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2D Collisional Transport in Nonneutral Plasmas

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These are preliminary lecture notes, intended only for distribution to participants.

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(3)

2D Collisional Transport in Nonneutral Plasmas

References:

(1) Anderegg, Huang, Driscoll, O'Neil, and Dubin, "Test Particle Transport due to Long Range Interactions," Phys. Rev. Lett. **78**, 2128 (1997).

(2) Dubin and O'Neil, "Cross-Magnetic Field Heat Conduction in Nonneutral Plasmas," Phys. Rev. Lett. **78**, 3868 (1997).

(3) Dubin, "Test Particle Diffusion and the Failure of Integration along Unperturbed Orbits," Phys. Rev. Lett. **79**, 2678 (1997).

(4) Hollmann, Anderegg, and Driscoll, "Measurement of Cross-Magnetic-Field Heat Transport in a Pure Ion Plasma," Phys. Rev. Lett. **82**, 4839 (1999).

test particle diffusion D — thermal conductivity κ

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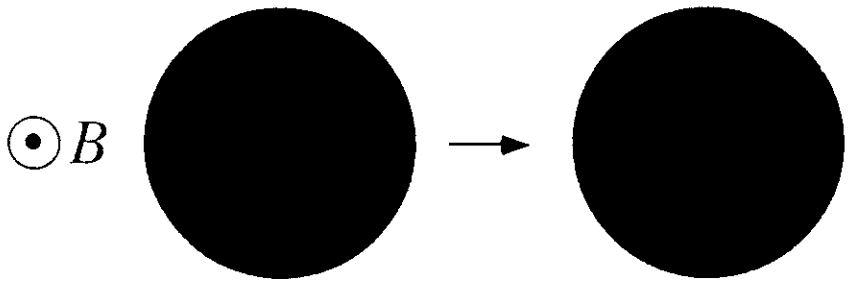
test particle diffusion
radial flux of test particles:

$$\Gamma_{test} = -D\, n_{total} \frac{\partial}{\partial r}\left(\frac{n_{test}}{n_{total}}\right)$$

$$D \sim cm^2 / \sec$$



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heat conduction
radial flux of heat:

$$\Gamma_Q = -\kappa \frac{\partial T}{\partial r}$$

$$\Rightarrow \kappa / n \sim cm^2 / \sec$$

↑

(thermal diffusivity χ)

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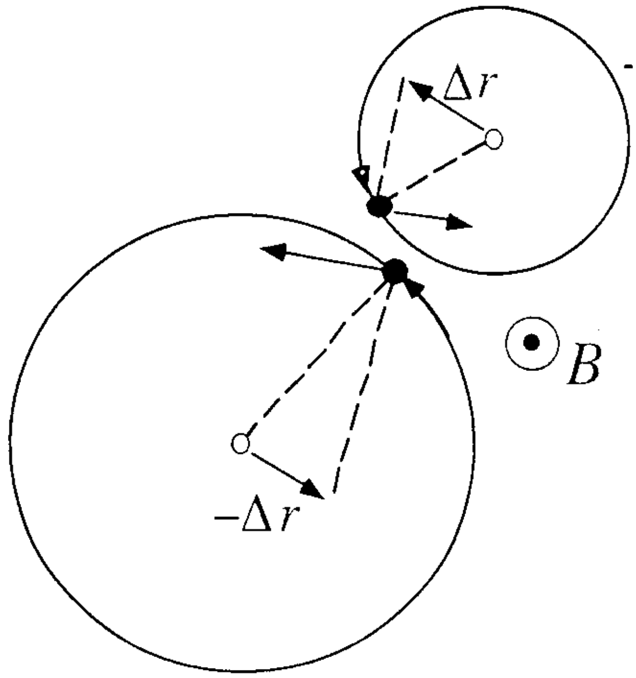
Classical Theory of Collisional Transport

implicitly assumes $r_c >> \lambda_D$



$$\Delta r \sim r_c$$

frequency of steps ~ collision frequency ν_c

$$D \sim \frac{< \Delta r^2 >}{\Delta t} \sim \nu_c r_c^2$$

$$\frac{\kappa}{n} \sim D \sim \nu_c r_c^2$$

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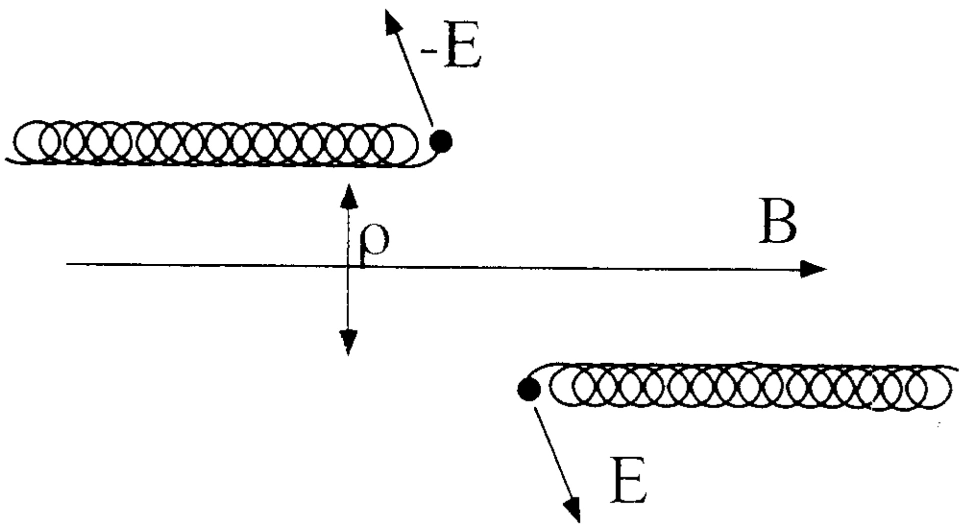
non-neutral plasmas

typically, $r_c << \lambda_D$

$\therefore$ most collisions are long range:

$r_c \lesssim \rho \lesssim \lambda_D$



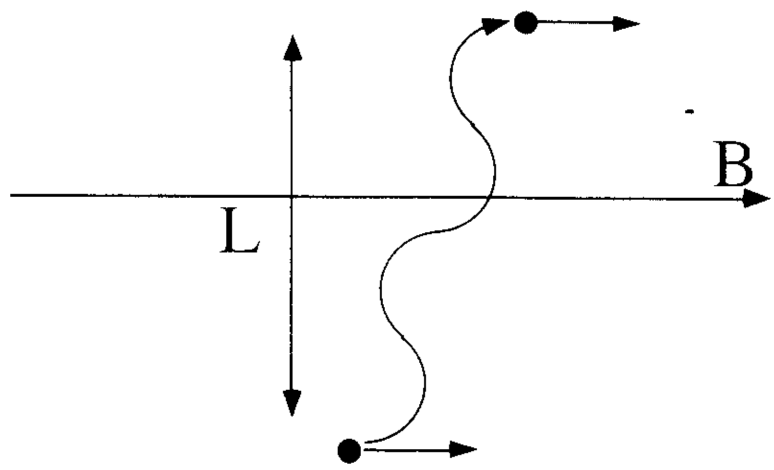
$$\kappa / n \sim \nu_c < \rho^2 >$$

$$\Rightarrow \kappa \sim n \nu_c \lambda_D^2 >> \kappa^{class}$$

Dubin + O'Neil, 97

Psimopoulos + Li, 92

emission and absorption of
lightly-damped plasma waves
enhances transport



ρ ~ plasma width L
(but $\nu < \nu_c$)

$$\kappa^{waves} > n\nu_c\lambda_D^2 \quad \text{if } L \gtrsim 10^2\,\lambda_D$$

Rosenbluth + Liu '76

Dubin + O'Neil '97

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Unified Theory of Heat Transport
(slab geometry; $x \leftrightarrow r$)

$$\Gamma_Q = -\sum_{j=1}^{\infty}\left(\kappa^{local} + \kappa_j^{waves}\right)\hat{T}_j \sin\frac{j\pi x}{L}$$

F.T. of dT/dx

$$\kappa^{local} = 1.2\, n\nu_c \lambda_D^2$$

$$\kappa_j^{waves} = n\nu_c\lambda_D \frac{L}{j}\; f\!\left(\frac{j\pi\lambda_D}{L}\right)$$

$$f(\varepsilon) \sim \frac{2}{3\pi(\ln[-\varepsilon/\ln(\varepsilon)])^2}, \quad \varepsilon << 1$$

$$\nu_c = n\bar{\mathrm{v}}b^2, \;\; b = e^2/T$$

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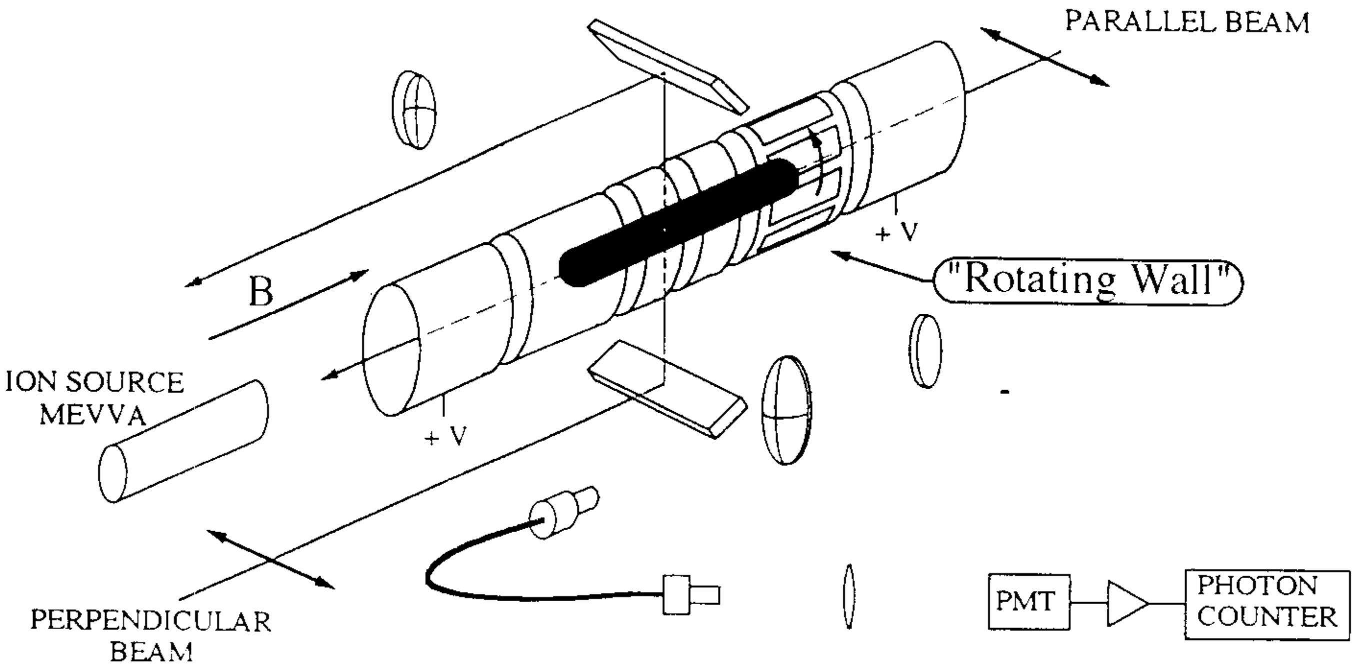

"Rotating wall" => plasma confined for weeks

Steady State plasma (no losses !)

Typical parameters:

B $0.8 \rightarrow 4\,T$

n $1 * 10^6 \rightarrow 2 * 10^8\ cm^{-3}$

T $0.001 \rightarrow 3\ eV$

$N_{tot} \approx 2 * 10^8$ ions

R_p $0.5\ cm \rightarrow 0.9\ cm$

$L_p \cong 10\ cm$

$r_c \cong \mathbf{0.1\ mm}\ T_{eV}^{1/2}\ B_{4T}$

$\lambda_D \cong \mathbf{2.\ mm}\ T_{eV}^{1/2}\ n_7^{-1/2}$

$\nu_{ii} \cong \mathbf{1\ sec^{-1}}\ n_7\ T_{eV}^{-3/2}$

$f_{bounce} \cong \mathbf{10\ kHz}\ T_{eV}^{1/2}$

$f_{rot} \cong \mathbf{15\ kHz}\ n_7$

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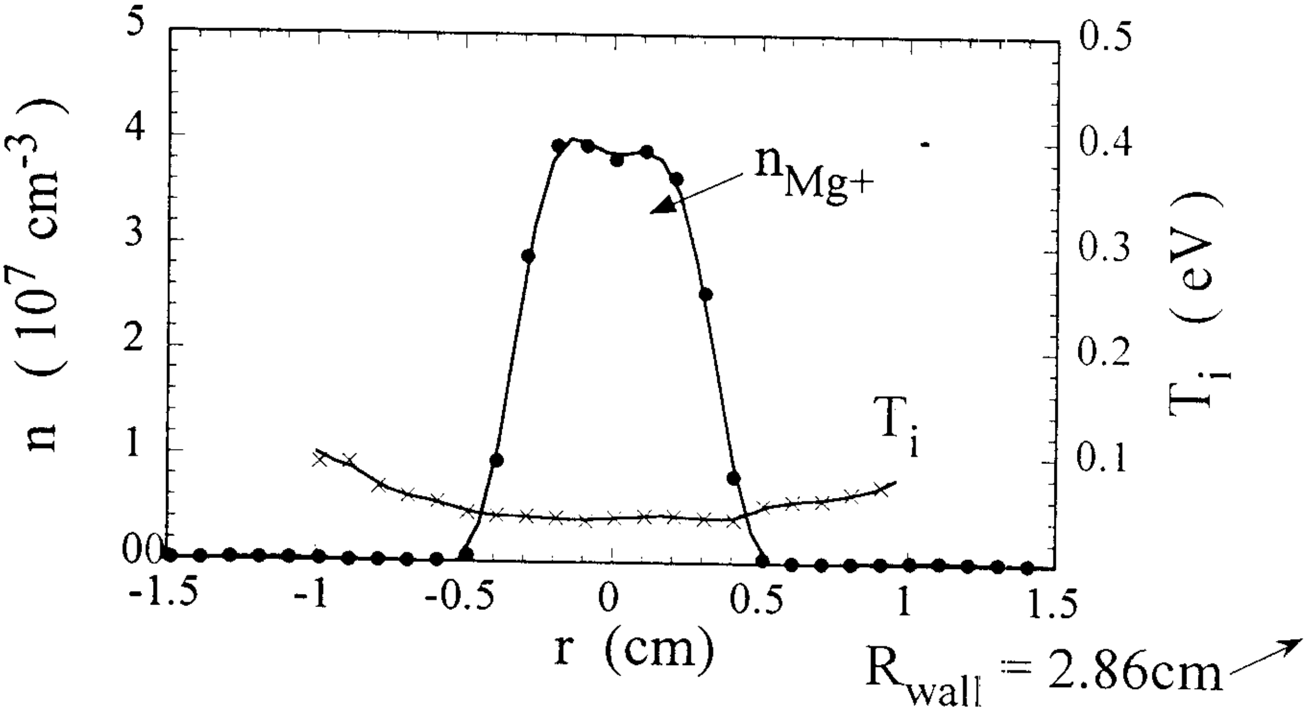
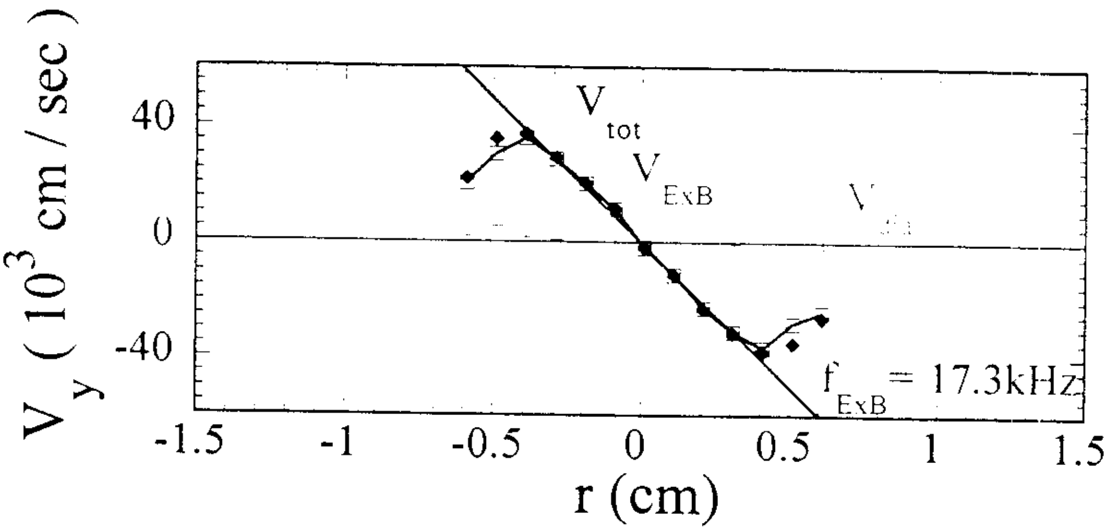
22 hours old plasma (same ions)





Relaxed to near-thermal equilibrium

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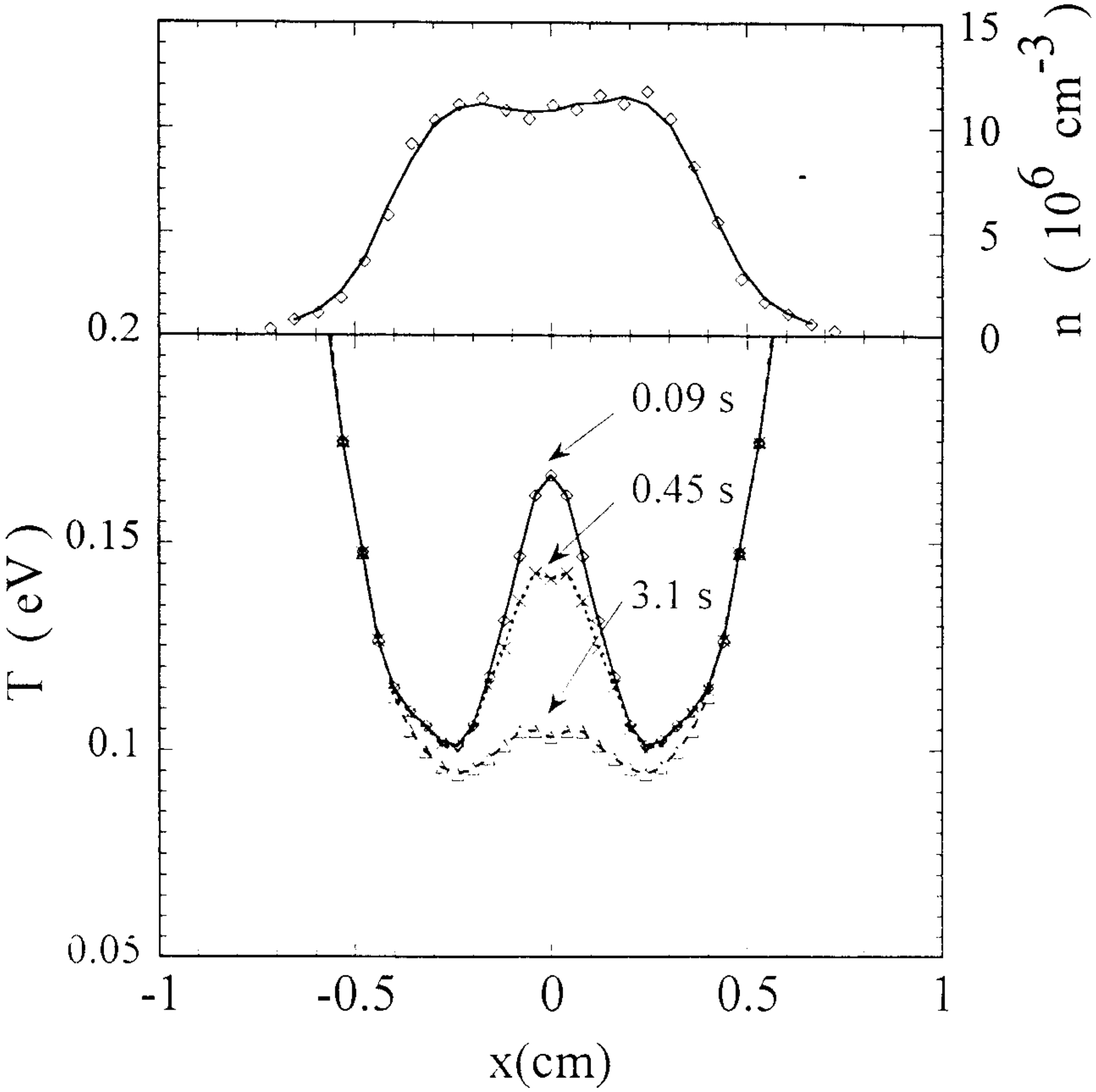
LASER HEATING of PLASMA CENTER TURNED OFF AT TIME t=0



F.A. April 97

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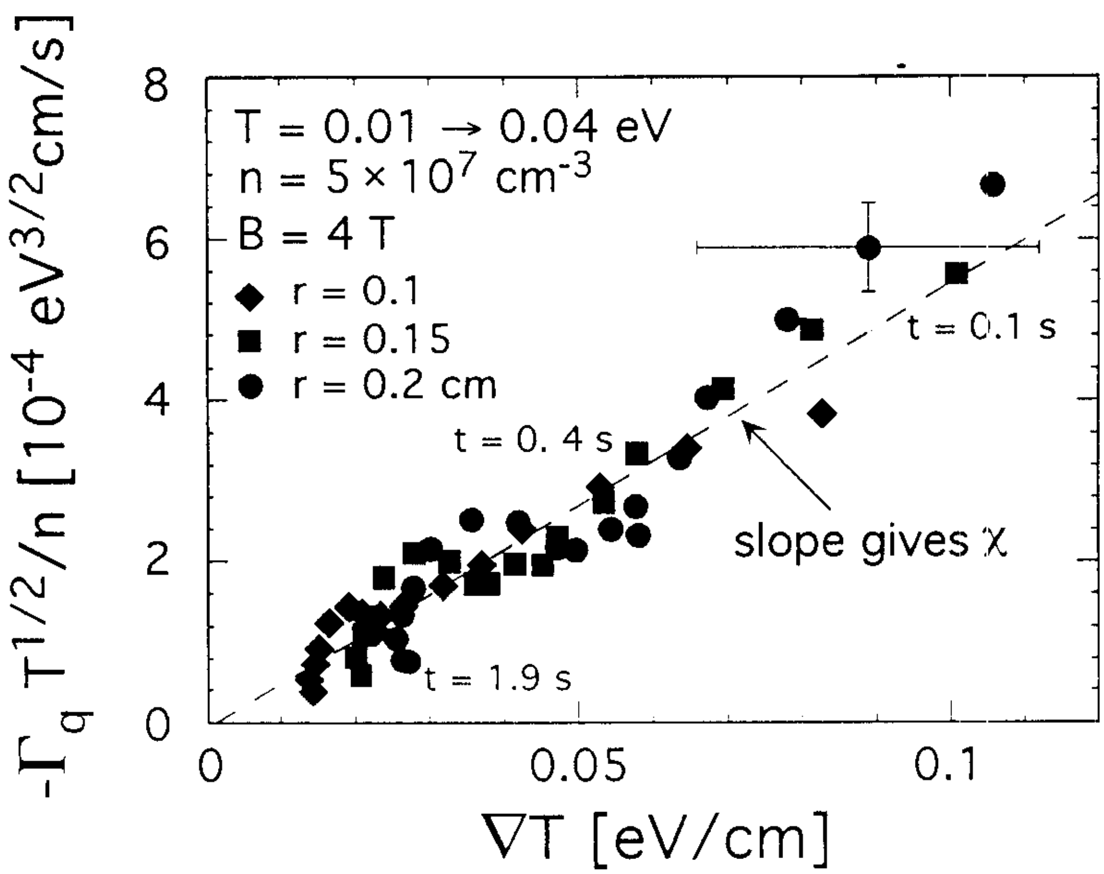
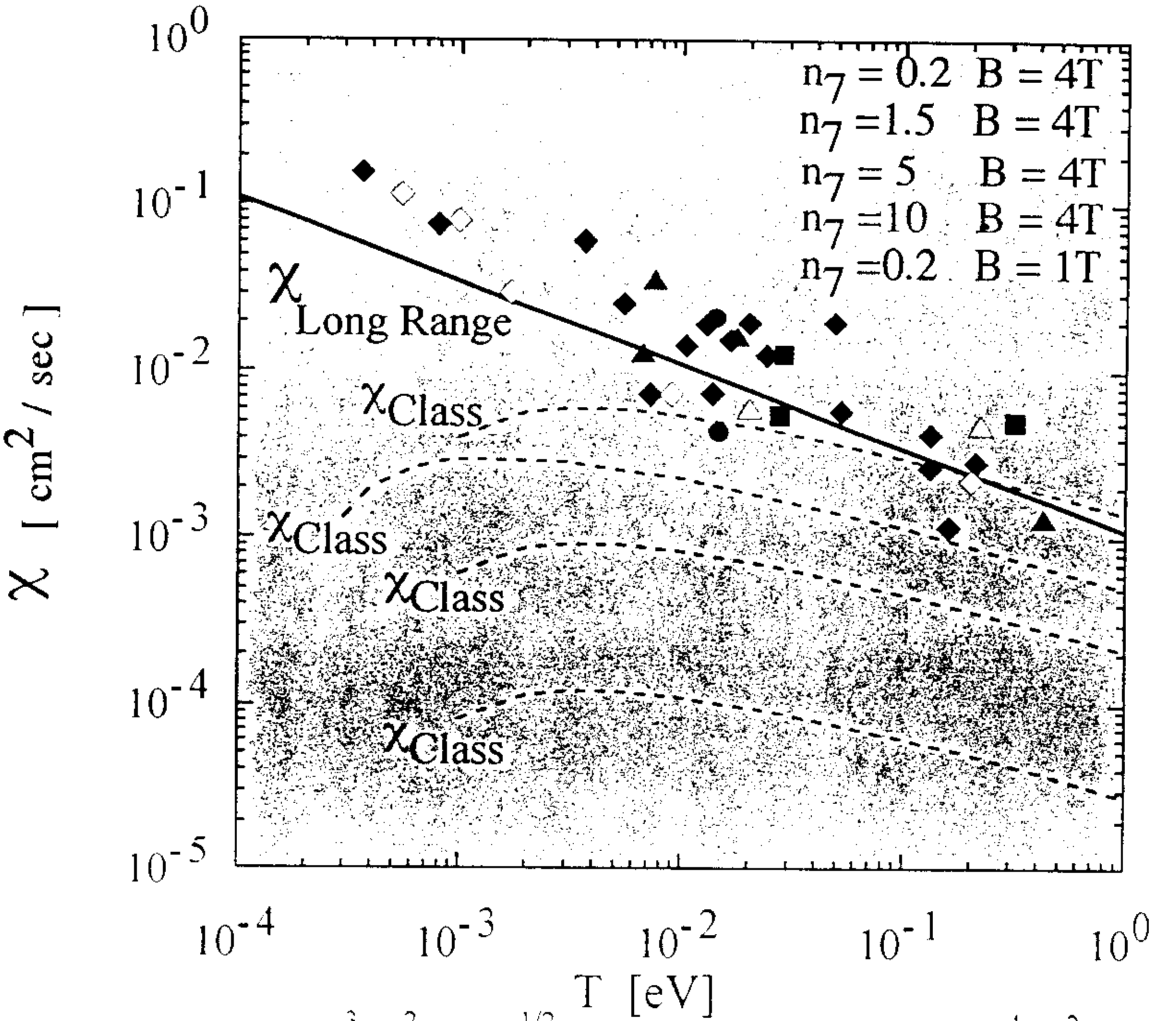
$$\Gamma_q \stackrel{?}{=} -\frac{5}{2} n \chi \nabla T$$



Heat flux is diffusive

Thermal diffusivity vs. Temperature



$\chi_{Long\ Range} \approx (1.1 \times 10^{-3}\ cm^2/s)\ T_{eV}^{-1/2}$ $\chi_{Class} \approx (1.4 \times 10^{-4}\ cm^2/s)\ T_{eV}^{-1/2} B_{4T}^{-2} n_7$

Data cover a range of 50 in density, 4 in magnetic field, and 10^3 in temperature.

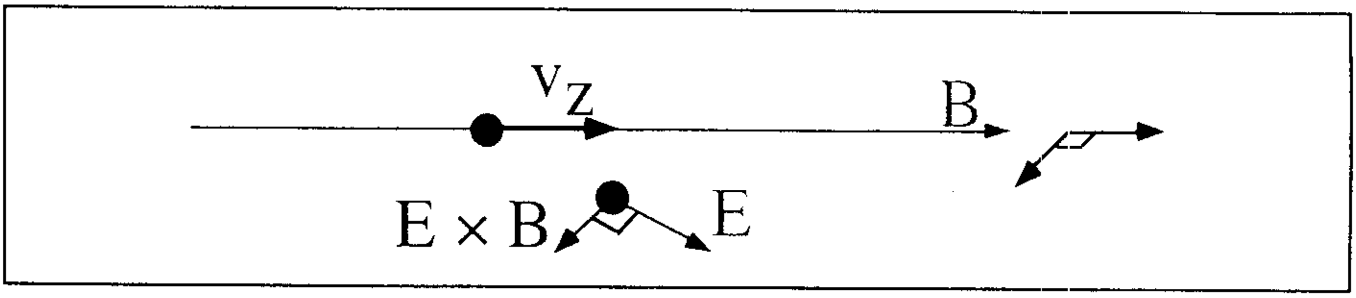
Heat transport is dominated by long range collisions.

χ independent of density and magnetic field

test particle diffusion:

$$D \sim \nu < \Delta x^2 > \quad \Rightarrow D^{class} \sim \nu_c r_c^2$$

Guiding Center Collisions*:



Coulomb Interaction

$$\Delta x = \int_{-\infty}^{\infty} dt\, E_y(t) \frac{c}{B}$$

integration along unperturbed orbits:

relative velocity v_z = constant

$$\Rightarrow D^{IUO} \sim \nu_c r_c^2 \; (\sim 3\, D^{class})$$

*Lifshitz and Pitaevskii *Physical Kinetics*, Anderegg et al. '97

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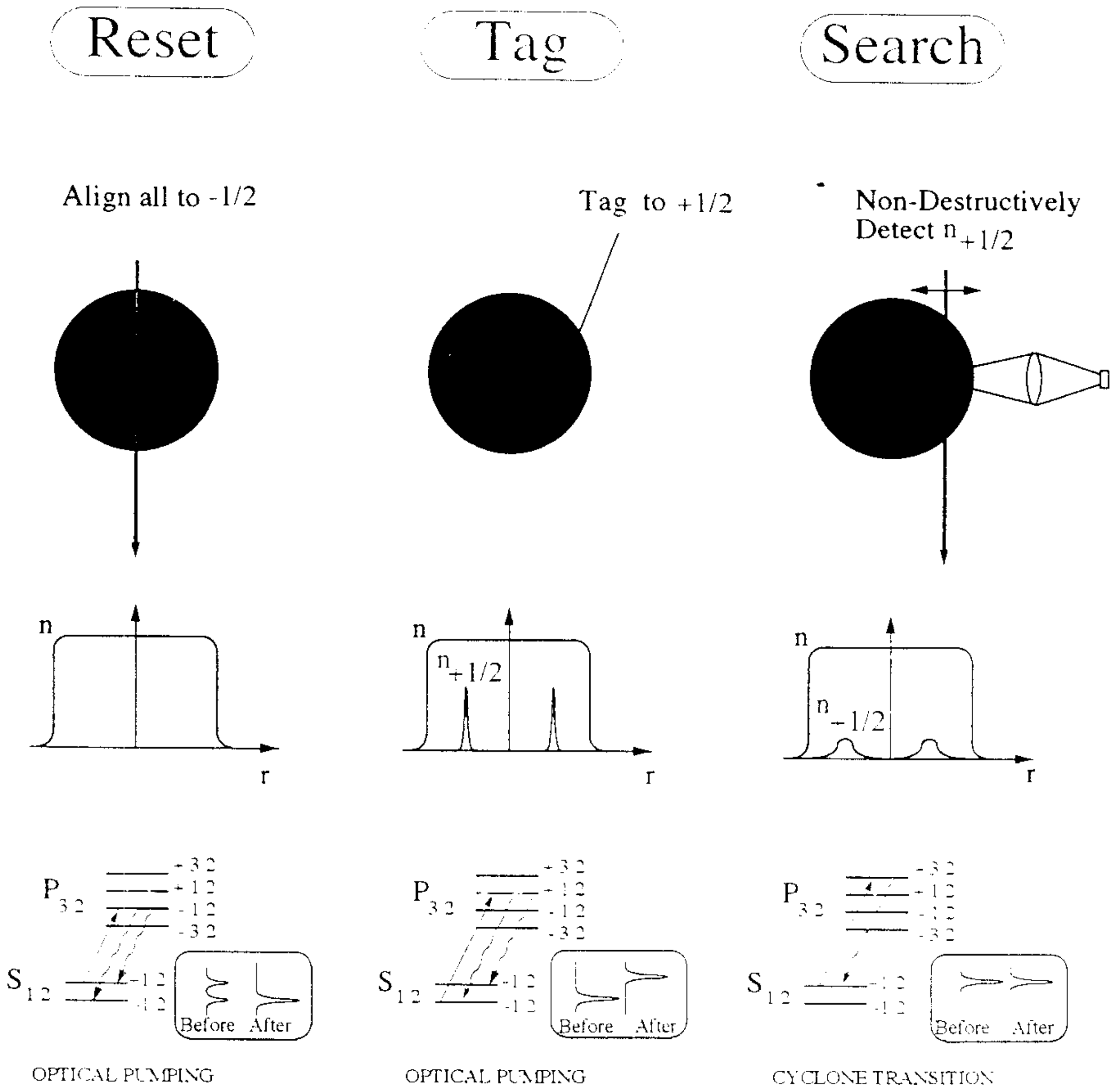
Ions are tagged by spin orientation



Spontaneous Spin Flip is slow

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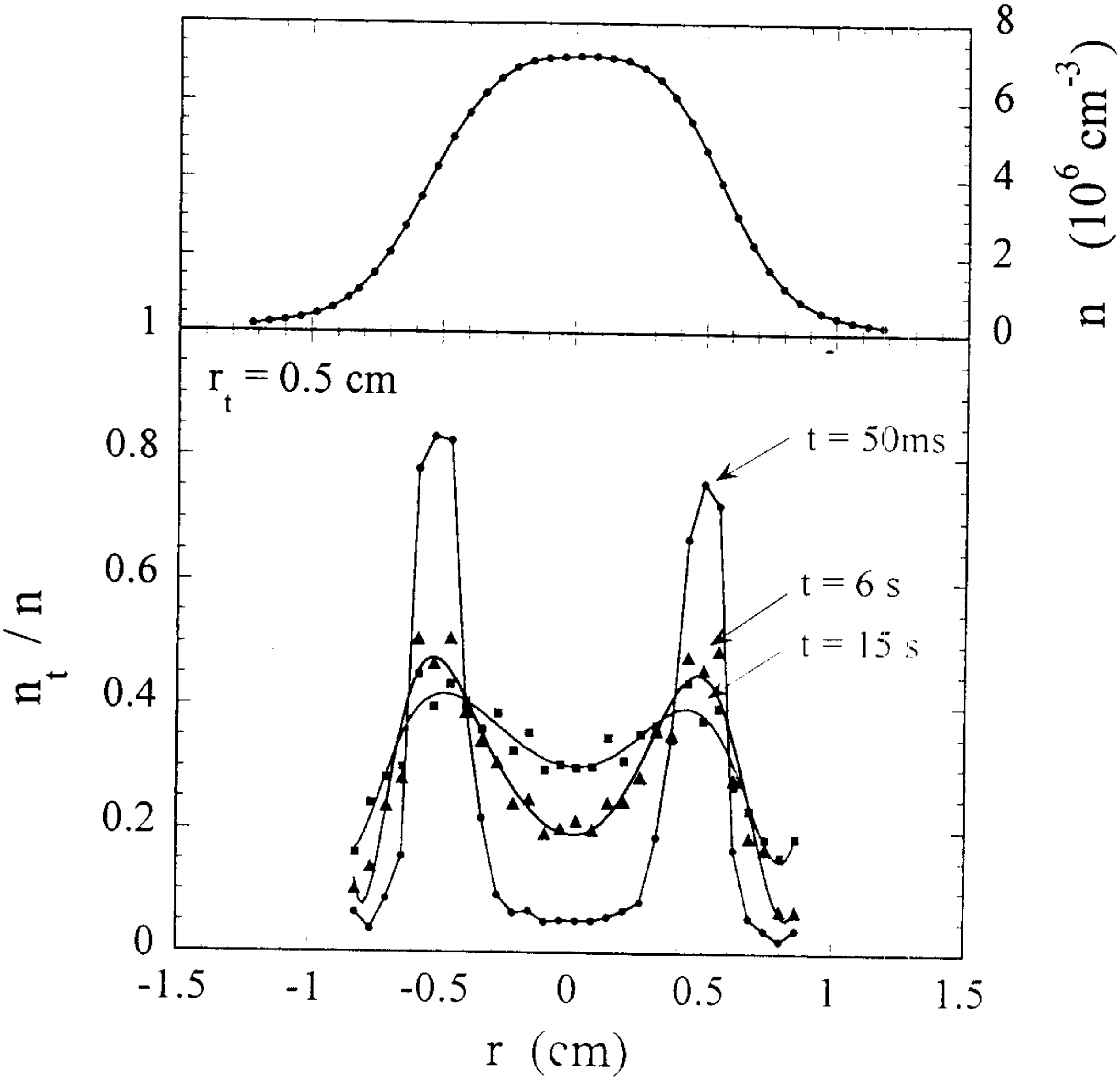

$N_t \cong$ constant to 10%

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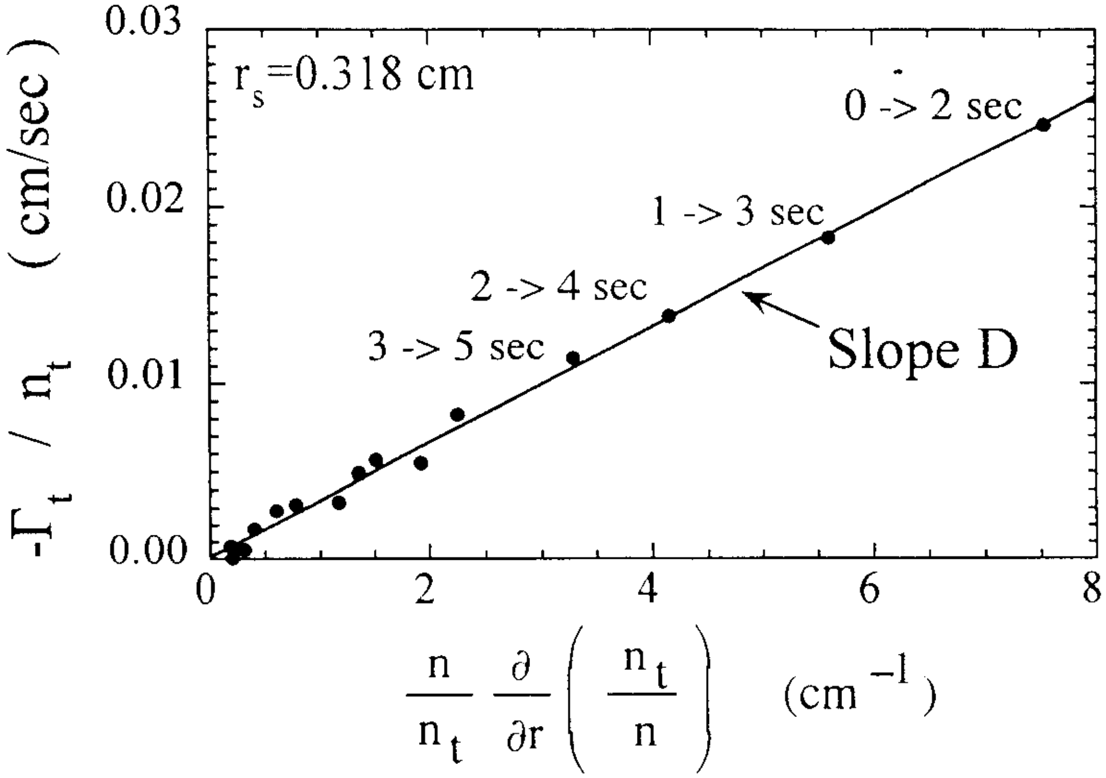
$$\text{Model :} \quad \Gamma_t(r,t) = -D(r)\, n(r) \frac{\partial}{\partial r}\left(\frac{n_t(r,t)}{n(r)}\right) + V(r) n_t(r,t)$$



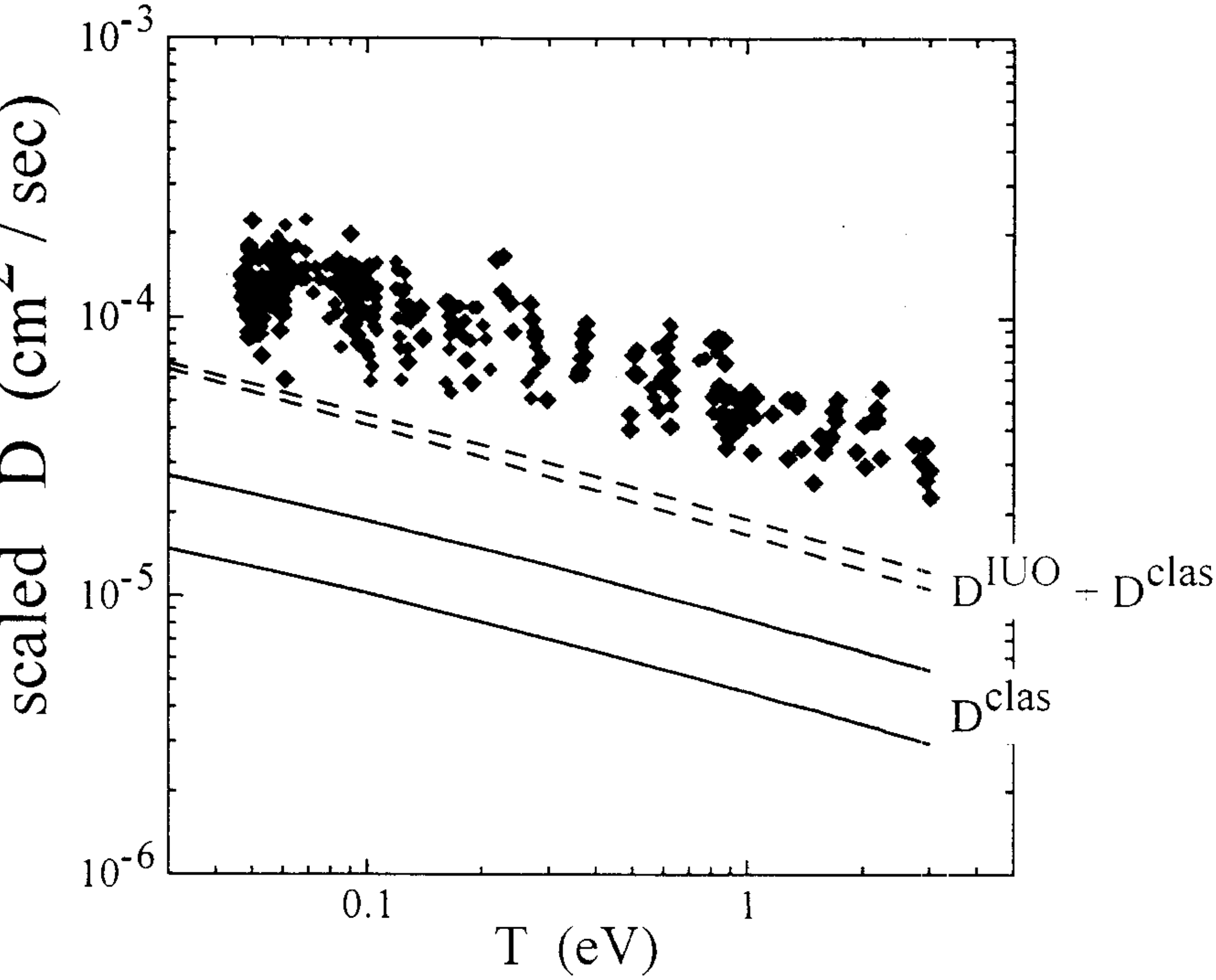
Linear dependance => Diffusion

Convective term V(r)=0 to experimental accuracy

(Steady state plasma can have no radial flow of test or non-test particles)

Flux is proportional to gradient of n_t/n

Diffusion coefficient versus Temperature



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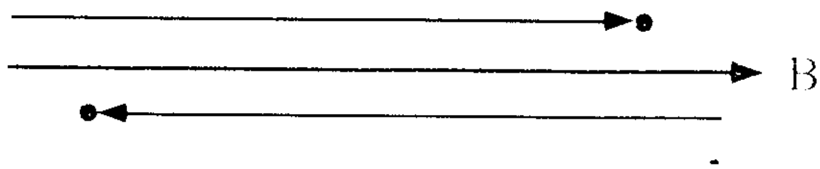
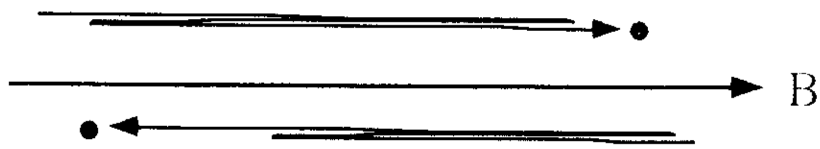
integration along unperturbed orbits:
particles collide only once



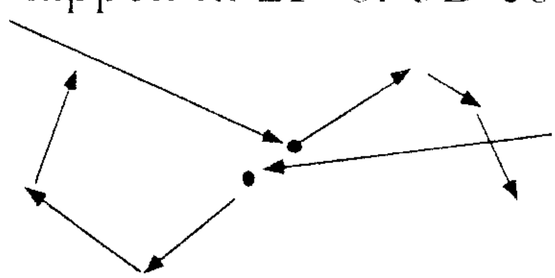
add VELOCITY DIFFUSION from
collisions with surrounding particles:

=> particles collide several times



$$\Rightarrow \lim_{\substack{\text{velocity} \\ \text{diffusion}} \to 0^+} \Delta x = 3\Delta x^{IUO}$$

doesn't happen in 2D or 3D collisions:



doesn't happen in plasma with large transverse shear
or if $\omega_b >> \omega_{E\times B}$

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evaluate diffusion coefficient*:

$$D \sim \nu < \Delta x^2 > \text{ and } \Delta x = 3\Delta x^{IUO}$$

$$\Rightarrow D \stackrel{?}{=} 9D^{IUO} \qquad \text{NO!}$$

Rate ν of multiple collision events that cause Δx:

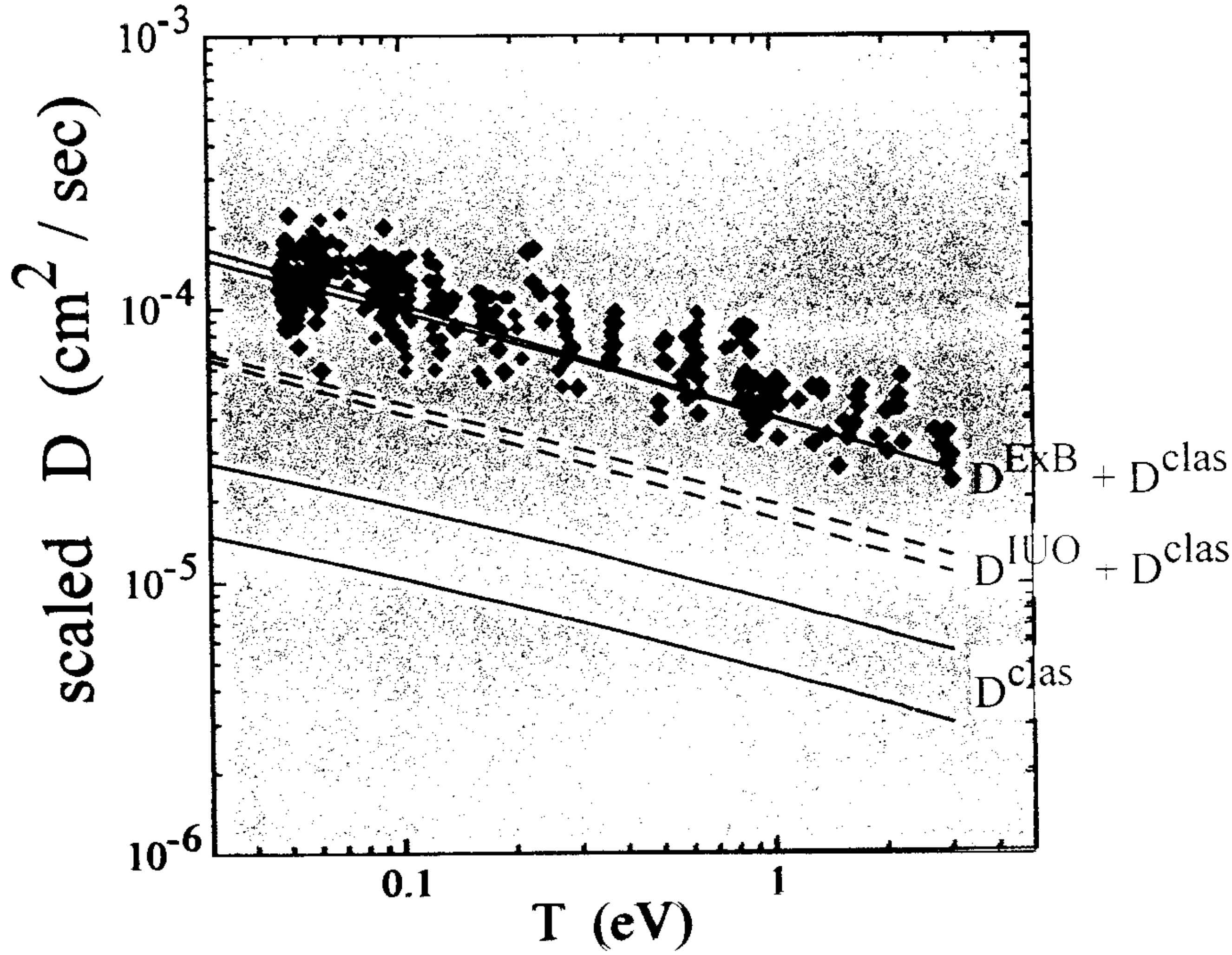
$$\nu = \frac{1}{3}\nu^{IUO}$$

$$\therefore D = 3D^{IUO}$$

*Dubin '97

7627

Diffusion coefficient versus Temperature



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Add velocity diffusion to orbits:

$$\Delta x = -\frac{ec}{\pi^2 B}\int d^3k \,\frac{ik_y}{k^2}\, e^{i\mathbf{k}\cdot\mathbf{r}}$$

velocity diffusion coefficient

$$\times \int_0^\infty dt\, e^{ik_z v_z t - k_z^2 D_v t^3/3}$$

$$\frac{\pi\delta(k_z)}{|v_z|} \;\; for \;\; D_v = 0;$$

$$\frac{3\pi\delta(k_z)}{|v_z|} \;\; for \;\; D_v \rightarrow 0$$

$$\int_0^\infty dt\, e^{i(kv-\omega)t - k^2 D_v t^3/3} = \pi\delta(kv-\omega),\;\; \frac{\omega v^2}{D_v} >> 1$$

7923

2D Vortex Dynamics and Turbulence in Nonneutral Plasmas

References:

(1) Fine, Driscoll, Malmberg, and Mitchell, "Measurements of Symmetric Vortex Merger," Phys. Rev. Lett. **67**, 588 (1991).

(2) Mitchell, Driscoll, and Fine, "Experiments on Stability of Equilibria of Two Vortices in a Cylindrical Trap," Phys. Rev. Lett. **71**, 1371 (1993).

(3) Fine, Flynn, Cass, and Driscoll, "Relaxation of 2D Turbulence to Vortex Crystals," Phys. Rev. Lett. **75**, 3277 (1995).

(4) Lansky, O'Neil, and Schecter, "A Theory of Vortex Merger," Phys. Rev. Lett. **79**, 1479 (1997).

(5) Jin and Dubin, "Maximum Entropy Theory of Vortex Crystal Formation," Phys. Rev. Lett. **80**, 4434 (1998).

Long electron column with the frequency ordering: $\Omega_c \gg \omega_B \gg \omega_R \sim \omega$

Bounce-average, 2D, $\underset{\sim}{E} \times \underline{B}$ drift dynamics:

$$\left.\begin{aligned} \underset{\sim}{V} &= \frac{c}{B}\hat{z} \times \underset{\sim}{\nabla_\perp} \varphi \\ \frac{\partial n}{\partial t} &+ \underset{\sim}{V} \cdot \underset{\sim}{\nabla_\perp} n = 0 \\ \nabla_\perp^2 \varphi &= 4\pi e n \end{aligned}\right\} \text{Drift-Poisson Equations}$$

Isomorphic to Euler equations for 2D flow of an ideal (incompressible and inviscid) fluid.

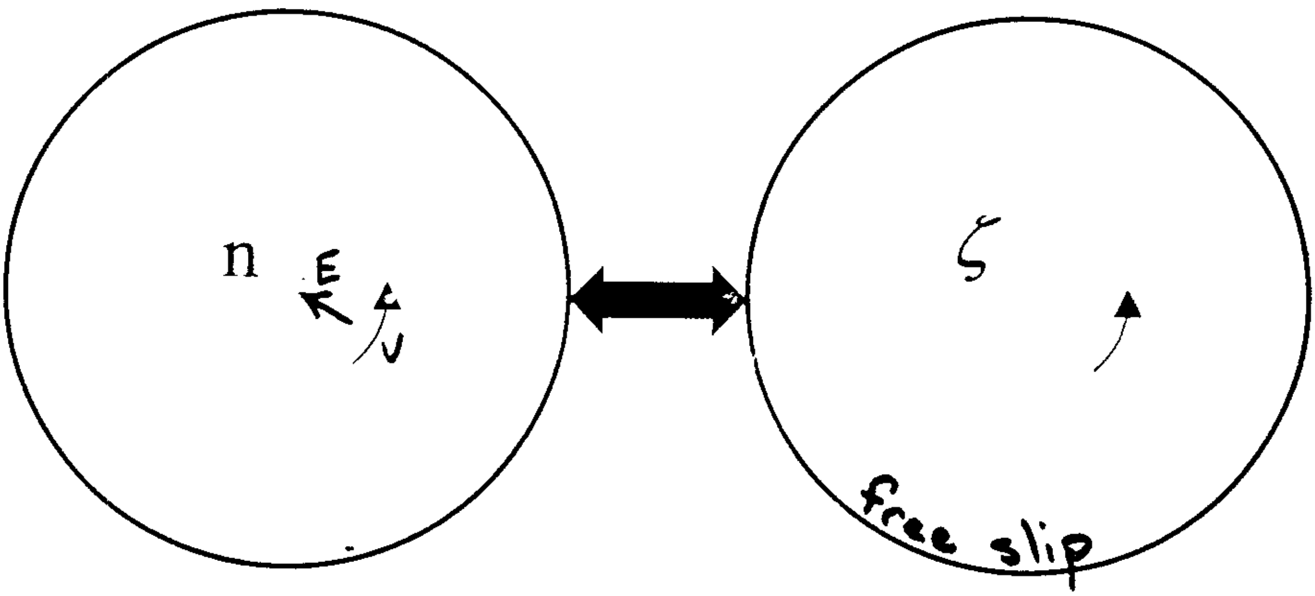
$$\varphi \longleftrightarrow \psi \quad \text{(stream function)}$$

$$n \longleftrightarrow \zeta = \hat{z} \cdot \underset{\sim}{\nabla} \times \underset{\sim}{V} \quad \text{(vorticity)}$$

Electron Plasma is a 2D Inviscid Fluid

ExB Drift Eqns ⟺ Euler Eqn

	v	$\Leftrightarrow$	v
$n,$	ζ	$\Leftrightarrow$	ζ
	ϕ	$\Leftrightarrow$	ψ



Electron Column in a Conducting Cylinder

Vortex in a Fluid within a Tank

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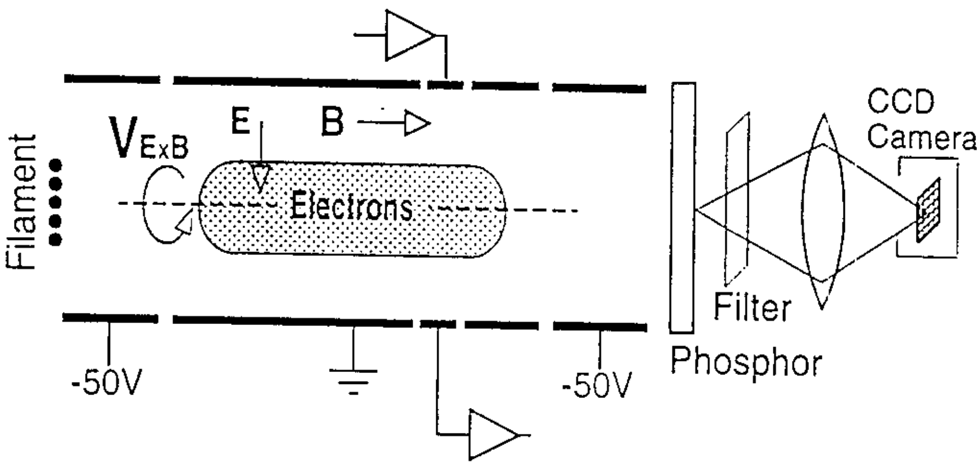
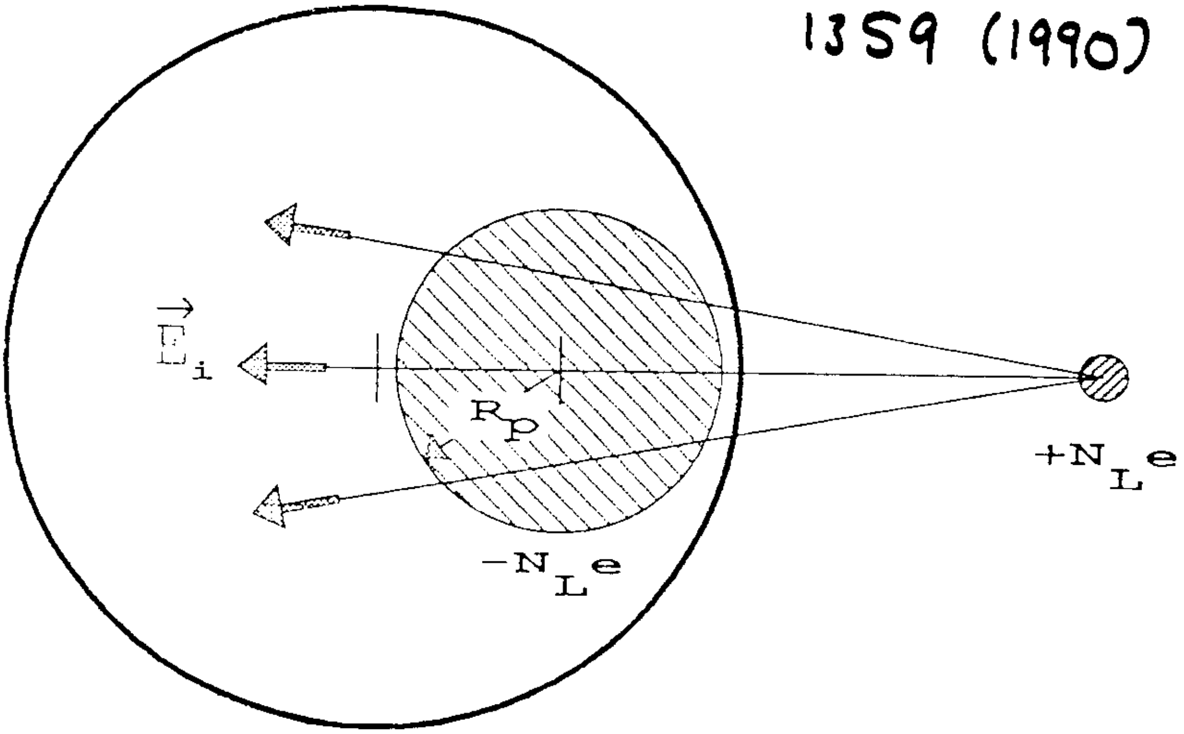


advantages

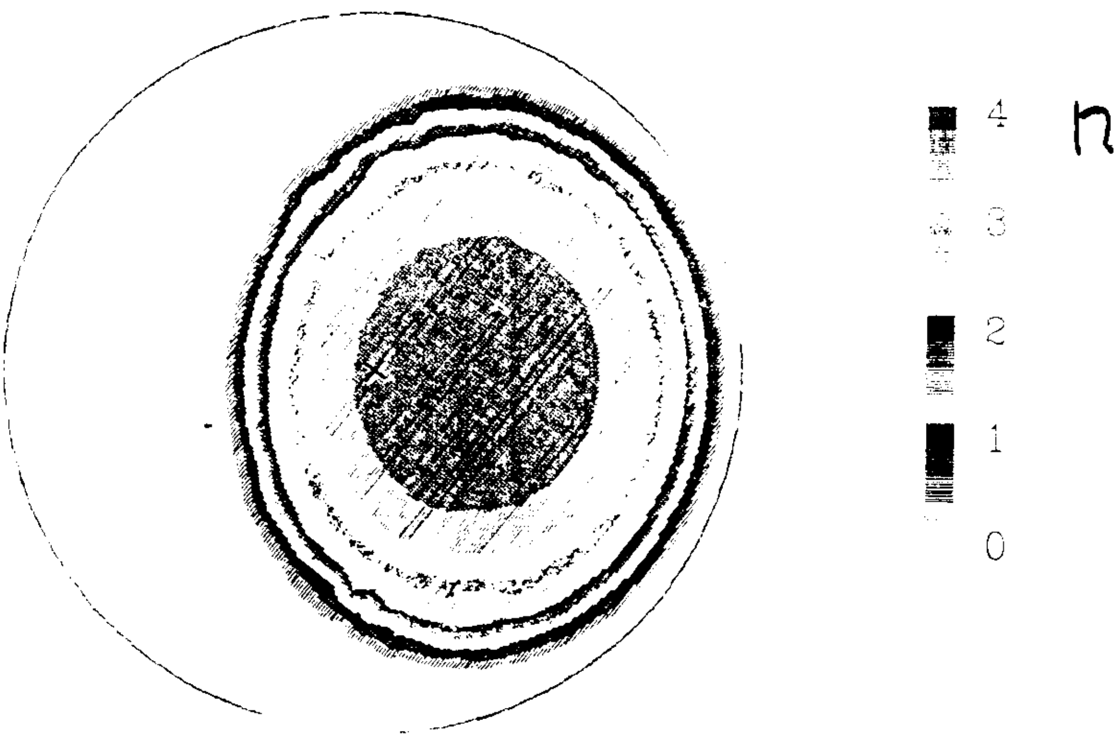
1. Image vorticity directly
2. Rigidly 2D
3. No boundary layer at edges and ends
4. Very low viscosity
5. Plasma remains confined

7980

Driscoll and Fine, Phys Fluids **B2** 1359 (1990)



$\tau \geq 10^5$ periods



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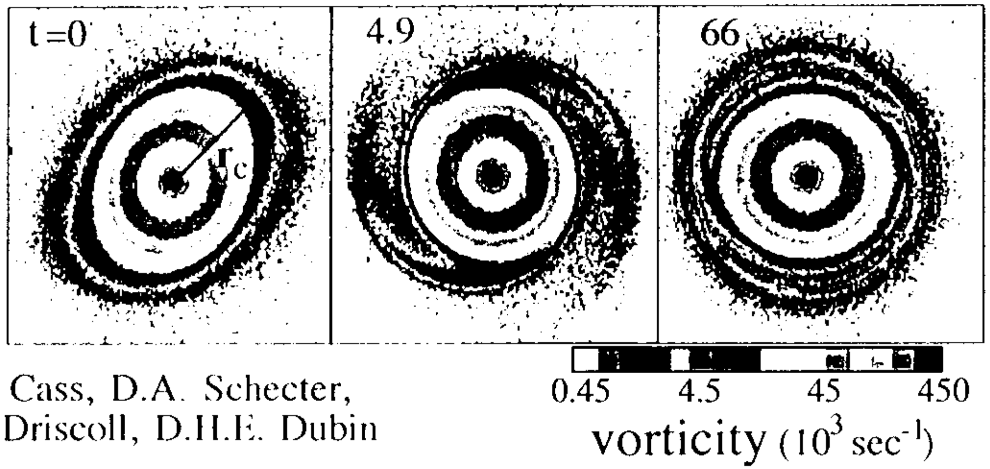
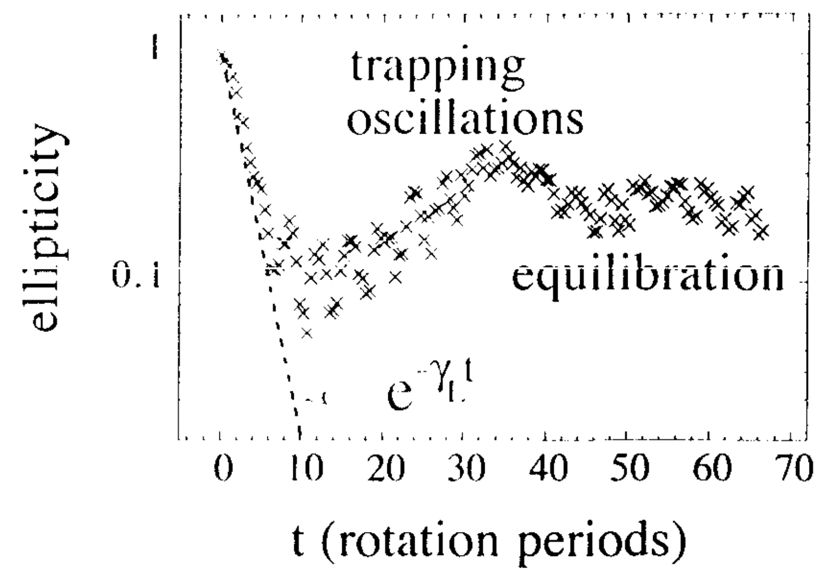
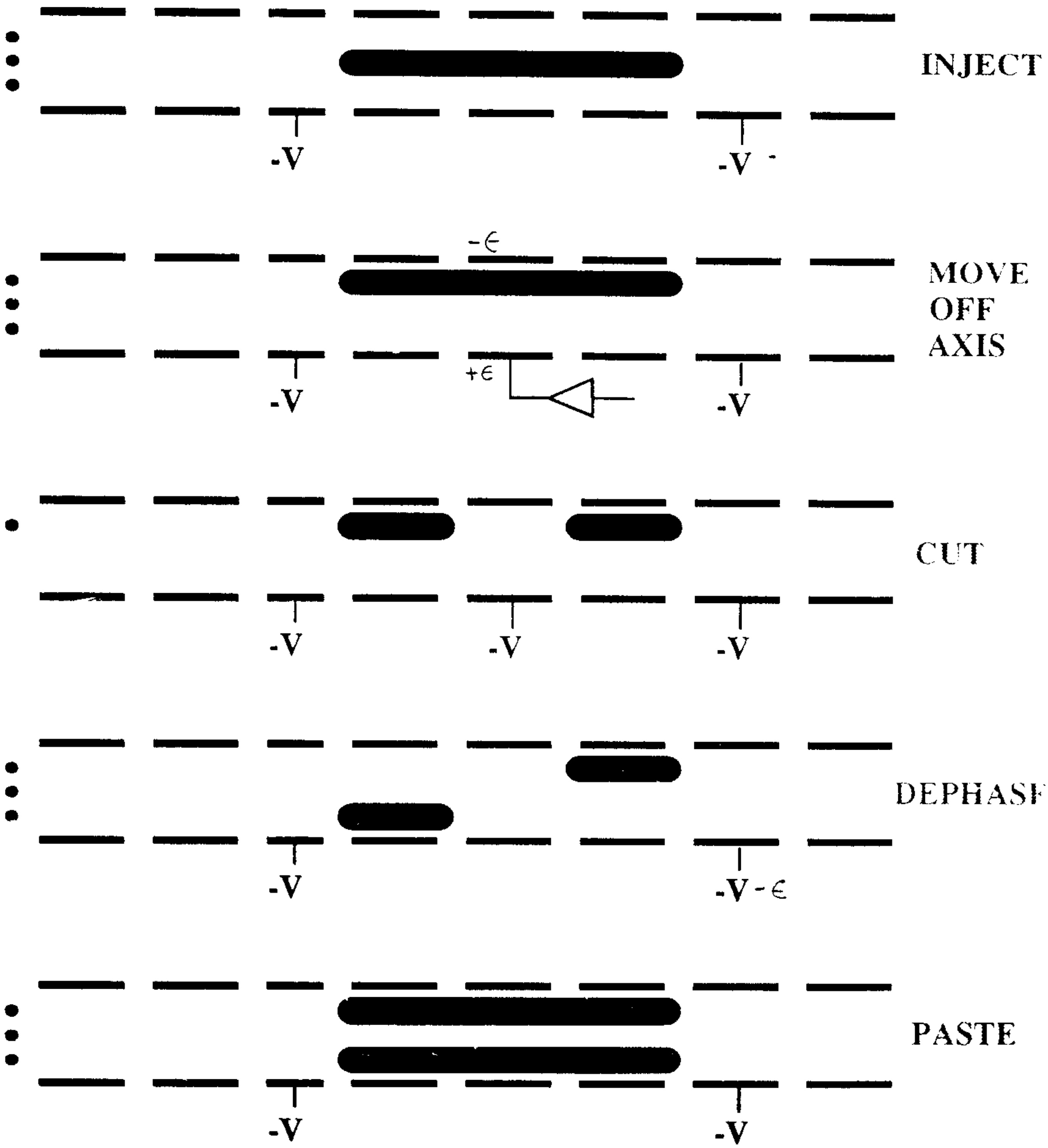
Landau Damping of m=2 Mode Due to Spatial Resonance $\omega = 2\omega_R(r_c)$



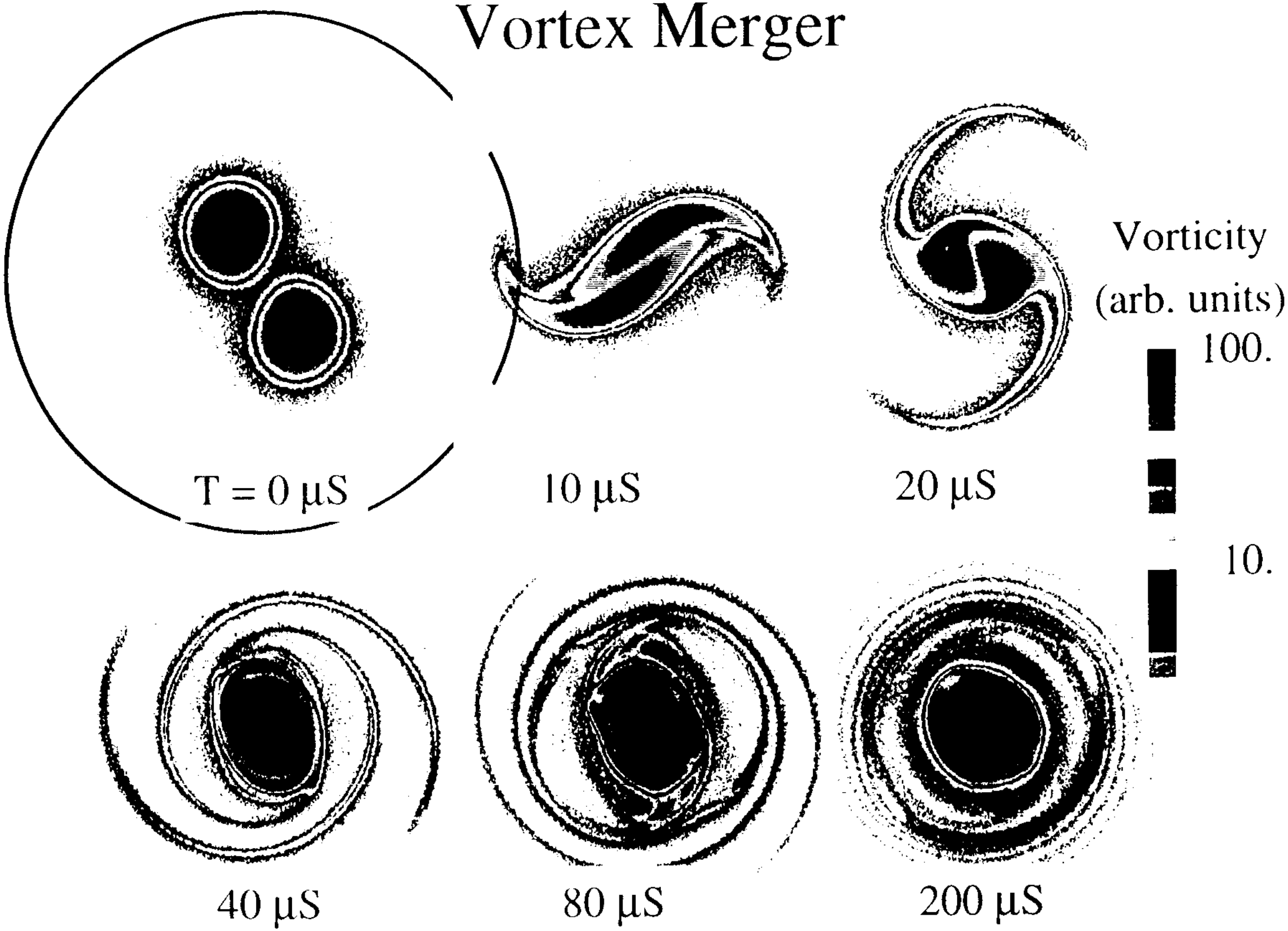
A.C. Cass, D.A. Schecter, C.F. Driscoll, D.H.E. Dubin



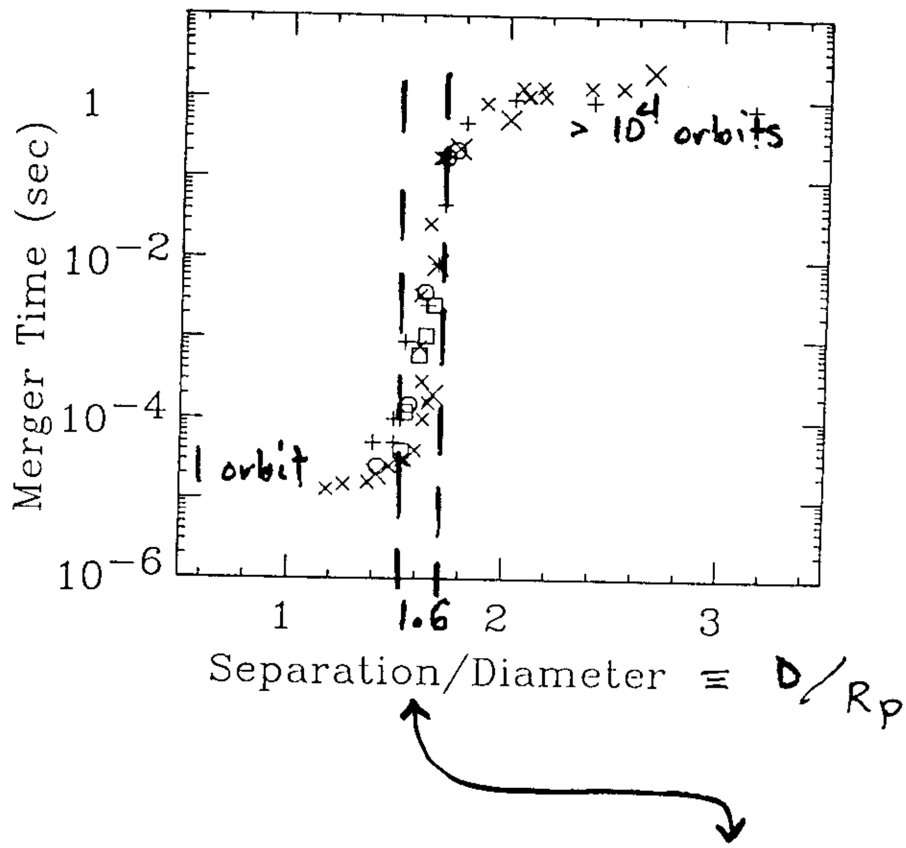
CREATION OF TWO VORTICES



Vortex Merger



2-Vortex Merger



Theory and Computational			Critical D/Rp
Roberts & Christiansen	'72	Cloud in a Cell	1.7
Moore & Saffman	'75	Elliptical Model	1.5
Rossow	'77	Point Vortices	1.7
Zabusky	'79	Contour Dynamics	1.7
Saffman & Szeto	'80	Numerical Soln of Exact Eqn	1.58
Experiment			
Griffiths & Hopfinger	'87	Water in Rotating Tank	~1.6

(Large non-ideal effects)

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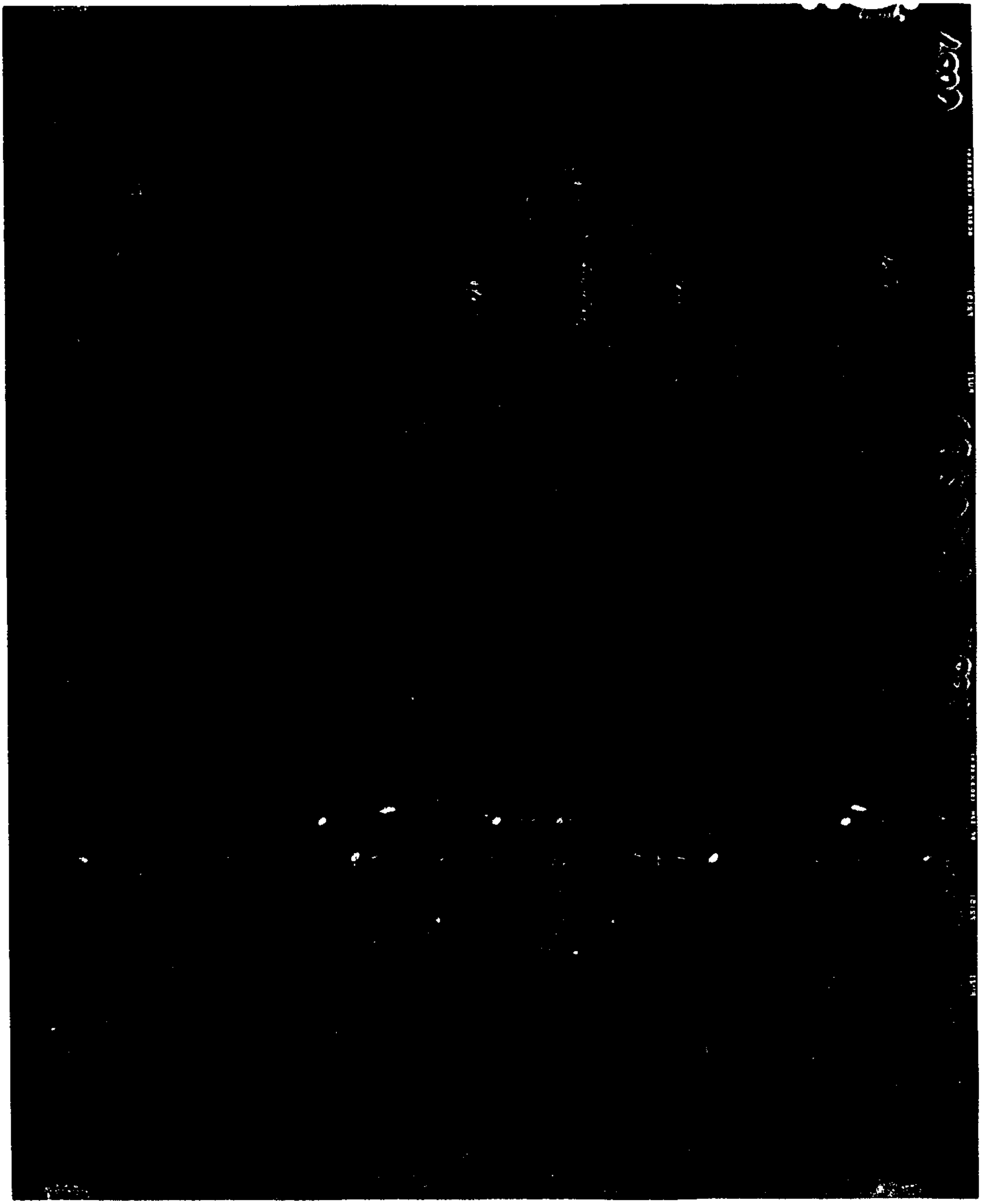
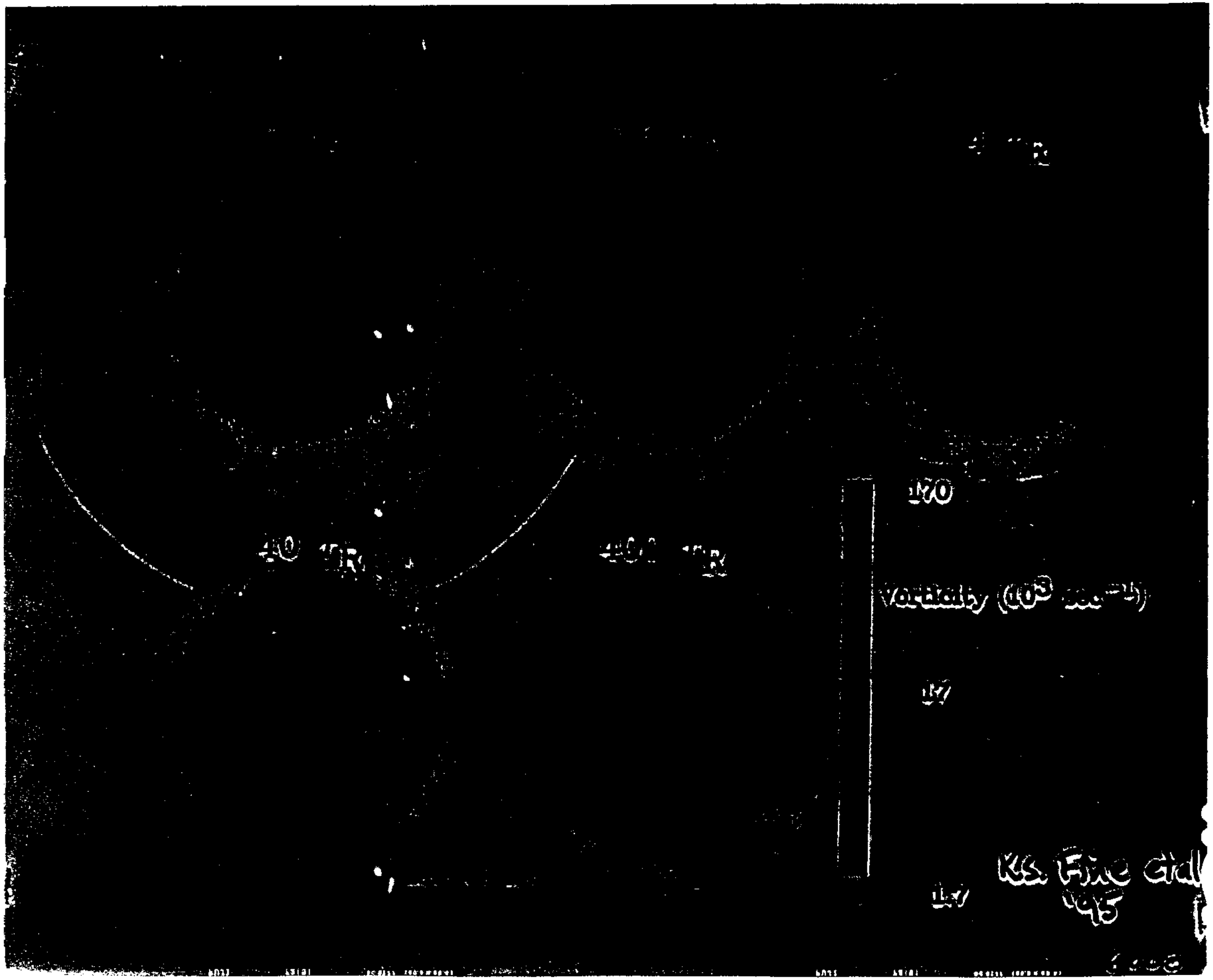
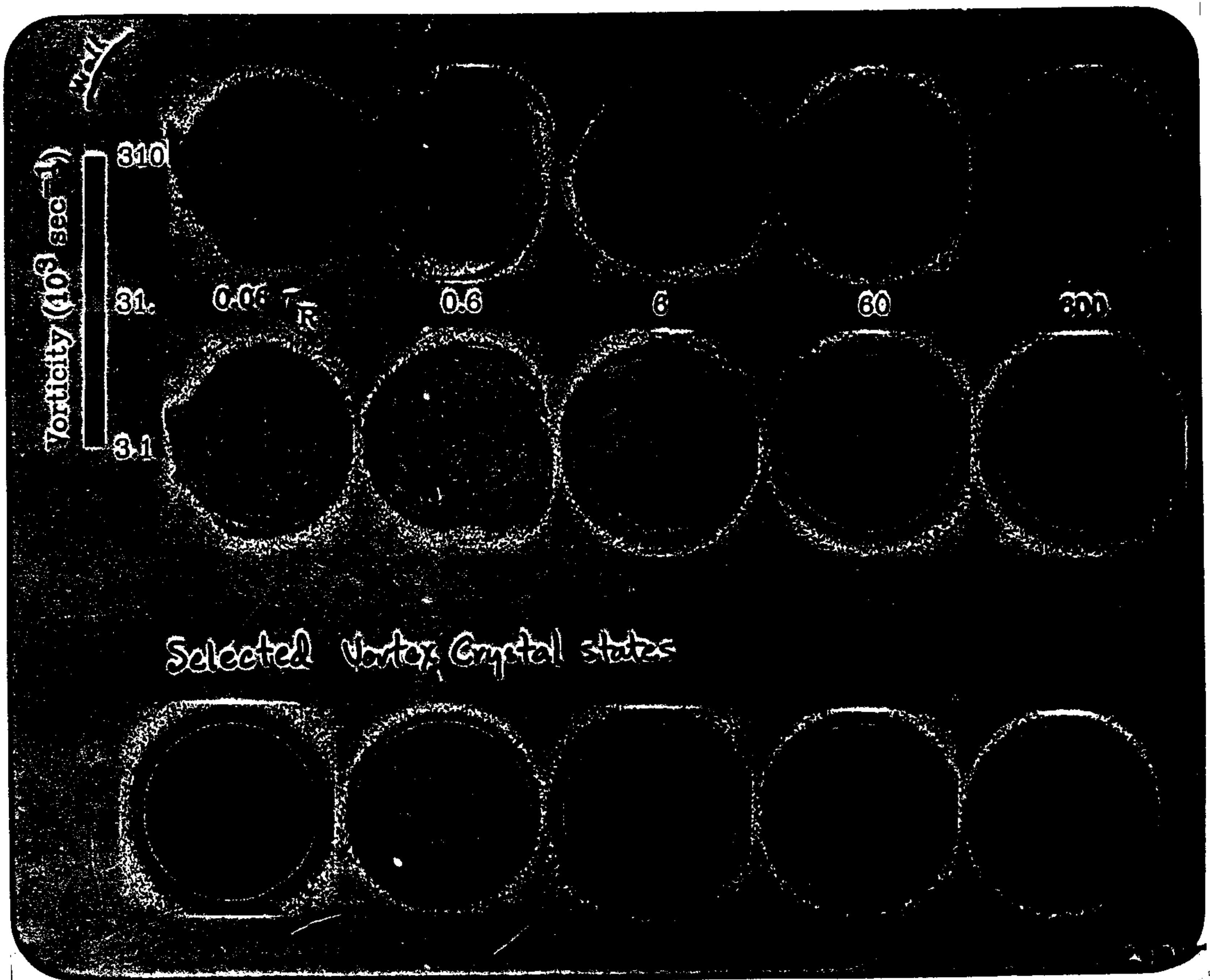
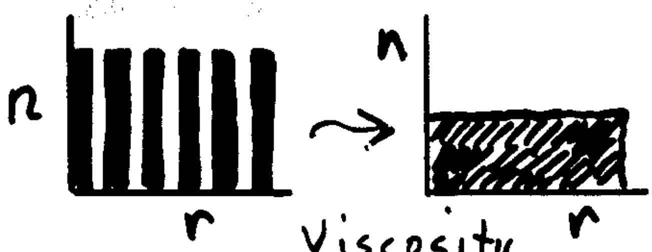



170
Vorticity (10^3 sec^-1)
17
1.7


Vorticity (10^3 sec^{-1})
310
31.
3.1
0.08 τ_R
0.6
6
60
300
Selected Vortex Crystal states



IDEAL INVARIANTS

Circulation

$$N_L = \int d^2r\, n$$

Angular Momentum

$$P_\theta = \int d^2r (1-r^2) \frac{n}{n_0}$$

Energy

$$H_\phi = -\frac{1}{2} \int d^2r\, \frac{\phi}{\phi_0} \frac{n}{n_0}$$

Entropy

$$S = -\int d^2r\, \frac{n}{n_0} \ln \frac{n}{n_0}$$

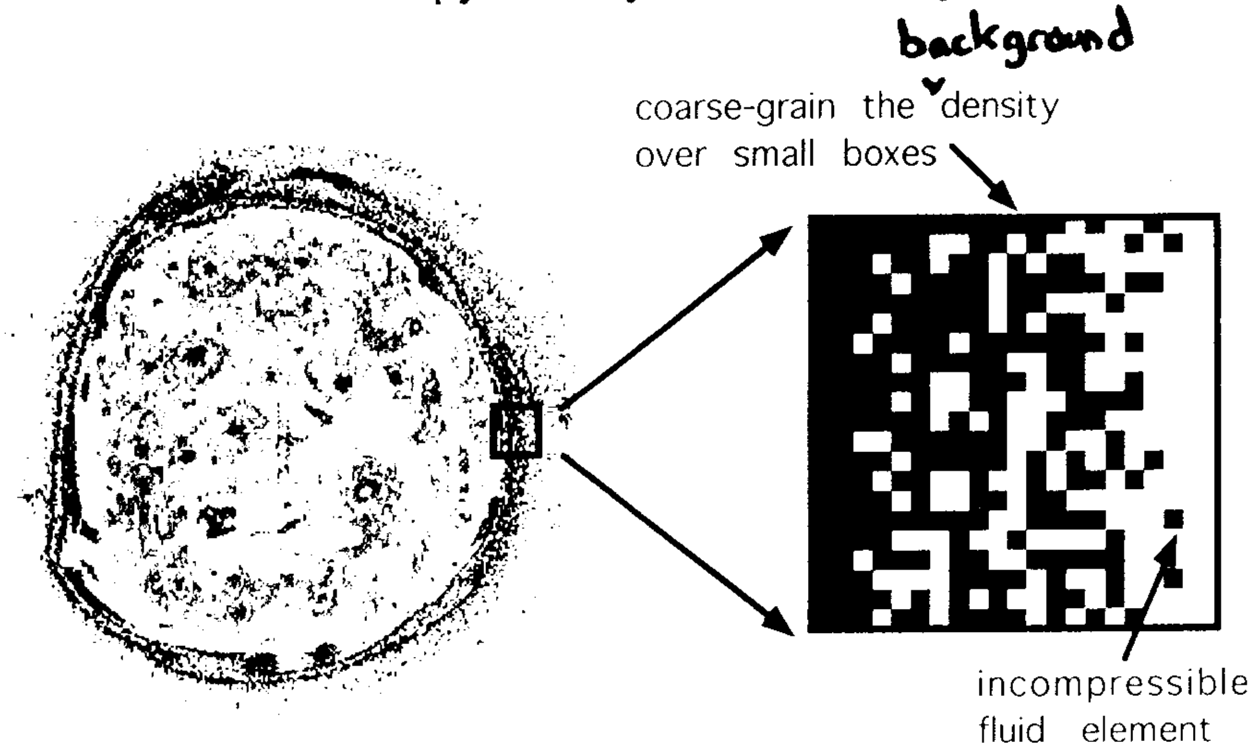
Enstrophy

$$Z_2 = \frac{1}{2} \int d^2r \left(\frac{n}{n_0}\right)^2$$

⋮

$$Z_m = \int d^2r\, n^m$$

Maximum Entropy Theory of Vortex Crystal State



$S = \mathrm{Log}$ [# of ways of arranging elements consistent with coarse-grained density]

$$\delta\left[S - \beta\left(E + \omega L + \mu N\right)\right] = 0$$

2nd law:

Order of vortex pattern increases ⇒ disorder in background increases to compensate

Jin and Dubin, Phys. Rev. Lett. 80, 4434 (1998)

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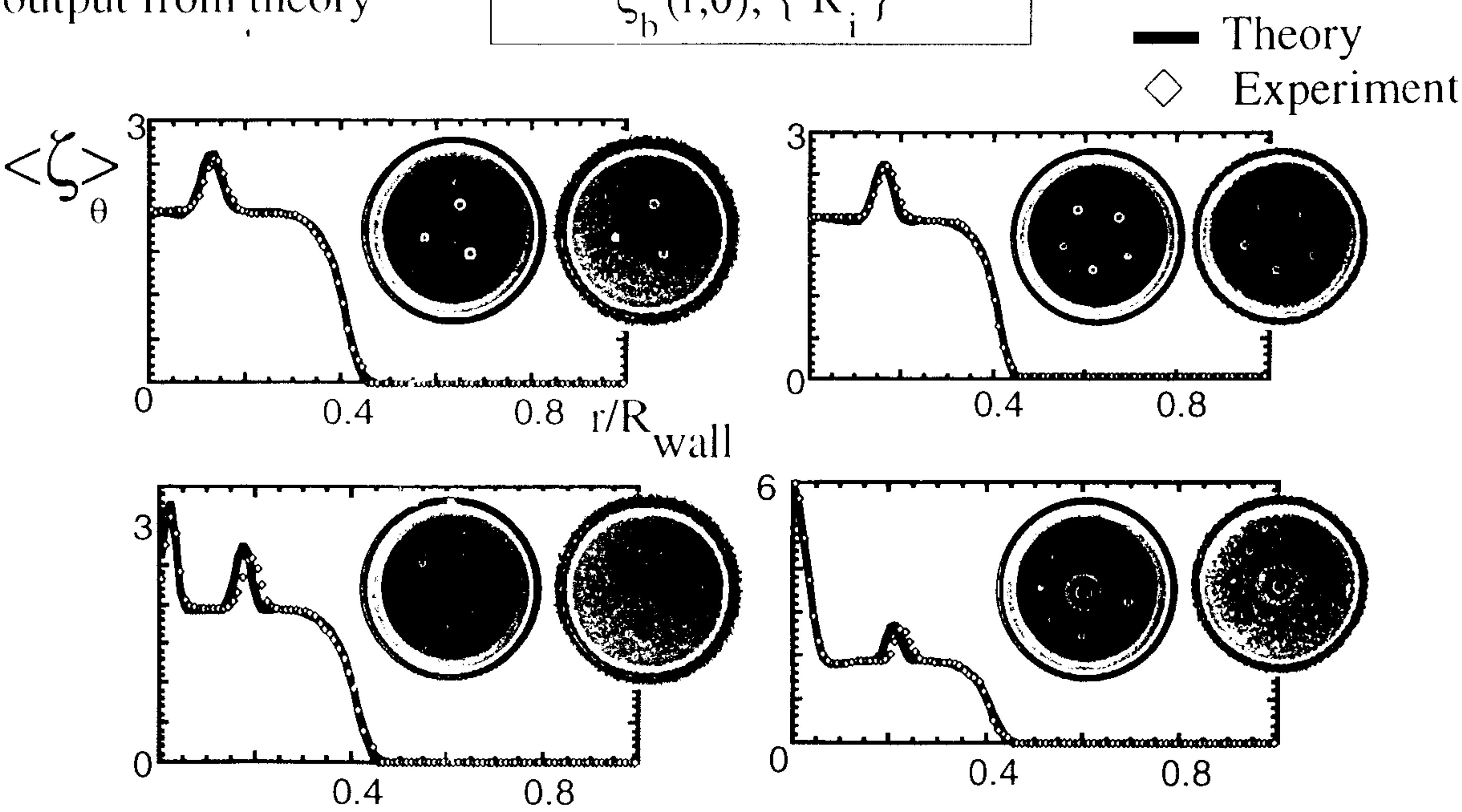
Predicted Maximum Entropy States Match Experiments with No Adjustable Parameters

input from experiment	$H, L, \Gamma, \zeta_f, \{ \zeta_i(\|\vec{r}-\vec{R}_i\|) \}$
output from theory	$\zeta_b(r,\theta), \{ \vec{R}_i \}$



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