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Fundamentals of Toroidal Gyrofluid Simulations

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Outline:

- Basic physics of key instabilities: ITG + TEM
- Derivation of Gyrofluid Eqns for ions + trapped electrons
- Important Results from gyrofluid simulations
- Toroidal simulation geometry

Motivation:

- Let's not solve the 5D nonlinear GK eqn $F(v_\parallel, \mu, x, y, z)$ (if we don't have to)
- Fluid models can be used for insight

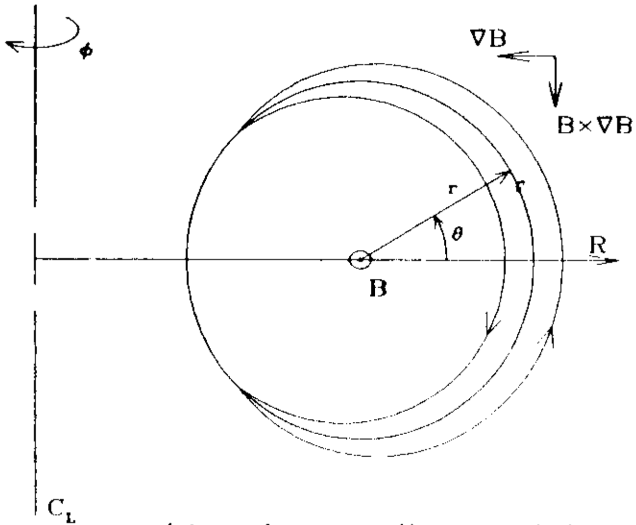
Guiding Center Drifts and Toroidal Precession

Averaged over the fast gyromotion ($\Omega \gg \omega$), particle guiding centers drift with the velocity:

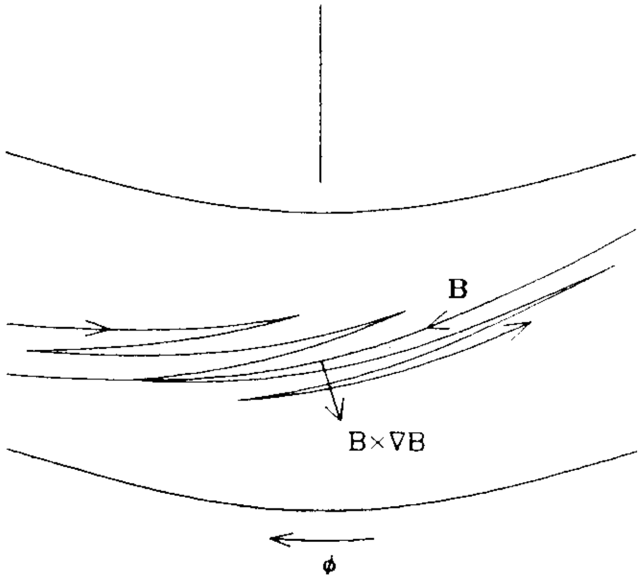
$$\mathbf{v}_{\mathbf{GC}} = v_{\|}\hat{\mathbf{b}} + \mathbf{v}_E + \mathbf{v}_d$$

$$\mathbf{v}_d = \frac{v_{\|}^2 + v_{\perp}^2/2}{\Omega B^2}\mathbf{B} \times \nabla B \qquad\qquad \mathbf{v}_E = \frac{c}{B^2}\mathbf{E} \times \mathbf{B}$$

Combination of $v_{\|}\hat{\mathbf{b}}$, $\mathbf{v_d}$ drifts, and parallel $\mu\nabla_{\|}B$ parallel force lead to trapped particle "banana" orbits.



Averaged over a bounce orbit, the small toroidal component of the curvature and ∇B drifts lead to toroidal "precession."



Length and Time Scales for Plasma Turbulence

A quantitative model of plasma turbulence and transport requires a fairly comprehensive model.

transit frequency for passing particles: $\omega_t = k_{\|} v_t = v_t / qR$

bounce frequency for trapped particles: $\omega_b = k_{\|} \sqrt{\epsilon} v_t = \sqrt{\epsilon} v_t / qR$

Ion species: $\omega_{bi} < \omega \sim \underset{k_\| v_{ti}}{\omega_{ti}} \ll \nu_{ii} \ll \Omega_i,$ $\qquad k_\perp \rho_i \sim 1$

- low collisionality ⇒ kinetic effects important
- $\omega_{bi} < \omega$ ⇒ trapped ions don't have time to bounce
- ion Finite Larmor Radius (FLR) effects important

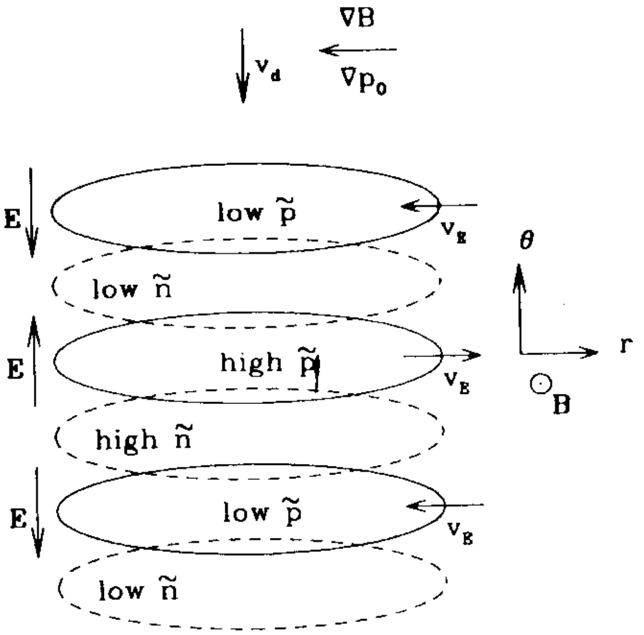
Electrons: $\omega \sim \nu_{ei} \ll \omega_{be} < \underset{k_\| v_{te}}{\omega_{te}} \ll \Omega_e,$ $\qquad k_\perp \rho_e \ll 1$

- collisions important, but collisionless resonances can persist
- fast parallel electron motion ⇒ adiabatic response: $\tilde{n}_e = e\tilde{\Phi}/T_e$
- trapped electrons dominate electron nonadiabatic response
- electron Finite Larmor Radius (FLR) effects negligible

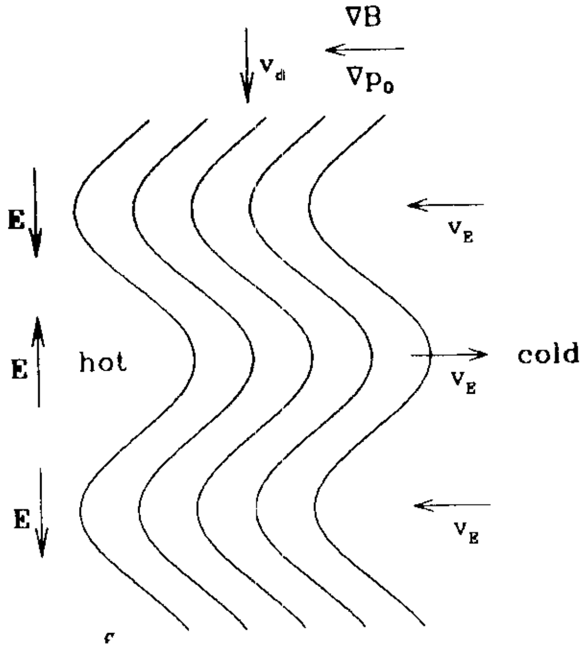
To describe this turbulence, we need ion "gyrofluid" and trapped electron fluid equations which capture kinetic effects, coupled by quasineutrality $\tilde{n}_i = \tilde{n}_e$, and solved in

- fully nonlinear 3D simulations
- in toroidal flux tube geometry ($\hat{s} - \alpha$ for simplicity)

Physical Picture of the Toroidal ITG Mode



Contours of perturbed $\tilde{p}$ and $\tilde{n}$



Contours of total $p=p_0+\tilde{p}$

- A pressure perturbation, $\tilde{p}$, causes a density perturbation, $\tilde{n}_i$, through the velocity dependence of

$$\mathbf{v}_d = \frac{v_\parallel^2 + v_\perp^2/2}{\Omega B^2}\mathbf{B} \times \nabla B.$$

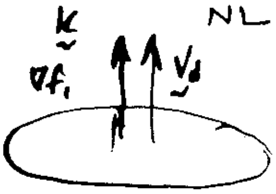
- This $\tilde{n}_i$ produces Φ through quasineutrality: $\tilde{n}_i = \tilde{n}_e = e\Phi/T_e$
- The potential leads to radial $\mathbf{E} \times \mathbf{B}$ convection of the equilibrium, amplifying $\tilde{p}$ on the outer midplane, and damping $\tilde{p}$ on the inner midplane.

Simple Derivation of Toroidal ITG Threshold : $\frac{R}{L_{T\,crit}}$

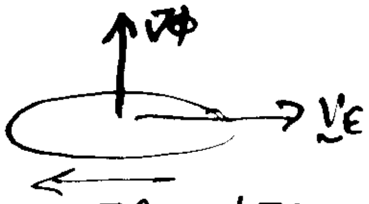
eqn : $$\frac{\partial f}{\partial t} + \left(v_\parallel \hat{b} + \underset{\sim}{v}_E + \underset{\sim}{v}_d\right)\cdot\nabla f + \left(\frac{q}{m}E_\parallel - \mu\nabla_\parallel B + v_\parallel(\hat{b}\cdot\nabla\hat{b})\cdot v_E\right)\frac{\partial f_0}{\partial v_\parallel} = 0$$

$$\underset{\sim}{v}_d = \frac{v_\parallel^2 + \frac{1}{2}v_\perp^2}{\Omega B^2}\underset{\sim}{B}\times\nabla B \qquad \underset{\sim}{v}_E = \frac{c}{B}\hat{b}\times\nabla\phi$$

$$\frac{\partial f_1}{\partial t} + v_E\cdot\nabla f_0 + \underbrace{v_E\cdot\nabla f_1}_{NL} + v_d\cdot\nabla f_1 - \frac{v_\parallel^2}{v_t^2}(\hat{b}\cdot\nabla\hat{b})\cdot\underset{\sim}{v}_E f_0 = 0$$



$\hat{b}$ ⊗



$$f_0 = \frac{n}{\left(2\pi\frac{T}{m}\right)^{3/2}}e^{-\frac{mv^2}{2T}} \qquad \nabla f_0 = -\frac{f_0}{L_T}\left(\frac{v^2}{2}-\frac{3}{2}\right)\hat{r}$$

$\frac{1}{B}\nabla B \approx \hat{b}\cdot\nabla\hat{b}$

$\nabla n_0 = 0$

$$\frac{\partial f_1}{\partial t} - i\omega_{*T} f_0\left(\frac{v^2}{2v_t^2}-\frac{3}{2}\right)\phi + i\omega_d\left(\frac{v_\parallel^2}{v_t^2} + \frac{\frac{1}{2}v_\perp^2}{v_t^2}\right)f_1 + 2i\omega_d f_0\phi\frac{v_\parallel^2}{v_t^2} = 0$$

$$\omega_{*T} = \frac{k_\theta\rho v_t}{L_T} \qquad \rho = \frac{v_t}{\Omega} \qquad \omega_d = \frac{k_\theta\rho v_t}{R}$$

$\Rightarrow\int d^3v f_1$: $\frac{\partial n}{\partial t} + 2i\omega_d p_\perp + 2i\omega_d\phi = 0$

assume $v_\parallel^2 \approx \frac{1}{2}v_\perp^2$ $\qquad \langle v^2 f_0\rangle = 3 \qquad \langle v_\parallel^2 f_0\rangle = 1$

$\Rightarrow\int d^3v \frac{v_\perp^2}{2}f_1$: $\frac{\partial p_\perp}{\partial t} - i\omega_{*T}\phi + i\omega_d\left(\langle v_\parallel^2 v_\perp^2 f_1\rangle + \frac{1}{2}\langle v_\perp^4 f_1\rangle\right) + 3i\omega_d\phi =$

$\langle v^4 f_0\rangle = 5 \qquad$ assume $\approx 6p - 3n \qquad \langle v_\parallel^4 f_0\rangle = 3$

if Maxwellian f_1

$$-i\omega n - 2i\omega_d p + 2i\omega_d\phi = 0$$

$$-i\omega p - i\omega_{*T}\phi + i\omega_d(6p-3n) + 3i\omega_d\phi = 0$$

Simple Derivation of Toroidal ITG Threshold (cont'd)

fast ∥ electron motion $\Rightarrow \frac{\tilde{n}_e}{n_0} = \frac{e\tilde{\phi}}{T_e}$

and Quasi-neutrality $\Rightarrow \tilde{n}_e = \tilde{n}_i = \phi$

n: $(\omega - 2\omega_d)\,\phi = 2\omega_d\, p \longrightarrow p = \frac{\omega - 2\omega_d}{2\omega_d}\phi$

p: $(\omega - 6\omega_d)\, p = -\omega_{*T}\,\phi$

$$(\omega - 6\omega_d)(\omega - 2\omega_d) = -2\omega_{*T}\,\omega_d$$ Disp. Reln.

$$\omega^2 - 8\omega_d\omega + 12\omega_d^2 + 2\omega_{*T}\omega_d = 0$$

$$\omega = 4\omega_d \pm \frac{1}{2}\sqrt{64\omega_d^2 - 4\left(12\omega_d^2 + 2\omega_{*T}\omega_d\right)}$$

$$= 4\omega_d \pm 2\omega_d\sqrt{1 - \frac{\omega_{*T}}{2\omega_d}}$$

Strongly unstable limit: $\omega_{*T} \gg \omega_d \Rightarrow \omega = i\sqrt{2\omega_{*T}\omega_d}$

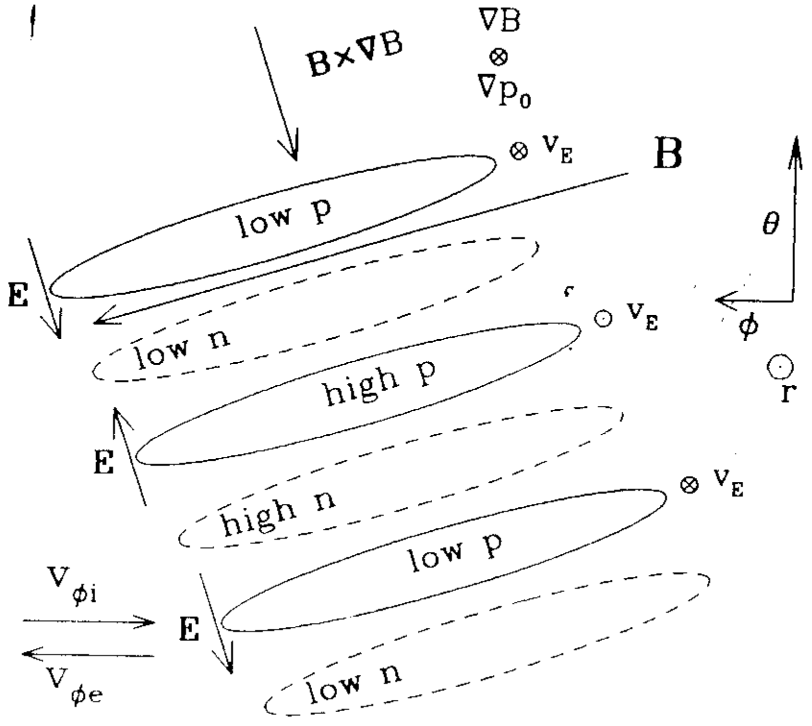
Threshold: $\frac{\omega_{*T}}{\omega_d} = 2 \qquad \frac{k_\theta \rho v_t}{L_T} \cdot \frac{R}{k_\theta \rho v_t} = 2 \Rightarrow \frac{R}{L_{Tc}} = 2$

Kinetic treatment $\Rightarrow \frac{R}{L_{Tc}} \approx 3$

Including ∥ ion dynamics, impurity dilution, density gradients $\Rightarrow \frac{R}{L_{Tc}} \approx 4\text{-}5$

Toroidal Precession Drifts Destabilize Trapped Electron Mode

- Side view of instablity on outer midplane
- Landau damping wipes out high $k_{\|}$ components so perturbations are elongated along field lines



Contours of perturbed $\tilde{p}$ and $\tilde{n}$

- Trapped electron precession drifts can resonate with and destabilize a wave which propagates in the opposite direction than the ITG mode, called the Trapped Electron Mode (TEM)

Closure Problem in the Fluid Moment Hierarchy

Generic Vlasov/Boltzmann Equation:

$$\frac{df}{dt} + \dot{q}_i \frac{\partial f}{\partial q_i} + \dot{p}_i \frac{\partial f}{\partial p_i} = C(f)$$

In classical fluids, $\lambda_{\rm mfp} \approx 10^{-4}{\rm cm}$ keeps $f(p_i, q_i)$ near **Maxwellian**, described by (n, u, T).

In plasmas, collisions are very weak $(\lambda_{\rm mfp} \sim 10^{3}{\rm m})$ and $f(p_i, q_i)$ can deviate dramatically from Maxwellian.

$$\frac{\partial f}{\partial t} + v_{\|} \frac{\partial f}{\partial z} + \frac{e}{m} E_{\|} \frac{\partial f}{\partial v_{\|}} = 0$$

$$n = \int d^3 v f : \quad \frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(n u_{\|}) = 0$$

$$n u_{\|} = \int d^3 v \, v_{\|} f : \quad \frac{\partial}{\partial t}(n m u_{\|}) + \frac{\partial p_{\|}}{\partial z} - e n E_{\|} = 0$$

$$p_{\|} = \int d^3 v \, v_{\|}^2 f : \quad \frac{\partial p_{\|}}{\partial t} + \frac{\partial q_{\|}}{\partial z} - 2 e n E_{\|} = 0$$

$$q_{\|} = \int d^3 v \, v_{\|}^3 f : \quad \frac{\partial q_{\|}}{\partial t} + \frac{\partial r_{\|}}{\partial z} - 3 e n E_{\|} = 0$$

$$r_{\|} = \int d^3 v \, v_{\|}^4 f : \quad \vdots$$

The $v_{\|}\frac{\partial f}{\partial z}$ free streaming term introduces higher moments into each exact fluid equation. These higher moments must be closed in terms of lower moments.

Closures $\propto |k_\parallel|$ Model Parallel Phase Mixing

Consider only the phase mixing from parallel free-streaming:

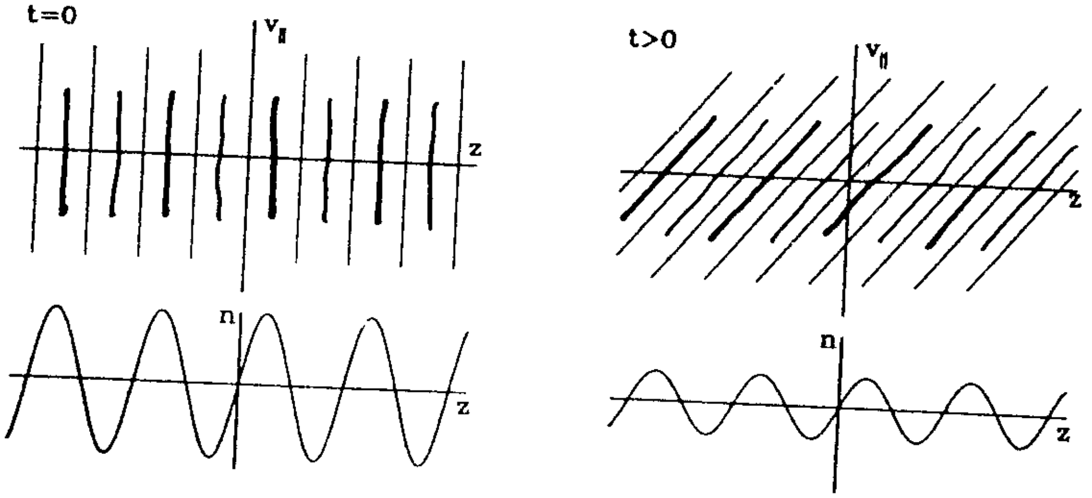
$$\frac{\partial f}{\partial t} + v_\parallel \frac{\partial f}{\partial z} = 0,$$

Solution: $f(z, v_\parallel, t) = f(z - v_\parallel t, v_\parallel, t = 0)$.
Start with a Maxwellian perturbation in f:

$$f_0 = e^{ik_\parallel z} f_M = e^{ik_\parallel z} \frac{n_0}{\sqrt{2\pi v_t^2}} e^{-v_\parallel^2/2v_t^2},$$

Free streaming will cause moments of f to phase mix away. For example, the density is:

$$n = \int d^3v\, f = \frac{n_0}{\sqrt{2\pi v_t}} \int dv_\parallel e^{ik_\parallel (z - v_\parallel t)} e^{-v_\parallel^2/2v_t^2} = n_0 e^{ik_\parallel z} e^{-k_\parallel^2 v_t^2 t^2/2}$$



Need to add damping proportional to $|k_\parallel| v_t$.

Closures $\propto |\omega_d|$ Model Toroidal Drift Phase Mixing

Consider only the phase mixing from toroidal drifts:

$$\frac{\partial f}{\partial t} + v_d \frac{\partial f}{\partial y} = 0, \qquad v_d = v_{d0} \frac{v_\parallel^2 + v_\perp^2/2}{v_t^2}, \qquad v_{d0} = \frac{\rho v_t}{R}.$$

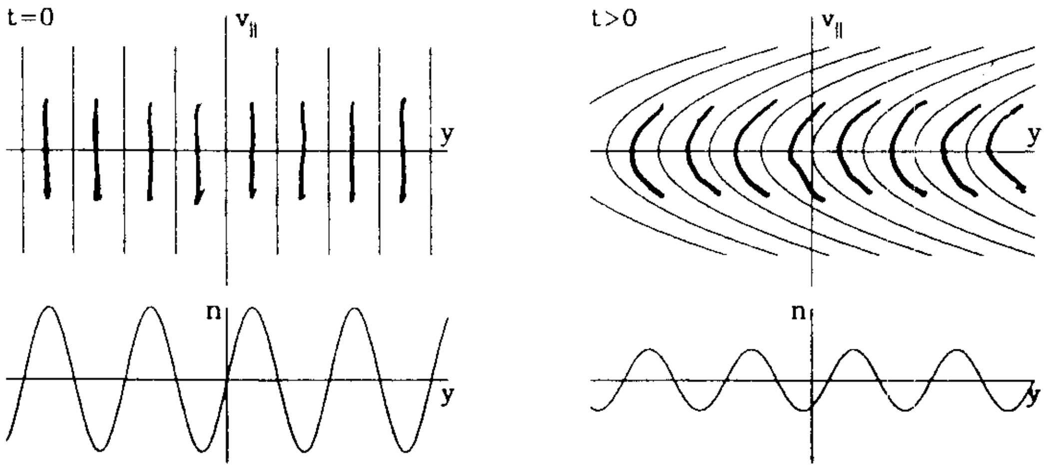
Solution: $f(y, v_\parallel, v_\perp, t) = f(y - v_d t, v_\parallel, v_\perp, t = 0)$.
Start with a Maxwellian perturbation in f:

$$f_0 = e^{ik_y y} f_M = e^{ik_y y} \frac{n_0}{(2\pi v_t^2)^{3/2}} e^{-(v_\parallel^2 + v_\perp^2)/2v_t^2},$$

Free streaming will cause moments of f to phase mix away. For example, the density is:

$$n = \int d^3v\, f = \frac{n_0}{\sqrt{2\pi v_t^3}} \int dv_\parallel dv_\perp v_\perp e^{ik_y(y - v_{d0} t \frac{v_\parallel^2 - v_\perp^2/2}{v_t^2})} e^{-\frac{v_\parallel^2 + v_\perp^2}{2v_t^2}}$$

$$= \frac{n_0 e^{ik_y y}}{\sqrt{1 + ik_y v_{d0} t}(1 + ik_y v_{d0} t/2)}.$$



Need to add damping proportional to $|k_y| v_{d0} = |\omega_d|$.

Toroidal Gyrofluid Equations for Ion Species

[Beer & Hammett, PoP 3, 4046 (1996)]

For ions, evolve moments of nonlinear electrostatic toroidal gyrokinetic eqn. $(n, u_\parallel, T_\parallel, T_\perp, q_\parallel, q_\perp)$: **[Frieman&Chen, Lee, Dubin, Krommes, Hahm]**

$$\frac{\partial f}{\partial t} + (v_\parallel \hat{\mathbf{b}} + \bar{\mathbf{v}}_E + \mathbf{v}_d) \cdot \nabla f + \left(\frac{e}{m} \bar{E}_\parallel - \mu \nabla_\parallel B + v_\parallel (\hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}}) \cdot \bar{\mathbf{v}}_E \right) \frac{\partial f}{\partial v_\parallel} = C(f)$$

$$\begin{aligned}
\frac{\partial n}{\partial t} \; & + \; \bar{\mathbf{v}}_E \cdot \nabla n + B \nabla_\parallel \frac{u_\parallel}{B} - \left(1 + \frac{\eta_\perp}{2} \hat{\nabla}_\perp^2 \right) i\omega_* \bar{\Phi} \\
& + \; \left(2 + \frac{3}{2} \hat{\nabla}_\perp^2 - \hat{\hat{\nabla}}_\perp^2 \right) i\omega_d \bar{\Phi} + i\omega_d (p_\parallel + p_\perp) = 0 \\
\frac{\partial u_\parallel}{\partial t} \; & + \; \bar{\mathbf{v}}_E \cdot \nabla u_\parallel + B \nabla_\parallel \frac{p_\parallel}{B} + \nabla_\parallel \bar{\Phi} + \left(p_\perp + \frac{1}{2} \hat{\nabla}_\perp^2 \bar{\Phi} \right) \frac{1}{B} \nabla_\parallel B \\
& + \; i\omega_d (q_\parallel + q_\perp + 4 u_\parallel) = 0 \\
& \;\; \vdots \\
\frac{\partial q_\parallel}{\partial t} \; & + \; \bar{\mathbf{v}}_E \cdot \nabla q_\parallel + (3 + \beta_\parallel) \nabla_\parallel T_\parallel + \sqrt{2} D_\parallel |k_\parallel| q_\parallel \\
& + \; i\omega_d (-3 q_\parallel - 3 q_\perp + 6 u_\parallel) + i |\omega_d| (\nu_5 u_\parallel + \nu_6 q_\parallel + \nu_7 q_\perp) = -\nu_{ii} q_\parallel
\end{aligned}$$

- each moment equation has $\mathbf{E} \times \mathbf{B}$ nonlinear term
- toroidal terms: $i\omega_d \equiv (cT/eB^3) \mathrm{B} \times \nabla B \cdot \nabla$
- H&P type parallel and toroidal closures: $|k_\parallel|$, $|\omega_d|$
- trapped ion CGL terms, ion-ion collisions (ν_{ii})
- FLR closures, $\hat{\nabla}_\perp$, $\hat{\hat{\nabla}}_\perp$

Quasineutrality:

$$n_e = n_i$$

$$n_{e0} \left(\frac{e\Phi}{T_e} - \langle \frac{e\Phi}{T_e} \rangle \right) = \bar{n}_i - n_{i0} (1 - \Gamma_0) \frac{e\Phi}{T_i}$$

$$\vdots$$

Bounce Averaged Trapped Electrons

Derived by taking moments of the nonlinear bounce averaged drift kinetic equation [**Gang and Diamond, PFB 1990**]:

$$\left(\frac{\partial}{\partial t} + i\left\langle\omega_{de}\right\rangle_b\right)\left\langle f_e\right\rangle_b + \hat{b}\times\nabla\left\langle\Phi\right\rangle_b\cdot\nabla\left\langle f_e\right\rangle_b$$
$$= \left\langle C\right\rangle_b\left(\left\langle f_e\right\rangle_b - \left\langle\Phi\right\rangle_b\right) - iF_M(\omega_{*e}^T - \left\langle\omega_d\right\rangle_b)\left\langle\Phi\right\rangle_b$$

$$f_e = F_M(\Phi - \left\langle\Phi\right\rangle_b) + \left\langle f_e\right\rangle_b$$

$\left\langle f_e\right\rangle_b(x, y, E, \kappa)$ (radial, toroidal, Energy, pitch-angle)

$$\left\langle\Phi\right\rangle_b(x, y, \kappa) = \left(\oint dz\,\Phi(x, y, z)/|v_\parallel|\right)/\left(\oint dz/|v_\parallel|\right)$$

- Gyrokinetic ions $f_i(x, y, z, \underbrace{v_\perp, v_\parallel}_{\text{moments}})$

  → 3-D Gyrofluid equations vs. (x, y, z)
- Drift-kinetic electrons $f_e(x, y, \underbrace{z}_{\text{bounce average}}, \underbrace{E}_{\text{moments}}, \kappa)$

  → 3-D Trapped-Fluid equations vs. (x, y, κ)

Bounce averaging eliminates the fast parallel electron motion, so fully nonlinear simulations are only about 2x slower than simulations assuming adiabatic electrons

Trapped Electron Fluid Equations

[**Beer & Hammett, PoP 3, 4018 (1996)**]

Evolve even moments over v of $\langle f_e \rangle_b$,

$$n_t(x, y, \kappa) = \frac{4\pi}{n_0} \int_0^\infty dv\, v^2 \left\langle f_e \right\rangle_b ,$$

with similar definitions for p_t, r_t, t_t.

$$\begin{aligned}
\frac{dn_t}{dt} &+ \frac{3}{2} i\omega_{de} p_t - \frac{3}{2} i\omega_{de} \left\langle \Phi \right\rangle_b + i\omega_{*e} \left\langle \Phi \right\rangle_b = \left\langle C \right\rangle_b \left(n_t - \left\langle \Phi \right\rangle_b\right) \\
\frac{dp_t}{dt} &+ \frac{5}{2} i\omega_{de} r_t - \frac{5}{2} i\omega_{de} \left\langle \Phi \right\rangle_b + i(1 + \eta_e)\omega_{*e} \left\langle \Phi \right\rangle_b = \left\langle C \right\rangle_b \left(p_t - \left\langle \Phi \right\rangle_b\right) \\
\frac{dr_t}{dt} &+ \frac{7}{2} i\omega_{de} t_t - \frac{7}{2} i\omega_{de} \left\langle \Phi \right\rangle_b + i(1 + 2\eta_e)\omega_{*e} \left\langle \Phi \right\rangle_b = \left\langle C \right\rangle_b \left(r_t - \left\langle \Phi \right\rangle_b\right)
\end{aligned}$$

$\omega_{de} \propto v^2$ introduces the usual closure problem of the coupled moments hierarchy. Extension of Landau-fluid closure approximation [Hammett & Perkins] to provide a 3-pole model of the precession resonance and phase mixing:

$$t_t = -i \frac{|\omega_{de}|}{\omega_{de}} (\nu_a n_t + \nu_b p_t + \nu_c r_t)$$

Keeping the pitch-angle dependence of ω_{de} and $\left\langle \Phi \right\rangle_b$ important for Advanced Tokamak regimes at high β or reversed shear, where the favorable precession drift of barely trapped particles can stabilize TEM

Bounce Averaged Pitch Angle Scattering Collision Operator

Since the electron moments are functions of pitch angle, we can keep a Lorentz collision operator:

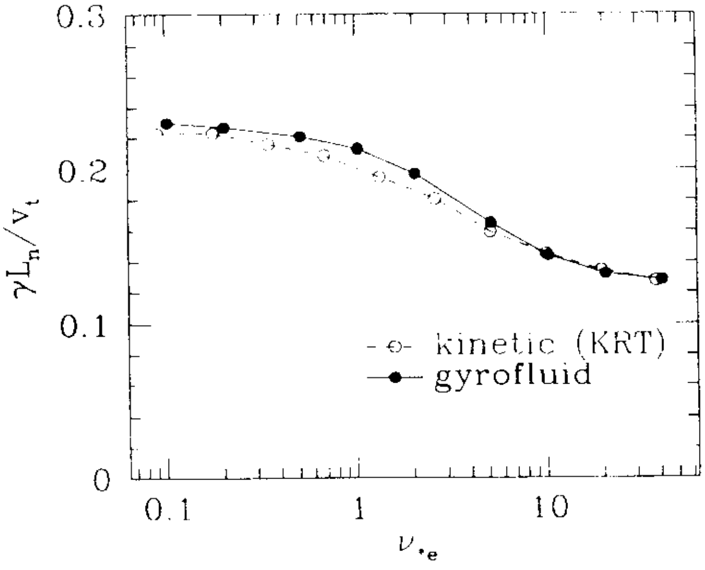
$$C = \frac{\nu_e(v)}{2}\frac{\partial}{\partial\xi}(1-\xi^2)\frac{\partial f_e}{\partial\xi},$$

Bounce average [**Cordey, NF 1976**]:

$$\langle C\rangle_b = \frac{\nu_e}{8\epsilon^2|\kappa|\tau_b}\frac{\partial}{\partial\kappa}\left[(1-2\epsilon\kappa^2)\frac{\tau_b}{|\kappa|}\left\{\left\langle\frac{B_{\min}}{B}\right\rangle_b - 1 + 2\epsilon\kappa^2\right\}\times \frac{\partial}{\partial\kappa}\left(\langle f_e\rangle_b - \langle\Phi\rangle_b\right)\right]$$

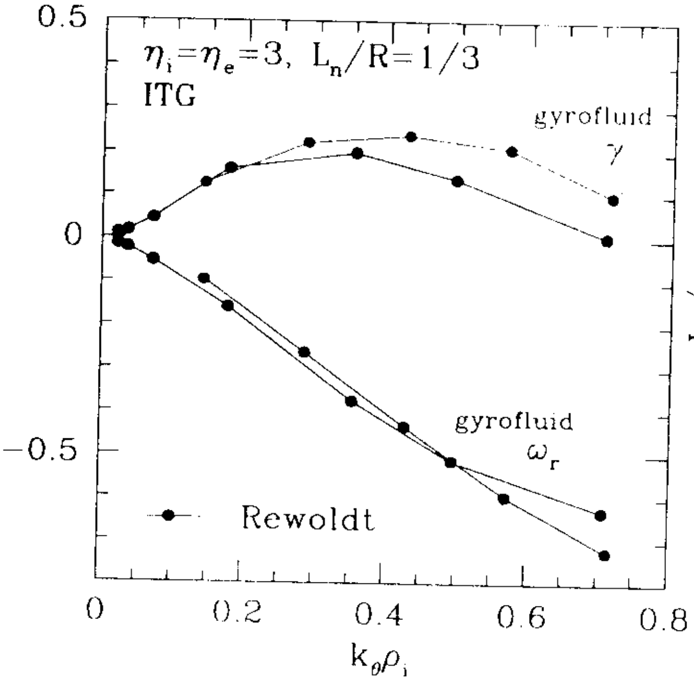
$$\kappa^2 = (1-\mu B_{\min}/E)/2\epsilon_B$$

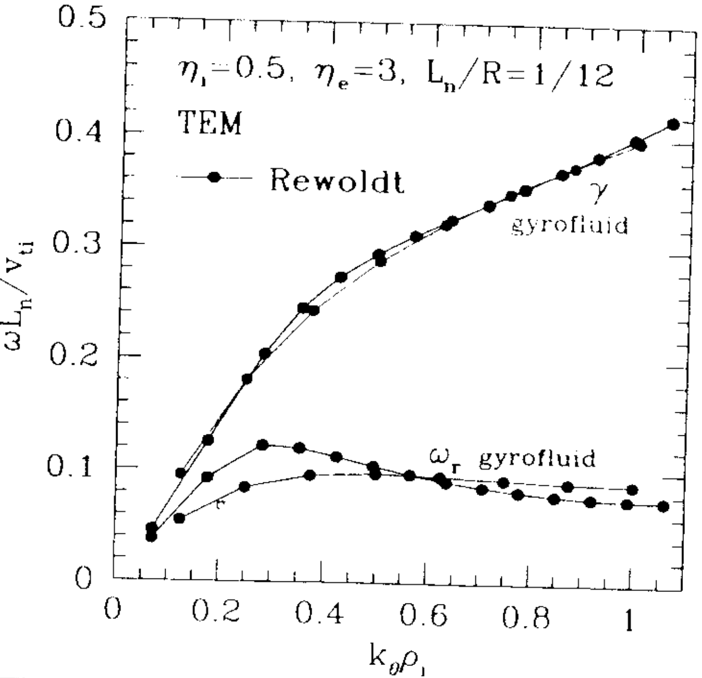
These electron equations are continuously valid from collisionless regime (destabilization from toroidal precession resonance) to collisional regime (where electrons become adiabatic)

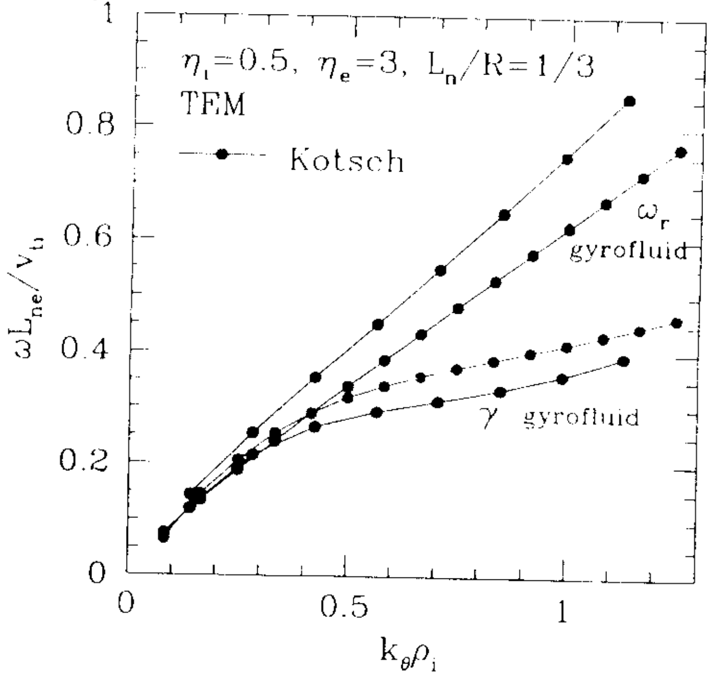


Linear Test Cases

Linear comparisons with fully kinetic calculations [**Rewoldt & Kotschenreuther**] are quite favorable.

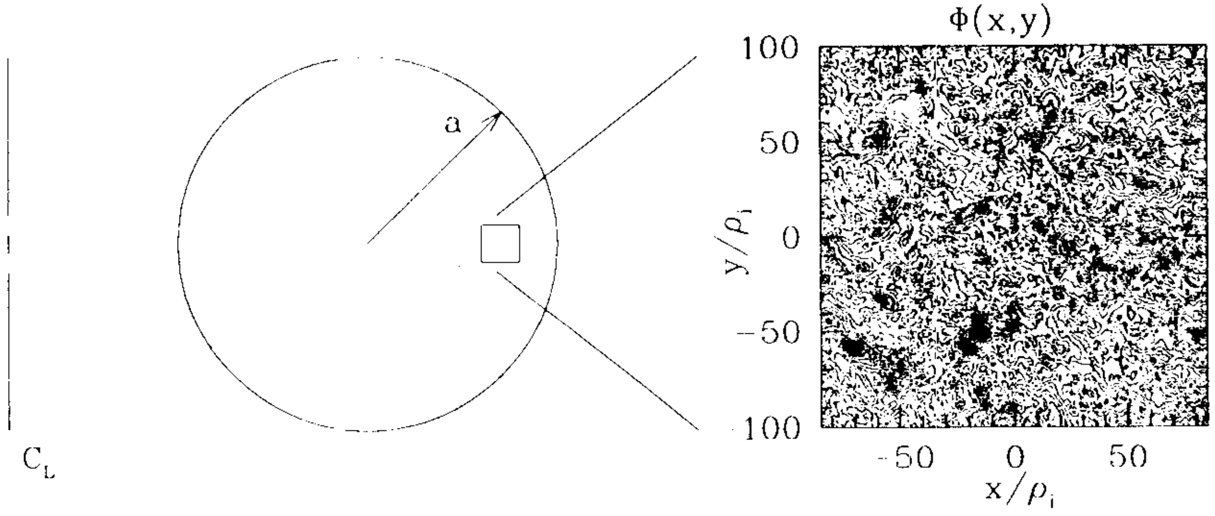






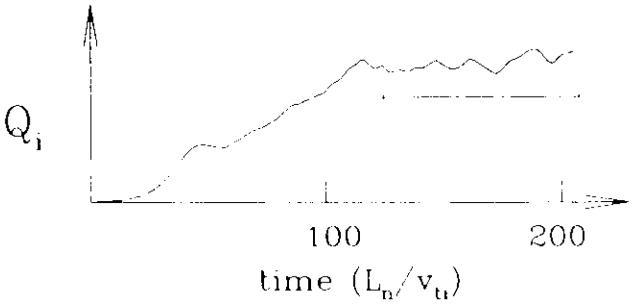
Flux Tube Simulation Model

Simulate small perpendicular cross section. Exploit separation between equilibrium scales $\sim a$, and fluctuations $\sim 10\rho_i \ll a$.



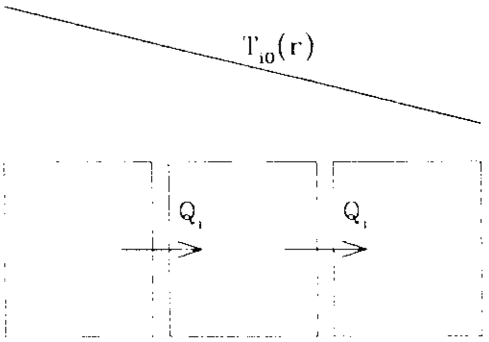
Equilibrium parameters used as inputs: q, $\hat{s}$, L_n, L_T, r/R, T_i/T_e, . . .

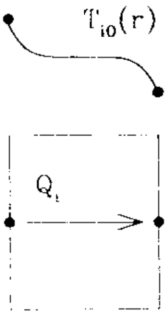
Gradients $\nabla n_0, \nabla T_0$ drive instabilities which evolve into turbulence.



$$Q_i = \langle \tilde{v}_{Er} \tilde{p}_i \rangle \equiv -n_0 \chi_i^{tot} \nabla T_{i0}$$

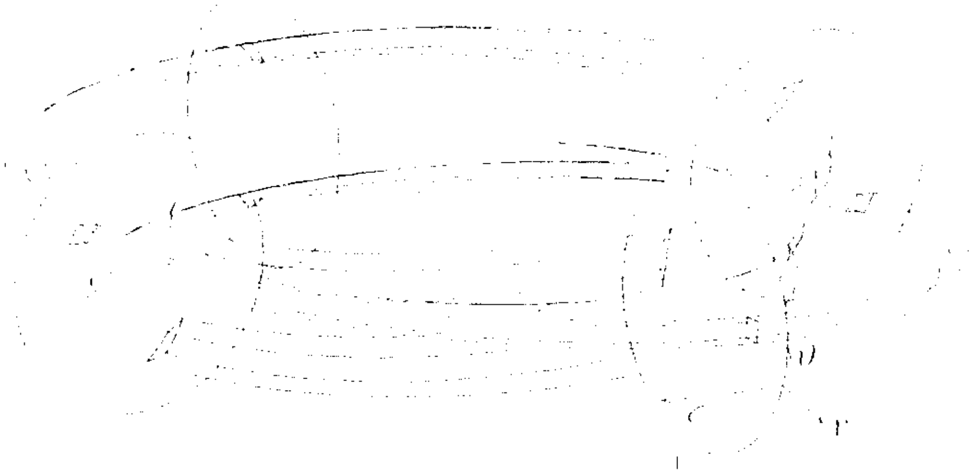
Radial periodicity inhibits flattening of equilibrium.

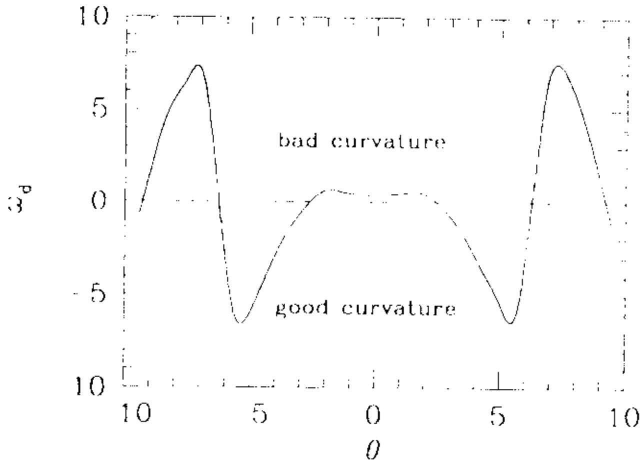


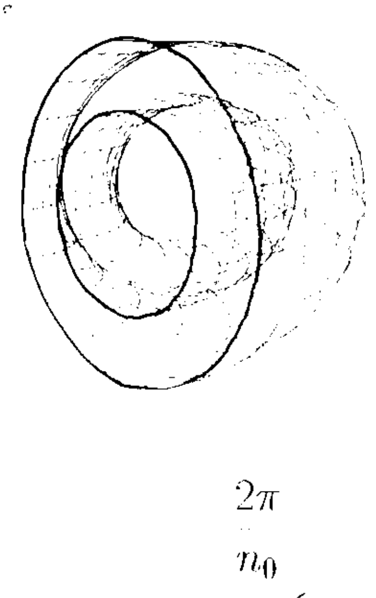


Geometric Effects Enter Through Variations along field line

Flux tube simulation domain aligned with sheared field lines.





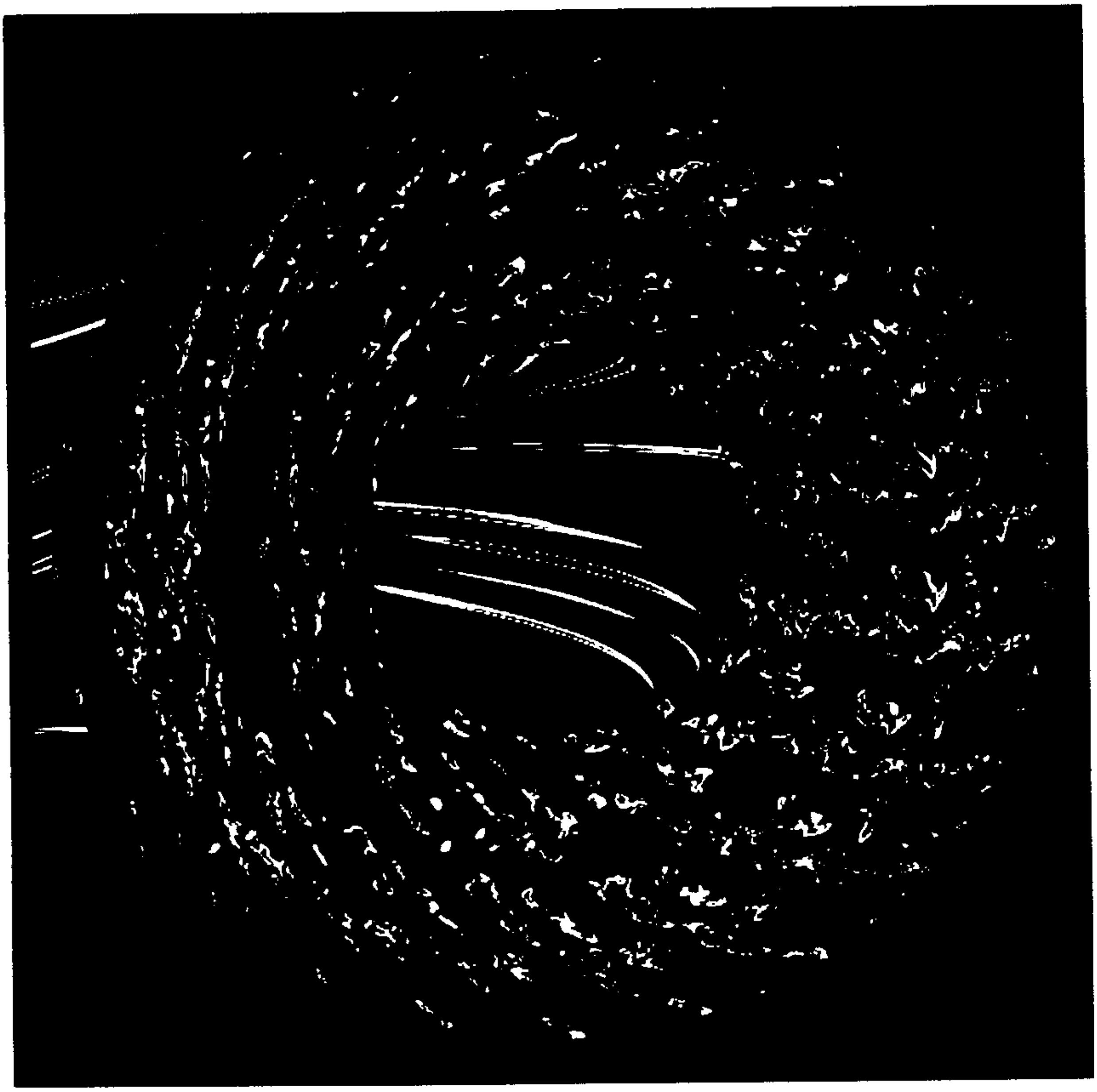


Equivalent to simulating a fraction $(1/n_0)$ of a toroidal annulus, with a coarse grid in toroidal mode number $n \in \{0, n_0, 2n_0, 3n_0, \ldots\}$

Variations of $\omega_d \propto \mathbf{B} \times \nabla B$ along field line contain effects of good and bad curvature

Gyrofluid Simulations of Tokamak Turbulence

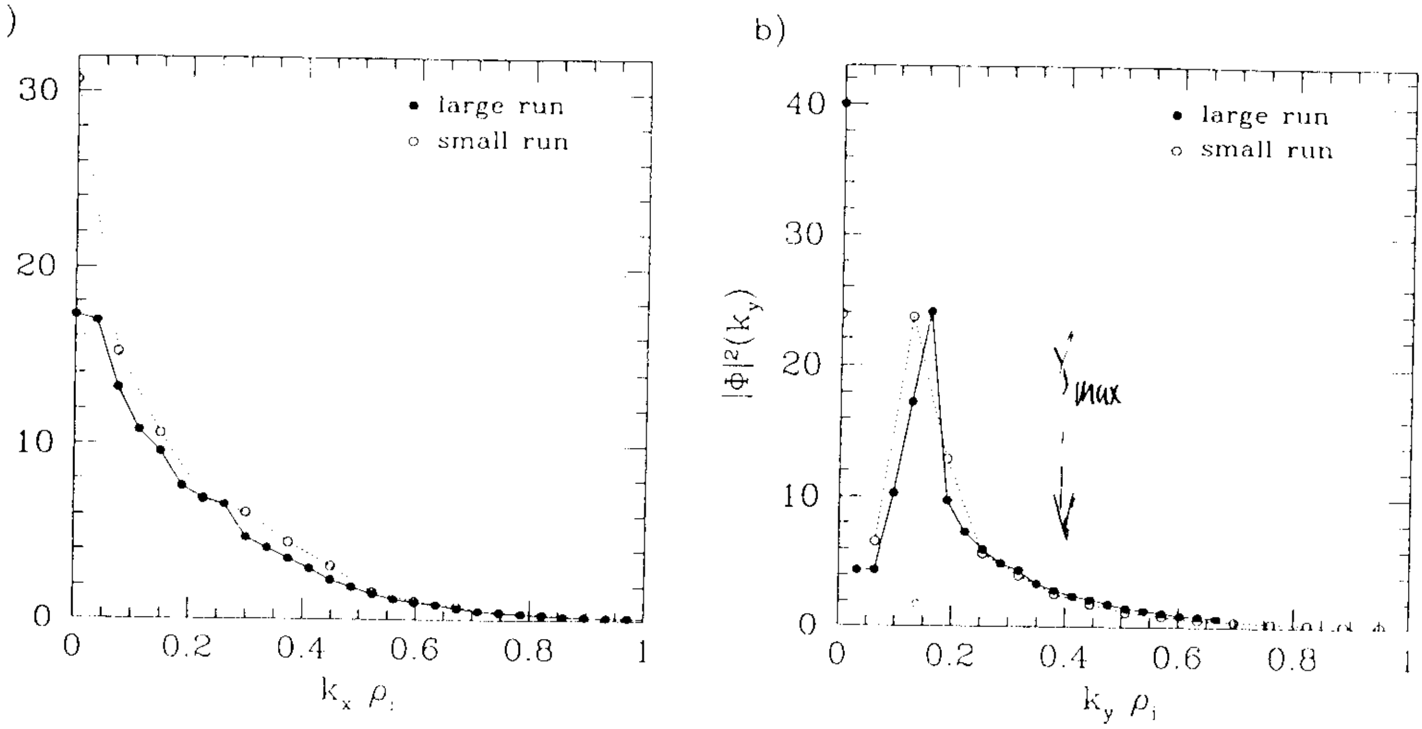
Contours of fluctuating electrostatic potential:



Courtesy G. Kerbel of the Numerical Tokamak Project

Fluctuation Spectra Resemble BES

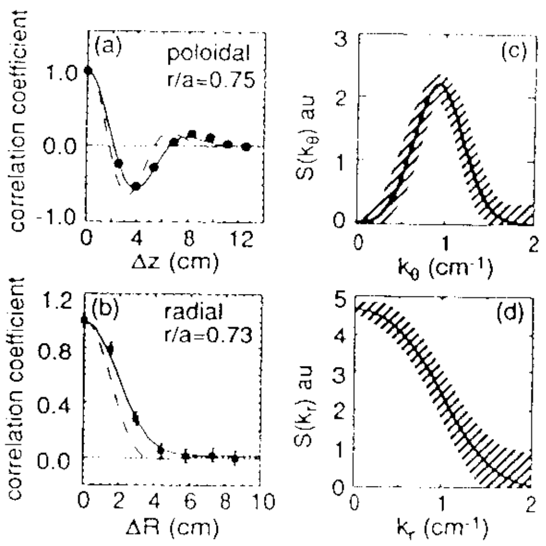
Toroidal gyrofluid simulations reproduce long-wavelength peak measured by BES [Fonck, *et al.*, PRL (1993)]. Growth rate peaks at higher k_θ: nonlinear downshift to long wavelengths.



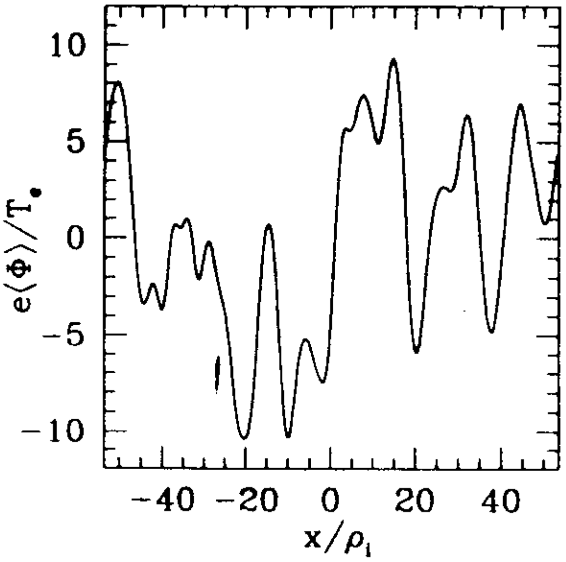
radial

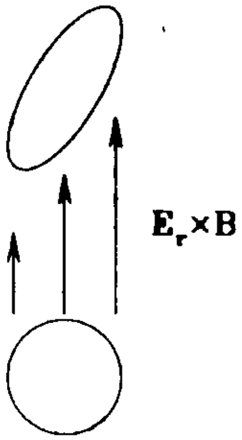
poloidal

Also seen in Gyrokinetic particle simulations [Parker, *et al.*, PRL (1993)].

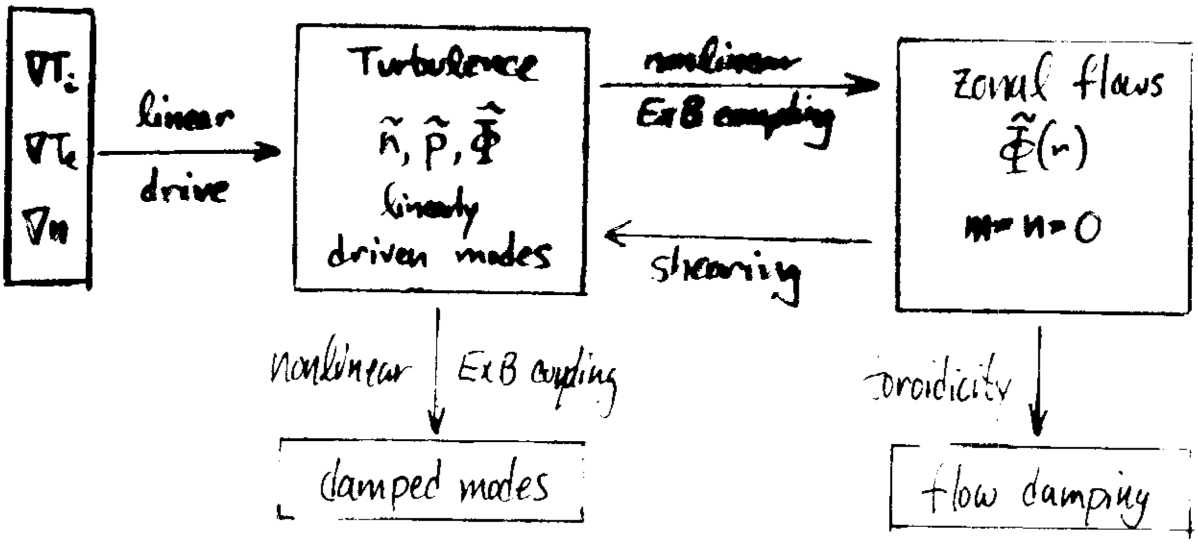


Sheared $\mathbf{E} \times \mathbf{B}$ Zonal Flows Play an Important Role in the Turbulent Dynamics

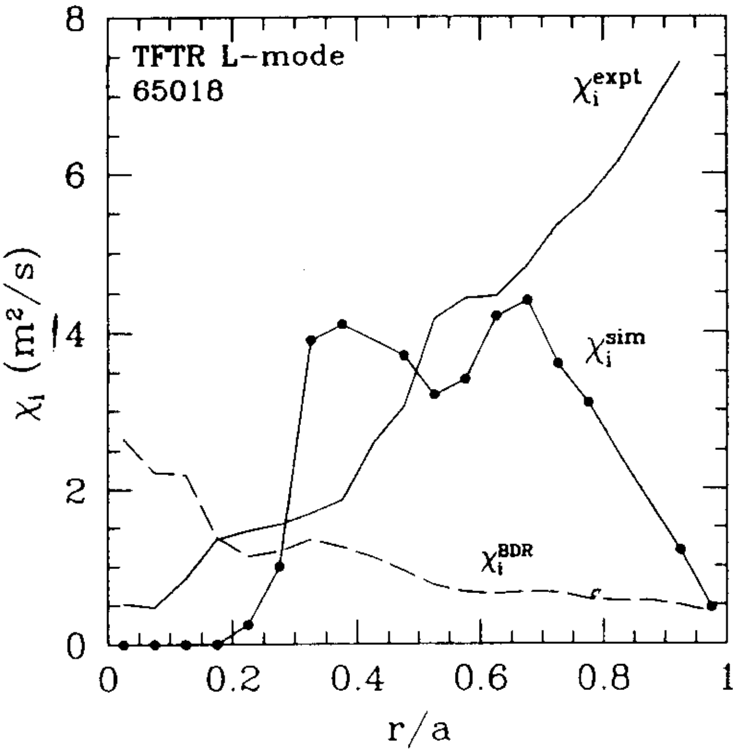




- Zonal flows are constant on a flux surface, but vary radially. **Also called radial** modes [Waltz, *et al.* (1994)]. Potential leads to radially **sheared** perpendicular $\mathbf{E} \times \mathbf{B}$ flow.
- Have small radial scales ~ turbulent scales, not equilibrium
- Flows are nonlinearly generated by the turbulence [Hasegawa & Wakatani (1987)], [Carreras, *et al.* (1991)], [Diamond & Kim, (1991)]
- Sheared flows stretch turbulent eddies, decreasing radial correlation and regulating the turbulence [Biglari, Diamond, Terry, (1990)]

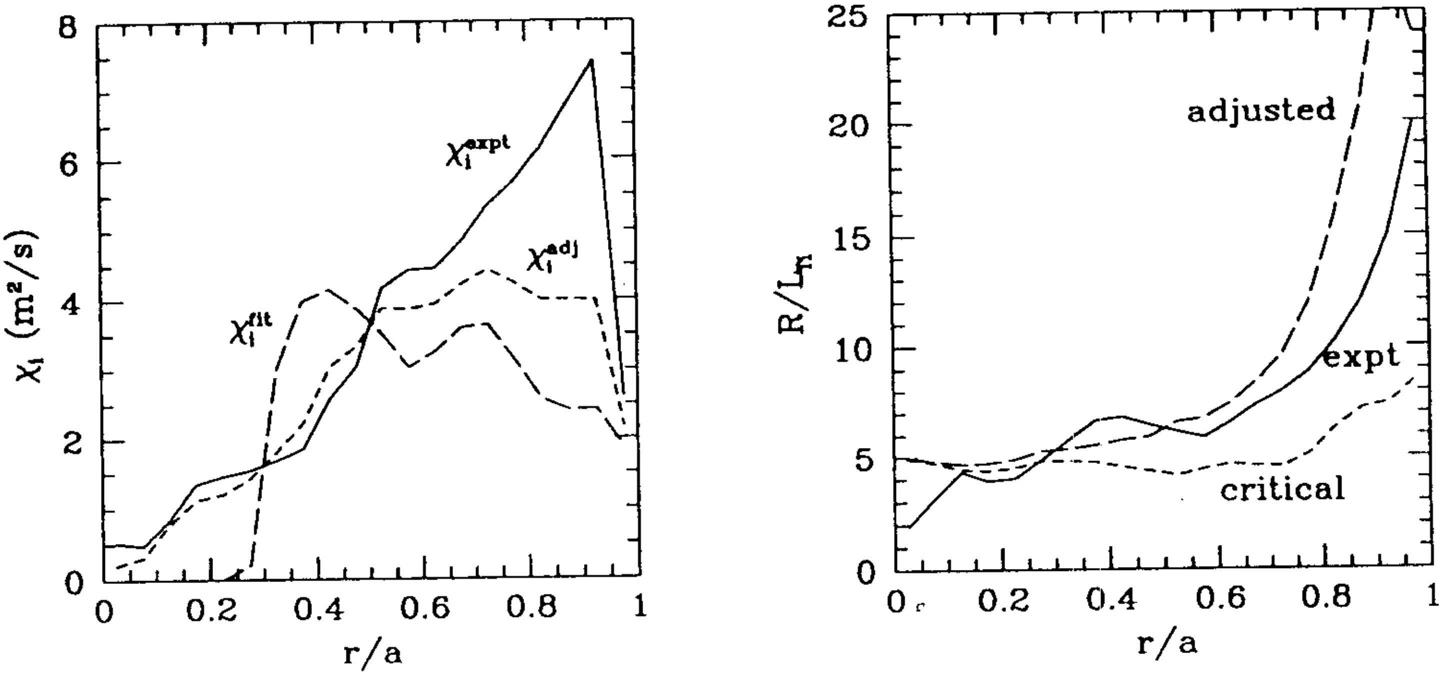


Comparison of Predicted and Measured Transport



- $\chi_i(r)$ directly calculated from ~ 20 nonlinear flux tube simulations, using measured parameters at each r as input.
- Also shown is the theoretical χ_i from Biglari, Diamond, Rosenbluth PFB (1989).
- Even with trapped electrons, core is stable for $r/a < 0.2$.
- Predicted edge transport is too low for $r/a > 0.8$.
- χ_i is very sensitive to small changes in ∇T_{i0}.

Small Adjustments to ∇T_i Improve Agreement in Core

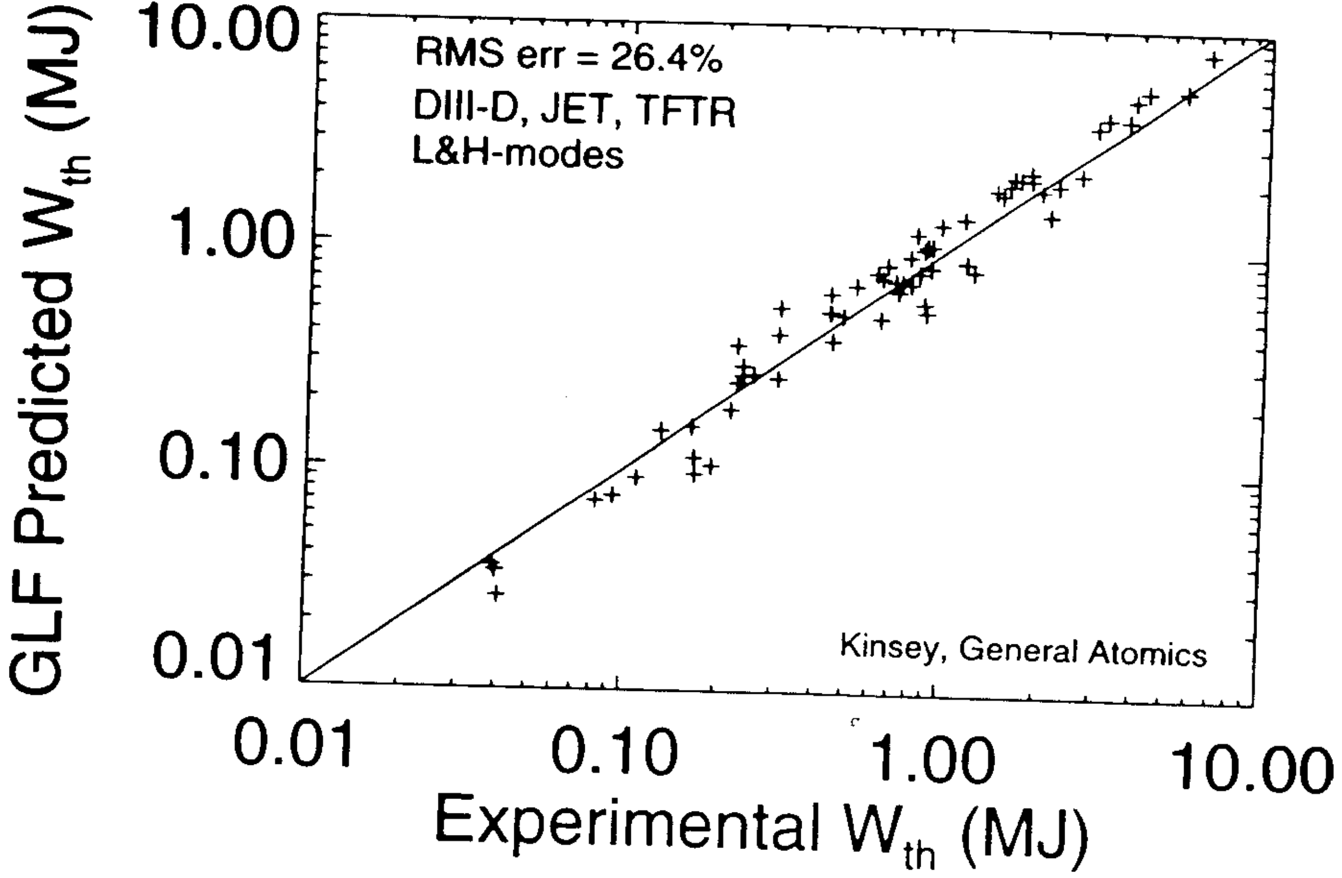


- (a) Comparison of experimental $\chi_i(r)$ (solid) and and the IFS-PPPL $\chi_i(r)$ (long dashes) [Dorland *et al.*, 1994]. Also shown is $\chi_i(r)$ using an $L_{Ti}(r)$ profile adjusted so the predicted heat flux matches the experimental heat flux (short dashes).

$$\chi_i = \frac{\rho_i^2 v_{ti}}{R} \left(\frac{R}{L_{Ti}} - \frac{R}{L_{Ti}^{\text{crit}}} \right)^{1,1/2} \frac{q}{2+\hat{s}} \cdots$$

- (b) Measured (solid) and adjusted (long dashes) R/L_{Ti}. In the core, the adjusted R/L_{Ti} is only slightly above the critical R/L_{ti} (short dashes).

Transport Model Based on Turbulence Simulations Follows Many Experimental Trends



GLF23 transport model by Waltz et.al fitted to Beer et.al. nonlinear -D gyrofluid simulations of ITG/trapped-electron turbulence.

Encouraging results so far, but many caveats: uses measured den-ity and rotation profiles, uses measured temperatures at $r/a = 0.9$, lectrostatic turbulence simulations need extension to magnetic fluctu-tions (~10×CPU time), gyrofluid/gyrokinetic discrepancy, etc... Much ture work needed to be more accurate over a wider range of plasma arameters.

Rescaled GLF23, $\downarrow \chi$ and $E\times B$ shear, improves to RMS error $\approx 19\%$.