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"Radiation from Electrons in a Synchrotron"

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Phys. Rev. 71, 829 (1947)

Radiation from Electrons in a Synchrotron

F. R. Elder, A. M. Gurewitsch, R. V. Langmuir, and H. C. Pollock
Research Laboratory, General Electric Company, Schenectady, New York
May 7, 1947

High energy electrons which are subjected to large accelerations normal to their velocity should radiate electromagnetic energy.[1-4] The radiation from electrons in a betatron or synchrotron should be emitted in a narrow cone tangent to the electron orbit, and its spectrum should extend into the visible region. This radiation has now been observed visually in the General Electric 70-Mev synchrotron.[5] This machine has an electron orbit radius of 29.3 cm and a peak magnetic field of 8100 gausses. The radiation is seen as a small spot of brilliant white light by an observer looking into the vacuum tube tangent to the orbit and toward the approaching electrons. The light is quite bright when the x-ray output of the machine at 70 Mev is 50 roentgens per minute at one meter from the target and can still be observed in daylight at outputs as low as 0.1 roentgen.

The synchrotron x-ray beam is obtained by turning off the r-f accelerating resonator and permitting subsequent changes in the field of the magnet to change the electron orbit radius so as to contract or expand the beam to suitable targets. If the electrons are contracted to a target at successively higher energies, the intensity of the light radiation is observed to increase rapidly with electron energy.

Maxwell's Equations:

$$\operatorname{div} \vec{E} = 4\pi \rho$$

$$\operatorname{div} \vec{B} = 0$$

$$\operatorname{curl} \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\operatorname{curl} \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t}$$

$\vec{E}$ = electric field

$\vec{B}$ = magnetic induction

c = speed of light

$\vec{J}$ = current density

ρ = charge density

$$\operatorname{div} \vec{A} = \frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y + \frac{\partial}{\partial z} A_z$$

$$(\operatorname{curl} \vec{A})_x = \frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y$$

y } by circular perm. x, y, z.

Radiation of accelerated charges.

Larmor's formula:

$$P = \frac{2 e^2}{3 c^3} |\dot{\vec{v}}|^2 \qquad (|\vec{v}| << c)$$

Total radiated power (energy per unit time of electromagnetic waves created) by particle of charge e, subject to acceleration $\dot{\vec{v}} \equiv \frac{d\vec{v}}{dt}$

Examples of accelerated motion:

* Simple harmonic motion

$x = x_0 \sin \omega t \qquad \dot{x} = \omega x_0 \cos \omega t = v_x$

$\dot{v}_x = -\omega^2 x_0 \sin \omega t$

* Circular motion

$|\dot{v}| = \frac{v^2}{r} \equiv \omega^2 r$

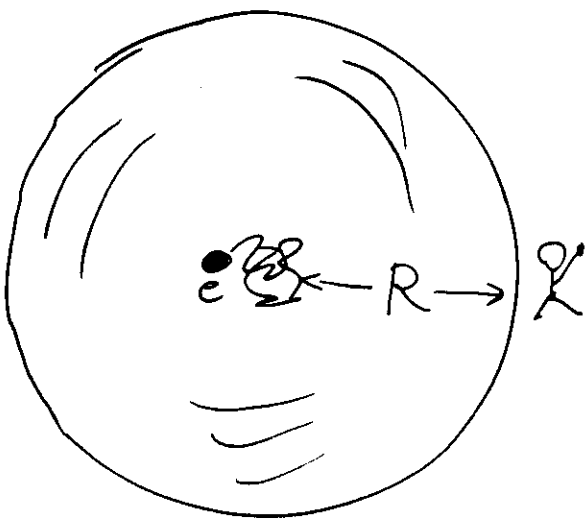
Relativistic generalization:
for $|\vec{v}|$ not so smaller than c.

Lienard formula (1898)

$$P = \frac{2}{3}\frac{e^2}{c^3}\gamma^6\left(|\dot{\vec{v}}|^2_{ret} - \left|\frac{\vec{v}\times\dot{\vec{v}}}{c}\right|^2_{ret}\right)$$

$$= \frac{2}{3}\frac{e^2}{c}\gamma^6\left(|\dot{\vec{\beta}}|^2_{ret} - |\vec{\beta}\times\dot{\vec{\beta}}|^2_{ret}\right)$$

"ret" means retarded, i.e. evaluated at time $t - R/c$, R: distance of charge to observer



$$\vec{\beta} = \frac{\vec{v}}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$E = \frac{m_0 c^2}{\sqrt{1-\beta^2}} = \gamma m_0 c^2$$

Circular Geometry:

$$\vec{\beta} \perp \dot{\vec{\beta}} \Rightarrow |\vec{\beta} \times \dot{\vec{\beta}}|^2 = \beta^2 \dot{\beta}^2$$

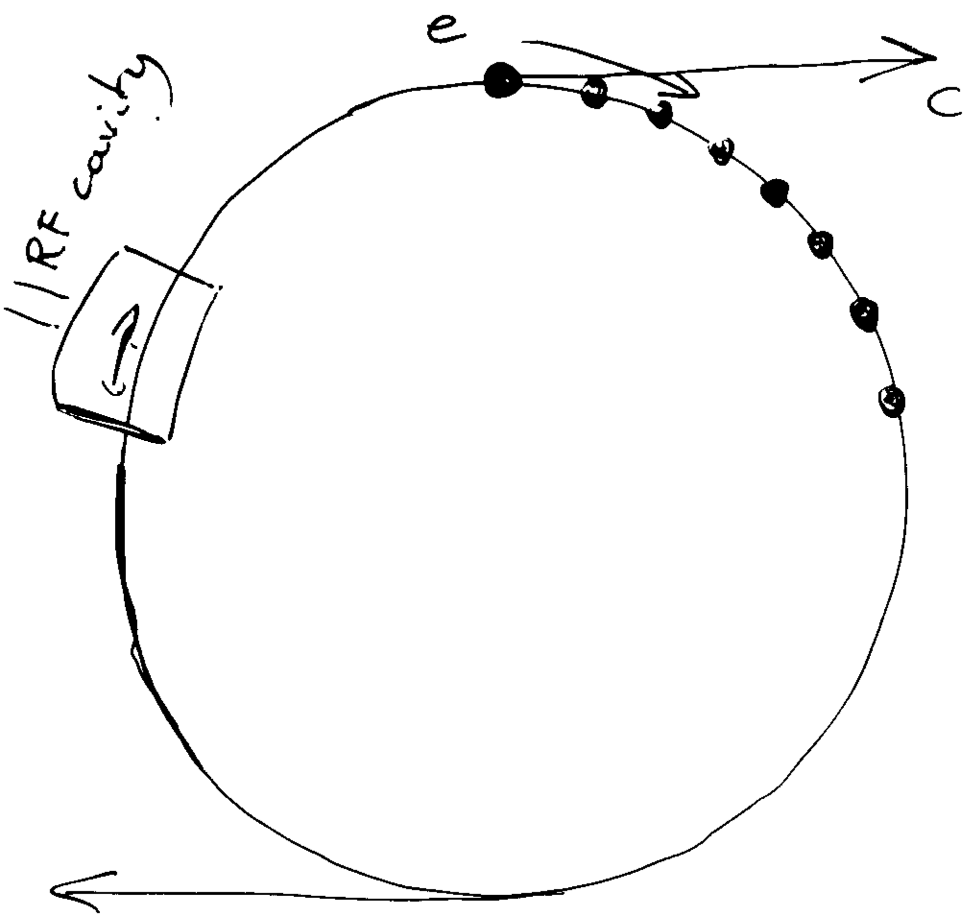
$$P = \frac{2}{3} \frac{e^2}{c} \gamma^6 \left(\dot{\beta}^2 - \beta^2 \dot{\beta}^2 \right) =$$

$$= \frac{2}{3} \frac{e^2}{c} \gamma^6 (1-\beta^2) \dot{\beta}^2 = \frac{2}{3} \frac{e^2}{c^3} \gamma^4 \dot{v}^2$$

$$|\dot{\vec{v}}| = \frac{c^2}{R}, \quad \dot{v}^2 = \frac{c^4}{R^2}$$

$$P = \frac{2}{3} \frac{e^2 c}{R^2} \gamma^4$$

Application to Circular Accelerator



- $E = \gamma m_0 c^2 = \dfrac{m_0 c^2}{\sqrt{1-\beta^2}}$

- Electrons or Positrons $m_0 c^2 = 511$ keV $\cong 0.5$ Me
  - Protons $m_0 c^2 \sim 1$ GeV

So a 1 GeV e^- ring has
$\gamma \sim 2 \times 10^3$

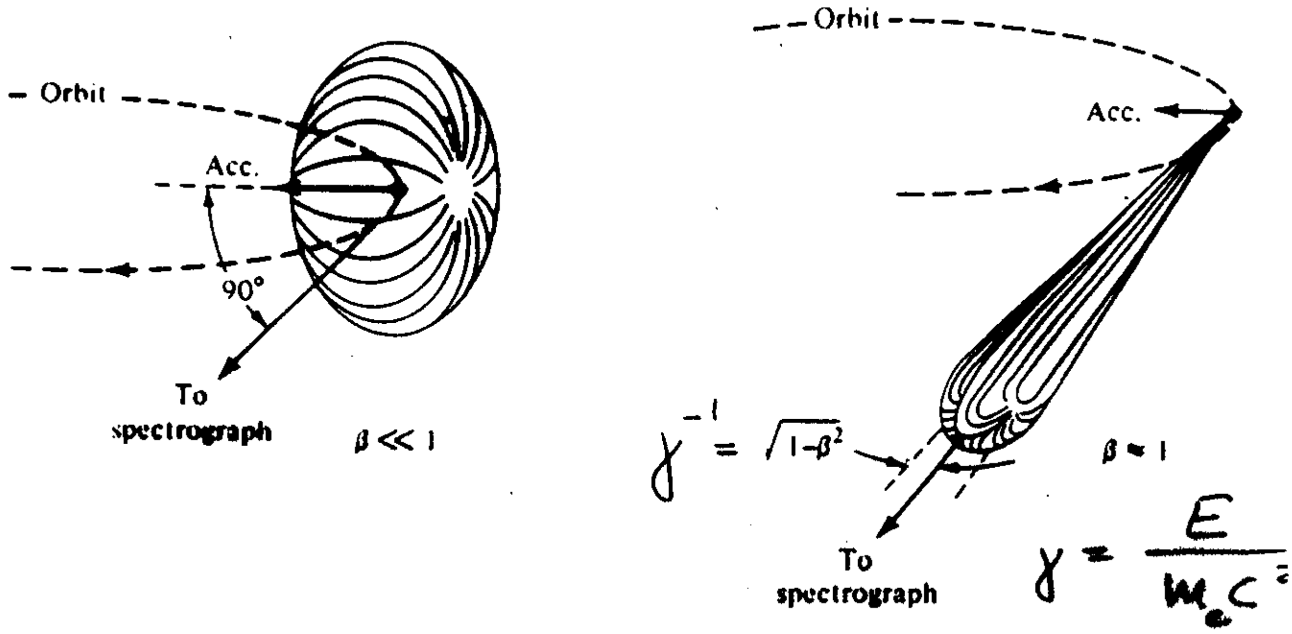
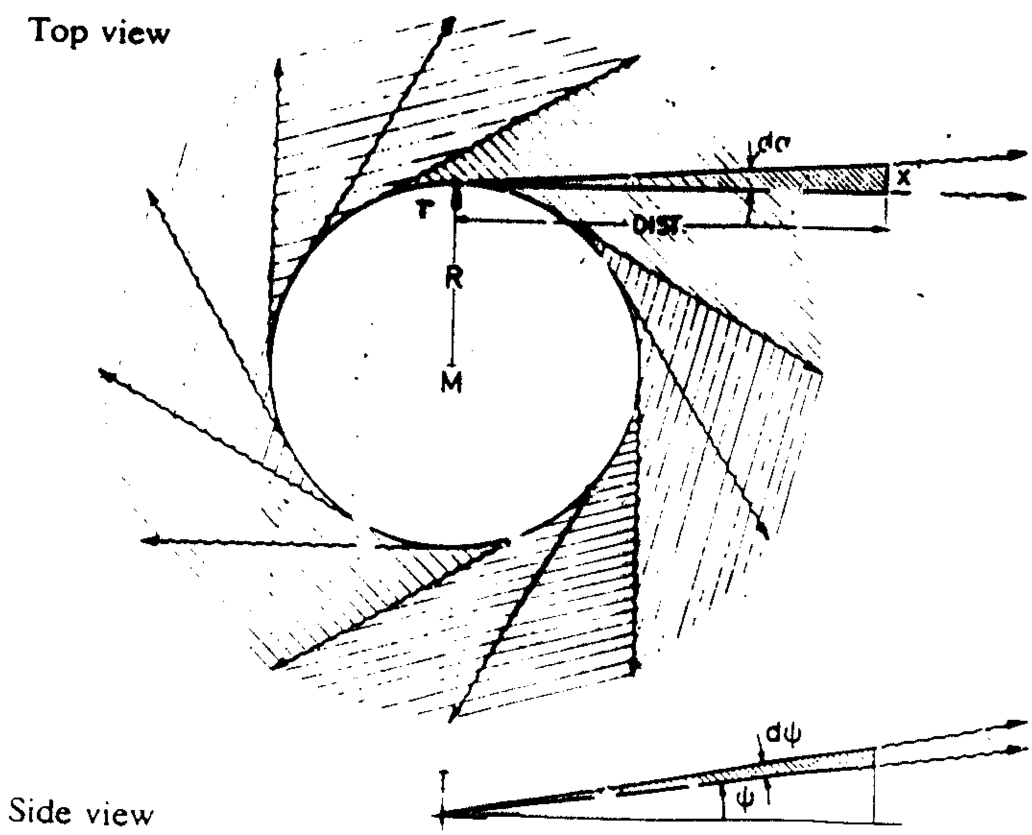
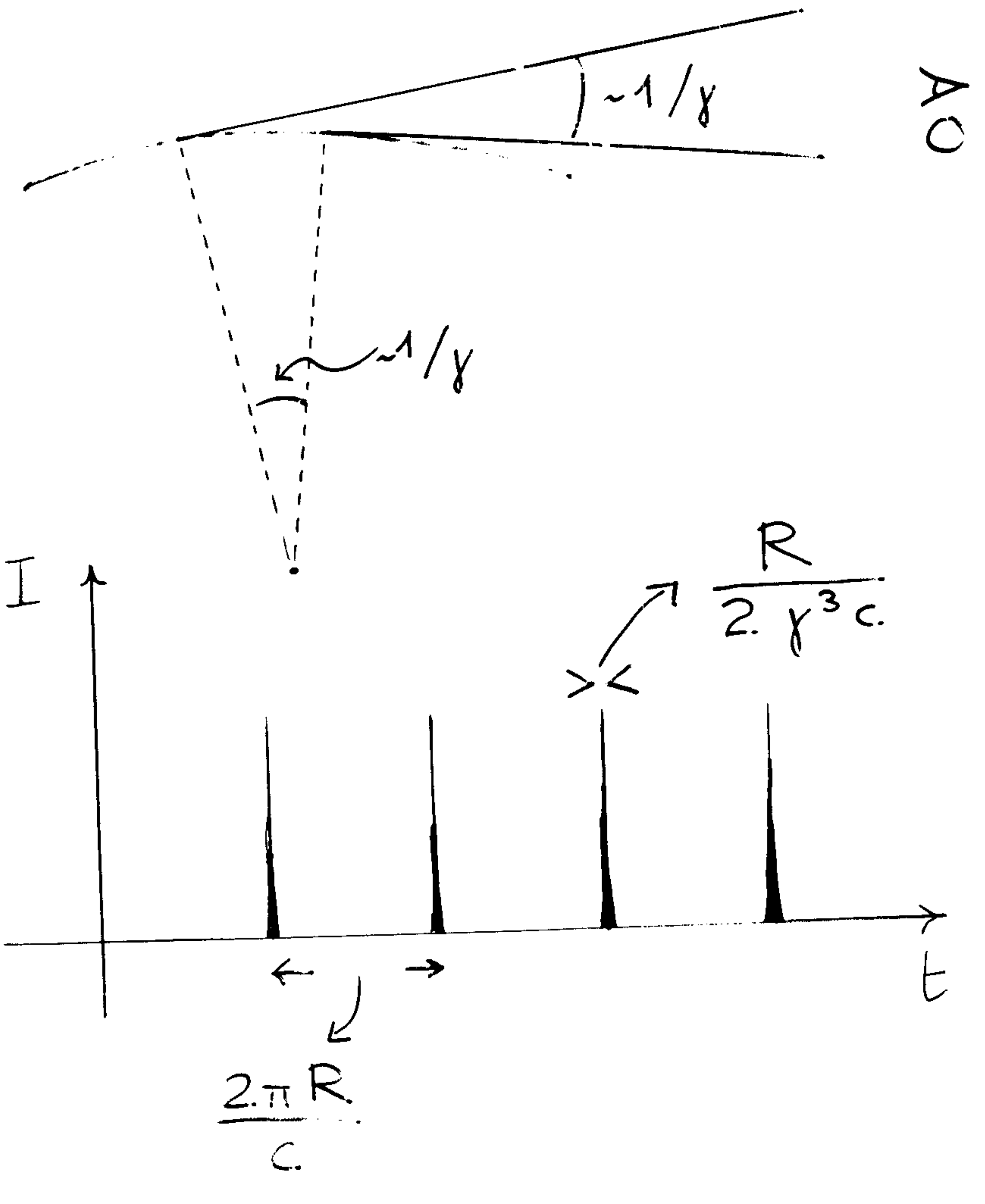


Fig. 5a. The "classical" picture for the geometry of synchrotron radiation emission (from Tomboulian and Hartman (1956)). The angular distribution (dipole pattern) of emitted intensity from a slow electron on a circular orbit (left) is distorted into a narrow cone in the instantaneous direction of motion for a relativistic electron moving with a velocity close to that of light ($\beta = v/c \approx 1$) (right).



Frequency distribution



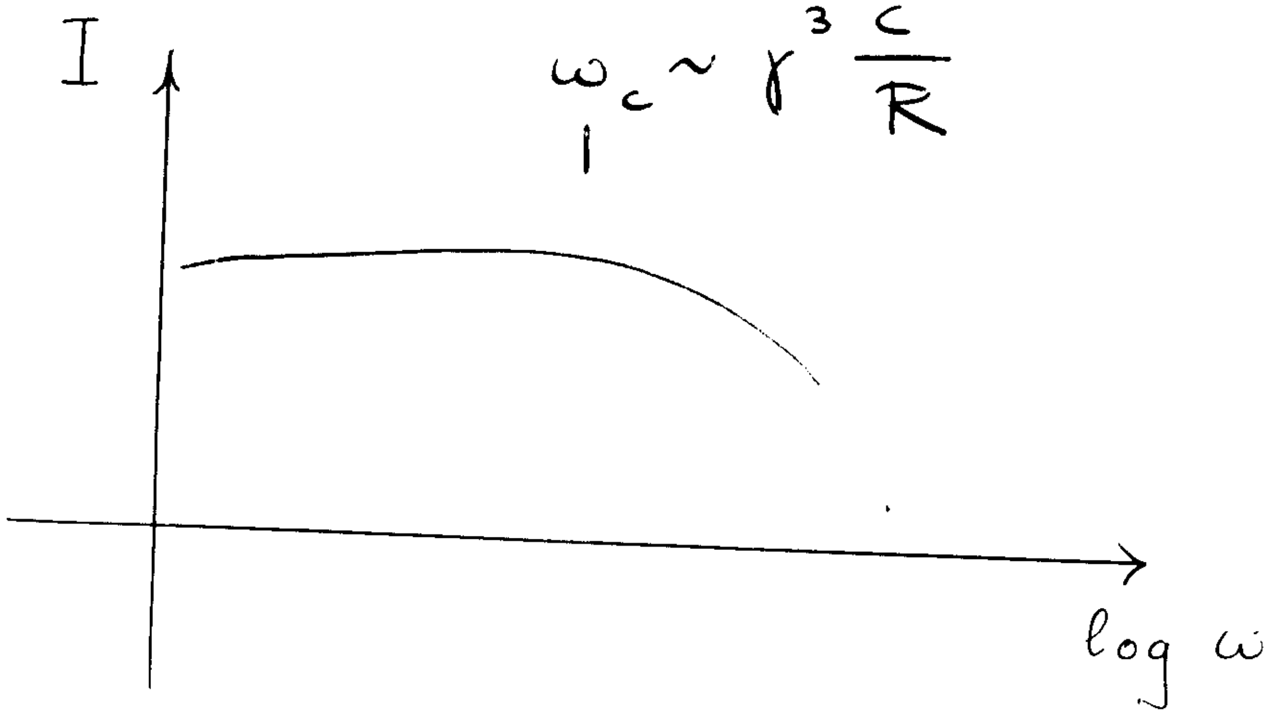
Broad (white) spectrum

from

$\omega_0 \sim \frac{c}{2\pi R}$ (~MHz)

to at least

$\omega_c \sim \gamma^3 \frac{c}{R}$ $(\sim 10^{10} - 10^{12}\,\omega_0)$



More precisely:

Critical frequency:

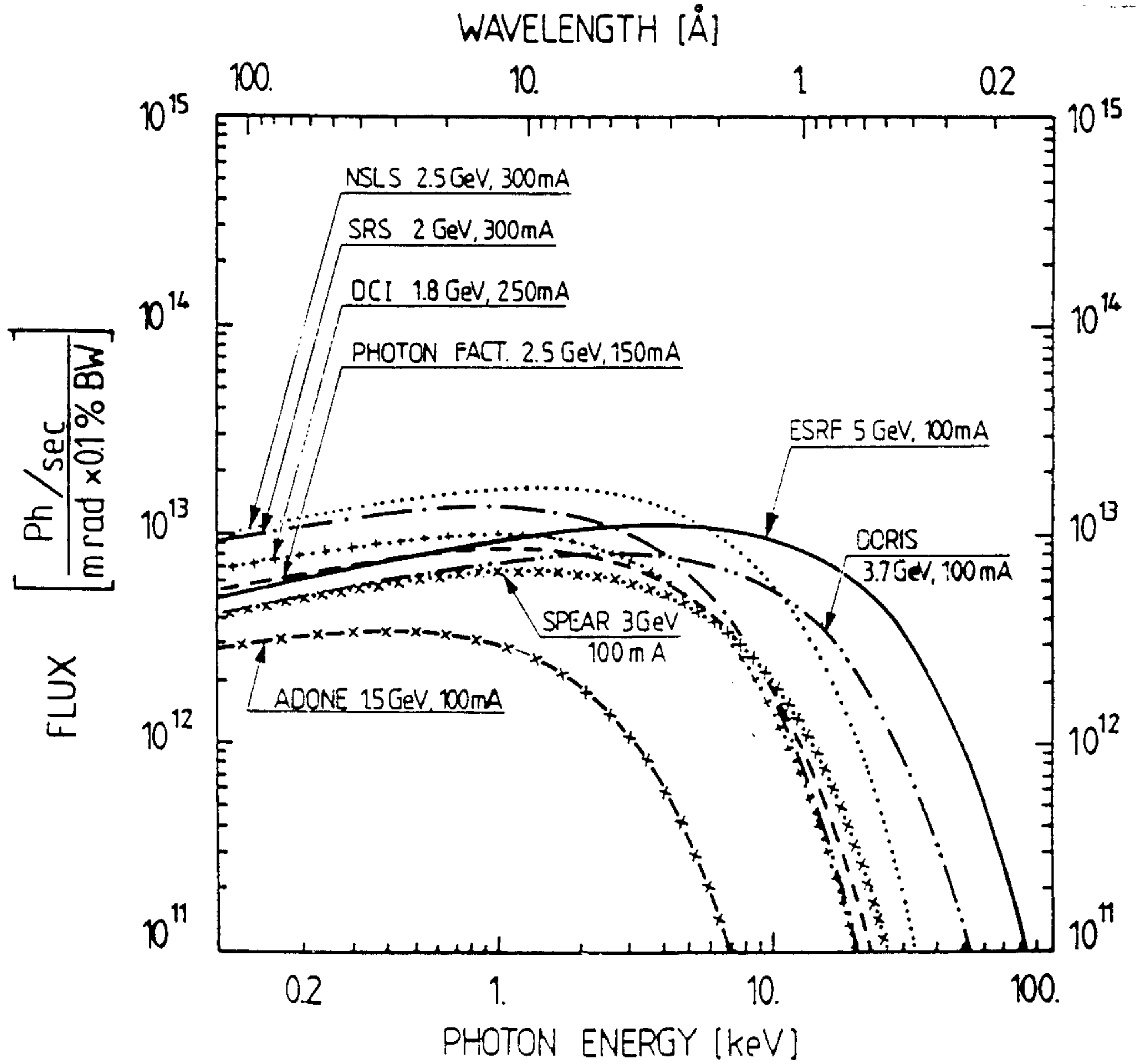
$$\omega_c = \frac{3}{2} \gamma^3 \frac{c}{R}$$

Half power emitted at $\omega < \omega_c$

" " " " $\omega > \omega_c$

$$\omega_c \sim \omega_0 \gamma^3$$

$$\gamma = \frac{E}{m_e c^2} \qquad m_e c^2 = 0.511 \text{ MeV}$$



SYNCHROTRON RADIATION : VERY BRIGHT EMISSION OF ELECTROMAGNETIC RADIATION FROM ELECTRONS ORBITING IN A STORAGE RING

IT IS THE BRIGHTEST AVAILABLE SOURCE OF ULTRAVIOLET AND X-RAY RADIATION

USED FOR RESEARCH IN :

- CONDENSED MATTER AND MATERIALS PHYSICS
- SURFACE AND INTERFACE PHYSICS AND CHEMISTRY
- STRUCTURAL BIOLOGY (30% OF WORLDWIDE USE)
- IMAGING (METALLURGY, MATERIALS SCIENCE, MEDICAL PURPOSES)
- MICROFABRICATION

THERE ARE ABOUT 45 S.R. SOURCES OPERATING WORLDWIDE, 8 OF THEM BEING OF THE MOST ADVANCED TYPE (INCLUDING ELETTRA IN TRIESTE)

BRILLIANCE / SPECTRAL BRIGHTNESS

$$\frac{\text{FLUX OF PHOTONS IN UNIT WAVELENGTH RANGE}}{\text{(SOURCE AREA) (BEAM DIVERGENCE)}}$$

UNITS:

Photons/s/mm^2/$mrad^2$/0.1% bandwidth

Technical Progress in SR sources

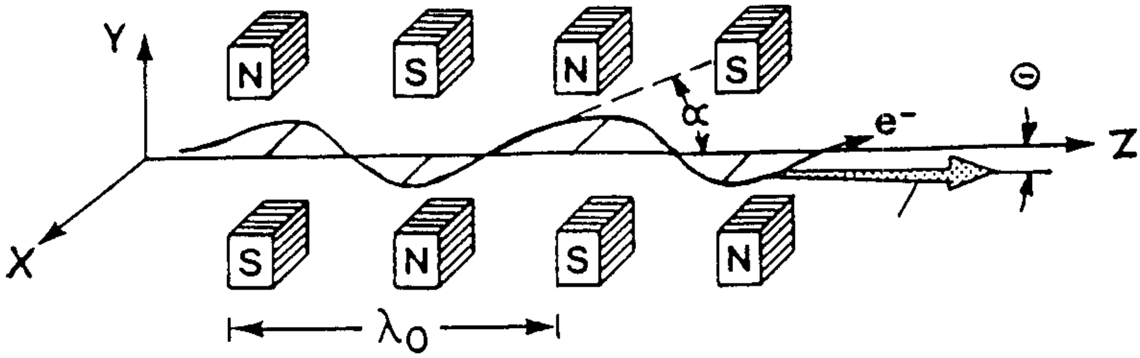
- Low emittance of storage ring

  emittance = size of particle beam X angular divergence

- "Insertion devices" for SR production

Horizontal Emittance of Synchrotron Radiation Sources

	E (GeV)	ε_H (nm.rad)
NSLS (USA)	2.5	80
Doris (FRG)	3.7 (5)	270 (500)
Bessy (FRG)	0.8	40
Photon Factory (Japan)	2.5	130
SRS-HBL (UK)	2.0	100
SUPER-ACO (France)	0.8	40
SPEAR (USA)	3	450
CESR (USA)	5.5	200
PEP-LEO (USA)	7	< 10
***		
APS (USA)	7	7
ESRF (France)	6	7 → 4
ALS (USA)	1.5	5
Trieste (Italy)	1.5-2.4	< 10 → 7
Bessy II (FRG)	1.7	5
Max II (Sweden)	1.5	7

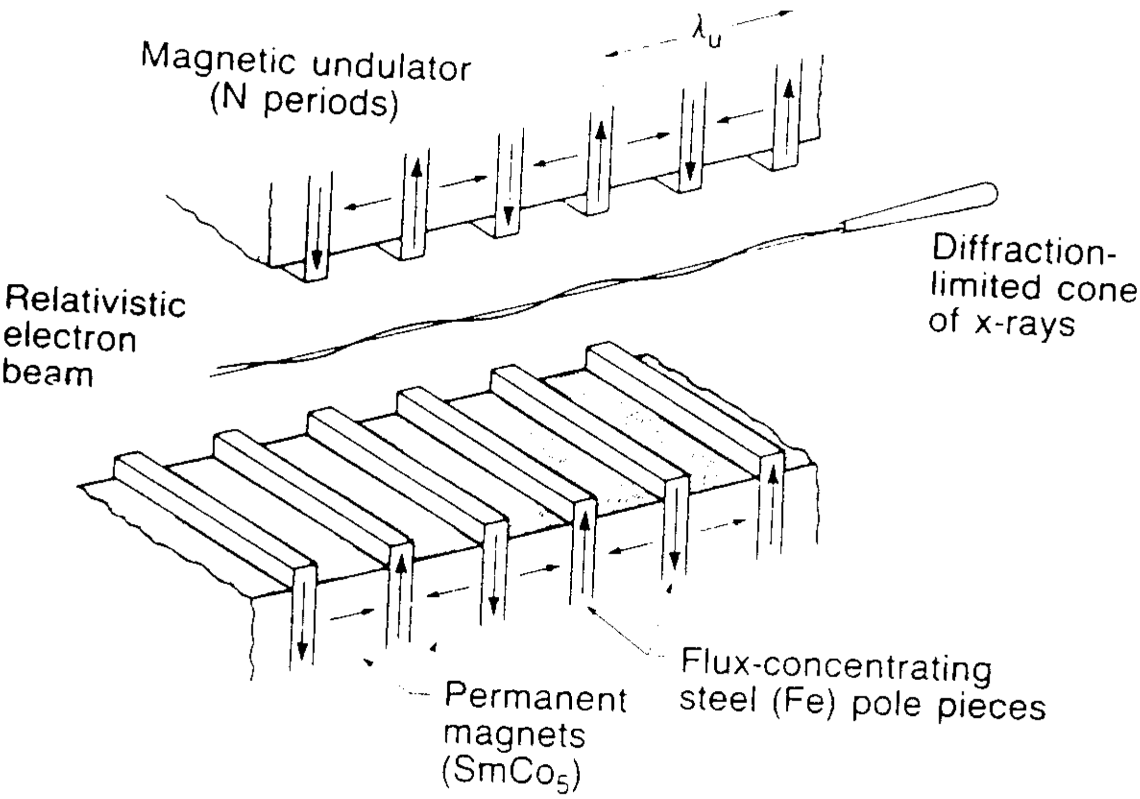


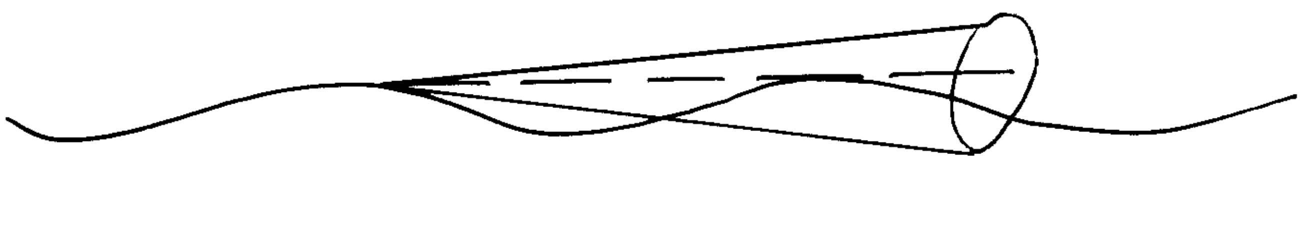
Insertion Devices

$$K = 93.4 \; \lambda_0[m] \; B_0(T)$$

$$K = \alpha \cdot \gamma = \alpha \frac{1}{\sqrt{1 - v^2/c^2}} \lesssim 1 \text{ Undulators}$$

$$\gg 1 \text{ Wigglers}$$





electron delay = $n \lambda$

Undulator Emission

On Axis

$$n\lambda = \frac{\lambda_0}{2\gamma^2}\left(1 + K^2/2\right)$$

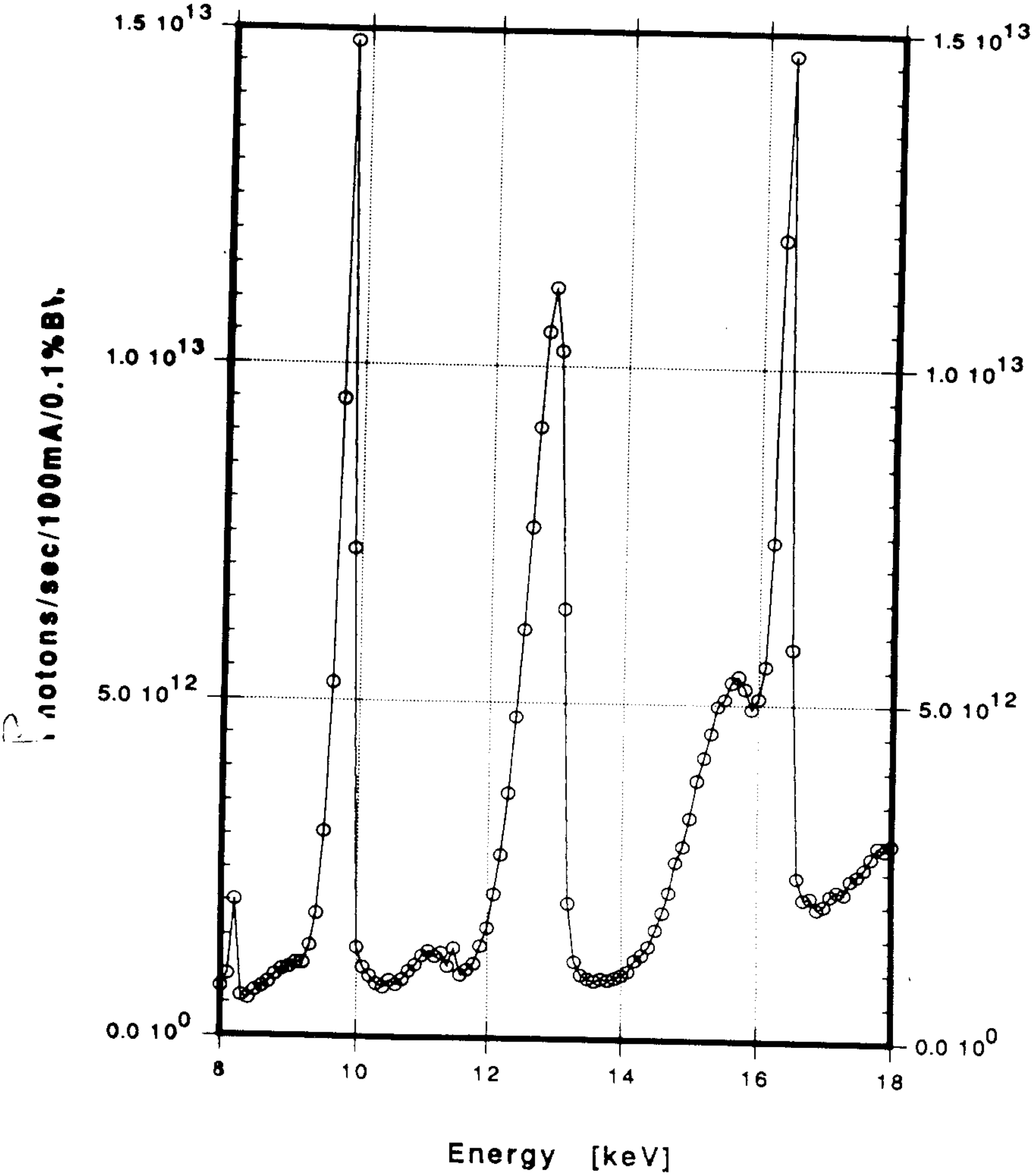
$n = 1, 3, 5, \ldots$

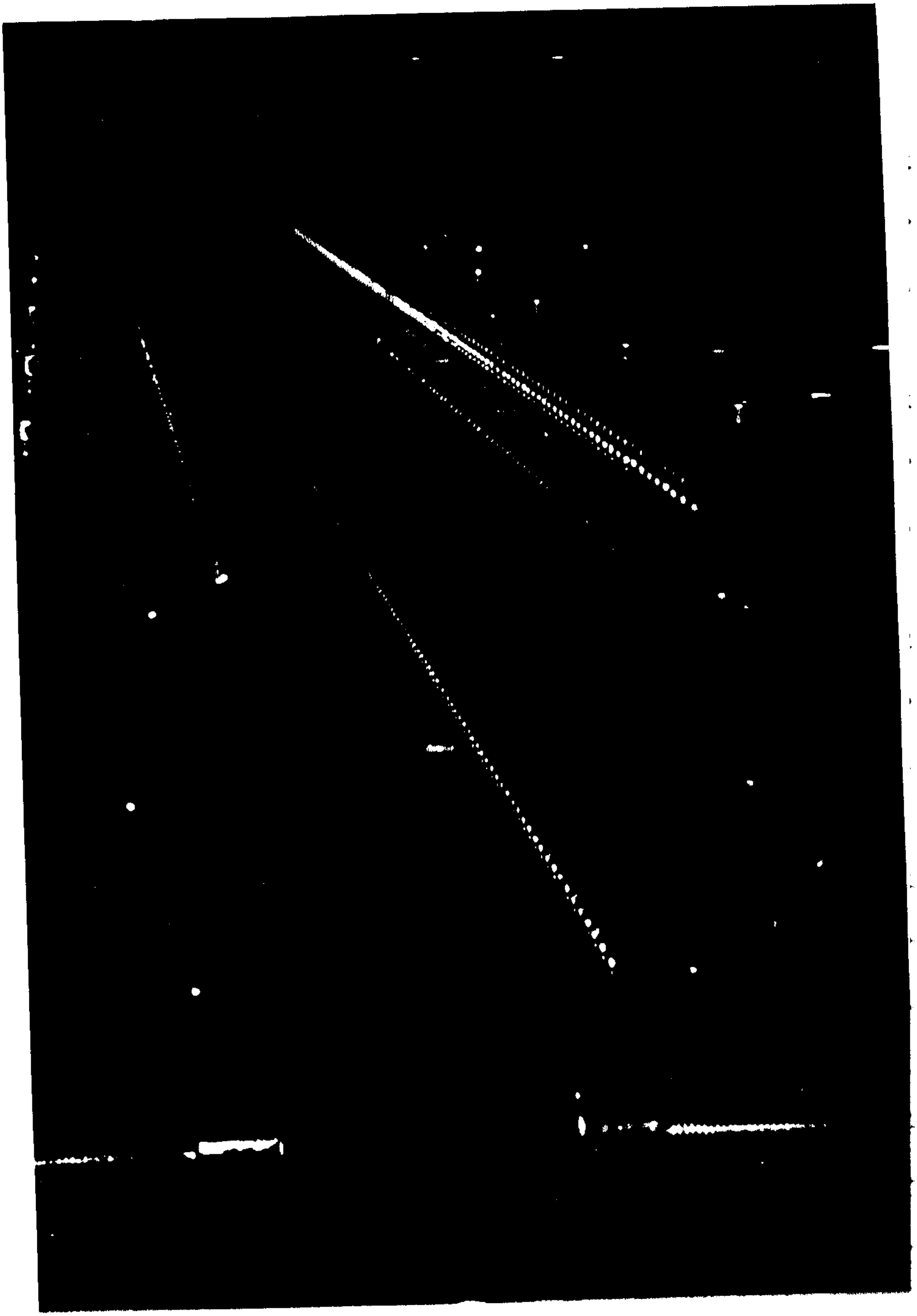
Off-axis

$$n\lambda = \frac{\lambda_0}{2\gamma^2}\left(1 + K^2/2 + \gamma^2\theta^2\right)$$

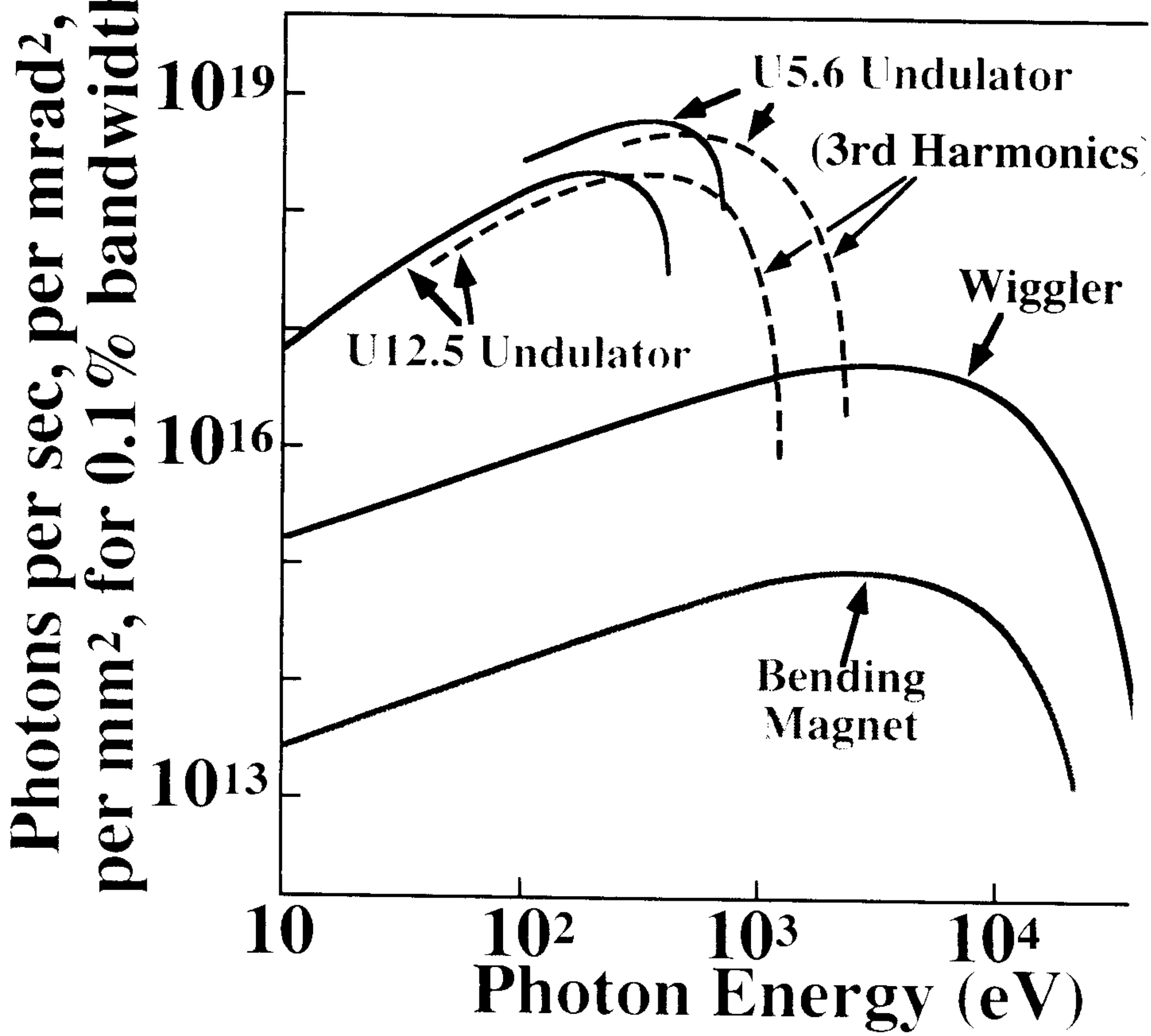
$n = 1, 2, 3, \ldots$

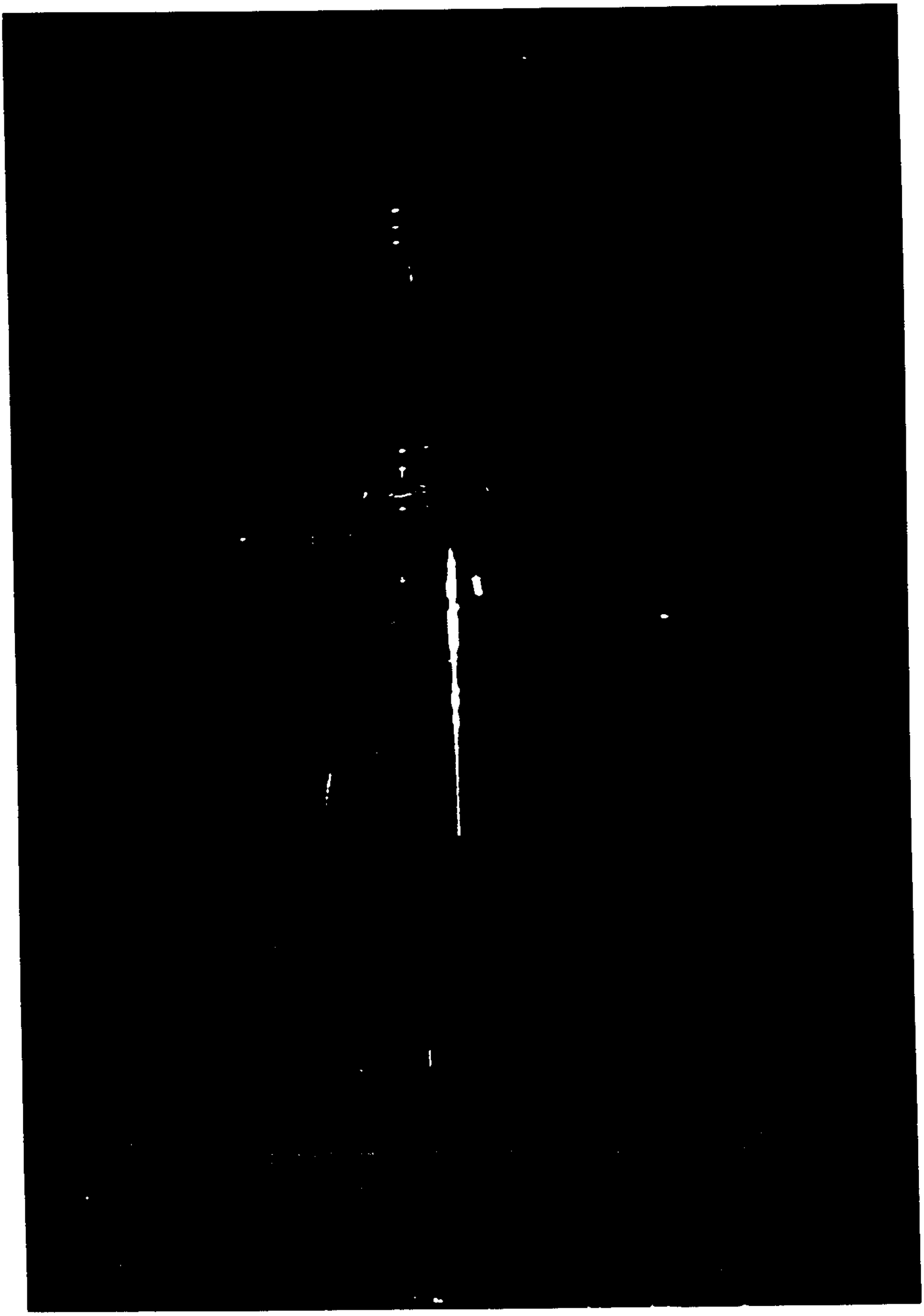
ID10 Undulator Spectrum at 22.8 mm Gap
Si(220) Monochromator at 28.5m from source
Primary Slits: 0.4mm(v), 0.5mm(h)
(measured with NaJ(Tl) and calib. Kapton foil, corrected for absorption)

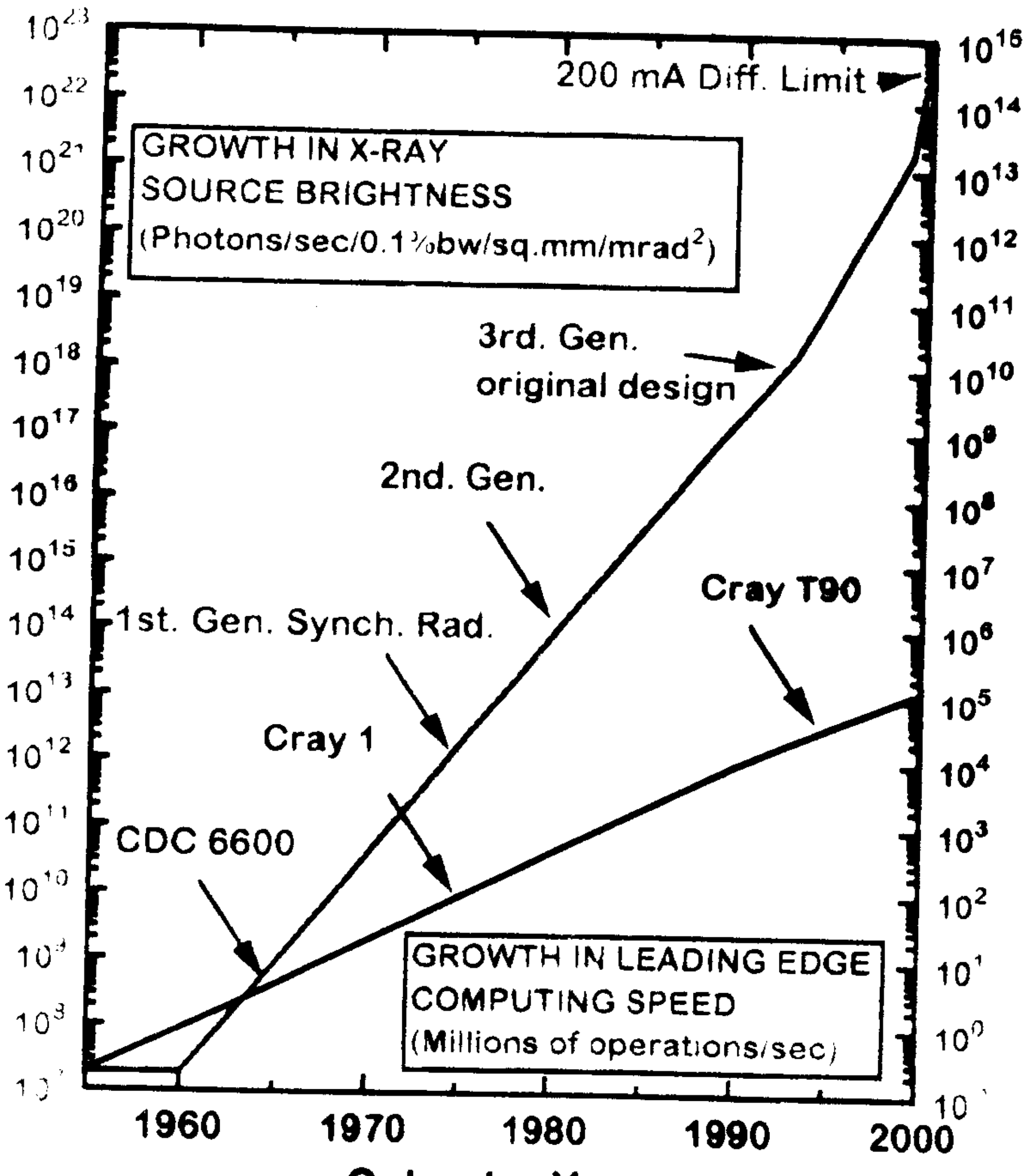




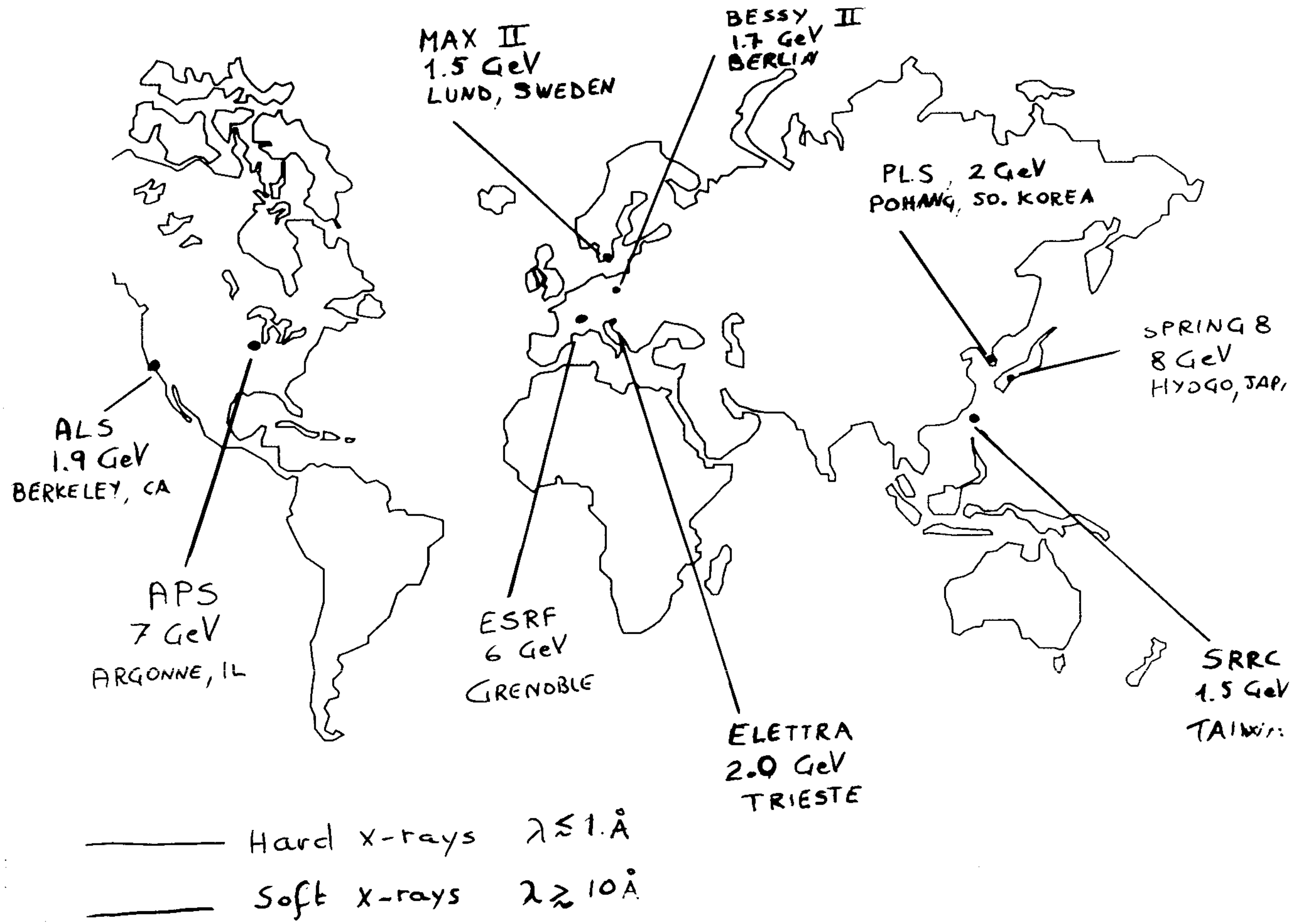
Brightness Curves of ELETTRA at 400 mA



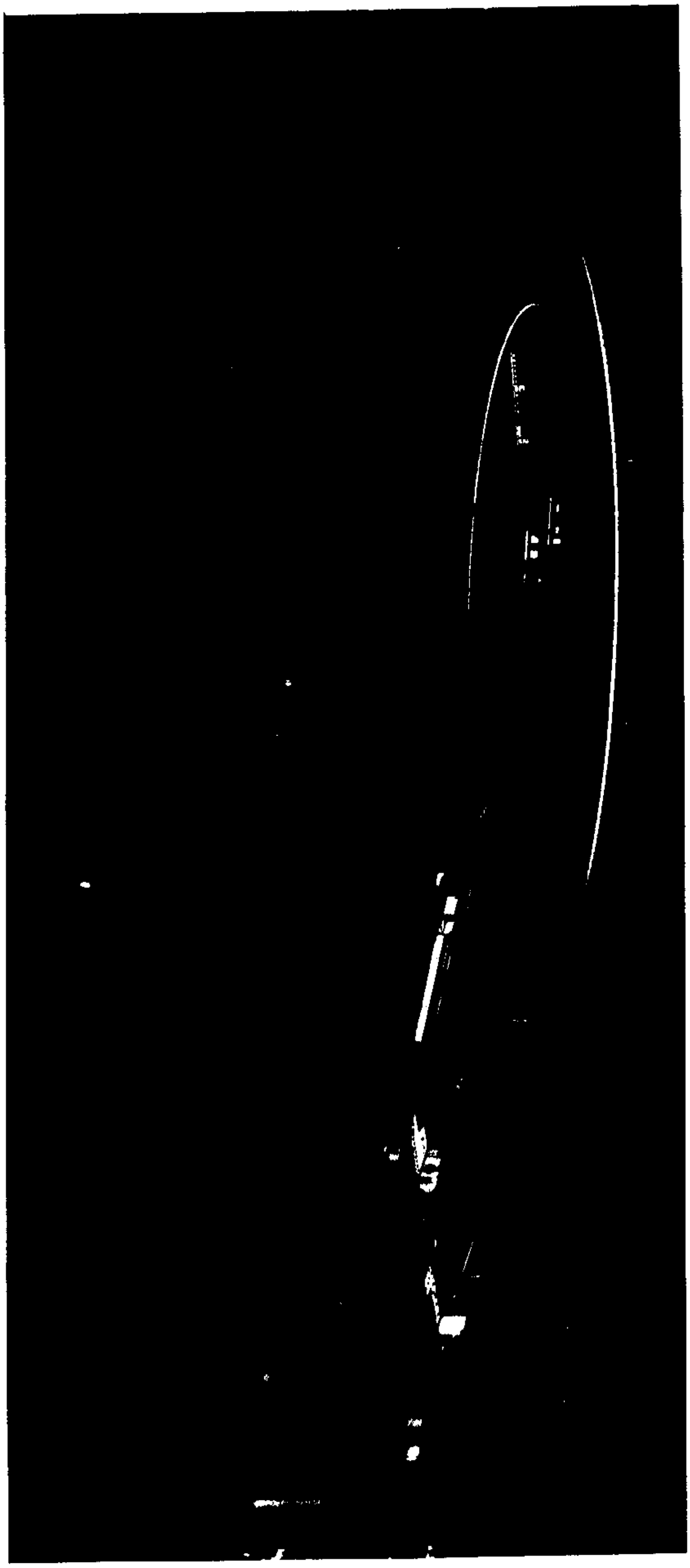




200 mA Diff. Limit
GROWTH IN X-RAY SOURCE BRIGHTNESS
(Photons/sec/0.1%bw/sq.mm/mrad^2)
3rd. Gen. original design
2nd. Gen.
Cray T90
1st. Gen. Synch. Rad.
Cray 1
CDC 6600
GROWTH IN LEADING EDGE COMPUTING SPEED
(Millions of operations/sec)
1960
1970
1980
1990
2000
Calendar Year


MAX II
1.5 GeV
LUND, SWEDEN
BESSY II
1.7 GeV
BERLIN
PL.S , 2 GeV
POHANG, SO. KOREA
SPRING 8
8 GeV
HYOGO, JAP,
ALS
1.9 GeV
BERKELEY, CA
APS
7 GeV
ARGONNE, IL
ESRF
6 GeV
GRENOBLE
ELETTRA
2.0 GeV
TRIESTE
SRRC
1.5 GeV
Hard x-rays $\lambda \lesssim 1.\text{Å}$
Soft x-rays $\lambda \gtrsim 10\text{Å}$

Geha



Präsentationshülle
Transparency jacket

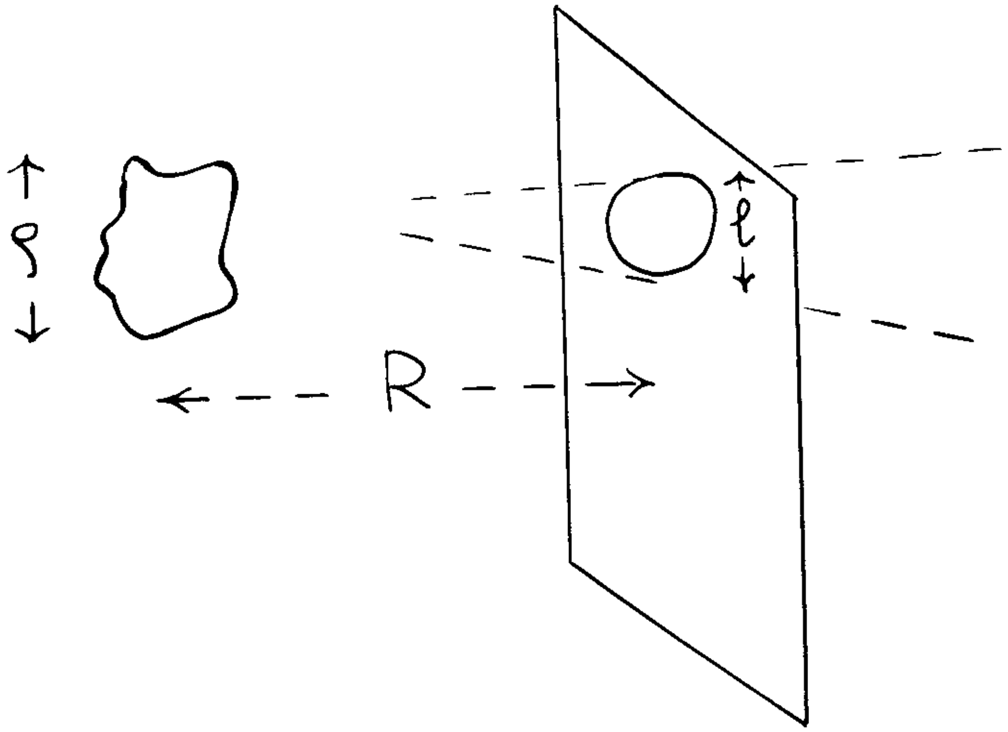
Properties of synchrotron radiation sources

TUNABILITY

BRILLIANCE

TRANSVERSE SPATIAL COHERENCE

POLARIZATION



Pinhole at distance R with:

$$\ell < \bar{\lambda} \frac{R}{2\rho} = \ell_{coh}$$

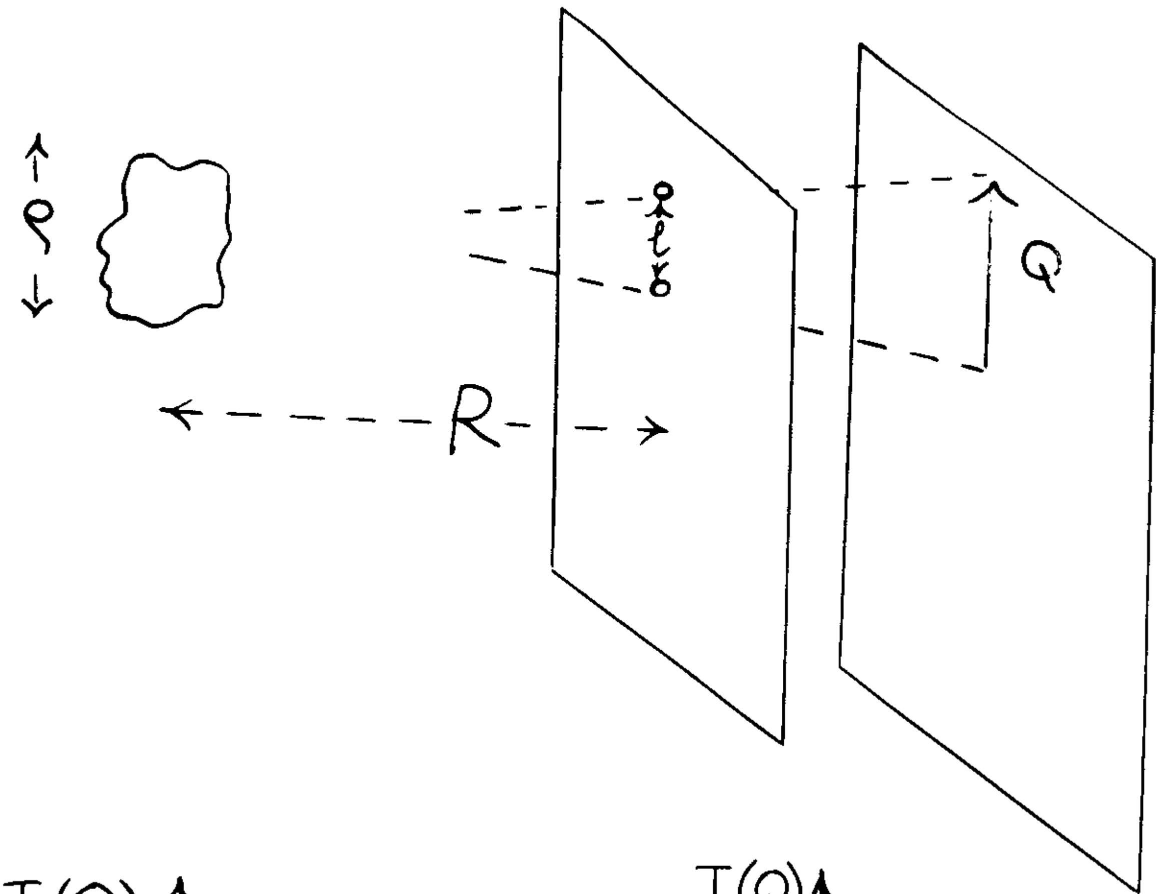
Coherent source.

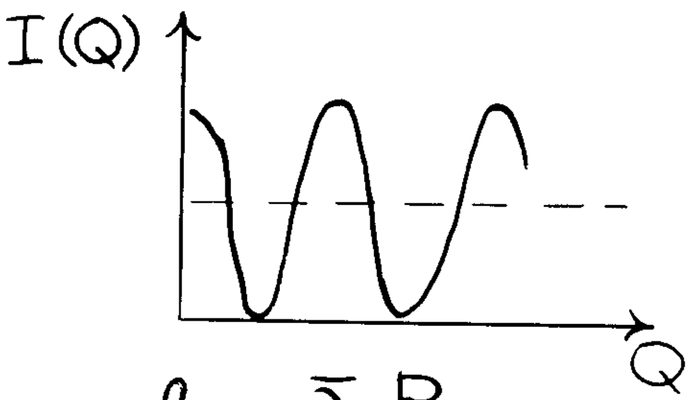
Sunlight: $\bar{\lambda} = 5.5 \cdot 10^{-5}$ cm

$\rho / R \simeq 10^{-2}$ $\ell_{coh} \sim 25 \mu m$

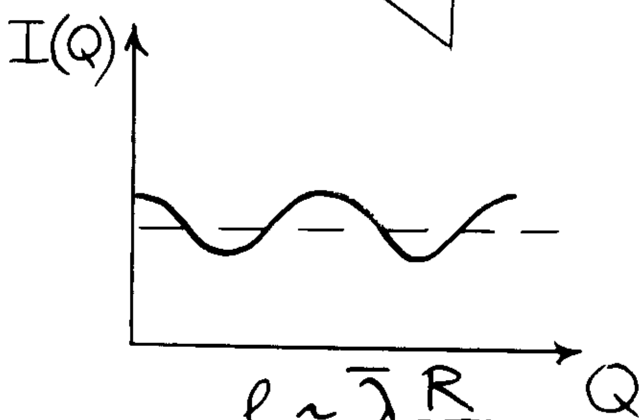
ESRF undulator: $\bar{\lambda} = 1.5 \times 10^{-8}$ cm

$\rho \sim 8 \cdot 10^{-2}$ cm $R \sim 50$ m $\ell_{coh} \sim 5 \mu m$

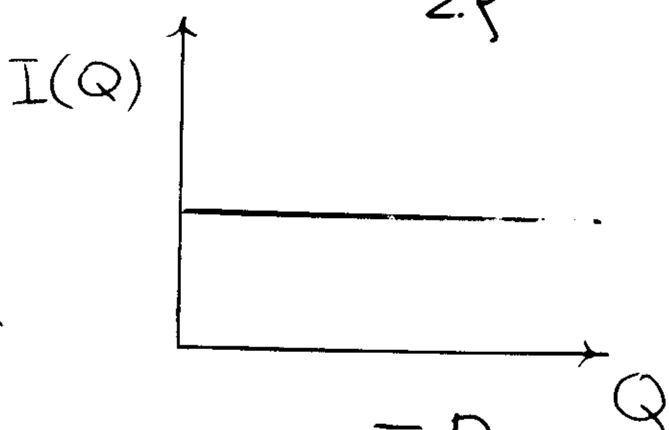




$\ell \ll \bar{\lambda} \frac{R}{2\rho}$



$\ell \sim \bar{\lambda} \frac{R}{2\rho}$

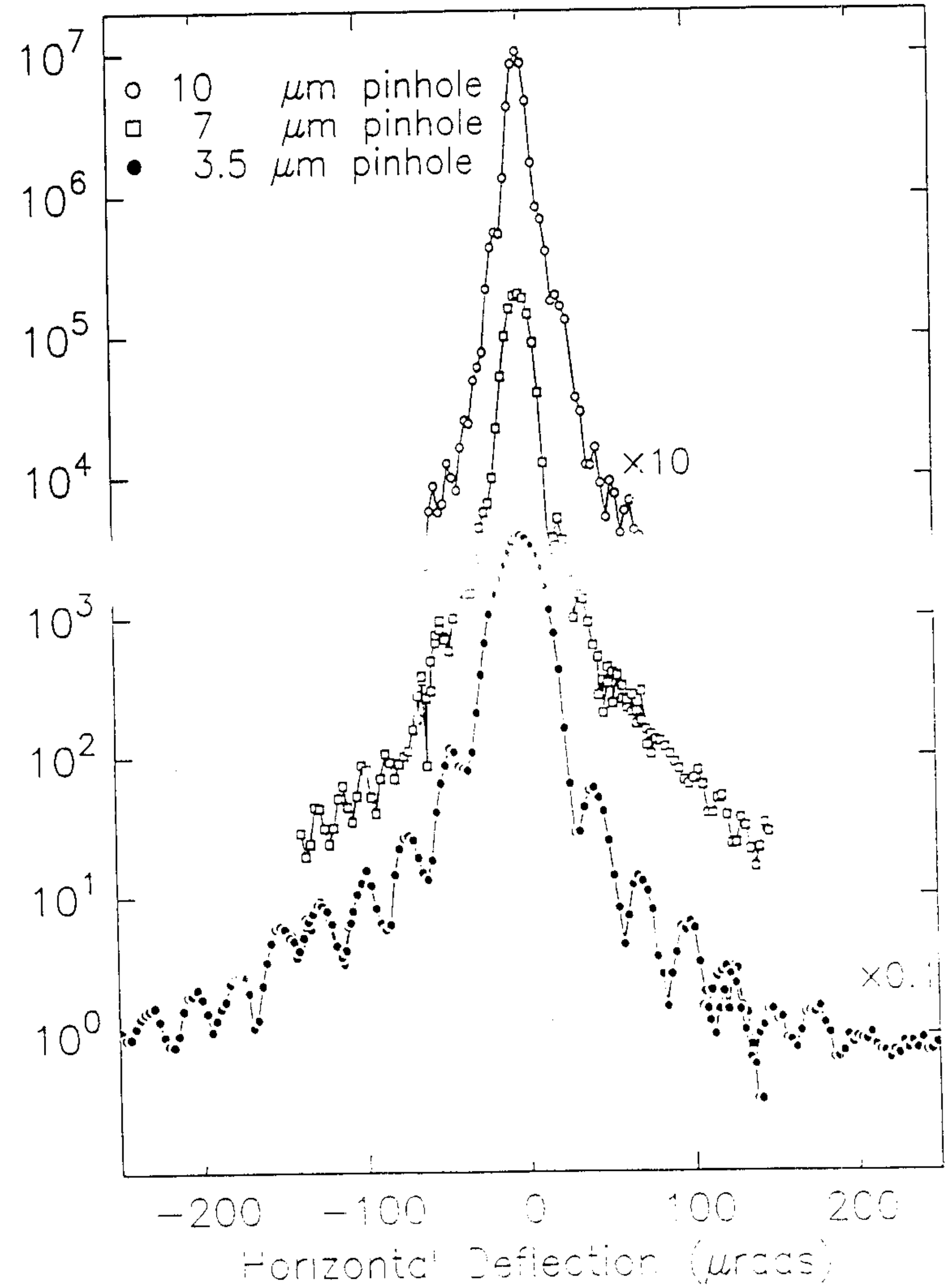


$\ell \gg \bar{\lambda} \frac{R}{2\rho}$

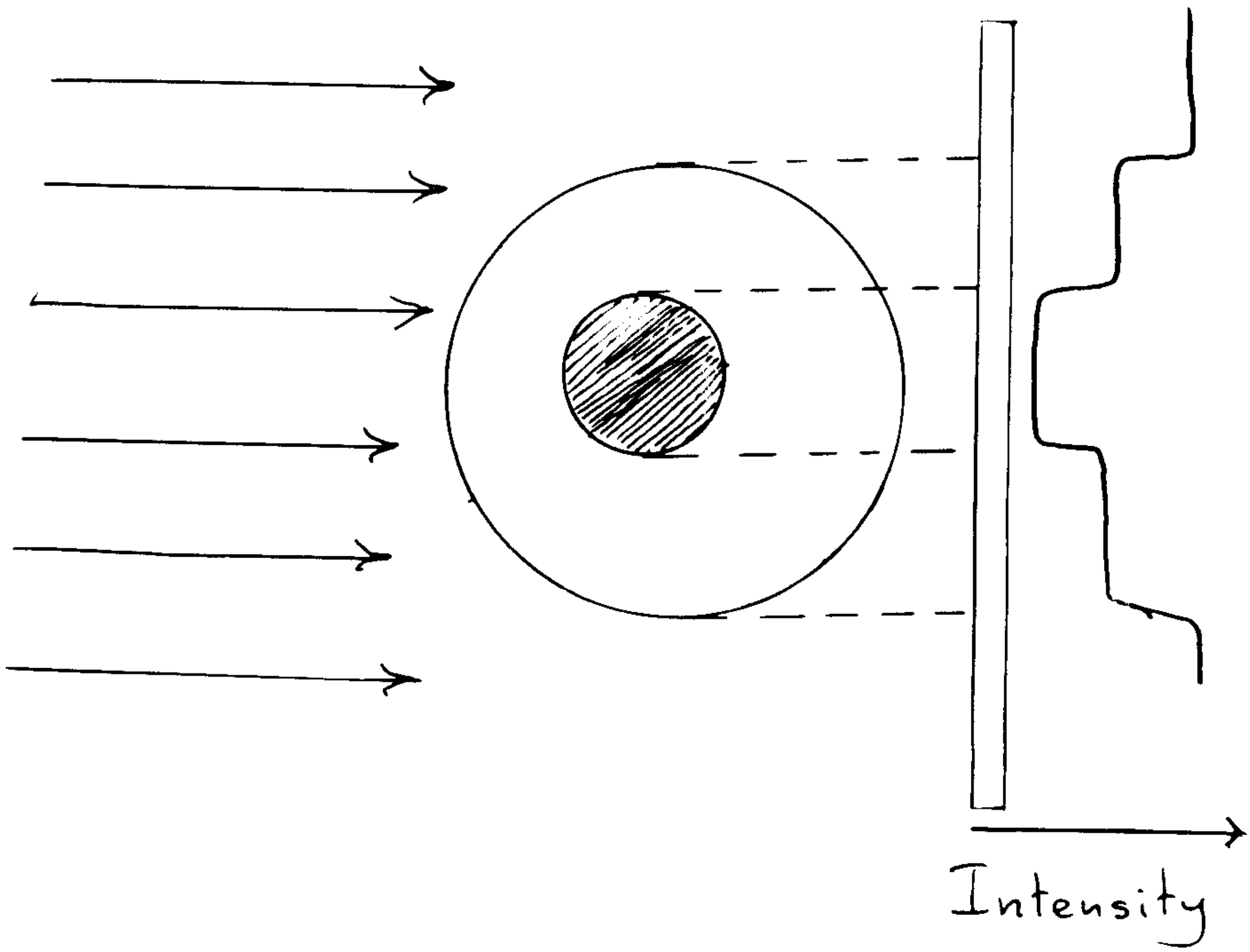
Transverse Coherence Length at distance R:

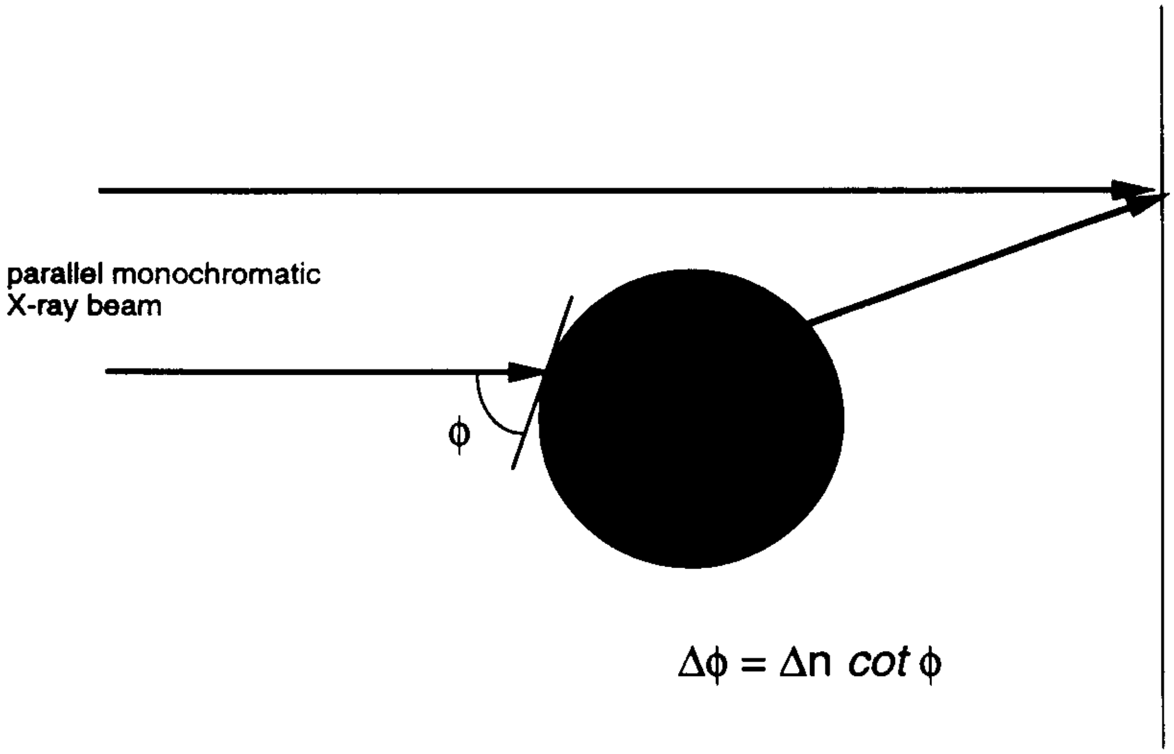
$$\ell_{coh} \sim \bar{\lambda} \frac{R}{2\rho}$$

G. Grübel
J. Als-Nielsen
USA + Canada team
Nov. 9, 1993



Radiography: Amplitude Contrast

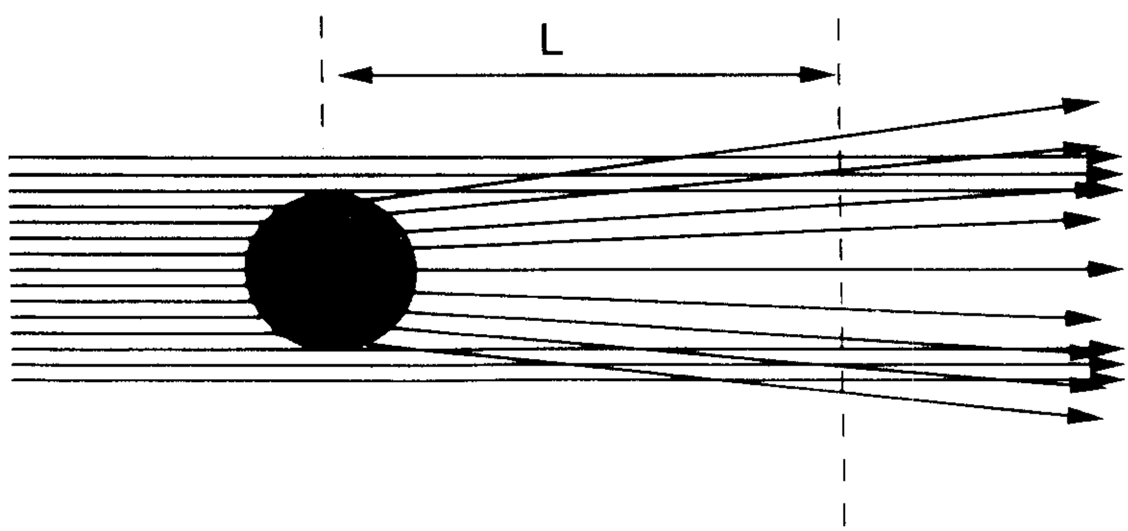




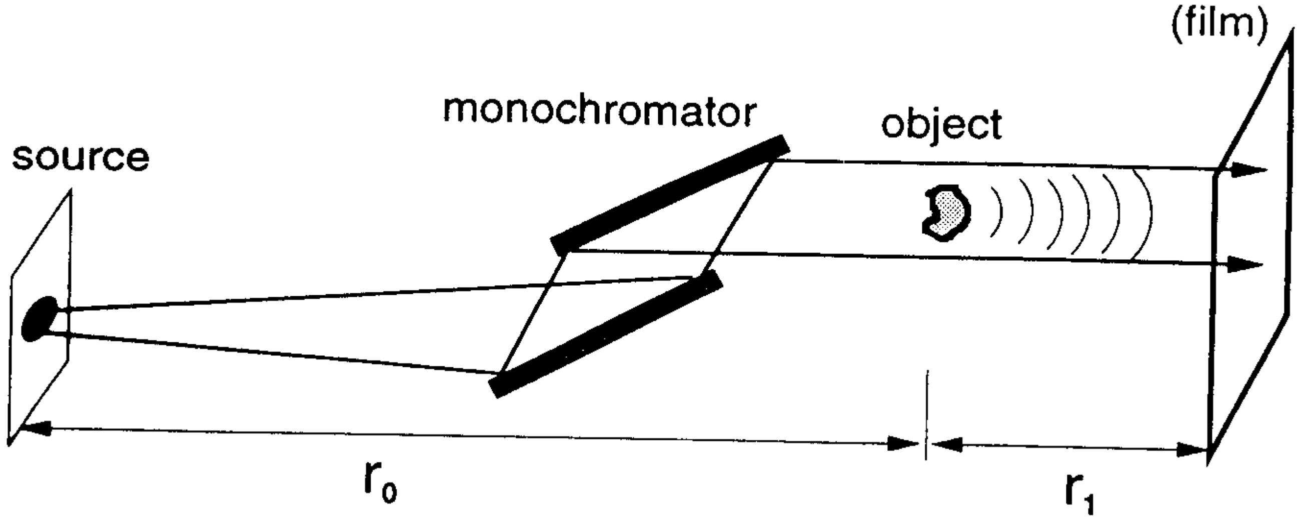
$$\Delta\phi = \Delta n \, cot\,\phi$$

$$\Delta n \sim 10^{-5} - 10^{-6}$$

$$cot\,\phi > 1,\ \phi > 45^{\circ}$$



Experimental conditions at the ESRF BL



source size: $s \sim 100\mu m$

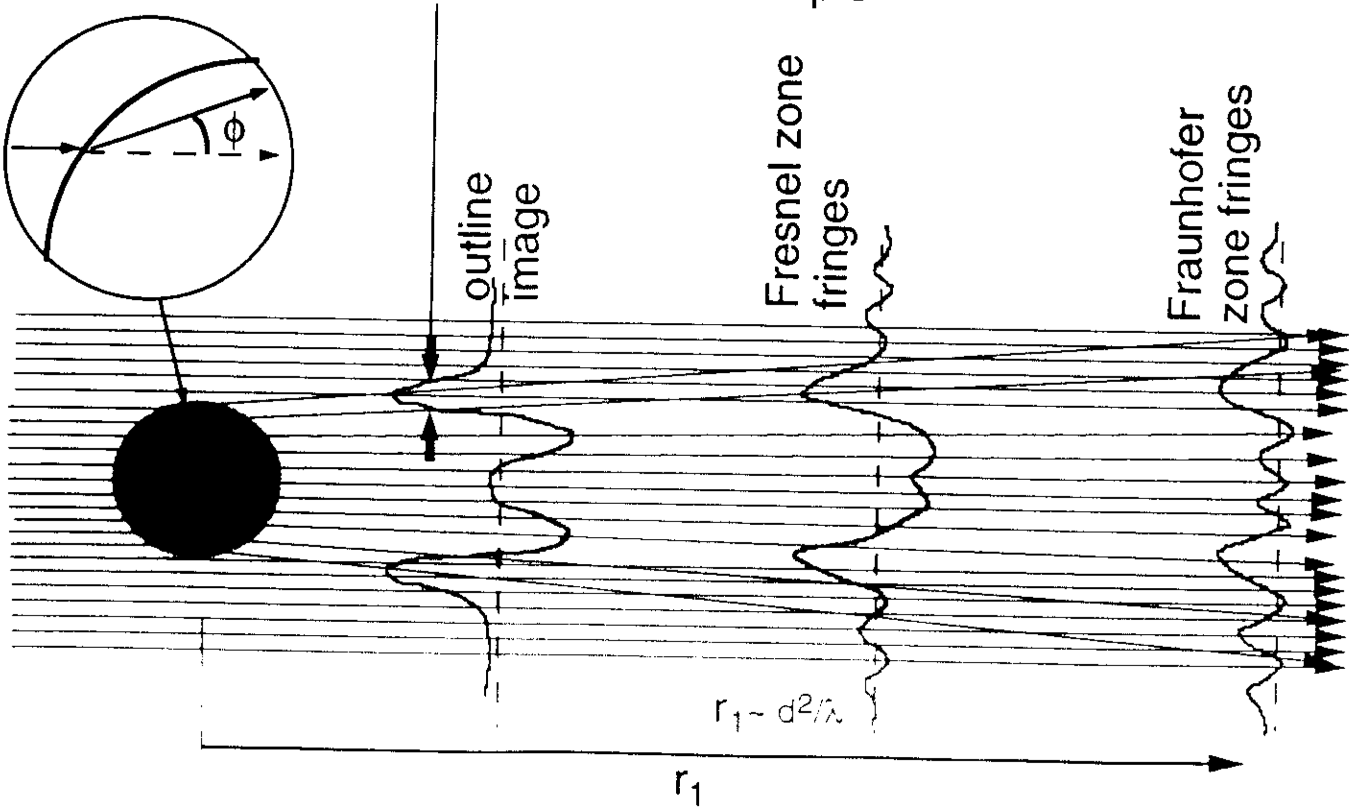
source-to-sample distance: $r_0 \sim 50m$ ⇒ spatial coherence

⇓

angular source size: ~ 2μrad

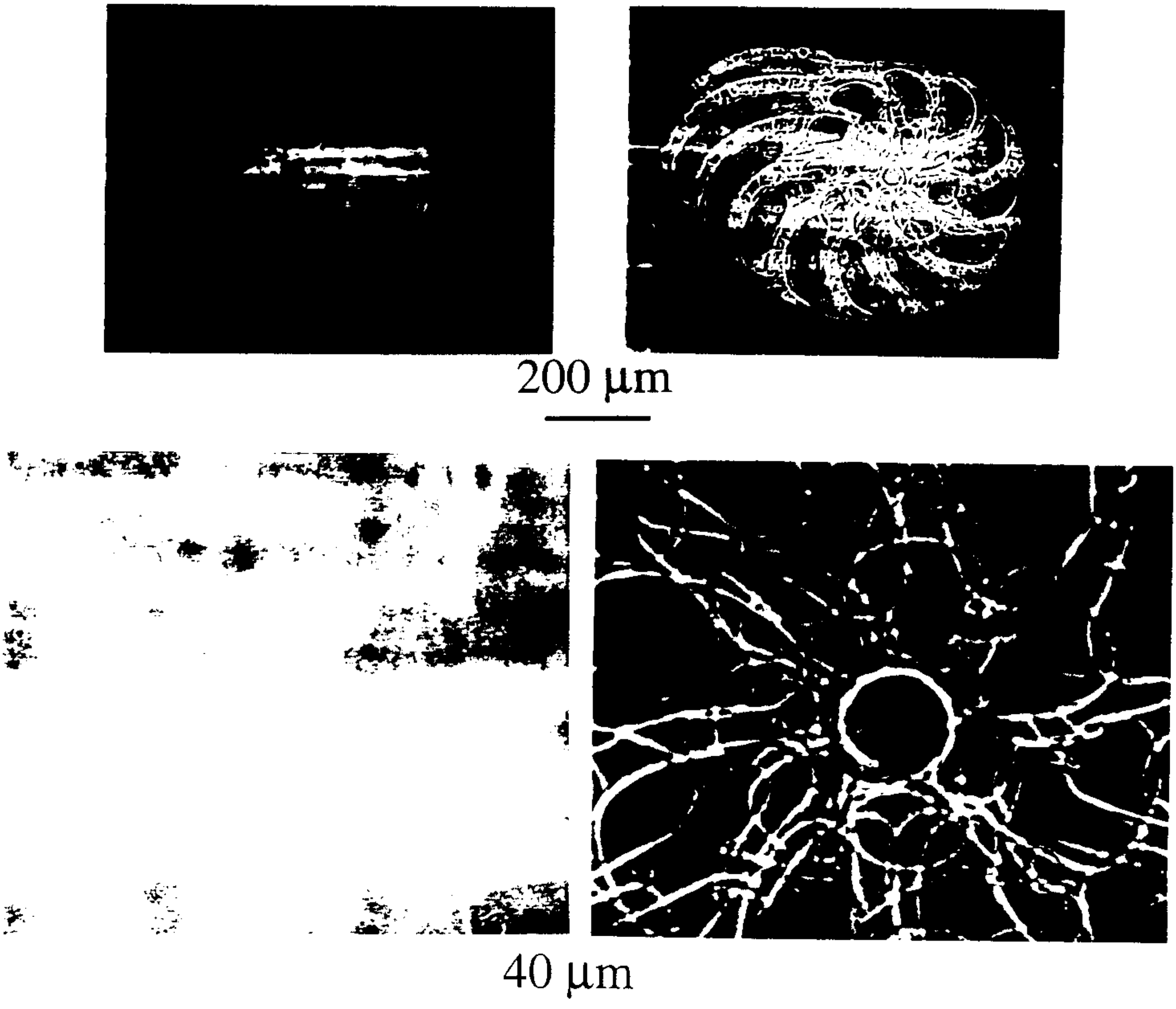
$\lambda r_0/s = 50\mu m$ (λ=1Å)

$\phi \sim \Delta n \sim 10^{-5}$ - 10^{-6} ⇒ edge contrast ~ 1-10μm at 1m from the sample



Snigirev A., SAC 4-5 May1995

X-ray imaging of the seashell "Peneroplis pertuses (Belize)" at 18 keV



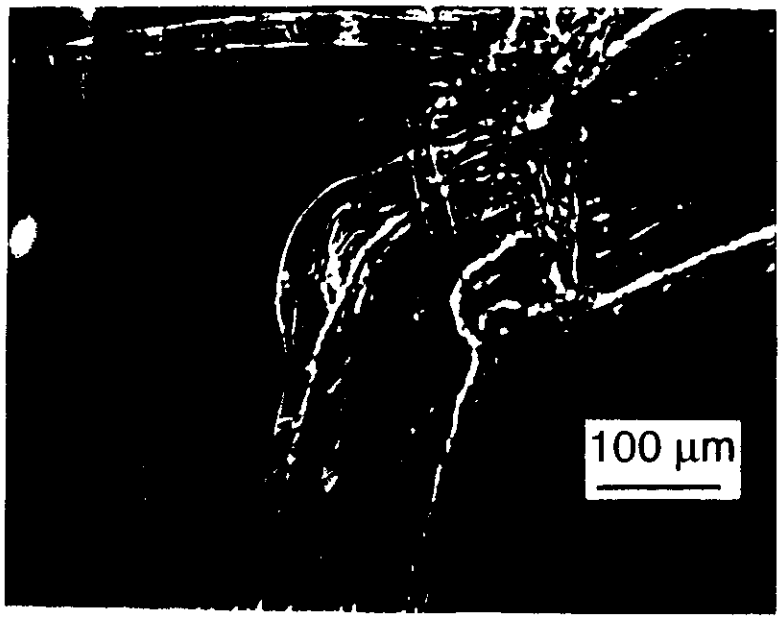
amplitude mode (0 cm)

phase contrast mode (10 cm)

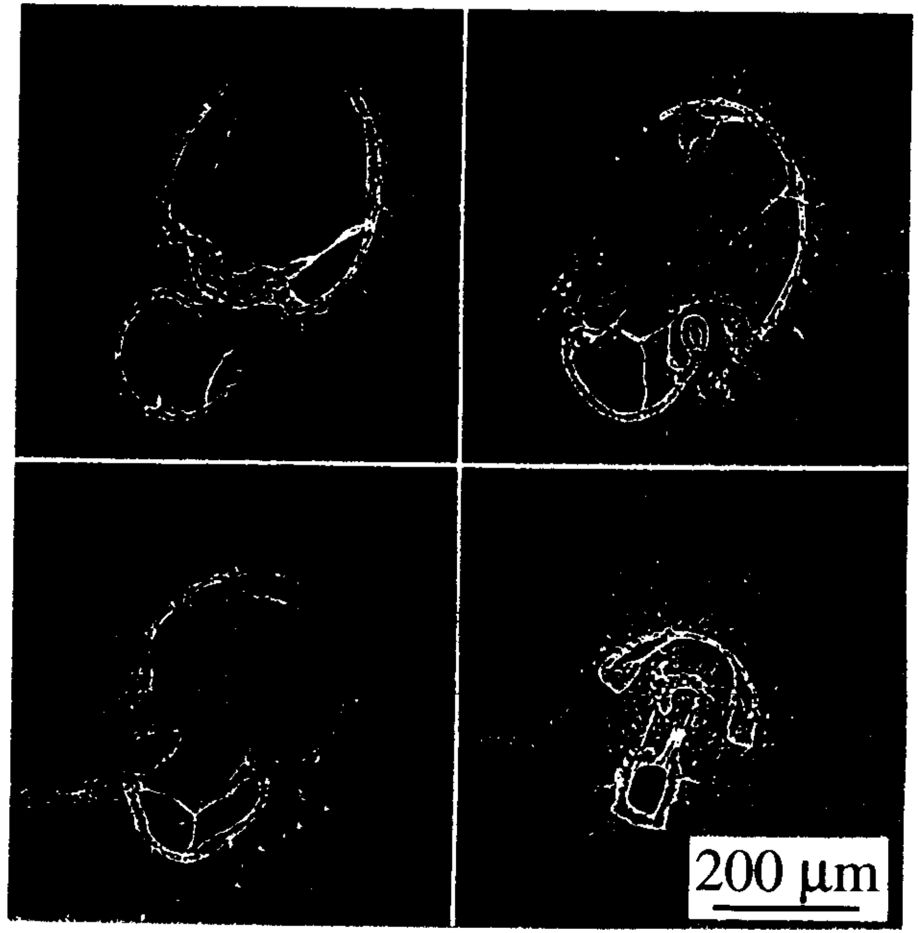
High Energy X - Ray Phase Contrast Tomography With µm - Resolution

E = 18 keV

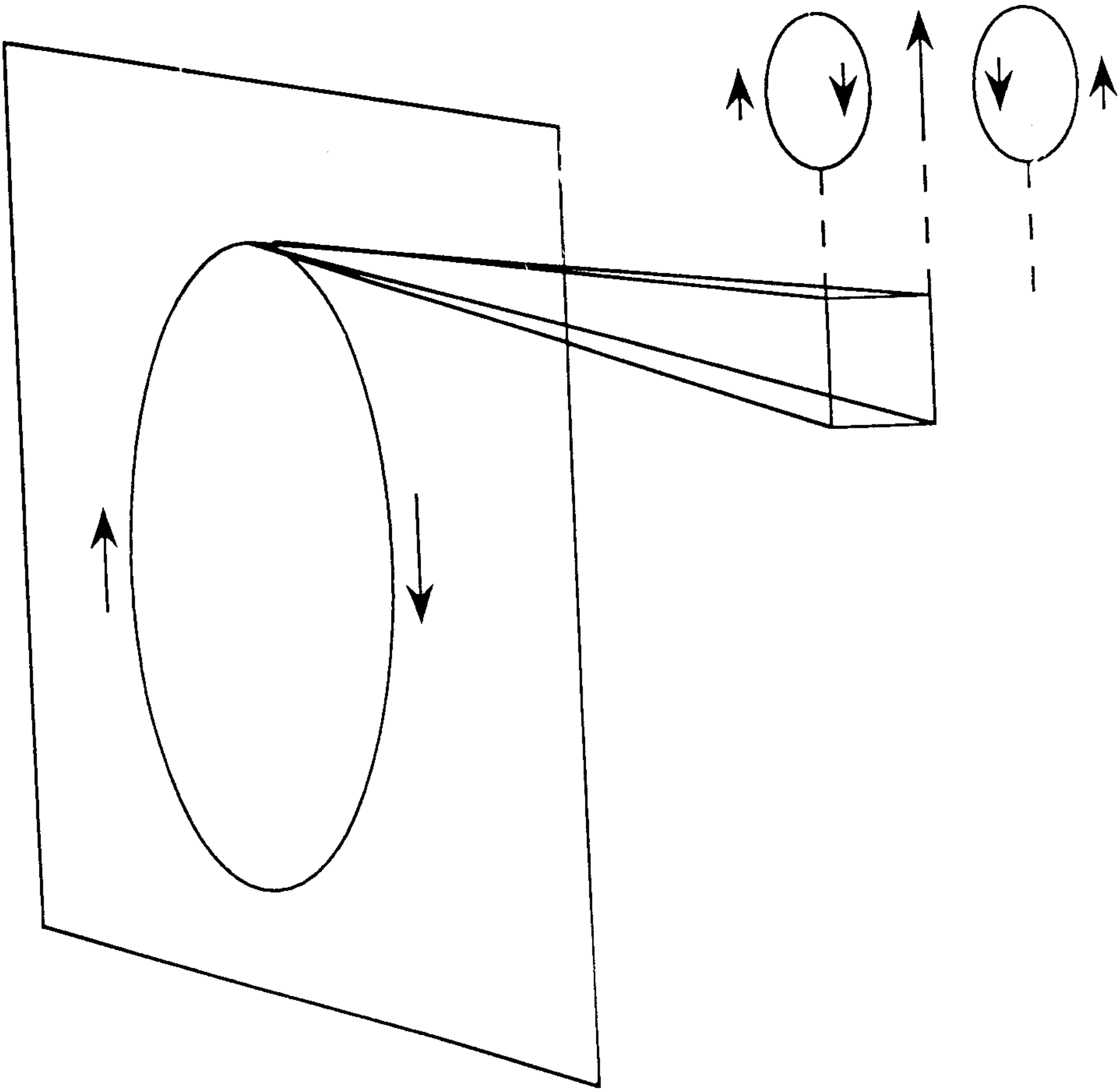
Mosquito knee



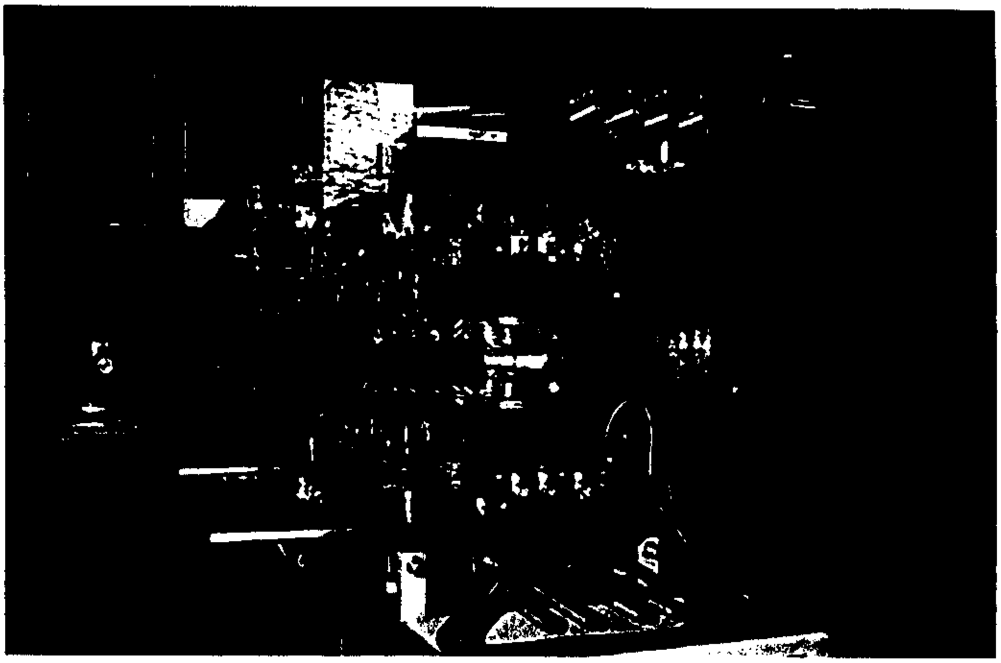
Phase contrast image



Cross sections through an insect's knee



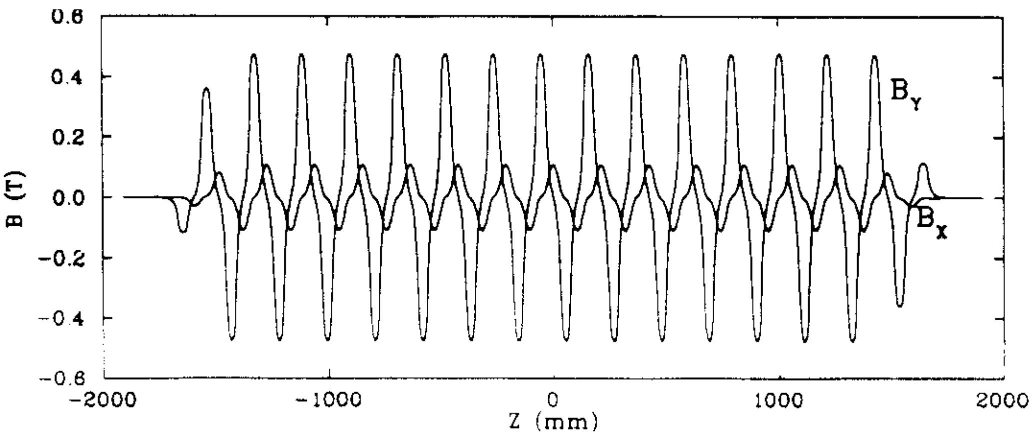
ELECTROMAGNETIC ELLIPTICAL WIGGLER



Main Parameters:

Period length	**212 mm**	
Total yoke length	**3.322 m**	
Pole gap	**18 mm**	
	Vertical field	**Horizontal field**
Maximum field amplitude	**0.59 T**	**0.11 T**
Number of poles	**32**	**31**

Measured field distributions at maximum current:



ELECTROMAGNETIC ELLIPTICAL WIGGLER

Special features:

- **double electromagnet: two field components contained in the same yoke**

- **open sided structure giving easy access for magnetic measurement, vacuum chamber installation etc.**

- **undulator and wiggler modes of operation, covering wide operating range, 5 eV - > 2 keV**

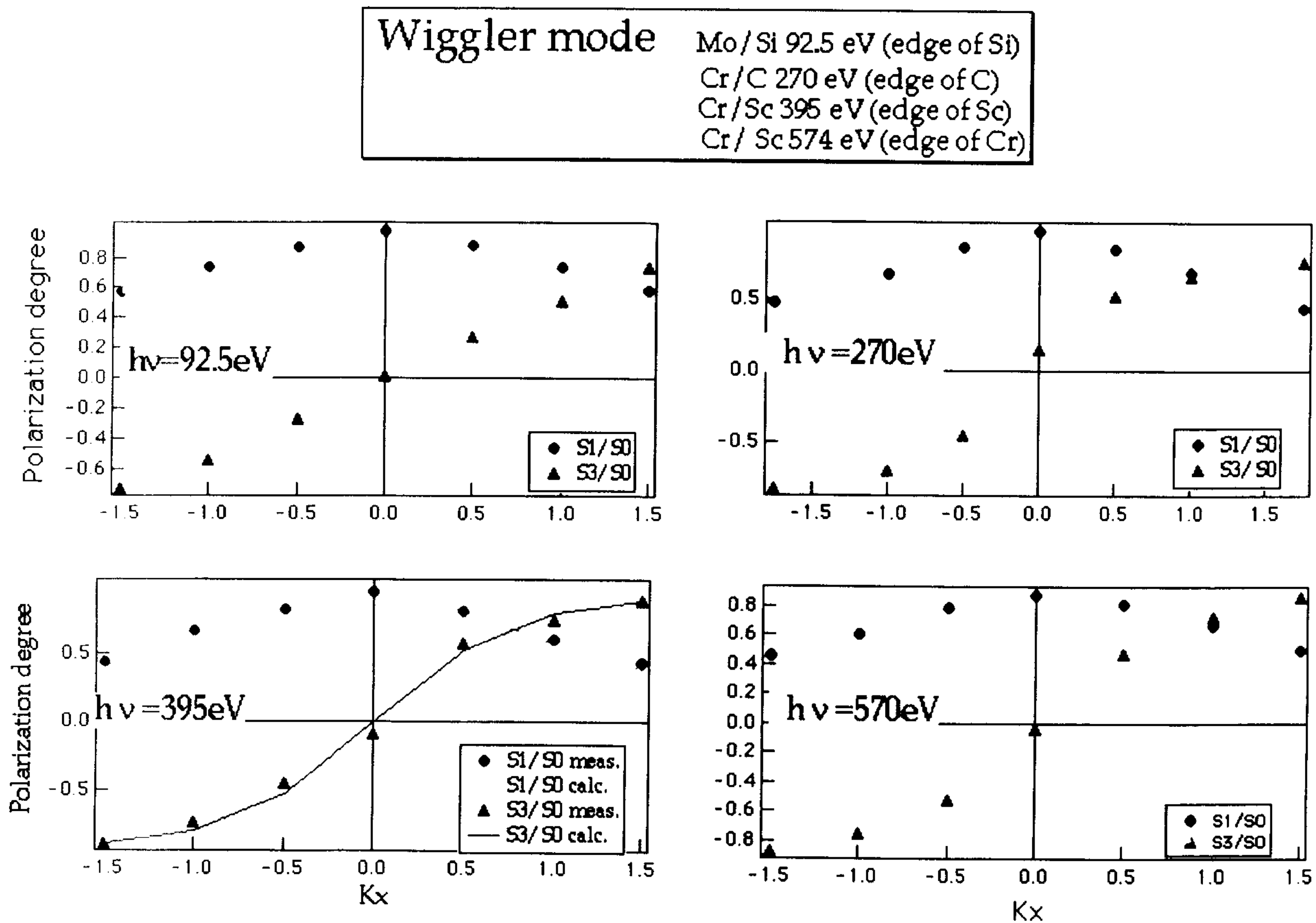
- **helicity of the circularly polarized component can be switched at up to 100 Hz**

- **programmable waveform for the horizontal field component:**

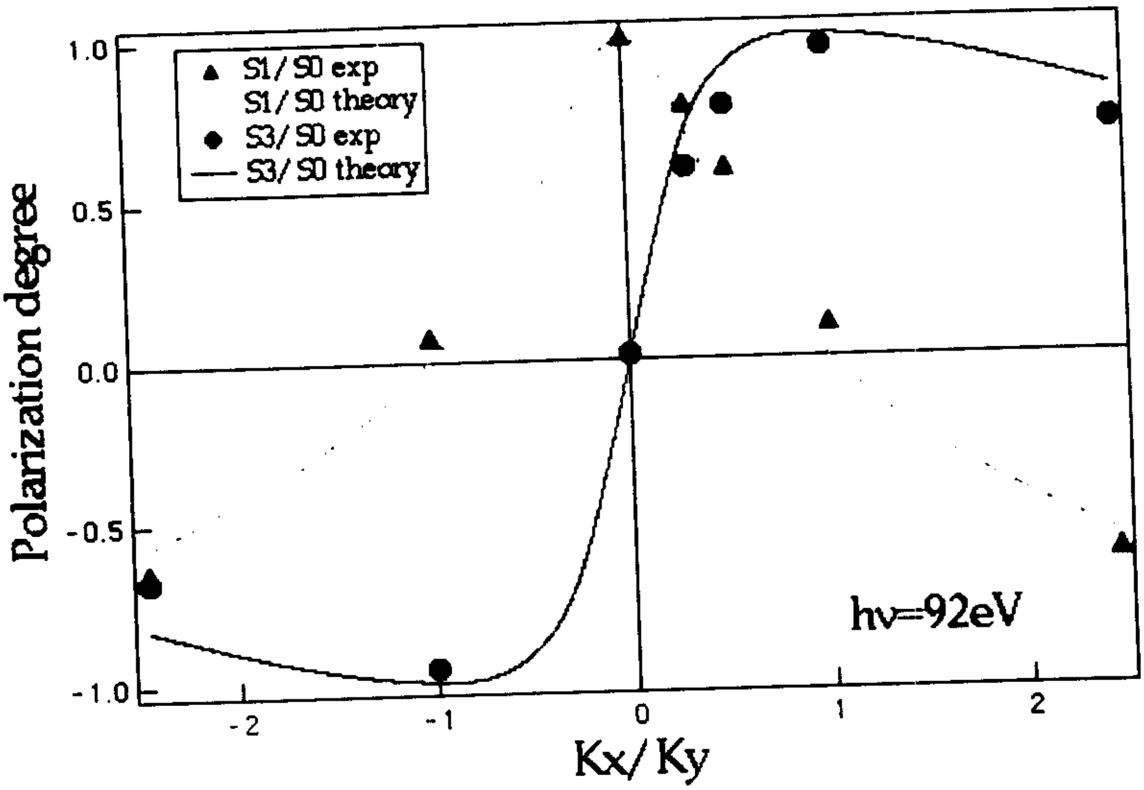
- d.c.

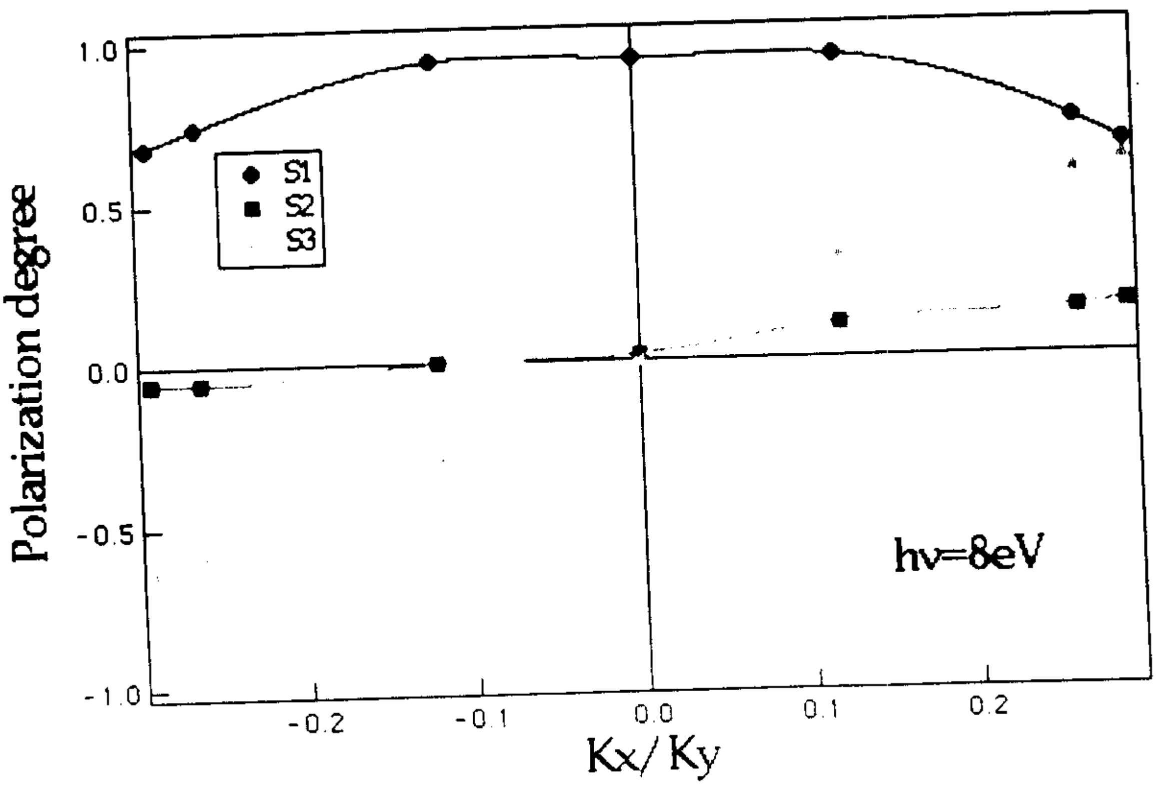
- trapezoidal, up to 1 Hz

- sinusoidal, up to 100 Hz


Wiggler mode
Mo/Si 92.5 eV (edge of Si)
Cr/C 270 eV (edge of C)
Cr/Sc 395 eV (edge of Sc)
Cr/ Sc 574 eV (edge of Cr)
hν=92.5eV
hν=270eV
hν=395eV
hν=570eV
Polarization degree
Kx
S1/S0
S3/S0
S1/S0 meas.
S1/S0 calc.
S3/S0 meas.
S3/S0 calc.

Undulator mode





Chen et al, Phys Rev. Lett. 75, 152 (95)

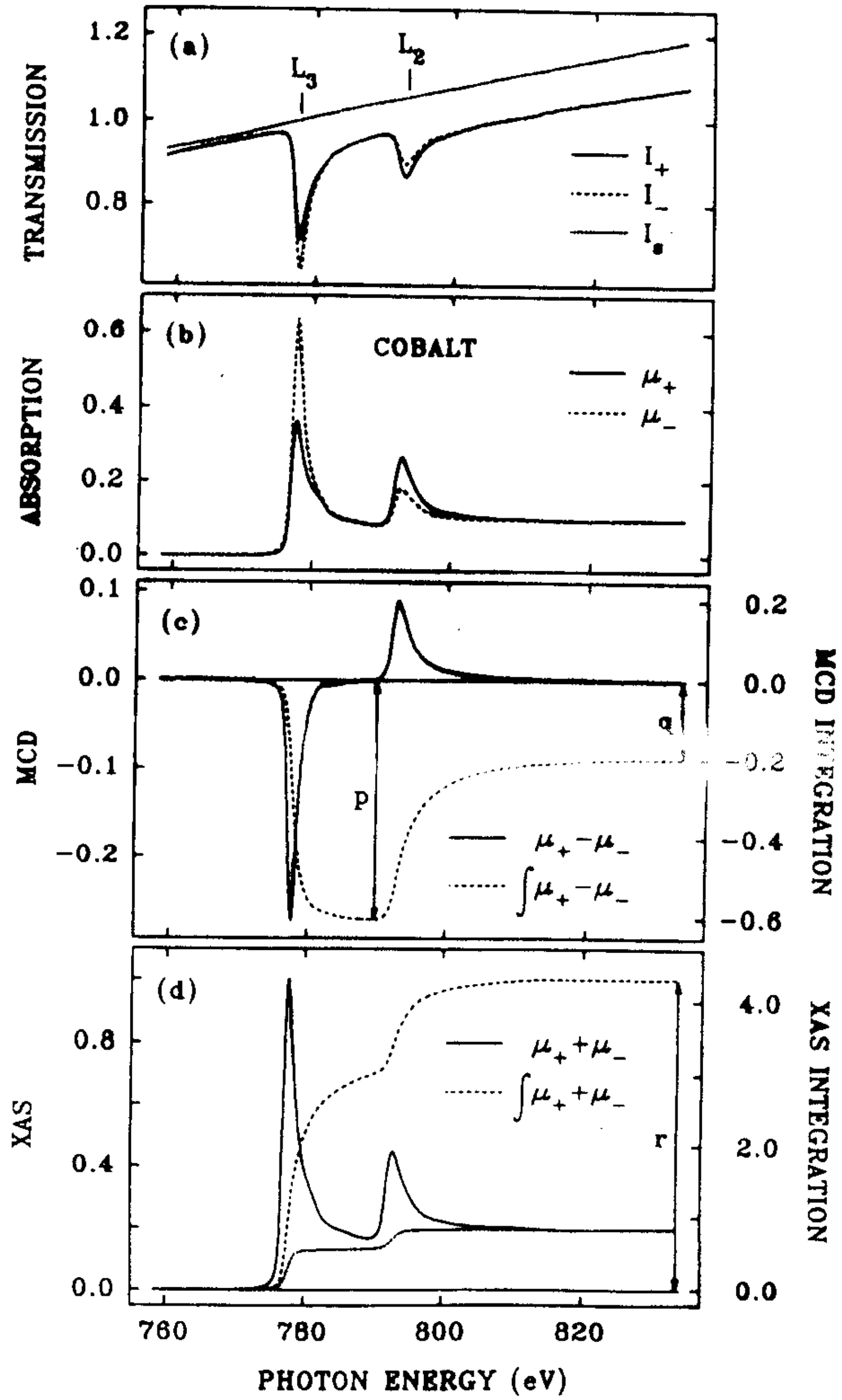


Fig. 2. $L_{2,3}$-edge XAS and MCD spectra of cobalt. The descriptions for figures (a)–(d) are the same as those of Fig. 1.