united nations educational, scientific and cultural organization

international atomic energy agency

the
abdus salam
international centre for theoretical physics

Winter College on Optics and Photonics
7 - 25 February 2000

1218-13

"Experimental Determination of the M^2 Quality Factor of Partially Coherent Gaussian Schnell-Model Sources"

R. BORGHI
Università Roma Tre
Dipt. Fisica
Italy

Experimental determination of the M^2 quality factor of partially coherent Gaussian Schell-model sources

Riccardo Borghi
Dipartimento di Ingegneria Elettronica, Univeristà degli Studi "Roma Tre"
Via della Vasca Navale, 84 I-00146 Rome, Italy

ICO/OSA Winter College on Optics and Photonics,
7-25 February 2000, ICTP, Miramare, Trieste, Italy.

1 Introduction

In this note, we describe the basic experimental scheme for measuring the M^2 beam quality factor of a light beam [1]. Such a scheme is based on the parabolic law of the propagation of the intensity second-order moment through a paraxial optical system. After briefly recalling the basic theoretical concepts, we describe the experimental setup for measuring the beam quality factor of partially coherent light beams emitted from Gaussian Schell-model sources[2].

2 Propagation of a light beam through a paraxial optical system

The starting point is the propagation law of the radial squared width of the transverse section of a quasimonochromatic (mean wavelength λ) paraxial light beam (coherent or partially coherent), which is formally similar to the quadratic law valid for the spot-size of a coherent fundamental Gaussian beam [1],

$$\sigma_z^2 = \sigma_{\min}^2 \left[1 + \left(\frac{M^2 \lambda}{2\pi\sigma_{\min}^2} \right)^2 (z-\zeta)^2 \right] , \qquad (1)$$

where σ_z denotes the standard deviation of the intensity[1] at the typical plane $z =$ const., $\sigma_{\min}$ its minimum value, achieved in corrispondence of the waist plane $z = \zeta$, and M^2 is the beam quality factor. In the case of a fundamental

[1] Here and in the following, we are considering the *radial* standard deviation which, in the case of a radially symmetric light beam, is $\sqrt{2}$ times the standard deviation along the x- or y-axis.

Gaussian beam the M^2 factor turns out to be one, whereas for any other kind of paraxial beam the beam quality factor is greater than one. Equation (1) suggests a way to obtain M^2 on measuring the width of the light beam during its propagation at different transverse planes, and then on fitting the resulting behavior with the parabolic law (1). An important aspect of the M^2 factor is that it is *invariant* after the passage of the beam through any paraxial optical system. In particular, we shall consider the focusing optical system shown in Fig. 1, whose $ABCD$ matrix[3] is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1-\frac{l_2}{f} & l_2 + l_1\left(1-\frac{l_2}{f}\right) \\ -\frac{1}{f} & 1-\frac{l_1}{f} \end{bmatrix}. \tag{2}$$

It is possible to show [1] that the standard deviation, say σ_2, of the beam after

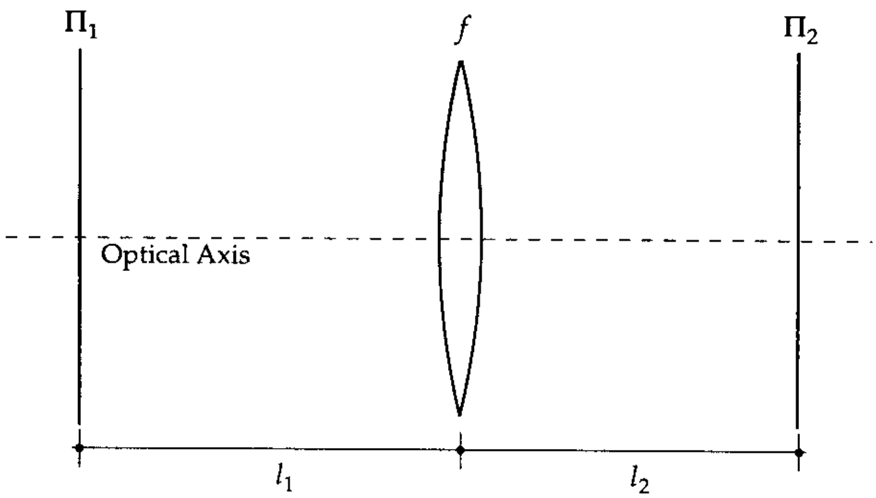

Figure 1: The focusing system.

the propagation through the optical system, i.e., at the plane Π_2, is given by

$$\sigma_2^2 = \frac{M^4}{k^2\sigma_1^2}\left[l_2^2 + \left(l_1^2 + \frac{k^2\sigma_1^4}{M^4}\right)\left(1-\frac{l_2}{f}\right)^2 + 2l_1l_2\left(1-\frac{l_2}{f}\right)\right], \tag{3}$$

where σ_1 is the standard deviation of the intensity profile at the input of the optical system, i.e., at the plane Π_1. Equation (3) can be read as a function of

l_2, i.e., on keeping fixed the focal length f and detecting σ_2 for different values of l_2. This corresponds to use the parabolic law of Eq. (1). Nonetheless, Eq. (3) can also be used on keeping fixed l_2 and changing the value of f. In the following, we will use both methods to obtain an estimate of M^2 factor of a so-called Gaussian Schell-model (GSM) beam [2].

3 GSM sources

A partially coherent GSM source is characterized by both a transverse intensity profile and a complex degree of coherence with Gaussian profiles [2]. More precisely, across the waist plane (i.e., the plane $z = 0$) we have

$$I(\vec{r}, 0) = I_0 \exp\left(\frac{r^2}{\sigma_I^2}\right), \tag{4}$$

$$\mu_0(\vec{r}_1, \vec{r}_2) = \exp\left[\frac{(\vec{r}_1 - \vec{r}_2)^2}{2\sigma_\mu^2}\right], \tag{5}$$

for the intensity distribution and the complex degree of coherence [4], respectively. Parameters σ_I and σ_μ completely characterize the source. In particular, in the limit $\sigma_\mu \gg \sigma_I$ we obtain the coherent fundamental Gaussian beam, whereas in the opposite limit, i.e., $\sigma_\mu \ll \sigma_I$, we have a spatially incoherent source with Gaussian profile. The fact that the complex degree of coherence [see Eq. (5)] depends only on the vectorial difference $\vec{r}_1 - \vec{r}_2$ suggests an easy practical procedure for synthesising a GSM source, as we will show in the following.

4 Measuring the M^2 factor

4.1 Description of the experimental setup

The experimental setup is shown in Fig. 2. Before describing in details this setup, it is worthwhile to spend some words about the synthesis of a GSM source. The particular structure of the spatial degree of coherence [see Eq. (5)] suggests the use of the van Cittert-Zernike (vCZ) theorem [4] for synthesizing the required μ on the source plane. This theorem says that it is possible to obtain a shift-invariant (i.e., depending on the sole difference $\vec{r}_1 - \vec{r}_2$) spatial degree of coherence starting from a spatially incoherent light source, through a simple Fourier transform operation. More precisely, if $I_{\text{inc}}(\vec{r})$ denotes the intensity distribution of the incoherent source, and the source plane is placed at a distance D from the former, then the spatial degree of coherence, say $\mu_0(\vec{r}_1, \vec{r}_2)$, on the plane of the synthesized source will be given by

$$\mu_0(\vec{r}_1, \vec{r}_2) = \mathcal{F}\left\{I_{\text{inc}}\right\}\left(\frac{\vec{r}_1 - \vec{r}_2}{\lambda D}\right) \exp\left[\frac{i\pi}{\lambda D}(r_1^2 - r_2^2)\right], \tag{6}$$

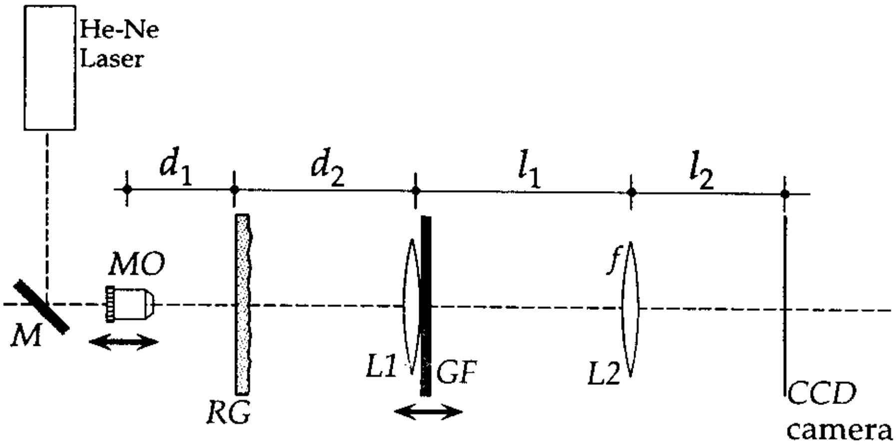

Figure 2: The experimental setup.

where $\mathcal{F}\{\cdot\}$ denotes the Fourier transform operator. Since the spatial degree of coherence of a GSM source is Gaussianly shaped [see Eq. (5)], we start from an incoherent source whose intensity profile is Gaussian. This corresponds to the first block of the setup in Fig. 2, composed by the microscope objective MO, by the rotating ground glass RG, and by the lens L1. The light emitted by the He-Ne laser is focused by MO and, after passing through RG, it produces an incoherent source with Gaussian profile

$$I_{\rm inc}(\vec{r}) = I_0 \exp\left(\frac{r^2}{\sigma_{\rm inc}^2}\right), \tag{7}$$

where $\sigma_{\rm inc}$ denotes the intensity standard deviation of the incoherent source and I_0 is a constant factor. The value of $\sigma_{\rm inc}$ can be adjusted on varying the distance d_1 by means of a translator. The vCZ theorem can now be applied for the free propagation through d_2. On substituting Eq. (7) into Eq. (6) we obtain a complex degree of coherence μ_0 of the form (5), where the parameter σ_μ turns out to be

$$\sigma_\mu = \frac{\lambda d_2}{\sqrt{2}\pi\sigma_{\rm inc}}. \tag{8}$$

The action of the lens L1, whose focal length equals d_2 is to eliminate the quadratic phase factor appearing in Eq. (7). In this way, the beam emerging ftom L1 is characterized by a spatial degree of coherence given by Eq. (5). Finally, in order to obtain a GSM source, we give the beam a Gaussian intensity profile with standard deviation σ_I by means of the Gaussian filter GF, placed immediately after L1. Thus, at the output of the filter, we are at the waist plane of our GSM beam. It should be stressed that, on varying the distance d_1, we can change the coherence features of the source.

Just to give an idea of typical values, a He-Ne laser beam ($\lambda = 632$ nm) focused on the RG by a '10x' MO produces an incoherent source with a standard deviation $\sigma_{\rm inc} \simeq 10$ μm. On choosing $d_2 \simeq 10$ cm (focal length of L1), the value of σ_μ turns out to be approximately 1.2 mm.

It is known [1] that the M^2 quality factor of a GSM source depends only on the ratio between σ_I and σ_μ through the following relation:

$$M^2 = \sqrt{1 + \frac{2\sigma_I^2}{\sigma_\mu^2}}, \tag{9}$$

which, in particular, in the limit of a purely coherent beam (i.e., when $\sigma_\mu \gg \sigma_I$), reduces to $M^2 = 1$. If $\sigma_I \simeq 1.4$ mm, as in our experimental setup, the estimated theoretical value of M^2, given by Eq. (9), is about 2.

The second block of the experimental setup realizes the focusing optical system shown in Fig. 1 of Sec. 2, while the data acquisition system is a beam analyzer Spiricon LBA-300.

4.2 Examples of experimental acquisitions

The measurement procedures for the estimate of the M^2 factor presented here are, as we said above, of two types.

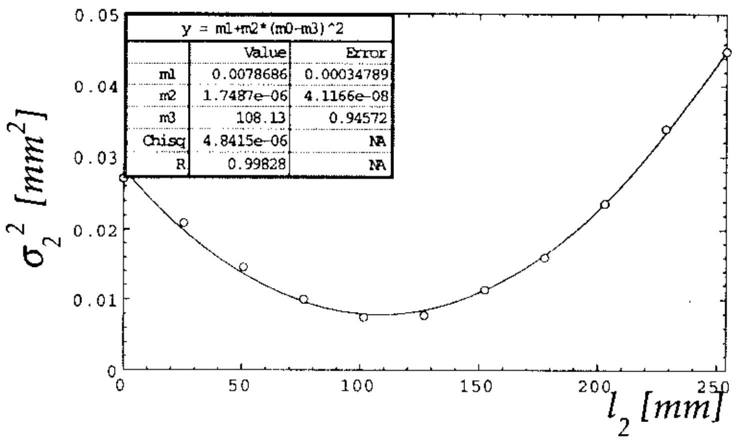

Figure 3: Experimental values (dots) of the intensity variance σ_2^2 for the light beam emitted from the He-Ne laser as function of the propagated distance z, together with the parabolic fit (solid line). $f = 300$ mm.

The first directly refers to Eq. (1). In practice, the focal length f of L2 is fixed, while the intensity profiles are detected at different propagation distances l_2. Furthermore, since in the case of the beams emitted by GSM sources such transverse intensity profiles are always Gaussianly shaped, we have used a Gaussian fitting procedure for recovering the width of the typical intensity profile, say W, from which the standard deviation σ_2 can be obtained as [1]

$$\sigma_2 = \frac{W}{2\sqrt{2}}. \tag{10}$$

In order to test the accuracy of our setup, in Fig. 3 the experimental values (dots) of the intensity variance σ_2^2, obtained for the beam directly emitted by the He-Ne laser and sent into a 300 mm focusing lens, are plotted. In this particular case, the first block of the setup (i.e., the microscope objective MO, the rotating ground glass RG, and the collimating lens L1) was eliminated. The solid curve represents the fit with the parabolic law (1). In the inset the fit parameters are also shown. In particular, the estimated M^2 factor turns out to be $M^2 \simeq 1.2$, near but not equal to the theoretical value 1, predicted for a pure, coherent Gaussian beam. There are some reasons for justifying such a discrepancy, the most important being the focusing lens aberrations, which strongly affect the M^2 factor of the resulting beam [1].

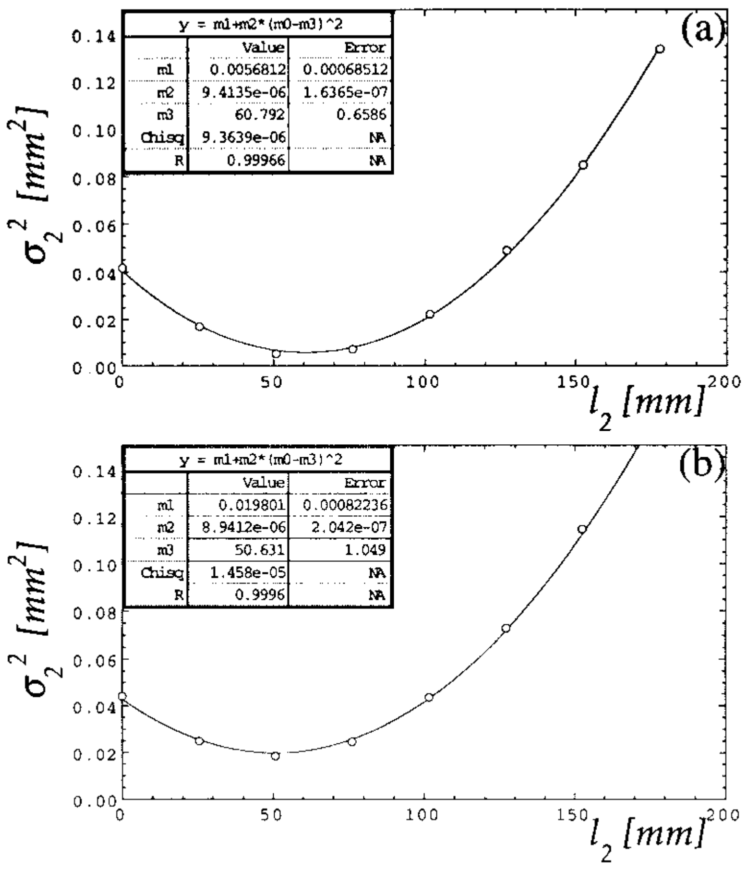

Figure 4: The same as in Fig. 3. but for two synthesized GSM sources. Figure (a) refers to the source having the highest (within the experimental conditions) complex degree of coherence. Figure (b) refers to a source having coherence features worse with respect to that in Figure (a).

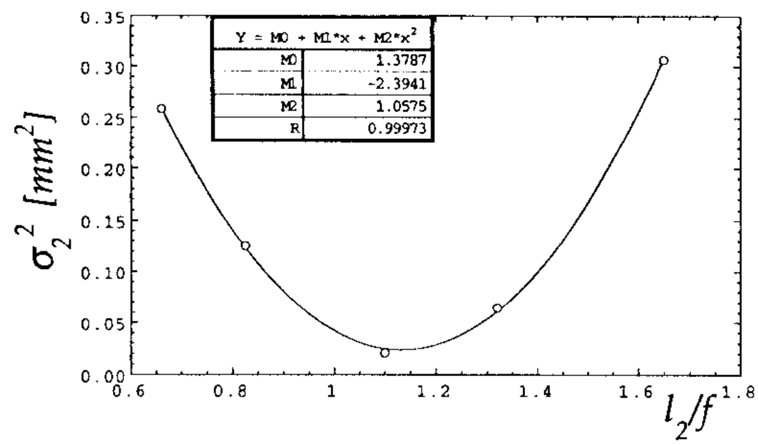

Figure 5: Behavior of the of the intensity variance σ_2^2 (dots) for the light beam emitted from the source of Fig. 4(b) as a function of l_2/f, with $l_2 = 340$ mm.

Let us now consider a partially coherent GSM source, synthesized as described above. On referring to Fig. 2, the light emitted from the He-Ne laser is expandend by means of MO and collimated by the lens L1 having focal length 10 cm. First, we consider the GSM with the best coherence features achievable with our setup, i.e., by placing the rotating ground glass RG at the focus plane of MO. Figure 4(a) shows the fitting (solid line) of the experimental data (dots) obtained by measuring σ_2^2 for $f = 300$ mm and several distances l_2. In particular, the estimate of the quality factor turns out ot be $M^2 \simeq 2.3$, which is in agreement, within the experimental errors, with the theoretical prediction obtained above. In order to show, at least in a qualitatively way, the effect of the coherence on the quality factor of the beam, Fig. 4(b) shows the same as in Fig. 4(a) but for a less coherent GSM source, obtained by moving the RG out of the focal plane of MO. The estimated M^2 in this case turns out to be about 4.2.

Finally, Figure 5 refers to the same beam as in Fig. 4(b), but in this case the measurement procedure is different. More precisely, the experimental values of σ_2^2 (dots) are plotted as a function of the dimensionless variable l_2/f, where $l_2 = 340$ mm, and $f = 200,\ 250,\ 300,\ 400,$ and 500 mm. The experimental data are then fitted (solid curve) with Eq. (3) and the M^2 factor turns out ot be $M^2 \simeq 4.6$.

It should be stressed that such a procedure is, from a practical viewpoint, simpler with respect to the previous one, since now the CCD camera remains at the same position, whereas in the other experimental configuration it has to be moved at different transverse planes. Nonetheless, the accuracy in the estimation of the M^2 seems poorer, due to the aberrations of the several lenses which have now to be used.

References

[1] M. Santarsiero, "Laser beam characterization," Lecture notes of *Winter College on Optics and Photonics 2000*, Internation Centre for Theoretical Physics, Miramare, Trieste, Italy, 2000.

[2] E. Collett and E. Wolf, "Is complete coherence necessary for the generation of highly directional light beams?," Optics Letters **2**, 27 (1978).

[3] R. Simon, M. Mukunda, and E. C. G. Sudarshan, "Partially coherent beams and a generalized ABCD law," Optics Communications **65**, 322 (1988)

[4] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics*, (Cambridge University Press, Cambridge, Massachusetts, 1995).