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Dynamics of Turbulence Driven Zonal ExB Flows

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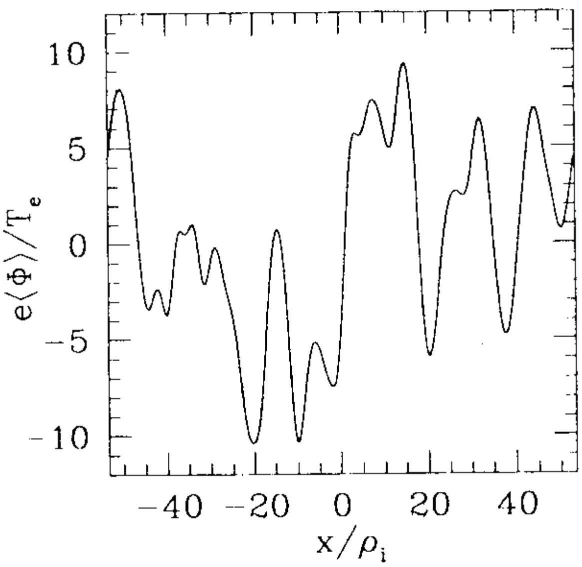
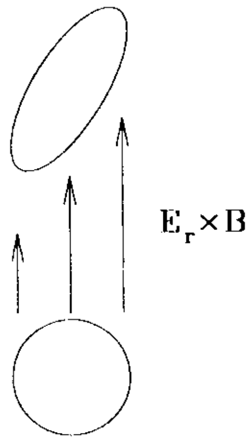
Acknowledgements: G. W. Hammett, W. Dorland

Outline:

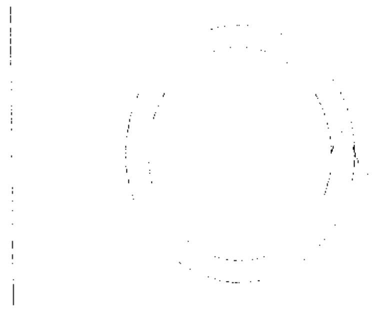
- Importance of zonal flows in saturation of ITG turbulence
- Linear damping of turbulence generated zonal flows
- Zonal flow dynamics in nonlinear simulations
- New closures which recover undamped component of zonal flows
- Nonlinear upshift of critical gradient $\frac{R}{L_{Tcrit}}$
- Bursty behavior near marginal stability
- Impact on predictions of reactor performance

Sheared $\mathbf{E} \times \mathbf{B}$ Zonal Flows Play an Important Role in the Turbulent Dynamics





- Zonal flows are constant on a flux surface, but vary radially. Also called radial modes [Waltz, *et al.* (1994)]. Potential leads to radially sheared perpendicular $\mathbf{E} \times \mathbf{B}$ flow.



$$\Phi(r) \sim e^{ik_r r} \Rightarrow v_{E\theta}(r)$$

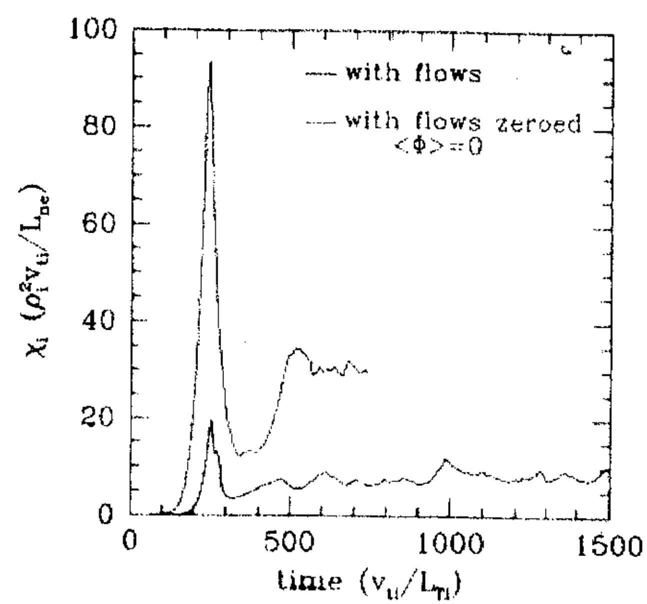
- Have small radial scales $\sim$ turbulent scales, not equilibrium
- Flows are nonlinearly generated by the turbulence [Hasegawa & Wakatani (1987)], [Carreras, *et al.* (1991)], [Diamond & Kim, (1991)]
- Sheared flows stretch turbulent eddies, decreasing radial correlation and regulating the turbulence [Biglari, Diamond, Terry, (1990)]

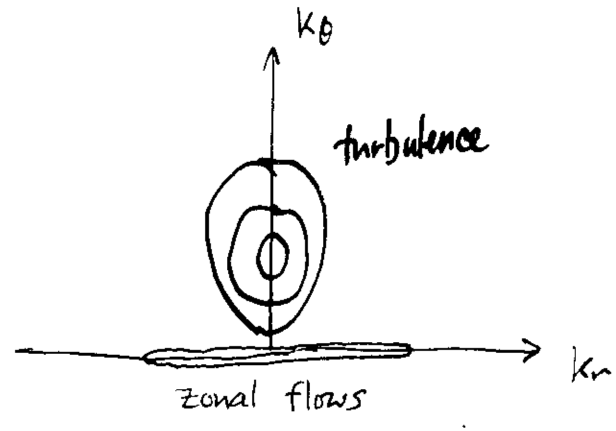
Turbulence-generated Zonal Flows

Hasegawa-Wakatani '87   3D resistive drift interchange

Carreras et.al. '91 }
Diamond + Kim '91 } Suggests this happens only very close to the edge

Dorland et.al. '93 }
Beer et.al. '93 } Show zonal flows very important for core ITG turbulence also.





Rosenbluth - Hinton show there should be a component of the zonal flows that are linearly undamped (missed by original gyrofluid closure) + $\chi \downarrow$ even more, as shown by Dimits [redacted] particle simulations.

An Incomplete History of Zonal Flows

Zonal flows are $k_\theta = 0$ convective cell, lots of early refs: Okuda, Cheng, Lee...?

1979 Hasegawa, et al., 2D sims of Hasegawa-Mima eqns

- predicted condensation of fluctn energy in zonal flow ($k_\theta = 0$)
- but Hasegawa-Mima eqns have incorrect adiabatic response

1987 Hasegawa & Wakatani

- 3D, better electron response, but still very reduced fluid eqns
- cylindrical plasma, so only weak classical flow damping

1991 Carreras et al., Diamond et al. resistive edge turbulence

- also see strong zonal flow generation, but suggest only very near the edge. Still cyclindrical.

1993 Hammett, et al., gyrofluid simulations

- Dorland: correct adiabatic electron response amplifies low-k_r flows, saw strong effect of flow generation in slab simulations
- Beer: flux-tube, toroidal effects introduce collisionless damping, zonal flows still very important

1994 Dimits, saw large effect of zonal flows in GKP flux-tube simulations

1995 Cummings et al., Sydora et al. (GKP full torus)

- added correct $k_{||} = 0$ adiabatic response, but saw little effect

1998 Rosenbluth & Hinton

- emphasized linearly undamped flow component
- predicted undamped flow would $\sim \sqrt{t}$

1998 Lin, et al. (GKP full torus)

- with broad profiles, recovers flux-tube limit
- with narrow profiles, recovers previous full torus limit
- including neoclassical collisional damping of "undamped" component

Three Important Characteristics of Zonal Flows

Aren't zonal flows just another component of the turbulence?

Three features make them unusual:

- Adiabatic electrons can't respond to $k_{\|} = 0$ zonal flow component, so polarization amplifies low k_r, $k_{\|} = 0$ component of potential relative to $k_{\|} \neq 0$ for the same n_i

$$n_e = n_i$$

$$n_{e0}\left(\frac{e\Phi}{T_e} - \langle\frac{e\Phi}{T_e}\rangle\right) = \bar{n}_i - n_{i0}(1 - \Gamma_0)\frac{e\Phi}{T_i}$$

In polarization density, $\Gamma_0 \approx 1 - k_r^2\rho_i^2$ for small $k_{\theta}\rho_i$

So for zonal flow, with $k_{\|} = 0$ and $n = 0$:

$$\frac{e\Phi}{T_i} \approx \frac{\bar{n}_i}{n_{i0}}\frac{1}{k_r^2\rho_i^2}$$

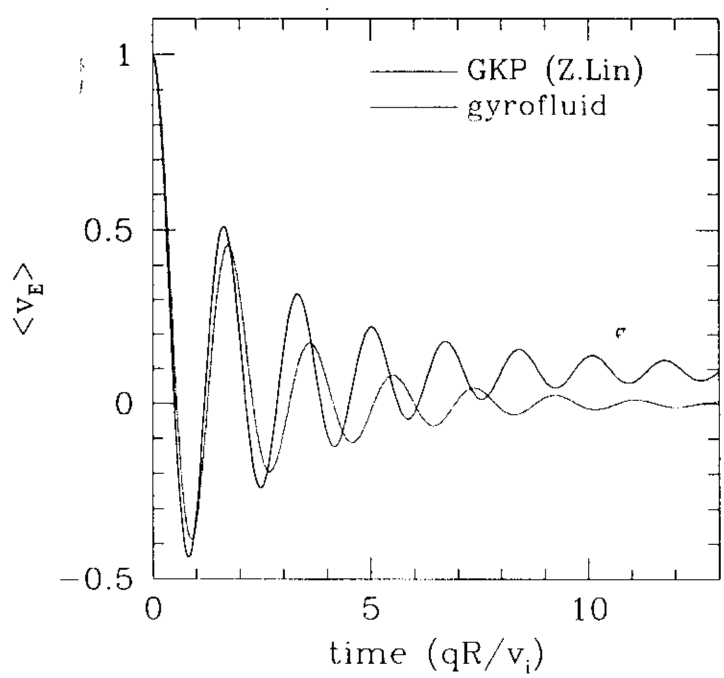
but for other modes $k_{\|} \neq 0$:

$$\frac{e\Phi}{T_i} \approx \frac{\bar{n}_i}{n_{i0}}\frac{1}{T_i/T_e + 1 - \Gamma_0}$$

- Other modes also cause convective decorrelation, but zonal flows do it over the whole flux surface, not locally
- The fact that $k_{\|} = 0$ and $k_\theta = 0$ makes their evolution somewhat simpler and very different (no other mode has $k_{\|} = 0$ and $\omega_* = 0$)

Importance of Linear Zonal Flow Damping

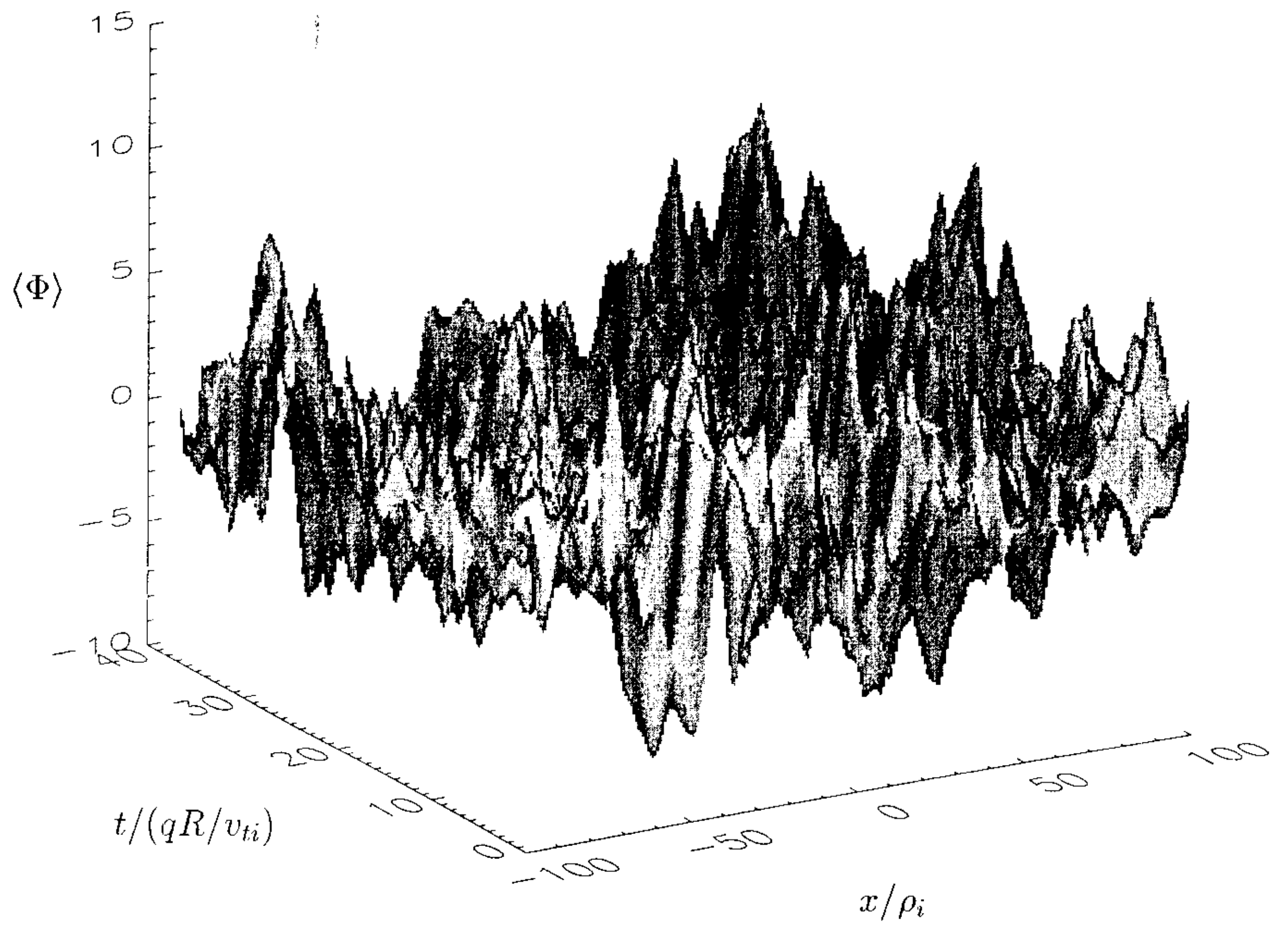
- Two phases: fast collisionless damping & slow collisional damping. Depends on initial flow conditions
- In [Beer, Ph.D. Thesis (1995)] showed that our gyrofluid equations accurately model the fast linear collisionless damping for $t < qR/v_{ti}\sqrt{\epsilon}$. Argued that long time linear flow dynamics are not important, nonlinear effects will dominate long term nonlinear flow evolution.



- [Rosenbluth & Hinton, PRL (1998)] emphasized a linearly undamped flow component. This "residual" flow damped by collisional effects. Argued that *nonlinearly*, residual component should grow in time $\sim \sqrt{t}$ in collisionless limit. Modeled nonlinear drive term as a white noise source.
- Since our original gyrofluid eqns underestimate residual component, if residual component is important nonlinearly, gyrofluid simulations would underestimate $\mathbf{E} \times \mathbf{B}$ flow levels and overpredict χ_i.

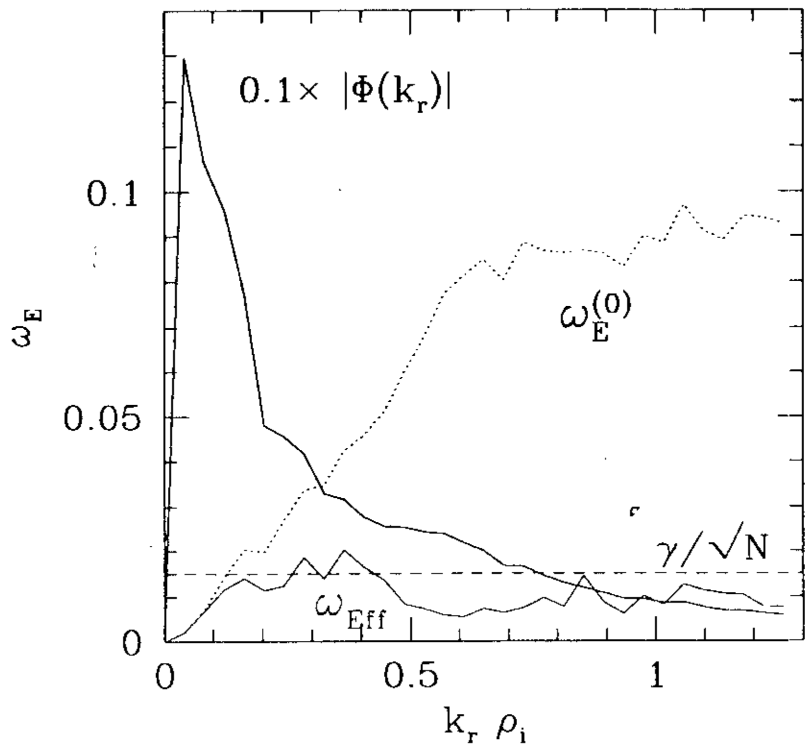
Zonal Flows Fluctuate on Turbulent Time and Space Scales

Time history of the flux surface averaged potential, $\langle \Phi(r,t) \rangle$, from the saturated phase of a nonlinear run for DIIID #81499 parameters at $\rho = 0.5$: $\hat{s} = .776$, $q = 1.4$, $\eta_i = 3.11$, $\epsilon_n = 0.45$, $T_i = T_e$.



Time Averaged Flow Spectrum

Spectrum of saturated flux surface averaged potential $|\Phi(k_r)|$ obtained by Fourier Transforming in r and averaging in t.



Shearing rate peaks at high k_r: $\gamma_{\text{shear}} = k_r^2|\Phi(k_r)|$ $\sim \frac{dv_E}{dr}$

While highest k_r shearing rates are large, they have small correlation times and thus small effect on turbulence (Hahm, Diamond).

Maximum $\gamma_{\text{lin}} \approx 0.1$

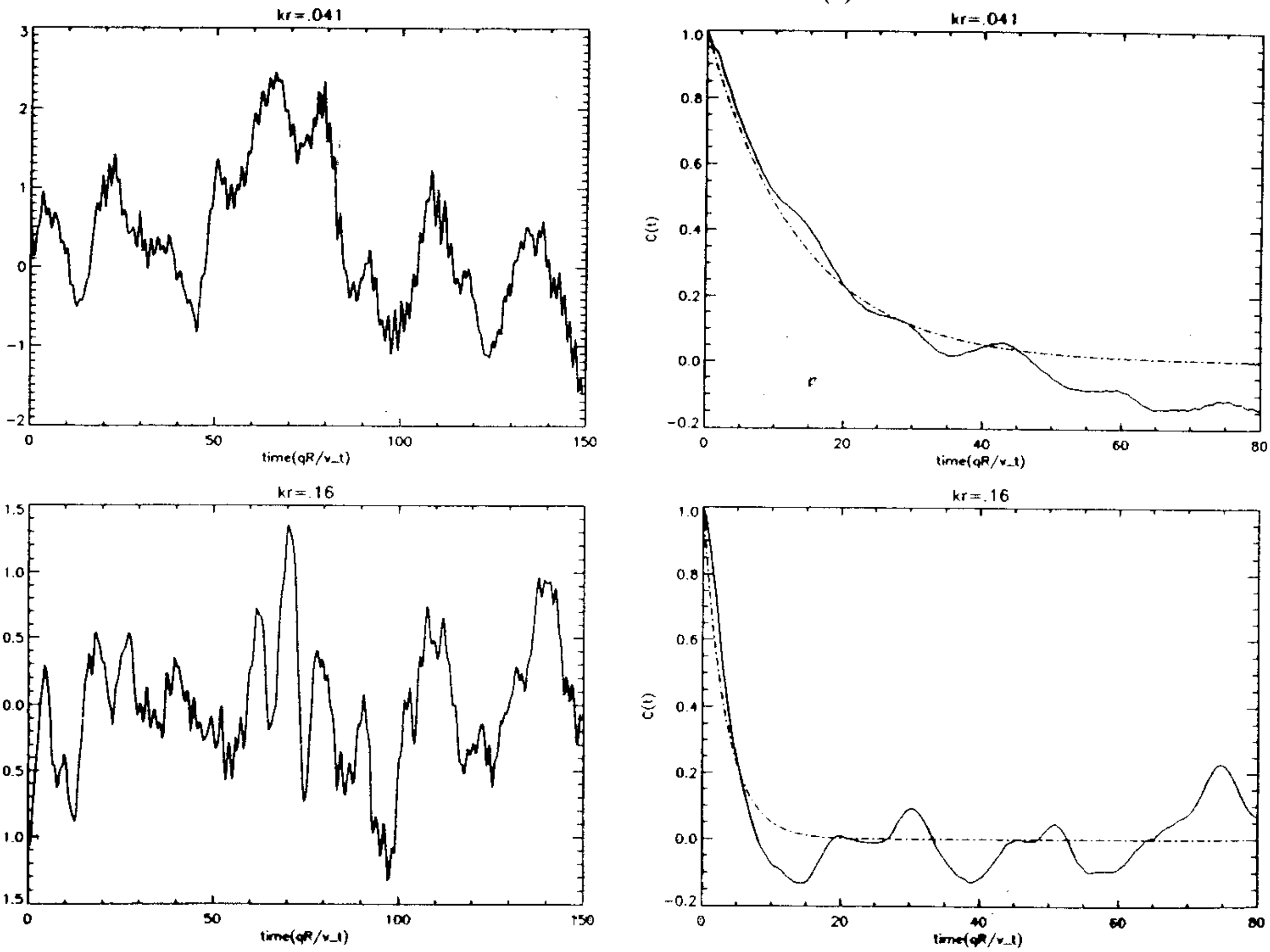
Using simulation zonal flow spectrum and using simulation zonal flow time history to calculate $\tau_{\text{corr}}(k_r)$, we estimate the time dependent effective shearing rate ω_{Eff} [Hahm] and find close to $\gamma_{\text{lin}}/\sqrt{N}$

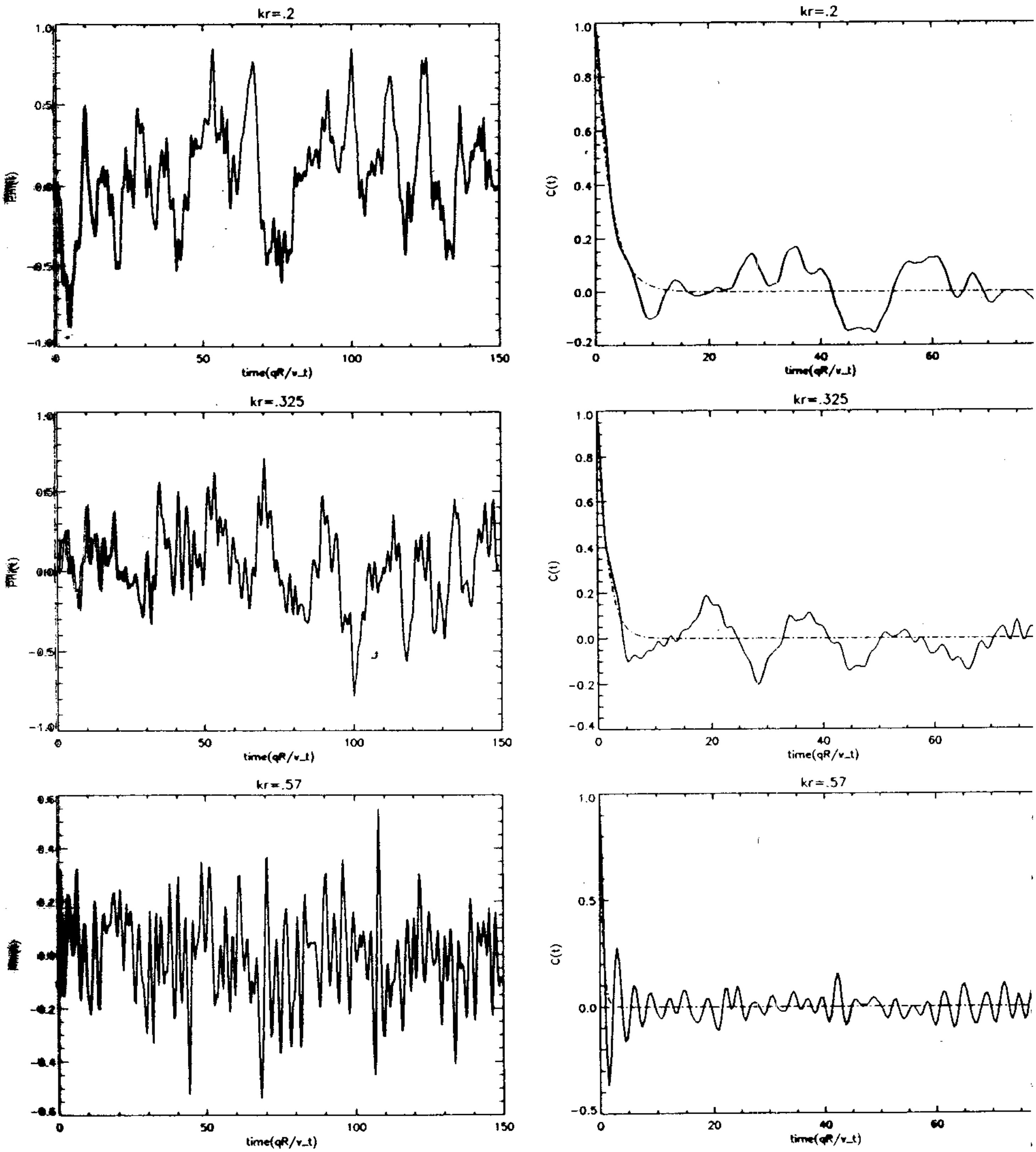
Flow Correlation Functions

After transforming in r, the correlation function can be obtained from the time series $\Phi(k_r, t)$:

$$C(t) = \int dt\, e^{-i\omega t} \Phi^* \Phi(\omega)$$

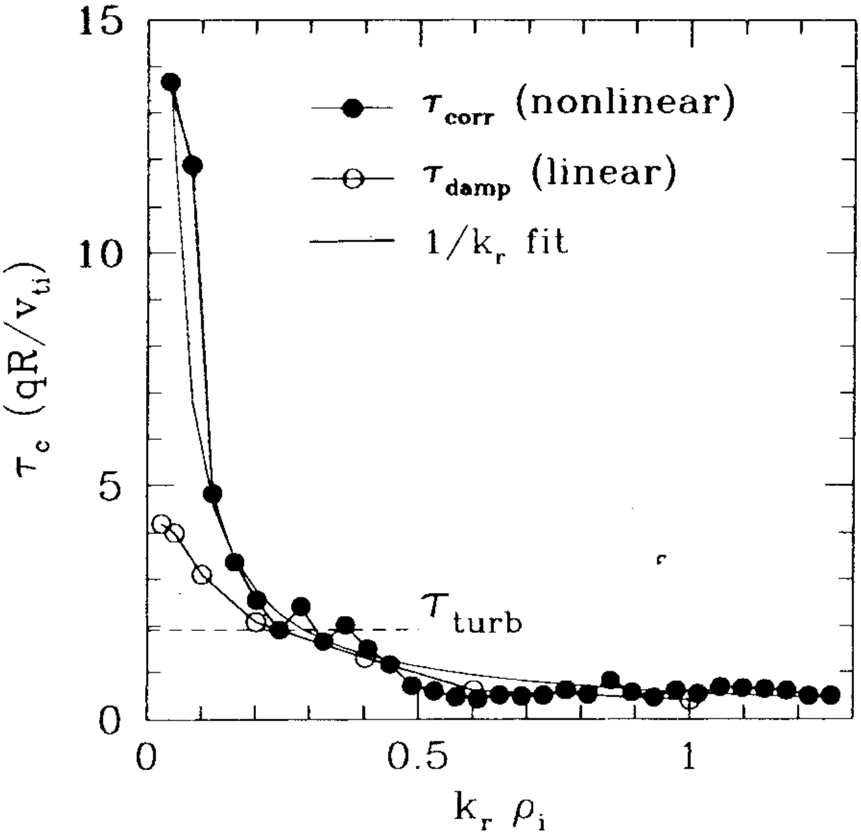
A least squares fit to the numerical data of the form $C(t) = e^{-t/\tau_c}$ is also shown




kr=.2
time(qR/v_t)
C(t)
kr=.2
time(qR/v_t)
kr=.325
time(qR/v_t)
C(t)
kr=.325
time(qR/v_t)
kr=.57
time(qR/v_t)
C(t)
kr=.57
time(qR/v_t)

Simulation $\tau_{\rm corr}$ Similar to Expt

$\tau_{\rm corr}$ vs. k_r similar to measurements by Coda [APS 1997]:



$\tau_c \approx \tau_{\rm damp}$ except for small k_r, where $\tau_c > \tau_{\rm damp}$.

$\tau_c > \tau_{\rm damp}$ implies that finite spectral width of nonlinear source $S(\omega)$ is dominating τ_c:

$$\frac{\partial \Phi}{\partial t} + i\omega_r \Phi = -\nu\Phi + S \quad \Rightarrow \quad |\Phi^2(\omega)| = \frac{|S^2(\omega)|}{(\omega - \omega_r)^2 + \nu^2}$$

$\Rightarrow$ NL Source is not white at low k_r.

Physics of the Undamped Flow Component

Since $\mathbf{v}_E \cdot \nabla F_0 = 0$ for the zonal flows, they obey a simplified collisionless electrostatic toroidal gyrokinetic equation:

$$\frac{\partial f}{\partial t} + v_\| \nabla_\| f + i\omega_d f + i(eF_0/T)\omega_d \Phi + v_\| (eF_0/T)\nabla_\| \Phi = 0,$$

where $i\omega_d = \mathbf{v}_d \cdot \nabla = i(k_r \rho_i / v_t R)(v_\|^2 + v_\perp^2/2)\sin\theta$.

Rosenbluth and Hinton found a general equilibrium solution:

$$f = -(e\Phi/T)F_0 + h(E,\mu)e^{-ik_r\rho_i(qB_0 v_\| / \epsilon B v_t)},$$

where $h(E,\mu)$ is arbitrary but satisfies $\partial h/\partial l = 0$. The $v_\|$ in the exponential keeps f non-Maxwellian.

In this equilibrium, parallel variations in f balance the velocity dependent cross field drifts.

Expanding for small banana width $k_r \rho_i q/\epsilon \ll 1$:

$$f = -(e\Phi/T)F_0 + h(E,\mu)[1 - ik_r\rho_i \frac{qB_0 v_\|}{\epsilon B v_t}],$$

we see that moments of f will be supported by radial gradients of higher moments, e.g. $u_\|$ is driven by $k_r p_\|$, analogous to Pfirsch-Schlüter flow:

$$n_0 u_\| = \int d^3v\, v_\| f = -ik_r\rho_i \frac{qB_0}{\epsilon B v_t} \int d^3v\, v_\|^2 h(E,\mu)$$

New Closures for Zonal Flows Which Retain Residual Component

If we choose $h(E,\mu)$ to be a perturbed Maxwellian with no n perturbation:

$$f = -(e\Phi/T)F_0 + F_0\left(\frac{mv^2}{2T_0} - \frac{3}{2}\right)\frac{\delta T}{T_0}\left[1 - ik_r\rho_i\frac{qB_0v_{\|}}{\epsilon B v_t}\right],$$

we can integrate this and find equilibrium $q_{\|}$ and $q_{\perp}$ moments:

$$q_{\|}^{(0)} = 3ik_r\rho_i\frac{qB_0}{\epsilon B}\delta T \qquad \text{and} \qquad q_{\perp}^{(0)} = ik_r\rho_i\frac{qB_0}{\epsilon B}\delta T.$$

Generalizing to non-isotropic h leads to:

$$q_{\|}^{(0)} = 3ik_r\rho_i\frac{qB_0}{\epsilon B}T_{\|} \qquad \text{and} \qquad q_{\perp}^{(0)} = ik_r\rho_i\frac{qB_0}{\epsilon B}T_{\perp}.$$

Our old parallel closures damped $q_{\|}$ and $q_{\perp}$ to zero, but now we replace:

$$\sqrt{2}D_{\|}|k_{\|}|q_{\|} \rightarrow \sqrt{2}D_{\|}|k_{\|}|(q_{\|} - q_{\|}^{(0)}) \quad \text{in the } q_{\|} \text{ eqn}$$
$$\sqrt{2}D_{\perp}|k_{\|}|q_{\perp} \rightarrow \sqrt{2}D_{\perp}|k_{\|}|(q_{\perp} - q_{\perp}^{(0)}) \quad \text{in the } q_{\perp} \text{ eqn}$$

We also have to modify the toroidal closures in the $p_{\|}$ and $p_{\perp}$ eqns to support this equilibrium. We have not found a completely satisfactory way to do this. Two possibilities are:

closure (a): $\nu_1 = \nu_2 = \nu_3 = \nu_4 = 0$, or

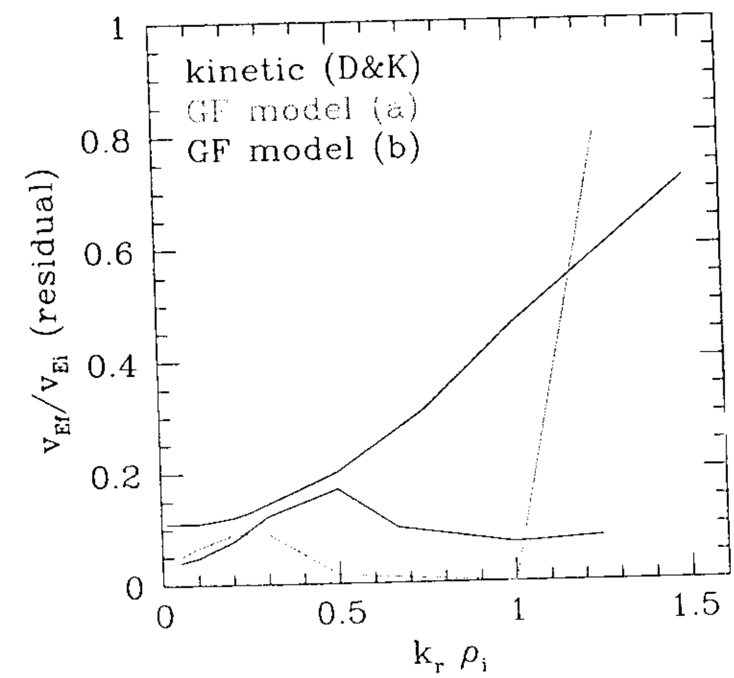
closure (b): $\nu_1 = (0,-3)$, $\nu_2 = (0,1)$, $\nu_3 = (0,0)$, $\nu_4 = (0,-3/2)$, and $q_{\|}^{(0)} = q_{\perp}^{(0)} = 0$, which makes less physical sense but doesn't do too poorly.

Both with ν_5-ν_{10}=0.

Because our flux-tube code is spectral, we can modify these evolution eqns for the zonal flows without changing the $k_\theta \neq 0$ components.

Comparison of Gyrokinetic and Gyrofluid Residual Component vs. k_r

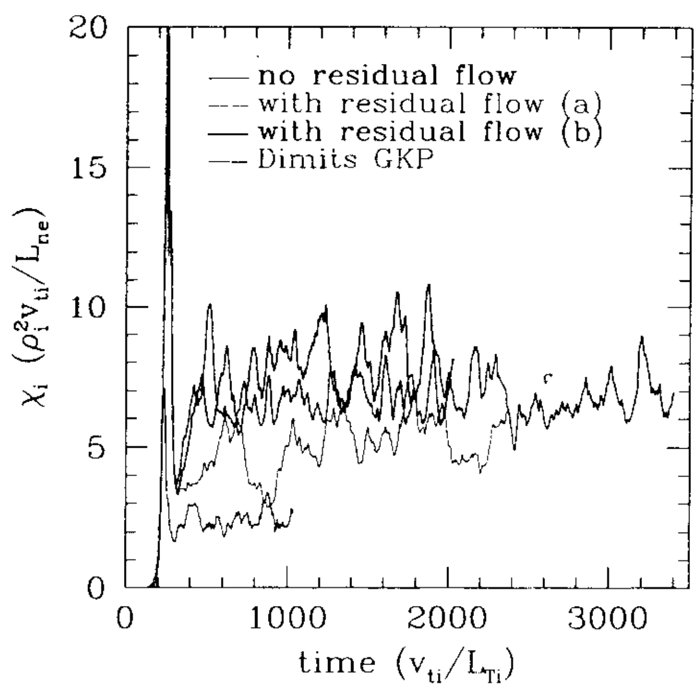
New closures don't do so well for other k_r's:



Closure (a) does slightly better at low k_r, which seems to be more important.

Nonlinear Tests of Importance of Residual Flow

For parameters from DIII-D shot 81499 (the Cyclone base case, with $R/L_{Ti} = 6.9$), we repeat nonlinear runs with the new closures (a) and (b), both including undamped components of the zonal flow.

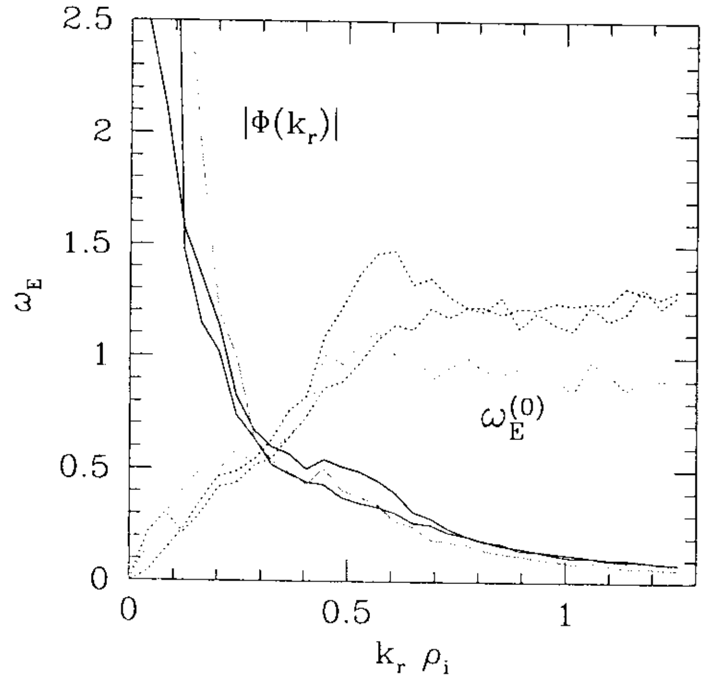


With residual flows, flux drops by up to about 35%, for this case

Nonlinear effects (*e.g.* turbulent viscosity) keep linearly undamped residual components from growing indefinitely

Time Averaged Zonal Flow Spectra with New Closures

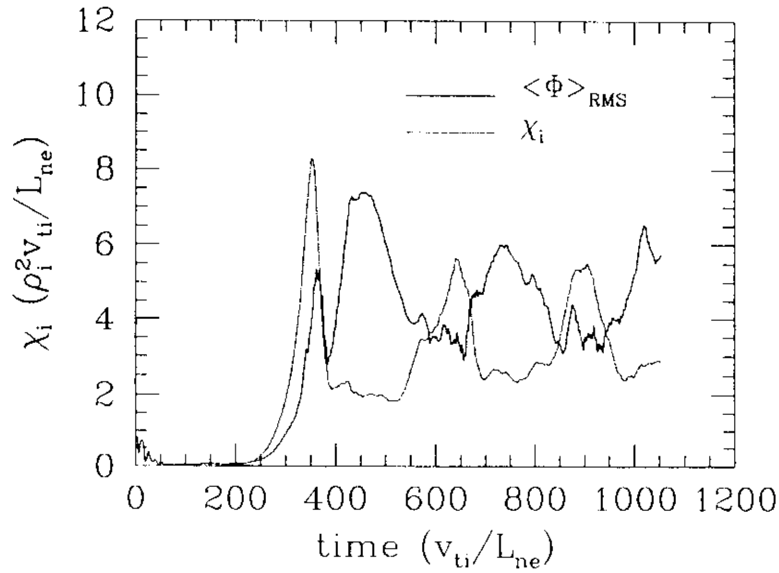
Low k_r zonal flows are larger with new closures



Since low k_r residual component is too small for our new closures, might expect more of an effect as we improve model further

Zonal Flows Can Cause Bursting Near Marginal Stability

Nearer marginal stability ($R/L_{Ti} = 4$), with the old closures we find intermittent behavior:



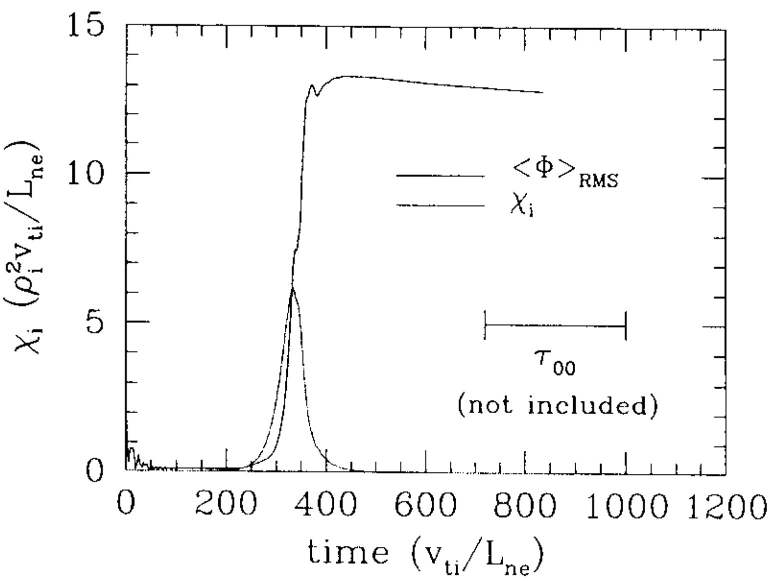
Turbulence (χ_i) drives zonal flows ($\langle\Phi\rangle_{RMS}$) which then damp turbulence. Flows then slowly damp and turbulence grows again.

Bursting is on 1ms time-scale, similar to Mazzucato's fluctuation measurements in RS, which are likely near marginal stability

Zhihong has now seen this bursting with a more realistic neoclassical damping of the flows.

Zero-flux State Near Marginal Stability With Undamped Residual Flow Component

Repeating this marginal stability case ($R/L_{Ti} = 4$) with the new closures, we find that the turbulence drives one burst of flow which is now undamped. Leads to nonlinear upshift in critical gradient (DIMITS, SHERWOOD 1998)



This is in the collisionless limit. A realistic amount of collisions would damp the zonal flows on a time scale $\tau_{00} = 1.5\epsilon/\nu_{ii}$ (ROSENBLUTH, APS 1997) and would likely lead to bursty behavior or a turbulent steady state

Possibly an artifact of initial conditions. We could initialize arbitrarily large flow and get zero flux for any R/L_{Ti}

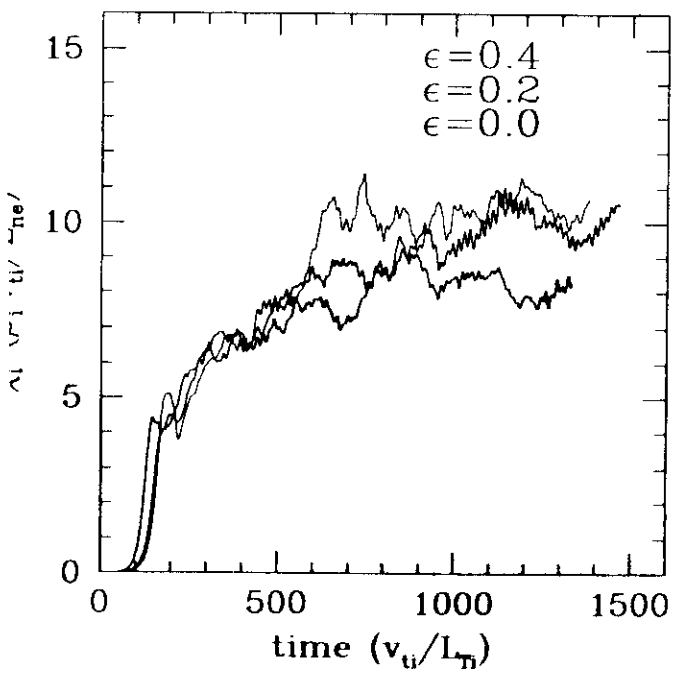
Gyrofluid/Gyrokinetic Comparisons: ϵ scan to test Residual Flow Effects

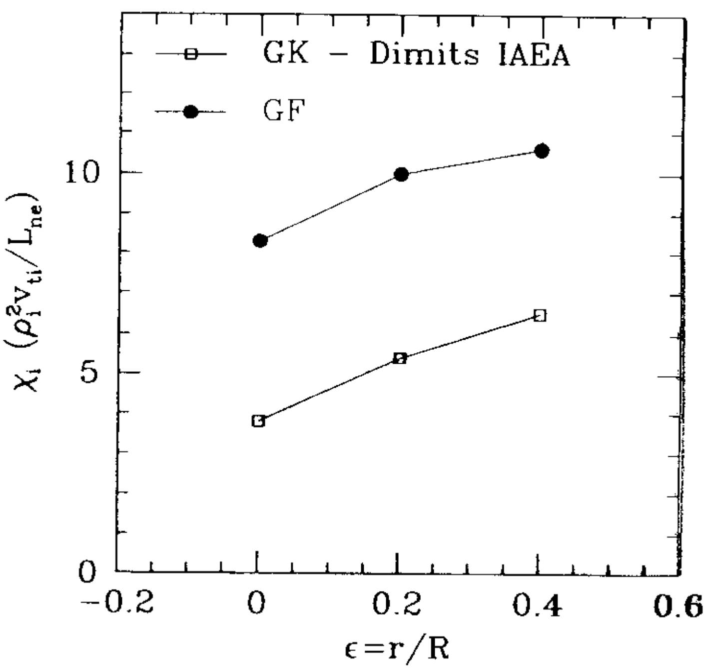
Amount of **residual flow after** an initial flow **perturbation has damped** away is controlled **by** $\epsilon = r/R$, as given by Rosenbluth **&** **Hinton and verified** by Dimits ($c = 0.625$):

$$\frac{\mathbf{v}_{Ef}}{\mathbf{v}_{Ei}} = \frac{c\sqrt{\epsilon}/q^2}{1 + c\sqrt{\epsilon}/q^2}$$

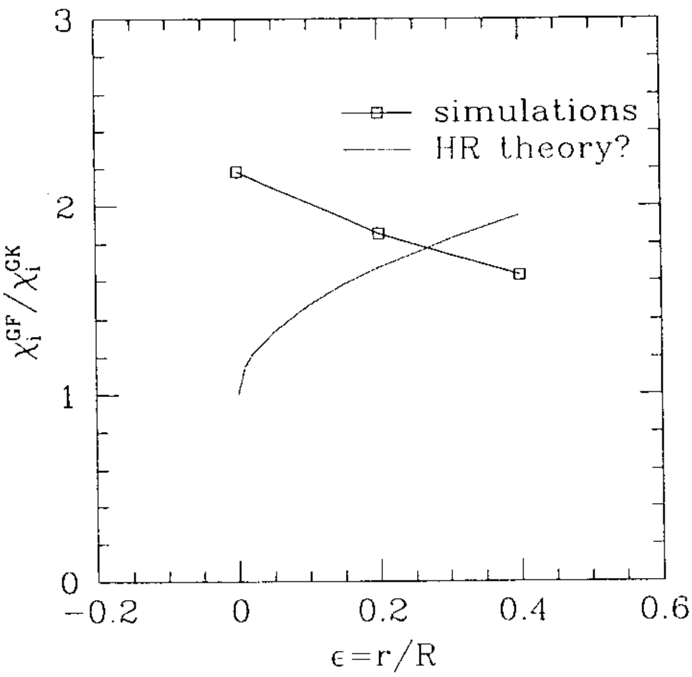
Residual flow component can be turned off by taking $\epsilon \to 0$.

Dimits reported an ϵ scan for the NTP test case parameters in his IAEA (1994) paper which we repeated with GF simulations.





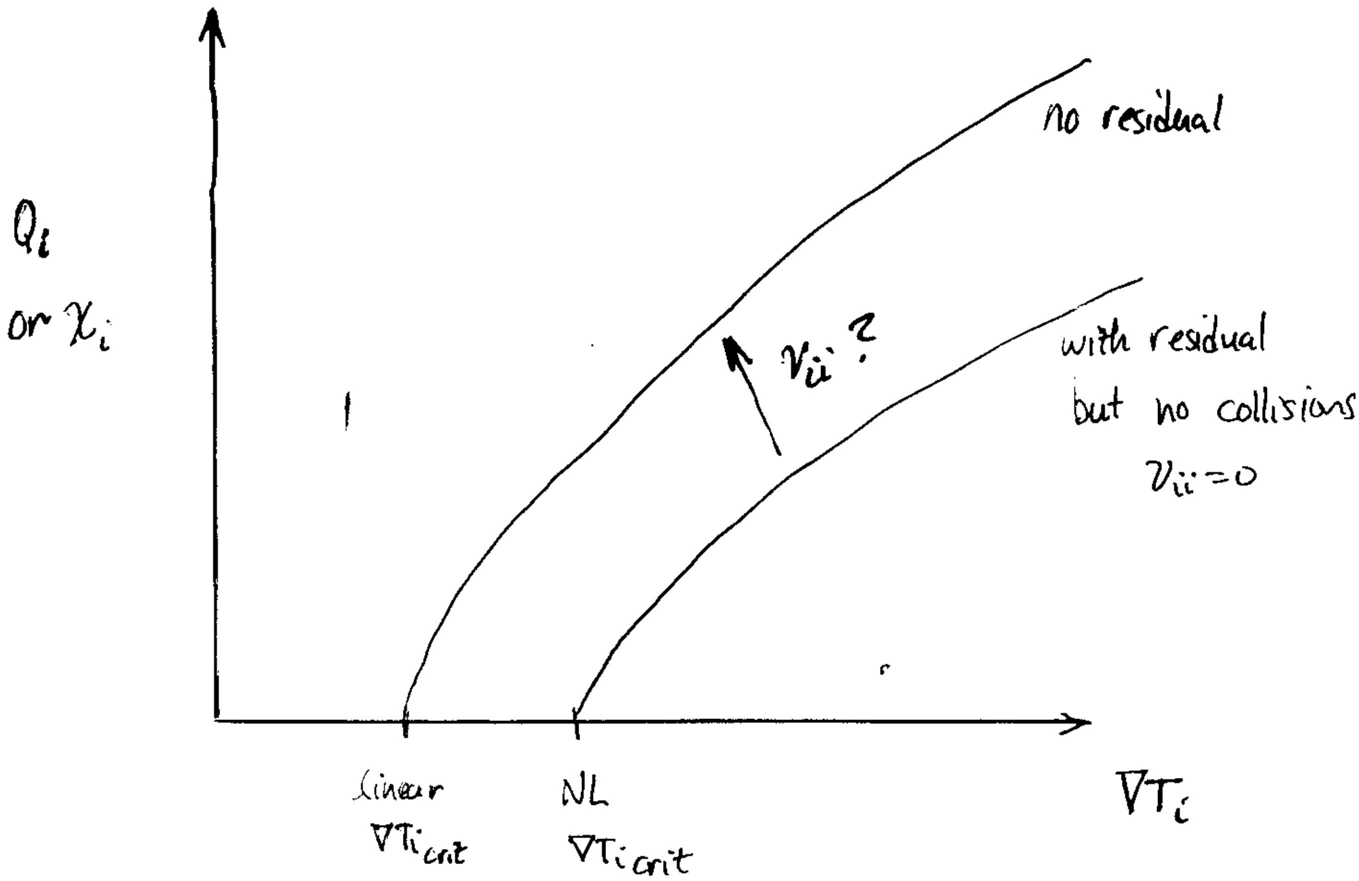
Gyrofluid/Gyrokinetic Comparisons: ϵ scan to test Residual Flow Effects



Ratio χ_{GF}/χ_{GK} does not change as residual flows are turned off.

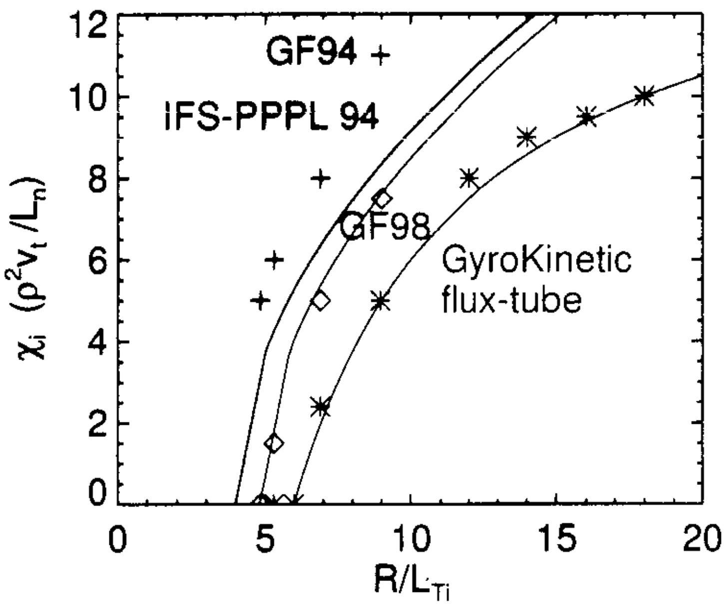
Evidence (?) that residual flows are not dominant source of GF vs. GKP discrepancy, and that turbulent viscosity is keeping residual components from growing to large amplitudes

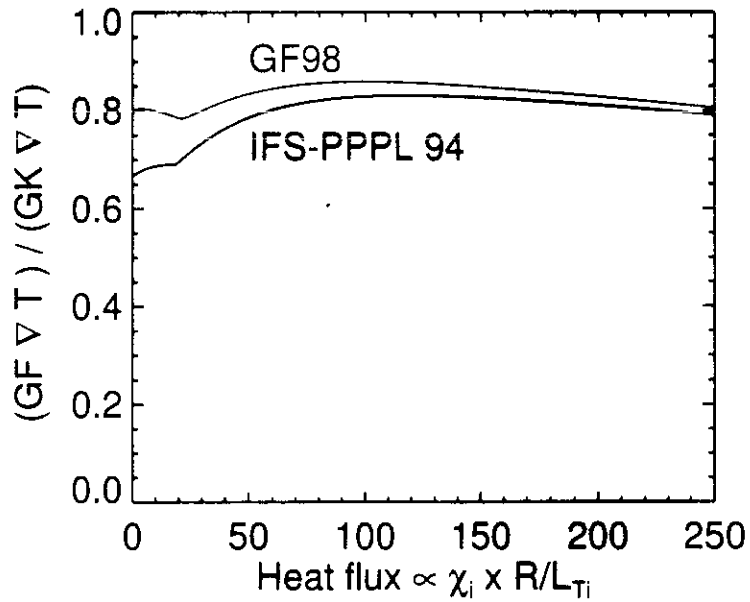
Effect of Residual Flow Component on χ_i vs. ∇T_i



- What sets NL $\nabla T_{i\,crit}$ − linear $\nabla T_{i\,crit}$ and how does it scale with ν_{ii} ?

Gyrofluid/gyrokinetic (GF/GK) simulation differences → 20-33% change in predicted temperature gradient





- **Dimits (LLNL): good convergence in his gyrokinetic particle simulations**
- **New neoclassical gyrofluid closure significantly improves GF/GK comparison.**
- **Turning this plot around, for a fixed amount of heat flux $\propto \chi\nabla T$, the temperature gradient predicted by the original gyrofluid-based IFS-PPPL model is 20-33% low. But $P_{fusion} \propto T^2$, and so may increase by $\times 2$ or more.**
- **Nonlinear upshift in critical gradient may depend on: Rosenbluth-Hinton undamped zonal flows ↑ with elongation (W. Dorland), ↓ with weak collisions (Z. Lin), ↓ ?? with non-adiabiatic electrons [may limit inverse cascade that drives zonal flows (Diamond, Liang, Terry-Horton, Waltz, ...) and ↑ turbulent viscosity].**

Conclusions

- New gyrofluid closures derived which retain linearly undamped residual zonal flow components
- Agreement at low k_r is not great, further closure modifications being investigated
- Nonlinear comparisons show that including residual component has 30%-40% effect for Cyclone DIII-D base case
- Might expect larger effect as low k_r behavior is improved
- Near marginal stability system can bifurcate into all flow, zero flux state
- When undamped flow effect is important, a small amount of collisions may increase χ_i.
  - collisional flow damping will be important here. Zhihong's results indicate that $\chi_i \sim \nu_{ii}$ near marginal stability.
  - intermittent or bursty behavior seen with some flow damping
- In strong turbulence regimes nonlinear effects appear to saturate residual flow component, (turbulent viscosity keeps residual components from growing indefinitely)
- Nonlinear GyroKinetic Particle (GKP) vs. GyroFluid (GF) comparisons:
  - GF/GKP discrepancy is typically 2-3.
  - Differences in linear zonal flow dynamics may account for some of the GF/GKP discrepancy, especially near marginal stability
  - Adding additional physics (e.g. TE's, collisions) may move system farther from marginal stability and improve agreement
- Future work
  - Investigate collisionality and IC dependence of flux near marginal stability
  - Perhaps move to more flexible frequency dependent closures (Mattor)