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Joint INFM - the Abdus Salam ICTP School on
"Magnetic Properties of Condensed Matter Investigated by Neutron Scattering and Synchrotron Radiation Techniques"

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SOLUTIONS TO EXERCISES

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1. ψ, ψ' continuous at interface $y = y_0$

$$(\psi' = \frac{\partial \psi}{\partial y})$$

This condition can be met if:

$$a_i + b_i = a_j + b_j$$

$$q_i (a_i - b_i) = q_j (a_j - b_j)$$

which we can rewrite as:

$$D(q_i) \begin{bmatrix} a_i \\ b_i \end{bmatrix} = D(q_j) \begin{bmatrix} a_j \\ b_j \end{bmatrix}$$

where the matrices are given by

$$\begin{bmatrix} 1 & 1 \\ q_i & -q_i \end{bmatrix} = D(q_i)$$

2. put $\left.\begin{array}{l} a_i = 1, \; b_i = r \\ a_j = t, \; b_j = 0 \end{array}\right\}$ as in lecture

put $D^{-1}(q_1) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 1 \\ +q_1 & -q_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \quad a - bq_1 = 1 \qquad a + bq_1 = 0$$

$$c + dq_1 = 0 \qquad c - dq_1 = 1$$

$$a = \frac{1}{2}, \quad +2bq_1 = 1 \Rightarrow b = +\frac{1}{2q_1}$$

$$c = \frac{1}{2}, \quad +2dq_1 = -1 \Rightarrow d = -\frac{1}{2q_1}$$

$$\therefore \quad M = D^{-1}(q_1)\,D(q_2)$$

$$= \frac{1}{2}\begin{bmatrix} 1 & +\frac{1}{q_1} \\ 1 & -\frac{1}{q_1} \end{bmatrix}\begin{bmatrix} 1 & 1 \\ q_2 & -q_2 \end{bmatrix}$$

$$M = \frac{1}{2}\begin{bmatrix} \left(1+\frac{q_2}{q_1}\right) & \left(1-\frac{q_2}{q_1}\right) \\ \left(1-\frac{q_2}{q_1}\right) & \left(1+\frac{q_2}{q_1}\right) \end{bmatrix}$$

$$r = \frac{q_1 - q_2}{q_1 + q_2}$$

$$q_2 = (q_1^2 - q_c^2)^{1/2} \approx q_1\left(1 - \frac{1}{2}\frac{q_c^2}{q_1^2}\right)$$

$$\therefore \quad r \approx \frac{q_1 - q_1\left(1 - \frac{1}{2}\frac{q_c^2}{q_1^2}\right)}{q_1\left(1 - \frac{1 q_c^2}{2q_1^2}\right) + q_1}$$

$$\approx \frac{\frac{1}{2} q_1 \cdot \frac{q_c^2}{q_1^2}}{2q_1} \approx \frac{1}{4}\frac{q_c^2}{q_1^2}$$

$$\Rightarrow \quad R(q) \text{ as } q \to \infty \text{ is } \sim \frac{q_c^2}{4q^2}$$

2)

3.

$$\psi_i(y = y_0) = \begin{pmatrix} a_i \\ b_i \end{pmatrix}$$

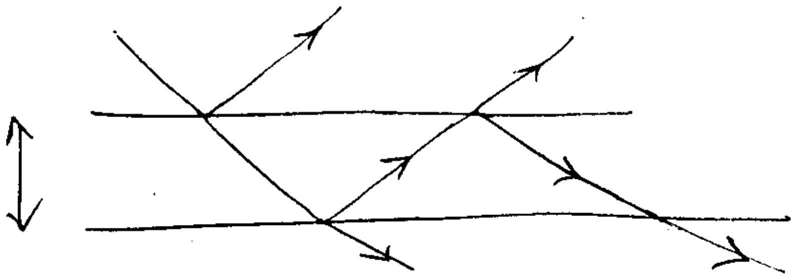
$$\psi_i(y = y_0 + d) = \begin{pmatrix} a_i e^{iq_i d} \\ b_i e^{-iq_i d} \end{pmatrix}$$

$$\therefore \begin{pmatrix} e^{-iq_j d} & 0 \\ 0 & e^{iq_j d} \end{pmatrix} \begin{pmatrix} a_j e^{iq_j d} \\ b_j e^{-iq_j d} \end{pmatrix}$$

$$P\psi_j(y_0+d) = \begin{pmatrix} a_j \\ b_j \end{pmatrix} = \psi_j(y_0)$$

where $P = \begin{pmatrix} e^{-iq_j d} & 0 \\ 0 & e^{iq_j d} \end{pmatrix}$

4.



$$r_{123} = r_{12} + t_{12} r_{23} t_{21} \phi_2 \big(1 + (r_{21} r_{23} \phi_2) + (r_{21} r_{23} \phi_2)^2 + \ldots \big)$$

from lecture. $\phi_2 = 2iq_2 d$

sum of geometric progression is

$$\frac{1 + (r_{21} r_{23} \phi_2)^N}{1 + (\quad)}$$

$$\therefore \quad r_{123} = r_{12} + \frac{t_{12} r_{23} t_{21} \phi_2}{1 + (r_{21} r_{23} \phi_2)}$$

$$= \frac{r_{12}(1 + r_{21} r_{23} \phi_2) + t_{12} r_{23} t_{21} \phi_2}{(1 + r_{21} r_{23} \phi_2)}$$

$$= \frac{r_{12} + r_{23}(r_{12} r_{21} + t_{12} t_{21}) \phi_2}{1 + r_{21} r_{23} \phi_2}$$

but $r_{12}r_{21} + t_{12}t_{21}$

$$r_{12} = \frac{q_1 - q_2}{q_1 + q_2}, \quad t_{12} = \frac{2q_1}{q_1 + q_2}$$

$$r_{12}r_{21} + t_{12}t_{21} = \frac{(q_1 - q_2)(q_2 - q_1) + 4q_1 q_2}{(q_1 + q_2)^2} = 1$$

$$\therefore \quad r_{123} = \frac{r_{12} + r_{23}\phi_2}{1 + r_{12}r_{23}\phi_2}$$

//<

last part of 4.

put $\phi_2 = \exp(2iq_2 d)$

$$r_{123} = \frac{r_{12} + r_{23}\phi_2}{1 + r_{12}r_{23}\phi_2}$$

$$r_{12} = \frac{q_1 - q_2}{q_1 + q_3}, \quad r_{23} = \frac{q_2 - q_3}{q_2 + q_3}$$

$$r_{13} = \frac{q_1 - q_3}{q_1 + q_3}$$

$$\therefore \quad \frac{q_1 - q_3}{q_1 + q_3} + \Delta = r_{123}$$

put $\phi_2 \simeq 1 + 2iq_2 d = 1 + i\delta$

$$r_{123} = \frac{\left(\frac{q_1 - q_2}{q_1 + q_2}\right) + \left(\frac{q_2 - q_3}{q_2 + q_3}\right)(1 + 2iq_2 d)}{1 + \left(\frac{q_1 - q_2}{q_1 + q_2}\right)\left(\frac{q_2 - q_3}{q_2 + q_3}\right)(1 + 2iq_2 d)}$$

but $r_{13} = \dfrac{r_{12} + r_{23}}{1 + r_{12}r_{23}}$ since $\phi_2 = 1$ for $d \to 0$

$$r_{123} = [r_{12} + r_{23}(1+i\delta)] / [1 + r_{12}r_{23}(1+i\delta)]$$

$$r_{123} \approx r_0 + \frac{i\delta r_{23}}{(1+r_{12}r_{23})} \left\{ 1 - \frac{i\delta r_{12} r_{23}}{(1+r_{12}r_{23})} \right\}$$

$\therefore$ first order correction is

$$\Delta = \frac{2iq_2 d r_{23}}{1 + r_{12}r_{23}}$$

i.e. the correction is imaginary in r to first order; real part $\sim$ Δ^2

last part, concerning roughness:

$$R = R_0 e^{-4q_1 q_2 \sigma^2}$$

$$\Rightarrow r = r_0 e^{-2q_1 q_2 \sigma^2}$$

$$\approx r_0 \exp(-2q^2 \sigma^2)$$

assuming $q_2 \approx q_1 \approx q$

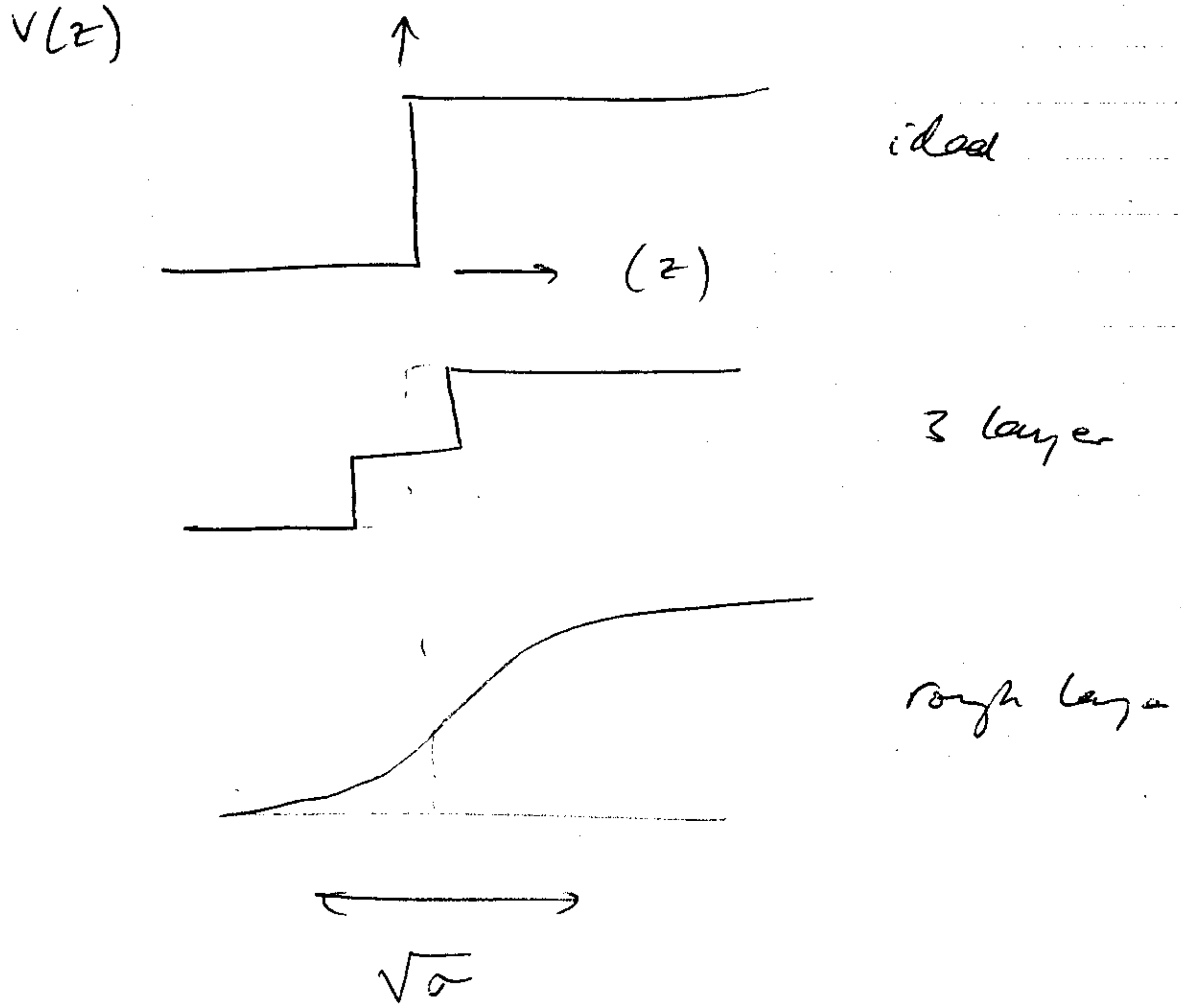
$$r \approx r_0(1 - 2q^2\sigma^2)$$

but the correction in $r \sim \Delta^2 \Rightarrow$

$$r_0 - r \approx \cancel{-\frac{4q^2 d^2 r_{23}^2}{(1 + r_{12} r_{23})^2}} \qquad \Delta^2 \propto q_2^2 \propto q^2$$

which is equivalent

Physically this is equivalent to the following picture for the interface scattering potential:



5. This is an extension of the matrix equation

$$D(q_1)\psi_1 = D(q_2)\psi_2$$

which operates on states of the form

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix}, \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

to the fully spin polarised version

If there is no spin mixing (when magnetic moments are aligned) then the equation is just

$$\begin{bmatrix} D(q_1^{\uparrow}) & 0 \\ 0 & D(q_1^{\downarrow}) \end{bmatrix} \begin{bmatrix} \psi_1^{\uparrow} \\ \psi_1^{\downarrow} \end{bmatrix}$$

$$= \begin{bmatrix} D(q_2^{\uparrow}) & 0 \\ 0 & D(q_2^{\downarrow}) \end{bmatrix} \begin{bmatrix} \psi_2^{\uparrow} \\ \psi_2^{\downarrow} \end{bmatrix}$$

when there is a change in the quantisation direction at the interface

$$\psi_1^{\uparrow} \rightarrow \alpha\psi_2^{\uparrow} + \beta\psi_2^{\downarrow}$$

where α, β are coeff.

similarly,

$$\psi_1^{\downarrow} \rightarrow \gamma\psi_2^{\uparrow} + \delta\psi_2^{\downarrow}$$

i.e. spin mixing occurs

(analagous to a change in polarization of light).

The problem is equivalent to finding the state $\psi_2^{\uparrow}, \psi_2^{\downarrow}$ which satisfies

$$(\underline{S}_x \cos\theta + \underline{S}_y \sin\theta)(\psi^{\uparrow,\downarrow}) = \pm(\psi^{\uparrow,\downarrow})$$

where S_x, S_y are Pauli spin matrices.

Solving this we find

$$\begin{pmatrix} \psi_1^{\uparrow} \\ \psi_1^{\downarrow} \end{pmatrix} = \begin{bmatrix} R_{11}(\theta) & R_{12}(\theta) \\ R_{21}(\theta) & R_{22}(\theta) \end{bmatrix} \begin{bmatrix} \psi_2^{\uparrow} \\ \psi_2^{\downarrow} \end{bmatrix}$$

where R is as in question.

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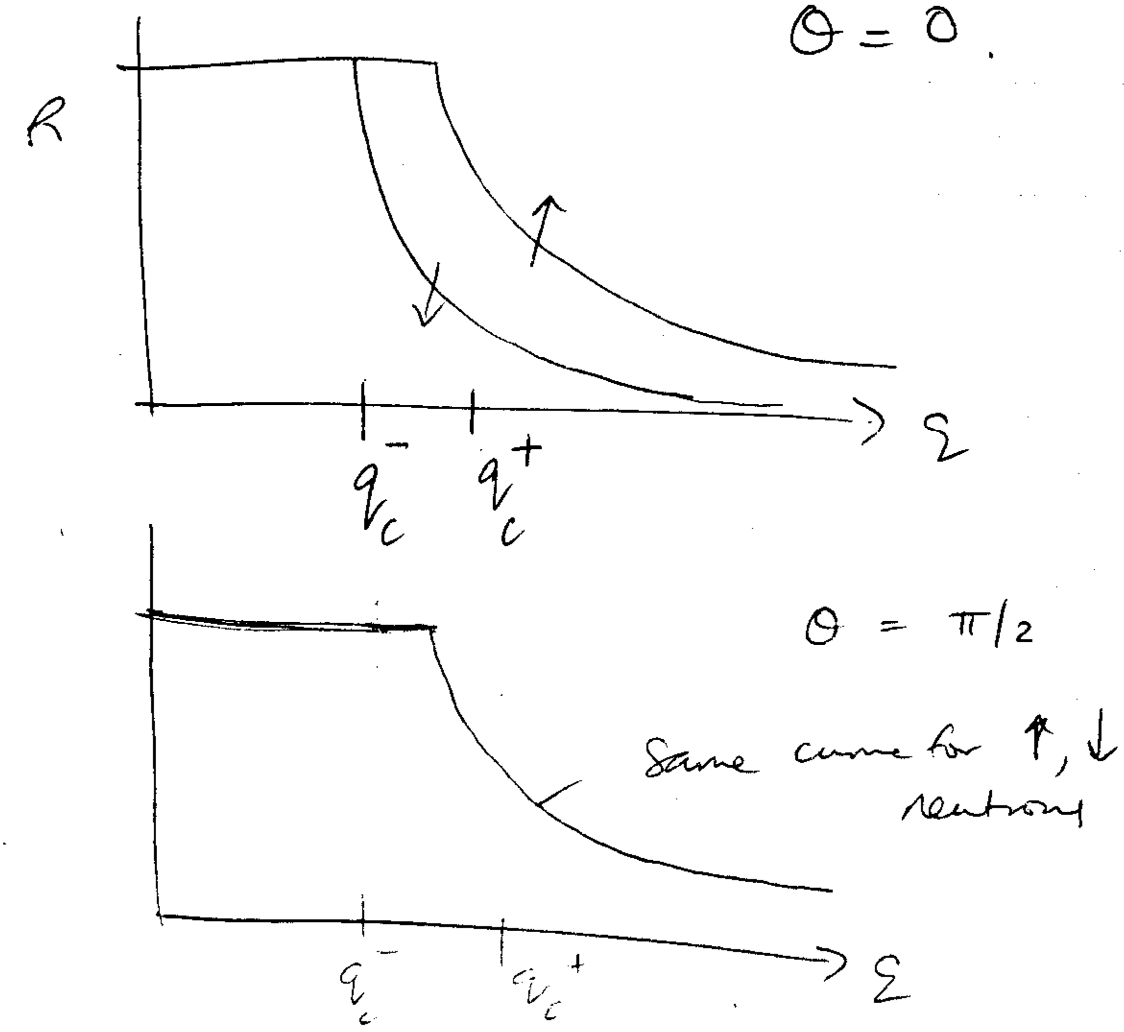
$\frac{\theta}{2}$ appears because wave satisfies 4π spinor symmetry.



but different from reflectivity for non-magnetic potential

6. $$b^{\pm} = b \pm 2cs$$

$$b = 9 \times 10^{-15}\ m$$
$$c = 0.2695 \times 10^{-15}\ m$$
$$s = 2.2$$

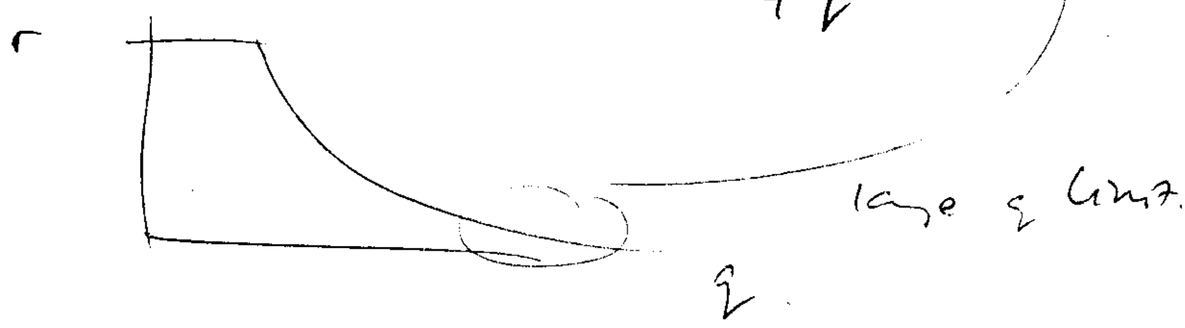
$$V^{\pm} = \frac{h^2}{2\pi m_n} \rho b^{\pm}$$

$$q_c^2 = \frac{2m_n V^{\pm}}{\hbar^2} = \frac{2m_n}{\hbar^2} \frac{h^2}{2\pi m_n} \rho b^{\pm}$$

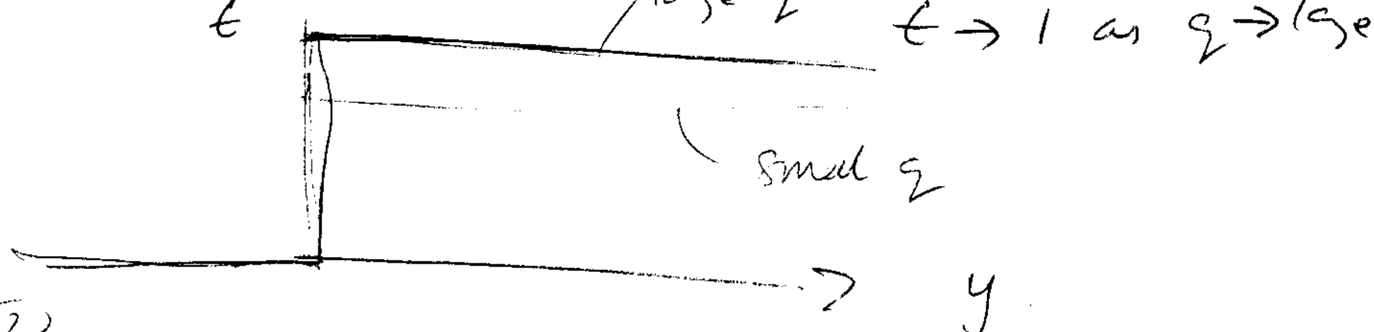
$$= 4\pi \rho b^{\pm}$$

can now sketch $r \sim \dfrac{q_c^2}{4q^2}$

$$t = 1 - r = 1 - \frac{q_c^2}{4q^2}$$



Form of wave intensity.



(12)

7. See lecture hand out.

$$q_c^2 = k^2(1-n^2)$$

$$\frac{\hbar^2 q_c^2}{2M} = V$$

Combine equations.