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Edge Turbulence and Drift Instabilities

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These are preliminary lecture notes, intended only for distribution to participants.

Edge turbulence and drift instabilities

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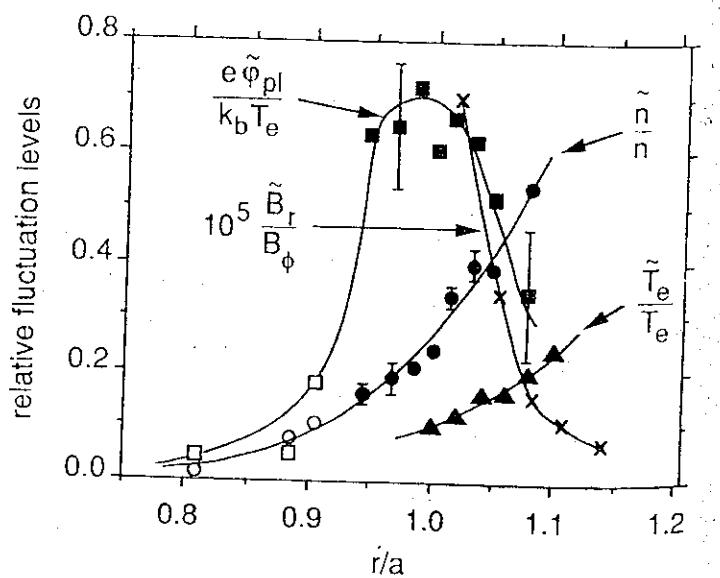
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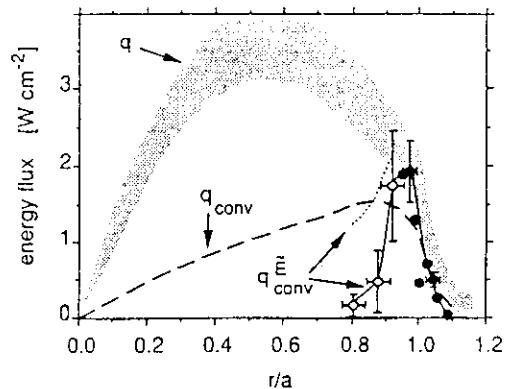
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Contents

- Resistive drift turbulence
- Resistive drift interchange turbulence
- Resistive drift Alfvén instabilities



Two-Fluid Model



Radial profiles of the total electron and ion energy flux $q = q_e + q_i$ from power balance (shaded area, defined by the standard deviation), the fluctuation-induced convected flux q_{conv}^E (filled circles from Langmuir probes, and open circles from HIBP; dotted line is upper bound in presence of η_i mode), and the total convected energy flux $q_{\text{conv}}(r)$ from a neutral-penetration code and H_α measurements.

図3 TEXTトカマクにおけるエネルギー束 q の径方向分布。 $r/a \gtrsim 0.9$ の edge plasma では、パワーバランスから求めた q とラングミュアプローブによる $q(\bullet)$ および HIBP による $q(o)$ がよい一致を示している。また、中性粒子密度から求めた q_{conv} もよく対応している。(文献[10]より)

{ conduction (heat)
convection (particle)

$$\frac{\partial n}{\partial t} = -\nabla \cdot (n\mathbf{v}) = -\nabla \cdot (n\mathbf{v}_e)$$

$$nm_i \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} + \nabla(P_e + P_i) + \nabla \cdot \overleftrightarrow{\Pi}_{gi} = \frac{1}{c} \mathbf{J} \times \mathbf{B}$$

($\overleftrightarrow{\Pi}_{gi}$ is viscosity tensor)

$$\mathbf{E} + \frac{1}{c} \mathbf{v}_e \times \mathbf{B} + \frac{1}{ne} \nabla P_e = \eta_{||} \mathbf{J}_{||} + \eta_{\perp} \mathbf{J}_{\perp}$$

$$\mathbf{v}_e = \mathbf{v} - \frac{\mathbf{J}}{ne}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c \nabla \times \mathbf{E}, \quad \nabla \cdot \mathbf{B} = 0$$

$$\frac{4\pi}{c} \mathbf{J} = \nabla \times \mathbf{B}$$

$$P_e = f_e(n), \quad P_i = f_i(n)$$

Assumptions

1. Cold ions, $T_e \gg T_i$

2. Two dimensional ion motion

$$|\mathbf{v}_{\perp i}| \gg |\mathbf{v}_{\parallel i}|$$

3. Isothermal electrons $T_e = \text{const.}$

4. $\nabla_{\parallel} P_e$ term is kept in Ohm's law

Vorticity equation:

$$\cdot \frac{d}{dt} \frac{\nabla_{\perp}^2 \hat{\phi}}{B_0 R_i} = \frac{1}{e n_0} \frac{\partial}{\partial z} J_2$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{\nabla \hat{\phi} \times \hat{\mathbf{z}}}{B_0} \cdot \nabla$$

$$\cdot \frac{dn}{dt} = \frac{1}{e} \frac{\partial J_2}{\partial z} \quad (\text{electron continuity equation})$$

$$\cdot J_2 = \frac{T_e}{e \eta} \frac{\partial}{\partial z} \left(\frac{n}{n_0} - \frac{e \hat{\phi}}{T_e} \right) \quad (\text{Ohm's law})$$

Model equations for resistive drift waves

$$\frac{d}{dt} \nabla_{\perp}^2 \hat{\phi} = - \frac{T_e}{e^2 n_0 \eta R_i} \frac{\partial^2}{\partial z^2} (\hat{\phi} - n)$$

$$n = n_0 e^{-\frac{z}{L}} \quad \frac{d}{dt} (n + \ln n_0) = - \frac{T_e}{e^2 n_0 \eta R_i} (\hat{\phi} - n)$$

Hasegawa-Mima equation:

$$\frac{d}{dt}(\nabla_i^2\phi - n) + \nabla \ln n_0 \times \nabla \phi \cdot \hat{z} = 0$$

$n = \phi$ (Boltzman relation)

$$" \frac{d}{dt}(\nabla_i^2\phi - \phi) + \nabla \ln n_0 \times \nabla \phi \cdot \hat{z} = 0 "$$

Normalization: $e\phi/T = \phi$, $n/n_0 = n$,
 $\omega_{ci}t = t$, $x/p_c \equiv x$

$$(1) (\frac{\partial}{\partial t} - \nabla \phi \times \hat{z} \cdot \nabla) \nabla^2 \phi = \bar{C}_1 (\phi - n) + C_2 \nabla^4 \phi + C_3 \phi$$

$$(2) (\frac{\partial}{\partial t} - \nabla \phi \times \hat{z} \cdot \nabla) (n + \ln n_0) = \bar{C}_1 (\phi - n)$$

$$\bar{C}_1 = - \frac{T_e}{e^2 n_0 \gamma \omega_{ci}} \frac{\partial^2}{\partial z^2}, \quad C_2 = \frac{\mu}{p_c^2 \omega_{ci}}$$

$$C_3 = \frac{\omega_x}{\omega_{ci}} \frac{T_e}{T_i}, \quad \text{if } \omega_x = \frac{v_{Ti}}{8R} \\ = 0 \quad , \quad \text{otherwise.}$$

extension of C_3 model:

gyrofluid model

Conservation laws

energy:

$$\frac{1}{2} \frac{\partial}{\partial t} \int [n^2 + (\nabla \phi)^2] dV$$

$$= -C_1' \int \left(\frac{\partial n}{\partial z} - \frac{\partial \phi}{\partial z} \right)^2 dV - C_2 \int (\nabla^2 \phi)^2 dV$$

$$- \int n (\hat{z} \times \vec{n}) \cdot \nabla \phi dV$$

enstrophy:

$$\frac{1}{2} \frac{\partial}{\partial t} \int (\nabla^2 \phi - n)^2 dV$$

$$= -C_2 \int (n - \nabla^2 \phi) \nabla^2 \phi dV - \int n (\hat{z} \times \vec{n}) \cdot \nabla \phi dV$$

Here $\vec{n} = -\rho_s v \hat{z} \times \vec{n}_0 > 0$. $C_1' = \bar{C}_1 / k_z^2$.

$$(\bar{P} = -\langle n \frac{\partial \phi}{\partial y} \rangle = - \int n (\hat{z} \times \vec{n}) \cdot \nabla \phi dV)$$

Linear Theory of Resistive Drift Waves

$$\omega^2 + i\omega(b + k^2 C_2) - ib\omega^* \\ - [k^2/(1+k^2)] b C_2 = 0,$$

where

$$b = C_1 [1 + k^2]/k^2,$$

$$\omega_* = k_y \bar{x} / (1 + k^2)$$

If C_2 is ignored,

$$\omega = \frac{1}{2} [-ib + i\sqrt{b(1 - 4i\omega_*/b)}]^{1/2}$$

If $b \gg \omega_*$,

$$\omega \approx \omega_* + i\omega_*^2/b$$

(growth rate)

Model Equations for ITG modes

$$(1) \frac{d}{dt}(\phi - \nabla_{\perp}^2 \phi) = -(1 - 2\epsilon_n + K \nabla_{\perp}^2) \frac{\partial \phi}{\partial y} +$$

$$2\epsilon_n \frac{\partial P}{\partial y} - D_n v$$

$$(2) \frac{dP}{dt} = -K \frac{\partial \phi}{\partial y}$$

$$(3) \frac{dv}{dt} = -D_n (\phi + P)$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V}_{\perp} \cdot \nabla \quad , \quad \mathbf{V}_{\perp} = \hat{\mathbf{z}} \times \nabla \phi$$

$$\epsilon_n = L_n/R, \quad K = (1 + \underline{\eta_i}) (T_i/T_e)$$

$$\underline{\eta_i} = L_n/L_T$$

dispersion relation

$$\omega^2(1 + K^2) - \omega k_y (1 - 2\epsilon_n - K k_y^2) + 2k_y^2 K \epsilon_n = 0$$

Drift Alfvén Turbulence (Braginskii equations)

$$(1) \left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla \right) \nabla^2 \phi = D_n J_n - \underline{\chi} (T + n)$$

$$(2) \hat{\beta} \frac{\partial \psi}{\partial t} + \hat{\mu} \left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla \right) J_{\parallel} = D_n (T + n - \phi) - \hat{\mu} \nu [J_n + \frac{0.71}{1.6} (\underline{g_n} + 0.71 J_n)] \text{ (ohm's law)}$$

$$(3) \left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla \right) n = -\omega_n \frac{\partial \phi}{\partial y} + D_n (J_n - U_n) - \chi (T + n - \phi)$$

$$(4) \frac{3}{2} \left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla \right) T = -\frac{3}{2} \omega_t \frac{\partial \phi}{\partial y} + D_{\parallel} (J_n - U_n - \underline{g_n}) - \chi (3.5 T + n - \phi)$$

$$(5) \hat{\mu} \left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla \right) \underline{g_n} = -\frac{5}{2} \Omega_B T_e - \underline{a_0 g_0} - \hat{\mu} \nu \frac{3.5}{1.6} (\underline{g_n} + 0.71 J_n)$$

$$(6) \epsilon_s \left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla \right) U_n = -D_n (n + T) - \mu_n D_n^2 U_n$$

Notations

$$\bar{J}_\parallel = -\nabla_\perp^2 \psi, \quad \mathbf{v}_E = \nabla s \times \nabla \phi$$

$$\nabla_\perp^2 = (\partial/\partial x - S^2 \partial/\partial y)^2 + (\partial/\partial y)^2$$

$$\nabla_N = (\partial/\partial s) - \hat{\beta} \nabla s \times \nabla \phi$$

$$x = w_B [\cos S(\partial/\partial y) + \sin S(\partial/\partial x)]$$

α_L : Landau damping

S^2 : magnetic shear

$$\omega_B = 2L_z/R, \quad \omega_t = L_z/LT.$$

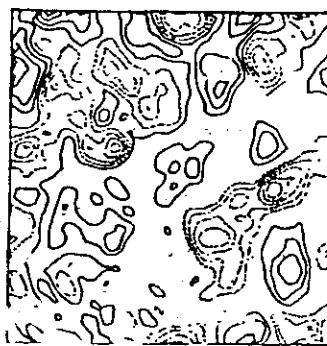
$$\omega_n = L_\perp/L_n$$

$$\alpha_n = \hat{\mu} (V_E L_\perp / G \delta R)$$

M. Wakatani and A. Hasegawa : Phys. Fluids
27 (1984) 611.



ϕ



N

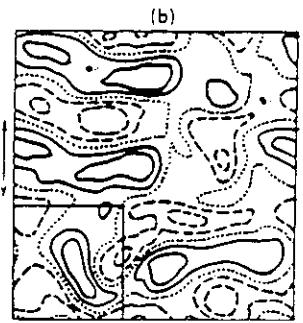
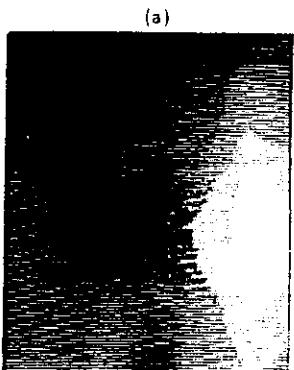
The potential contour (ϕ) and number density contour (N) in x - y plane.

$$\tilde{n} \neq \tilde{\phi}$$

(Resistive Drift Wave Turbulence)



Bahm type diffusion



Structure of density turbulence in the $r-\theta$ plane.
(a) Density fluctuations in the edge regions of the Caltech Research Tokamak measured using an array of 64 probes (see Section 2.2.3); (b) density fluctuations calculated from a theoretical drift wave model due to Waltz. Size of Caltech array corresponds to box shown in lower left hand corner.

図 7 Caltech Research Tokamak で測定された (r, θ) 平面における密度ゆらぎの構造、(a) ランダムアーブロープ測定の結果。(b)Waltz[39]によるドリフト波の乱流シミュレーション結果。(文献 [41] より)

Inertial Range Cascade

Single invariant \rightarrow cascade to small scale

Two invariants \rightarrow dual cascade

Usually invariant with higher power of k goes to small scale.

Example: energy = $\int v_k^2 dV$ and enstrophy = $\int k^2 v_k^2 dV$.

**Problem III : $E_r(r)$ Effects on Resistive Drift Wave and Interchange Turbulence
in a Cylindrical Plasma with Magnetic and Velocity Shear**

$$\begin{aligned}
 (\text{III}-1) \quad & \frac{d}{dt} \nabla_{\perp}^2 \phi = \frac{1}{\nu} \nabla_{\parallel}^2 (n - \Phi) + \nabla n \times \nabla \Omega \cdot \hat{z} + \mu \nabla_{\perp}^2 \Phi \\
 (\text{III}-2) \quad & \frac{d}{dt} (n + \bar{n}) = \frac{1}{\nu} \nabla_{\parallel}^2 (n - \Phi) + \nabla (n - \Phi) \times \nabla \Omega \cdot \hat{z} + D_{\perp} \nabla_{\perp}^2 n
 \end{aligned}$$

Normalizations : $\Phi \equiv e\Phi/T_e$, $n \equiv n/n_0$, $t = \Omega_{it}$,

$$r \equiv r/\rho_s, z \equiv z/\rho_s, \nu \equiv \nu_e/\Omega_o, \mu \equiv \mu/(\rho_s^2 \Omega)$$

$\rho_s = C_s/\Omega_i$. \bar{n} : background density

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} - \nabla \Phi \times \hat{z} \cdot \nabla$$

$$\nabla^2 \perp \equiv \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}$$

$$\nabla_{\parallel} \equiv \frac{\partial}{\partial z} + \nabla \Psi_h \times \hat{z} \cdot \nabla$$

$$\Psi_h = - \int_0^r r \iota(r) dr.$$

Linear eigenmode equation

$$\begin{aligned}
 & \left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{m^2}{r^2} \right) \tilde{\phi} \\
 &= \frac{1}{k_{\parallel}^2 - i\nu(\omega - \omega_g)} \left[\left(1 - \frac{\omega_{*e} - \omega_g}{\omega} \right) k_{\parallel}^2 - i\nu \frac{\omega_g(\omega_{*e} - \omega_g)}{\omega} \right] \tilde{\phi}
 \end{aligned}$$

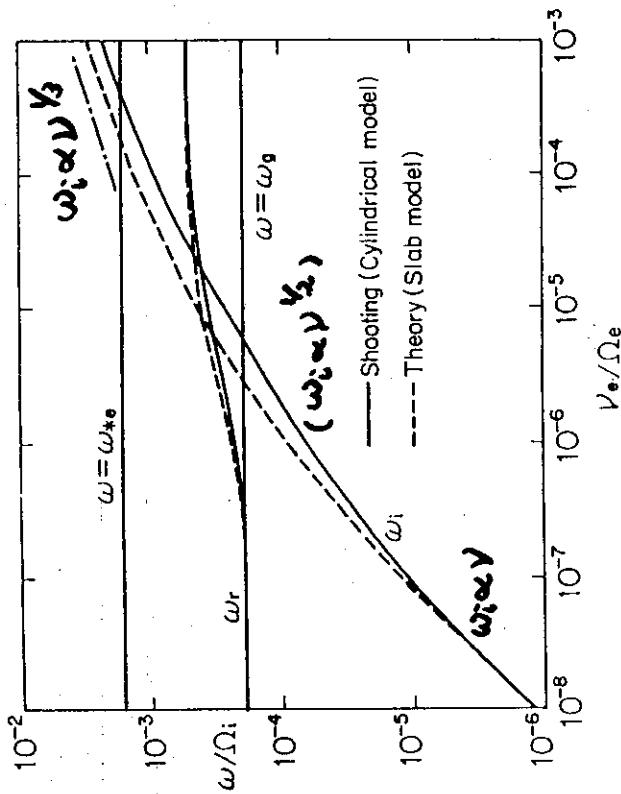
$$\begin{aligned}
 k_{\parallel} &= \frac{\rho_s}{R} (m\iota - n) , \\
 \omega_{*e} &= - \frac{m d \bar{n}}{r dr} \\
 \omega_g &= \frac{m d \Omega}{r dr}
 \end{aligned}$$

Analytic eigenvalue for $\nu \rightarrow 0$,

$$\omega = \omega_g + i \frac{\pi^2}{4} \frac{\nu}{k_{\parallel}^2} \frac{\omega_g(\omega_{*e} - \omega_g)^2}{\omega_{*e} - \omega_g(m^2/r_0^2 + 2)} .$$

(resistive interchange branch)

FKR
 $(\omega \propto k^2 v_s)$



Parameters of nonlinear calculations

$$\rho = 1/40, (\rho_s/a) \quad \epsilon = 1/13, (z_0/R)$$

$$\mu (\equiv \mu / \Omega_i \rho_s^2) = 4 \times 10^{-4}, \quad D_\perp (\equiv D_\perp / \Omega_i \rho_s^2) = 4 \times 10^{-4}$$

background density profile :

$$n(r) = 0.9 \exp(-2r^2/a^2) + 0.1$$

or

$$n(r) = 0.9(1 - r^2/a^2) + 0.1 .$$

rotational transform :

$$\iota(r) = 0.51 + 0.39r^2/a^2 .$$

$$10^{-2} \leq \nu \leq 10^{-4} \text{ (or } 10^2 \lesssim S \lesssim 10^4\text{)}$$

energy :

$$E = \int \frac{1}{2} [n^2 + (\nabla_\perp \phi)^2] dv$$

enstrophy :

$$E_n \quad E_k$$

$$u = \int \frac{1}{2} (\nabla_\perp^2 \phi - n)^2 dv .$$

Adiabatic parameter :

$$\bullet \quad \frac{\Omega_e}{\nu_e} \frac{\rho_s^2}{R^2} \frac{1}{\kappa \rho_s}$$

Hydrodynamic layer width :

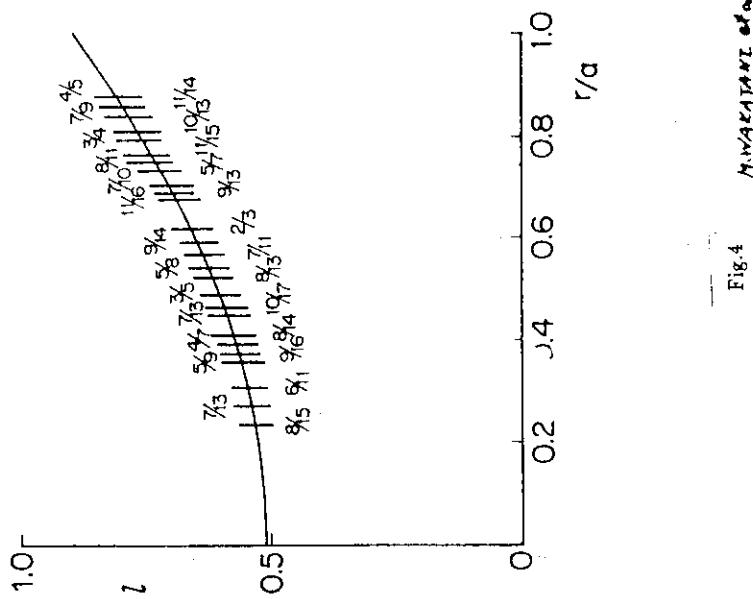
$$\Delta_D/a = \left(\frac{\omega \nu_e}{\Omega_i \Omega_e} \right)^{1/2} \frac{\rho_s}{a} \frac{1}{k l_{||} \rho_s^2}$$

$$\left(\frac{\Delta_D}{a} \sim \frac{0.1}{\sqrt{m}} \text{ for } \frac{\nu}{\Omega_e} = 10^{-4} \right)$$

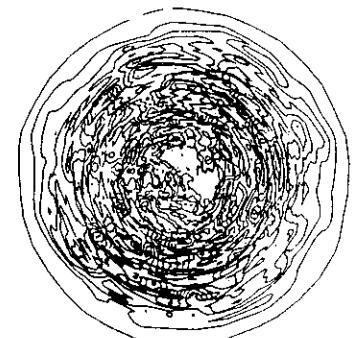
Magnetic Reynolds number :

$$S = \frac{\Omega_e}{\nu_e} \frac{\omega_{pi} a}{c} \frac{a}{R}$$

$$(S \simeq 10^2 - 10^4)$$



TIME=3.0



TIME=4.5



TIME=6.0

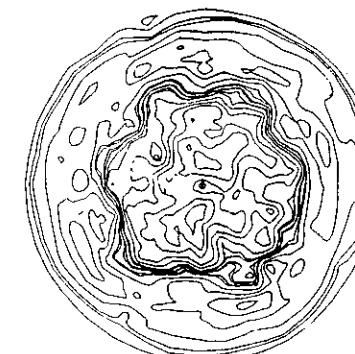


DENSITY CONTOUR

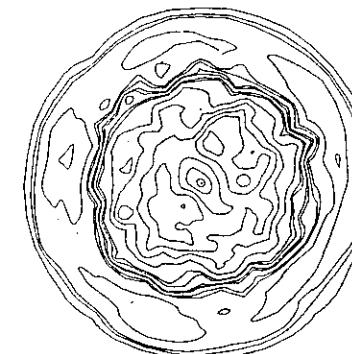
TIME=3.0



TIME=4.5



TIME=6.0



POTENTIAL CONTOUR

"zonal flow"
↓
Improvement of
particle
confinement

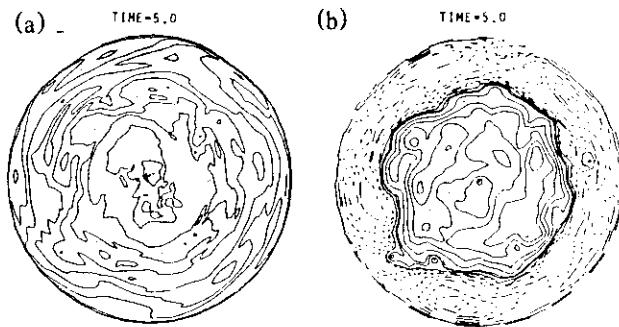


FIG. 1. (a) The density contour and (b) the potential contour from the three-dimensional computer simulation of electrostatic plasma turbulence in a cylindrical plasma with magnetic curvature and shear. In (b) the solid (dashed) lines are for the positive (negative) potential contours. Note the development of closed potential contours near the $\phi=0$ surface.

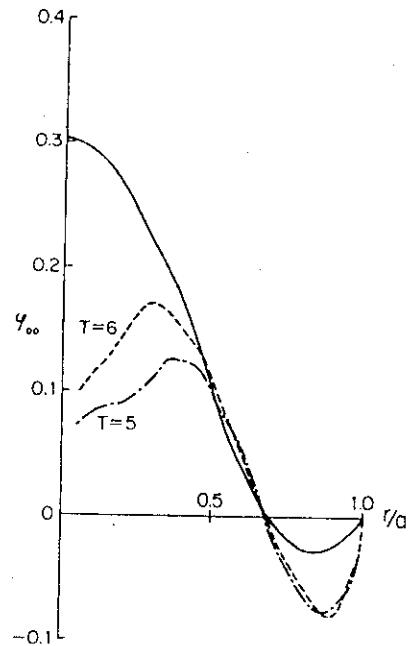


FIG. 2. Profiles of $\phi(r)$ for $m=0, n=0$ mode at two different time steps (dashed and dash-dotted lines) as compared with the predicted profile (solid line) based on the self-organization conjecture. The predicted curve is fitted at $r/a=0.5$.

• Self-organization •

Poloidal Flow Profile Evolution

$$\frac{\partial \langle V_\theta \rangle}{\partial t} = -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \langle \tilde{v}_r \tilde{v}_\theta \rangle) - \frac{1}{\rho_0 \mu_0} r^2 \langle \tilde{B}_r \tilde{B}_\theta \rangle$$

$$+ \mu \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial}{\partial r} (r \langle V_\theta \rangle) \right]$$

Reynolds Stress:

$$S_{ij} \equiv \langle \tilde{v}_i \tilde{v}_j \rangle - \frac{1}{\rho_0 \mu_0} \langle \tilde{B}_i \tilde{B}_j \rangle$$

ES part EM part

$$\frac{d}{dt} \int_0^a \langle V_\theta \rangle r^2 dr = 0.$$

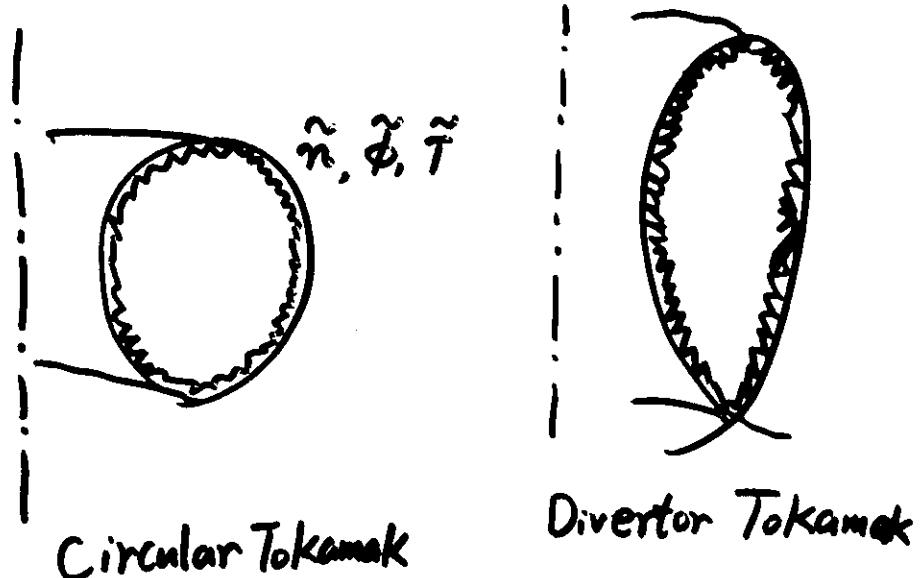
This means Reynolds stress does not create momentum; however, it does redistribute momentum along the radial direction.

Note: $\langle \cdot \rangle$ denotes averaging over magnetic surface.

Summary II

- Resistive drift turbulence is a candidate for explaining edge turbulence observed in medium size tokamaks.
- Shear flow (or zonal flow) is generated by Reynolds stress due to resistive drift interchange turbulence.

Edge Turbulence



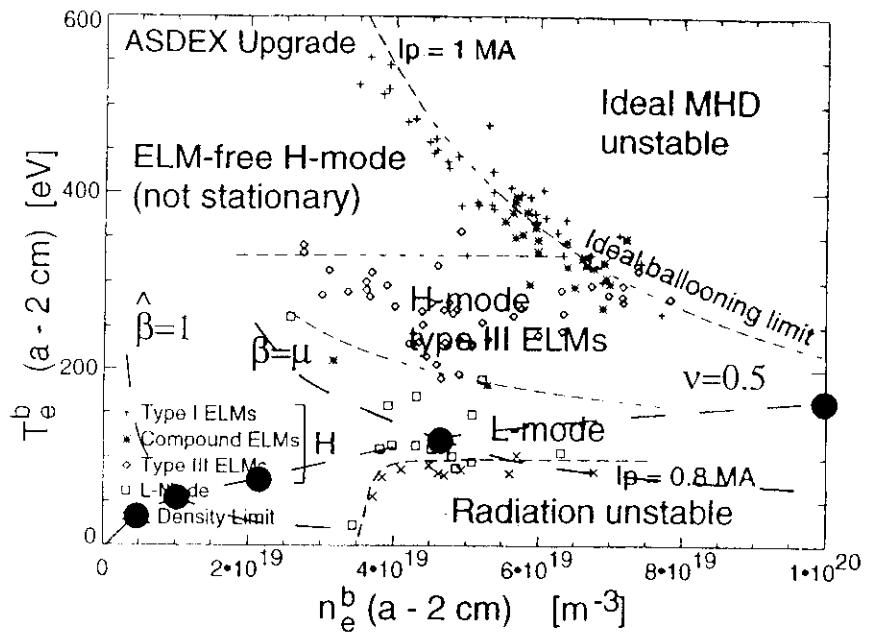
Circular Tokamak

Divertor Tokamak

Origin: resistive drift turbulence.
Is "finite β effect" stabilizing or destabilizing?

Coupling between drift wave mode,
shear Alfvén, and the gyrokinetic
drift Alfvén modes.

Laboratory plasma experiment
(high density collisional plasmas)



Cylindrical plasma

$$\beta \approx 0.1 \text{ T}, \quad (\beta r_0) \lesssim 0.1$$

Wave characteristics were studied for resistive drift-Alfvén instabilities. Experimental results were compared with local linear theory.

Here we study global mode structures by solving eigenmode equations.

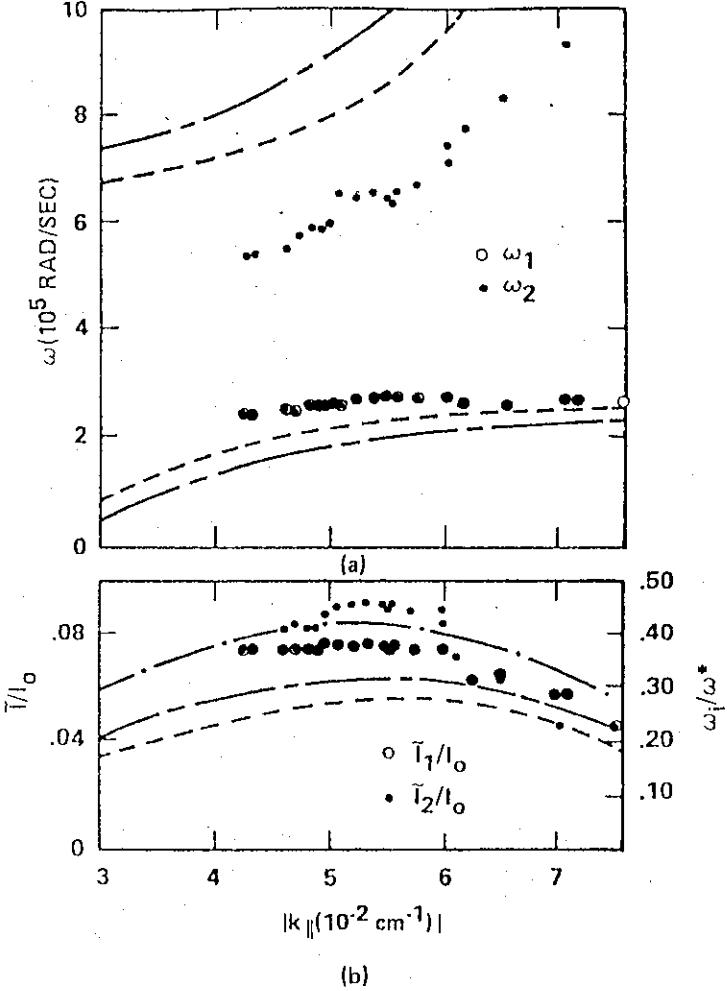


FIG. 9. (a) Dispersion of the drift-Alfvén wave showing the coupling near $\omega^* \approx k_z V_A$ compared with the theoretical dispersion calculated from Eq. (17) for $n_0 = 5 \times 10^{13}$ cm $^{-3}$, $B_0 = 1.6$ kG, $T_e = 4$ eV, and $k_\perp = [(1/n_0)/(dn_0/dr)] = 1.25$ cm $^{-1}$. The dashed--line represents the theoretical frequency and growth rate for the case of no axial current. Only the lower branch is unstable. (b) The segmented lines --- and -·- represent theoretical values for the case of no axial current ($v_{ez0} = V_A$). Referring to the growth rate ω_t/ω^* diagram, the --- curve is 0.2 times ω_{t1}/ω^* while the -·- curve is ω_{t2}/ω^* .

Two-Fluid Equations for Resistive Drift-Alfvén Wave

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{V}_\alpha) = 0 \quad (1)$$

$$m_\alpha n_\alpha \frac{d\mathbf{V}_\alpha}{dt} = n_\alpha q_\alpha (\mathbf{E} + \mathbf{V}_\alpha \times \mathbf{B}) - \nabla P_\alpha + \mathbf{R}_{\alpha\beta} \quad (2)$$

$$\mathbf{R}_{\alpha\beta} = -m_\alpha n_\alpha \nu_{\alpha\beta} (\mathbf{V}_\alpha - \mathbf{V}_\beta) \quad (3)$$

$$\mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t}, \quad (4)$$

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (5)$$

$$\mu_0 \mathbf{j} = \nabla \times \mathbf{B} \quad (6)$$

$$\mathbf{V}_{de} = -\frac{k_B T_e}{e B_0} \frac{1}{n_0} \frac{dn_0}{dr} \hat{\theta}, \quad (7)$$

$$\underline{T_e = \text{const.}}, \quad \underline{T_i \simeq 0} \quad (8)$$

Linearized Equations

$$i(\omega - \omega_*) \frac{\tilde{n}}{n_0} = \frac{1}{n_0} \nabla_{\perp} \cdot (n_0 \tilde{\mathbf{V}}_{e\perp}) + \nabla_{\parallel 0} \tilde{V}_{e\parallel} \quad (9)$$

$$+ \frac{\tilde{\mathbf{B}}_{\perp}}{B_0 n_0} \cdot \nabla_{\perp} \cdot (n_0 \bar{V}_{e\parallel}) + \frac{1}{n_0} \bar{V}_{e\parallel} \cdot \nabla_{\parallel 0} \tilde{n}$$

$$-k_B T_e \nabla_{\perp} \tilde{n} - n_0 e (\tilde{\mathbf{E}}_{\perp} + \tilde{\mathbf{V}}_{e\perp} \times \mathbf{B}_0 + \mathbf{V}_{de} \times \tilde{\mathbf{B}}_{\perp}) \quad (10)$$

$$- \tilde{n} e (\mathbf{V}_{de} \times \mathbf{B}_0) = 0$$

$$-k_B T_e \nabla_{\parallel 0} \tilde{n} - k_B T_e \frac{\tilde{\mathbf{B}}_{\perp}}{B_0} \cdot \nabla_{\perp} n_0 - n_0 e \tilde{\mathbf{E}}_{\parallel} \quad (11)$$

$$- n_0 m_e \nu_{ci} \tilde{V}_{ez} = 0$$

$$i\omega \frac{\tilde{n}}{n_0} = \frac{1}{n_0} \nabla_{\perp} \cdot (n_0 \tilde{\mathbf{V}}_{i\perp}), \quad (12)$$

$$n_0 m_i \frac{\partial \tilde{\mathbf{V}}_{i\perp}}{\partial t} = n_0 e (\tilde{\mathbf{E}}_{\perp} + \tilde{\mathbf{V}}_{i\perp} \times \mathbf{B}_0). \quad (13)$$

$$\underline{j_{z0} = -n_0 e \tilde{V}_{ez}} \quad (14)$$

$$\tilde{j}_z = -\tilde{n} e \tilde{V}_{ez} - n_0 e \tilde{V}_{ez}. \quad (15)$$

Eigenmode Equations

$$(\rho_s^2 \omega_A^2 \nabla_{\perp}^2 + \omega \omega_1) \frac{e \tilde{A}}{T_e} + \frac{\omega_3}{j_{z0}} \frac{d j_{z0}}{dr} \frac{m}{r} \tilde{A} = \omega_1 k_{\parallel} \frac{e \tilde{\phi}}{T_e} \quad (16)$$

$$(\omega_1 + b\omega) k_{\parallel} \frac{e \tilde{\phi}}{T_e} - \omega \omega_1 \frac{e \tilde{A}}{T_e} - \frac{\omega_2}{j_{z0}} \frac{d j_{z0}}{dr} \frac{m}{r} \tilde{A}$$

$$= \rho_s^2 \omega \frac{k_{\parallel}}{r n_0} \frac{d}{dr} \left\{ r n_0 \frac{d}{dr} \left(\frac{e \tilde{\phi}}{T_e} \right) \right\} \quad (17)$$

$$\omega_1 = \frac{\omega - \omega_*}{1 - i\omega/\nu_{\parallel}}$$

$$\omega_2 = \frac{i\omega k_{\parallel} \tilde{V}_{e\parallel}/\nu_{\parallel}}{1 - i\omega/\nu_{\parallel}}$$

$$\omega_3 = \frac{k_{\parallel} \tilde{V}_{e\parallel}}{1 - i\omega/\nu_{\parallel}}$$

$$\nu_{\parallel} = k_{\parallel}^2 V_{Te}^2 / \omega_{ci}$$

Eigenmode equations are solved with a shooting method.

B.C.

$$\tilde{A}(r) = a_0 r^m$$

$$\tilde{\phi}(r) = b_0 r^n$$

$$\tilde{A}(a) = 0$$

$$\tilde{\phi}(a) = 0$$

(a : plasma radius)

Parameters

$$n(r) = n_0 \exp[-4(r/a)^2]$$

$$T_e = \text{const.}$$

$$a$$

$$B_0$$

$$(m, k_z)$$

Local Dispersion Relation (for $J_{z0} = 0$)

$$(-b\omega_A^2 + \omega\omega_1)\tilde{A} = \omega_1 k_z \tilde{\phi}, \quad (18)$$

$$(\omega_1 + b\omega)k_z \tilde{\phi} = \omega\omega_1 \tilde{A}, \quad (19)$$

where $d\omega_*/dr = 0$, $\bar{V}_{ez} = 0$.

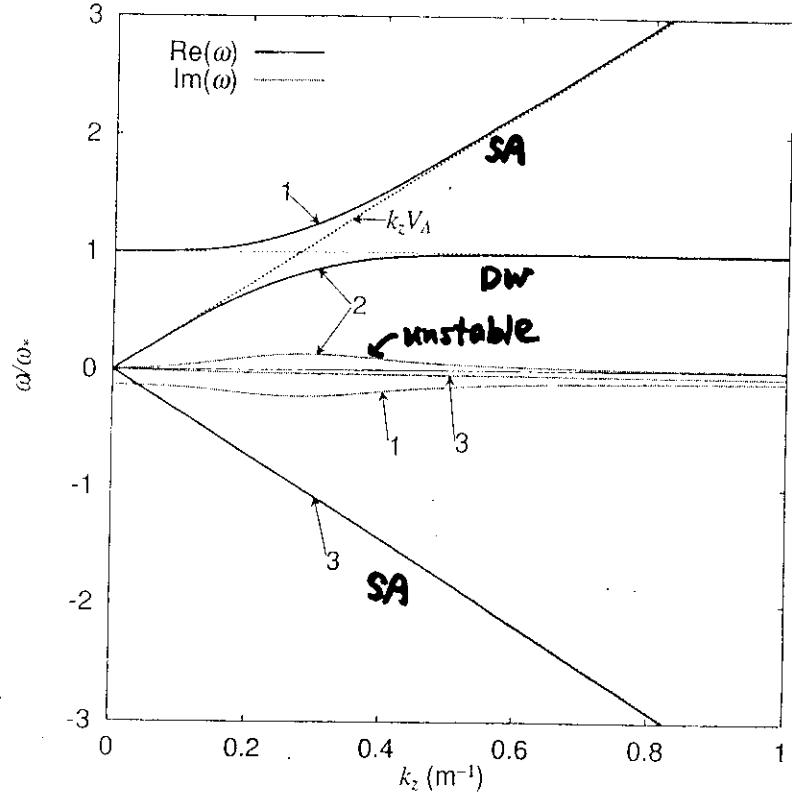
$$(\omega^2 - \omega_A^2)(\omega - \omega^*) - b\omega_A^2 \omega \left(1 - i\frac{\omega}{\nu_{||}}\right) = 0, \quad (20)$$

$$\text{where } \omega_A = k_z V_A, \underline{b = m^2 \rho_s^2 / r^2}, \\ \rho_s^2 = T_e / m_i \omega_{ci}^2.$$

Solution of (20),

$$\omega = \pm\omega_A, \quad \omega = \omega_* + ib\omega_*^2/\nu_{||} \quad (21)$$

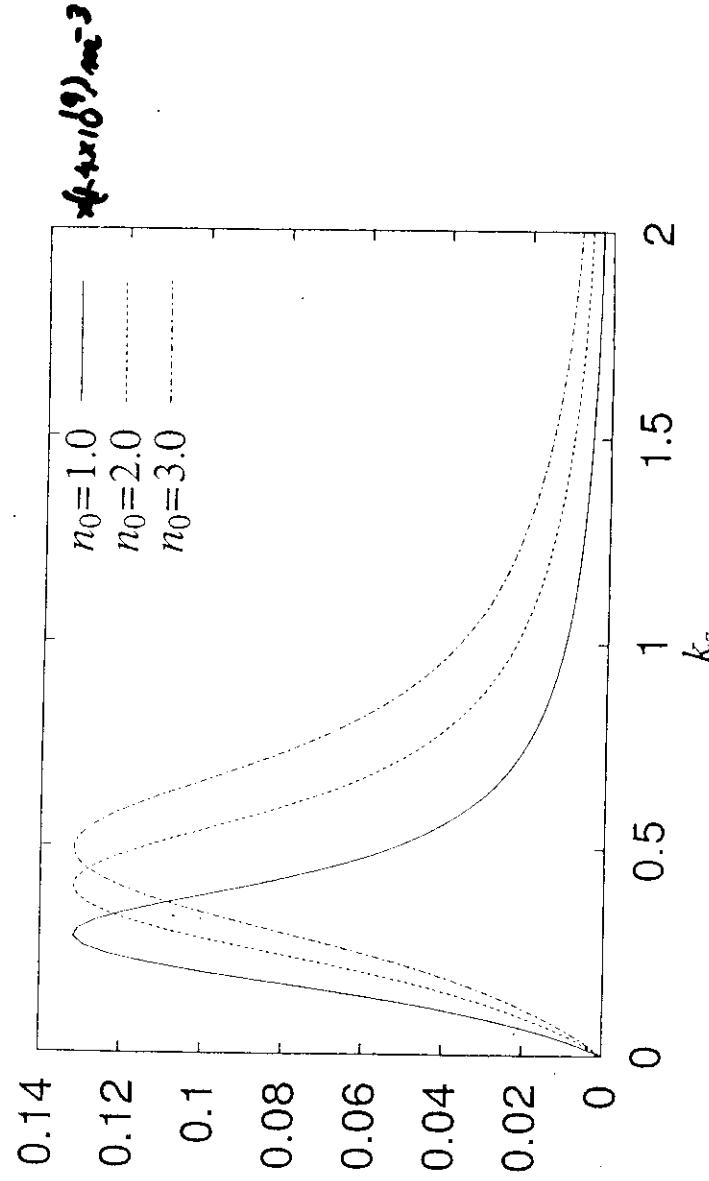
$$\omega = k_z^2 V_{Te}^2 / \nu_{ei}$$



$$B_0 = 0.1 \text{ T}, \bar{T}_e = 100 \text{ eV}, n_0 = 1.4 \times 10^{19} \text{ m}^{-3}$$

$$m=2, \gamma_a = 0.7, q = 0.1/m$$

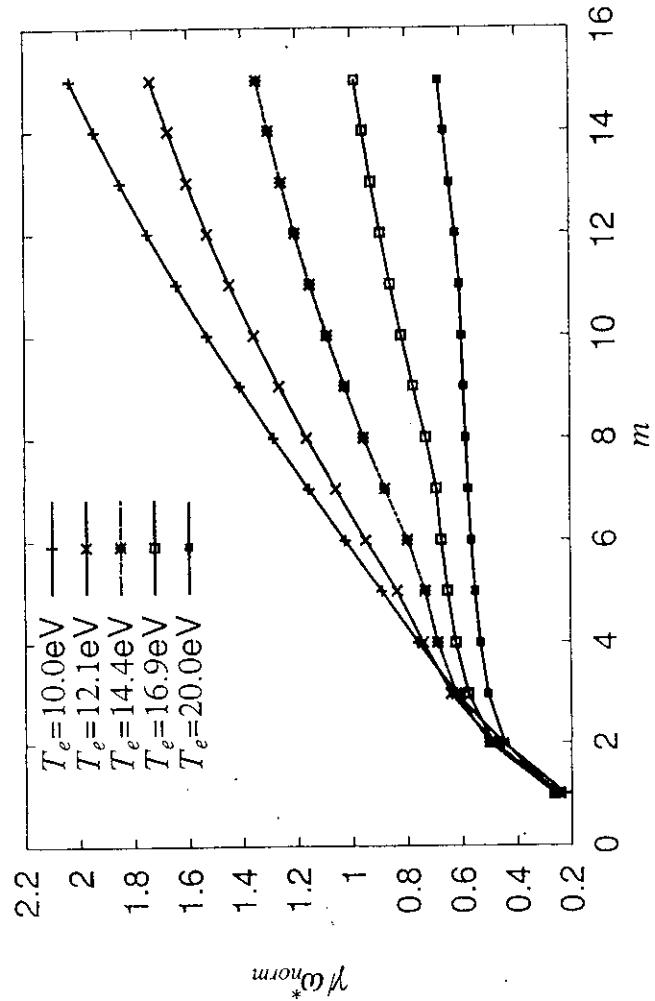
Local analysis



$*\omega/\lambda$

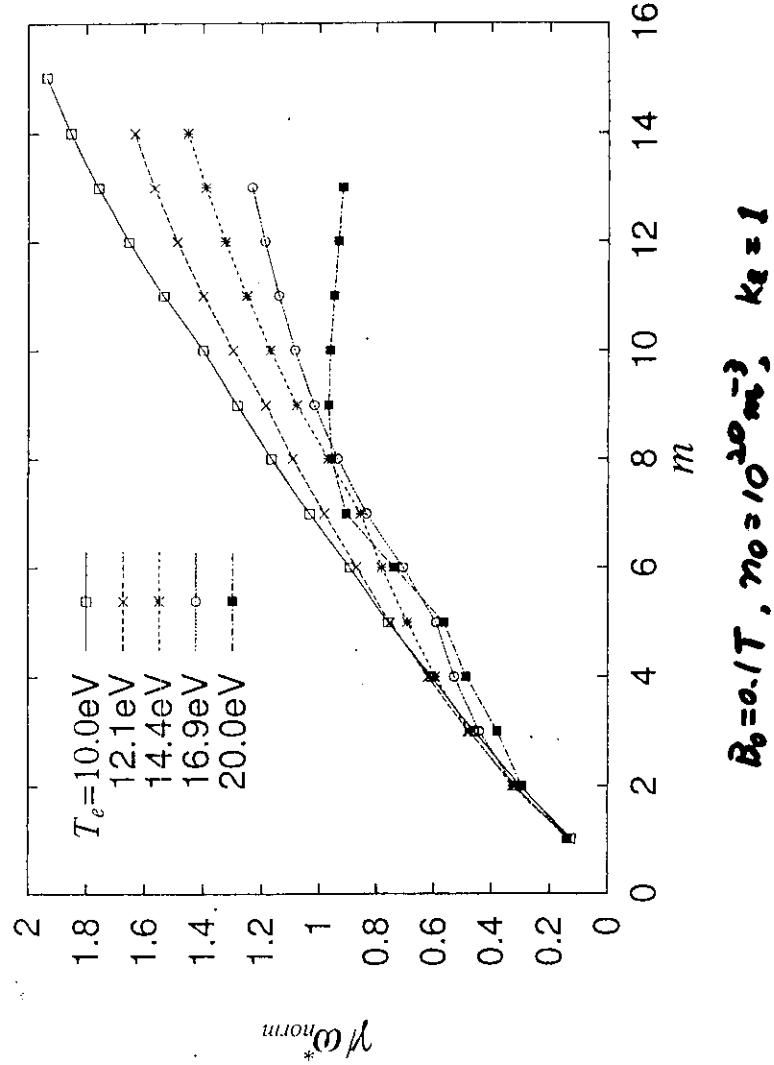
$$\text{Bo} = 0.1/T_e, T_e = 100 \text{ eV}, m = 2, \gamma_a = 0.7, q = 0.1/m$$

Local analysis



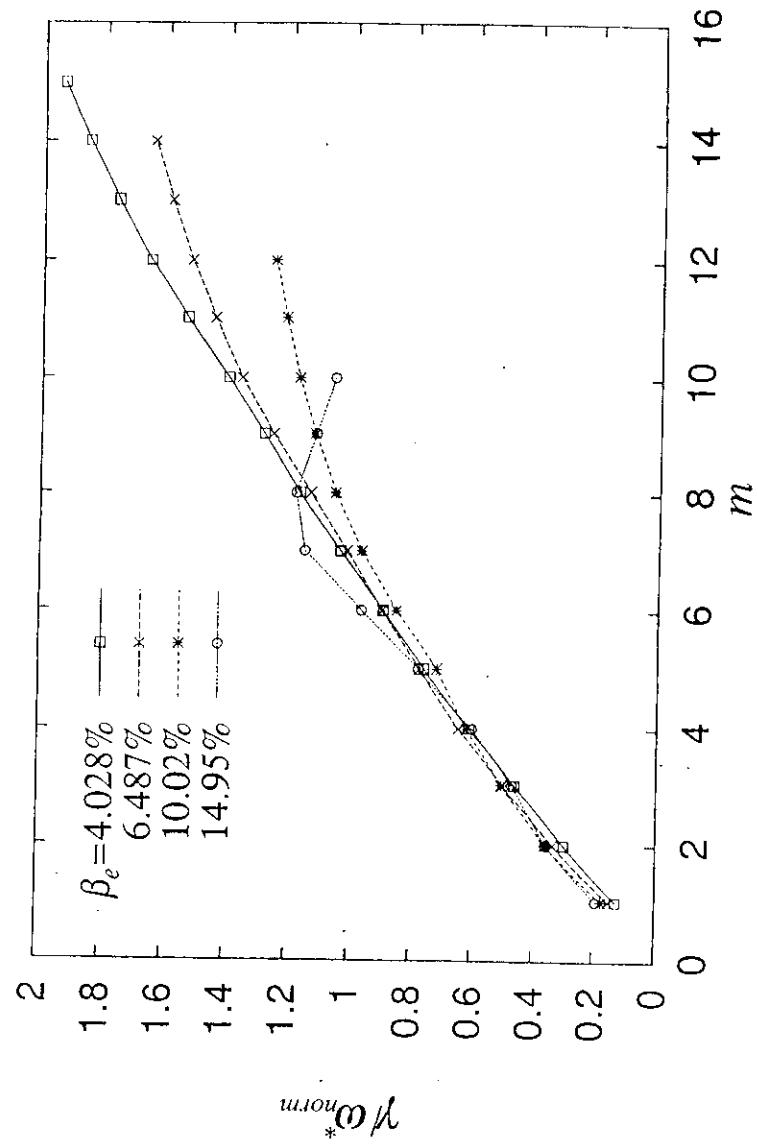
$B_0 = 0.1 T$, $k_z = 1$, $n_0 = 1.4 \times 10^{19} \text{ m}^{-3}$, $a = 0.1 \text{ m}$

Non-local model

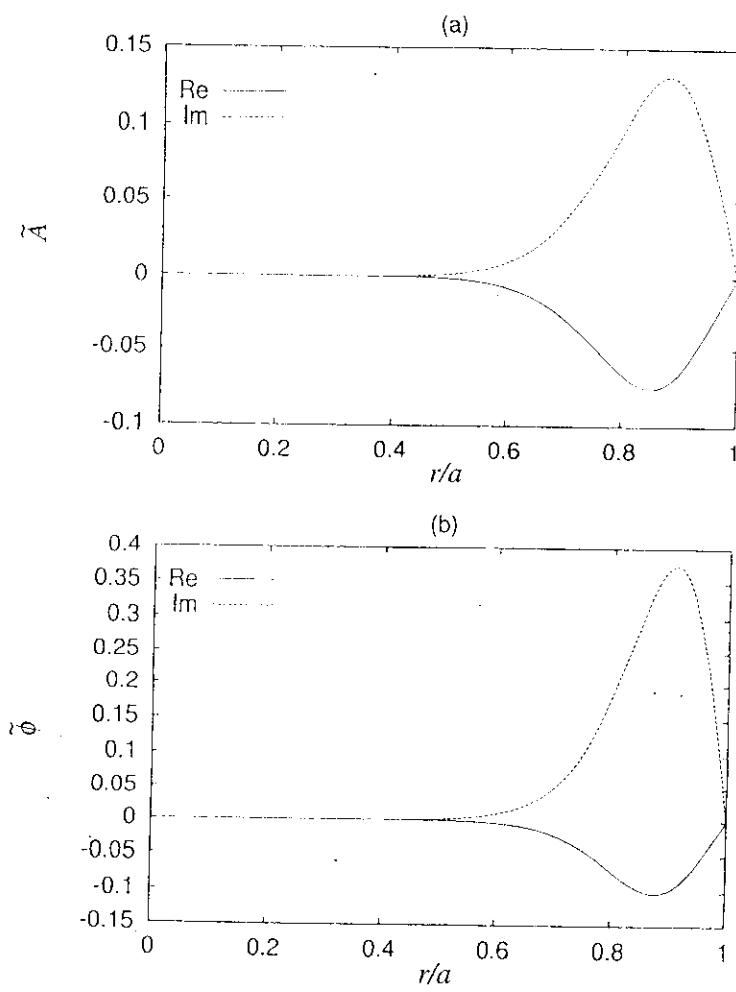


$B_0 = 0.1 T$, $n_0 = 10^{20} \text{ m}^{-3}$, $k_z = 1$

Non-local model

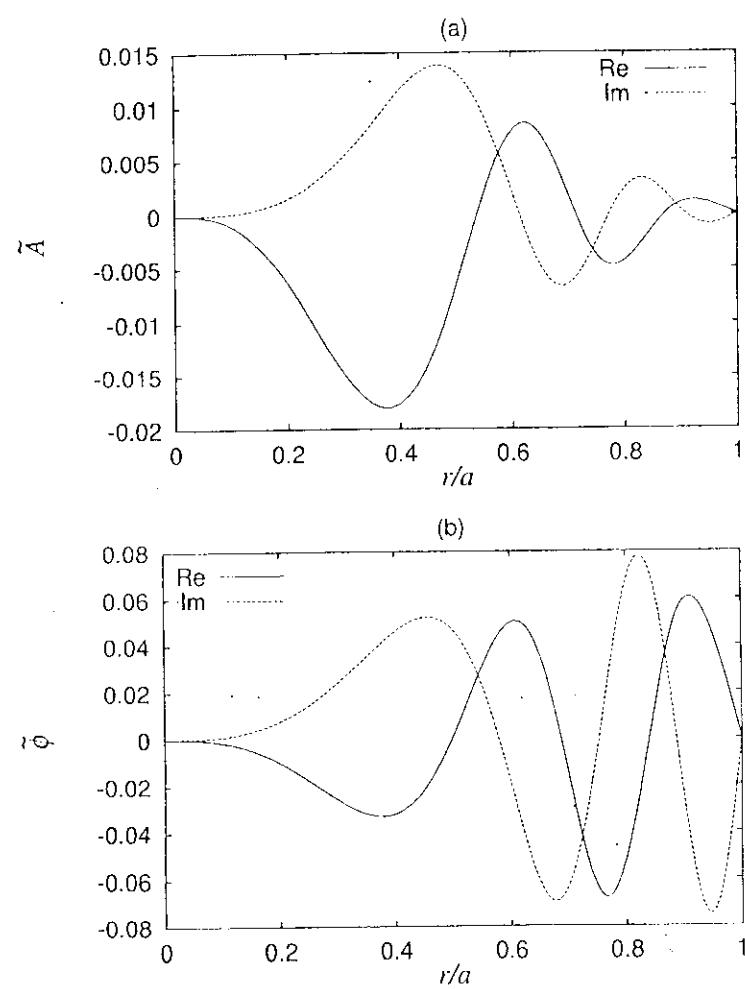


$B_0 = 0.1 \text{ T}, k_z = 1, a = 0.1 \text{ m}$



$m=10$

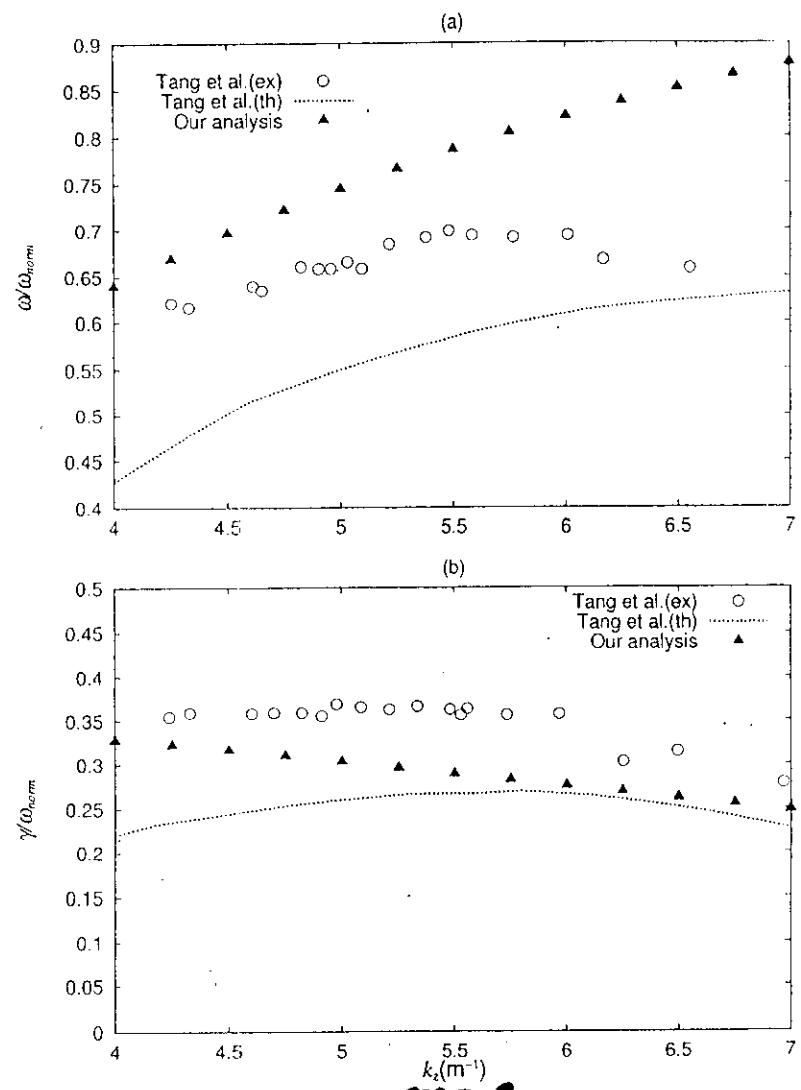
$B_0 = 0.1 \text{ T}, k_z = 1, T_e = 10 \text{ eV}$
 $a = 0.1 \text{ m}, n_0 = 10^{20} \text{ m}^{-3}$



$m=3$

$B_0=0.1T, K_2=1, T_e=10\text{eV}$.

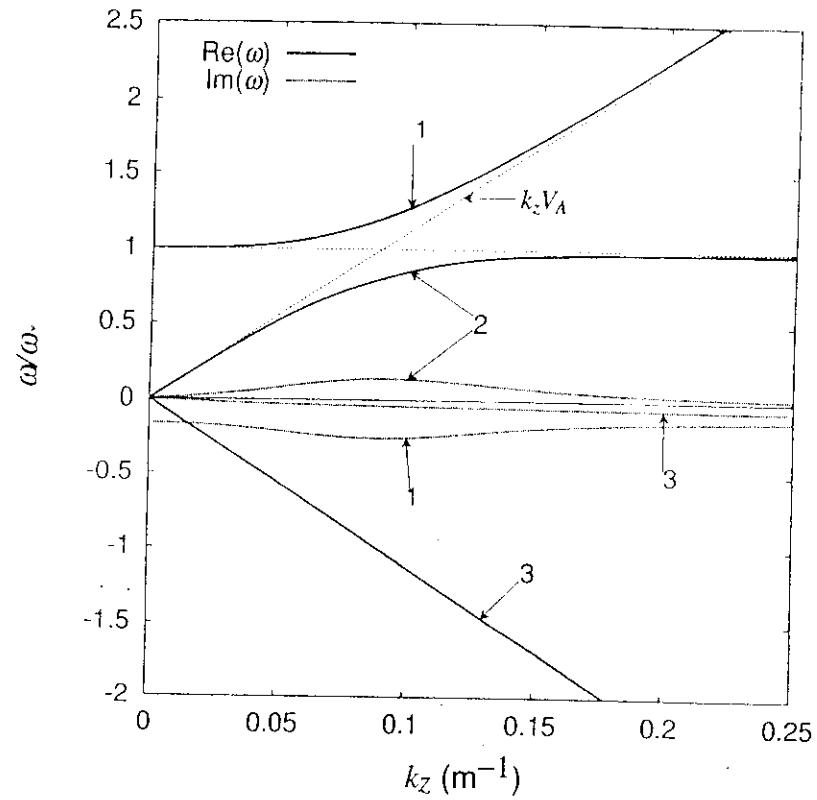
$a=0.1m, n_0=10^{20}\text{m}^{-3}$



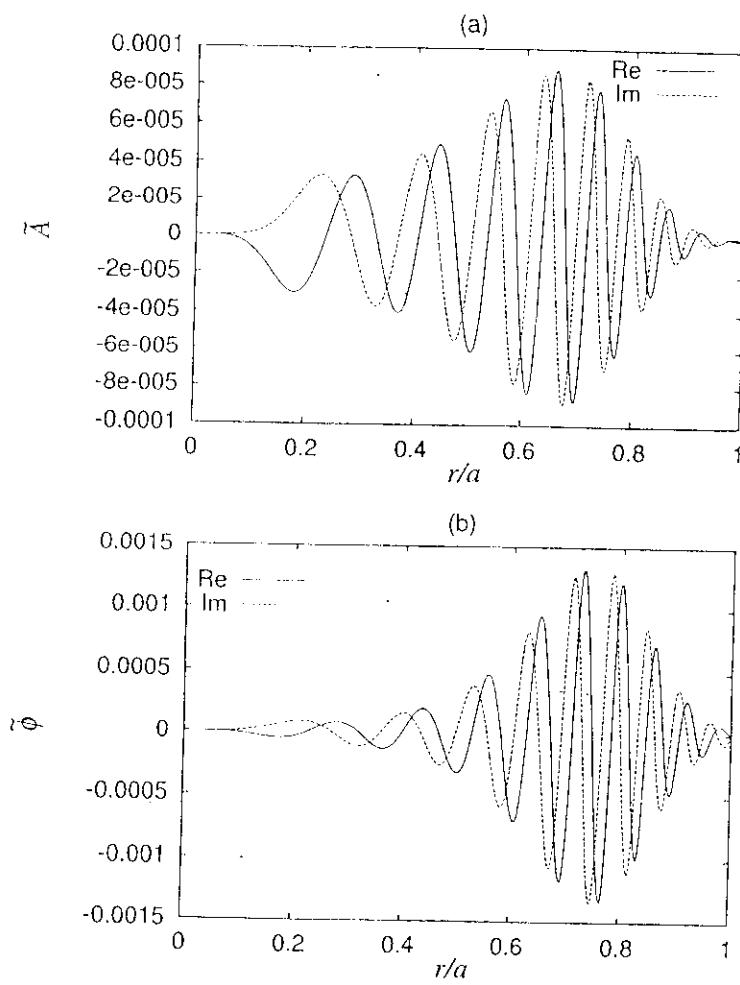
$m=2$

$B_0=0.16T, T_e=T_i=4\text{eV}, a=0.018m$

$n_0=7\times 10^{19}\text{m}^{-3}$

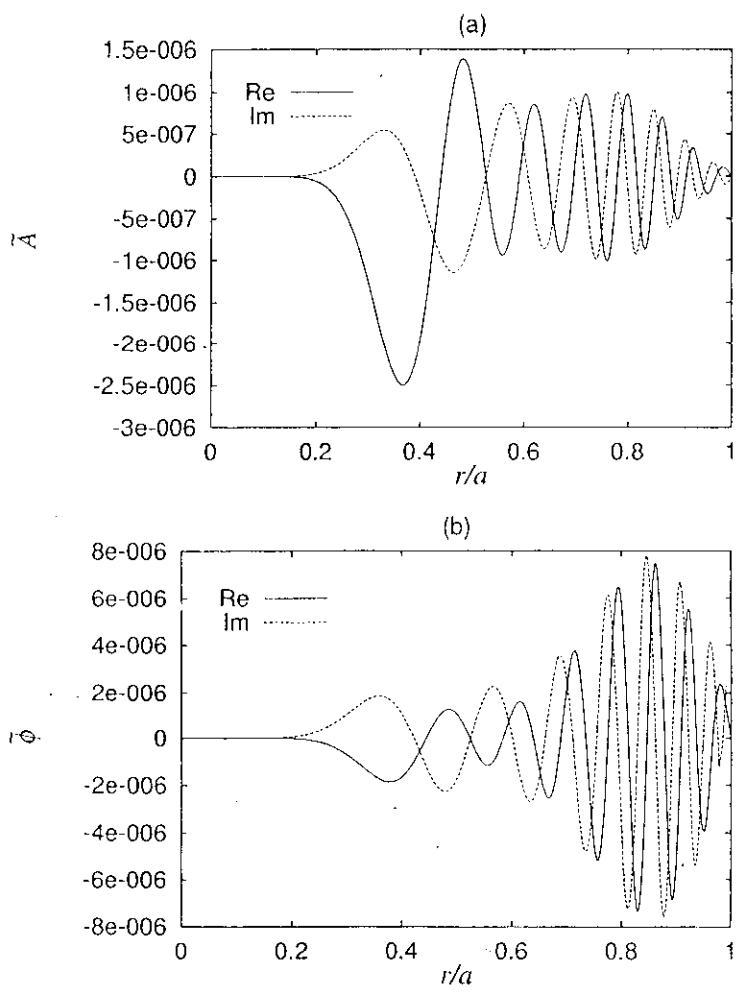


$$B_0 = 1 \text{ T}$$



$$m=5$$

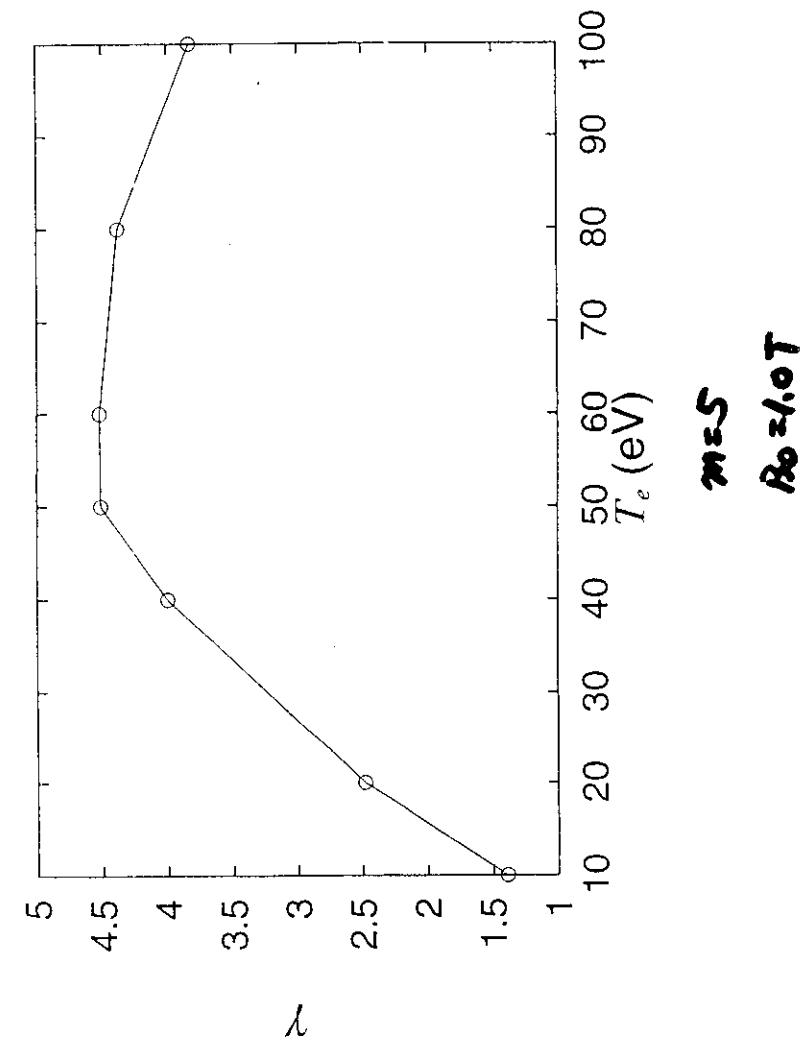
$$\begin{aligned} k_z &= 0.1, \quad T_e = 50 \text{ eV}, \quad a = 0.1 \text{ m} \\ n_e &= 10^{20} \text{ m}^{-3}, \quad B_0 = 1.0 \text{ T} \end{aligned}$$



$m=10$

$k_z=0.1$, $T_e=50$ eV, $a=0.1m$

$n_0=10^{20} m^{-3}$, $\beta_0=1.0T$



Concluding Remarks

1. Resistive drift-Alfvén(RDA) instabilities may be relevant to the edge physics of tokamaks including the L-H transition.
2. When T_e or β_e is increased, growth rates of RDA instabilities decrease.
3. ∇T_e , ∇T_i and J_{\parallel} effects will be studied soon.

