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"Erbium Doped Fibre Amplifiers"

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ERBIUM DOPED FIBRE AMPLIFIERS

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1. INTRODUCTION

In optical fibre communications systems, the optical signal pulses are subjected to power loss and pulse spreading due to attenuation and dispersion in the optical fibre. These signal degradation mechanisms limit the transmission distance of any system. In long haul links, such limitations are overcome most often by the use of optoelectronic repeaters, which are essentially back to back combinations of receiver and transmitter units. The receiver detects the degraded optical pulses, regenerates the original electronic digital signal, restoring its amplitude and pulse width, and feeds it as the modulation current to the laser driver. In this way the optical signal is fully restored to its original form. Optoelectronic repeaters are designed for operation at a specified bit rate. They will not allow higher bit rates and hence, links using this technology cannot be upgraded by advances in the terminal equipment. Furthermore, in any wavelength division multiplexing (WDM) scheme, each wavelength channel would require its own dedicated repeater making such systems prohibitively expensive.

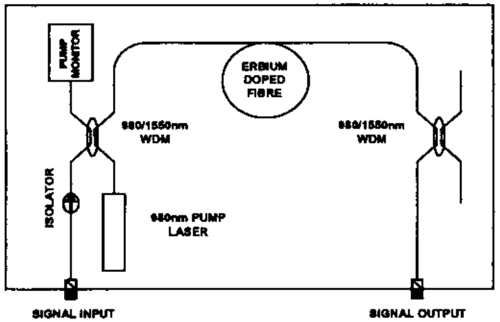


Figure 1.1: Schematic of EDFA

Full regeneration of the original signals in both amplitude and time is essential in systems limited by dispersion. However, in attenuation limited systems only amplification of the signal is required and many advantages may be gained if this is achieved optically without conversion of the signals back to the electrical domain. After many years of R & D effort, the currently favoured technology for optically amplified systems utilises the erbium doped fibre amplifier or EDFA for short (Figure 1.1). In this approach, a length of erbium doped fibre is simply spliced into the transmission line. When energised by the pump laser, the erbium doped fibre provides up to 40dB of optical power gain for a single pass. Use of these devices as amplifiers in long haul transmission systems offers many advantages relative to optoelectronic repeater technology. For instance, amplification is independent of bit rate and is available over a useful range of wavelengths (1540-1565nm), giving rise to the terms bit rate and wavelength transparency associated with these amplifiers. This implies that optically amplified systems may be upgraded in capacity by using higher bit rate terminal equipment and / or by adopting WDM techniques, again only requiring

appropriate terminal equipment. In addition, EDFAs require less power and are less complex and more reliable than optoelectronic repeaters.

Two major disadvantages must be noted. EDFAs do not restore the pulse width in time and hence the effects of dispersion accumulate over the system. In addition, optical noise is added to the signal at each amplifier. Given that the technologies of very low dispersion fibre and dispersion compensating fibre are now mature, EDFAs may be used effectively in very long haul, even transoceanic, systems. The added optical noise does present a problem, but, if it is properly managed in the design and operation of the system, useful and technically superior systems may still be realised.

In these notes, we examine the basic principles of optical amplifiers[1,2] in general before addressing the principles, characteristics and applications of EDFAs. Precise analysis of the performance characteristics of EDFAs is mathematically tedious and only necessary if we require absolute simulation accuracy. Although some design and operational features of EDFAs are significantly different from those of bulk optical amplifiers, they exhibit all of the same relative performance characteristics defined, in general, by the same physical phenomena. Therefore, considerable insight into understanding the performance characteristics of EDFAs may be gained from the less tedious analysis of bulk optical amplifiers. In particular, analysis of the noise characteristics and signal to noise ratios arising from the use of bulk optical amplifiers is very useful and indeed, directly applicable to many systems involving EDFAs. Also, analysis of the gain and power saturation characteristics of bulk optical amplifiers predicts the same general performance trends and features as found in EDFAs. In these notes we develop a simplified analysis of the operation of bulk optic amplifiers before describing the structure, principles and characteristics of EDFAs in relation to our model of the bulk optics case. The difference between EDFAs and bulk amplifiers are discussed and particular system applications are described and analysed.

2. PRINCIPLES OF ATOMIC RADIATION

2.1 Photon - Material Interactions

To understand the basics of optical amplifiers we must firstly examine the three ways in which photons of light may interact with materials. Consider a system (Figure 2.1) in which a beam of light passes through a volume of material containing atoms which have two allowed energy levels denoted E_i and E_j. N_i and N_j, respectively, are the numbers of atoms per unit volume in these energy states, usually referred to as the populations or population densities. As we know from quantum theory, atoms and molecules in any material system can only exist at certain discrete allowed energy levels and can only change their energy by discrete jumps between these allowed levels. In most cases involving the emission or absorption of visible / near infra-red light, the energy exchange process involves the transition of an electron between two allowed orbits of fixed, discrete energy. Energy may be exchanged between light and such an atomic system in three ways: spontaneous emission, absorption and stimulated emission (Figure 2.2) described in detail below.

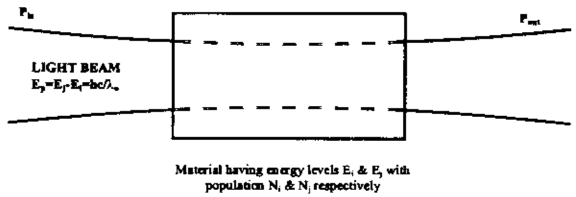


Figure 2.1: *Light of photon energy $E_p=hc/\lambda_o$ passing through a potential gain material having 2 energy levels E_j & E_i (E_j-E_i=E_p)*

2.1.1 Spontaneous emission

An electron occupying an orbit of energy level E_j within an atom may randomly make the transition to another orbit of energy E_i (Figure 2.2a) by giving up a photon of energy $E_p = E_j - E_i = h\nu = hc/\lambda$, where ν and λ are the frequency and wavelength of the emitted photon, h is Plank's constant (6.63 x10^{-34}) and c is the speed of light. Each photon produced results from one atom making the transition from state E_j to E_i Hence, the rate of photon emission is equal to $-dN_j/dt$ which is directly proportional to the number density of atoms, N_j (no. per m^3), in the energy level E_j and we can write:

$$Rate_{[SpE]} = -\mathrm{d}N_j/\mathrm{d}t_{[SpE]} = A_{ji}N_j \tag{2.1}$$

where A_{ji} is the proportionality constant.

In this totally random process, referred to as spontaneous emission, the photons are emitted randomly in time and direction implying that light of random polarisation is emitted into a 4π steradian sphere with no phase correlation from photon to photon (i.e. random phase from photon to photon).

Equation 2.1 represents the instantaneous rate of decline of the population N_j of energy level E_j given no other influences. Hence, if we pump a large number of atoms, ΔN_j, from the ground state up into energy level E_j and then abruptly switch off the pumping source, the population N_j and the spontaneous emission intensity will decay according to equation 2.1. We can readily solve equation 2.1 to give N_j as a function of time as follows:

$$N_j = \Delta N_j \exp(-A_{ji}t) = \Delta N_j \exp(-t/\tau_{ji}) \tag{2.1a}$$

Equation 2.1a indicates that a population perturbation, in excess of thermal equilibrium, decays exponentially with a time constant $\tau_{ji} = 1/A_{ji}$ which is referred to as the spontaneous emission lifetime of state j (i.e. the time for the population to reach 1/e of its initial peak perturbation).

Note that equation 2.1a applies only to transitions between 2 specific energy levels. More generally, spontaneous emission may occur to a number of energy levels below state j and we have:

$$Rate_{[SpE]} = -\mathrm{d}N_j/\mathrm{d}t_{[SpE]} = A_{j0}N_j + A_{j1}N_j + ...A_{jn}N_j$$

and

$$N_j = \Delta N_j \exp{-A_jt} = \Delta N_j \exp(-t/\tau_j) \tag{2.1b}$$

where A_{jn} are the spontaneous emission rate constants for transitions between states j and n, τ_{jn} are the spontaneous emission time constants for these transitions, $A_j = \sum_n A_{jn}$ and $\tau_j = 1/A_j$.

If the population of state j decays by additional, perhaps non-radiative, mechanisms, such as atomic collisions or lattice phonon interactions, with rate constants k_{jn} to states n, then equation 2.1b must be written as:

$$N_j = \Delta N_j \exp[-(A_j + k_j)t] = \Delta N_j \exp[-t/(A_j + k_j)] \tag{2.1c}$$

where the lifetime, τ_j^*, is now equal to $1/[A_j + k_j]$. Spontaneous emission to additional energy levels or further non-radiative decay mechanisms may be accounted for by adding the appropriate rate constants to the expression within the square brackets of equation 2.1b.

2.1.2 Absorption

In the system depicted in Figure 2.1, photons of energy $E_p = E_j - E_i = h\nu = hc/\lambda$ may be absorbed by atoms in energy level E_i (Figure 2.2b) which make the transition to energy level E_j as their electrons move between the corresponding orbits. Since each transition involves the absorption of one photon into one atom, the rate of absorption (i.e. the rate of photons removed from the incident beam) is equal to $-dN_i/dt$. Intuitively, therefore, the rate of absorption is proportional to the population density of atoms, N_i, in energy level E_i and to the photon energy density at frequency ν. Hence, we can write:

$$Rate_{[ABS]} = \mathrm{d}N_i / \mathrm{d}t_{[ABS]} = B_{ij}N_i\rho(\nu) \tag{2.2}$$

Equations 2.18 to 2.22 clearly indicate that the emission of light from a specific atomic transition is not a single frequency but occurs over a spread of frequencies with a FWHM linewidth which can be defined in terms of the rate constant or lifetime of the transition. However, the analysis as presented above is a little over simplistic. We must consider that the population of state j will decay by spontaneous emission to all states below it at a rate , $A_j = 1/\tau_j$ and further, that state i decays to all states below it at a spontaneous emission rate, $A_i = 1/\tau_i$. The interpretation of this situation and of expressions such as equation 2.21 is quite clear. There is an uncertainty in the time that an atom can remain in state j implying that there is a corresponding uncertainty in the energy of that state. The same argument applies to state i. Hence, if we are dealing with a transition from state j to state i we must consider the spread in energy associated with each state which results in a spread in photon energies and frequencies associated with transitions between the two states. Given this argument, the normalised spectral distribution of the frequencies emitted by the ji transition is given by equations 2.21 and 2.22, but the line width is given by

$$\Delta\nu = \frac{1}{2\pi(\tau^{*}_{ji})} = \frac{1}{2\pi}\left(\frac{1}{\tau_j} + \frac{1}{\tau_i}\right) = \frac{A_j + A_i}{2\pi} \qquad (2.24)$$

where $\tau_{ji}^{*} = 1/[A_j + A_i]$.

Equation 2.24 represents the theoretical lower limit on the FWHM spread of frequencies emitted or absorbed by the ji transition as defined by the spontaneous emission decay processes only. As such they are referred to as the natural lineshape and the natural linewidth of the transition respectively.

As we have seen, other decay processes, such as collisions between atoms or molecules in a gas, shorten the lifetimes of excited states (see equation 2.1c). Taking these into consideration and substituting the appropriate shorter time constants into equation 2.24 results in a broadening of the linewidth. In addition, collisions during the emission process interrupt the phase of the emitted waveforms, further and more significantly broadening the frequency content of the emission. For these reasons such processes are known as broadening mechanisms. There are many types of broadening process such as interactions between the emitting atoms and lattice vibrations or phonons in solids and the analysis of their effects can be very complicated and often inaccurate in predicting the resultant lineshape. The broadening mechanisms and the precise analysis of their effects on the line shape are often unimportant, provided we can measure the lineshape and understand its behaviour under the operating conditions of any process we are investigating. It is, however, important to distinguish and fully understand the differences between two very specific categories of broadening viz. homogeneous and inhomogeneous line broadening.

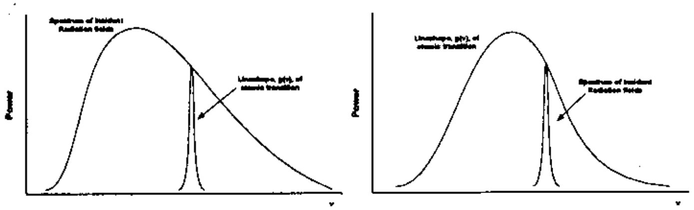
For materials in which the probability of a transition (spontaneous, stimulated or absorption) as a function of wavelength is the same for all atoms, then the linewidth or gain curve is said to be ***homogeneously broadened***. This implies that all of the active atoms in the gain medium are subjected to the same environment, external influences and homogeneous broadening mechanisms (mechanisms which affect all atoms equally). One such mechanism is collisional broadening in a gas. All atoms in a gas at uniform temperature have the same probability of suffering collisions during the emission process.

If the distribution of transition probability with wavelength varies for different groups of atoms, the linewidth or gain curve is said to be ***inhomogeneously broadened*** (see "Optical Amplifiers"). Inhomogeneous broadening occurs in gain materials in which the active atoms occupy different local environments within the host material, such as the different possible doping sites in a crystal. Alternatively, it occurs if the broadening mechanism affects different groups of atoms in a different way, such as in Doppler broadening in a gas where the Doppler shift in wavelength for any atom making a transition depends on the vector component of the atom's velocity in the direction of the beam axis. Hence, different atoms travelling in different directions relative to the optical axis and at different velocities give rise to a distribution of wavelengths in the Doppler broadened lineshape. For materials with inhomogeneously broadened lines each atom (or group of similar atoms) has a homogeneously broadened transition probability and the entire lineshape or gain curve for the material is the summation of the homogeneously broadened lineshapes of all atoms.

2.4 Transition Rates For Narrow Band Radiation

The frequency dependence of the relative probability of emission and absorption must be taken into account appropriately in the transition rate expressions 2.2 and 2.3. Consider firstly the situation depicted in Figure 2.4a in which the atomic transition is interacting with a broadband radiation field as in the black body radiator. To determine the rate of absorption or stimulated emission the energy density, $\rho(\nu)$, in each frequency interval $d\nu$ must be weighted by the probability that it will lead to a transition as determined by the line shape function, and the total weighted energy density function is found by integrating with respect to frequency over the limits from 0 to infinity. Hence the rate of absorption is given by:

$$Rate = B_{ij} N_i \int_0^{\infty} \rho(\nu) g(\nu) d\nu \qquad (2.25)$$



a) b)

***Figure 2.4**: Relative spectra of incident radiation fields and the atomic transitions they interact with. (a) Broadband radiation interacting with narrow linewidth atomic transition, (b) Narrowband radiation field interacting with broader band atomic transition*

Over the narrow frequency interval for which $g(\nu)$ is significant, we can assume that $\rho(\nu)$ is constant (Figure 2.4a). Therefore, we can take $\rho(\nu)$ out of the integral and set ν

= v^* (where v^* is the centre frequency of g(v)). Using the normalisation condition (equation 2.16), 2.27 becomes:

$$Rate = B_{ij} N_i \rho(v^*) \quad (2.26)$$

Similarly, for stimulated emission we obtain:

$$Rate = B_{ji} N_j \rho(v^*) \quad (2.27)$$

Equations 2.26 and 2.27 are consistent with equations 2.2 and 2.3 which were proposed for the interaction of an atomic transition with a broadband black body radiation field.

In applications of optical amplifiers and certainly for lasers, the interacting radiation field is monochromatic and its linewidth is much narrower than the line shape function of the atomic transition (Figure 2.4b). The rate of absorption is again given by equation 2.25, but in this case g(v) may be considered as constant over the narrow frequency range of the monochromatic radiation field and it can thus be brought out from the integral. The remaining integral of ρ(v) with respect to dv is simply the photon energy density, ρ_v, in the monochromatic light wave of centre frequency v. ρ_v can be measured. Hence, the rate of absorption for a monochromatic light wave is:

$$Rate = B_{ij} N_i \rho_v g(v) \quad (2.28)$$

and the rate of stimulated emission is:

$$Rate = B_{ji} N_j \rho_v g(v) \quad (2.29)$$

For most applications it is more useful to express equations 2.28 and 2.29 in terms of the intensity of the incident light wave, I_v, using the substitution:

$$\rho_v = \frac{I_v . n}{c} \quad (2.30)$$

where n is the refractive index of the gain medium

3. OPTICAL AMPLIFICATION - SMALL SIGNAL GAIN

Given the photon material interaction processes described above, it is obvious that only stimulated emission can lead to optical amplification. In the system depicted by Figure 2.1, stimulated emission competes with absorption to determine whither the incident beam is amplified or attenuated. Spontaneous emission results in background light emitted randomly into a 4π steradian sphere, a proportion of which reaches the detector as background noise. To achieve amplification, the rate of stimulated emission must be greater than the rate of absorption and hence $B_{ji}N_j$ must be greater than $B_{ij}N_i$ (see equations 2.2 & 2.3) which generally means that N_j must be greater than N_i. This situation ($N_j > N_i$) is referred to as a population inversion, since at thermal equilibrium the populations are highest for lower order states as defined by the Boltzmann distribution (equation 2.6). For example, the relative population, N_j/N_i, at 295°K of two states differing in energy equivalent to the photon energy of light at a wavelength of 1μm is 6.0 $\times 10^{-22}$ (assuming $g_j = g_i$). Hence, at thermal equilibrium absorption completely dominates and the beam is attenuated. The challenge, given the numbers involved, is to achieve a population inversion allowing stimulated emission to dominate (see Section 4).

Given that we can manipulate the relative population densities of two states of a chosen gain medium to achieve a population inversion, we now wish to develop an expression for the measured output light intensity of the experiment depicted by Figure 2.1. In this way we will be able to determine the extent of amplification or attenuation of that gain medium. Using the rate equations 2.1, 2.28 and 2.29, we can derive such an expression in terms of the population densities of the states involved and the incident light intensity. Let us firstly consider the process of stimulated emission in a slab of material of thickness Δz somewhere in the body of the gain medium (see Figure 3.1).

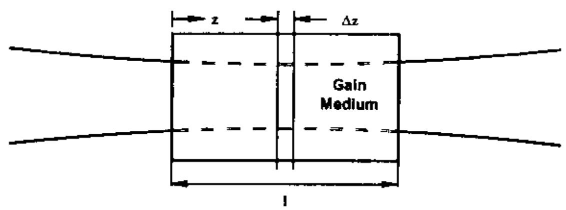


Figure 3.1: *Geometry for calculating gain from slab of thickness Δz within the body of the gain medium*

The photon energy density in the slab is $\rho_v(z)$ and the incident light intensity is $I_v(z)$ (where $\rho_v(z) = I_v(z)n/c$, as given by equation. 2.30). Applying equations 2.29 and 2.30, the rate of reduction of the number of atoms in energy level E_j due to stimulated emission in the slab is:

$$\frac{dN_j}{dt}.A.\Delta z = -B_{ji} N_j . \frac{I_v(z)n}{c}.g(v).A.\Delta z \quad (3.1)$$

where A is the cross-sectional area of the beam.

Each transition adds a photon of energy hv to the beam. Hence, we simply multiply 3.1 by hv to get an expression for the incremental power, ΔP, added to the beam by stimulated emission in the slab, and divide by the cross-sectional area to get the incremental intensity, $\Delta I_v(z)$:

$$\Delta I_v(z) = -\frac{dN_j}{dt}.hv.\Delta z = B_{ji}N_j.\frac{I_v(z)n}{c}.g(v).hv.\Delta z \tag{3.2}$$

Similarly, each absorption transition from state i to state j annihilates a photon from the beam and by analogy we can derive the incremental reduction in intensity, $-\Delta I_v(z)$, due to absorption as:

$$-\Delta I_v(z) = \frac{dN_i}{dt}.hv.\Delta z = B_{ij}N_i.\frac{I_v(z)n}{c}.g(v).hv.\Delta z \tag{3.3}$$

In addition, we must consider the contribution of spontaneous emission from the atoms of the slab to the total radiation field. With respect to the detected signal, only the spontaneous emission collected by the detector is of interest. Since, spontaneous emission is random and omni-directional only a fraction of the light emitted by any element of the slab is collected at the detector, that fraction being $\Omega/4\pi$, where Ω is the solid angle subtended by the detector at the plane of the slab. Hence, the incremental intensity provided by the slab to the detected beam from spontaneous emission is:

$$\Delta I_v(z) = \frac{dN_j}{dt}.hv.\frac{\Omega}{4\pi}\Delta z = A_{ji}N_j.hv.\frac{\Omega}{4\pi}\Delta z \tag{3.4}$$

In the formulation of equation 3.4, it is assumed that spontaneous emission contributes over the entire line shape function of the atomic transition and equation 2.12 has been used. If a narrow band filter is used in front of the detector to reduce the level of spontaneous emission then we must multiply equation 3.4 by $g(v)\Delta v$, where Δv is the linewidth of the filter.

The total contribution of the slab to the signal intensity is simply the summation of the incremental intensities contributed by stimulated and spontaneous emission minus the absorbed intensity. Hence, the rate of change of intensity with distance z through the gain medium is given by:

$$\frac{dI_v(z)}{dz} = B_{ji}N_j.g(v).hv.I_v(z).\frac{n}{c} + B_{ij}N_i.g(v).hv.I_v(z).\frac{n}{c} + A_{ji}N_j.\frac{\Omega}{4\pi}.hv$$

$$= \frac{hv}{c}.n.g(v).[B_{ji}N_j - B_{ij}N_i]I_v(z) + A_{ji}N_j.\frac{\Omega}{4\pi}.hv \tag{3.5}$$

The second term in equation 3.5 is the contribution of spontaneous emission to the collected signal. It is basically a source of noise. It is essential to the operation of any laser and it must be taken account of in any analysis of the signal to noise ratio in amplifier applications. However, for the moment we are only interested in the process of amplification. Neglecting the noise term and using the relationships, 2.9 and 2.10, between the Einstein coefficients, we obtain the most widely used expression to describe the process of amplification / attenuation arising from the competing processes of stimulated emission and absorption respectively:

$$\frac{dI_v}{dz} = \left[A_{ji}\frac{\lambda_0^2}{8\pi n^2}g(v)\left(N_j - \frac{g_j}{g_i}N_i\right)\right]I_v(z) = \gamma_0(v)I_v(z) \tag{3.6}$$

In the derivation of equation 3.6 it is implicitly assumed that the populations N_j and N_i and hence $\gamma_0(v)$ are independent of $I_v(z)$. Clearly if $I_v(z)$ is sufficiently large the resulting intense stimulated emission will drive down the population inversion changing $\gamma_0(v)$. Equation 3.6 is thus only valid for $I_v(0)$ sufficiently small to ensure negligible perturbation of N_j and N_i. For this reason $\gamma_0(v)$ is referred to as the small signal gain coefficient which is frequency dependent through the line shape function g(v). Clearly the condition for amplification is that the term $[N_j - N_i(g_2/g_1)]$, referred to as the population inversion, is greater than 0, i.e. $N_j > N_i(g_2/g_1)$.

Integrating 3.6 with respect to z, we get the intensity as a function of z for an input signal of $I_v(0)$:

$$I_v(z) = I_v(0)\exp[\gamma_0(v)z] \tag{3.7}$$

For a gain medium of length l equation 3.7 becomes

$$I_v(l) = G_0(v).I_v(0) \tag{3.8}$$

where

$$G_0(v) = \exp[\gamma_0(v)l] \tag{3.9}$$

$G_0(v)$ is referred to as the small signal gain of an amplifier of length l.

It is often convenient to specify the gain coefficient in terms of the stimulated emission or absorption cross-sections, σ_{SE} and σ_{ABS}, where these are given by:

$$\sigma_{SE} = A_{ji}.\frac{\lambda_0^2}{8\pi n^2}.g(v) \quad and \quad \sigma_{ABS} = A_{ij}.\frac{\lambda_0^2}{8\pi n^2}.g(v).\frac{g_2}{g_1} \tag{3.10}$$

Hence

$$\gamma_o(v) = \sigma_{se}(v).\left[N_j - \frac{g_2}{g_1}N_i\right] \tag{3.11}$$

where the second term on the right is referred to as the population inversion.

Indeed, equation 3.11 can be generalised to provide the gain coefficient, $\gamma(v)$, for any level of input signal provided the population inversion used is that for the prevailing signal conditions.

As we increase the pump power the population of the upper gain state increases linearly (eqn. 2.28) as does the population inversion and the gain coefficient. It is customary to express the gain of an amplifier in decibels. Therefore, taking $10\log_{10}$ of both sides of the expression for the overall gain of the amplifier (eqn. 3.9), we get:

$$Gain(dB) = 10\log_{10}G_0(v) = 10\log_{10}e^{\gamma(v)l} = 4.34\gamma(v)l \tag{3.12}$$

Equation 3.12 shows that the small signal gain (in dB) of an optical amplifier increases linearly with the gain coefficient and hence the pump power. At very low pump powers the population inversion is insufficient to provide gain and the signal is attenuated (by an amount depending on the population of the lower gain state). As the pump power, the population inversion and stimulated emission increase the attenuation decreases and the system becomes transparent. Beyond the point of transparency (the gain threshold) the gain (in dB) increases linearly with pump power according to equation 3.12. It must be noted that the above model only applies under weak pumping conditions for which we can assume insignificant depletion of the ground state. For strong pumping, the ground state becomes severely depleted and further increases in pump power result in minimal improvements to the gain and the output power. In this region the amplifier is in pump saturation and the gain gradient decreases and flattens out with increasing pump power. From plots of gain versus pump power (see later the laboratory exercises), we can determine the gain threshold and the gain gradient (from the linear part of the plot) which in turn can be used to determine the small signal gain coefficient for a uniformly pumped amplifier. A useful performance parameter which can also be determine from such a plot is the gain efficiency i.e. the gradient of the line drawn through the origin and tangent to the gain / pump curve before it flattens out (see laboratory exercises). The gain efficiency contains information about the threshold as well as the gain gradient.

4. PUMPING MECHANISMS

To achieve optical amplification we need to realise a population inversion between the two energy levels involved in the process. In the above analysis we assumed that a given population inversion could be obtained by appropriate pumping techniques without considering the details. Under thermal equilibrium conditions most of the atoms of any particular material system are in the ground state and the relative populations of the higher allowed states are given by the Boltzmann distribution (eqn. 2.6, Figure 4.1). For amplification of a particular wavelength, the energy levels involved in the gain process must be separated in energy by an amount equal to the photon energy at that wavelength. At visible and near infra red wavelengths (400nm to 2000nm), the photon energies imply enormous differences in the thermal populations of useful energy levels in the potential gain media. For example, the relative population, N_j/N_i, at 295°K, of two states suitable for amplifying light at a wavelength of 1μm (photon energy = 1.99×10^{-19}J) is 6.0×10^{-22} (assuming $g_j = g_i$). This means that to attain a population inversion atoms must be "pumped" (raised in energy) in huge numbers from the ground state to some appropriate higher level state.

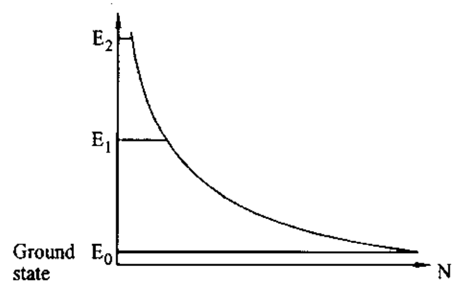


Figure 4.1: *The Boltzmann distribution showing the relative populations of discrete energy level of a system under thermal equilibrium*

With the exception of semiconductor and gas systems, most laser and amplifier gain media are pumped optically. In this process, atoms in the ground state (Energy level E_o) of the material are raised to a higher energy level (E_j) by the absorption of photons of energy, $E_j - E_o$, supplied from an external light source which may be another laser or a high power arc or flash lamp. In this way the population of the ground state decreases while that of the upper state increases. If only two energy levels were involved in the process, then the population of the upper state would increase until the rates of absorption and stimulated emission of pump photons (eqns. 2.28 & 2.29) were equal. Hence, a population inversion cannot be created by pumping from the ground state in a strictly 2 level system.

Most optical amplifiers and lasers are based on either a 3 or a 4 level gain medium and pumping system. Figure 4.2 shows the simplified energy level diagram and pumping scheme for a typical 3 level system. Atoms are pumped, by photons of energy, $E_2 - E_o$, from the ground state to some higher energy level, E_2, from where they make rapid transitions into energy level E_1. Provided that the rate of decay of the atomic population in the E_1 level (by spontaneous emission or other means) is slow relative to the pumping rate (i.e. the E_1 energy level is metastable), the population of E_1 will increase to exceed that of the ground state, thus creating a population inversion. Light of a wavelength satisfying the relationship $E_{photon} = E_1 - E_0 = hc/\lambda$ may then be amplified by this gain medium.

The rates of pumping and decay of the various populations are also indicated in Figure 4.2. R_2 is the rate ($dN_2/dt_{[pump]}$) at which atoms are being pumped into state E_2 from the ground state as a result of absorption of the pump light. Since the transition rate, N_2/τ_{21}, to state E_1 is very rapid (τ_{21} is short), R_2 is also the pumping rate ($dN_1/dt_{[pump]}$) of the upper lasing level, E_1. In the absence of stimulated emission the population of the metastable upper lasing state decays slowly at a rate $dN_1/dt = N_1/\tau_{10}$, where τ_{10} is the lifetime of the E_1 level (see equations 2.1a & b). The erbium doped fibre amplifier is a good example of a 3 level system and will be discussed in detail in section 7.

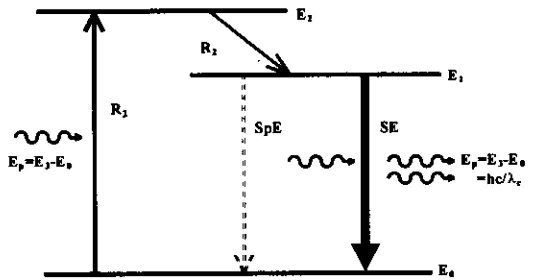


Figure 4.2: Simplified energy level diagram of a 3 level gain system showing the pumping scheme and transition rates.

In a 4 level system (Figure 4.3), ground state atoms are pumped, by photons of energy $E_3 - E_0$, to energy level E_3 from where they rapidly make the transition to the metastable state, E_2. Due to the long lifetime / slow decay of the atomic population in the metastable E_2 level, its population builds up creating an inversion relative to level E_1 and providing amplification of light of a wavelength satisfying the relationship $E_{photon} = E_2 - E_1 = hc/\lambda$. In efficient gain media the E_1 level is sufficiently higher than the ground state that its thermal equilibrium population is negligible. In addition, its transition rate to the ground state, by spontaneous emission or otherwise, is usually very fast to ensure that its population remains negligible even under high rates of stimulated emission from the E_2 level.

Again the pump and decay rates are indicated on the energy level diagram. R_3 is the pump rate of level E_3 and the upper lasing level, E_2, since the transition rate, N_3/τ_{32}, is rapid. In the absence of stimulated emission the population of the metastable E_2 level decays slowly by spontaneous emission to E_1 and to the ground state at the rates N_2/τ_{21} and N_2/τ_{20} respectively. The decay rate of the E_1 population, N_1/τ_{10}, is very rapid thus maintaining low N_1.

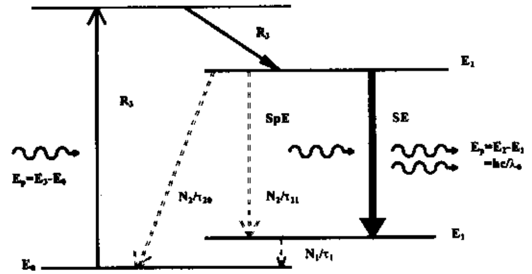


Figure 4.3: Simplified energy level diagram of a 4 level gain system showing the pumping scheme and transition rates.

Neodymium doped YAG (yttrium aluminium garnet) is an excellent example of an optically pumped four level gain medium predominantly used in lasers of the same name. Neodymium ions are the active species with YAG as the dopant host material. Figure 4.4 shows the partial energy level diagram of neodymium. The pumping transition is from the ground state to a band of levels denoted 3 in the diagram leading to absorption over a broad wavelength band centred in the visible spectrum at around 500-600nm. Pump photons provided by a high power Xenon flash tube with a centre wavelength around 500nm and a broad spectral output providing efficient pumping of the broad transition. Atoms put into the upper pump states decay rapidly to the metastable $F_{3/2}$ level creating a population inversion relative to the $I_{11/2}$, $I_{13/2}$ and $I_{15/2}$ levels which are short lived. The transitions to the former two of these are the most often used providing optical amplification and laser action at 1064nm and 1315nm. It is also possible to create a population inversion with the ground state providing gain at 972nm from a 3 level system.

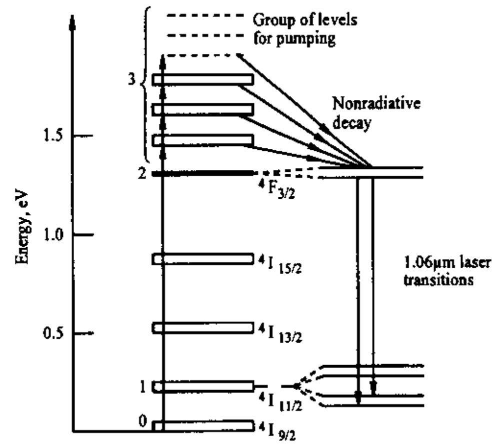


Figure 4.4: Energy level diagram for the 4 level Nd:YAG gain medium.

5. OPTICAL AMPLIFICATION - LARGE SIGNAL GAIN

5.1 Introduction

In the analysis of small signal gain carried out in section 3, it is implicitly assumed that the population inversion is constant, remaining unperturbed by the low levels of stimulated emission arising from amplification of a weak input signal. As the signal strength increases, the stimulated emission process begins to significantly reduce the population inversion and the gain decreases, a phenomenon referred to as gain saturation. To analyse large signal gain and gain saturation, we must consider the coupled rate equations for all of the transitions which influence the populations of the two energy levels involved in the amplification process. The analysis and results are different for 3 and 4 level systems [1, 2] and hence, they are dealt with separately below.

5.2 Four level systems

With the aid of Figure 4.3, we can readily identify the main transitions which affect the population inversion and establish their rates:

1. R_3 is the rate at which atoms are pumped from the ground state into E_2, via E_3, assuming that the transition of atoms from E_3 to E_2 is very rapid. R_3 is given by the rate of absorption of pump photons into the ground state atoms:

$$R_3 = B_{03} I_v^P . n . g(v) N_0 / c \tag{5.1}$$

   where I_v^P is the pump intensity.

   Using equations 2.10 and 3.10, we can express equation 5.1 in terms of the pump absorption cross-section, ρ_p:

$$R_3 = \frac{\sigma_p I_v^P}{hv} \tag{5.2}$$

2. N_2/τ_{21} is the transition rate associated with all the processes (mainly spontaneous emission) which decrease the population of E_2 whilst simultaneously increasing that of E_1.
3. N_2/τ_{20} is the transition rate associated with all processes which decrease the population of E_2 and eventually feed the ground state but without passing through E_1.
4. The total rate of population decay of state E_2 is the summation of the above two transition rates given by $N_2/\tau_{21} + N_2/\tau_{20} = N_2/\tau_2$ where τ_2 is the lifetime of state 2 determined by all possible decay mechanisms. τ_2 is defined by $1/\tau_2 = 1/\tau_{21} + 1/\tau_{20}$
5. Using equations 2.28 and 2.29, we can readily write an expression for the rate of population decay of the E_2 level due to stimulated emission offset by the rate of increase due to signal absorption i.e. $Rate_{21} = g(v)\rho_v[B_{21}N_2 - B_{12}N_1]$. The final result may then be expressed in terms of the signal intensity and the stimulated emission cross-section using equations 2.30 and 3.10 respectively:

$$Rate_{21} = \frac{\sigma_{SE} I_v}{hv}(N_2 - \frac{g_2}{g_1} N_1) \tag{5.3}$$

6. N_1/τ_1 is the rate of population decline from state E_1 where τ_1 is the lifetime of state 1 determined by all possible decay mechanisms but largely by spontaneous emission.

It is now a simple matter to write the coupled differential rate equations describing the population exchanges between the two levels involved in the amplification process:

$$\frac{dN_2}{dt} = R_3 - \frac{N_2}{\tau_2} - \frac{\sigma_{SE} I_v}{hv}(N_2 - \frac{g_2}{g_1} N_1) \tag{5.4a}$$

and

$$\frac{dN_1}{dt} = \frac{N_2}{\tau_{21}} - \frac{N_1}{\tau_1} + \frac{\sigma_{SE} I_v}{hv}(N_2 - \frac{g_2}{g_1} N_1) \tag{5.4b}$$

In a rigorous solution to these equations we must impose the condition that $N_{tot} = N_0 + N_1 + N_2$. However, this leads to considerable mathematical complexity and tedium. In order to simplify matters in the following solution of these equations, it is assumed that the populations N_2 and N_1 are very small in comparison to N_0 and hence that N_0 is independent of the pump rate. In addition, we shall also set $g_1 = g_2$. Although this simplified approach is only strictly valid in the limited case of weak pumping, it enables us to make useful general deductions about performance trends and characteristics of optical amplifiers. However, it must be remembered that this assumption is invalid for strong pumping and for 3 level systems where by definition $N_1 >> N_0$.

For our present purposes, we are interested in the solution to the above coupled differential equations in the steady state condition that $dN_1/dt = dN_2/dt = 0$. Without detailing the mathematical process, which is of little interest to the scientist or engineer, the solution in terms of the population inversion $(N_2 - N_1)$ is:

$$N_2 - N_1 = \frac{R_3\tau_2(1 - \tau_1/\tau_2)}{1 + (\tau_1 + \tau_2 - \tau_1\tau_2/\tau_{21})\left(\frac{\sigma_{SE} I_v}{hv}\right)} \tag{5.5}$$

Multiplying by the stimulated emission cross-section, σ_{SE}, gives the gain coefficient:

$$\gamma(v) = \frac{\sigma_{SE} R_3\tau_2(1 - \tau_1/\tau_2)}{1 + (\tau_1 + \tau_2 - \tau_1\tau_2/\tau_{21})\left(\frac{\sigma_{SE} I_v}{hv}\right)} \tag{5.6}$$

For small input signals (i.e. small I_v), the second term of the denominator is negligible and the numerator is thus the small signal gain coefficient. Hence we can rewrite equation 5.6 as:

$$\gamma(v) = \frac{\gamma_0(v)}{1 + (\tau_1 + \tau_2 - \tau_1\tau_2/\tau_{21})\left(\frac{\sigma_{SE} I_v}{hv}\right)} \tag{5.7}$$

In a good four level system, $\tau_1 << \tau_2$ and $\tau_2 \approx \tau_{21}$ (i.e. state E_2 is metastable - long lived - and decays primarily by spontaneous emission) and equation 5.7 can be simplified to:

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1+\left(\frac{\tau_2 . \sigma_{SE} . I_\nu}{h\nu}\right)} \tag{5.8}$$

The signal intensity at which the gain coefficient drops to half of the small signal gain coefficient is referred to as the saturation intensity, I_s, which occurs when the second term in the denominator of equation 5.8 equals 1. This means that I_s is given by:

$$I_s = \frac{h\nu}{\sigma_{SE}\tau_2} \tag{5.9}$$

and equation 5.8 may be rewritten as:

$$\gamma(\nu) = \frac{1}{I_\nu} . \frac{dI_\nu}{dz} = \frac{\gamma_0(\nu)}{1+\left(\frac{I_\nu}{I_s}\right)} \tag{5.10}$$

This equation is solved by integrating with respect to dI_ν and dz, giving:

$$\int_{I_\nu(0)}^{I_\nu(l)} dI_\nu \left[\frac{1}{I_\nu} + \frac{g(\nu)}{I_s}\right] = \int_0^l \gamma_0(\nu) dz \tag{5.11}$$

Hence

$$\ln \frac{I_\nu(l)}{I_\nu(0)} + \frac{g(\nu)}{I_s}\left(I_\nu(l) - I_\nu(0)\right) = \gamma_0(\nu) l \tag{5.12}$$

$$\Rightarrow \qquad \ln G + g(\nu)\frac{I_\nu(0)}{I_s}(G-1) = \gamma_0(\nu) l \tag{5.13}$$

Intuitively, increasing signal strength for a given pump power eventually results in significant depletion of the population inversion and the gain, therefore, falls off accordingly. In the region of decreasing gain the amplifier is said to be in a state of gain saturation. This effect is confirmed by equation 5.10 which clearly shows the gain coefficient decreasing as the signal intensity, I_ν, approaches and then exceeds the saturation intensity, I_s. When $I_\nu = I_s$, the gain coefficient is halved providing a practical definition of I_s. Equation 5.13 cannot be solved analytically, but a numerical analysis leading to a plot of G versus $I_\nu(0)$ shows that, as expected intuitively, the gain does indeed drop off with increasing input intensity becoming half of its initial low signal level at $I_\nu = I_s$.

5.3 Three level systems [2]

Consulting Figure 4.2 we can easily determine the transition rates which control the relative populations of the ground and E_1 energy levels:

1. The rate of increase in the population of the E_1 energy level due to the optical pumping (absorption) into E_2 followed by very rapid transition to E_1 is given by:

$$R_2 = B_{02} I_\nu^P . n . g(\nu) N_0 / c \tag{5.14}$$

Using equations 2.10 and 3.10, we can express equation 5.14 in terms of the absorption cross-section:

$$R_2 = \frac{\sigma_p I_\nu^P}{h\nu} \tag{5.15}$$

2. N_1/τ_{10} is the rate of decline in the population of energy level E_1 predominantly due to spontaneous emission. τ_{10} is the spontaneous emission lifetime of the E_1 level.
3. Following the logic described in item 5 of the transition rate analysis for 4 level systems the rate of decline in the population of level E_1 arising from stimulated emission off-set by absorption of the signal photons by ground state atoms is given by:

$$Rate_{10} = \frac{\sigma_{SE} I_\nu}{h\nu}(N_1 - \frac{g_1}{g_0} N_0) \tag{5.16}$$

Amplified spontaneous emission plays a significant role in depleting the population of the upper gain state. However, we can gain a useful insight into the amplifier characteristics by neglecting it whilst simplifying the mathematics considerably. The differential rate equation for the population of the upper gain state can now be written as:

$$\frac{dN_1}{dt} = R_2 - \frac{N_1}{\tau_{10}} - \frac{\sigma_{SE} I_\nu}{h\nu}(N_1 - \frac{g_1}{g_0} N_0) \tag{5.17}$$

By definition, in a 3 level system, the ground state population, N_0, is significantly depleted and, unlike the 4 level system where we neglected the condition of conservation of the total population, we must impose the condition that $N_t = N_1 + N_2$, where N_t is the total population density. Given this condition, the steady state ($dN_1/dt = 0$) solution of equation 5.17 is given by:

$$N_2 = \frac{(I_\nu^* + I_p^*)N_t}{1 + 2I_\nu^* + I_p^*} \tag{5.18}$$

where $I_\nu^* = I_\nu / I_s$ and $I_p^* = I_p / I_{ps}$, where I_{ps} is the saturation pump power.

Given that $dI_\nu/dz = \sigma_{SE}(N_2 - N_1)I_\nu$, $dI_p/dz = \sigma_{ABS\_p} N_1$ and substituting $N_1 = N_t - N_2$ and N_2 from equation 5.18, we obtain:

$$\frac{dI_\nu}{dz} = \frac{(I_p^* - 1)\sigma_{SE} N_t I_\nu}{1 + 2I_\nu^* + I_p^*} \tag{5.19}$$

and

$$\frac{dI_p}{dz} = \frac{(I_s^* + 1)\sigma_{ABS\_p} N_t I_p}{1 + 2I_\nu^* + I_p^*} \tag{5.20}$$

Clearly from equation 5.19, the gain per unit length and hence the overall gain (in dB) increases linearly with increasing pump intensity, I_p, but then flattens out as I_p approaches and exceeds I_{ps}, increasing I_p^* in the denominator. Intuitively such behaviour is expected (and observed experimentally, see laboratory exercises) since a large pump power will result in severe depletion of the ground state leading to reduced absorption and pumping rate. This phenomenon, referred to as pump saturation, prevails in all operating 3 level systems where the lower gain state is the ground state which by definition must be at least 50% depleted simply to obtain a population inversion. It is also evident in 4 level systems under strong pumping conditions. It should also be noted that signal gain saturation is also predicted by equation 5.19.

5.4 Issues of homogeneous and inhomogeneous line broadening

With respect to the operation of lasers or optical amplifiers, the significance of whether a gain curve is homogeneously or inhomogeneously broadened lies in the behaviour of the gain curve under intense radiation, such as for an amplifier approaching saturation or the gain medium of a laser in equilibrium oscillation. For homogeneously broadened gain media under intense radiation at any wavelength under the gain curve, the high level of stimulated emission simply depletes the population of the upper state and the entire gain curve diminishes but maintains its shape (see Figure 5.1). This means that the gain for all wavelengths under the gain curve is reduced uniformly. For gain media with inhomogeneously broadened transitions under intense radiation, the population of the upper state only decreases for that group of atoms whose homogeneous lineshape overlaps the radiation wavelength. Hence, the gain is only diminished in a narrow range of wavelengths (the homogeneously broadened linewidth for these atoms) around the radiation wavelength (see Figure 5.2). This phenomenon is referred to as spectral hole burning. The gain for wavelengths under the gain curve but outwith this region is unaffected.

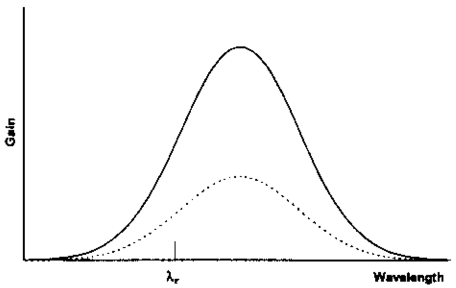


Figure 5.1: *General form of a typical gain curve under small signal conditions (solid line) and under internal radiation at λ_r for homogeneously broadened transition*

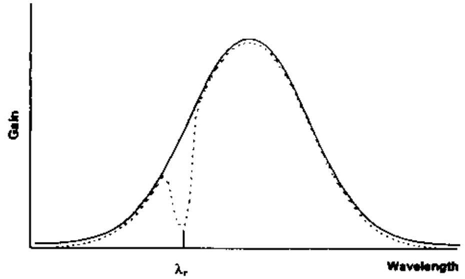


Figure 5.2: *General form of a typical gain curve under small signal conditions (solid line) and under internal radiation at λ_r for inhomogeneously broadened transition*

6. NOISE IN OPTICALLY AMPLIFIED SIGNALS

6.1 Noise in Optically Amplified Signals

As well as providing optical amplification for an incident signal, any gain medium in which there is a population inversion is a source of significant levels of spontaneous emission. Some of that spontaneous emission from any given volume element in the material travels co-axially with the signal (see equations 3.3-3.5) and is amplified by the remaining gain medium between its source and the amplifier output facet[3, 4]. Such amplified spontaneous emission (ASE), when mixed with the signal on the detector, is a source of noise[3, 4]. Indeed, noise associated with the ASE is the limiting factor in determining the ultimate signal to noise ratio in any system using optical amplifiers [4], particularly in long haul periodically amplified systems using EDFAs in which the ASE accumulates through the system [5, 6].

Let us consider the spontaneous emission from a cylindrical gain medium of cross-sectional area A and length l (Figure 6.1). A cross-sectional slab of material of infinitesimal thickness, dz, will spontaneously emit a total power of $A_{21}N_2.g(v)dv.hv.A.dz$ in the frequency range v to v + dv. Generally, we are only concerned with power emitted through the end face of the cylinder and confined within a given solid angle, dΩ, given by:

$$dP_{ASE}(d\Omega) = A_{21}N_2 g(v)dv.hv.\frac{d\Omega}{4\pi}.Adz \qquad (6.1)$$

For example, dΩ, may be the angle subtended at the centre of the cylinder by a remote receiver (removed far enough that we do not need to consider the variation of dΩ with length along the cylinder). Alternatively, if the gain medium is in the form of a waveguide as in optical fibre amplifiers, dΩ, is the solid angle associated with the numerical aperture of the guide i.e. $d\Omega = NA^2/4$.

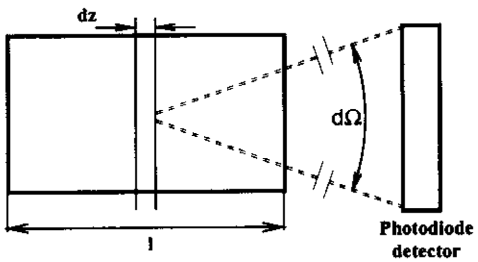


Figure 6.1: Geometry for calculating spontaneous emission power collected by a remote detector from a slab of thickness dz within the body of the gain medium.

For small dΩ, power emitted spontaneously from a slab at position z along the length of the cylinder will be amplified by the remaining gain medium of length l-z by a factor $e^{\gamma(l-z)}$ before it leaves the exit face. Hence, the ASE power, in the frequency

interval dv, emerging from the cylinder end face is the summation of the contributions from each slab of thickness dz making up the entire gain medium:

$$P_{ASE} = \int_0^l dz.\frac{dP_{ASE}}{dz}.e^{\gamma(l-z)} \qquad (6.2)$$

Evaluation of the integral gives:

$$P_{ASE} = \frac{[G-1]}{\gamma}.A_{21}N_2 g(v).hv.A\frac{d\Omega}{4\pi}.dv \qquad (6.3)$$

where G is the total gain of the amplifier given as $G = \exp[\gamma l]$

Substituting $\gamma(v) = \sigma_{SE}(v).\left[N_2 - \frac{g_2}{g_1}N_1\right]$ and $\sigma_{SE}(v) = A_{21}.\frac{\lambda_0^2}{8\pi n^2}.g(v)$ gives:

$$P_{ASE} = \frac{N_2}{N_2 - \frac{g_2}{g_1}N_1}.[G-1].hv.2A\frac{n^2}{\lambda_0^2}d\Omega.dv \qquad (6.4)$$

$$= 2\mu[G-1]hv.\frac{An^2}{\lambda_0^2}.d\Omega.dv \qquad (6.5)$$

where $\mu = N_2/[N_2 - (g_2/g_1)N_1]$ is known as the population inversion factor.

The term $An^2 d\Omega/\lambda_o^2$ characterises the geometry of the light emission and collection system relative to the wavelength. Generally, to minimise the noise, it is desirable to collect only the minimum possible amount of ASE when detecting the signal. Here we now consider the limits. Consider a laser beam passing through a gain medium collimated such that it forms a beam waist at the output facet (Figure 6.1).

To minimise the ASE travelling with the beam to the detector we can place an aperture stop at the output facet with a radius, a, equal to the beam radius. The output beam diverges by diffraction at a half angle, θ, given by:

$$\theta = \frac{\lambda_o}{\pi n a} \qquad (6.6)$$

For small angles, θ, this corresponds to a solid angle, Ω_{min},:

$$\Omega_{min} = \pi\sin^2\theta = \pi\theta^2 = \frac{\lambda_o^2}{\pi n^2 a^2} = \frac{\lambda_o^2}{n^2 A} \qquad (6.7)$$

where A is the area of the aperture and Ω_{min} is the minimum solid angle of collection subtended by the receiver at the gain medium to ensure that the total power of the signal is detected. For small angles and assuming transmission through a linear polariser, the minimum ASE power incident on the receiver with the signal is obtained by substituting Ω_{min} for dΩ in equation 6.5 and dividing by 2 to give [3]:

$$P_{ASE} = \mu[G-1].hv.dv = \rho_{ASE}dv \qquad (6.8)$$

where ρ_{ASE} (= μ[G - 1]hv) is the spectral power density of the ASE contained within the amplified signal beam and reaching the detector via a linear polariser. We have restricted the calculation of the ASE power to a single linear polarisation state because the primary noise terms arise from beating between the ASE and a linearly polarised signal. Only the E Field components in the ASE which are polarised parallel with the signal E Field result in non zero beat terms. For unpolarised ASE power the right hand side of 6.8 is simply doubled.

At first sight equation 6.8 is rather confusing. The population inversion factor μ increases with decreasing population inversion. Equation 6.8, therefore, at first sight, indicates that ASE increases with decreasing population inversion, whereas, intuitively, we would expect the opposite. However, as μ is decreasing with improving population inversion, G = exp[γl] (γ is given by equation 3.9) is increasing much faster. Hence, P_{ASE} does indeed increase with improving population inversion.

For a single mode optical fibre the collection angle is equal to the angle of divergence of the emergent beam due to diffraction, as given by equation 6.6, corresponding to a solid angle given by equation 6.7. This means that spontaneous emission in an optical fibre amplifier is collected by the fibre with a solid angle, Ω_{min} given by equation 6.7. Hence, the ASE power per frequency interval dv propagating in the fibre with the signal, polarised in the same direction as the signal and inseparable from it, is given by equation 6.8.

At the output of an optical amplifier, the total optical power, P_R, incident on a receiver is thus the summation of the received signal power, P_S, and the total ASE power, $\rho_{ASE}.B_o$, which has accumulated from the amplifier:

$$P_R = P_S + \rho_{ASE} B_o \quad (6.9)$$

where B_o is the optical bandwidth of the system or of an optical filter placed in front of the receiver.

The presence of the ASE gives rise to additional optical noise terms at the receiver output, over and above signal shot noise current. The ASE has its own shot noise, and it beats both with itself and the signal in the square law detector to generate ASE-ASE beat noise and signal-ASE beat noise. These optical noise components must be considered in addition to the intrinsic noise of the receiver which is usually dominated by thermal noise from the load resistance.

Noise is characterised by the variance of the current fluctuations, σ, equivalent to the mean square current fluctuation, $\langle i^2 \rangle$. The noise sources discussed above are uncorrelated and the total variance of the receiver current fluctuation, σ_N, is simply the sum of the variances associated with each noise source i.e.:

$$\sigma_N = \sigma_T + \sigma_S + \sigma_{ASE} + \sigma_{S-ASE} + \sigma_{ASE-ASE} \quad (6.10)$$

where the terms on the right are, in order of appearance, the mean square current fluctuations (the current variance) associated with thermal noise in the receiver, signal shot noise, ASE shot noise, signal-ASE beat noise and ASE-ASE beat noise. The optical noise terms are given by the following expressions [4]:

$$\sigma_S = 2eI_S B_e = 2eRP_S B_e = 2eRGP_0 B_e \quad (6.11)$$

$$\sigma_{ASE} = 2eI_{ASE} B_e = 2eRP_{ASE} B_e \quad (6.12)$$

$$\sigma_{S-ASE} = 4R^2 GP_0 \rho_{ASE} B_e = 4R^2 GP_0 P_{ASE}^{B_e} \quad (6.13)$$

$$\sigma_{ASE-ASE} = 2R^2 \rho_{ASE}^2 B_o B_e = 2R^2 P_{ASE}^{B_o} P_{ASE}^{B_e} \quad (6.14)$$

where e is the electronic charge, R is the photodiode responsivity (R = ηq/hv, η being the quantum efficiency), B_e is the receiver bandwidth, B_o is the optical bandwidth, I_S and I_{ASE} are the photodetector currents arising from the signal and ASE respectively, P_0 is the signal input power to the amplifier, P_S is the received signal power and $P_{ASE}^{B_o}$ ($\rho_{ASE}.B_o$) and $P_{ASE}^{B_e}$ ($\rho_{ASE}.B_e$) are the single polarisation ASE powers in the optical and electrical (receiver) bandwidths respectively.

The shot noise and thermal noise expressions are well known and their derivations may be found in most text books on optical communications[7]. The two beat noise terms[4, 5] are more particular to systems using optical amplifiers and are less familiar. These terms[5] are thus derived in Appendix A.

Clearly from the expressions 6.11 - 6.14, for any significant level of gain, G, and input signal, P_0, the signal-ASE beat noise and / or the ASE-ASE beat noise terms represent the largest optical contributions to the total noise. In most applications of optical amplifiers, one or other or both of these noise components limits the overall performance of any system. Hence, for most applications the received signal to noise ratio, SNR_{out}, at the output of an optical amplifier is:

$$SNR_{out} = \frac{(RGP_0)^2}{4R^2 GP_0 P_{ASE}^{B_e} + 2R^2 P_{ASE}^{B_o} P_{ASE}^{B_e}} \quad (6.15)$$

For some applications, particularly for low signal levels and when the amplifier is not in saturation, the ASE-ASE beat noise becomes important or even dominant and we can neglect the first term in the denominator. However, in many more applications the amplifiers have significant output signal levels and operate in or near saturation, implying that we can neglect the second term. In such cases the received SNR is given by:

$$SNR_{out} = \frac{I_S^2}{\langle i_{S-ASE}^2 \rangle} = \frac{I_S^2}{\sigma_{S-ASE}} = \frac{(RGP_0)^2}{4R^2 GP_0 \rho_{ASE} B_e} = \frac{GP_0}{4P_{ASE}^{B_e}} \quad (6.16)$$

6.2 The Noise Figure

Often it is convenient to characterise the noise and SNR of optically amplified systems using a parameter known as the amplifier noise figure, NF [4]. NF is the ratio of the optical SNR at the amplifier input to the optical SNR at the output, as detected by a receiver whose intrinsic noise level (thermal noise) is less than the optical noise in both cases. The optical noise at the input is simply the signal shot noise and the SNR (using eqn. 6.11 and R = e/hv, η =1) is given by:

$$SNR_{in} = \frac{(RP_0)^2}{2eRP_0 B} = \frac{P_0}{2hvB_e} \quad (6.17)$$

Using 6.16 the noise figure is given by:

$$NF = \frac{SNR_{in}}{SNR_{out}} = \frac{P_0}{2h\nu B_e} \Big/ \frac{GP_0}{4P_{ASE}^{B_e}} = \frac{2P_{ASE}^{B_e}}{Gh\nu B_e} \qquad (6.18)$$

Substituting $P_{ASE}^{B_e} = \rho_{ASE}.B_o$, using equation 6.8 and assuming significant gain, G>>1, gives:

$$NF = 2\mu \qquad (6.19)$$

This implies that the minimum possible noise figure is 2 (3dB) for an ideal amplifier having a complete population inversion (i.e. $N_1 = 0$, $\mu = 1$). That is, even for an ideal amplifier, the output SNR is degraded by 3dB relative to the input SNR. Typically, in practice, amplifiers operate with a noise factor greater than 3dB, and it can be as high as 7-8dB. If the noise figure is known under the conditions at which the amplifier is operated then we can use it to calculate the output SNR. Applying 6.17 and 6.18, SNR_{out} in terms of the noise figure is:

$$SNR_{out} = \frac{P_0}{2h\nu B_e . NF} \qquad (6.20)$$

It must be noted that the noise figure varies as a function of the precise operating conditions of the amplifier, particularly as regards the signal power relative to the saturation power, but also as regards the pump power relative to the saturation pump power. To be useful in expressions such as 6.20, the variation of the noise figure must be known as a function of these parameters.

The noise figure, of course, can be measured simply by measuring the input and output SNR as a function of the amplifier operating conditions. However, it can also be determined by measuring the full optical output spectrum of the amplified signal plus ASE and carrying out the appropriate analysis. From equation 6.18, assuming that Signal-ASE beat noise is dominant, NF is given by:

$$NF = \frac{2P_{ASE}^{B_e}}{Gh\nu B_e} \qquad (6.21)$$

From measurements of the total ASE power and the ASE spectrum (Figure 6.2) we can readily calculate $P_{ASE}^{B_e}$ (i.e. the ASE power within the receiver bandwidth). Measurement of the Gain then gives all the necessary information to calculate the noise figure.

In many systems the ASE-ASE beat noise is significant and must be included in the measurements of noise figure and expressions for SNR_{out}:

$$SNR_{out} = \frac{(RGP_o)^2}{4R^2GP_0P_{ASE}^{B_e} + 2R^2P_{ASE}^{B_e}P_{ASE}^{B_e}} \qquad (6.22)$$

and

$$\begin{aligned} NF &= \frac{P_o(4R^2GP_0P_{ASE}^{B_e} + 2R^2P_{ASE}^{B_e}P_{ASE}^{B_e})}{2h\nu B_e(RGP_o)^2} \\ &= \frac{2P_{ASE}^{B_e}}{Gh\nu B_e} + \frac{P_{ASE}^{B_e}P_{ASE}^{B_e}}{G^2P_oh\nu B_e} \end{aligned} \qquad (6.23)$$

Again, measurement of the total ASE power, plus the ASE spectrum and the gain allow the noise figure to be calculated.

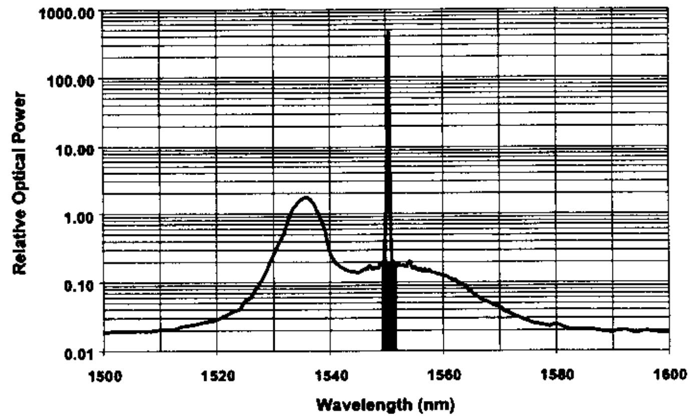


Figure 6.2: *ASE power spectrum showing the ASE falling within the bandwidth of the receiver, B_e. P_{ASE}^{Be} is integral of the ASE spectrum over the shaded area.*

7. THE ERBIUM DOPED FIBRE AMPLIFIER - EDFA

7.1 Structure and Principles

With 1550nm becoming the preferred choice of wavelength for long haul optical communications systems in the late 1980s, optical amplifiers at 1550nm [4, 8] became an obvious R & D goal. Spectroscopic studies had shown that Erbium atoms may be suitable as an active 3 level species for optical amplification at 1550nm. Figure 7.1 shows a partial energy level diagram for erbium atoms doped into a glass host. The broad band of levels denoted $^4I_{13/2}$ are metastable with long spontaneous emission lifetimes in the region of a few ms and transitions to the ground state produce photons in the wavelength range 1520-1580nm providing the possibility of optical amplification centred on 1550nm. It was found that the population of the $^4I_{13/2}$ levels could be pumped by irradiation at 980nm or 1480nm. Photons at 980nm are absorbed by ground state atoms which make the transition to energy level $^4I_{11/2}$.

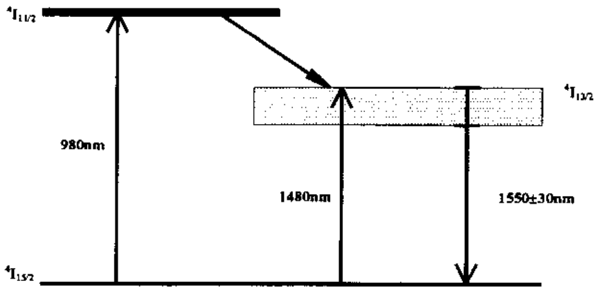


Figure 7.1: Energy level diagram and pumping scheme for erbium doped silica glass - a three level system giving gain at 1550±30nm.

Further non-radiative transitions from the $^4I_{11/2}$ level to the $^4I_{13/2}$ level are very rapid and the population of the $^4I_{13/2}$ metastable levels builds up. Alternatively, irradiation by 1480nm light allows direct pumping into the upper levels of the $^4I_{13/2}$ band with rapid transitions to the lower, long lived levels allowing this pumping scheme to operate as a quasi 3 level system. Three level systems with the ground state as the lower gain state require very strong pumping to achieve a population inversion and the erbium doped glass system is further impaired by the inability to achieve high doping concentrations, implying the need for long lengths of material to achieve significant gain. Consequently, amplifier configurations using bulk optic gain media failed to produce significant levels of gain for any practical levels of pump power. The key to success arrived with the development of techniques for doping significant concentrations of erbium into the core of single mode silica fibre. Longitudinal pumping by launching 980nm or 1480nm light into the end of the fibre resulted in useful gain at practical pump powers and fibres were developed to provide single mode guiding at the pump and signal wavelengths. The vastly improved efficiency relative to bulk systems arises from the high pump intensity, low extraneous signal and pump power loss, strong confinement of the pump light within a single guided mode over long lengths of gain medium and strong overlap of the single mode signal

light with the gain volume. With such advantages, gain efficiencies in excess of 3dB/mW were found to be readily available. Following such proof of practical operation, semi-conductor lasers tailor made for efficient launching of 980nm and 1480nm light into single mode erbium doped fibres were developed and optimised. In addition, fibre designs evolved to provide optimum pump / signal overlap as well as mode matching to standard telecommunications fibre for low loss splicing. Recognition of the advantages of EDFAs and rapid development of the design features discussed above ensured the deployment of EDFAs in long haul systems as early as 1994 which is remarkable considering proof of principle as recently as the late 1980s.

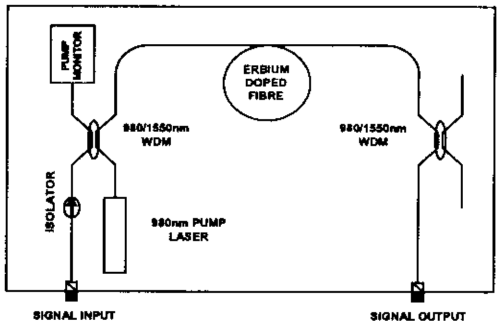


Figure 7.2: EDFA diagram

The basic structure of modern EDFAs[4, 8] is shown in Figure 7.2. Standard fibre pigtails are spliced onto the erbium doped fibre and the necessary ancillary components are then spliced onto the ends. The pump light (usually 980nm as shown here) is introduced using a 980 / 1550nm fused fibre wavelength division multiplexer. Most of the pump light is absorbed into the erbium fibre length providing the population inversion and amplification of the 1550nm signal. Excess pump light is removed by the 980/1550nm demultiplexer at the output end. As shown in Figure 7.1 single end pumping is usually co-directional with the signal as this geometry minimises the ASE propagating in the signal direction. With single pass gains of up to 40dB being readily available and often used, it is necessary to include at least one optical isolator in the system to prevent amplification of any down stream feedback from splices, connectors, terminals etc. In addition, isolators prevent the strong tendency of even low levels of feedback from within the components of the amplifier to induce oscillation (laser action), which serves to increase the noise and reduces the net signal gain. In some systems, low coupling ratio fused fibre taps are included to enable monitoring of the amplifier output, input and pump, with feedback control to the pump power to ensure constant signal output power. Also, narrow band optical filters centred on the signal are sometimes included to limit the optical bandwidth of the ASE reducing the total ASE power and its resultant noise. For very high gain or high output power applications two lasers are used to implement bi-directional pumping.

7.2 Gain Characteristics of EDFAs

The waveguide geometry of EDFAs provides extremely high pumping efficiency and near optimum signal interaction with the pumped volume over long lengths, resulting in high gain and high saturated output power relative to the pump power. Stark splitting and atom phonon interactions in the erbium doped glass system result in significant broadening of the $^4I_{13/2}$ level in the erbium ions. This results in broad band spontaneous emission and broad band gain over a range of wavelengths from 1520nm to 1580nm. Figure 7.3 shows the small signal gain versus wavelength for an EDFA pumped by 50mW at 980nm. It is this broadened gain characteristic which makes EDFAs suitable for wavelength division multiplexing. At the peak of the gain characteristic, single pass gains in excess of 45dB have been achieved and practical amplifiers are deployed routinely with gains in the region of 20 to 35dB at around 1550nm. For 980nm pumping, saturated output powers in excess of 30% of the pump power are possible and at 1480nm this figure is in excess of 50%. (**Note:** the saturated output power is defined as the output power for which the gain of the amplifier has dropped by -3dB relative to the small signal gain).

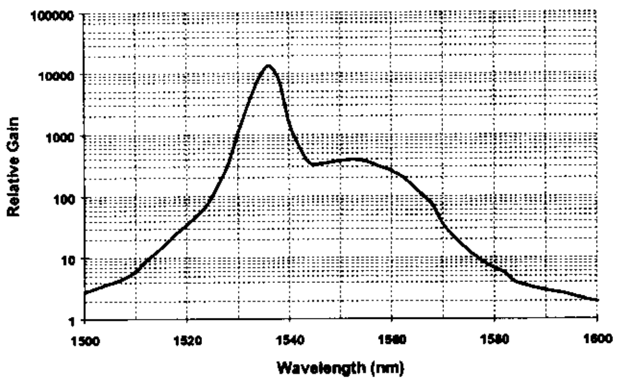


***Figure 7.3:** Gain spectrum of EDFA*

The variation of the gain, output power and saturated output power with pump power and input signal level are important characteristics of EDFAs. A useful insight into the general forms of these characteristics may be gained from the analysis presented above for bulk optical amplifiers, whilst remembering that longitudinal pumping is the primary operational difference. One immediately obvious consequence of longitudinal pumping is the existence of an optimum length of gain fibre for a given pump power. In a longitudinal pumping geometry the pump power decreases exponentially along the length of the fibre as the power is absorbed according to Beer's law. Hence, the gain coefficient is distance dependent. If the doped fibre is too long the power at the end furthest from the pump source is insufficient to create a population inversion and that part of the fibre acts as an attenuator. Reducing the length from this condition results in increasing gain until we reach the optimum length at which the pump power at the end of the fibre is just sufficient to achieve transparency. Here, the gain is optimum and further reductions in length lead to decreasing gain. Once the optimum, or near, but slightly less than optimum length is chosen, the amplifier must be fully characterised.

In the laboratory exercises associated with these notes you will investigate the gain and power characteristics of an optical amplifier measuring parameters such as the gain gradient, the gain efficiency, pump and signal saturation effects etc. In addition, you will be asked to explain the form of each characteristic in terms of the physical principles of optical amplifiers as described above. To complete these notes add your measured characteristics and your discussions of their forms as Appendix B

7.3 Noise Characteristics and SNR

The theory of noise in optically amplified systems developed in Section 6 and in Appendix B is directly applicable to systems using EDFAs in which the detection process is optical noise limited (i.e. receiver thermal noise is small). In such systems, the noise at the receiver output is determined by the summation of ASE-ASE and Signal-ASE beat noise which are the dominant sources of noise in optically amplified signals. Hence the SNR at the receiver output is given by (see Section 6 and Appendix A):

$$SNR_{out} = \frac{(RGP_o)^2}{4R^2GP_0P_{ASE}^{B_e} + 2R^2P_{ASE}^{B_e}P_{ASE}^{B_e}} \qquad (7.1)$$

For low output signal levels and / or broad optical bandwidth (no optical filtering) the noise is dominated by the second term in the denominator (ASE-ASE beat noise). With growing signal levels and / or reduction of the optical bandwidth the Signal-ASE beat noise becomes dominant and the receiver SNR is given by:

$$SNR_{out} = \frac{(RGP_0)^2}{4R^2GP_0\rho_{ASE}B_e} = \frac{GP_0}{4P_{ASE}^{B_e}} \qquad (7.2)$$

Equations 7.1 and 7.2 clearly indicate the importance of reducing the level of ASE arriving at the receiver, and anything we can do to minimise it improves the SNR_{out}. The most common approaches to ASE reduction are the use of a narrow band optical filter centred on the signal wavelength and, where possible the insertion of a polariser aligned to the signal polarisation state, thus rejecting 50% of the unpolarised ASE. Lowering the gain also reduces the ASE level (equation 6.8) and in some systems where it is necessary to limit the levels of ASE it is better to operate with larger input signal levels and lower gains to achieve the required output power.

Increasing the signal input level to an optical amplifier eventually leads to significant depletion of the population inversion and the gain. This is referred to as gain saturation. As the amplifier operation moves into gain saturation, depletion of the population inversion leads to reduction of the ASE. Under such operating conditions the optical noise for single amplifiers is dominated by Signal-ASE beat noise and the output SNR is given by equation 7.2. In systems using multiple amplifiers (amplifier cascades) this also reduces the build up of ASE power which, if unchecked in large systems, ultimately outgrows the signal and takes up more of the gain from subsequent amplifiers. Another advantage of operating in gain saturation is the self correcting effect it imposes on the output signal power for variation in the input signal power. Larger input signals experience less gain and vice versa. Hence, for small variations in the signal input power the output remains constant. For these reasons optically amplified systems are often designed such that the amplifiers operate in gain saturation.

As noted in Section 6, the noise figure, NF, can be useful in calculations of the output SNR of a system using optical amplifiers, provided that it is known for the precise operating conditions of the amplifiers(s) i.e. signal and pump levels relative to saturation powers. If indeed, the noise figure is known then the output SNR of a single amplifier may be determined from:

$$SNR_{out} = \frac{P_0}{2h\nu B_e.NF} \quad (7.3)$$

For this purpose noise figures are either measured directly as a function of the operating conditions or indirectly by measuring the ASE power spectrum and the gain followed by a simple calculation using equation 6.21 or 6.23. The theoretical lower limit on the noise figure is 3dB, as noted in Section 6, and measurements on EDFAs have realised figures close to this level under small signal gain conditions far from saturation.

7.4 Noise in Amplifier Cascades

In long distance systems or systems in which there is a great deal of loss, such as branched distribution networks, multiple amplifiers are inserted at strategic intervals between the transmitter and receiver(s) to compensate for the losses. In such systems each amplifier adds its ASE to the signal and amplifies the ASE (reduced by intervening loss) from preceding amplifiers. Hence, the ASE level builds up through the system and ultimately determines the SNR and detection limits at the system output through its participation in Signal-ASE and ASE-ASE beat noise. To establish the SNR at the output we need to determine the total gain (loss) of the entire system to calculate the output signal and then we need to determine the accumulation of ASE through the system in order to calculate the optical noise at the output[4, 6]. The total gain of the system, G_T, is the product of all of the amplifier gains times the product of the total line losses from the system input to the output of the last amplifier. Each amplifier generates its own ASE of a given spectral density, ρ_{ASE-m}, where m is the amplifier number sequenced from the system input. That spectral density experiences a net gain equal to the product of the total amplifier gains and the total line losses between that particular amplifier output and the output of the last amplifier. Hence, the spectral density of ASE, ρ_{ASE-R}, at the output of the final amplifier is:

$$\rho_{ASE-R} = \rho_{ASE-1}.G_1^{**} + \rho_{ASE-2}.G_2^{**} + ...\rho_{ASE-n}.G_n^{**} \quad (7.4)$$

where there are a total of n amplifiers in the system and G_m^{**} is the net gain between the output of amplifier m and the output of the final amplifier i.e.

$$\rho_{ASE-R} = \sum_{m=1}^{n} \rho_{ASE-m} G_m^{**} \quad (7.5)$$

Applying equation 7.1, the signal to noise ratio, SNR_R, at the final amplifier output is given by:

$$SNR_R = \frac{(RG_T P_o)^2}{4R^2 G_T P_0 \rho_{ASE-R} B_e + 2R^2 \rho_{ASE-R}^2 B_o B_e} \quad (7.6)$$

In many systems the amplifiers are equally spaced and the gain of each amplifier exactly offsets the line loss of the link from the previous amplifier. In such cases, referred to as unity gain systems, $\rho_{ASE-R} = n\rho_{ASE}$ (assuming that all of the amplifiers have the same gain) and G_T and G_m^{**} are 1, provided that P_0 is the launch power from the transmitter. Hence equation 7.6 becomes:

$$SNR_R = \frac{P_o^2}{4P_0 n\rho_{ASE} B_e + 2n^2 \rho_{ASE}^2 B_o B_e} = \frac{P_o^2}{4P_0 n P_{ASE}^{B_e} + 2n^2 P_{ASE}^{B_o} P_{ASE}^{B_e}} \quad (7.7)$$

where ρ_{ASE} is the spectral density of ASE emitted by each amplifier in the chain and $P_{ASE}^{B_e}$ and $P_{ASE}^{B_o}$ are the ASE powers emitted in the receiver and optical bandwidths respectively.

Since the signal and the ASE are attenuated equally by the fibre, equations 7.6 and 7.7 give the signal to noise ratio at the receiver irrespective of its position relative to the final amplifier provided that the detected signal is optical noise limited (i.e. the final fibre section is not long enough to attenuate the optical beat noise terms to such an extent that they are less than the receiver thermal noise0.

When the Signal-ASE beat noise is dominant equation 7.6 becomes:

$$SNR_R = \frac{G_T P_o}{4\rho_{ASE-R} B_e} \quad (7.8)$$

and for unity gain systems we have:

$$SNR_R = \frac{P_o}{4n\rho_{ASE} B_e} = \frac{P_o}{4nP_{ASE}^{B_e}} \quad (7.9)$$

In the above equations, n = L/L_o, where L is the total link length and L_o is the distance between successive amplifiers and ρ_{ASE} is given by equation A.1. Taking unity gain systems limited by Signal ASE beat noise as an example, this means that the received SNR may be written:

$$SNR_R = \frac{P_o}{4n\rho_{ASE} B_e} = \frac{P_o}{4(L/L_o).h\nu.\mu.(G_m - 1)B_e} \quad (7.10)$$

where G_m is the gain of each individual amplifier given by $G_m = \exp[\alpha L_o]$ in a unity gain system, α being the loss coefficient of the fibre link. Hence, rewriting equation 7.10 we get:

$$SNR_R = \frac{P_o}{4(L/L_o).h\nu.\mu.(\exp[\alpha L_o] - 1)B_e} \quad (7.11)$$

Let us examine the dependence of SNR_R on the distance between amplifiers. As we reduce L_o, n increases linearly with $1/L_o$, but $\exp[\alpha L_o]$ decreases much faster. Assuming all other parameters remain constant (although, μ increases with decreasing gain but only very slowly), equation 7.11 indicates that the received SNR at a given fixed link length improves as we increase the number of amplifiers and reduce their gains. This is consistent with the fact that reducing the gain of an amplifier reduces the ASE level emitted and hence the noise at the receiver. Alternatively, for a required SNR_R at the receiver, this means that we can transmit further if we reduce the amplifier spacing. The amplifier spacing and gain setting are thus crucial parameters in achieving the desired SNR_R at the required link length. Although we have developed this argument for a unity gain system limited by Signal-ASE beat noise it also applies to all other conditions.

7.4.1 Noise Figures for Amplifier Cascades

An effective noise figure, NF_{eff}, for an amplifier cascade may be readily determined by substituting equation 7.6 into 6.20. In this instance, however, we will define the system input power (denoted here as P_0^*) as that immediately before the first amplifier in keeping with the definition of noise figure. This is perfectly valid as equation 7.6 applies generally with any definition of the input power. The result is:

$$NF_{eff} = \frac{4R^2 G_T P_o^2 P_{ASE-R}^{B_e} + 2R^2 P_o P_{ASE-R}^{B_e} P_{ASE-R}^{B_e}}{2h\nu B_e.(RG_T P_o)^2} \tag{7.12}$$

$$= \frac{2\rho_{ASE-R}.B_e}{h\nu G_T.B_e} + \frac{\rho_{ASE-R}^2.B_o}{h\nu P_o G_T^2} \tag{7.12a}$$

$$= \frac{2}{h\nu}.\sum_{m=1}^{n} \frac{\rho_{ASE-m}}{G_m G_m^*} + \frac{B_o}{h\nu P_o}.\sum_{m=1}^{n} \frac{(\rho_{ASE-m})^2}{G_m^2 (G_m^*)^2} \tag{7.12b}$$

where G_m is the gain of each individual amplifier and G_m^* is the accumulated gain (including losses) from the input of the first amplifier to the input of the m^{th} amplifier. In a unity gain system (described above) G_m^* is 1 and equation 7.12b becomes:

$$NF_{eff} = \frac{2n\rho_{ASE-m}}{h\nu G_m} + \frac{B_o n^2 \rho_{ASE-m}^2}{h\nu P_o G_m^2} \tag{7.13}$$

For systems in which the noise is dominated by Signal-ASE beat noise, we can write:

$$NF_{eff} = \frac{2\sum_{m=1}^{n} \rho_{ASE-m}}{h\nu G_m G_m^*} \tag{7.14}$$

and by comparison with equation 6.21, we obtain:

$$NF_{eff} = NF_1 + \frac{NF_2}{G_2^*} + \frac{NF_3}{G_3^*} \ldots\ldots\ldots\ldots \frac{NF_n}{G_m^*} \tag{7.15}$$

For unity gain systems dominated by Signal-ASE beat noise, G_m^* is 1 and the effective noise figure is simply n times the noise figure for an individual amplifier ($N_{eff} = nNF$).

Given that we can establish the noise figures of individual amplifiers under their prevailing operating conditions and the cumulative gains across the appropriate sections of the system, then we can establish the effective noise figure for the entire system. By analogy with equation 6.20 it is then a simple matter to determine the received signal to noise ratio, SNR_R, as:

$$SNR_R = \frac{P_0^*}{2h\nu B_e.NF_{eff}} \tag{7.16}$$

where generally N_{eff} is given by equation 7.15.

For unity gain systems the received signal to noise ratio is then given by:

$$SNR_R = \frac{P_0^*}{2h\nu B_e.n.NF} \tag{7.17}$$

where P_0^* is the input power to the first amplifier in the chain.

Again, as for single amplifiers, we can thus use the effective noise figure to determine the output SNR of an amplifier cascade.

8.CONCLUSIONS

Following initial demonstrations of principle in the late 1980s, it was quickly appreciated that EDFAs were highly compatible with 1550nm fibre optic telecommunications systems and that their potential performance could significantly enhance such systems by providing high gain, excellent reliability and low power consumption. In addition, they proved to be bit rate transparent and wavelength transparent (within their optical bandwidth), implying that system capacities could be increased by increasing the bit rate capabilities of terminal equipment or by introducing wavelength division multiplexing. For these reasons EDFAs were developed from proof of principle to fully engineered products for deployment under the ocean in about 6 to 8 years. Of course, EDFAs provide only gain but do not reshape the signals which spread and degrade by dispersion. However, with the advent of dispersion shifted and dispersion compensating fibre, the dispersion limits have been extended dramatically even to transoceanic distances. EDFAs have thus found numerous applications, turning previously attenuation limited systems into much higher performance dispersion limited systems. They have found use as:

- Periodic in line amplifiers using multiple amplifiers in long haul trunk and transoceanic systems
- Pre-amplifiers in front of receivers to enhance their sensitivity
- Power amplifiers in front of transmitters to increase the launched power
- In line amplifiers in branched networks to compensate for splitting losses in higher order branches and
- various combinations of the above in a single system.

As such EDFAs represent one of the most significant single developments in fibre optic telecommunications in the last decade.

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