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Solar Wind: Dynamics of the Flow and MHD Fluctuations

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1 Hydrodynamics of the solar wind expansion: why the solar wind is supersonic

Although knowledge of a solar influence at the earth's orbit dates back from Lord Carrington's observations that aurorae often occurred several hours after white light solar flares in the second half of the last century, the first direct indication of a continuous outflow of fast particles from the sun came from Biermann's investigation in the fifties of the shape of the cometary ion tails, from which he deduced an average speed of around 475 km/sec for this flow. In 1957 Chapman showed how a static conductive corona starting at 10^6 K at the sun should maintain a high density out to far distances (in fact, after an initial decrease, the density should increase outward again!), and in 1958 Parker argued, on the basis of the unreasonably high pressures that static solutions yielded at large distances, that "probably it is not possible for the solar corona, or, indeed, perhaps the atmosphere of any star, to be in complete hydrostatic equilibrium out to large distances". He then proceeded to show that a viable solution yielding negligible pressures at infinity consisted in a flow accelerating continuously and becoming supersonic at large distances. Consider hydrostatic balance for a spherically symmetric atmosphere with gravity

$$\frac{\partial p}{\partial r} = -m_p n \frac{g}{r^2}, \tag{1}$$

where g/R_0 is the gravitational acceleration at the solar surface (R_0 the solar radius) and we have normalized distances with solar radius. Also m_p is the proton mass so that the mass density $\rho = m_p n$. Recalling that $p = n\kappa T$ we may integrate to find that

$$n\kappa T = n_0 \kappa T_0 \exp\Big(-\int_1^r dr\, \frac{m_p}{\kappa T}\frac{g}{r^2}\Big) \tag{2}$$

so that a static spherically symmetric extended atmosphere with a temperature profile decreasing with distance less rapidly than $1/r$ requires a finite pressure at infinity to be confined, the same being true if the atmosphere 'evaporates' a subsonic flow, or breeze. The subsequent Parker - Chamberlain debate on supersonic/subsonic evaporation was cut short by the in situ measurement of the steady, supersonic wind by the Luna 2 (1959) and Explorer (1961) spacecraft Hundhausen, (1972). However, Mestel (quoted in Roberts and Soward, 1972), first remarked that it would not take a large fall in coronal temperature for the pressure of the local interstellar medium (ISM) to be sufficient to suppress the solar wind entirely. Indeed, the pressure of the ISM, $p_{ISM} \simeq 1.24\ 10^{-12}$ dyne/ cm^2 would suffice to confine a 4 10^5 K static corona with base density 10^9 cm^{-3}. So although correct, the argument for a supersonic wind does not appear to be as strong on the basis of pressure arguments only. In reality, the dependence of spherically symmetric, non-rotating atmospheres with flows on changes of the external conditions is somewhat more subtle, as shown by Velli (1994), and it is worthwhile to discuss the problem in detail. For the sake of analytical simplicity, we will consider only isothermal flows, i.e. a flows for which the temperature may be considered constant out to great radial distances.

1.1 Stationary isothermal flows: breezes, winds, accretion

The equations of motion for one-dimensional, spherically symmetric, stationary isothermal flow neglecting self-gravity may be written in the form

$$\frac{\partial}{\partial r}(\rho v r^2) = 0, \qquad p = c^2\rho \tag{3}$$

$$v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{g}{r^2} \tag{4}$$

where v is the velocity, c the constant sound speed. For a static atmosphere, the pressure profile is given by $p = p_0 \exp(-g/c^2 + g/rc^2)$ which, as discussed above, implies a non-vanishing asymptotic value for the pressure at large distances $p^s_\infty = p_0 e^{-g/c^2}$. In terms of the mach number $M = v/c$ the flow equations may be written (a prime denoting radial derivatives throughout this section)

$$\left(M - \frac{1}{M}\right)M' = \frac{2}{r} - \frac{g}{r^2c^2} \tag{5}$$

which may be integrated and expressed in two equivalent ways

$$\frac{1}{2}(M^2 - M_0^2) - \log\left(\frac{M}{M_0}\right) = 2\log r + \frac{g}{rc^2} - \frac{g}{c^2}. \quad (6)$$

$$M^2/2 + \log p - \frac{g}{rc^2} = M_0^2/2 + \log p_0 - \frac{g}{c^2}, \quad (7)$$

where M_0 is the base Mach number. The second form is essentially the conservation of energy flux, where for an isothermal atmosphere the enthalpy is expressed as $\log p$ instead of $\gamma p/(\gamma - 1)$. Eq.(5) has a singular point at the sonic point, $r = g/2c^2$, $M = 1$. Solutions to the above equations may be represented in the (M, r) phase plane illustrated in fig.(1), which, following the symmetry of eq.(5) is symmetric in the sign of M. The diagram is divided into 4 (8, considering positive and negative M) areas (labeled I-IV) by the two critical (transonic) solutions which cross at the sonic point $r = g/2c^2$, $M = 1$. Single valued continuous flow profiles $M(r)$ which are subsonic for all r, the breezes, lie below both transonic curves (region I). Among flows which are subsonic at the atmospheric base the accelerating transonic has the special property that density and pressure tend to zero at large distances: because of the small but finite values of the pressure of the ambient 'external' medium, a terminal shock transition, connecting to the lower branch of the double valued solutions filling region II will in general be present (see e.g. Holzer and Axford, 1970). The jump conditions across such a shock are found from conservation of mass and momentum across the shock, which read (superscripts -,+ denote the solution immediately upstream and downstream from the shock respectively)

$$\rho^- M^- = \rho^+ M^+, \quad (8)$$

$$\rho^+ (M^+)^2 + \rho^+ = \rho^- (M^-)^2 + \rho^-, \quad (9)$$

from which one find immediately $M^+ M^- = 1$. This gives a way to graphically construct shock transitions; it is sufficient to plot the curves corresponding to $1/M$ for the transonics (dashed lines in fig. (1)), and connect the transonic with the double valued curve in region II where the dashed line intersects them: such solution is given by curve W. The downward transonic in itself is not a possible solution for outflows, since a continuous transition from supersonic to subsonic flows is unstable. However, it plays the same role the Parker wind solution plays for inward directed accretion flows: for negative M solutions, the same construction leads to accretion shocks in the flow (McCrea, 1956), this time in the region labeled IV, and one such transition is shown in fig.(1) (curve A). For given base values of the pressure, the position of the shock is uniquely determined by the pressure of the interstellar medium, and the distance from the critical point to the shock decreases as the pressure increases: conservation of mass across the shock immediately gives the asymptotic pressure in terms of the upstream Mach number M^- as

$$p_\infty = p_0 M^- \exp\left(M_*{}^2 - 2\frac{g}{c^2} - M^{-2} + \frac{1}{M^{-2}}\right)/2, \qquad (10)$$

where M_* is the base Mach number of the upward transonic. p_∞ is a monotonically decreasing function of M^-, which is itself, obviously, a monotonically *increasing* function of the shock position r_s, so that increasing p_∞ decreases r_s. When p_∞ reaches a value $p_\infty^c = p_0 \exp(M_*{}^2/2 - g/c^2)$, the shock distance $r_s = r_c = g/2c^2$ and the discontinuity in the flow velocity reduces to a discontinuity in the derivative of the profile. This is the fastest possible, or critical, breeze, made up of the section of upward transonic below r_c and the section of downward transonic beyond r_c. For the breeze solutions, with a base Mach number M_0 such that $M_* > M_0 \geq 0$, the asymptotic pressure is easily calculated to be

$$p_\infty = p_0 \exp\left(M_0^2/2 - g/c^2\right) \geq p_\infty^s. \qquad (11)$$

It follows that the pressure required to confine a breeze *increases* with increasing base Mach number and is greater, if only slightly,than that of a static atmosphere. The limiting value of p_∞ is again p_∞^c. For a given base pressure and asymptotic pressures $p_\infty^c > p_\infty \geq p_\infty^s$ it therefore appears that *two* possible stationary outflow solutions exist, a supersonic shocked wind and a subsonic breeze.

When such conditions occur, it is frequently the case that one of the solutions may be unstable, i.e. that any small perturbation may lead the flow to evolve away from stationarity. In this case, it is the breeze solutions that are unstable: to prove this we introduce small perturbations (sound waves) and linearize the equations of motion around the stationary state. We will apply boundary conditions which allow the configuration to evolve from one to the other of the stationary solutions we have found: i.e., the perturbing sound waves will leave the pressure (and density) unperturbed at the atmospheric base and infinity, i.e. they will be standing waves. It is convenient

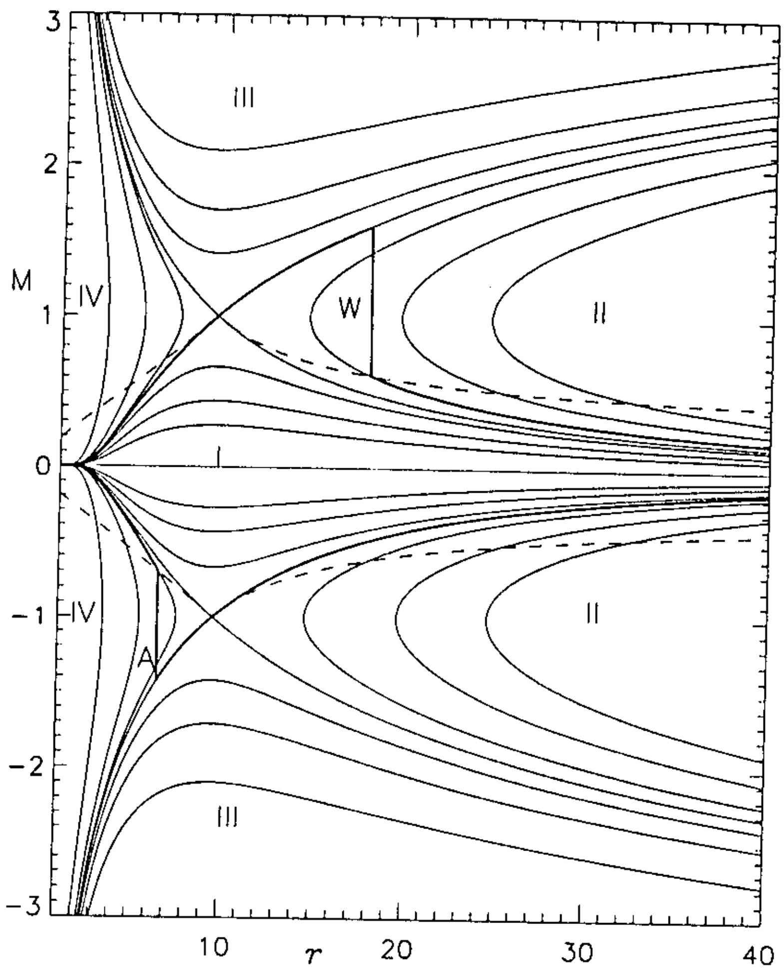

Fig. 1. The (M,r) phase plane. The continuous curves are appropriate both for positive and negative M. The dashed line intersection with double valued curves defines the shock position for winds (region II, curve W) or accretion flows (region IV, curve A).

to introduce characteristic variables $y^{\pm} = \hat{m} \pm \hat{p}$, where $\hat{p} = \tilde{p}/p$ is the adimensional normalized pressure perturbation and $\hat{m}$ is the mach number (velocity) fluctuation. In these variables an outward (inward) propagating sound wave has $y^{-} = 0$ ($y^{+} = 0$). Assuming a time dependence $y^{\pm} = y^{\pm}(r)exp(-i\omega + \gamma)t$ the linearized equations become

$$(M \pm 1)y^{\pm\prime} - i(\omega + i\gamma)y^{\pm} + \frac{1}{2}(y^{\pm} + y^{\mp})\frac{M'}{M}(M \mp 1) = 0. \qquad (12)$$

Problem : prove eq.(12), and show that in a uniform flow one has stable propagating waves with dispersion relation $\omega^{\pm} = (M \pm 1)k$ where k is a radial wave-number, i.e. a spatial dependence $\sim exp(ikr)$ is assumed.

In the presence of a nonuniform but stable flow, eq.(12) describes wave propagation and reflection, and a conserved flux, the wave action flux, exists (in a static medium, the wave energy flux is conserved; when there are mass motions, this is replaced by the wave action flux: see, e.g. Kadomtsev, (1983) §4 for a discussion of wave energy and Velli, 1993 and references therein for a discussion of wave-action conservation). When $\gamma \neq 0$ the wave action evolution equation becomes

$$\left[\frac{(M+1)^2}{M}|y^+|^2 - \frac{(M-1)^2}{M}|y^-|^2\right]' + 2\frac{\gamma}{M}\left[(M+1)|y^+|^2 - (M-1)|y^-|^2\right] = 0, \quad (13)$$

the first square bracket being proportional to the wave action.
Problem : prove eq.(13). Hint: one must multiply the equations for inward and outward waves by the appropriate factor and then sum to remove the y^+, y^- coupling terms.

Notice that for $|M| < 1$ the term in the second square bracket is positive definite. Integrating this equation between 1 and r and imposing the boundary condition that the pressure perturbation vanish at the extremes, we find the following estimate for γ:

$$\gamma = \frac{2\left(|y^+|_0^2 - |y^+|_r^2\right)}{\int_1^r dr\, M^{-1}\left[(M+1)|y^+|^2 - (M-1)|y^-|^2\right]}, \quad (14)$$

where $|y^+|_0^2, |y^+|_r^2$ are the fluctuation amplitudes at the atmospheric base and r respectively. It follows then that if the perturbation amplitude is non vanishing at the base but tends to 0 at great distances, the flow is unstable. Now for $\omega = 0$ and large r eqs. (12) have, for breeze velocity profiles, leading order asymptotic solutions

$$y^+ \sim \frac{e^{\mp\gamma r}}{r}\left(1 \pm \frac{1}{2\gamma r}\right), \qquad y^- \sim \pm\frac{e^{\mp\gamma r}}{2\gamma r^2} \quad (15)$$

so the boundary conditions are satisfied either by the first solution, if γ is positive, or the second, if γ is negative. In both cases the amplitudes tend to zero at great distances, the numerator of eq.(14) is always positive, γ is also positive, and, provided eigenmodes exist, breeze solutions are unstable. Numerical solutions show that this is indeed the case: in fig.(2a) we plot the growth rate of the instability as a function of base Mach number. The growth rate is largest for high values of the base Mach numbers but both the static atmosphere ($M_0 = 0$) and the critical breeze ($M_0 = M_*$) are marginally stable. In the latter case the perturbation equations become singular at the sonic point, because the phase speed of the inward propagating wave vanishes there: an additional regularity condition must be imposed in the stationary equations, effectively isolating the region below the sonic point from the region beyond it. This is the mathematical reason behind the stability of flows with a continuous subsonic/supersonic transition.

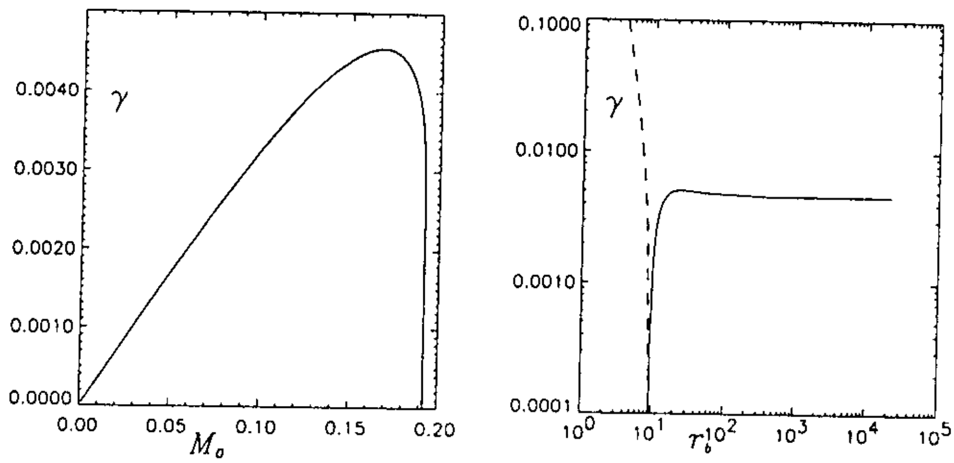

Fig. 2. left: Growth rate γ of breeze instability as a function of base Mach number ($g/c^2 = 5.0$). Marginal stability is obtained for $M_0 = 0$, $M_0 = M_*$. Right: Maximal growth rate for breezes (continuous line) and subsonic accretion (dotted line) as a function of the position of the outer boundary r_b The two lines join in $r_b = r_s$, where $\gamma = 0$.

The breeze instability is driven by the unfavorable stratification (eq.(11)). Imagine a static atmosphere, and let the pressure at infinity increase: clearly an *inflow*, not an outflow, is expected to result. That accretion breezes are stable follows immediately from the analysis presented above; the denominator in eq.(14) changes sign, so the only consistent way to satisfy the boundary conditions is to choose the second solution in eq.(15), implying a negative value for γ. In fact, the stationary equations are symmetrical in M, while the perturbation equations are invariant under a change in sign of both M *and* γ.

Given that breezes are unstable, we see that even in the pressure range $p^c_\infty > p_\infty \geq p^s_\infty$ the only stable outflow is a supersonic wind with a terminal shock. What happens if the pressure difference between the coronal base and the distant medium varies? As p_∞ increases, the shock moves inward, decreasing in amplitude as the critical point is approached. When the critical breeze is reached, there is no neighbouring outflow solution capable of sustaining a higher pressure at infinity. The only possibility for the flow is to collapse into its symmetrical ($M \rightarrow -M$) critical breeze accretion profile, which is also marginally stable. As the pressure is increased further, an accretion shock is formed *below* the sonic point, connecting the symmetrical of the downward transonic to one of the double valued curves in region IV, (as shown by the curve labelled A in fig. (1)). For $p_\infty > p^c_\infty$ there is a unique shocked accretion flow (McCrea, 1956), and the shock position moves inward from the critical point as the pressure is increased beyond p^c_∞ (if the pressure is too high, the shock may occur below the normalization radius chosen). Consider now what happens if, starting from a shocked accretion flow, the pressure at the surface increases, or alternatively the pressure of the ISM decreases. The shock moves outwards, but this time, as p_∞ decreases below p^c_∞, the flow can evolve with continuity into subsonic accretion. As p_∞ decreases further, the accretion-breeze velocities decrease, but when p_∞ decreases below p^s_∞, the flow must accelerate again into a supersonic shocked wind.

The stratification produced by breezes, though globally unstable, is not locally unstable everywhere: for example, below the critical point the pressure in breezes decreases with height more rapidly than in the static case. Inspection of eq.(7) actually shows that this is true out to the radius r_s where the Mach number of the flow has decreased to the same level as the base Mach number, which may be calculated by imposing $M = M_0$ in eq.(7), i.e. $2\log r_s + g/r_s - g = 0$. This equation is independent of base mach number, which also means that at this height the pressure is the same for all breezes, while below this radius, the pressure at a given height is a monotonically *decreasing* function of base Mach number. As the the boundary conditions are imposed at closer and closer distances r_b the growth rate of the instability is reduced, and marginal stability is obtained when $r_b = r_s$. Imposing boundary conditions below this radius stabilizes the breezes, but consequently destabilizes subsonic accretion, as is shown in fig. (2b), where the maximal growth rate for breezes (continuous line) and accretion (dashed line) as a function of r_b is plotted. For large values of g this marginal stability radius depends exponentially on g as $r_s \simeq \exp(g/2) - g/2$.

When the boundary is at $r_b < r_s$ the equilibrium flows still present an hysteresis-type cycle in terms of the enthalpy jump between r_b and the coronal base, but in a reversed order with respect to that previously described: supersonic accretion is blown into supersonic winds as the base pressure is increased beyond a critical value, while an outflow breeze phase exists before the collapse to accretion as the pressure at the outer boundary is increased beyond the value appropriate to a static atmosphere.

In generalizing to polytropic or other more realistic equations for the energy, some attention is necessary since the density and pressure may fall to 0 at a finite distance, and transonic flows do not exist for all polytropic indices ($\gamma \leq 3/2$ below the sonic point is a necessary condition Parker (1963)). With these caveats, the discussion of the isothermal case is easily generalized. The energy equation now becomes (as the sound speed varies, c_0 is its base value)

$$v^2/2 + c^2/(\gamma - 1) - g/r = v_0^2/2 + c_0^2/(\gamma - 1) - g.$$

For breezes the asymptotic behavior $v \sim 1/r^2$ still holds, so that in fact we may write

$$c_\infty^2/(\gamma - 1) = v_0^2/2 + c_0^2/(\gamma - 1) - g$$

which shows that the temperature at great distances from the central object increases with the base Mach number, up to the value which, for a given base density and pressure, gives a transonic flow (Holzer and Axford, 1970). Conservation of energy across the shock then implies that independently of the asymptotic pressure, c_∞ is always the same. It is still true that given the base density and pressure, for a range of pressures at great distances between that of the static atmosphere and that of the critical breeze there are two solutions, an unstable breeze (or stable accretion) and a shocked wind, but now the thermodynamic state of the distant medium is different, the breeze having a higher density and lower temperature.

2 Kinetic models of the solar wind expansion

In fluid models of the solar wind, a fundamental role is played by transport coefficients such as the heat conduction, whose properties are well defined only if there are small gradients in the system and the distribution function do not depart significantly from a Maxwellian. For example, the conditions for the applicability of the $\kappa \sim \kappa_0 T^{5/2}$ thermal conduction law, which relies on collisions, is that the mean free path be much smaller than the characteristic scale of the temperature gradient, a condition which is verified below about 15 solar radii. Also, we have seen that the conditions for the electric field to become greater than the Dreicer electric field are easily overcome in the solar corona. Finally, distribution functions in the solar wind, show significant anisotropies in temperature as well as accelerated populations (see e.g. Marsch 1991).

To begin the study of kinetic models of coronal evaporation consider first the separate behavior of electrons and protons with identical temperatures in the solar gravitational fields. From hydrostatic equilibrium one finds that

$$\frac{\partial p_e}{\partial r} = -m_e n_e \frac{g}{r^2}, \tag{16}$$

$$\frac{\partial p_i}{\partial r} = -m_i n_i \frac{g}{r^2}, \tag{17}$$

from which it follows that the electron scale height is much larger than the proton scale height

$$n_e = n_{e0} \exp\Big(-\frac{g m_e}{\kappa T}(1 - \frac{1}{r})\Big), \tag{18}$$

$$n_i = n_{i0} \exp\Big(-\frac{g m_i}{\kappa T}(1 - \frac{1}{r})\Big), \tag{19}$$

and therefore quasi-neutrality is violated. What happens is that the charge separation creates a polarization electric field, known as the Pannekoek-Rosseland (PR) field, which lifts the protons and pulls the electrons back in order to ensure quasi-neutrality. Problem : show that the PR field is given by

$$E_{PR} = \frac{m_p - m_e}{2eR_0} \frac{g}{r^2}. \tag{20}$$

Because the mean-free path of particles increases with height due to the decrease in density, one may estimate a radius above which collisions will no longer be important. This leads to the separation of the solar corona into two regions: a collisional barosphere and a collisionless exosphere, whose base is taken to be at that distance r_0 where the mean free path λ is equal to the scale height H. For typical coronal values, this exobase is located between 2 and 10 solar radii. One may now use Vlasov's equation and a Maxwellian distribution function at the exobase to determine how the distribution evolves with height.

Problem : show that the solution is given trivially by the fact that F is only a function of the constants of the motion, so that $F(r,v) = F_0(r_0, v_0)$.

Using the PR field and a 10^6 K temperature Chamberlain obtained a mean flow speed at the earth's orbit of around 10 km/sec., and this was the main reason he defended the "breeze" hypothesis against the Parker "wind" model. Subsequent modifications which allowed for an energy-dependent exobase, yielded an increase of the speed to about 140 km/sec but still much less than the speed observed. A major breakthrough was however made by Lemaire and Scherer, (1971) and Jockers (1970), who realized that the polarization field in a flowing atmosphere must be much larger than the PR field. The reason is that otherwise the sun would continuously charge itself: compare the proton and electron escape velocities

$$V_p = \left(2\frac{g}{r_0} - 2\frac{e\phi_E(r_0)}{m_p}\right)^{1/2}, \tag{21}$$

$$V_e = \left(2\frac{g}{r_0} - 2\frac{e\phi_E(r_0)}{m_e}\right)^{1/2}, \tag{22}$$

where the potential from the PR electric field (which vanishes at infinity) is given by

$$\phi_E(r) = \frac{m_p - m_e}{2e}\frac{g}{r}. \tag{23}$$

It follows that $m_e V_e^2 = m_p V_p^2$ in $r = r_0$; one may then integrate over the Maxwellian distribution function to find the flux of particles which have $v > V$ and therefore contribute to the mass flux in the wind. The result is that the ratio of electron to proton flux is $F_e/F_p = \sqrt{m_p/m_e} \simeq 41$ and the sun would become positively charged.

Problem : prove the result above.

In reality one must adjust the electric field so as to satisfy both quasi-neutrality and conservation of current, instead of taking as a given the PR field (which was derived assuming a static stratification). The Vlasov equation is again a function only of the constants of motion, which, taking the radial magnetic field into account, are the total energy (sum of kinetic, polarization potential, and gravitational) and the magnetic moment. Charge conservation and the Vlasov equation are solved iteratively until convergence is achieved. Lemaire and Scherer showed that for an exobase at $3.5R_0$

and the usual 10^6 K corona the actual potential was about twice as large as the PR potential (600 Volts vs 270 Volts), and that with such a potential in fact the effective escape speed for protons falls to zero, i.e., all outgoing protons escape, while electrons are divided into three categories: escaping, ballistic, and trapped. They obtained an outflow velocity of 240 km/sec at earth's orbit. The major pitfall of such models as compared to observations is the dramatic decrease in perpendicular to parallel proton and electron temperatures, which comes from the conservation of magnetic moment in a radially expanding magnetic field ($B \sim 1/r^2$). In reality, the same problem would arise also in the absence of magnetic fields because of conservation of angular momentum, which would also lead for individual particles to $v_\perp \sim 1/r$. Nonetheless, Hundhausen (1972) was able to conclude his discussion by saying that the predictions of the exospheric models "are in better agreement with observations than the predictions of basic fluid models". Unfortunately, while in the subsequent twentyfive years much attention has been given to fluid modeling, little has been done to overcome the temperature anisotropy problem in exospheric models, a problem which must find its solution in the interaction of the plasma turbulence observed in the wind with the distribution functions.

Recently, the Lemaire and Scherer (1971) model has been updated by Maksimovic et al. (1997a), in that a distribution function for protons and electrons more closely resembling those observed *in situ* has been taken as initial condition for the Vlasov equation. This class of distribution functions, the κ distribution, has, with respect to a Maxwellian, an extended high energy tail. Interest in this kind of distribution has been sparked by the possibility of explaining the question of the heating of the corona and expansion of the wind via a very straightforward process known as velocity-filtration (Scudder, 1992 a,b). For a Maxwellian distribution function, which seen as a function of kinetic and potential energies is separable as well as self-similar, the temperature (second order moment of the distribution) does not change with height (gravitational or electric potential). If on the other hand there is an excess of high energy particles, these will climb through the gravitational potential and preferentially survive at greater heights, leading to a wider (hotter) distribution. Though the question of whether such distributions may be created at chromospheric levels remains an open one, it is true that κ function distributions fit the solar wind electron distributions remarkably well, as shown in Maksimovic et al. (1997b), and the correlation distribution-width wind speed has the proper sign, i.e. wider distributions "go faster", as predicted by the theory, which can easily produce speeds at 1 AU in excess of 500 km/sec, with all the drawbacks and simplifications discussed above. It therefore seems that kinetic solar wind models are reaching a competitive stage, though the ultimate aim should be to construct a theory which converges to fluid closures and is both in agreement with present observations, and predictive as concerns possible future missions, such as the solar probe.

3 Alfvénic Fluctuations and the Solar wind

Fluctuations in the high-speed solar wind streams with periods below a few hours, and down to periods of minutes and less are found to be dominated by what is known as Alfvénic turbulence, that is a well developed turbulence spectrum which has all the properties of a flux of large amplitude, constant magnetic field magnitude Alfvén waves propagating away from the sun. The properties of such fluctuations have been summarized in Grappin et al. (1993) as far as Helios observations are concerned, while the observations within the high speed flow at polar latitudes by the Ulysses spacecraft are described in Horbury et al. (1996) . Denoting the magnetic fluctuations and velocity fluctuations by $\mathbf{b}$ and $\mathbf{v}$ respectively, and defining $\mathbf{z}^{\pm} = \mathbf{v} \mp \mathrm{sign}(\mathbf{B})\mathbf{b}/\sqrt{4\pi\rho}$, (we have incorporated changes in the sign of the average field in the definition of Alfvén waves), we may characterize Alfvénic turbulence by the relations $\delta|\mathbf{B}|^2 << |\mathbf{b}|^2$, i.e. small total magnetic intensity fluctuations; $|\mathbf{z}^+| >> |\mathbf{z}^-|$, i.e. outward propagating waves dominate ;$|\delta\rho/\rho|^2 << |\mathbf{v}/C_s|^2 = M_T^2$, where C_s is the sound speed and M_T the turbulent Mach number. In standard MHD turbulence on the other hand, all the $<<, >>$ above become $\simeq$. With little exceptions, at at least at solar minimum, solar wind turbulence varies continuously between the Alfvénic state (in the polar wind and in trailing edges of high speed streams in the ecliptic plane) and the standard state (slow wind at magnetic sector crossings). Incompressible MHD turbulence predicts Alfvénic turbulence as the asymptotic outcome when initial conditions have $u \simeq \delta b/\sqrt{(4\pi\rho)}$ (u being the absolute value of velocity fluctuations). There is some indication that this result is also valid in compressible MHD, while the observed evolution with heliocentric distance is such that Alfvénic turbulence decays towards "standard": the power index of the transverse magnetic field spectrum is typically $\alpha \simeq -1$ for lower frequencies close to the sun, decreasing to the Kolmogorov value $\alpha \simeq -1.6$ at higher frequencies. The bend in the spectrum moves to lower frequencies with increasing distance from the sun, the evolution being somewhat faster within high-speed streams in the ecliptic plane and slower in the polar wind. Together with the evolution in the shape of the spectrum, the specific energy in the fluctuations also varies with distance from the sun, in a way which is roughly consistent, $e \sim r^{-1}$ (r being heliocentric distance, normalized to the solar radius), with the conservation of wave action at the lowest frequencies ($\simeq 10^{-3}$Hz). Suggestions to solve this paradox have included nonlinear evolution due to the in situ generation of inward modes in the solar wind (such modes are necessary, in incompressible MHD, to have nonlinear interactions) and the interaction of the waves with the large scale magnetic field and velocity shears in the current sheet and between fast and slow streams, as will be discussed further below in the section on turbulence.

3.1 Propagation through a static atmosphere

The basic equations for transverse magnetic field (**b**) and incompressible velocity (**v**) fluctuations may be written in terms of the Elsasser variables (defined above) which in a homogeneous medium describe Alfvén waves propagating in opposite directions along the average magnetic field $\mathbf{B}_0$:

$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} \pm \mathbf{V}_a \cdot \nabla \mathbf{z}^{\pm} \mp \mathbf{z}^{\mp} \cdot \nabla \mathbf{V}_a \pm \frac{1}{2}(\mathbf{z}^{\mp} - \mathbf{z}^{\pm}) \nabla \cdot \mathbf{V}_a = 0, \qquad (24)$$

where $\mathbf{V}_a$ is the mean (large-scale) Alfvén velocity. The first two terms in eq.(24) describe wave propagation; the third term describes the reflection of waves by the gradient of the Alfvén speed along the fluctuations (which vanishes for a vertical field in a planar atmosphere, but is different from zero in the more realistic case of a spherically or supraspherically diverging flux tube); the fourth term describes the WKB amplitude variation (which occurs because energy flux must be conserved in the medium with variable wave speed) and the isotropic part of the reflection. In eq.(24) gravity and terms involving the gradients of the average density along the fluctuation polarisation are absent: this is because the average magnetic field and gravity are assumed to be collinear. Eq.(24) then describes the parallel propagation of fluctuations in the plane perpendicular to **B**, or in the case of spherical or cylindrical symmetry, the propagation of toroidal fluctuations in the equatorial plane. In more general cases the magnetic, velocity and density fluctuations are coupled together via magnetoacoustic modes, a process we neglect here but will come back to in subsequent sections. Conservation of net upward energy flux may be written as

$$S^+ - S^- = S_\infty, \qquad S^\pm = FV_a|z^\pm|^2/8, \tag{25}$$

where S_∞ is the constant flux and $F = \rho r^\sigma$ (r is the normalized radial distance from the base of the atmosphere and σ is the infinitesimal flux tube expansion factor: σ=0, 2 in a plane and spherical atmosphere respectively). One may then define the

transmission coefficient across an atmospheric layer bounded by regions of constant Alfvén speed by applying the boundary condition that only an outward propagating wave should exist above the layer in question: T is then given by $T = S_\infty/S_0^+$, where S_∞ coincides with the energy flux carried by the outwardly propagating wave, while S_0^+ is the outward propagating energy flux at the atmospheric base. Note that for waves of frequency ω and wavevector $k = \omega/V_a$, eq.(24) becomes, after elimination of the systematic amplitude variation of $z^\pm$ through the renormalization $z^\pm = \rho^{1/4}z^\pm$,

$$z^{\pm\prime} \mp ikz^\pm - \frac{1}{2}\frac{k'}{k}z^\mp = 0, \tag{26}$$

(a prime denotes differentiation with respect to r). With the propagation equation written in this form, it becomes obvious that the relative importance of reflection (the term coupling of z^+ and z^-) and propagation are determined by the non-dimensional ratio $\epsilon_a = |k'/2k^2| = |V_a'/2 \;|$. Velli (1993) Velli, 1993 discusses the properties of eq.(26) in detail, and develops a general formalism for obtaining approximate analytical solutions by dividing the region of propagation into intervals where $\epsilon_a < 1$ (propagation dominates over reflection), and regions where $\epsilon_a > 1$ (reflection dominates). An important point to recall is that for very long wave-length waves propagating over a region with varying Alfvén speed the transmission coefficient may be simply written as

$$T = \frac{4V_{al}V_{ar}}{(V_{al} + V_{ar})^2}, \tag{27}$$

where $V_{al,r}$ indicate the Alfvén speed on either side of the layer.

For an isothermal, static, spherical corona and a radial magnetic field the Alfvén speed depends on radius as

$$V_a = \frac{V_{a0}}{r^2}\exp\left(\left(\frac{\alpha}{2}(1-\frac{1}{r}\right)\right), \qquad \alpha = \frac{GM_0}{R_0 C_s^2}, \qquad 1 < r < \infty, \tag{28}$$

where C_s is the isothermal sound speed, R_0 the coronal base raiuds. The parameter α, for the sun, typically lies in the range $4 \leq \alpha \leq 15$ for coronal temperatures between $8.0\ 10^5$ – $3.0\ 10^6$ °K. For this family of profiles, the Alfvén speed first increases exponentially, has a maximum in $r = \alpha/4$ and then decreases asymptotically as $V_a \sim r^{-2}$. The general behaviour of T may be gleaned from the low frequency approximation eq.(27) if one is careful to remember that the thickness of the reflection-dominated layer depends on the frequency and extends from $r = 1$ (for frequencies such that $\epsilon(r=1) \ll 1$ to the distance r_ω where $\epsilon = 1$. Writing $\Omega = \omega R_0/V_{a0}$ one has

$$\epsilon = 1 \rightarrow \Omega \simeq 1/r_\omega^3 \exp(\alpha/2). \tag{29}$$

In other words the corona becomes transparent to Alfvén waves at a distance $r_\omega \simeq \exp(\alpha/6)\Omega^{-1/3}$, where the value of the Alfvén speed is

$$V_a = V_{a0}\Omega^{2/3}\exp\left(\frac{\alpha}{6}\right). \tag{30}$$

Substitution into eq.(27) then shows that transmission for very low frequencies should increase like $\Omega^{2/3}$, reach a maximum where $V_a = V_{a0}$ and then decrease again as $\Omega^{-2/3}$. This was shown in Velli, 1993, where the value of the transmission at the maximum was computed both numerically and analytically (its value is not 1, but about 0.6). The above result holds true provided α is large enough ($\alpha \geq 8$) so that the region where the Alfvén speed reaches a maximum (and where propagation always dominates) is small enough (in fact, it is the presence of the maximum and hence of two distinct non-propagating regions on either side of the maximum to yield the factor 0.6 mentioned above). For smaller values of α one only sees a monotonic increase of the transmission with frequency going as $\Omega^{2/3}$. For the range of temperatures compatible with the solar corona wave periods below 15 minutes are completely transmitted, and even for periods of a few hours transmission is above 50%.

3.2 Propagation in the solar wind

In the above discussion the presence of the solar wind, which becomes fundamental above a few solar radii. In this case the wave propagation equation becomes (Heinemann and Olbert, 1980),

$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} + (\mathbf{U} \pm \mathbf{V}_a) \cdot \nabla \mathbf{z}^{\pm} + \mathbf{z}^{\mp} \cdot \nabla(\mathbf{U} \mp \mathbf{V}_a) + \frac{1}{2}(\mathbf{z}^{-} - \mathbf{z}^{+})\nabla \cdot \left(\mathbf{V}_a \mp \frac{1}{2}\mathbf{U}\right) = 0, \quad (31)$$

where $\mathbf{U}$ is the average wind velocity. The wind and Alfvén speed profiles corresponding to the previous isothermal atmosphere are now (introducing the mach number $M = U/C_s$)

$$(M - \frac{1}{M})M' = \frac{2}{r} - \frac{\alpha}{r^2}, \qquad V_a = V_{a0}/r(U/U_0)^{1/2}. \quad (32)$$

For waves propagating in the wind, the energy flux is no longer conserved since the wave pressure does work in the expansion. The wave-action however is still conserved,

$$S^{+} - S^{-} = S_{\infty}, \qquad S^{\pm} = F\frac{(U \pm V_a)^2}{V_a}|z^{\pm}|^2/8, \quad (33)$$

where F is the geometrical factor defined previously. With a conserved flux (in this case the wave action) one can still associate a transmission coefficient as long as there is a position where there is no "inward propagating" wave. From eq.(33) it follows that the inward flux vanishes at the Alfvénic critical point where the solar wind speed equals the Alfvén speed, so that $T = S_{\infty}/S_0^{+}$, where now however the wave action flux is determined by the amplitude of the outwardly propagating wave at the critical point as $S_{\infty} = S_c^{+} = F|\mathbf{z}^{+}|_c^2/2$. Remarkably, the transmission T for moderate to high frequencies in this case parallels the static computation exactly, and the conclusions of the previous paragraph remain valid. At the lowest frequencies instead one finds a transmission coefficient which is significantly enhanced. This may be understood by rewriting eq.(31) in the low frequency case in terms of $y^{\pm} = (U \pm V_a)z^{\pm}$:

$$y^{\pm\prime} - \frac{1}{2}\frac{V_a'}{V_a}y^{\mp} - \frac{1}{2}\left(\frac{U'}{U} + \frac{V_a'}{V_a}\right)y^{\pm} = 0 \quad (34)$$

which has the two solutions (as in the static case) $y^{+} = \pm y^{-}$. Imposing that y^{-} vanish at the critical point then gives (Heinemann and Olbert, 1980,Leer E. et al. 1982, Hollweg, J.V., & Lee, M.A., 1989)

$$\mathbf{z}^{\pm}(r) \sim \frac{1}{U \pm V_a}\left(\frac{U}{U_c}\right)^{1/2}\left(\frac{V_a}{V_{ac}} \pm 1\right),$$

and the subscript denotes quantities calculated at the Alfvénic critical point. For the low frequency limit of the transmission we obtain

$$T = \frac{4U_0 V_{a0}}{(U_0 + V_{a0})^2} \frac{\mathbf{z}^{+2}_c}{\mathbf{z}^{+2}_0} = \frac{4V_{a0}V_{ac}}{\left(V_{a0} + V_{ac}\right)^2}.$$

Thus we see that it is possible to have perfect transmission at low frequencies, if the Alfvén speed at the coronal base and the critical point are 'tuned' close to the same value. We remark that the above result is independent of the position of the coronal base, provided the geometry allows for the propagation of a pure Alfvén-type wave.

The meaning of the transmission coefficient into the wind requires some discussion however since we have seen that it is calculated exactly at the Alfvén critical point. Beyond this point both "inward" and "outward" modes are carried together outwards because the wind speed is greater than the mode propagation speed, so that in this sense it is a good definition. However it is not true that at greater distance the amplitude of the inward mode vanishes, on the contrary, the normalized cross-helicity

$$\sigma_c = \frac{z^{+2} - z^{-2}}{z^{+2} + z^{-2}} \tag{35}$$

which in the static case is by definition equal to one when there is only an outward propagating wave (in which case the specific energy in velocity and magnetic field fluctuations is the same) continues to evolve. In the spherically expanding case, the behaviour of σ_c beyond the critical point depends on the frequency. This may be seen using a particularly simple model for wind velocity and Alfvén speed at large distances: $U = U_\infty$, a constant, so the radial Alfvén speed goes as $V_a = V_{a\infty}/r$. Eq.(31) may now be rewritten as

$$(U_\infty \pm \frac{V_{a\infty}}{r})z^{\pm\prime} - i\omega R_0 z^{\pm} + \frac{1}{2r}(z^+ + z^-)(U_\infty \mp \frac{V_{a\infty}}{r}) = 0. \tag{36}$$

An eikonal expansion which treats the boundary condition at the critical point correctly (see e.g. Barkhudarov (1991) , Velli et al. (1991)) then shows that at large distances

$$z^+ \simeq -\left(\left(1 - 4\omega^2 V_{a\infty}^2 R_0^2/U_\infty^4\right)^{\frac{1}{2}} - 2i\omega V_{a\infty}R_0/U_\infty\right)^{-1} z^- + O(1/r)z^-. \tag{37}$$

For all frequencies greater than $\omega_0 = U_\infty^2/2V_{a\infty}R_0$ the normalized cross helicity σ_c increases with distance beyond the Alfvén critical point to a frequency-dependent limiting value which tends to one at high frequencies as $1-(\omega_0/\omega)^2$. At frequencies below ω_0 however σ_c decreases with distance and tends asymptotically to 0, i.e., we have total reflection at infinity. This critical frequency has a straightforward physical interpretation, in terms of the relative strength of the wave coupling and gradient, or rather expansion effects. An Alfvén wave is a coupling of transverse magnetic and velocity fluctuations in which the underlying field-line tension provides the restoring force. In the presence of a wind, the equations are modified by the outward flow, but the angular momentum and magnetic flux must be conserved. This translates into the appearance of a decaying term in u/r for the transverse velocity fluctuation u in the momentum equation, and a decaying term b/r for the transverse magnetic field b in then induction equation, which disappears if b is renormalized with the square root of the density (i.e. one writes it in terms of the transverse alfven velocity $b=b/\sqrt{(4\pi\rho)}$ as was done earlier in defining the Elsasser variables). The equations for the fluctuations then become

$$\frac{\partial u}{\partial t}+U_\infty u'-\frac{V_{a\infty}}{r}b'+\frac{U_\infty}{r}u=0, \tag{38}$$

$$\frac{\partial b}{\partial t}+U_\infty b'-\frac{V_{a\infty}}{r}u'=0, \tag{39}$$

which are the same as eq.(36) where we have neglected the gradients of the Alfvén speed with respect to the divergence of the bulk velocity field in the last term in parentheses of eq.(36).

At large distances and to lowest order oscillations with frequency ω have a wave-number given by $kR_0=\omega/U_\infty$ (first two terms in eqs.(38,39). If the Alfvén speed is vanishingly small (i.e. low-frequency oscillations) the magnetic and velocity fields are decoupled entirely, the transverse velocity decays as $u\sim 1/r$ while the magnetic field is constant. This translates into a cross-helicity which tends to zero at great distances. If the Alfvén speed is not negligible, one may substitute for the terms of type $V_{a\infty}b'/r$ the value obtained with the wave-number kR_0 to get $V_{a\infty}R_0\omega b/U_\infty r$. This term depends on r in the same way as the angular momentum conservation term $U_\infty u/r$ in eq.(38), the relative magnitude of the two being given dimensionally by $\omega V_{a\infty}R_0/U_\infty^2=\omega/2\omega_0$. Therefore, for frequencies much larger than the critical one, the Alfvénic coupling is important, and in this regime u,b are constrained to evolve together (i.e. reflection may be neglected), both fields decaying asymptotically $u, b \sim 1/\sqrt(r)$. For frequencies below the critical one, the fields of course decouple as shown more rigorously by the expansion eq.(37).

The critical frequency is a number of some importance: in the high-speed solar wind streams, where $U \simeq 800$ km/sec, and assuming a typical value for the Alfvén speed $V_a \simeq 50$ km/sec at $R = 1$ AU, we obtain $\omega_0 \simeq 4.27\ 10^{-5}$sec, corresponding to a period of about 41 hours. This is quite a long period, while Alfvénic turbulence is seen at periods substantially lower, from several hours to a few minutes, and indeed the specific energy in this range appears to fall as r^{-1}, which is consistent consistent with $u, b \sim 1/\sqrt(r)$.

4 Turbulence in the solar wind

The shape of the velocity and magnetic field spectra observed in situ are strongly suggestive of a nonlinear cascade, where one expects to find energy on all possible wavevectors, although not uniformly distributed: a specific property of turbulence is the scale invariance of the energy spectrum, which manifests itself in the form of a power-law spectrum. This viewpoint was first adopted by Coleman (1968). The large scales show a dominance of kinetic energy over magnetic energy ($f \leq 10^{-4}$ Hz) while at smaller scales both kinetic and magnetic energies are approximately of the same order of magnitude. In this last range, the spectral index is between 1.5 and 1.7: these values are compatible with the spectral slopes that one expects on the basis of either fluid or MHD turbulence (as will be shown below). Moreover, density fluctuations are usually weak, $\delta\rho/\rho \ll \delta u/c_s$ (where c_s is the proton thermal speed): this means that the amplitude of compressible waves is small. It is this range, between 10^{-4} Hz and 1 Hz, which we shall call the Alfvénic domain. On the other hand, Belcher and Davis (1971), observed that, during a substantial portion of the time (the so-called "Alfvénic periods"), the velocity $\mathbf{u}$ and the magnetic field fluctuations $\mathbf{b}$ not only are of the same magnitude, but are almost completely correlated: $\mathbf{z}^{\pm} = \mathbf{u} \pm \mathbf{b}/\sqrt{4\pi\rho}$ is often very small, depending on the polarity of the average field $\mathbf{B}_0$, so that the waves propagating away from the Sun appear to dominate. This suggests that the fluctuations consist of linearly propagating waves, a possibility confirmed (Dobrowolny, Mangeney, Veltri

1980a) by inspection of the incompressible MHD equations in a uniform field in static equilibrium which may be written formally as

$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} \mp \mathbf{V}_a \cdot \nabla \mathbf{z}^{\pm} = -\frac{1}{\rho}\nabla p^T - \left(\mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} -\right), \tag{40}$$

where $\mathbf{V}_a = \mathrm{B}_0/(4\pi\rho)$ is the Alfvén velocity and p^T is the fluctuation in the total (kinetic and magnetic) pressure; introducing the Fourier amplitudes of $\mathbf{z}^{\pm}$, $\mathbf{z}^{\pm}_{\mathbf{k}}$, the Fourier transform of equ. (40) may be written as

$$\left(\frac{\partial \mathbf{z}^{\pm}_{\mathbf{k}}}{\partial t} \mp i\mathbf{k}\cdot\mathbf{V}_a \mathbf{z}^{\pm}_{\mathbf{k}}\right) = \int_{\mathbf{p}+\mathbf{q}=\mathbf{k}} \mathbf{A}_{\mathbf{kpq}} \mathbf{z}^{\mp}_{\mathbf{p}} \mathbf{z}^{\pm}_{\mathbf{q}} \mathrm{d}^3\mathrm{q}, \tag{41}$$

where the tensor $\mathbf{A}_{\mathbf{kpq}}$ has components

$$\mathbf{A}_{\mathbf{kpq}ilm} = k_m\left(\delta_{il} - \frac{k_i k_l}{k^2}\right) + k_l\left(\delta_{im} - \frac{k_i k_m}{k^2}\right) + k_l\delta_{im} - k_m\delta_{il} \tag{42}$$

It is clear from eq.42 the nonlinear interactions vanish when either z^+ or z^- is zero. The existence in the solar wind of a well established, scale-invariant spectrum made up of non-interacting waves is problematical, as remarked by Dobrowolny et al. (1980a): given a solar source of outgoing waves, the observed spectrum should reflect its properties, as well as the filtering of the intervening medium, and hence gaps in the spectrum or peaks at some typical generation frequencies (or harmonics) should appear, which is not observed in the data.

However the standard situation is that of a mixture of both types of Alfvén waves, with moderate dominance of outward propagating waves, so that the nonlinear couplings are not vanishingly small. For example, on the average, the ratio z^+/z^- calculated between 10^{-4} and 10^{-2} Hz, was about 0.6 during the first three months of the Helios mission (at heliocentric distance between 0.3 and 1 AU and at solar minimum). Consider first this quasi-symmetric regime ($z^+ \sim z^-$), which one may attempt indeed to describe by "standard" MHD theory,as first proposed by Coleman (1968). Just as in any wind tunnel, one can not expect to observe fully developped turbulence too close to tunnel entry; some time must be allowed to let the non- linear effects to develop significantly. This time is the turnover-time τ_{nl}, which for an eddy of size l depends on the rms energy contained around this scale, about u^2 if u is the velocity fluctuation amplitude within the eddy: $\tau_{nl} \sim l/\mathrm{u} = (\mathrm{U}/\mathrm{u})\mathrm{T}$, if $\mathrm{T} = l/\mathrm{U}$ is the typical timescale measured in the spacecraft frame. At a given heliocentric distance R, the transport time by the average flow is $\tau_{tr} = R/U$. A third time scale, $\tau_{ad} \sim 1/(\nabla \cdot U) = -(1/\rho)D\rho/Dt$, describes the rate of change of the plasma specific volume $1/\rho$ associated with the geometry of the expansion, and does not depend on the spatial scale of the eddies. The adiabatic and transport times are of the same order of magnitude in the supersonic region of the wind (they are identical for a spherical expansion with constant speed) For example at R =0.3 AU, the distance of closest approach of the Sun by the Helios spacecrafts, τ_{tr} = R/U =35h; an inspection of figure 1 shows that in the Alfvénic range of periods 1 h > T > 3 mn, $u/U \simeq 0.05$, and thus $\tau_{nl} < \tau_{tr} \sim t_{ad}$. If we follow a plasma parcel which is convected with the wind, nonlinear effects will strongly affect only the part of the spectrum for which the average turnover time is smaller than the transport time. Therefore the width of the inertial range, if one does exist, must depend on the radial distance. Except if the spectrum is very steep, the nonlinear time decreases with the scale; hence there will always be a critical scale $L(R) \sim (u/U)R$ below which nonlinear effects dominate (Tu et al. 1984). As heliocentric distance R increases, the adiabatic time also increases, and so does L(R), as long as u does'nt decrease as fast as R^{-1}, i.e., as long as the turbulent specific energy does not decrease as fast as R_{-2}. The nonlinear interactions are thus free to redistribute the energy among the degrees of freedom available between the scale L(R) and a dissipation scale $l_d \sim l_g$. A fully developed turbulent state is expected when several orders of magnitude separate the two scales ($L(R) \gg l_d$). According to the Kolmogorov (1941) theory, two properties characterize such a state. First, the energy dissipation rate is independent from the viscosity of the fluid, i.e., it reaches a finite value in the limit of zero viscosity.

Since the nonlinear interactions respect the conservation of energy, the dissipation rate must be given by the energy injection rate $\Pi^{\odot} = \epsilon$, which either comes from an external source, or from the largest, energy-containing eddies (here at scale L(R)).

Second, the energy is not transfered directly from the largest scale down to the dissipation scale, but instead is transfered via successive interactions between smaller and smaller (but at each step comparable) wavenumbers (i.e., in equ.(1b), the dominant contribution to A_{kpq} comes from interactions with $|p| \sim |q| \sim |k|$) whence the name "energy cascade". During this cascade, eddies of a given size l break into smaller eddies, but are regenerated by the larger eddies, and so on; since all scales are in energetic equilibrium, the energy dissipation rate of eddies of size l, $\Pi(l)$, is independent of the scale l, i.e., it is equal to ϵ:$\Pi(l) = \Pi^{\odot} = \epsilon$ Let $\tau^*(l)$ be the characteristic time for an eddy of size l to transfer its energy $E(l)$, so that

$$\Pi(l) \sim E(l)/\tau^*(l). \tag{43}$$

Since the energy transfer in an ordinary fluid results essentially from the self-distortion of the eddy, the transfer time is simply equal to the turnover-time τ_{nl}. Assuming that the interactions are local in wavenumber space, it may be written as $\tau_{nl} \sim l/u(l)$. Using $E(l) \sim u^2(l)$, we finally obtain that $E(l) \sim (\epsilon l)^{2/3}$, or since (with $k \sim 1/l$) $E(l) \sim \int k E_{k'} dk' \sim kE_k$:

$$\Pi_k \sim k^{5/2} E_k^{3/2} = \epsilon \tag{44}$$

and the scale-invariance of the flux leads then to $E_k \sim \epsilon^{2/3} k^{-5/3}$, which is well known the Kolmogorov spectrum. Now, consider the case of a conducting fluid imbedded within a uniform magnetic field , and assume that there are incompressible fluctuations, in the form of Alfvén waves. The essence of the Iroshnikov's (1963) and Kraichnan's (1965) theory is to recognize that the self-distortion, in a large-scale magnetic field, is replaced by weaker interactions between propagating Alfvén waves. The propagation introduces an additional timescale, the Alfvén time $\tau_A \sim l/V_a$, and the effective energy transfer time τ^* is no longer equal to the eddy-turnover-time τ_{nl}. Indeed, noting that, to lowest order, the nonlinear terms couple linear solutions (i.e. Alfvén wavepackets) propagating in opposite directions, the coherent interaction time is reduced to τ_A, which is smaller than the eddy-turnover-time by the factor $\delta B/B_0$. The amplitude change during a typical "collision" is proportional to $\delta B/B_0$. Assuming that successive collisions of wavepackets are independent, it is found that the time τ^* for a full interaction is such that $\tau_A : \tau_{nl} : \tau^*$ or also: $\tau^* \sim (\tau_{nl}/\tau_A)\tau_{nl} \sim (B_0/\delta B)\tau_{nl}$, which may be much longer than τ_{nl}. Now, replacing the turnover-time by the effective transfer time τ^* in the expression of the energy transfer rate Π of equ.(43) , we obtain

$$\Pi_k \sim kE^2/V_a = k^3 E_k^2/V_a \tag{45}$$

and again imposing a constant dissipation rate ϵ, this leads to $E(l) \sim (\epsilon V_a l)^{1/2}$, or $E_k \sim (\epsilon V_a)^{1/2} k^{-3/2}$, which is the Iroshnikov-Kraichnan spectrum.

Although the solar wind is neither incompressible, nor isotropic, or homogeneous, the observed slopes are close to those inferred by the above arguments. While Coleman (1968) argued in favor of an Iroshnikov-Kraichnan spectrum (-3/2 slope), most of the observational evidence seems to be for spectral slopes in the solar wind turbulence very near the Kolmogorov slope, as soon as the heliocentric distance is larger than about 1 AU (see for example Bavassano B., and E.J., Smith, Radial variation of interplanetary Alfvenic fluctuations: Pionnier 10 and 11 observations between 1 and 5 AU, J. Geophys. Res., 91, 1706, 1986). The present state of numerical simulations does not help to clarify the situation: even when the conditions are favcrable (an incompressible, homogenous fluid with large scale magnetic fields) it is difficult, because of the limited resolution available, to measure spectral slopes accurately enough, and thus to determine which of the Kolomogorov or Iroshnikov-Kraichnan phenomenology is valid (see,e.g., Biskamp and Welter, 1989).

The preceding arguments assume implicitely that both field amplitudes z^+ and z^- are comparable. As remarked before, in "Alfvénic " situations this is not the case. From the MHD equations (1a), one sees that the turnover times for z^+ and z^- eddies in reality depends on the amplitude of the other field (see Dobrowolny et al 1980b):

$$\tau_{nl}^{\pm} = l/z^{\mp}. \tag{46}$$

Nonlinear interactions conserve separately the energies $E^{\pm}$ in both fields. It is thus legitimate to consider the possibility of separate energy cascades, via distinct fluxes Π^+ and Π^-. If one assumes that the Iroshnikov-Kraichnan decorrelation effect holds, then equal Π^+ and Π^- fluxes are obtained:

$$\Pi_k^+ = \Pi_k^- = k^3 E_k^+ E_k^- / V_a. \tag{47}$$

This formula again leads to the Iroshnikov-Kraichnan spectrum (proportional to $k^{-3/2}$) when both amplitude are comparable . When the $z^{\pm}$ asymmetry becomes large, eq.(46) does not provide a unique determination of the spectral slopes m^+, m^-; it only predicts that their sum should be equal to 3 (Grappin et al, 1983):

$$m^+ + m^- = 3. \tag{48}$$

In order to obtain a relation between the slopes and the fluxes a more elaborate theory is needed: we shall come back to this point in the next section. On the other hand if one assumes the interactions to be coherent as in Kolmogorov's theory (see Matthaeus et al. 1983), i.e., assumes the transfer times $\tau^{*\pm}$ to be equal to the turnover times, then the transfer rates are distinct:

$$\Pi_k^{\pm} = k^{5/2} E_k^{+} E_k^{-} / (E_k^{\mp})^{1/2} = \epsilon^{\pm}. \tag{49}$$

In this case, the constraint of constant fluxes leads to the Kolmogorov spectra for both fields, whatever the asymmetry in the fluxes $\epsilon^{\pm}$.

As seen in the paragraphs on linear evolution of waves in the solar wind, the expansion effects appear as supplementary terms in the equation, which involve the "average" Alfvén and advection velocities V_a and U and their radial derivatives. The definition of the timescale T over which the full equations are averaged in order to separate the large-scale wind expansion from the small-scale fluctuations is somewhat arbitrary. The basic requirement is that T^{-1} should be less than the lowest frequency considered. The amplitude can then be split in two parts, an average $< z >$ and a fluctuating z. Upon substraction of the time average from the original MHD equations and assuming incompressible fluctuations, one obtains now the full equations (Whang, 1980):

$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} + (\mathbf{V}_g{}^{\pm}) \cdot \nabla \mathbf{z}^{\pm} + \mathbf{z}^{\mp} \cdot \nabla (\mathbf{V}_g{}^{\mp}) + \frac{1}{2}(\mathbf{z}^{\pm} - \mathbf{z}^{\mp})\left(\frac{1}{2}\nabla \cdot \mathbf{U} \pm \nabla \cdot \mathbf{V}_a\right) = \tag{50}$$

$$= -\frac{1}{\rho}\nabla p^T - \left(\mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} - < \mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} >\right),$$

where $\mathbf{V}_g{}^{\pm} = \mathbf{U} \mp \mathbf{V}_a$ is the linear group velocity . An equation for the evolution of the quadratic quantities, such as the energy spectrum, may be obtained by multiplying equ.(50) by $\mathbf{z}^{\pm}(\mathbf{x}', t')$, averaging and then Fourier transforming with respect to the "fast" variables $\mathbf{x} - \mathbf{x}', t - t'$. Assuming isotropy and time stationarity of the spectral quantities, one obtains equations for the spectra which depend both on wavenumber k (fast variable) and distance r (slow variable) (Marsch and Tu 1989):

$$\nabla \cdot (\mathbf{V}_g^{\pm}\ E_k^{\pm}(\mathbf{r})) + E_k^{\pm}(\mathbf{r})\frac{1}{2}\nabla \cdot \mathbf{U} + M_k^{\pm}(\mathbf{r}) = \frac{\partial \Pi_k^{\pm}}{\partial k}. \tag{51}$$

where the derivatives in the lhs denote derivatives with respect to the slow variable r. In eq.(51) the first term is the wave energy flux, the second term is the work done by the waves in accelerating the bulk flow. The third term $M^{\pm}$ represents the linear coupling between the two wave species, due to the large scale inhomogeneities:

$$M^{\pm}(\mathbf{k},\mathbf{r}) = -\mathbf{z}_{\mathbf{k}}^{+}\mathbf{z}_{\mathbf{k}}^{-} :: \left(\nabla \mathbf{V}_g^{\pm} - \frac{1}{2}\delta_{ij}(\nabla\cdot\mathbf{V}_a \pm \frac{1}{2}\nabla\cdot\mathbf{U})\right) \qquad (52)$$

and effectively couples the $\pm$ equations with that for the residual energy $E^r = \mathbf{z}^+ \cdot \mathbf{z}^- = (u^2 - b^2)$. Its order of magnitude may be estimated as $\approx E^r/\tau_{ad}$. In the linear case of transverse Alfvén waves, $M^{\pm}$ and the nonlinear terms may be neglected,(the distinction between energy spectral densities and energies at a given scale may in this case be dropped without altering the equations), and eq.(51,52) reduces to the equation of conservation of the adiabatic invariant $S^{\pm} = E^{\pm}/(k.V_a)$:

$$\nabla\cdot(\mathbf{V}_g^{\pm} S^{\pm}) = 0.$$

In the solar wind the Alfvén speed may be neglected compared to the wind speed, so that we obtain from eq.(52) (Jacques, 1977):

$$\nabla\cdot(\mathbf{U}\rho E^{\pm}) + \rho E^{\pm}\nabla\cdot\mathbf{U}/2 = 0$$

which in a spherical expansion at constant speed gives $E^{\pm} \propto r^{-1}$ (recall that $E^{\pm}$ is the specific energy; the result usually quoted is $\rho E^{\pm} \propto r^{-3}$). Both spectra E^+ and E^- thus decrease with distance in a self-similar way, i.e., without changing their shape. The observations confirm that this simple linear description is valid at low frequencies, between $(2\cdot 10^{-4}, 2\cdot 10^{-3})$ Hz, while at larger frequencies they show that total turbulent energy decays more rapidly than simply predicted by the adiabatic change, suggesting again that turbulent dissipation (and thus nonlinear interactions) are at work (Bavassano et al, 1982; Schwenn 1983).

These considerations apply only in the inner heliosphere. Indeed, at larger distances from the sun, the energy decay rate appears to be roughly frequency independent (Bavassano and Smith, 1986). The radial dependence, however, is $\propto r^{-}3.5$, which is slightly steeper than the WKB dependence. In order to explain the changes in spectral shapes in the inner heliosphere, Tu et al (1984) and Tu (1988) developed a model in which the nonlinear flux on the rhs of eq.(51) is calculated following dimensional analysis as described in the previous section (eq.47) or eq.(49): isotropy in k is necessarily assumed, so that the flux depends on the wavevector amplitude only). To close their equations, they assume the residual energy to be negligible, ($M^{\pm} = 0$). Moreover, the ratio $\alpha = E^+/E^-$ is chosen to be a constant, fitting an average observed value, and a spherically symmetric background wind is assumed, so that they finally obtain from eq.(51) a single equation for the spectral evolution of E^+, with the nonlinear flux term given by one of the two expressions:

$$\Pi_k^+ = A\frac{\alpha k^3 E_k^+(r)^2}{V_a} \quad (a), \qquad \Pi_k^+ = B\alpha k^{5/2} E_k^+(r)^{3/2} \quad (b), \tag{53}$$

for the cases of strong magnetic fields (IK) and Kolmogorov's theory respectively (A and B are constants of order unity). The model describes the two regimes on either side of the critical scale $L(r)$ at which $\tau_{nl} \approx \tau_{ad}$. At small wavevector the nonlinear effects are negligibly small, the only modifications are limited to the WKB decay $u^2 \propto r^{-1}$; at larger wavevector they dominate so that locally an equilibrium spectrum in $k^{-5/3}$ is maintained (or $k^{-3/2}$, depending on the form (a) or (b) of the nonlinear flux (11)) the WKB effects acting to fix the energy level at a given frequency. In this way, when starting with a flat (k^{-1}) spectrum as similar to what is observed at 0.3 AU, one obtains the steepening towards equilibrium spectrum, first at high frequencies, and later on at low frequencies.

The model by Tu et al (1984) and Tu (1988) approximates the ratio E^+/E^- by an observational constant. However, both E^+ and E^- obey coupled evolution equations, the coupling occuring through expansion and nonlinear effects. Hence there is no reason to expect that the ratio remain constant. Then one may speculate on whether the situation found in the Alfvénic periods, where $z^- \ll z^+$, is the result of the evolution of the turbulence, or is a distinct property of high speed winds. A particular MHD mechanism, called "dynamic alignment" was proposed by Dobrowolny et al (1980b) to explain why the spectra were well-developed even when the inward mode energy E^- is very small. They suggested that in MHD turbulence, the velocity and magnetic field become more and more aligned due to nonlinear interactions, thus enhancing with time any initial imbalance between both inward and ouward component. The argument, which follows directly from the IK theory, is that the turbulent dissipation of both energies is the same (eq.(47)), so that the energy difference, i.e. the $\mathbf{v}.\mathbf{b}$ correlation $C = (E^+ - E^-)/2$ stays constant while the total energy $E = (E^+ + E^-)/2$ decays, and the proportion $\gamma = C/E$ of the initially dominant (z^+) species indeed grows with time during the nonlinear interactions. However, the evolution towards alignment is very slow, in fact too slow to take place in the solar wind between, say, 0.3 and 1 AU (Grappin et al., 1982). Second, and most importantly, the evolution actually seems to proceed in the reverse way in the solar wind: Roberts et al. (1987) found that in the average, the outward dominance tends to decrease rather than increase with heliocentric distance in the 0.3 AU-20 AU range. The explanation of the decrease of outward dominance at long distances may be that, in the solar wind, most of the energy is initially stored in the velocity shears due to the coexistence of high speed and low speed streams (Sturrock and Hartle, 1966): the nonlinear transfer of this pure kinetic energy towards small scales should sweep out any initial outward/inward imbalance of smaller scale eddies (see the closure calculations by Grappin et al. 1983, the simulations by Roberts et al. (1990), and the analysis by Roberts et al. (1987)).

Another difficult question is to understand why the relative density fluctuations are an order of magnitude lower in the Alfvénic periods than in the non-Alfvénic periods, while the turbulent Mach numbers are comparable or higher. This contradicts the known fact that a compressible flow naturally develops density fluctuations whose amplitude depends essentially on the turbulent Mach number (see Passot and Pouquet 1987 for numerical experiments in the hydrodynamic case, and Montgomery et al., 1987, Shebalin and Montgomery, 1988 in the MHD case). One often invokes the larger damping of magnetosonic waves to explain the low level of density fluctuations, in the Alfvénic periods, but then why is it that the same damping does not work in the slow winds? Or is it because there is not enough time to couple compressible and solenoidal fluctuations in the fast wind?

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