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Trapped Nonneutral Plasmas, Liquids and Crystals

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three lectures:

(1) Trapped Nonneutral Plasmas, Liquids, and Crystals (The thermal equilibrium states)

(2) Thermodynamics of Trapped Nonneutral Plasmas

(3) Collisional Transport and 2D Vortex Dynamics in Nonneutral Plasmas

references:

1. Thomas O'Neil, Trapped Plasmas with a Single Sign of Charge, <u>Physics Today</u>, February 1999

2. D. H. E. Dubin and T. M. O'Neil, Trapped Nonneutral Plasmas, Liquids, and Crystals (the thermal equilibrium states), Rev. Mod. Phys. <u>71</u> 87 (1999)

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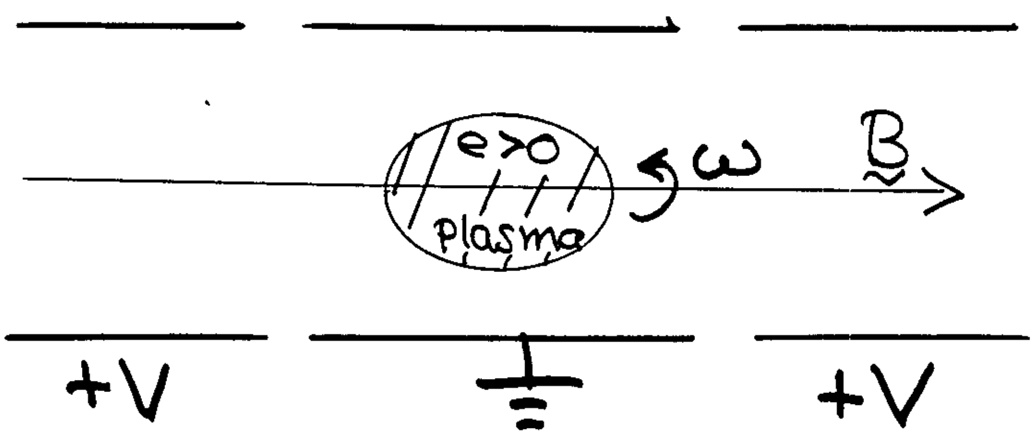
Malmberg-Penning Trap



- radial force balance

$$-\frac{nmv_\theta^2}{r} = ne\left[E_r + \underbrace{\frac{v_\theta B}{c}}_{\substack{\text{balances} \\ \text{other terms}}}\right] - \frac{\partial p}{\partial r}$$

- rotation through a magnetic field is like neutralization by background charge.

Constants of the Motion

For an ideal trap, the N-particle Hamiltonian is characterized by two symmetries:

(1) H does not depend explicitly on t.

(2) H ~~is~~ invariant under rotation,

$\therefore$ H = const.

$$P_\theta = \underbrace{\sum_{j=1}^{N} m v_{\theta_j} r_j + \frac{e}{c} A_\theta(r_j) r_j}_{\text{Total Canonical Angular Momentum}} = \text{const}$$

Confinement Theorem

For sufficiently large V, the axial confinement is guaranteed on energetic grounds (since $H =$ const.).

To understand the radial confinement, we use the constancy of the total canonical angular momentum

$$P_\theta = \sum_{j=1}^{N} m v_{\theta_j} r_j + \frac{e}{c} \underbrace{A_\theta(r_j)}_{} r_j$$

for uniform axial $\underline{B}$ field $\longrightarrow \frac{B r_j}{2}$

For sufficiently large $\underset{\sim}{B}$, the vector potential term dominates

$$P_\theta \simeq \sum_{j=1}^{N} \frac{eB}{2c} r_j^2 = \frac{eB}{2c} \sum_{j=1}^{N} r_j^2$$

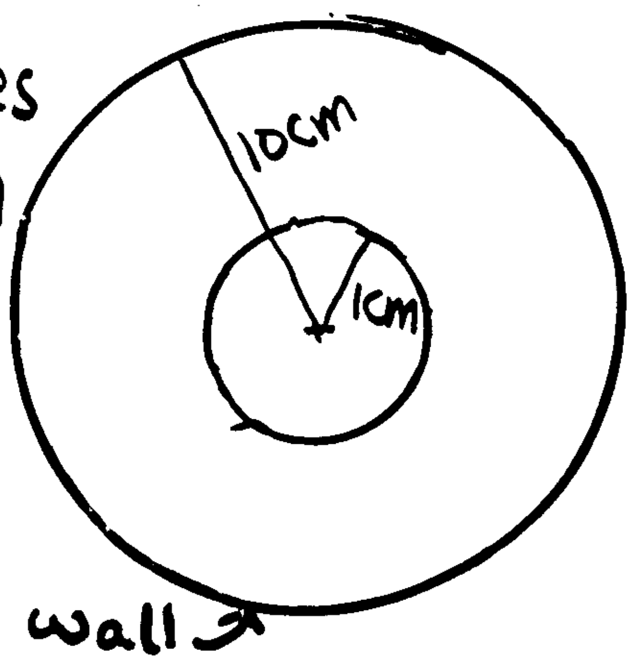
$$\therefore P_\theta = \text{const.} \Longrightarrow \sum_{j=1}^{N} r_j^2 = \text{const.}$$

This is <u>powerful constraint</u>.

Let $r_j(t=0) = 1\,\text{cm}$ for $j = 1 -- N$.

Only 1% of particles can ever reach wall, at $r_j = 10\,\text{cm}$, provided that

$$\sum_j r_j^2(t) = \text{const.}$$



$\therefore$ 99% of particles remain confined <u>forever</u>, provided that

$$\frac{eB}{2c} \sum_j r_j^2 \simeq P_\theta = \text{const.}$$

For a neutral plasma, the constraint is replaced by

$$\sum_j e_j r_j^2 = \text{const.},$$

so electrons and ions can go to the wall together.

Of course, for a real plasma in a real trap, H and P_θ are <u>not</u> conserved exactly.

(1) The charges slowly radiate energy and angular momentum.

(2) Collisions with neutrals cause slow change in values of H and P_θ.

(3) Small field errors break the cylindrical symmetry of the confinment apparatus (i.e., of H) and allow P_θ to change slowly.

However, with care (good vacuum, high degree of cylindrical symmetry, etc.) a trap can be constructed for which H and P_θ are nearly constant on time scale for Coulomb collisions to bring charges into state of global thermal equilibrium.

We discuss thermal equilibrium states using model of ideal trap ($H, P_\theta = \text{const.}$).

Return later to include effect of slow changes in H and P_θ.

Boltzmann distribution for a system characterized by the two constants of the motion H and P_θ:

$$f = \frac{N \exp\left[-\frac{1}{T}(h+\omega P_\theta)\right]}{\int d^3\underline{r}\, d^3\underline{v}\, \exp\left[-\frac{1}{T}(h+\omega P_\theta)\right]}$$

$$h = \frac{mv^2}{2} + e\varphi_T(r,z) + e\varphi_p(r,z)$$

trap potential

plasma space charge potential

$$P_\theta = mv_\theta r + eBr^2/2c$$

T is temperature

$-\omega$ is rotation frequency

$$(H, P_\theta) \Longrightarrow (T, \omega)$$

Substituting for h and P_θ yields:

$$f = n(r,z)\left(\frac{m}{2\pi T}\right)^{3/2} \exp\left[\frac{-m}{2T}\left(\underset{\sim}{v} + \omega r \hat{\theta}\right)^2\right]$$

$$n(r,z) = \frac{N \exp\left\{-\frac{1}{T}\left[e\phi_p(r,z) + e\phi_R(r,z)\right]\right\}}{\int d^3\underline{r} \exp\left\{ \qquad \right\}}$$

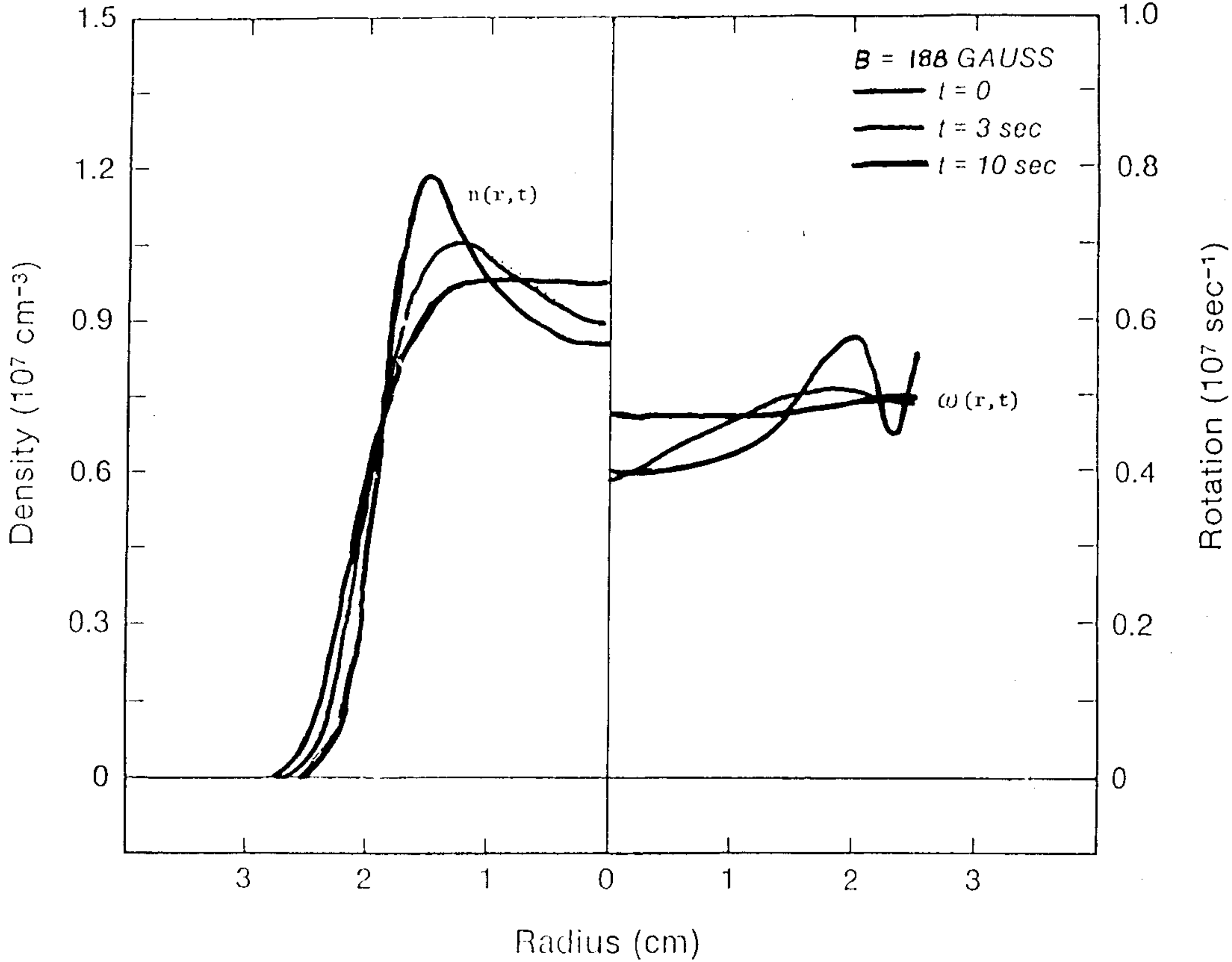
where

$$e\phi_R(r,z) = e\phi_T(r,z) + m\omega(\Omega_c - \omega)\frac{r^2}{2}$$

effective trap potential in the rotating frame

- shear-free (rigid rotor) flow
- plasma is confined for sufficiently large V and B

Driscoll, Malmberg, and Fine, Phys. Rev. Lett. 60, 1290 (1987)



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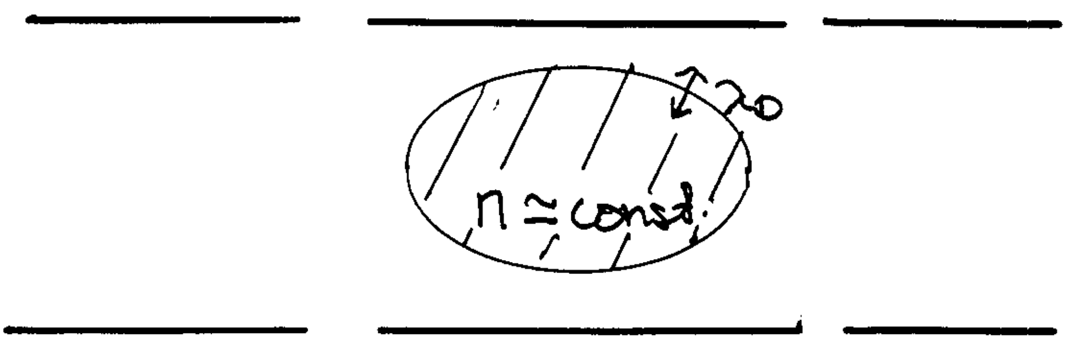
Self-consistent density + potential

$$\nabla^2 \phi_p(r,z) = -4\pi e\, n(r,z)$$

$$\phi_p(r=R,z) = 0 \quad \text{on wall}$$



- Debye shielding (in rotating frame) forces

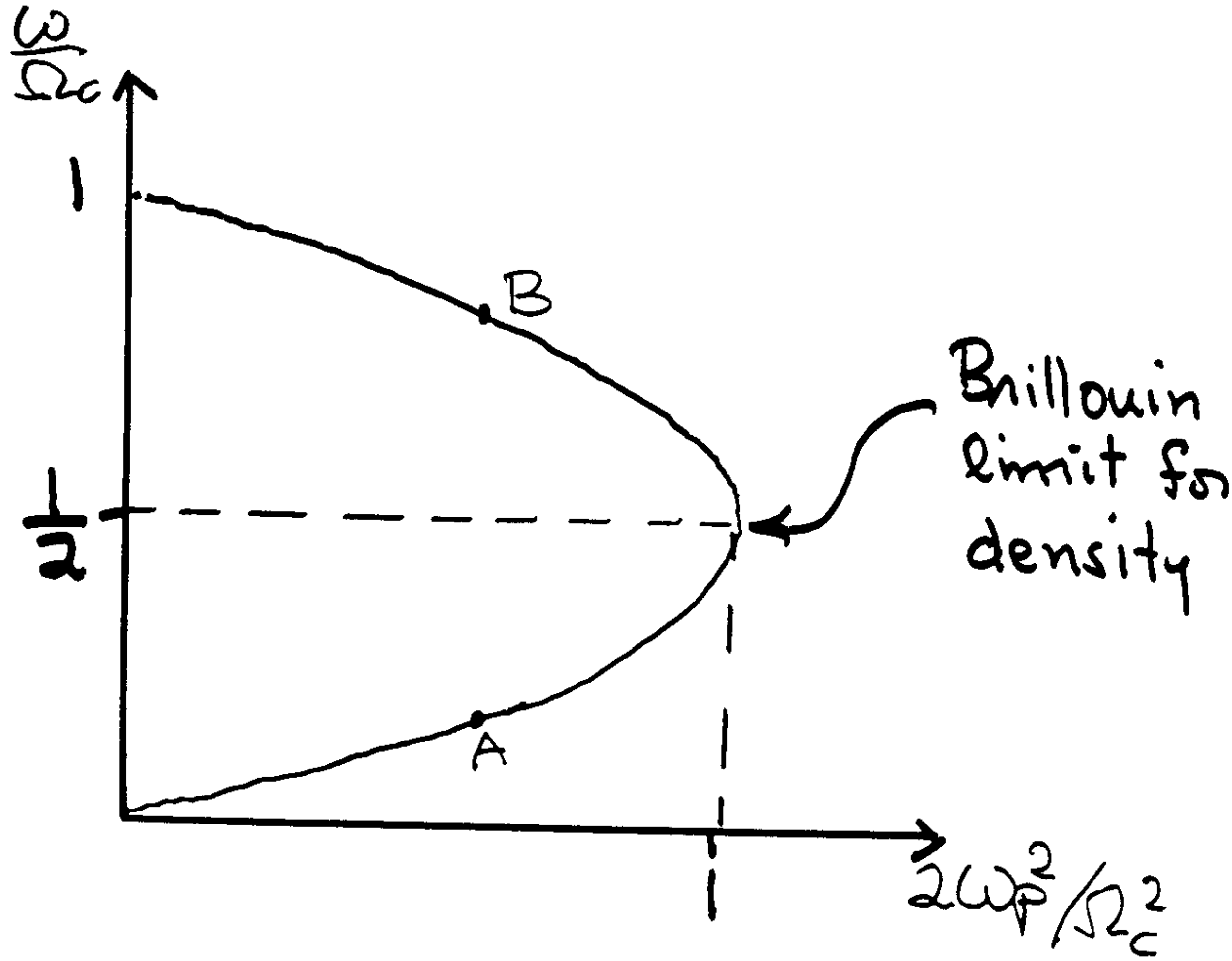
$$\left.\begin{aligned} &\phi_p(r,z) + \phi_R(r,z) \simeq \text{const.} \\ &\therefore\ n(r,z) \simeq \text{const.} \end{aligned}\right\} \text{inside plasma}$$

- Imaginary neutralizing charge

$$-4\pi e^2 n_- = \nabla^2 e\phi_R = \underbrace{\nabla^2 e\phi_T}_{\simeq 0} + \underbrace{\nabla^2 m\omega(\Omega_c-\omega)\frac{r^2}{2}}_{2m\omega(\Omega_c-\omega)}$$

$$n \simeq n_-$$

$$\omega_p^2 = 2\omega(\Omega_c - \omega)$$



- $\Phi_R(r, z; \omega = \omega_A) = \Phi_R(r, z; \omega = \omega_B)$
- vortex frequency : $\Omega_v = \Omega_c - 2\omega$

$$\Omega_v^{(A)} = -\Omega_v^{(B)}$$

Small plasma in quadratic trap potential

$$e\varphi_T(r,z) = C + \frac{m\omega_z^2}{2}\left(z^2 - \frac{r^2}{2}\right) + -$$

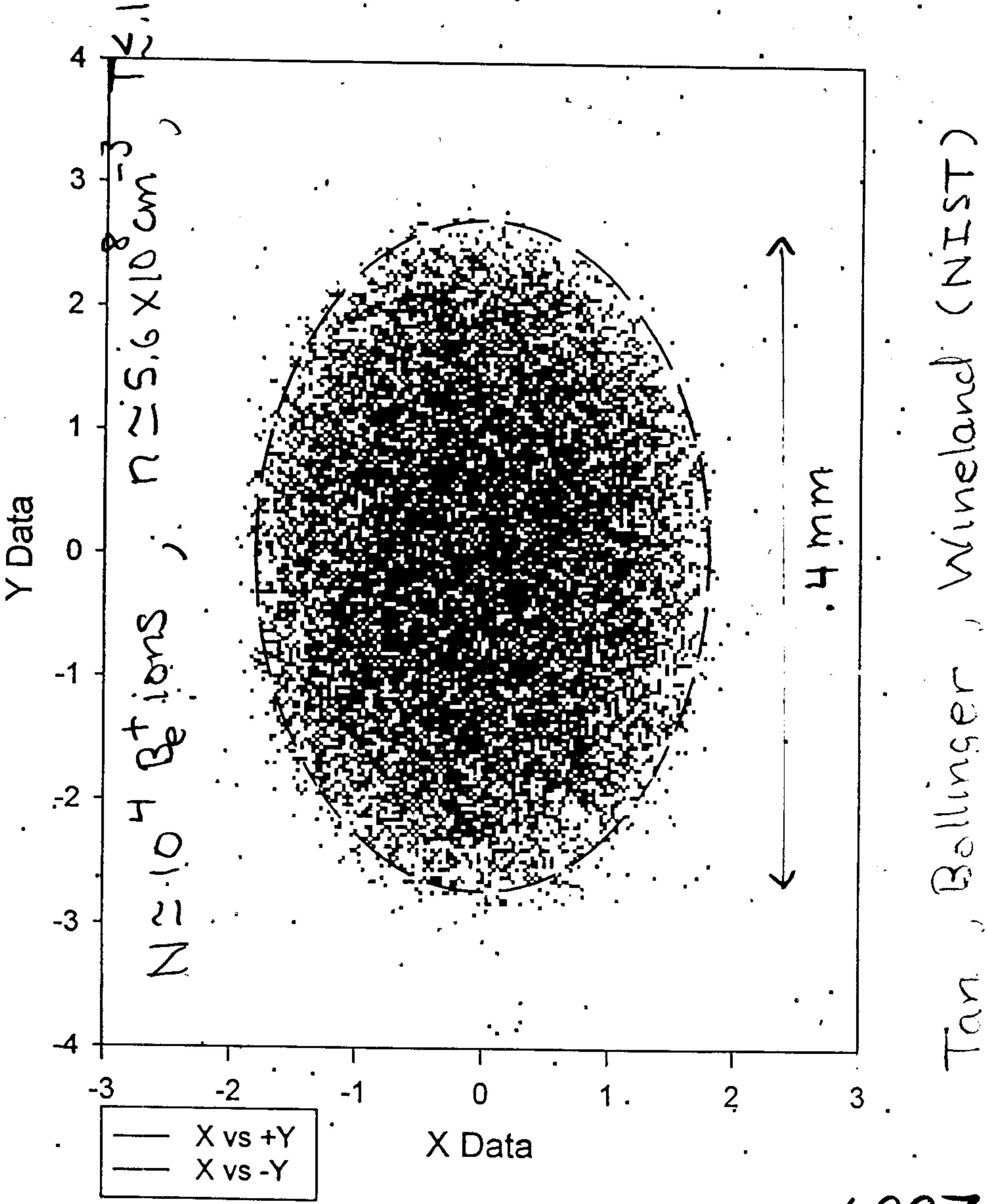
$$e\varphi_R(r,z) = e\varphi_T(r,z) + m\omega(\Omega_c - \omega)\frac{r^2}{2}$$

also is quadratic.

$\varphi_p(r,z) + \varphi_R(r,z) = \text{const.}$ inside plasma implies that $\varphi_p(r,z)$ is quadratic inside plasma.

$\therefore$ Plasma is uniform density spheroid (ellipse of revolution)

2D Graph 4



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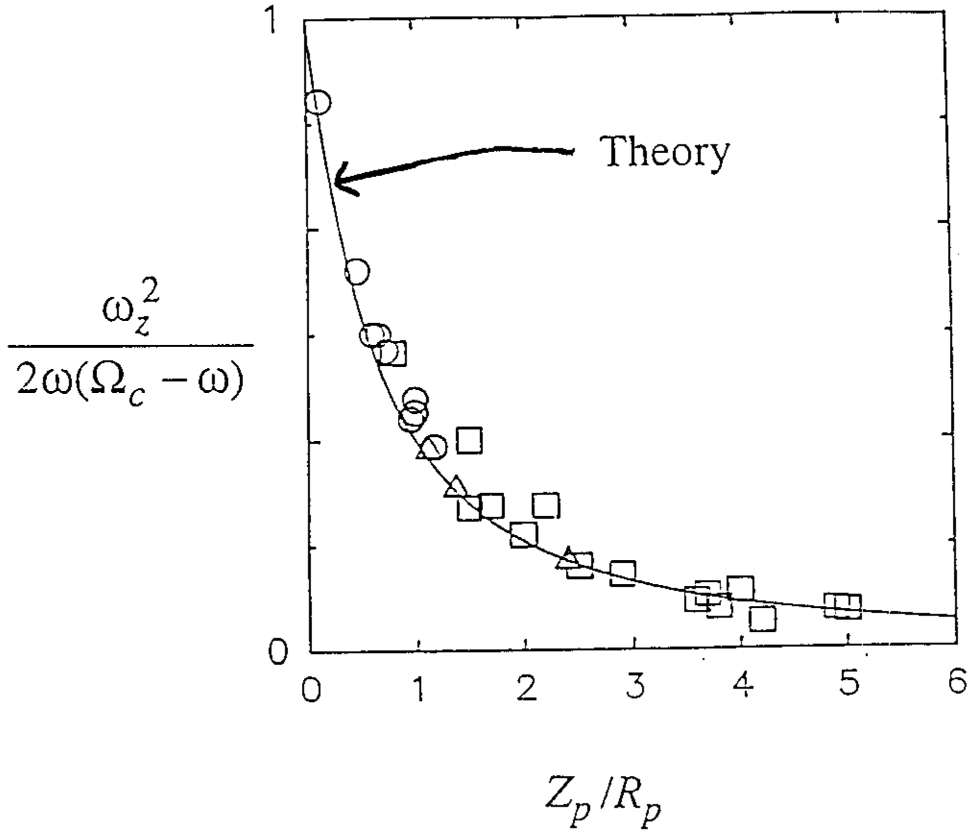
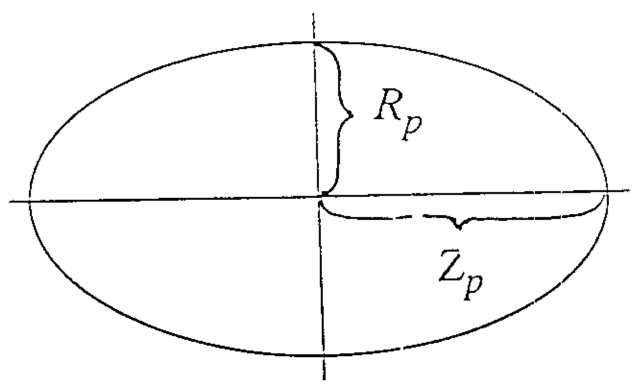
$$e\,\phi_T = \frac{m\,\omega_z^2}{2}\left(z^2 - \frac{r^2}{2}\right)$$





Brewer, Prestage, Bollinger, Itano, Larson and Wineland, Phys. Rev. A **38**, 859 (1988).

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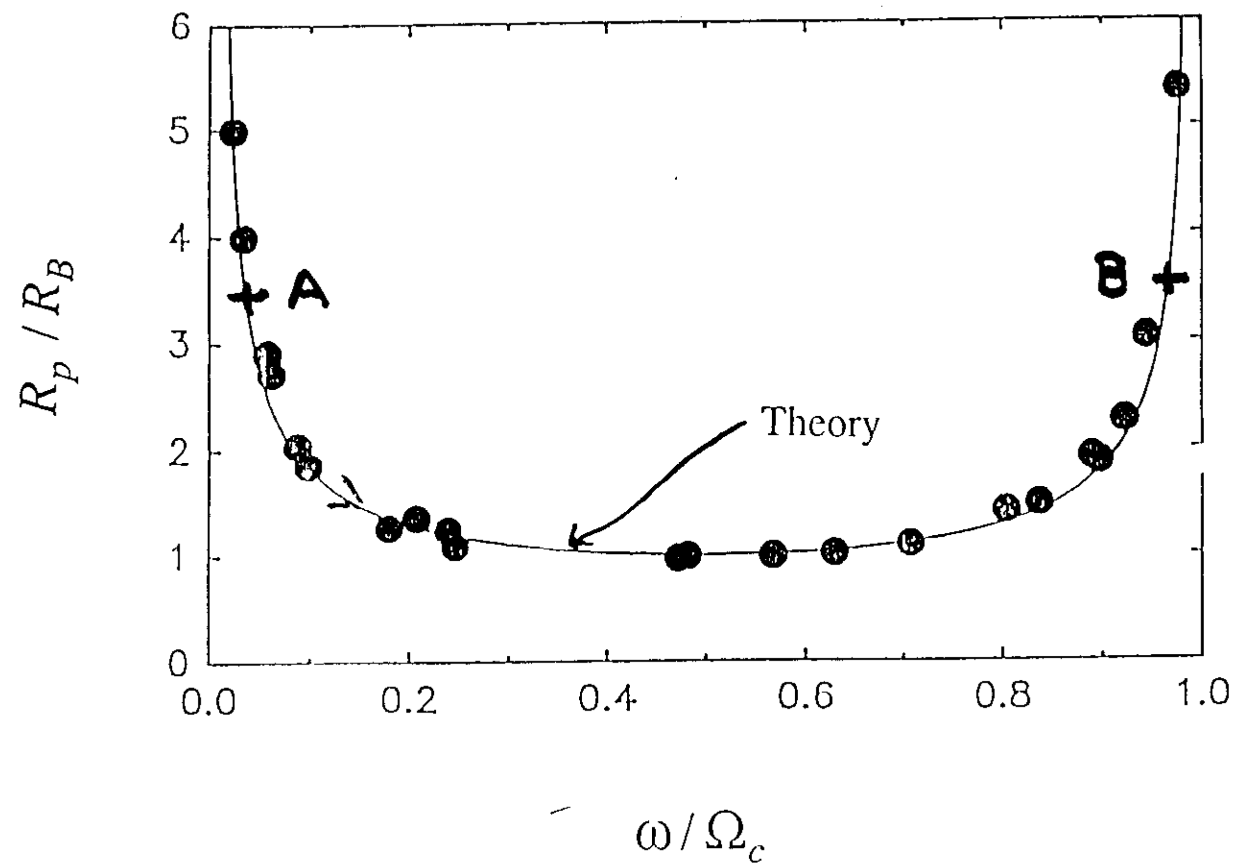
$$e\,\phi_R = e\,\phi_T + m\,\omega\,(\Omega_c - \omega)\,\frac{r^2}{2} = e\,\phi_T + m\left[\left(\frac{\Omega_c}{2}\right)^2 - \left(\frac{\Omega_c}{2} - \omega\right)^2\right]\frac{r^2}{2}$$

$$(\Omega_c)_R = \Omega_c - 2\omega$$ due to Coriolis force



Bollinger, Heinzen, Moore, Itano, Wineland and Dubin, Phys. Rev. A **48**, 525 (1993).

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Modes for a small spheroidal plasma

$$\delta\phi(x,y,z,t) = \delta\phi(x,y,z)\, e^{-i\omega t}$$

mode freq

$$\underset{\sim}{\nabla} \cdot \underset{\approx}{\epsilon}(\underset{\sim}{r},\omega) \cdot \underset{\sim}{\nabla}\delta\phi = 0$$

$\delta\phi = 0$ on wall (at $\underset{\sim}{r} \to \infty$).

$$\underset{\approx}{\epsilon} = \begin{bmatrix} \epsilon_1 & -i\epsilon_2 & 0 \\ i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix}$$

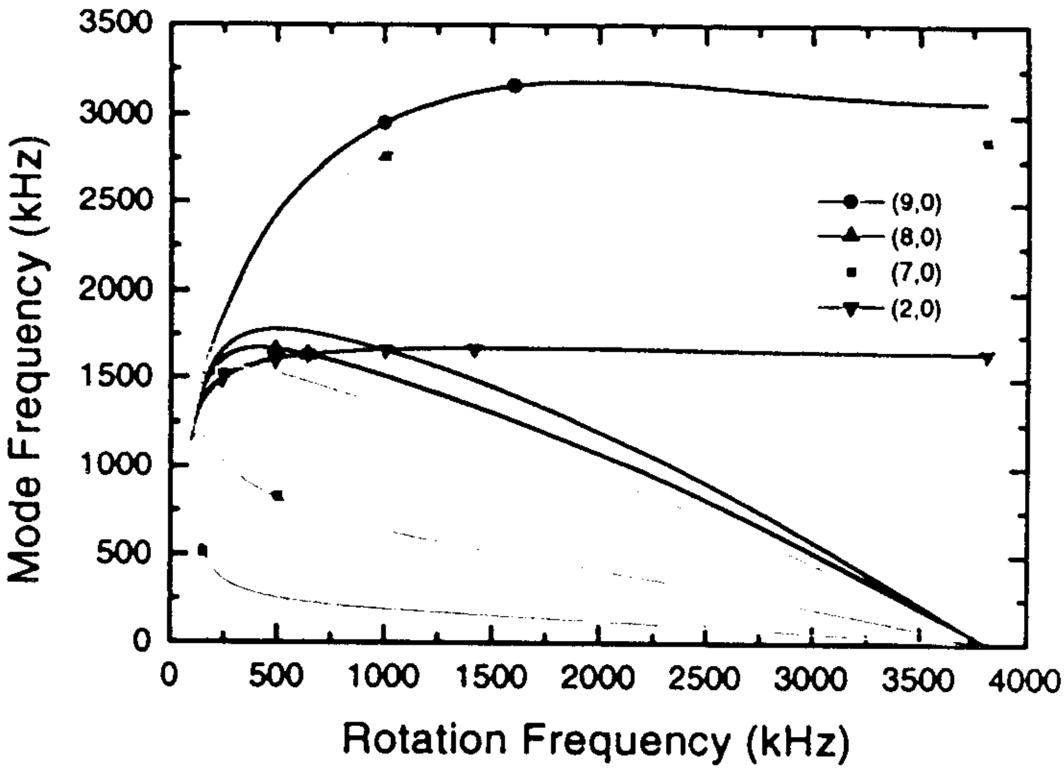
$$\epsilon_1 = 1 - \frac{\omega_p^2(\underset{\sim}{r})}{(\omega - \omega_R)^2}, \quad \epsilon_2 = \frac{\Omega_c \omega_p^2(\underset{\sim}{r})}{\omega(\omega^2 - \omega_R^2)}$$

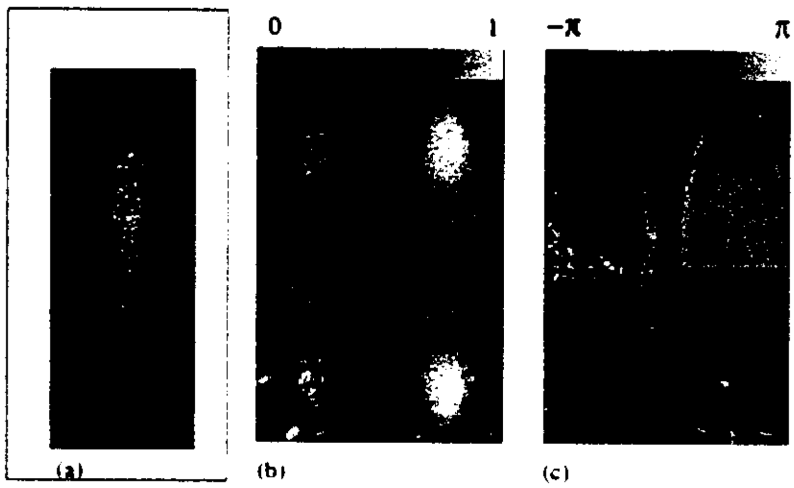
$$\epsilon_3 = 1 - \omega_p^2/\omega^2$$

rotation freq.

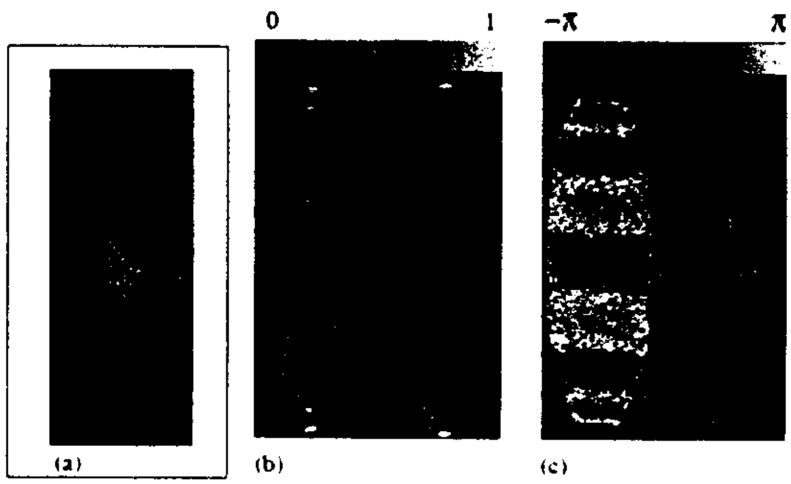
Dubin found exact soln using spheroidal coordinates [Phys. Rev. Lett. 66 2076 (1991)]

T.B. Mitchell et al., Optics Express 2, 314 (1998)





(2,0) mode
at 1.656 MHz



(9,0) mode
at 2.952 MHz

Plasmas with a single sign of charge can be cooled to the cryogenic temperature range without the occurrence of recombination.

Correlation strength

$$\Gamma = \frac{e^2/a}{T} \quad , \quad \frac{4}{3}\pi a^3 n = 1$$

For an infinite homogeneous one component plasma (OCP):

$\Gamma \ll 1$ weakly correlated plasma

$\Gamma \gtrsim 2$ fluid

$\Gamma > 174$ b.c.c. crystal

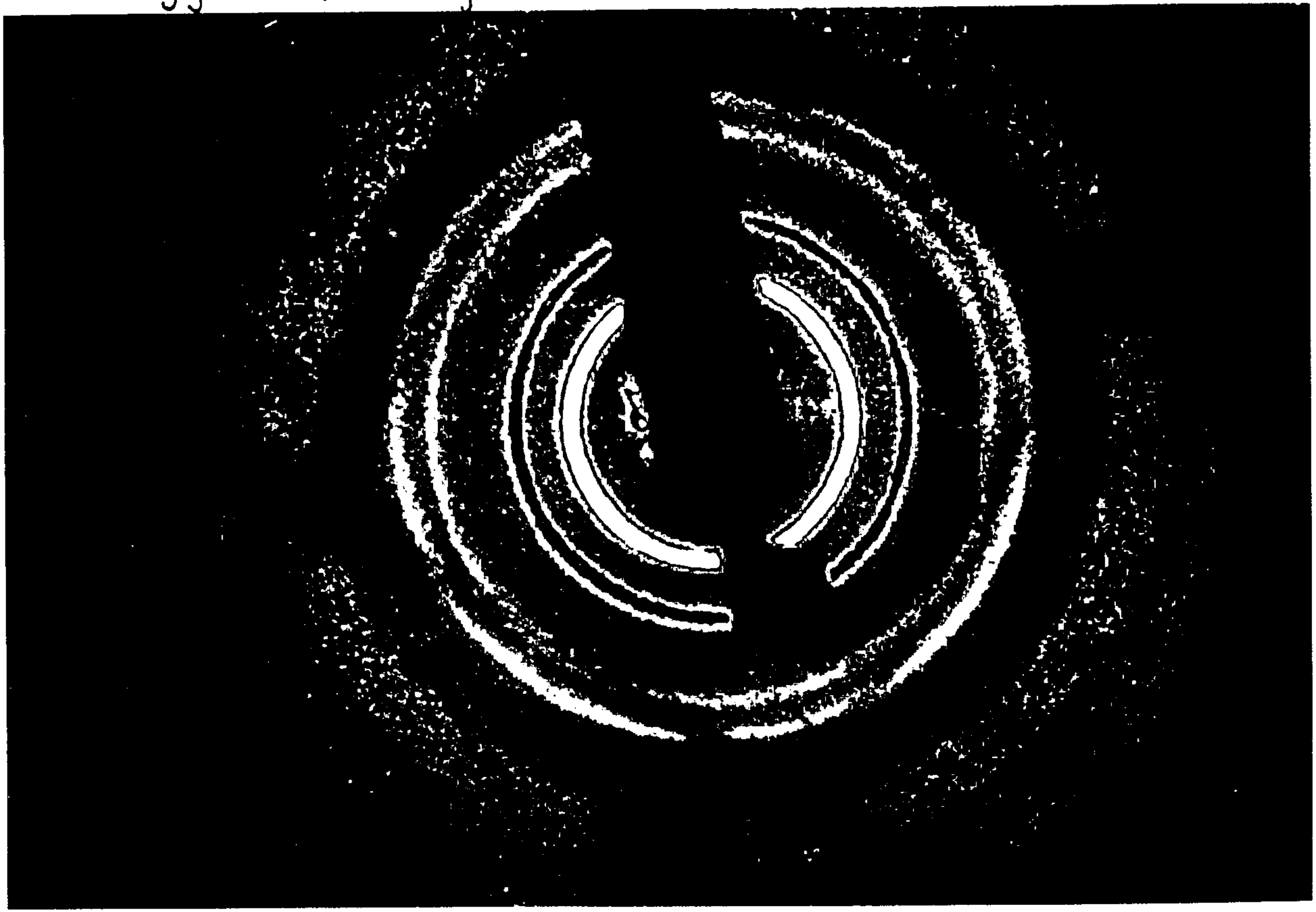
To describe correlated plasma we need the Gibbs distribution:

$$\rho = C \exp\left[-\frac{1}{T}(H + \omega P_\theta)\right]$$

The Gibbs distribution for a magnetically confined plasma differs only by rotation from the Gibbs distribution for a plasma that is confined by a cylinder of uniform neutralizing background charge (i.e., an OCP).

∴ For sufficiently large plasma, can use previous results obtained for infinite homogeneous OCP.

Bragg scattering from $N \simeq 5\times10^{5}$ $^{9}Be^{+}$ ions



Tan, Bollinger, Jelenkovic, and Wineland (NIST

[illegible], Bollinger, Jelenkovic, Itano, and Wineland (NI

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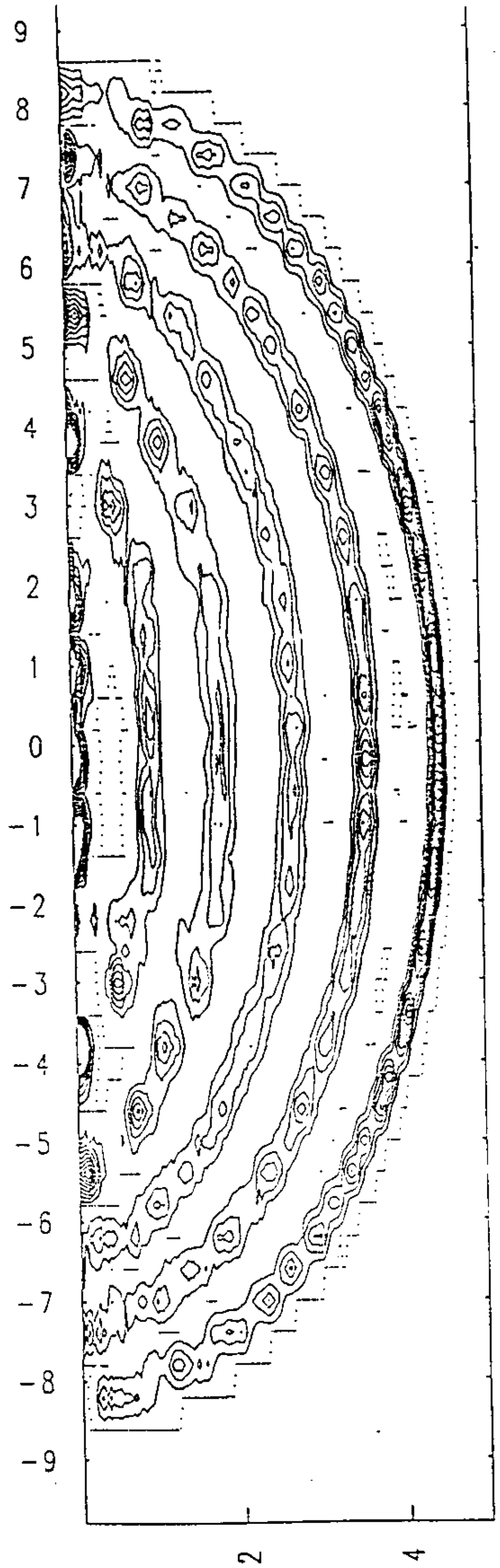

Density contours

$\Gamma = 120$, $N = 1028$

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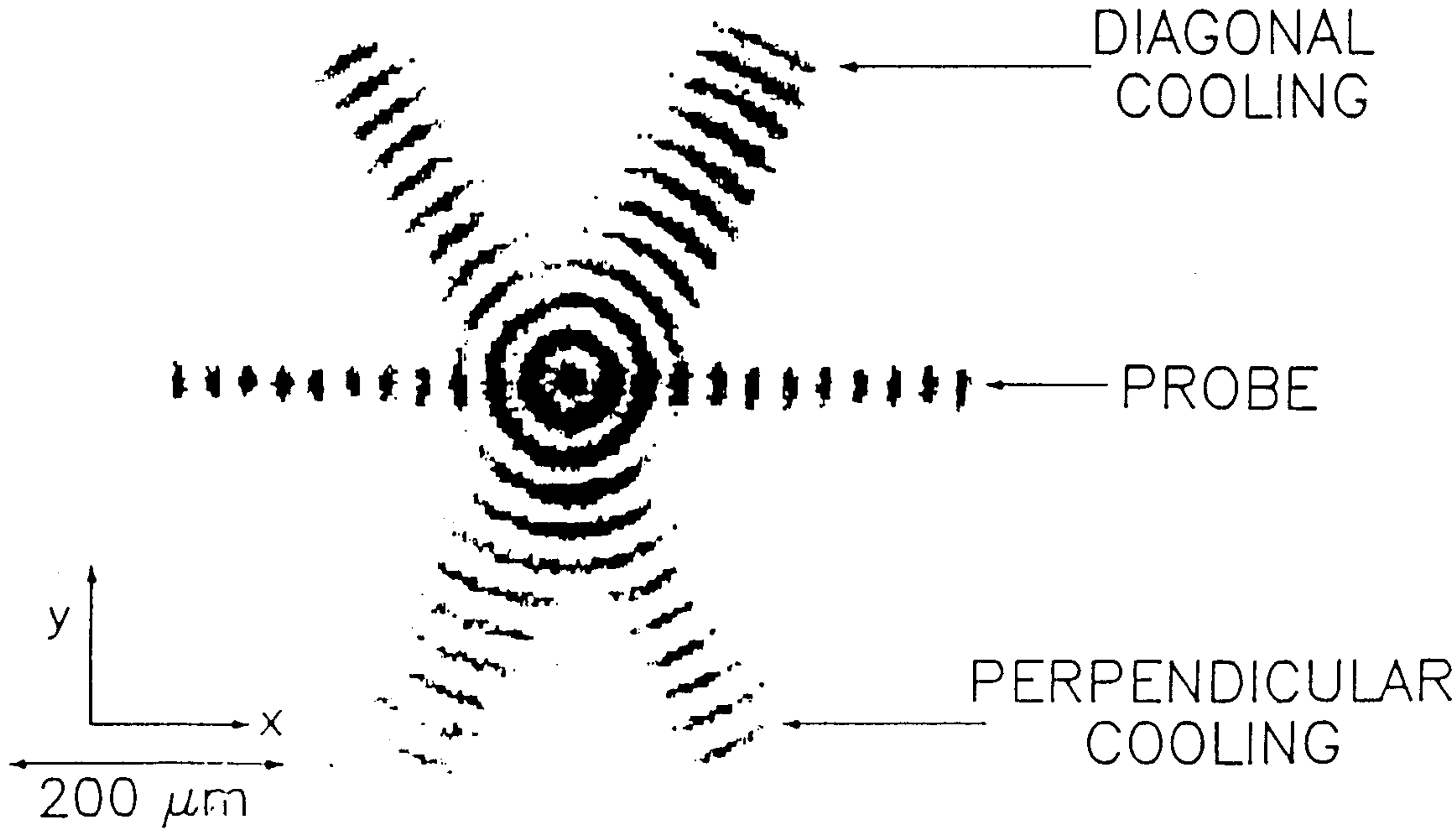
Dubin and O'Neil



J.J. Bollinger and D.J. Wineland, Scientific American p. 124 (January 1990)