a star $\approx 10^4$ °K. As a black hole radiates, it loses mass, but "g" increases $\rightarrow$ temperature increases $\rightarrow$ radiates even more. Typical life-time $\sim 10^{71} (M_\odot/M)^{-3}$ sec. Black holes with $M < 10^{15}$ gm have evaporated by now by this Hawking radiation. ($\tau_{universe} \sim 10^{17}$ sec.). In the last 0.1 seconds of its existence, it loses .

10^{31} ergs of energy

$\equiv 10^{6}$ Megatons of Nuclear Explosion.

Such hot dense stars are characterized by high energy density plasmas with explosive reactions — collective and reactive systems far from chemical equilibrium and often from thermodynamic equilibrium. Such systems are usually nonlinear and dissipative and can undergo non-equilibrium phase-transitions to new stable states with striking behavior! These new stable states may be steady states in which:

(a) density of different species vary in space <u>or</u>

(b) spatially homogenous states exhibit time-varying densities <u>or</u>

(c) space-time varying density propagate as Nonlinear Travelling waves.

Spatially varying steady states, temporally oscillating homogenous states and nonlinear travelling waves — all have been observed in chemical and biological systems

EXPLOSIVE REACTIONS IN HIGH ENERGY DENSITY "PLASMAS" OF COSMIC NUCLEO-SYNTHESIS

Explosive reactions in high energy density plasmas, presumed to be prevalent in the era of cosmic nucleosynthesis, can be modelled by nonlinear chemical reactions that exhibit phase transitions. Consider a chemical reaction of the form:

$$\nu_A A + \nu_B B \underset{k_2}{\overset{k_1}{\rightleftarrows}} \nu_C C + \nu_D D$$

where $\{\nu_A, \cdots, \nu_D\} \equiv$ Stoichometric coefficients, ν_j denoting the number of molecules of type 'j' needed for the reaction to take place; k_1 and k_2 being the 'rate constants' — the probability per unit time that a chemical reaction takes place — for the 'forward' and 'backward' reactions.

$$\frac{dN_A}{dt} = -k_1 N_A^{|\nu_A|} N_B^{|\nu_B|} + k_2 N_C^{|\nu_C|} N_D^{|\nu_D|}$$

⇒ $|\nu_A|$ molecules of 'A' and $|\nu_B|$ molecules of 'B' must collide to destroy molecules of species 'A' while $|\nu_C|$ molecules of 'C' and $|\nu_D|$ molecules of collide to create 'A' molecules.

We consider a nonlinear chemical reaction introduced by Schlogl:

$$A + 2X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} 3X \quad \text{and} \quad X \underset{k_4}{\overset{k_3}{\rightleftharpoons}} B$$

This system exhibits a first-order phase transition analogons to a van der Wall's fluid. Thermodynamic equilibrium occurs when:

$$k_1 n_A^{eq} (n_X^{eq})^2 = k_2 (n_X^{eq})^3 \quad \underline{\text{and}} \quad k_3 n_X^{eq} = k_4 n_B^{eq}$$

where n_A^{eq}, n_B^{eq} and n_X^{eq} are the equilibrium molar densities of molecules A, B and X respectively.

At "chemical equilibrium", the ratio of equilibrium densities n_A^{eq}, n_B^{eq} is fixed and determined by the rate constants entirely:

$$\frac{n_A^{eq}}{n_B^{eq}} = R_{eq} = \left(\frac{k_4 k_2}{k_3 k_1}\right) .$$

Let us now hold n_A and n_B fixed at some nonequilibrium values, $n_A \equiv n_A^0 \neq n_A^{eq}$ and $n_B \equiv n_B^0 \neq n_B^{eq}$ and allow n_X to vary.

Thus $R = n_A^0 / n_B^0 \neq R_{eq.}$ and the steady state concentration $\bar{n}_X$ of the intermediate species X need not be a state of thermodynamic equilibrium even, not to speak of chemical equilibrium. The rate equation of n_X can be written :

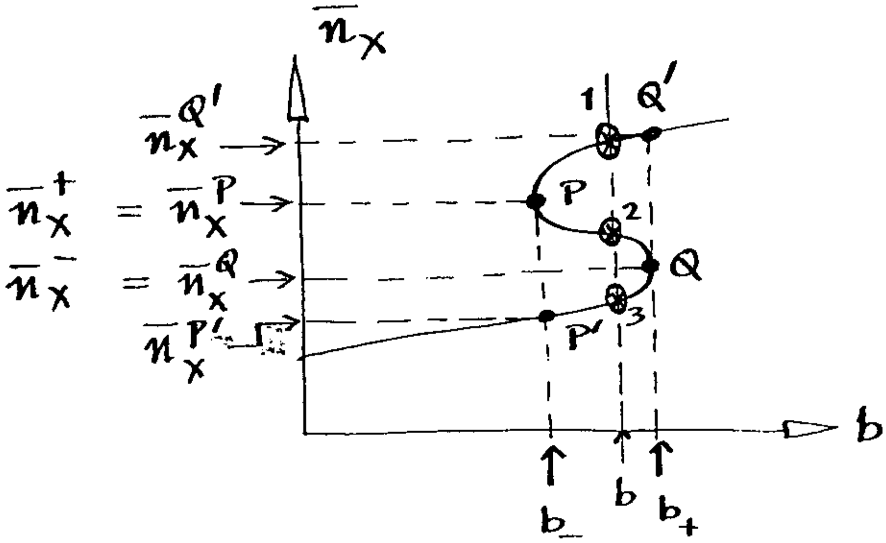
$$\frac{dn_X}{dt} = k_1 n_A^0 n_X^2 - k_2 n_X^3 - k_3 n_X + k_4 n_B^0$$

and the steady-state solution $\bar{n}_X$, satisfying $\dot{\bar{n}}_X = d\bar{n}_X/dt = 0$ is given by the solution(s) of a cubic equation :

$$n_X^3 - a n_X^2 + K n_X - b = 0$$

where $a = k_1 n_A^0 / k_2$, $K = k_3/k_2$ and $b = k_4 n_B^0 / k_2$. This is analogous to van der Waal's equation and has similar properties.

We now consider $\bar{n}_X$ as a function of 'b' for fixed values of 'a' and 'K' and draw the line of 'steady state' solutions of the Schlogle Model.



$\boxed{a > \sqrt{3K}}$

($a < \sqrt{3K}$, curve is 'monotonic' in both 'b' and $\bar{n}_X$ and only one steady state solution for each value of 'b' possible)

The $(\bar{n}_X^{+}, \bar{n}_X^{-})$ can be easily found from the extrema of the curve $b \equiv b(\bar{n}_X)$ as the roots of the equation:

$$\frac{db}{d\bar{n}_X} = 3\bar{n}_X^{2} - 2a\bar{n}_X + K = 0$$

$$\Longrightarrow \bar{n}_X^{\pm} = \frac{1}{3}\left(a \pm \sqrt{a^2 - 3K}\right)$$

and the values $b_{\pm} = b(\bar{n}_X^{\pm})$ are found easily as:

$$b_{\pm} = (\bar{n}_X^{\pm})^3 - a(\bar{n}_X^{\pm})^2 + K\bar{n}_X^{\pm}$$

Thus for $a > \sqrt{3K}$, there is a range of values of b, $b_- \leq b \leq b_+$, for which the system can exist in three different steady states, in principle! However, they can be realized in nature only if they are stable
$\Longrightarrow$ requires Linear Stability Analysis around

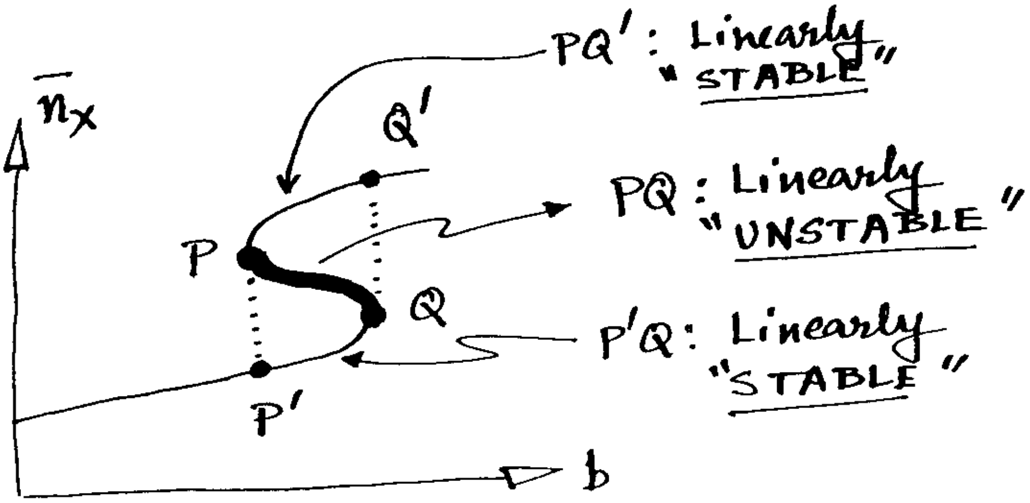
For linear stability around stationary solution $\bar{n}_X$, let

$$n_X(t) = \bar{n}_X + \delta n_X(t)$$

where $\delta n_X(t)$ is a small fluctuation about the steady state, $\bar{n}_X$. We substitute this $n_X(t)$ in

$$n_X^3(t) - a\, n_X^2(t) + K\, n_X(t) - b = 0$$

and keep only terms of first order i.e. linear in $\delta n_X(t)$. We obtain a linearized equation for $\delta n_X(t)$ $\Rightarrow$ Eigensolution of of stability for this linear equation shows that $\delta n_X(t)$ decays exponentially in regions $P'Q$ and PQ', but grows exponentially in region PQ. Since fluctuations are inevitable, the system will be driven away from steady state in the region PQ and we cannot find it occurring naturally.

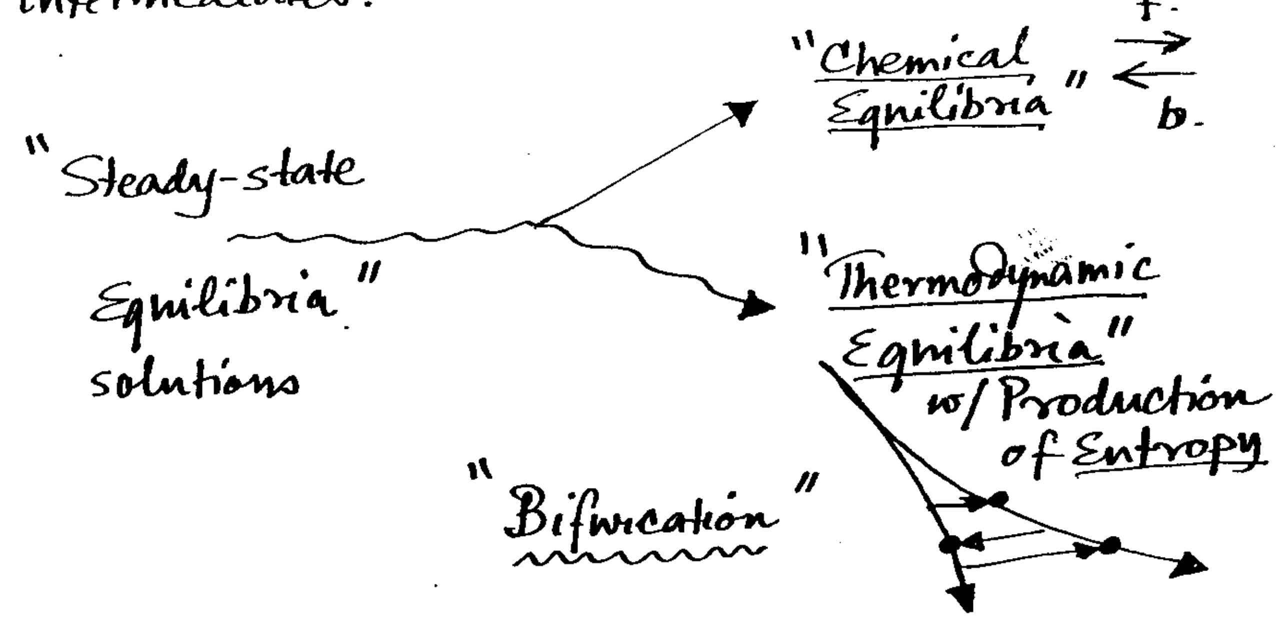


Let $P' \to Q$ as 'b' increases. When we reach Q, it
$Q \to Q'$ from steady with molar

APPEARENCE OF "DISSIPATIVE STRUCTURES" IN REACTIVE HIGH ENERGY DENSITY SYSTEMS

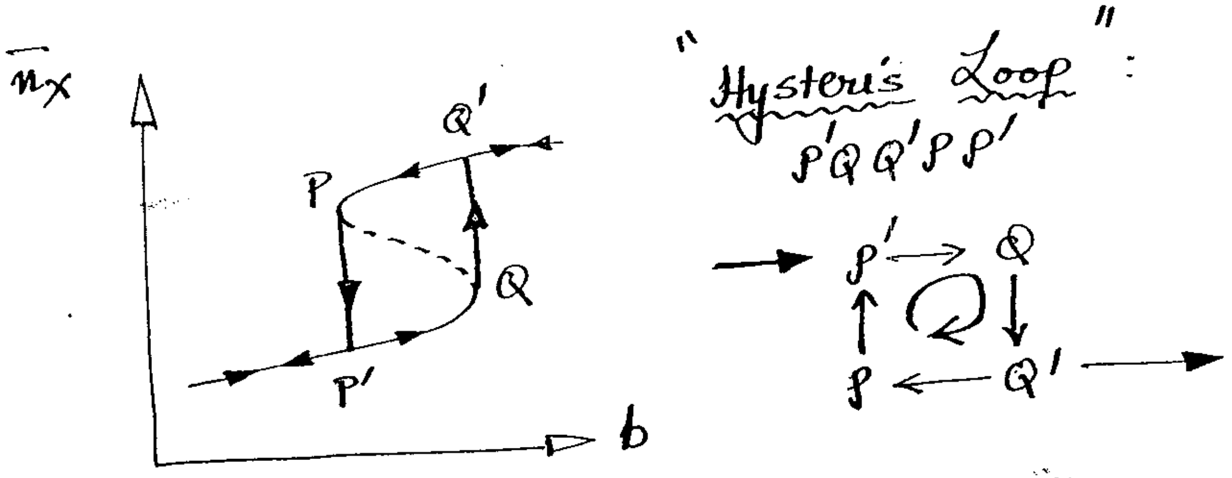
Reactive high energy density systems can be in local thermodynamic equilibrium and yet held far from chemical equilibrium. An example is the Belousov-Zhabotinski reaction (B-Z reaction).

The qualitative behavior of the B-Z type reaction occurs in simpler models such as the Brusselator, first introduced by Prigogine and Lefever which involve "two-variable intermediates."

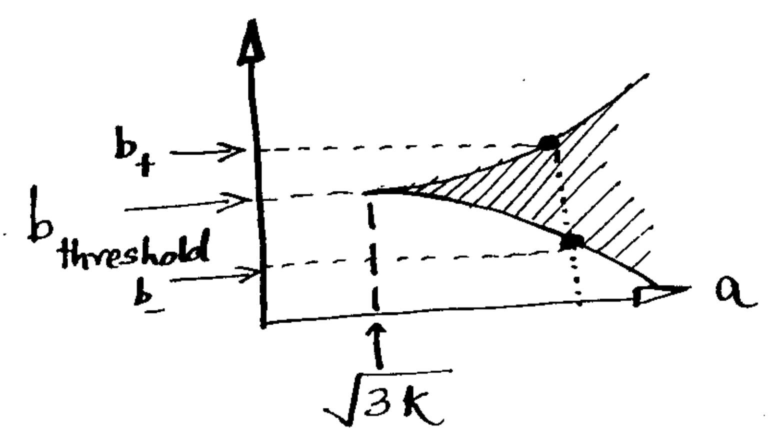


Prigogine has called these fascinating equilibrium states far from chemical equilibrium — "Dissipative Structures".

density $\bar{n}_X^Q$ to one with molar density $\bar{n}_X^{Q'}$, for values $b \geqslant b_+$. Thus the chemical system appears to undergo an abrupt transition at $b = b_+$. In the reverse direction, it also exhibits the behavior of "HYSTERESIS" — as we decrease 'b', we remain on curve $Q'P$ until we reach $b = b_-$. Then the system goes through an abrupt change $P \to P'$, a transition from a steady state with molar density $\bar{n}_X^P$ to one with molar density $\bar{n}_X^{P'}$.



We can plot the hysteresis region as a function of 'a' and 'b'. Such behavior has been used as a model for "EXPLOSIVE REACTIONS" in plasma nucleosynthesis in cosmology.



" ... STRUCTURES"

Spatially varying steady states, temporally oscillating homogenous states and nonlinear travelling waves — have been observed in chemical and biological systems.

e.g. Artist's impression of travelling waves in the chemical concentrations of the <u>Belousov–Zhabotinski</u> reaction (B-Z)

"Cerium ion catalyzed oxidation of malonic acid by bromate in a sulphuric acid medium"

3-variable intermediates model: "<u>OREGONATOR</u>"

Nonlinear reaction involving "<u>auto</u>-<u>catalytic</u> <u>steps</u>". When well-stirred, can behave like a "<u>chemical clock</u>".

$\Rightarrow$ Periodic change in the concentration of Br^- and of relative concentration Ce^{4+}/Ce^{3+}. Blue $\rightarrow$ Red $\rightarrow$ Blue $\rightarrow \ldots$ with time-period $\sim$ 1 minute. Travelling waves of concentration observed in shallow unstirred dishes"

Their existence depend upon dissipative processes, such as chemical reactions far from equilibrium or diffusion (if spatial structures occur). These spatio-temporal structures are maintained by "production of entropy" and by a related flow of energy and matter from the world.

Autocatalytic reactions of the type that produce "dissipative structures" are abundant in living systems $\Longrightarrow$ important role of dissipative structures in the maintainance of life processes in living systems $\Longrightarrow$

- Chemical Clock
- Biological cycle
- Recurring states in astronomy ...

Example: BRUSSELATOR

$$A \xrightarrow{k_1} X$$

$$B + X \xrightarrow[k_2]{} Y + D$$

$$2X + Y \xrightarrow[k_3]{} 3X$$

and

$$X \xrightarrow[k_4]{} E$$

[Implement^{r}:
- A, B present in excess
- D, E removed as soon as they appear]