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"Optoelectronic Repeaters"

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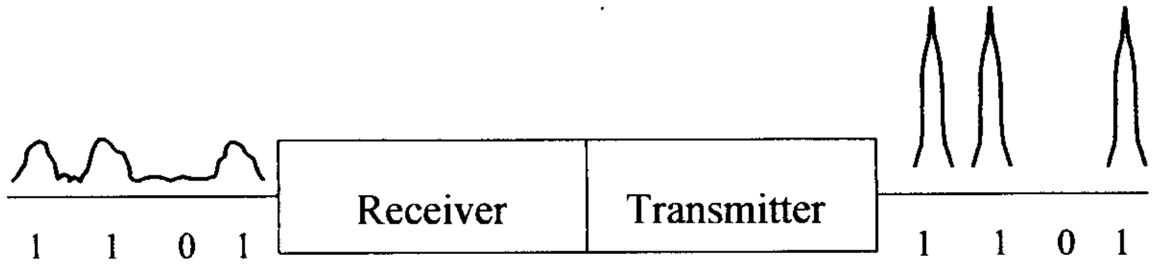
Please note: These are preliminary notes intended for internal distribution only.

OPTOELECTRONIC REPEATERS



- Restores pulses to their original amplitude (power) and width

Disadvantages

- High bit rate is expensive
- Cannot upgrade bit rate
- Single wavelength operation
- Power, cost, reliability & maintenance issues

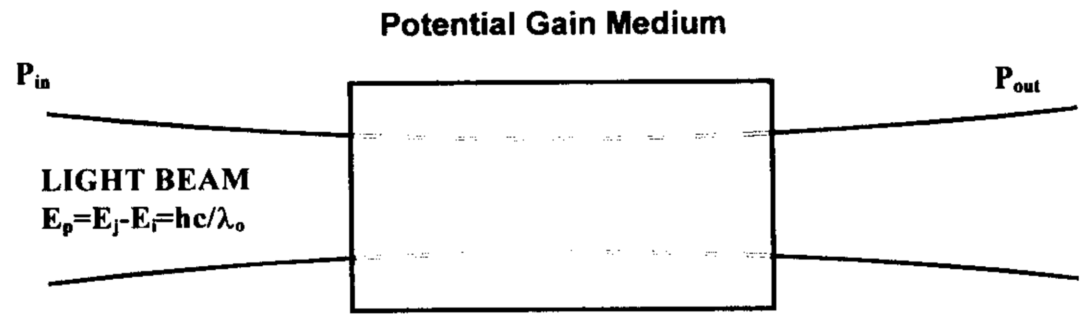
OPTICAL AMPLIFIERS / REPEATERS

- No need for optical to electrical to optical conversion
- Bit rate transparency????
- Wavelength Transparency????
  - Multiple wavelength amplification and WDM????
- Low power consumption, cost, maintenance etc???

PRINCIPLES OF LASERS

A LASER IS AN OPTICAL OSCILLATOR - AN OPTICAL AMPLIFIER WITH PARTIAL POSITIVE FEEDBACK

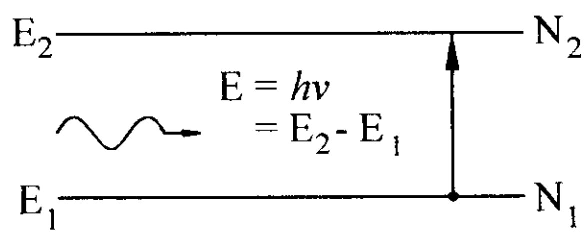
Principles of optical amplification:



Gain medium has 2 energy levels E_j & E_i ($E_j-E_i=E_p$)

Consider the material photon interactions for the above system

ABSORPTION



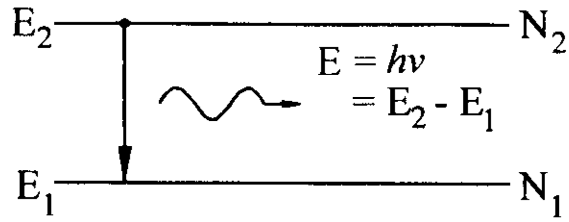
Rate of absorption = $N_i\ \rho_\nu B_{12}$

N_1 = No of atoms in energy state 1 per unit volume ie. the population of state I

ρ_ν = The photon energy density -

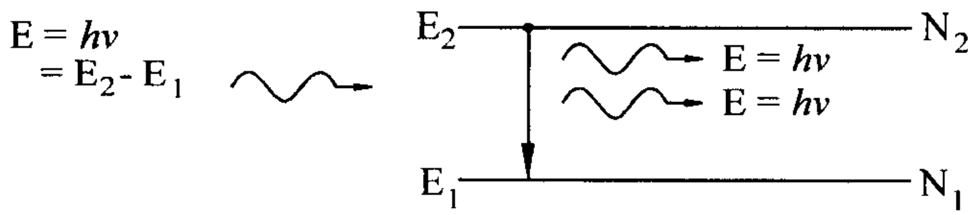
B_{12} = Proportionality constant - probability of absorption per unit time

SPONTANEOUS EMISSION



Rate of spontaneous emission = $A_{21}N_2$

STIMULATED EMISSION



Rate of stimulated emission = $N_2\rho_v B_{21}$

Rate of absorption = $N_1\rho_v B_{12}$
Rate of stimulated emission = $N_2\rho_v B_{21}$

$B_{12} = B_{21}$ Hence relative levels of stimulated emission or absorption depend on N_1 and N_2

$N_1 > N_2 \rightarrow$ Net absorption $\rightarrow$ Loss
$N_2 > N_1 \rightarrow$ Net stimulated emission $\rightarrow$ Gain

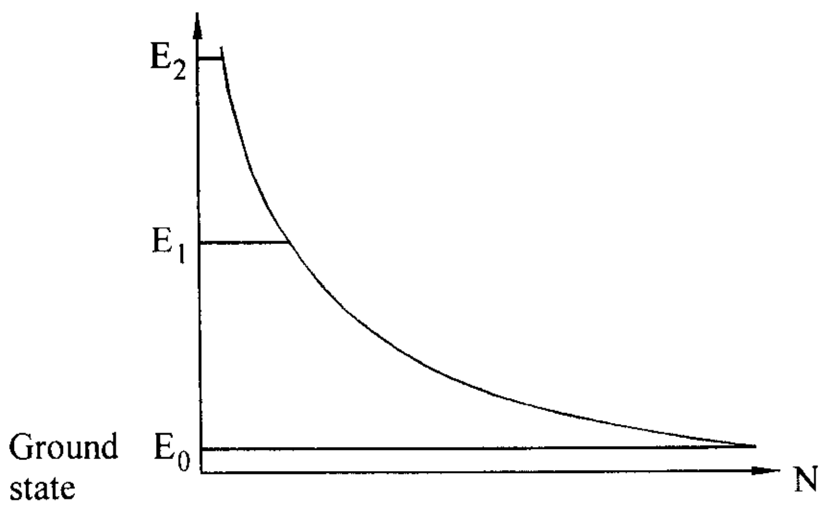
$N_2 > N_1 \rightarrow$ POPULATION INVERSION

The Boltzmann distribution of Population at thermal equilibrium:

$$\frac{N_j}{N_i} = \frac{g_j}{g_i} e^{-(E_j - E_i)/kT}$$

$$= e^{-(E_j - E_i)/kT}$$

$$= e^{-h\nu/kT}$$

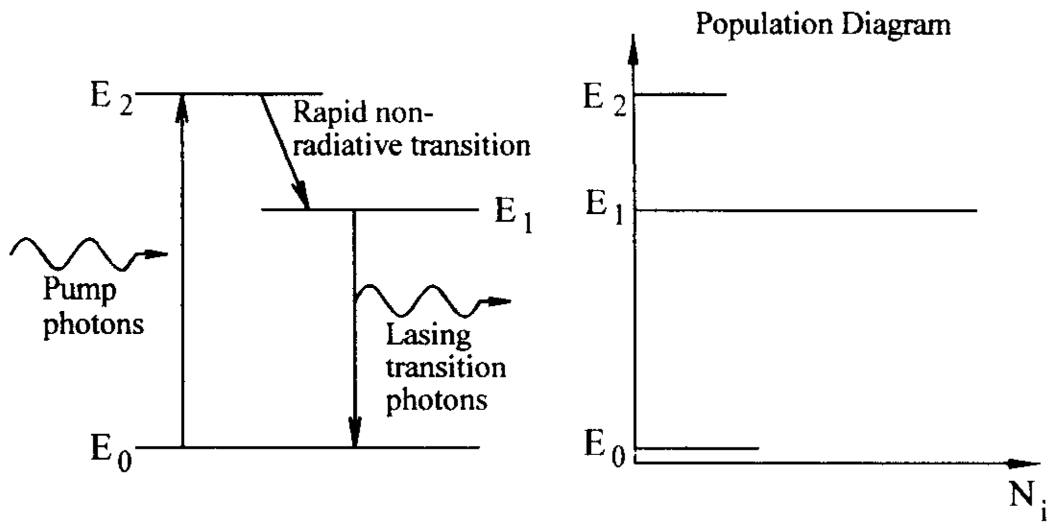


The Boltzmann distribution

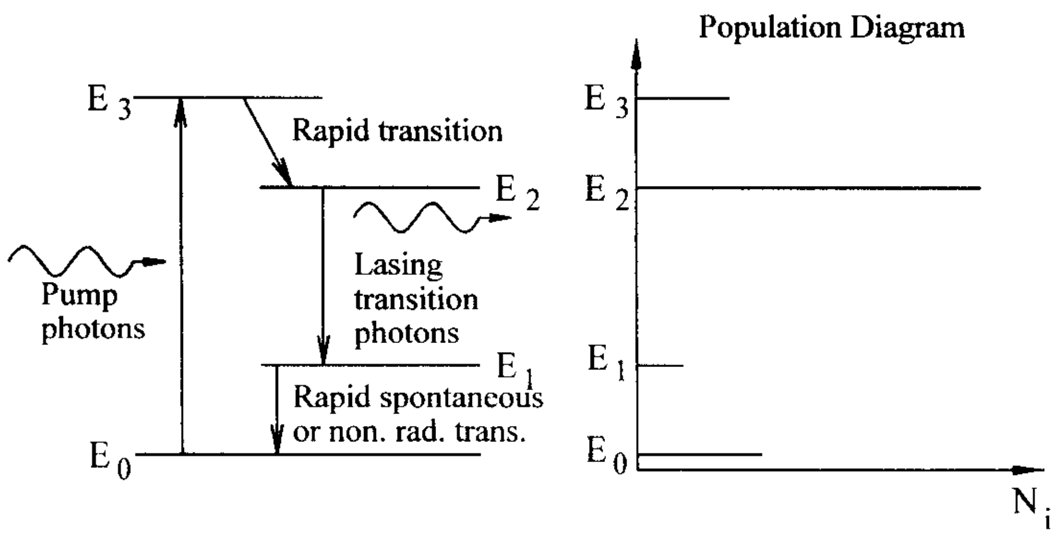
For $E_j - E_i = 1.99 \times 10^{-19}$J $N_j/N_i = 6.0 \times 10^{-22}$

Photon energy = 1.99×10^{-19}J, λ is 1μm

POPULATION INVERSION - THREE LEVEL SYSTEM



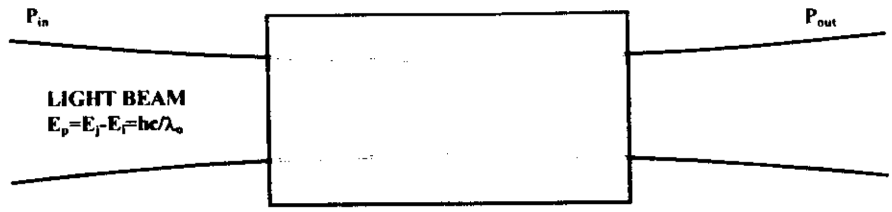
POPULATION INVERSION - FOUR LEVEL SYSTEM



USING 3 & 4 LEVEL SYSTEMS WITH SUITABLE PUMPING WE CAN REALISE OPTICAL AMPLIFIERS → ALL WE NEED IS FEEDBACK

4 LEVEL SYSTEM IS BETTER

OPTICAL AMPLIFICATION



Material having energy levels E_i & E_j with population N_i & N_j respectively

Light of photon energy $E_p=hc/\lambda_o$ passing through a potential gain material having 2 energy levels E_j & E_i ($E_j-E_i=E_p$)

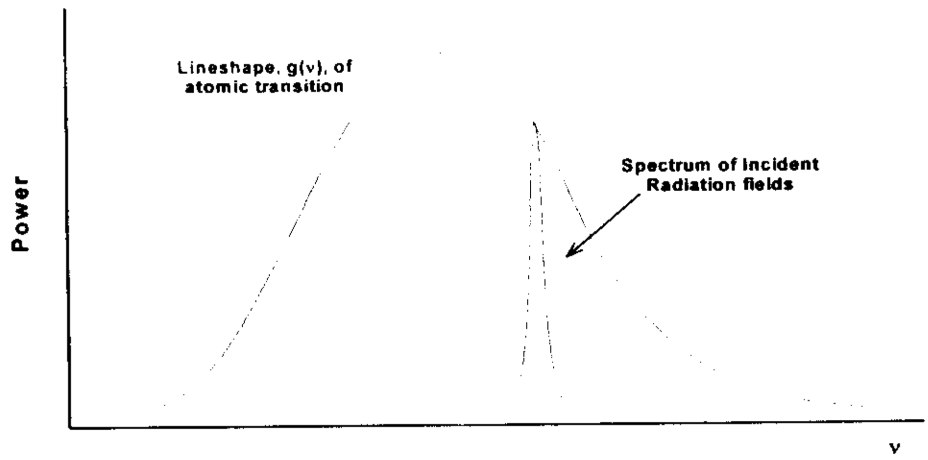
$$Rate_{[SpE]} = -\mathrm{dN_j/dt}_{[SpE]} = A_{ji}N_j \tag{1}$$

$$Rate_{[ABS]} = \mathrm{dN_i/dt}_{[ABS]} = B_{ij}N_i\rho(\nu) \tag{2}$$

$$Rate_{[SE]} = \mathrm{dN_j\,/\,dt_{[SE]}} = B_{ji}N_j\rho(\nu) \tag{3}$$

Rate equations 2 & 3 are not quite correct due to the interaction of a narrow band radiation field with the broader lineshape functions of atomic transitions

Narrow band incident light field interacting with broader atomic transition



g(ν) is the relative probability of a transition occurring for incident light frequency ν across the lineshape function of an atomic transition

$$\int_0^\infty g(\nu)d\nu = 1 \tag{4}$$

$$Rate = B_{ij}N_i \int_0^\infty \rho(\nu)g(\nu)d\upsilon \tag{5}$$

Hence, the rate of absorption for a monochromatic light wave is:

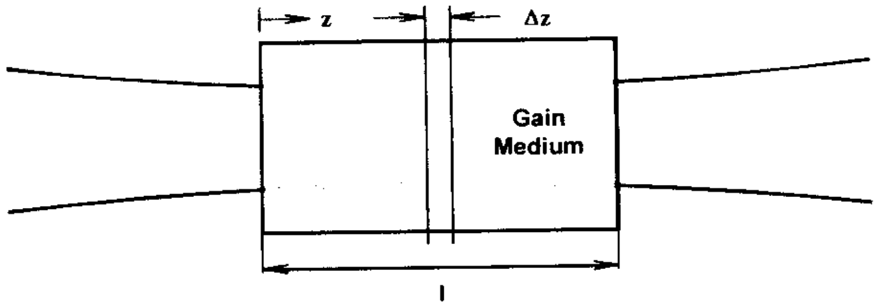
$$Rate = B_{ij}N_i\rho_\nu g(\nu) \tag{6}$$

and the rate of stimulated emission is:

$$Rate = B_{ji}N_j\rho_\nu g(\nu) \tag{7}$$

Often we make the substitution: $\rho_\nu = \dfrac{I_\nu . n}{c}$ (8)

DERIVATION OF GAIN EQUATIONS



Geometry for calculating gain from slab of thickness Δz within the body of the gain medium

Stimulated emission rate:

$$\frac{dN_j}{dt}.A.\Delta z = -B_{ji}N_j.\frac{I_\nu(z)n}{c}.g(\nu).A.\Delta z \qquad (9)$$

where A is crossection of beam & I(z) is the light intensity at position z

Each transition adds a photon of energy hν:

$$\Delta I_\nu(z) = B_{ji}N_j.\frac{I_\nu(z)n}{c}.g(\nu).h\nu.\Delta z \qquad (10)$$

Similarly for absorption:

$$-\Delta I_\nu(z) = B_{ij}N_i.\frac{I_\nu(z)n}{c}.g(\nu).h\nu.\Delta z \qquad (11)$$

Hence the rate of change of intensity with z

$$\frac{dI_\nu(z)}{dz} = B_{ji}N_j.\frac{I_\nu(z)n}{c}.g(\nu).h\nu. - B_{ij}N_i.\frac{I_\nu(z)n}{c}.g(\nu).h\nu.$$

$$= \left[\frac{n}{c}.g(\nu).h\nu B_{ji}\left(N_j - N_i\right)\right]I_\nu(z) \qquad (12)$$

$$= \gamma_o(\nu).I_\nu(z)$$

where $B_{ji} = B_{ij}$, $\gamma_o(\nu)$ is called the gain coefficient and it is assumed that N_I & N_j are independent of $I_\nu(z)$.

making the substitution, $A_{ji} = \frac{8\pi n^3 h\upsilon^3}{c^3}.B_{ji}$, (see notes) we get the more common expression:

$$\frac{dI_\nu}{dz} = \left[A_{ji}\frac{\lambda_0^2}{8\pi n^2}g(\nu)\left(N_j - \frac{g_j}{g_i}N_i\right)\right]I_\nu(z) = \gamma_0(\nu)I_\nu(z) \qquad (13)$$

Integrating (13) wrt z we get $I_\nu(z)$ for an input signal $I_\nu(0)$:

$$\mathrm{I}_\nu(z) = \mathrm{I}_\nu(0)\exp[\gamma_0(\nu)z] \qquad (14)$$

For a gain medium of length l, $G_0(\nu) = I_\nu(l)/I_\nu(0)$, where

$$G_0(\nu) = \exp[\gamma_0(\nu)l] \qquad (15)$$

Expressing the gain in the customary way in dB we get

$$Gain(dB) = 10\log_{10}G_0(\nu) = 10\log_{10}e^{\gamma(\nu)l} = 4.34\gamma_o(\nu)l \qquad (16)$$

It is often convenient to specify the gain coefficient in terms of the stimulated emission or absorption cross-sections, σ_{SE} and σ_{ABS}, where these are given by:

$$\sigma_{SE} = A_{ji} \cdot \frac{\lambda_0^2}{8\pi n^2} \cdot g(\nu) \quad and \quad \sigma_{ABS} = A_{ij} \cdot \frac{\lambda_0^2}{8\pi n^2} \cdot g(\nu) \cdot \frac{g_2}{g_1} \qquad (17)$$

Hence

$$\gamma_o(\nu) = \sigma_{se}(\nu) . \lfloor N_j - N_i \rfloor \qquad (18)$$

where the second term on the right is referred to as the population inversion.

Equations (12) to (16) assume that the populations N_j and N_l are independent of the pump power and the signal power. For this reason $\gamma_o(\nu)$ is called the small signal gain coefficient.

Equations 13 & 16 predict that the gain and the output signal should increase linearly with pump power and that the gain is constant with signal power for a given pump.

In practice we get **signal saturation** and **pump saturation**

A full analysis, beyond the scope of this course (see notes), shows that for a 3 level system:

$$N_2 = \frac{(I_\nu^* + I_p^*)N_t}{1 + 2I_\nu^* + I_p^*} \qquad (19)$$

where $I_\nu^* = I_\nu / I_s$ and $I_p^* = I_p / I_{ps}$, with I_s and I_{ps} are the signal and pump saturation powers and N_t is the total population density of atoms.

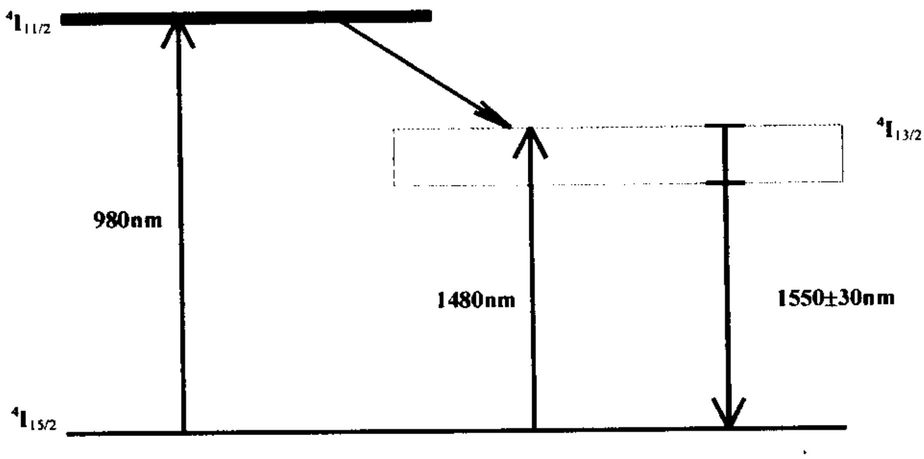
Given that $dI_\nu/dz = \sigma_{SE}(N_2 - N_1)I_\nu$, $dI_p / dz = \sigma_{ABS\_p}N_1$ and substituting $N_1 = N_t - N_2$ and N_2 from equation (17), we obtain:

$$\frac{dI_\nu}{dz} = \frac{(I_p^* - 1)\sigma_{SE}N_t I_\nu}{1 + 2I_\nu^* + I_p^*} \qquad (20)$$

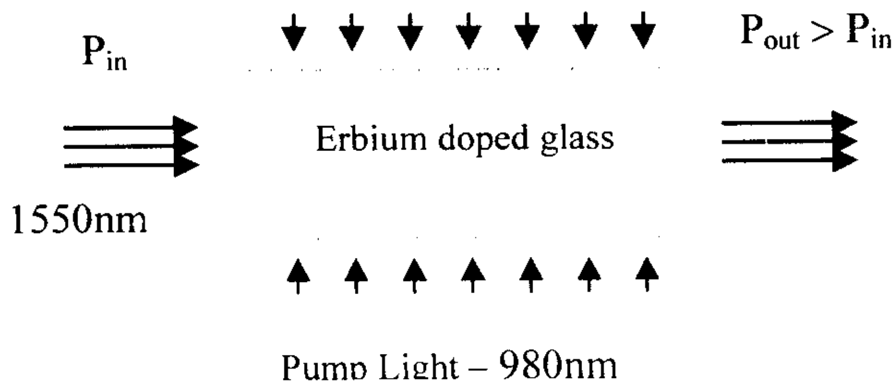
It must also be remembered that optical amplifiers also spontaneously emit light and we get high levels of Amplified Spontaneous Emission (ASE) at the output. This leads to noise on an amplified signal (see later).

ERBIUM DOPED FIBRE AMPLIFIERS (EDFAS)

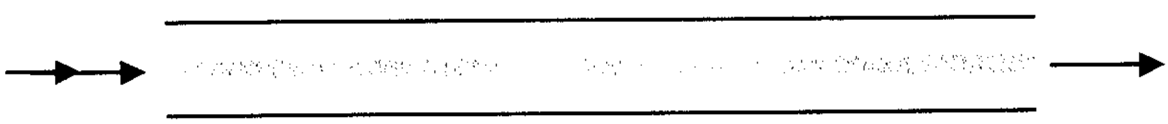
The requirement in communications systems is for again at 1550nm or 1300nm.



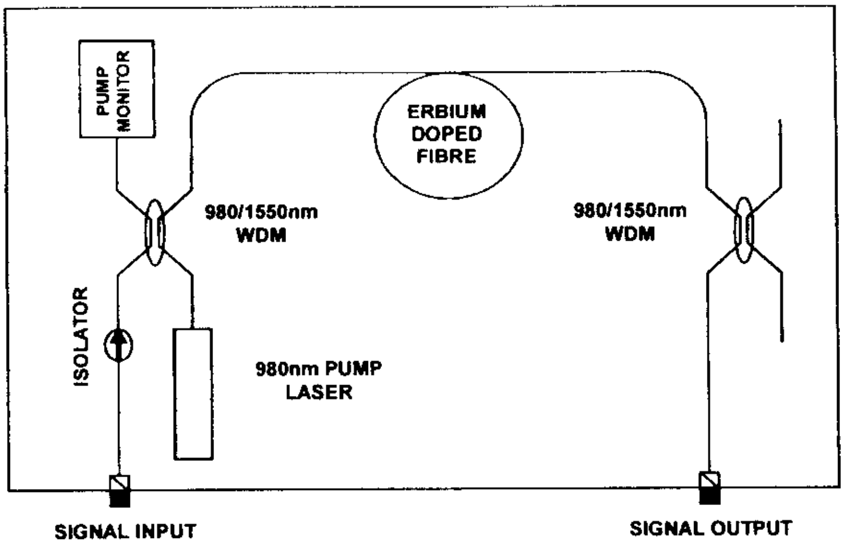
Energy level diagram and pumping scheme for erbium doped silica glass - a three level system giving gain at 1550±30nm.



Erbium doped optical fibre:

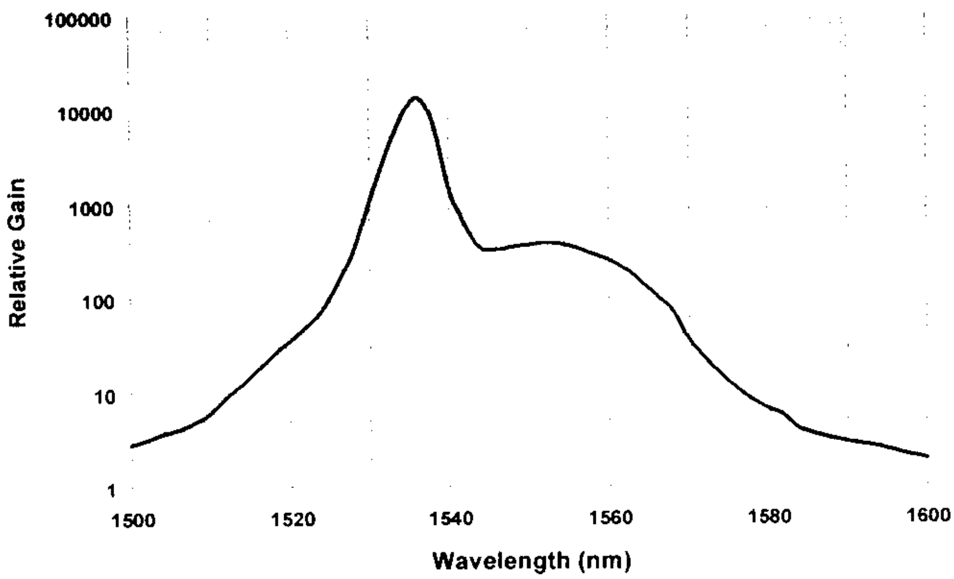


EDFAs



Often a second isolator is used and a band pass filter is inserted at the output to reduce the amplified spontaneous emission (ASE)

Gain versus wavelength:



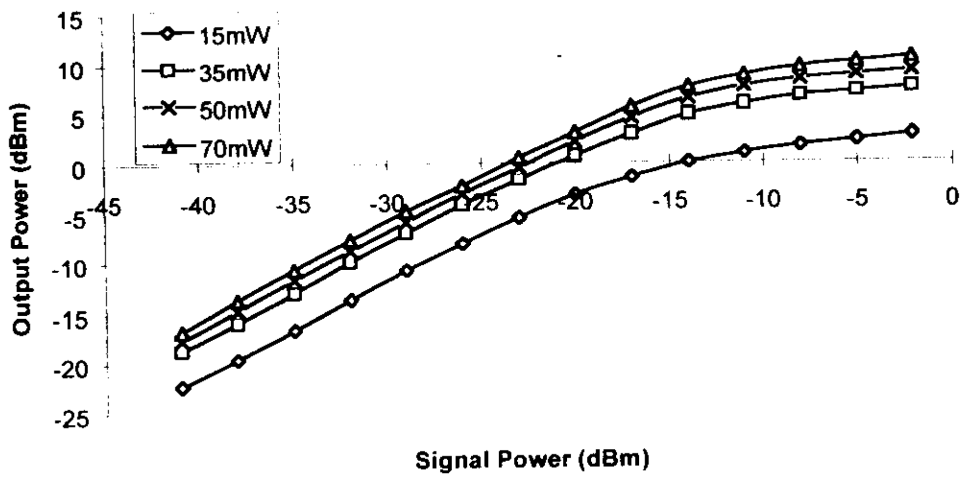
GAIN CHARACTERISTICS OF EDFAs

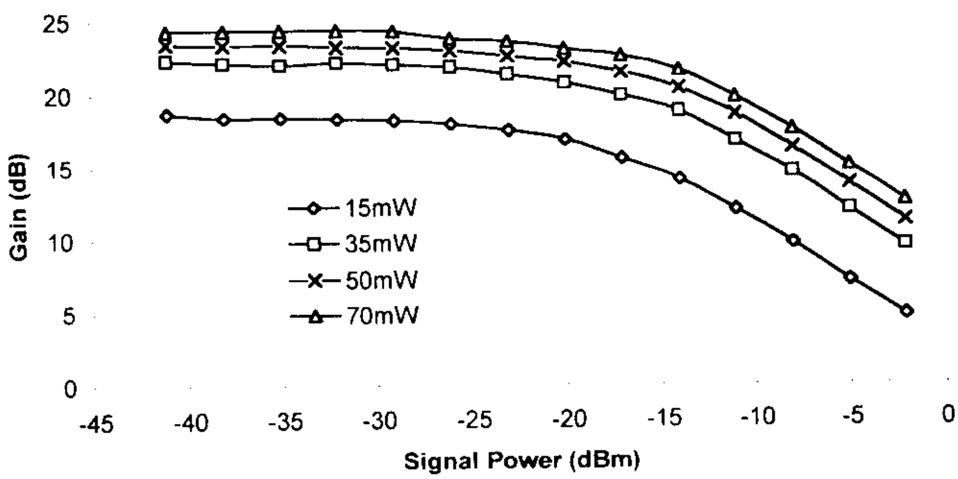
Bearing in mind the small signal gain expression

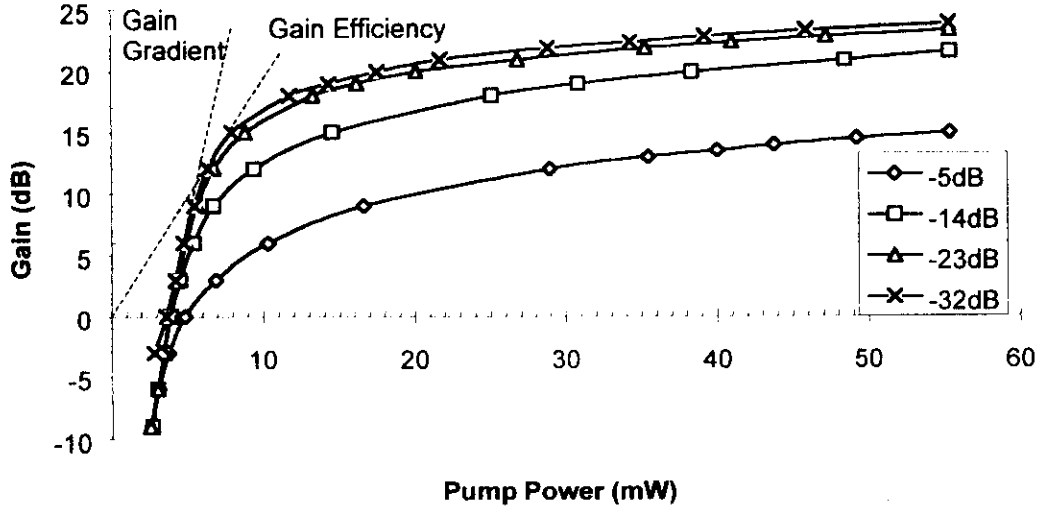
$$Gain(dB) = 10\log_{10} G_0(\nu) = 10\log_{10} e^{\gamma(\nu)l} = 4.34\gamma_o(\nu)l \quad (19)$$

and the expression tking account of saturation effects

$$\frac{dI_\nu}{dz} = \frac{(I_p^* - 1)\sigma_{SE} N_t I_\nu}{1 + 2I_\nu^* + I_p^*} \quad (20)$$

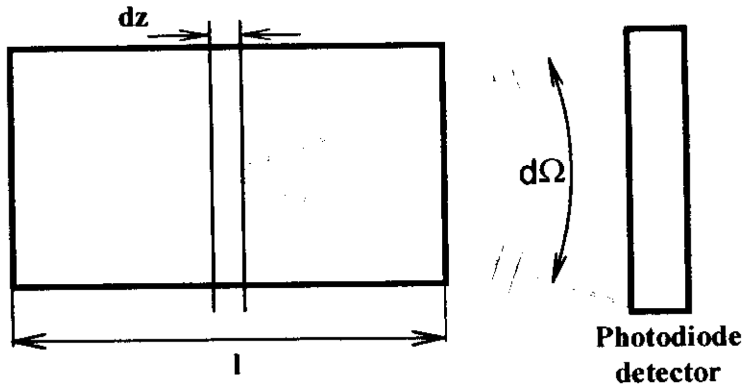






Gain
Gradient
Gain Efficiency
25
20
15
10
5
0
-5
-10
Gain (dB)
10
20
30
40
50
60
Pump Power (mW)
-5dB
-14dB
-23dB
-32dB

ASE & NOISE CHARACTERISTICS OF EDFAs



Geometry for calculating spontaneous emission power collected by a remote detector from a slab of thickness dz within the body of the gain medium.

SE power emitted through end face into solid angle dΩ from thin slab of thickness dz:

$$dP_{ASE}(d\Omega) = A_{21}N_2 g(\nu)d\nu.h\nu.\frac{d\Omega}{4\pi}.Adz \qquad (21)$$

For a waveguide, dΩ, is the solid angle associated with the numerical aperture of the guide i.e. $d\Omega = NA^2/4$.

This power is amplified by a factor $e^{\gamma(l-z)}$ and the total ASE is the summation of the contributions from all of the slabs i.e.

$$P_{ASE} = \int_0^l dz.\frac{dP_{ASE}}{dz}.e^{\gamma(l-z)} \qquad (22)$$

Evaluation of the integral gives:

$$P_{ASE} = \frac{[G-1]}{\gamma}.A_{21}N_2 g(\nu).h\nu.A\frac{d\Omega}{4\pi}.d\nu \qquad (23)$$

where G is the total gain of the amplifier given as G = exp[γl]

Substituting $\gamma(\nu) = \sigma_{SE}(\nu).[N_2 - N_1]$ and $\sigma_{SE}(\nu) = A_{21}.\frac{\lambda_0^2}{8\pi n^2}.g(\nu)$ gives:

$$P_{ASE} = \frac{N_2}{N_2 - N_1}.[G-1].h\nu.2A\frac{n^2}{\lambda_0^2}d\Omega.d\nu \qquad (24)$$

$$= 2\mu[G-1]h\nu.\frac{An^2}{\lambda_0^2}.d\Omega.d\nu \qquad (25)$$

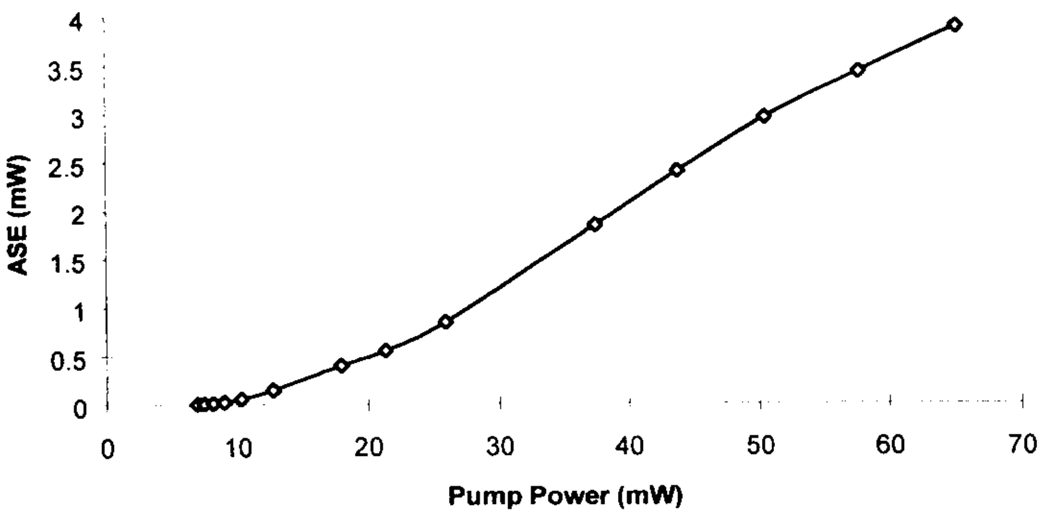
where $\mu = N_2/(N_2-N_1)$ is the poulation inversion factor.
In the diffraction limit the output beam diverges at a half angle, θ, given by:

$$\theta = \frac{\lambda_o}{\pi n a} \qquad (26)$$

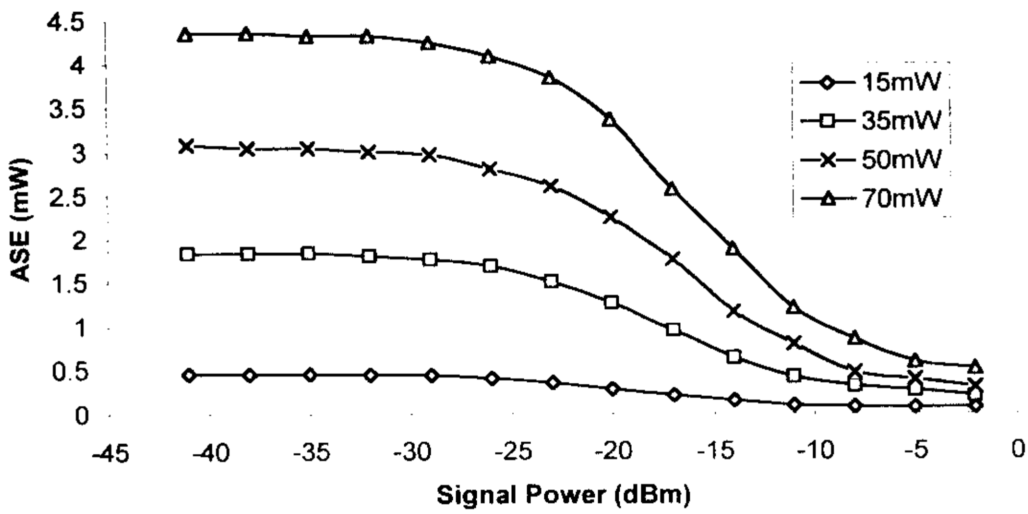
For small angles, θ, this corresponds to a solid angle, Ω_{min},:

$$\Omega_{min} = \pi \sin^2\theta = \pi\theta^2 = \frac{\lambda_o^2}{\pi n^2 a^2} = \frac{\lambda_o^2}{n^2 A} \qquad (27)$$

and hence: $P_{ASE} = \mu[G-1].h\nu.d\nu = \rho_{ASE}d\nu$ (28)



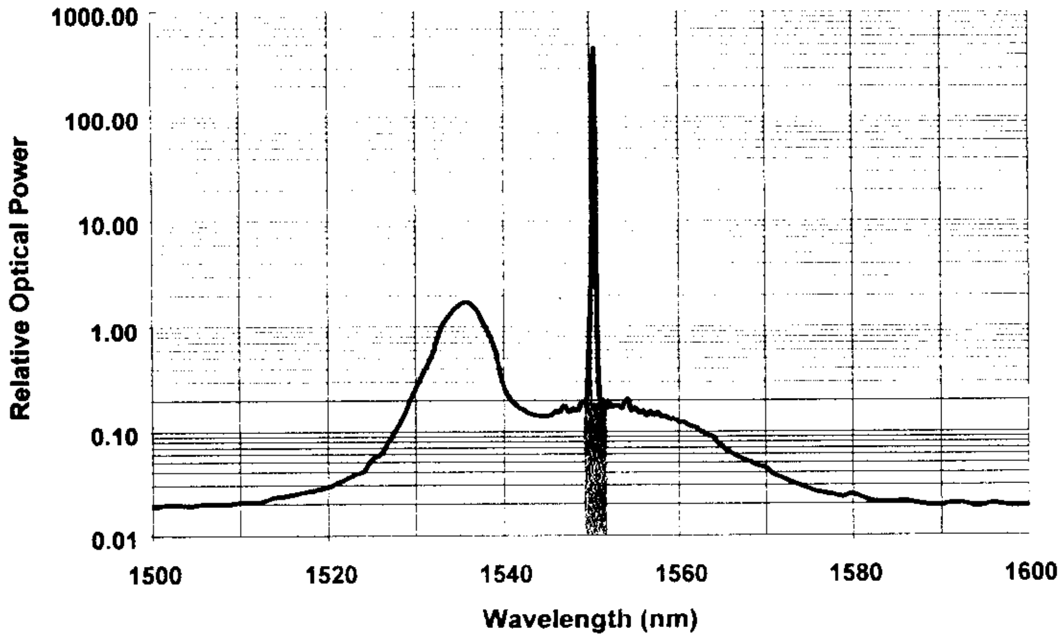
ASE power versus pump power with no signal present



ASE power versus signal power for several values of pump power

EDFA NOISE ANALYSIS

Both the signal and the ASE reach the receiver giving rise to additional noise terms. The ASE has its own shot noise and it beats with the signal and itself to give Signal-ASE beat noise and ASE-ASE beat noise.



The total noise at the receiver is therefore given by:

$$\sigma_N = \sigma_T + \sigma_S + \sigma_{ASE} + \sigma_{S-ASE} + \sigma_{ASE-ASE} \qquad (29)$$

and the optical noise terms are given by:

$$\sigma_S = 2eI_S B_e = 2eRP_S B_e = 2eRGP_0 B_e \qquad (30)$$

$$\sigma_{ASE} = 2eI_{ASE} B_e = 2eRP_{ASE} B_e \qquad (31)$$

$$\sigma_{S-ASE} = 4R^2 GP_0 \rho_{ASE} B_e = 4R^2 GP_0 P_{ASE}^{B_e} \qquad (32)$$

$$\sigma_{ASE-ASE} = 2R^2 \rho_{ASE}^2 B_o B_e = 2R^2 P_{ASE}^{B_o} P_{ASE}^{B_e} \qquad (33)$$

For most applications the output signal is kept large and the latter two optical noise terms dominate. Hence, the signal to noise ratio is given by:

$$SNR_{out} = \frac{(RGP_0)^2}{4R^2 GP_0 P_{ASE}^{B_e} + 2R^2 P_{ASE}^{B_o} P_{ASE}^{B_e}} \qquad (34)$$

In many applications optical filtering is used to remove most of the ASE and the Signal –ASE beat noise becomes dominant giving:

$$SNR_{out} = \frac{I_S^2}{\langle i_{S-ASE}^2 \rangle} = \frac{I_S^2}{\sigma_{S-ASE}} = \frac{(RGP_0)^2}{4R^2 GP_0 \rho_{ASE} B_e} = \frac{GP_0}{4P_{ASE}^{B_e}} \qquad (35)$$

<u>Noise Figure</u>

A useful way to characterise the effects of noise is to use the Noise Figure (NF = SNR_{out}/SNR_{in}). SNR_{in} is given by:

$$SNR_{in} = \frac{(RP_0)^2}{2eRP_0 B} = \frac{P_0}{2h\nu B_e} \qquad (36)$$

and substituting (35) to give NF:

$$NF = \frac{SNR_{in}}{SNR_{out}} = \frac{P_0}{2h\nu B_e} \Big/ \frac{GP_0}{4P_{ASE}^{B_e}} = \frac{2P_{ASE}^{B_e}}{Gh\nu B_e} \qquad (37)$$

The NF is given by:

$$NF = \frac{2P_{ASE}^{B_e}}{Gh\nu B_e}$$

Substituting $P_{ASE}^{B_e} = \rho_{ASE}.B_e$, using equation 28 $P_{ASE} = \mu[G-1].h\nu.d\nu = \rho_{ASE}d\nu$ and assuming significant gain, G>>1, gives:

$$NF = 2\mu \qquad (38)$$

This implies that the minimum possible noise figure is 2 (3dB) for an ideal amplifier having a complete population inversion (i.e. $N_1 = 0$, $\mu = 1$).

The Noise Figure can be very useful in calculating the SNR at the output. Using the definition of NF and equation (36):

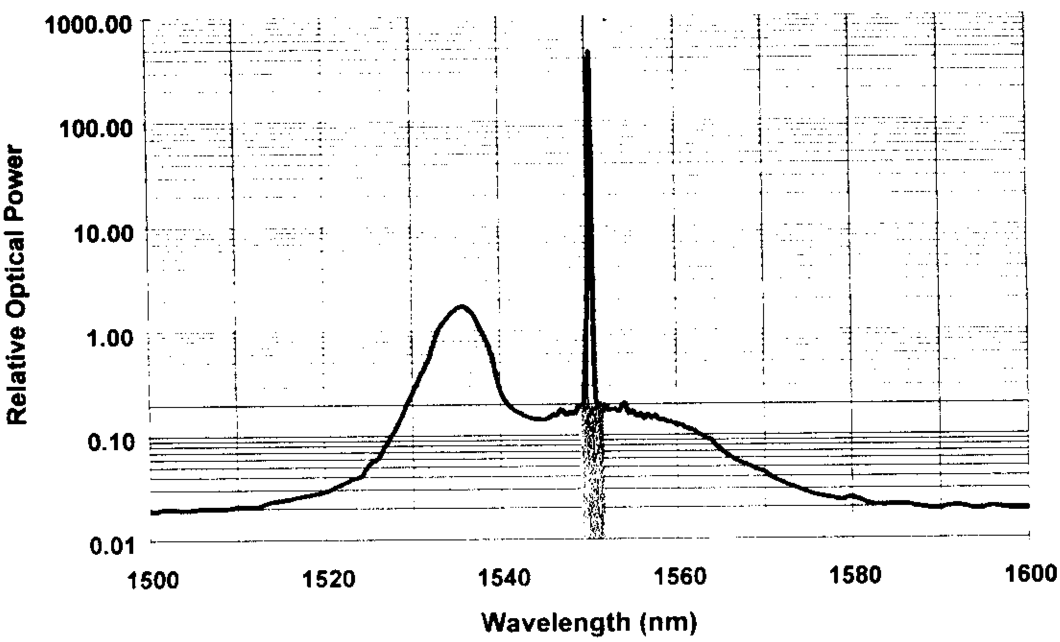
$$SNR_{out} = \frac{SNR_{in}}{NF} = \frac{P_0}{2h\nu B_e.NF} \qquad (39)$$

MEASUREMENT OF NOISE FIGURE

The Noise Figure is given by:

$$NF = \frac{2P_{ASE}^{B_e}}{Gh\nu B_e}$$

$P_{ASE}^{B_e}$ can be readily obtained from spectral measurements:



ASE power spectrum showing the ASE falling within the bandwidth of the receiver, B_e. P_{ASE}^{Be} is integral of the ASE spectrum over the shaded area.

NOISE IN EDFA CASCADES

We can find the SNR if we know the output Signal and ASE:

The spectral density of ASE, ρ_{ASE-R}, at the output of the final amplifier is:

$$\rho_{ASE-R} = \rho_{ASE-1}.G_1^{**} + \rho_{ASE-2}.G_2^{**} + \ldots\ldots \rho_{ASE-n}.G_n^{**} \qquad (40)$$

where there are a total of n amplifiers in the system and G_m^{**} is the net gain between the output of amplifier m and the output of the final amplifier i.e.

$$\rho_{ASE-R} = \sum_{m=1}^{n} \rho_{ASE-m} G_m^{**} \qquad (41)$$

Hence, assuming that Signal-ASE noise dominates, the signal to noise ratio, SNR_R, at the final amplifier output is given by:

$$SNR_R = \frac{(RG_T P_{in})^2}{4R^2 G_T P_{in} \rho_{ASE-R} B_e} \qquad (42)$$

For unity gain systems, $\rho_{ASE-R} = n\rho_{ASE}$ (assuming that all of the amplifiers have the same gain) and G_T and G_m^{**} are 1, provided that P_{in} is the launch power from the transmitter. Hence equation 7.6 becomes:

$$SNR_R = \frac{P_{in}^2}{4P_{in} n \rho_{ASE} B_e} = \frac{P_{in}}{4nP_{ASE}^{B_e}} \tag{43}$$

and

$$P_{ASE}(o/p) = nP_{ASE} \tag{44}$$

where ρ_{ASE} is the spectral density of ASE emitted by each amplifier in the chain and $P_{ASE}^{B_e}$ and $P_{ASE}^{B_e}$ are the ASE powers emitted in the receiver and optical bandwidths respectively.

Noise Figure in Amplifier Cascades

$$SNR_{out} = \frac{P_{in}}{4nP_{ASE}^{B_e}} = \frac{GP_o}{4nP_{ASE}^{B_e}}$$

$$SNR_{in} = \frac{P_0}{2h\nu B_e}$$

Hence the effective noise figure, N_{eff} for a unity gain EDFA cascade EDFA cascade is

$$\begin{aligned} NF &= \frac{P_o 4nP_{ASE}^{B_e}}{2h\nu B_e G P_o} = \frac{2n\rho_{ASE} B_e}{h\nu B_e G} \\ &= \frac{2n\mu(G-1)h\nu}{Gh\nu} \\ &= 2n\mu \end{aligned} \tag{45}$$

The noise figure for an individual amp is 2μ. Hence

$$NF_{eff} = nNF \tag{46}$$

and as before the output SNR is given by

$$\begin{aligned} SNR_{out} &= \frac{P_0}{2h\nu B_e . NF_{eff}} \\ &= \frac{P_0}{2h\nu B_e . n . NF} \end{aligned} \tag{47}$$

EDFAs

Advantages:

- Bit rate transparency- upgrade bit rate by adding changing terminal equipment
- Greater Bit Rates at lower cost
- Wavelength Transparency – WDM
- Low power consumption – cost & reliability
- High reliability/Low maintenance Min operational costs

Disadvantages

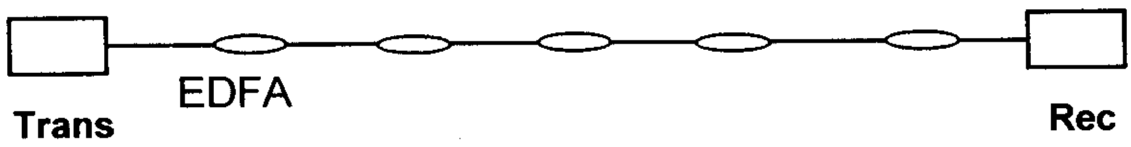
- Only amplifies signal
- Does not restore pulse widths

Systems become pulse spread / dispersion limited

→ Use with low dispersion systems. After about 550km dispersion compensating fibre is used

APPLICATIONS

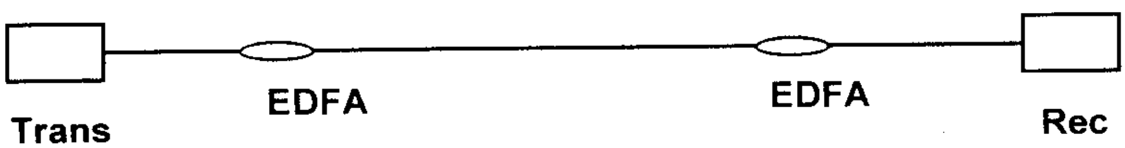
1. Long Haul Systems – In-line amplifiers



2. Preamplifiers to boost receiver sensitivity

Receiver sensitivity boosted by up to 12dB

3. Long range single spans with remotely pumped power amp and preamps



Single spans of up to 400km can be achieved

USEFUL EQUATIONS

Individual EDFAs:

$$P_{ASE} = \mu[G-1].h\nu.d\nu = \rho_{ASE} d\nu \qquad (28)$$

$$\sigma_{S-ASE} = 4R^2 G P_0 \rho_{ASE} B_e = 4R^2 G P_0 P_{ASE}^{B_e} \qquad (32)$$

$$\sigma_{ASE-ASE} = 2R^2 \rho_{ASE}^2 B_o B_e = 2R^2 P_{ASE}^{B_o} P_{ASE}^{B_e} \qquad (33)$$

$$SNR_{out} = \frac{GP_0}{4P_{ASE}^{B_e}} \qquad (35)$$

$$SNR_{in} = \frac{P_0}{2h\nu B_e} \qquad (36)$$

$$NF = \frac{2P_{ASE}^{B_e}}{Gh\nu B_e} = 2\mu \qquad (37)$$

$$SNR_{out} = \frac{SNR_{in}}{NF} = \frac{P_0}{2h\nu B_e.NF} \qquad (39)$$

Cascades of n EDFAs in a unity gain arrangement:

$$P_{ASE}(o/p) = nP_{ASE}$$

$$SNR_{out} = \frac{P_{in}^2}{4P_{in}n\rho_{ASE}B_e} = \frac{P_{in}}{4nP_{ASE}^{B_e}} \quad (43)$$

$$NF_{eff} = nNF = 2n\mu \quad (46)$$

and as before the output SNR is given by

$$SNR_{out} = \frac{P_0}{2h\nu B_e.NF_{eff}} = \frac{P_0}{2h\nu B_e.n.NF} \quad (47)$$

EDFA Tutorial Solutions

1. A particular EDFA is operating under the following conditions:

Signal Wavelength -	1550nm	Input power –	0.15μW
Receiver Bandwidth –	1GHz	Total ASE Power -	3.5mW
Gain -	23dB	Inversion factor, μ -	1.5
Receiver sensitivity -	0.72mA/mW		

a) What are the Signal-ASE and ASE-ASE beat noise currents and what is the optical signal to noise ratio at the receiver?
b) What filter bandwidth is required to reduce the ASE-ASE beat noise to the same level as the Signal-ASE beat noise?
c) What ASE power is transmitted by the filter in section b)?
d) An optical filter with a bandwidth of 1.5nm is used at the amplifier output to ensure that Signal-ASE beat noise is dominant. Determine the noise figure and use it to calculate the output optical SNR.

Solution:

a) $$\rho_{ASE} = \mu(G-1)h\nu = 1.5\times 199\times h\nu = 3.83\times 10^{-17}J$$

(@1550nm, hν = 1.283 × 10^{-19})

$$P_{ASE}^{B_e} = \rho_{ASE}\times B_e = 3.83\times 10^{-17}\times 1\times 10^{-9} = 38nW$$

$$P_{ASE}^{B_o} = 3.5mW$$

$$\sigma_{sA} = 4R^2GP_oP_{ASE}^{B_e} = 4\times 0.7^2\times 200\times 0.15\times 10^{-6}\times 38\times 10^{-9} = 2.364\times 10^{-12}A^2$$

Signal-ASE beat noise current, $i_{sa} = \sqrt{\sigma_{sa}} = 1.54\mu W$

$$\sigma_{AA} = 2R^2P_{ASE}^{B_e}P_{ASE}^{B_o} = 2\times 0.72^2\times 38\times 10^{-9}\times 3.5\times 10^{-3} = 1.38\times 10^{-10}A^2$$

ASE-ASE beat noise current, $i_{aa} = 11.74\mu A$

$$SNR = \frac{(GRP_o)^2}{\sigma_{SA}+\sigma_{AA}} = \frac{(200\times 0.72\times 0.15\times 10^{-6})^2}{2.364\times 10^{-12}+1.38\times 10^{-10}} = 3.3$$

b) $$\sigma_{SA} = \sigma_{AA}$$

$$4R^2GP_oP_{ASE}^{B_e} = 2R^2P_{ASE}^{B_e}\rho_{ASE}B_o$$

$$B_o = \frac{2GP_o}{\sigma_{ASE}} = \frac{2\times 200\times 0.15\times 10^{-6}}{3.83\times 10^{-17}} = 1.57\times 10^{12}\,Hz = 12.5nm \qquad (125\text{GHz} = 1\text{nm})$$

c) ASE power transmitted, $\rho_{ASE}\times 1.57\times 10^{12} = 60\mu W$

d) NF =2μ =3

$$SNR_{out} = \frac{P_o}{2h\nu B_e.NF} = \frac{0.15\times 10^{-6}}{2\times h\nu\times 1\times 10^{9}\times 3} = 195$$

Alternatively:

$$SNR_{out} = \frac{GP_o}{4P_{ASE}^{B_e}} = \frac{200\times 0.15\times 10^{-6}}{4\times 38\times 10^{-9}} = 197.36$$

2. The total ASE power emitted by a particular amplifier was measured at 2mW while it was operating with a gain of 27dB for a signal input power of 4μW at 1550nm. From an analysis of the measured ASE spectral density distribution, it was found that the fraction of the ASE power contained in a bandwidth of 1nm at 1550nm is 0.008. The amplifier output is optically filtered to a bandwidth of 1.8nm. The receiver bandwidth is 5GHz and the responsivity is 0.72mA/mW.

   a) Determine whether the noise is dominated by Signal-ASE or ASE-ASE beat noise.
   b) Determine the noise figure and use it to calculate the optical SNR at the output.

Solution:

a) $P_{ASE}^{B_o} = 0.008\times 2\times 1.8 = 28.8\mu W$

$$P_{ASE}^{B_e} = 0.008\times 2\times \frac{5}{125} = 28.8\mu W = 0.64\mu W \qquad (125\text{GHz} = 1\text{nm})$$

$$\sigma_{SA} = 4R^2GP_oP_{ASE}^{B_e} = 4\times 0.72^2\times 500\times 4\times 10^{-6}\times 0.64\times 10^{-6} = 2.654nA^2$$

$$\sigma_{AA} = 2R^2P_{ASE}^{B_e}P_{ASE}^{B_o} = 2\times 0.72^2\times 0.64\times 10^{-6}\times 28.8\times 10^{-6} = 1.911\times 10^{-2}\,nA^2$$

Hence, Signal-ASE beat noise dominates.

b) $$NF = \frac{2P_{ASE}^{B_e}}{Gh\nu B_e} = \frac{2\times 0.64\times 10^{-6}}{500\times 1.283\times 10^{-19}\times 5\times 10^{9}} = 3.991$$

$$SNR_{out} = \frac{P_o}{2h\nu B_e.NF} = \frac{4\times 10^{-6}}{2h\nu\times 5\times 10^{9}\times 3.991} = 781$$

Alternatively:

$$SNR_{out} = \frac{GP_o}{4P_{ASE}^{B_e}} = \frac{500\times 4\times 10^{-6}}{4\times 0.64\times 10^{-6}} = 781$$

3. A particular variable gain EDFA is operating with a fixed gain of 27dB and an inversion factor of 1.4 at a signal wavelength of 1550nm. The signal bandwidth is 2GHz and optical filtering is used to ensure that the noise is dominated by Signal-ASE beat noise.

   a) Calculate the noise figure and the output SNRs for input powers of 0.01, 0.1 and 1μW.
   b) For the same amplifier with a fixed input power of 0.5μW calculate the output SNR for gains of 20, 23, 27, and 30dB.
   c) For a fixed output power of 1 mW, calculate the output SNR for an input signal 1, 5, 10 and 50μW.
   d) Discuss the above results.

Solution:

a) $$NF = 2\mu = 2.8$$

$$SNR_{out} = \frac{P_o}{2h\nu B_e.NF}$$

@P_0 = 0.01μW, SNR_{out} = 6.96
@P_0 = 0.1μW, SNR_{out} = 69.6
@P_0 = 1μW, SNR_{out} = 696

b) $$P_{ASE}^{B_e} = \mu(G-1)h\nu B_e = 1.4(G-1)\times 1.283\times 10^{-19}\times 2\times 10^{9}$$

$$SNR_{out} = \frac{GP_o}{4P_{ASE}^{B_e}}$$

Using the above:

@G = 20dB, SNR_{out} = 350
@G = 23dB, SNR_{out} = 350
@G = 27dB, SNR_{out} = 350
@G = 30dB, SNR_{out} = 350

c) $$SNR_{out} = \frac{GP_o}{4P_{ASE}^{B_e}} = \frac{1\times10^{-3}}{4\times1.4\times(G-1).h\nu\times2\div10^{9}}$$

Fixed input power of 1mW and variable gain.

@G = 30dB, SNR_{out} = 696
@G = 23dB, SNR_{out} = 3480
@G = 20dB, SNR_{out} = 6960
@G = 13dB, SNR_{out} = 34800

4. **An optical fibre telecommunications system operating at 1550nm with a signal bandwidth of 1.2GHz requires a received optical SNR of 144 to achieve the minimum acceptable bit error rate of 10^{-9}. The receiver sensitivity is enhanced by a variable gain EDFA pre- amplifier with a noise figure of 4.5dB. What is the minimum detectable power assuming that Signal–ASE beat noise is dominant.**

Solution:

$$SNR_{out} = \frac{P_o}{2h\nu B_e.NF}$$

$$P_{min} = SNR\times2h\nu.B_e.NF = 144\times2\times h\nu\times12\times10^{9}\times2.82 = 0.125\mu W$$

5. What is the minimum optical SNR at the output of a pre-amp to achieve a received SNR of 144, assuming that the minimum possible signal will be incident on the photo-diode.

Solution:

The minimum incident signal power must create a signal to noise ratio of 144 taking account of receiver noise as well as the optical noise. Hence, the minimum signal can be defined as that for which the optical noise is equal to the receiver noise (i.e. the total noise is $2\sigma_{S\text{-}ASE}$ and

$$SNR = \frac{(GRP_o)^2}{2\sigma_{SA}} = 144 \qquad \text{i.e.} \qquad \text{The optical } SNR = \frac{(GRP_o)^2}{\sigma_{SA}} = 288$$

$\& (GRP_o)^2 \geq 288 \times \sigma_R$ (σ_R = the receiver noise)

6. A particular receiver has a thermal noise limited sensitivity of –25 dBm at a bandwidth of 2.5GHz. By how much can we enhance the sensitivity using an ideal EDFA preamp ($\mu = 1$) with variable gain up to 30dB. Assume that the required signal to noise ratio is 144 to achieve a bit error rate of 10^{-9}.

Solution:

$$SNR_{out} = \frac{P_o}{2h\nu B_e.NF}$$

$$P_{min} = SNR \times 2h\nu.B_e.NF = 144 \times 2 \times h\nu \times 2.5 \times 10^9 \times 2 = 0.185\mu W = -37.3dBm$$

The improvement in sensitivity is 12.3dB, on the assumption that the signal is amplified by about 20 – 30dB and the optical noise dominates the SNR.

7. For a particular EDFA design used in a long distance telecommunications link, the measured spectral power density around the signal wavelength of 1550nm is 10μW/nm. This amplifier design is used in a unity gain system with the following specifications:

Input power -	2mW	EDFA Gain -	26dB
Signal bandwidth -	2.5GHz	Required SNR -	288
Fibre attenuation -	0.26dB/km	Distance –	640km
Optical Bandwidth –	2.0nm	Receiver responsivity-	0.65mA/mW
Receiver sensitivity -	-25dBm		

a) What are the ASE power, the ASE-ASE beat noise, the Signal-ASE beat noise and the optical SNR at the output? Which source of noise dominates and what is the SNR for this source alone?
b) What is the noise figure of each amplifier and use it to confirm the output SNR? Assume that Signal-ASE noise is dominant.
c) Assuming that Signal-ASE beat noise dominates, what are the output SNRs for input powers of 10mW and 500μW?
d) For an input power of 2mW, what are the output SNRs for amplifier gains of 20dB and 13dB (with the inter amplifier span adjusted accordingly)? Again assume that Signal-ASE noise is dominant and that the noise figure remains constant.
e) Assuming that Signal-ASE noise dominates and that the ASE power remains negligible relative to the signal, estimate the maximum transmission lengths for all of the systems described in a), c) & d) above?

Solution:

Individual amplifier output:

$$P_{ASE}^{B_o} = B_o \times \rho_{ASE} = 2 \times 10 = 20\mu W$$

$$P_{ASE}^{B_{eo}} = B_e \times \rho_{ASE} = 2.5 \times 0.008 = 0.2\mu W$$

$$NF = \frac{2P_{ASE}^{B_e}}{Gh\nu B_e} = \frac{2 \times 0.2 \times 10^{-6}}{400 \times h\nu \times 2.5 \times 10^9} = 3.117$$

a) Unity gain system output:

Span between amps = 26dB ÷ 0.26dB/km = 100km

No of amps required, n = 640km ÷ 100km = 6

$$P_{ASE}^{B_o}(o/p) = n \times P_{ASE}^{B_o} = 6 \times 20\mu W = 120\mu W$$

$$P_{ASE}^{B_e}(o/p) = n \times P_{ASE}^{B_e} = 6 \times 0.2\mu W = 1.2\mu W$$

$$\sigma_{AA} = 2R^2 P_{ASE}^{B_o}.P_{ASE}^{B_e} = 2 \times 0.65^2 \times 120 \times 10^{-6} \times 1.2 \times 10^{-6} = 1.217 \times 10^{-10} A^2$$

$$\sigma_{SA} = 4R^2 P_{out}.P_{ASE}^{B_e} = 4 \times 0.65^2 \times 2 \times 10^{-3} \times 1.2 \times 10^{-6} = 4.056 \times 10^{-9} A^2$$

$$SNR_{o/p} = \frac{(RP_{out})^2}{\sigma_{AA} + \sigma_{SA}} = \frac{(2 \times 10^{-3} \times 0.65)^2}{1.217 \times 10^{-10} + 4.056 \times 10^{-9}} = 404.5$$

Hence, σ_{SA} is dominant:

$$SNR_{o/p} = \frac{(RP_{out})^2}{\sigma_{SA}} = \frac{(2 \times 10^{-3} \times 0.65)^2}{4.056 \times 10^{-9}} = 416$$

Alternatively:

$$SNR_{o/p} = \frac{GP_o}{4P_{ASE}^{B_e}} = \frac{P_{out}}{4P_{ASE}^{B_e}} = \frac{2 \times 10^{-3}}{4 \times 1.2 \times 10^{-6}}$$

b) $$SNR_{out} = \frac{P_o}{2h\nu B_e.NF_{eff}} = \frac{5 \times 10^{-6}}{2h\nu \times 2.5 \times 10^9 \times 6 \times 3.117} = 416.7$$ ($NF_{eff} = nNF$)

where P_o is the input power to the first amplifier.

c) As for b) above

@ $P_{out} = P_{in}$ = 10mW, SNR_{out} = 2083 and
@ $P_{out} = P_{in}$ = 500μW, SNR_{out} = 104.2

c) For 20dB gain per amp:

Distance between amps	= 20dB÷0.26dB/km	= 77km
No of amps in link, n	= 640km÷77km	= 8

$$SNR_{out} = \frac{P_o}{2h\nu B_e.NF_{eff}} = \frac{20\times10^{-6}}{2h\nu\times2.5\times10^9\times8\times3.117} = 1250$$

d) For 13dB gain per amp:

Distance between amps	= 13dB÷0.26dB/km	= 50km
No of amps in link, n	= 640km÷50km	= 12

$$SNR_{out} = \frac{P_o}{2h\nu B_e.NF_{eff}} = \frac{100\times10^{-6}}{2h\nu\times2.5\times10^9\times12\times3.117} = 4167$$

e) $$SNR_{out} = \frac{P_o}{2h\nu B_e.NF_{eff}} = \frac{P_o}{2h\nu B_e.n.NF}$$

Hence, the maximum number of amplifiers is given by

$$n = \frac{P_o}{2h\nu.B_e.NF.SNR} = \frac{P_o}{2.h\nu.2.5\times10^9\times3.117\times288} = \frac{P_o}{5.76\times10^{-7}}$$

assuming that an optical SNR of 288 is required.

1) For 26dB gain, 100km span and P_{in} = 2mW (i.e. P_o = 5μW).

n given by the above expression is 8

For a receiver sensitivity of –25dBm, a received signal power of –22dBm will contain optical noise equal to or less than the receiver noise. Hence with a +3dBm input a good estimate of the maximum distance to the receiver from the final amplifier is ≈25dB÷0.26dB/km = 96km

∴Maximum distance for this system structure = (8×100)+96km = 896km

2) For 26dB gain, 100km span and P_{in} = 10mW (i.e. P_o = 25μW).

n given by the above expression is 43 and

The maximum distance $= (43\times100)+96\text{km} = 4396$

3) For 26dB gain, 100km span and $P_{in} = 500\mu W$ (i.e. $P_o = 1.25\mu W$).

   n given by the above expression is 2 and

   The maximum distance $= (2\times100)+96\text{km} = 296$

4) For 20dB gain, 77km span and $P_{in} = 2mW$ (i.e. $P_o = 20\mu W$).

   n given by the above expression is 34 and

   The maximum distance $= (34\times77)+96\text{km} = 3496$

5) For 13dB gain, 50km span and $P_{in} = 2mW$ (i.e. $P_o = 100\mu W$).

   n given by the above expression is 173 and

   The maximum distance $= (173\times50)+96\text{km} = 8746$

8. A 1550nm sub-sea telecommunications link spanning a distance of 292km uses a remotely pumped power amplifier with a gain of 20dB (NF = 2.6) situated 40km from the input and a remote preamp with a gain of 30dB (NF = 2.2) situated 80km from the receiver. The receiver sensitivity is –28dBm, its bandwidth is 2GHz and its responsivity is 0.7mA/mW. Filtering is used to achieve an optical bandwidth of 2nm. The fibre loss is 0.25dB/km and the input power from the transmitter is 2mW.

   Carry out an appropriate analysis to determine if this system can achieve a bit error rate of 10^{-9}.

Solution:

Calculate the output signal power:

Input power	= 2mW	= +3dBm	(2mW)
Loss of 40km span to power amp	= 40×0.25dB	= -10dB	
Signal power at input to power amp		= -7dBm	(0.2mW)
Output from power amp	= (-7+20)dBm	= +13dBm	(20mW)
Loss of 172 km central span	= 172×0.25dB	= -43dB	

Power at input to preamp	= (+13-43)dBm	= -30dBm (1μW)
Output from preamp	= (-30+30)dBm	= 0dBm (1mW)
Loss of 80km span to receiver	= 80×0.25dB	= 20dB
Received power	= (0-20)dBm	= -20dBm (10μW)

The receiver sensitivity is –28dBm, ∴the received signal is OK in terms of power.

Calculate optical SNR:

Firstly calculate the accumulated ASE in the system:

Power Amp: $\mu = \frac{NF}{2} = 1.3$

$$\rho_{ASE} = \mu(G-1)h\nu = 1.3 \times 99 \times h\nu = 1.652 \times 10^{-17} J$$

ASE output from power amp:

$$P_{ASE}^{B_o} = \rho_{ASE} \times B_o = \rho_{ASE} \times 250 \times 10^9 = 4.13\mu W$$
$$P_{ASE}^{B_e} = \rho_{ASE} \times B_e = \rho_{ASE} \times 2 \times 10^9 = 33nW$$

These contributions undergo a –43dB loss in the central section and are then boosted by 30dB at the preamp giving a net loss of –13dB to the output of the preamp. Hence contributions to the accumulated ASE at the preamp output are:

$P_{ASE}^{B_o} = 0.206\mu W$ & $P_{ASE}^{B_e} = 1.65nW$

Preamp: $\mu = 1.1$

$$\rho_{ASE} = \mu(G-1)h\nu = 1.1 \times 999 \times h\nu = 1.4 \times 10^{-16} J$$

$$P_{ASE}^{B_o} = \rho_{ASE} \times B_o = \rho_{ASE} \times 250 \times 10^9 = 35.25\mu W$$
$$P_{ASE}^{B_e} = \rho_{ASE} \times B_e = \rho_{ASE} \times 2 \times 10^9 = 0.282nW$$

Total outputs including contributions from the power amp:

$P_{ASE}^{B_o} = 35.46\mu W$ & $P_{ASE}^{B_e} = 0.284\mu W$

With a 20dB loss in the final span to the receiver, the received ASE powers are:

$P_{ASE}^{B_o} = 0.355\mu W$ & $P_{ASE}^{B_e} = 0.003\mu W$

The Signal-ASE and ASE-ASE beat noise terms are thus given by:

$$\sigma_{SA} = 4R^2 P_R . P_{ASE}^{B_e} = 4\times 0.7^2 \times 10\times 10^{-6} \times 0.003\times 10^{-6} = 6.3\times 10^{-14} A^2$$
$$\sigma_{SA} = 2R^2 P_{ASE}^{B_o} . P_{ASE}^{B_e} = 4\times 0.7^2 \times 0.003\times 10^{-6} \times 0.355\times 10^{-6} = 2.2\times 10^{-15} A^2$$

$$\text{Hence } SNR = \frac{(RP_R)^2}{\sigma_{SA}+\sigma_{AA}} = \frac{(0.7\times 10\times 10^{-6})^2}{6.32\times 10^{-14}} = 816$$

The optical SNR is considerably greater than the minimum required 288. Therefore this system will operate effectively.

9. The 1550nm trans-Atlantic optical fibre transmission link, TAT 12/13, spans a total length of 5985km and operates at a signal bandwidth of 5GHz. EDFAs are used to boost the signal power every 45km in a unity gain arrangement. The amplifiers have a noise figure of 3.8dB and are optically filtered to a 1.5nm bandwidth to ensure that the SNR is dominated by Signal-ASE beat noise. The fibre loss is 0.28dB/km and the receiver responsivity is 0.68mA/mW.

   a) Assuming an input power of 2mW, determine if the output SNR is compatible with achieving a bit error rate of 10^{-9}.
   b) Calculate the accumulated ASE power at the output and show that the system SNR is indeed dominated by Signal-ASE beat noise and that the calculation in a) is therefore valid.
   c) Discuss whether or not your conclusions above are valid given the level of ASE power at the output.

Solution:

a) No of amplifiers in link = 5985/45 = 133
Hence the gain of each amplifier is 45×0.28dB = 12.6dB (i.e. G = 18.2)
The noise figure is 3.8dB (i.e. 2.4) and hence μ = 1.2

Calculation of output SNR:

$$ASE_{out} = 133 P_{ASE}^{B_o}$$

$$P_{ASE}^{B_e} = \rho_{ASE} \times B_e = \mu(G-1)h\nu B_e = 1.2\times 17.2\times h\nu \times 5\times 10^9 = 0.013\mu W$$

Hence $P_{ASE}^{B_o}(o/p) = 133\times 0.013\mu W = 1.773\mu W$

Assuming that ASE-ASE beat noise is negligible, the optical SNR at the output is given by:

$$SNR_{out} = \frac{P_{out}}{4.P_{ASE}^{B_e}} = \frac{2\times10^{-3}}{4\times1.773\times10^{-6}} = 282$$

The required optical SNR is only about 144 to achieve a BER of 10^{-9}, provided that the received signal power is much larger than the receiver sensitivity. This is likely for this system. Hence this system is compatible with achieving its required BER.

b) Calculate the accumulated ASE and the noise terms:

$$P_{ASE}^{B_o} = \rho_{ASE} \times B_o = \mu(G-1)h\nu B_o = 1.2\times17.2\times h\nu\times187.5\times10^9 = 0.5\mu W$$

Hence $P_{ASE}^{B_o}(o/p) = 133\times0.5\mu W = 66.5\mu W$ and

$$\sigma_{AA} = 2R^2 P_{ASE}^{B_e}.P_{ASE}^{B_o} = 2\times0.68^2\times1.773\times66.5\times10^{-12} = 1.09\times10^{-10}A^2$$

$$\sigma_{SA} = 4R^2 P_R P_{ASE}^{B_e} = 4\times0.68^2\times2\times10^{-3}\times1.773\times10^{-6} = 6.56\times10^{-9}$$

Hence, ASE-ASE beat noise can be regarded as negligible relative to Signal-ASE beat noise and the above calculation is valid.

c) Use the noise figure to calculate the output optical SNR:

$$SNR_{out} = \frac{P_o}{2h\nu B_e.n.NF}$$

P_o is the input power to the first amplifier = (+3-12.6)dBm =111μW

$$SNR_{out} = \frac{111\times10^{-6}}{2h\nu\times5\times10^9\times133\times2.4} = 271$$

10. The Rioja 1550nm telecommunications system, linking Spain, The UK and Belgium spans a total distance of 676 km using EDFAs with a near ideal noise figure of 3.0dB spaced at 94km in a unity gain arrangement. The signal and receiver bandwidth is 5GHz and the receiver responsivity is 0.7 mA/mW. The average cable loss is 0.3dB/km. Optical filtering is used to provide an optical bandwidth of 2.5nm and it can be assumed that Signal-ASE beat noise is dominant.

a) Calculate the total ASE power contained in the optical bandwidth and in the receiver bandwidth at the output of the last amplifier.
b) Use the results of a) to show that the SNR is compatible with achieving a BER of 10^{-9} for an input power of 4mW.
c) Use the noise figure to verify the answer to b).

Solution:

a) No. of amplifiers in link = 676/94 = 7

Gain of each amp = 94km × 0.3dB/km = 28.2dB (i.e. G = 661)

For a noise figure of 3dB, μ = 1.

$$P_{ASE}^{B_o} = \mu(G-1)h\nu B_o = 660.h\nu \times 312.5\times 10^9 = 26.47\mu W$$ and

$$P_{ASE}^{B_o}(o/p) = 7\times 26.47 = 185.3\mu W$$

Similarly,

$P_{ASE}^{B_e} = 0.424\mu W$ and $P_{ASE}^{B_e}(o/p) = 2.965\mu W$

b) The optical SNR at the output is given by:

$$SNR_{out} = \frac{4\times 10^{-3}}{4\times 2.965\times 10^{-6}} = 337.2$$

The output optical SNR is greater than 144. Therefore the system is compatible of achieving a BER of 10^{-9}.

c) For an input of power of 4mW and a loss of 28.2dB to the first amplifier, the input power, P_o, to the first amp is 6.03μW. Using the noise figure:

$$SNR_{out} = \frac{6.03\times 10^{-6}}{2.h\nu.5\times 10^9\times 7\times 2} = 336$$