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13 March - 14 April 2000

Miramare - Trieste, Italy

Reactor Lattice Transport Calculations

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1. Reactor lattice

1.1. A unit cell concept

In thermal reactors fuel is arranged in lumps of rods or plates separated by a material such as graphite, water or heavy water, in which neutrons are slowed to thermal energy with a minimum of capture. The fuel has a cladding separating the fission products from the cooling water. Thus, every thermal reactor, of research as well as of power type, is heterogeneous. The fuel elements are arranged in a regular manner. The cylindrical fuel elements with circular horizontal intersection are arranged in squares (cf.Fig. 1), hexagons or rings. The fuel plates are arranged in parallel bundles. In any case the fuel elements surrounded by moderator (coolant) form a *reactor lattice* which in the first step of reactor calculations is assumed infinite. We speak about the square lattice if fuel elements are arranged in squares, hexagonal if fuel elements are situated in corners of hexagons etc. In any type of reactor lattice we are able to identify a repetitive fragment composed of a single fuel element surrounded by a portion of adjacent moderator. Thus a fictitious boundary is introduced in the middle of moderator dividing the nearest fuel elements. The fuel rod (or plate) with its cladding and adjacent moderator portion form *a unit cell,* as shown in Fig. 1.

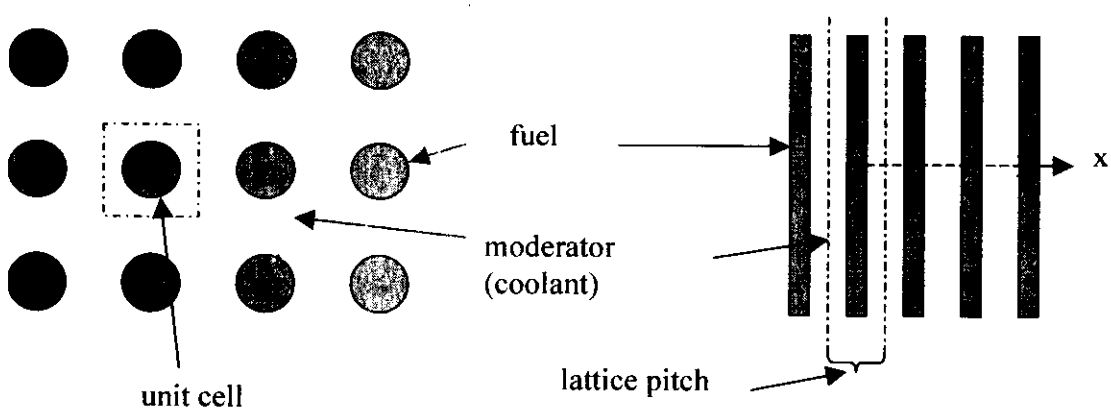

Figure 1: Fragment of a horizontal intersection of a square and plane lattice with a unit cell.

The form of the unit cell depends on the reactor type. For instance a typical unit cell of a PWR type reactor is square with a cylindrical fuel rod in its centre (cf. Fig. 1). The unit cell for TRIGA is most often hexagonal with a cylindrical rod, the MTR unit cell is a fuel plate, cladded on both sides and surrounded by water. Typical shapes of unit cells are shown in Fig. 2.

In the concept of the unit cell it is assumed that such a cell is a repetitive fragment of the large reactor lattice and under this assumption a zero current boundary condition can be imposed on its boundary. The outer boundary of the unit cell, in case of a cylindrical fuel rod, is transformed from the rectangle, hexagon etc. into a cylinder as shown in Fig.2.

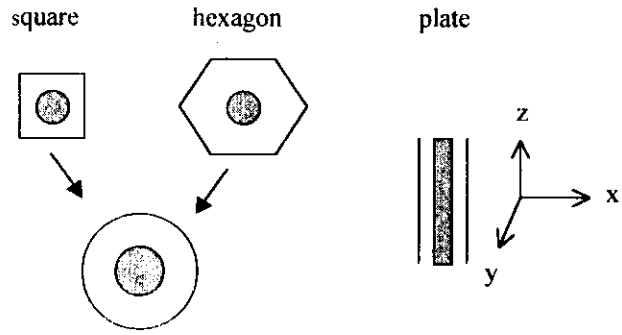

Figure 2: Typical shapes of unit cells.

The transformation of the outer boundary is carried out on the basis of preservation of the volumes of all materials. For a rectangle the outer radius of the equivalent unit cell is $R = a/\sqrt{\pi}$ with a denoting the *lattice pitch* (distance between centres of direct neighbours of fuel rods). The white boundary condition, introduced in section 2.3 of "Introduction to the Neutron Transport Phenomena", at the cylindrical unit cell boundary is applied.

In the plate unit cell concept it is usually assumed that the plates are infinite in both y and z directions (cf. Fig.2), which reduces the problem of solution of the transport equation to a one-dimensional one with constant flux (or zero current) boundary condition. With this assumption the plane unit cell does not need to be transformed. Similarly, it is assumed that for a cylindrical unit cell, the cell is infinite in the vertical direction. This again reduces the transport equation to a one-dimensional case in cylindrical geometry.

1.2. Definition of a macrocell

Unfortunately, the fuel elements are not the only heterogeneity in the reactor core. In power reactors the fuel elements are combined into fuel assemblies. This is not a serious problem, as the number of fuel elements in the assembly is large enough to assume an infinite lattice of unit cells. The real difficulty is connected with the presence of strongly absorbing control elements. In research reactors besides control elements (plates or rods) there exist other types of heterogeneity as, e.g., various non-multiplying media inserted for irradiation.

To account for various types of strong heterogeneity a concept of a *macrocell* has been created. A macrocell is again a repetitive fragment of the reactor lattice but composed of several unit cells. By 'repetitive' it is understood that a constant flux (zero current) boundary condition is justified at the outer boundary of the macrocell. It is just left to the reactor physicist to decide which region of a given reactor core can be chosen as a macrocell. Typical shapes of macrocells are shown in Fig. 3.

The typical approach applied in reactor macrocell calculations is to solve first the neutron transport equation for a unit cell and then use the obtained results in the second solution of the transport equation over the macrocell. This two-step procedure can be carried out by one code or by two codes with automatic transfer of information. The two

steps can use the same method of solution or different methods and/or different approximations of the neutron transport equation. For instance, in case of a PWR assembly the second step can be carried out for the whole, or a quarter of, the fuel assembly using an improved diffusion theory approximation.

Sometimes for the macrocell the transport equation is solved in a rectangular geometry with the fuel cross section area transformed into a square with volume preservation. In that case there arise doubts if such a change of interface and outer boundary shapes does not introduce an additional error. The effect can be more pronounced for reactors with a large lattice pitch.

In order to clarify this point there have been considered four model cells composed of a homogeneous fuel element region and beryllium moderator with the lattice pitch equal 13 cm. The number densities for the homogenised fuel region are given in Table 1.

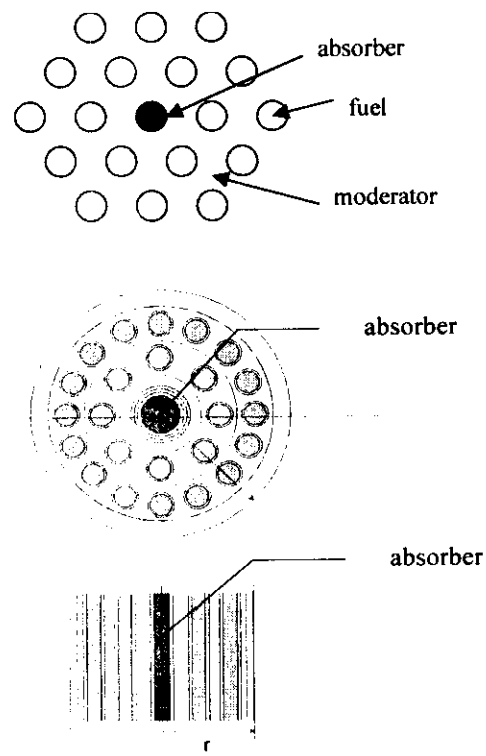

Figure 3: Possible macrocells with fuel rods and fuel plates with an absorber.

Table 1
Number densities of the homogenised fuel region.

Isotope	Number density x 1.0E-24
hydrogen	4.1271E-02
oxygen	2.0638E-02
uranium-235	1.5254E-04
uranium-238	3.7649E-05
aluminium	2.2310E-02

The MCNP-4A [1] calculations have been performed for all the four combinations of square and circular boundary as shown in Fig.4. The first case corresponds to the real geometry of the cell. The second is equivalent to the unit cell geometry, the third to macrocell or whole reactor geometry and the fourth has been added for completeness.

The results are given in Table 2. It may be concluded that the change of geometry from unit cell to square macrocell gives discrepancies of less than 2mk. However, the unit cell geometry introduces approximately 6mk discrepancy as compared to the actual one. Thus, calculations in rectangular geometry are closer to reality due to the error cancellation.

The transport equation over the unit cell should be solved to get the neutron flux distribution and eigenvalue. The diffusion approximation is not recommended here as it can be used only in case of low neutron flux gradients. At the fuel-moderator interface this is never the case, nor it is in the presence of strongly absorbing control elements.

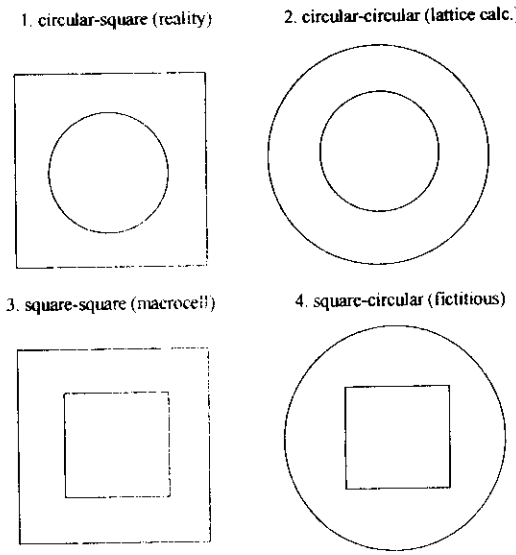

Figure 4: Possible schemes of fuel element and outer cell boundaries.

Table 2
MCNP k-inf values for a cell with homogenised fuel region
with different shapes of interfaces.

No	Type of boundaries	*k-inf*
1	cylindrical-square, reality	1.7090
2	cylindrical-cylindrical, (unit cell)	1.7031
3	square- square, (macrocell or whole reactor)	1.7094
4	square-cylindrical, fictitious	1.7054

1.3. Energy dependence

The energy dependence in lattice calculations is treated through the multigroup approach. The number of groups dipends strongly on the actual computer code but with current computers it usually approaches hundred and, very often, even several thousands of groups are used to treat, eg., the resonance phenomena. However, in the analysis of the physics for a particular type of the reactor lattice it is convenient to distinguish several energy intervals characterised by special physical phenomena:

1. Fast energy region in which the fission neutrons emerge and the neutron energy dependence (spectrum) follows approximately the fission spectrum.
2. The slowing-down region with the energy dependence of $1/E$.
3. Resonance region in which the heavy nuclei exhibit a resonance character.
4. Thermal region where the thermalization of neutrons takes place and both up- and down-scattering of neutrons are possible.

The neutrons are born with energies corresponding to fast energy region. They interact with the medium nuclei and reduce their energy in collisions. Some of these neutrons produce fission as they are slowed down, but in a thermal reactor it happens (by definition) with a rather low probability. The total number of neutrons generated in fission, divided by the number of neutrons produced by the thermal fission is denoted by ε. In a thermal reactor ε slightly exceeds unity.

In the upper energy region where the slowing-down is the main process and no significant number of neutrons emerge from fission, it is convenient to introduce a variable called *lethargy*, related to energy:

$$u = ln(E_{max}/E) \quad \rightarrow \Phi(u)du = -\Phi(E)dE, \tag{2.1}$$

A maximum energy loss of a neutron suffering a collision is:

$$(1-\alpha)E_i \text{, it is } E_f \geq E_f \geq \alpha E_i$$

where E_i is the initial and E_f final neutron energy, and $\alpha = (A-1)^2/(A+1)^2$, with A being the nucleus mass.

An *average energy loss*, or lethargy gain, *per collision*, ξ, is then defined and using the just introduced quantities can be expressed as:

$$\xi = 1 - \frac{\alpha}{1-\alpha}\ln\alpha \cong \frac{2}{A+\frac{2}{3}}. \tag{2.2}$$

As neutrons slow down through the resonance region, the resonance cross sections change their magnitude dramatically in a small energy interval of few eV. In this energy region special methods are employed in lattice calculations to take into account the rapid changes of the coefficients in the neutron transport equation. The basic quantity for the resonance region is the escape probability which has the meaning of the probability for a neutron to escape the resonance absorption and in the simplified form is equal to:

$$P = exp\,(-(\Sigma^t)_e/(\xi\Sigma^s)) \tag{2.3}$$

with $(\Sigma_r)_e$ the *effective resonance integral*, ξ defined by Eq. (2.2) and Σ_s the macroscopic scattering cross section.

There are several effects that should be taken into account in the resonance region, non-existent or negligible for other energies.

The most important effect is due to the fuel lumping. The neutron born in the fuel rod or plate has to get out of the fuel area to reach the moderator and to get a possibility of collision with its nuclei. On its way to the fuel-moderator interface it can enter into collision with a fuel nucleus and get absorbed. Thus the fuel lumping decreases the probability of neutrons of being slowed down. The probability of absorption in the fuel increases with the fuel dimensions and fuel number density. The effect is called *self-shielding*. The *Bell factor* is introduced, to relate a resonance integral of a lumped fuel to that for fuel and moderator forming a homogeneous mixture.

If a neutron leaves a fuel rod/plate of his birth it can still enter another rod/plate of the lattice without a collision (cf. Fig.1). The *Dancoff factor* is introduced to take into account the fact that the fuel element in the reactor lattice is not isolated. Namely, the resonance integral for the lattice of fuel rods of radius R is the same as that of an isolated fuel pin of radius γR, where γ is the Dancoff factor. It can be also defined [2] as the reduction factor of the fuel escape probability compared to that of an isolated fuel pin when all fuel pins are black. The correction to the resonance escape probability, responsible for this effect, is called the *Dancoff correction*.

Then still there exists a flux depression caused by a resonance and the interference of resonances of different resonance isotopes. The algorithms applied for all these corrections vary for various authors.

The thermal region is the one where the majority of fission reactions take place. It is characterised by existence of upscattering of neutrons as a slow neutron entering into a collision with a nucleus can not only loose but also gain the energy. The thermal neutron flux is a quantity of prime importance in the thermal reactor physics and several quantities are introduced useful for the description of physical properties of various lattices. Some of them are introduced below.

1.4. Thermal flux distribution in a unit cell

The neutron thermal flux in the fuel region is always lower than in the moderator because of a high absorption of neutrons by the fuel nuclei. Typical shape of the thermal flux is shown in Fig. 5, where the dotted lines represent the average flux levels in the fuel and in the moderator

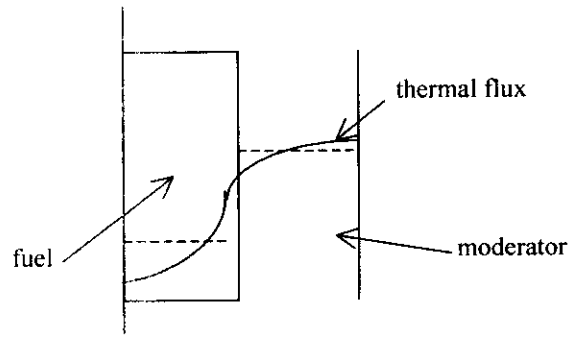

Figure: 5: Thermal flux distribution in a unit cell.

The quantity called *flux disadvantage factor* is used to compare the flux level in the fuel element to the average flux in the fuel cell and is calculated as a ratio of average flux values in the fuel and in the cell:

$$f_{dis} = \frac{\overline{\phi_{fuel}}}{\overline{\phi_{cell}}}. \tag{2.4}$$

1.5. A 'four-factor formula' for the infinite multiplication factor

In a thermal reactor it is convenient to divide the whole energy region into two parts: the slowing-down energy region, where only down-scattering takes place, and the energy region with up-scattering – the thermal region. The cross sections of any type, x, for each of those energy regions, G, are then understood as averages over respective energy groups, $g \in G$:

$$\Sigma_G^x = \frac{\sum_{g \in G} \Sigma_g^x \phi_g}{\sum_{g \in G} \phi_g}$$

where κ is the number of neutrons per fission. We introduce the number of neutrons per absorption in fuel (denoted by index U)

$$\eta = \frac{\left(\Sigma^f\right)_U}{\left(\Sigma^a\right)_U} \kappa . \tag{2.5}$$

Here the index a denotes the thermal absorption and f – thermal fission

We introduce also a quantity called the *thermal utilisation* factor that is equal to the fraction of all thermal neutrons absorbed that are absorbed in uranium. With the assumption of a negligible absorption in the moderator the thermal utilisation factor is given by a formula:

$$f = \frac{\left(\Sigma^a\right)_U}{\Sigma^a} \tag{2.6}$$

Otherwise Eq.(2.6) has to be multiplied by the fuel disadvantage factor defined in section 1.4. Thus, the number of fast neutrons produced by one thermal neutron absorbed in the lattice can be written as ηf. This quantity has to be corrected by the quantity ε to account for a possibility of fission caused by fast neutrons. In thermal reactors ε is very close to unity and is often neglected in rough estimations. The probability that neutrons are not captured during the slowing-down process is labelled p, and called the resonance escape probability defined in section 1.3.

Then the number of fission neutrons obtained from one thermal neutron absorbed in the lattice is $\varepsilon \eta f$ and the number of neutrons becoming thermal and ending the neutron generation is

$$k = \varepsilon p f \eta \tag{2.7}$$

This is by definition the infinite multiplication factor, k_∞, as no leakage has been taken into account.

1.6. Neutron leakage and buckling concept

The neutron cycle in the thermal reactor starts with fission, which proceeds at a rate

$$S = \phi \Sigma^f \kappa , \tag{2.8}$$

Combining Eqs. (2.5 , 2.6, 2.8) the fast source is

$$S = \phi \Sigma^a f \eta .$$

Of the fast neutrons a fraction F, which can be called the fast neutron non-leakage probability, will slow down in the reactor, without escaping from the core and of those the fraction εp will escape the fast and resonance absorption. Thus, of the S fast neutrons only $SF\varepsilon p$ get to thermal, and the resulting thermal source is equal

$$S_{th} = \phi \Sigma^a p \varepsilon f \eta F = \phi \Sigma^a k F \tag{2.9}$$

The diffusion equation derived in Chapter 1 is:

$$D \nabla^2 \phi - \Sigma^a \phi + Q = 0 , \tag{2.10}$$

where for the thermal reactor the source term can be substituted by Eq. (2.9) leading to:

$$D \nabla^2 \phi = \Sigma^a \phi (kF - 1). \tag{2.11}$$

The constants depending on reactor materials are usually grouped into a single one, called the *buckling:*

$$B^2 = \frac{\Sigma^a}{D}(kF - 1) , \tag{2.12}$$

so that Eq. (2.11) simplifies to a form of the so-called wave equation,

$$\nabla^2 \phi - B^2 \phi = 0 \tag{2.13}$$

The flux distribution with position is given by its solution. But to solve this equation it is necessary to select the reactor core shape. In spherical coordinates the general solution is a linear combination

$$\phi = A \frac{\sin Br}{r} + C \frac{\cos Br}{r}, \tag{2.14}$$

where A and C are arbitrary constants to be determined by information on the flux condition at the boundary and the centre of the core. Since an infinite flux is not allowed on physical grounds, C must be equal to zero, leaving

$$\phi = A\frac{\sin Br}{r} \,. \tag{2.15}$$

A reasonable condition is the flux going to zero at some distance d beyond the core boundary, i.e., at $R' = R+d$. Applying this boundary condition, Eq. (2.15) gives

$$A\frac{\sin BR'}{R'} = 0 \,.$$

Since R' is finite, A cannot be zero without incurring a meaningful solution, so that $\sin(BR')=0$, $BR'=\pi$, or

$$\begin{aligned} B^2 &= \left(\frac{\pi}{R'}\right)^2, \\ R &= R'-d = \frac{\pi}{B} - d. \end{aligned} \tag{2.16}$$

Thus the reactor size is determined by the constant B^2. The buckling from Eq.(2.12) is sometimes referred to as *'material buckling'*, $(B^2)_m$, as it is defined by the material properties of the medium. The expression for B^2 in Eq.(2.16) is usually called the geometric buckling $(B^2)_g$ dependent only on the size and shape of the core. The critical condition can be written as $(B^2)_m = (B^2)_g$.

1.7. The boundary with vacuum

It has been mentioned in the previous section that the typical boundary condition, introduced together with the diffusion approximation, is the neutron flux going to zero at some distance from the outer boundary of the system considered. Let us consider an idealised case of an infinite plane reactor core surrounded by vacuum. The distance at which the flux drops off to zero is called then the *extrapolation distance*, and it is shown in Fig. 6.

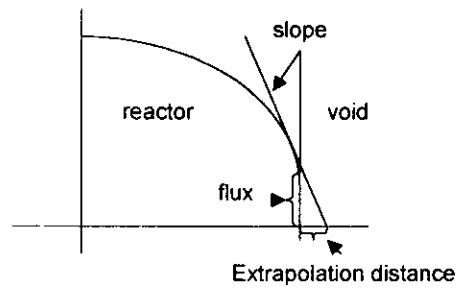

Figure 6: Extrapolation distance.

By diffusion theory, using Eqs. (1.55), the extrapolation distance, d, is found to be equal to:

$$d = \frac{2}{3}\lambda_{tr} \,. \tag{2.17}$$

A calculation based on the transport theory gives approximately:

$$d = -\frac{\phi}{d\phi / dx} = 0.71\ \lambda_{tr} \ . \tag{2.18}$$

where λ_{tr} is the transport mean free path.

1.8. Fuel burn-up

The lattice calculations are made for a steady-state reactor and do not involve the time variable explicitly. Also the coefficients of the transport equation, i.e., the macroscopic cross sections are considered constant in time. But in practice, the slow time evolution has to be included to account for the fuel burn-up.

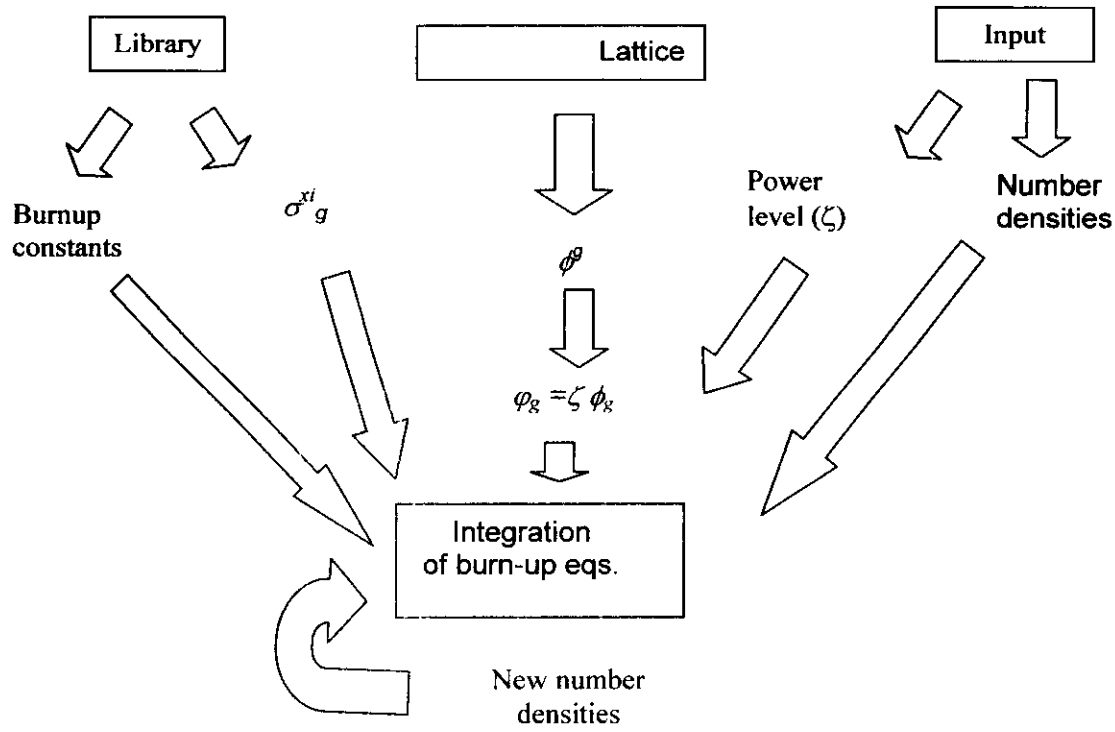

Figure 7: General flow of depletion calculations.

The burn-up changes the number densities, and hence the macroscopic cross sections of the nuclides undergoing the depletion or the build-up process. Thus, the neutron transport equation loses its linearity. To cope with the problem in an efficient way a repetition of the sequence shown schematically in Fig. 7 is applied:

1. Full solution of the transport equation with starting neutron densities or those from 3.
2. Normalisation of the neutron flux to a given power level (ζ – normalisation factor in Fig.7).
3. Solution of the equation for isotopic transformation, establishing new number densities.
4. Calculation of new macroscopic cross sections.

5. Go to 1.

Usually several burn-up steps can be carried out without accounting for the neutron spectrum modification, as shown in Fig. 7. However, after a sufficiently long time the change in macroscopic cross sections gets significant and the repetition of lattice spectrum calculations is necessary. Thus, the flow chart from Fig. 7 has to be repeated many times until the desired burn-up level is reached

In practice, the procedure gets much more complicated to make it more accurate, more efficient or take into account additional phenomena, as e.g., burnable poisons.

2. Discrete ordinates method

2.1. Discretisation of independent variables

The discrete ordinate method, referenced as S_N [3] or DSN [4], belongs to the most often used numerical methods for solving the steady state neutron transport equation. In this method the finite difference approach is applied in all three independent variables:

1. The multigroup approach is used for the energy dependence.
2. Angular integrals are replaced by sums over discrete directions.
3. Angular derivatives are transformed into finite differences.
4. Discrete space mesh is imposed on the spatial region.
5. Spatial derivatives are transformed into finite diffrences finite differences.

A number of discrete directions is introduced denoted by Ω_m to each of which a weight w_m is associated. Each weight represents a segment $\Delta\Omega_m$ on the unit directional sphere, in stereradians, with the normalisation condition:

$$w_m = \Delta\Omega_m / 4\pi; \quad \Rightarrow \quad \sum_m w_m = 1. \tag{2.19}$$

The subscript m on angular flux (source) means 4π times its value at Ω_m.

The neutron flux and neutron current are approximated by the basic equations of the method:

$$\begin{aligned} \phi_g(r) &= \sum_m w_m \phi_{mg}(r) \\ J_g(r) &= \sum_m w_m \Omega_m \phi_{mg}(r) \end{aligned} \tag{2.20}$$

for g=1,2,.....,G and m=1,2,...,M.

The effective realisation of the method has encountered a set of problems, which had to be solved. The most important of them are:

1. the choice of a particular discrete directions,
2. the approximation of the integrals over the direction variable,
3. the approximation of the derivatives of the neutron angular flux with respect to the components of Ω appearing in the transport equation in curved geometries.

Applying the approximation defined by Eqs. (2.20), to the multigroup neutron transport equations with linearly anisotropic scattering we get:

$$\Omega_m \cdot \nabla \phi_{mg}(r) + \Sigma_{mg}(r)\phi_{mg}(r) = S_{mg}(r) + \frac{1}{k}\chi_g(r)F(r), \tag{2.21}$$

where

$$F(r) = \sum_{g'} \kappa\Sigma^f_{g'}(r)\phi_{g'}(r),$$

$$S_{mg}(r) = \sum_{g'} \Sigma^s_{0,g'\to g}(r)\phi_g(r) + 3\Omega_m \cdot \sum_{g'} \Sigma^s_{1,g'\to g}(r)J_{g'}(r)$$

$$\phi_{mg} = \int_g \phi_m dE; \quad \phi_g = \int_g \phi dE; \quad J_g = \int_g J dE; \quad \chi_g = \int_g \chi dE,$$

$$\kappa\Sigma^f_g = \frac{\int_g \kappa\Sigma^f(E')\phi(E')dE'}{\phi_g},$$

$$\Sigma^s_{0,g'\to g} = \frac{\int_{g'}\int_g \Sigma^s_0(E'\to E)\phi(E')dE'\,dE}{\phi_{g'}},$$

$$\Sigma^s_{1,g'\to g} = \frac{\int_{g'}\int_g \Sigma^s_1(E'\to E)J(E')dE'\,dE}{J_{g'}},$$

There are several possibilities of defining the directional total cross section [3]. The simplest, often chosen definition is:

$$\Sigma_{mg} = \Sigma_g = \frac{\int_g \Sigma\phi dE}{\phi_g}.$$

2.2. *The discrete ordinate form of the neutron transport equation*

The streaming term has to be defined, separately for each geometry. In rectangular geometry it is:

$$\Omega\nabla\phi = \mu\frac{\partial\phi}{\partial x} + \eta\frac{\partial\phi}{\partial y} + \varsigma\frac{\partial\phi}{\partial z} \tag{2.22}$$

The spatial mesh is imposed which for a 2-dimensional case, (x,y), is shown in Fig.8.

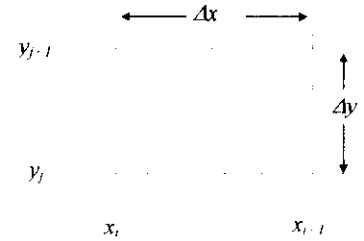

Figure 8. Spatial mesh in (x,y) geometry.

The finite difference equations based on that mesh become:

$$\mu_m \Delta y(\phi_{i+1,m} - \phi_{i,m}) + \eta_m \Delta x(\phi_{j+1,m} - \phi_{j,m}) = V(Q_m - \Sigma\phi_m), \qquad (2.23)$$

where $V=\Delta x \cdot \Delta y$, is the mesh area.

The Gauss quadrature set of directions and weights has been found an adequate choice, but various computer codes use different approaches to particular problems. More information on the method can be found, e.g., in Ref. 4.

3. Method of collision probabilities

3.1. Probability of a neutron to travel of a distance s without collision

The method of collision probabilities, or more adequately the method of the first flight collision probability, is applied for the solution of the neutron transport equation at least as frequently as the S_N one. Before defining the basic equation solved in this method, two basic quantities are introduced.

Let $\Sigma(E)$ be the total cross section, i.e., the probability of neutron interaction with a nucleus per unit distance, s the distance measured in the direction of the neutron travel, Ω, and $p(s)$ the probability of a neutron to travel of a distance s without collision. Then the decrease of $p(s)$ is:

$$d\,p(s) = -\,p(s)\,\Sigma(E)ds.$$

and hence:

$$p(s) = \exp[-\Sigma(E)s] \qquad (2.24)$$

3.2. Escape probability

Let us consider a square unit cell composed of a fuel rod surrounded by moderator.. Neutrons, which escape from the fuel, will be slowed down in the moderator. Let us consider a neutron produced at position $\boldsymbol{r}$ with direction Ω, and let $R(\boldsymbol{r}, \Omega)$ be the distance from the point of the neutron birth to the boundary of the region in the direction Ω. The probability that a neutron will escape from the region without making a collision by Eq. (2.24) is $\exp[-\Sigma(E)R(r,\Omega)]$.

Probability that a neutron will be generated in the direction $d\Omega$ about Ω and position in the volume element dV about $\boldsymbol{r}$ is $(d\Omega/(4\pi))\cdot(dV/V)$. Hence, the escape probability, P_{esc}, for neutrons born in the whole region V is:

$$P_{esc} = \frac{1}{4\pi}\iint exp(-\Sigma(E)R(r,\Omega))d\Omega dV\,. \qquad (2.25)$$

If dimensions of the body are large compared to the mean free path, $1/\Sigma$, then Eq.(2.25) can be approximated by $P_{esc}=1/(4\Sigma V)$. For small bodies P_{esc} must approach unity. A *rational approximation* proposed by Wigner for bodies of all sizes is $P_{esc} = 1/(1+4\Sigma V)$, or with a *mean chord*, $R=(4V)/A$, where A is a surface of the fuel intersection:

$$P_{esc} = 1/(1+\Sigma RA) \tag{2.26}$$

3.3 Transfer probabilities

Let us consider now a region divided into a finite number of subregions, as shown in Fig. 9. It is assumed that neutrons are produced uniformly and isotropically in each of these subregions. The problem is then to determine the probability that neutrons born in one of the regions make their next collision in the source region or in one of the other regions.

All the probabilities from Fig. 9 can be calculated as functions of the macroscopic total cross sections and the P_c probability, which is a nonescape probability from an infinite cylinder and has been determined by Case, de Hoffman, Placzek [5]. Thus, under the assumption of constant material properties in each in the subregions, it is possible to calculate the transfer probabilities for all the subregions.

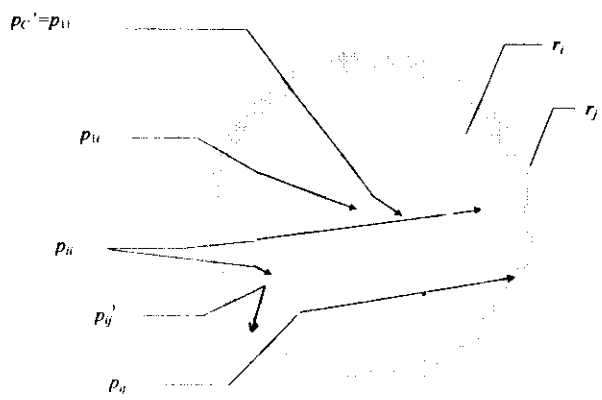

Figure 9: A circular region divided into a set of subregions.

3.4. Equations solved in the collision probability method

We recall the steady state transport equation in its integral form and with the assumption of isotropic scattering and a constant cross section in the region:

$$\phi(r,E) = \int \frac{e^{-\Sigma(E)R}}{4\pi R^2} \left[\int \Sigma(r';E' \rightarrow E)\phi(r',E')dE' + Q(r',E) \right] dr',$$

with $R = |\boldsymbol{r}-\boldsymbol{r}'|$.

To this equation the multigroup approach is applied, which allows for substitution of the integral over energy by a sum over groups. Integrating over each subregion volume gives a set of equations, with i,j the region indices and g a group index:

$$\phi_{g,i} = \sum_j \frac{V_j}{V_i} M_{g,ij} \left(\Sigma^s_{g,j} \phi_{g,j} + Q_{g,j} \right), \tag{2.27}$$

where:

$$\phi_{g,i} = \frac{1}{V_i} \int_{V_i} \phi_g(r) dr ,$$

$$Q_{g,i} = \sum_{g'} \left(\Sigma^s_{g'g,i} \phi_{g',i} + \chi_g \frac{\left(\kappa \Sigma^f\right)_{g',i}}{k} \phi_{g',i} \right),$$

and $\mathbf{M}_g$ is a square matrix with the dimension equal to the number of subregions, $\mathbf{Q}_g$ and Φ_g are vectors for each energy group g. $\mathbf{Q}$ is here an energy transfer matrix including also any transfer of neutron energy due to fission.

Hence, with $\mathbf{P}$ the group collision probability matrix averaged over the emission spectrum we get a set of two equations for the emission rate of neutrons produced from all sources in group g:

$$\Psi_{g,i} = \sum_{g'=1}^{G} \mathrm{Q}_{gg',i} \Phi_{g',i} + \mathrm{S}_{g,j} , \tag{2.28a}$$

and for the total collision rate in group g, defined as $\Phi_{g,i} = V_i \Sigma_{g,i} \phi_{g,i}$:

$$\Phi_{g,i} = \sum_{j=1}^{J} P_{g,ij} \Psi_{g,j} \tag{2.28b}$$

These equations are effectively solved on the computer.

More details on both, DSN and Collision Probability methods, can be found in Ref.2.

References

1. J. Briesmeister, *MCNP - A General Monte Carlo N-Particle Transport Code Version 4A, LA*-12625, (1993).
2. R. J. J. Stammler and M. J. Abbate: *Methods of Steady State Reactor Physics in Nuclear Design*, (Academic Press, 1983).
3. B. Carlson, *Solution of the Transport Equation by S_n Approximation*, LA-1891, (1955).
4. K. P. Lathrop and B.G. Carlson, *Discrete Ordinate Angular Quadrature of the Neutron Transport Equation*, LA-3186, (1965).
5. K. M. Case, F de Hoffman, G. Placzek, *Introduction to the Theory of Neutron Diffusion, I,Los Alamos Sci. Laboratory*, (1953).