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SMR/1220-16

Workshop on

Nuclear Reaction Data and Nuclear Reactors: Physics, Design and Safety

13 March - 14 April 2000

Miramare - Trieste, Italy

Statistical Nuclear Reactions

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STATISTICAL NUCLEAR REACTIONS

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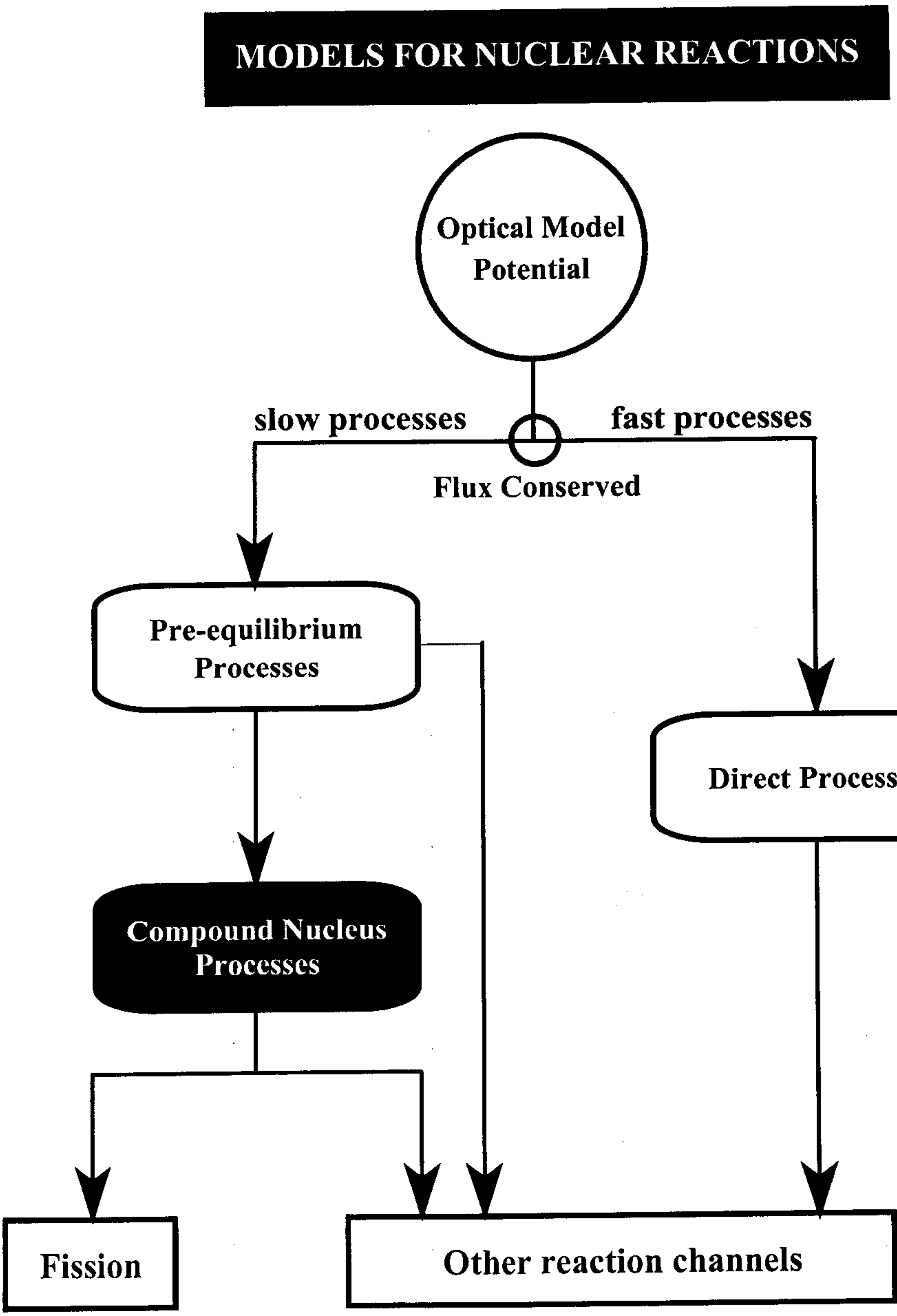
ICTP
21 March 2000

INTRODUCTION



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STATISTICAL MODEL

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Principle

Theoretical approach

MODEL BASES

Statistical Model hypothesis

- A compound Nucleus has been created
- Continuum of CN levels
- Independence between entrance and outgoing channels

Channel definition

$$a + A \rightarrow (CN^{*}) \rightarrow b + B$$

Incident channel $\alpha = (\vec{l}_a, \vec{s}_a, \vec{I}_A, \pi_a, \pi_A)$

with : $\vec{l}_a$ = projectile orbital angular momentum

$\vec{s}_a$, π_a = projectile intrinsic spin and parity

$\vec{I}_A$, π_A = target spin and parity

Outgoing channel $\beta = (\vec{l}_b, \vec{s}_b, \vec{I}_B, \pi_b, \pi_B)$

Conservation equations

Total energy : $E_a + E_A = E_b + E_B$

Total momentum : $\vec{p}_a + \vec{p}_A = \vec{p}_b + \vec{p}_B$

Total angular momentum : $\vec{I}_A + \vec{l}_a + \vec{s}_a = \vec{I}_B + \vec{l}_b + \vec{s}_b$

Total parity : $\pi_a \ \pi_A (-1)^{l_a} = \pi_b \ \pi_B (-1)^{l_b}$

HAUSER-FESHBACH EXPRESSION

Simplest situation : spinless particles

See Hodgson, Nuclear reactions and nuclear structure, Clarendon Press Oxford (1971) p 284.

Consider a reaction from a initial channel α to a final channel β without any spin consideration

Diffusion theory $\Rightarrow \sigma_{\alpha\beta} = \frac{\pi}{k_\alpha^2} \left\langle \left| \delta_{\alpha\beta} - S_{\alpha\beta} \right|^2 \right\rangle$ **(1)**

Optical Model Potential $\Rightarrow S_{\alpha\beta}$ **and** $T_\alpha = 1 - \left| S_{\alpha\alpha} \right|^2$

$\Rightarrow$**Compound nucleus formation cross section** $\sigma_\alpha^{(CN)} = \frac{\pi}{k_\alpha^2} T_\alpha$

Independence hypothesis $\Rightarrow$ $\sigma_{\alpha\beta} = \sigma_\alpha^{(CN)} P_\beta$ **(2)**

Reciprocity theorem + (1) + (2) $\Rightarrow$ $P_\beta = \frac{T_\beta}{\sum_\alpha T_\alpha}$

$\Rightarrow$ Hauser-Feshbach formula

$$\sigma_{\alpha\beta} = \frac{\pi}{k_\alpha^2} \frac{T_\alpha T_\beta}{\sum_c T_c}$$

HAUSER-FESHBACH EXPRESSION

Angle integrated cross section

In realistic calculations, all possible reaction channels have to be considered

$\Rightarrow$ summations over all possible quantum numbers

Denoting by a and b the incident and outgoing channels characteristics (Z_a, N_a, E_a, s_a, E_A, π_A), one can write

$$\sigma_{ab}=\frac{\pi}{k_a^2}\sum_{J=|I_A-s_a|}^{I_A+s_a+l_a^{max}}\sum_{\pi=\pm}\frac{2J+1}{(2I_A+1)(2s_a+1)}\sum_{j_a=|J-I_A|}^{J+I_A}\sum_{l_a=|j_a-s_a|}^{j_a+s_a}\sum_{j_b=|J-I_B|}^{J+I_B}\sum_{l_b=|j_b-s_b|}^{j_b+s_b}\delta_\pi(a)\,\delta_\pi(b)\frac{T^J_{a,l_a,j_a}\,T^J_{b,l_b,j_b}}{\sum_c T^J_{c,l_c,j_c}}$$

With $\delta_\pi(c)=1$ if $(-1)^{l_c}\pi_c\pi_C=\pi$ and 0 otherwise.

HAUSER-FESHBACH EXPRESSION

Angular distributions

The compound angular distribution cross section reads

$$\sigma_{ab}(\theta) = \sum_{\text{even } L} C_L P_L(\cos\theta)$$

where the P_L's are Legendre Polynomials and the coefficients C_L are given by

$$C_L = \frac{\pi}{k_a^2} \sum_{J,\pi} \frac{2J+1}{(2I_A+1)(2s_a+1)} \sum_{j_a} \sum_{l_a} \sum_{j_b} \sum_{l_b} \delta_\pi(a)\,\delta_\pi(b) \frac{T^J_{a,l_a,j_a}\, T^J_{b,l_b,j_b}}{\sum_c T^J_{c,l_c,j_c}} A^J_{I_A,l_a,j_a,I_B,l_b,j_b:L}$$

with $$A^J_{I_A,l_a,j_a,I_B,l_b,j_b:L} = \frac{(-1)^{I_B-s_b-I_A+s_a}}{4\pi}(2J+1)\; Z(I_A, j_a, l_a, J, L)\; Z(I_B, j_b, l_b, J, L)$$

and $$Z(I, j, l, J, L) = (2j+1)(2l+1) \underbrace{(l\,l\,0\,0\,|\,L\,0)}_{\text{Clebsch-Gordan}}\, \underbrace{W(j\,j\,l\,l;\,L\,s)}_{\text{Racah}}\, W(J\,j\,J\,j\,;\,L\,0)$$

This expressions is restricted to the coupling scheme J=I+(l+s)

WIDTH FLUCTUATION CONCEPT

Principle

HF too simple because the independence hypothesis not completely true, especially for low projectile energy. Corrections must be addec to HF formula.

Theoretical approach (1/2)

Breit-Wigner formula for one isolated resonance of spin J in the CN

$$\Rightarrow \sigma_{\alpha\beta} = \frac{\pi}{k_\alpha^2} \frac{\Gamma_\alpha \Gamma_\beta}{(E-E_0)^2 + \frac{1}{4}\Gamma^2}$$

where E_0 is the resonance energy and $\Gamma = \sum_i \Gamma_i$, the width of all CN resonances with spin J.

Averaging BW formula over an energy interval and all CN resonance: with spin J in this intervall gives

see Lane and Lynn, Proc.Phys. Soc. 70 (1957) 557.

$$\langle \sigma_{\alpha\beta} \rangle = \frac{\pi}{k_\alpha^2} \frac{2\pi}{D} \left\langle \frac{\Gamma_\alpha \Gamma_\beta}{\Gamma} \right\rangle$$

where D is the mean resonance spacing

WIDTH FLUCTUATION CONCEPT

Theoretical approach (2/2)

It can be shown that

$$\langle S_{\alpha\alpha}\rangle = e^{2i\delta}\left(1 - \frac{\pi\langle\Gamma_\alpha\rangle}{D}\right)$$

$$\Rightarrow \quad T_\alpha = 1 - \left|\langle S_{\alpha\alpha}\rangle\right|^2 \approx \frac{2\pi\langle\Gamma_\alpha\rangle}{D}$$

Formal expression for WFF

We introduce the ratio of T. coeff. appearing in HF formula and obtain

$$\sigma_{\alpha\beta} = \frac{\pi}{k_\alpha^2}\frac{T_\alpha T_\beta}{\sum_c T_c} W_{\alpha\beta}$$

Where the width fluctuation factor $W_{\alpha\beta}$ reads

$$W_{\alpha\beta} = \left\langle \frac{\Gamma_\alpha \Gamma_\beta}{\Gamma} \right\rangle \frac{\langle\Gamma\rangle}{\langle\Gamma_\alpha\rangle\langle\Gamma_\beta\rangle}$$

MODEL INGREDIENTS

MODEL INGREDIENTS

LEVEL DENSITIES

See specific lecture

PARTICLE TRANSMISSION COEFFICIENTS

Single channel transmission coefficients

Effective transmission coefficients

GAMMA TRANSMISSION COEFFICIENTS

Gamma ray transmission coefficients

Gamma ray strength functions

Gamma ray selection rules

Renormalisation procedure

FISSION TRANSMISSION COEFFICIENTS

The fission model

Transition states concept

Multiple humped barriers

Class II states

PARTICLES TRANSMISSION COEFFICIENTS

Single channel transmission coefficients

⇒ **Given by "good" optical model potentials**

In principle one OMP per type of emitted particle

Effective transmission coefficients

If the compound nucleus decays to a residual nucleus, such that the excitation energy of the residual nucleus is high enough, it is impossible to take separatly into account each residual nucleus level concerned by a given decay channel b.

⇒ **Summation over all residual levels found in a given energy bin of the channel transmission coefficients**

$$\left\langle T^{J,\pi}_{b,\, l_b,\, j_b} \right\rangle = \int \rho(E, I_B, \pi_B)\ \delta_\pi(b)\ T^{J,\pi}_{b,\, l_b,\, j_b}\ dE$$

where ρ is the residual nucleus level density

GAMMA TRANSMISSION COEFFICIENTS

See Chrien, Neutron radiative capture, Pergamon Press (1984).
RIPL handbook, IAEA Vienna (http://www-nds.iaea.or.at/ripl/), chapter 6.

Gamma ray transmission coefficients

Given by
$$T^{xl}(\varepsilon_\gamma) = 2\pi\, f(x,l)\, \varepsilon_\gamma^{2l+1}$$

where xl = multipolarity, ε_γ = gamma ray energy and f(x,l) = strength function

Gamma ray strength functions

Many models for f(x,l) available

- Weisskopf model ⇒ f(x,l) = constant
- Brink-Axel model ⇒ f(x,l) = standart Lorentzian

$$f(x,l) = K_{xl} \frac{\sigma_0\, \varepsilon_\gamma^{3-2l}\, \Gamma_0^2}{\left(\varepsilon_\gamma^2 - E_0^2\right)^2 + \varepsilon_\gamma^2 \Gamma_0^2} \quad \left(\sigma_0, \Gamma_0, E_0 \text{ free parameters}\right)$$

- Kopecky-Uhl model ⇒ generalized Lorentzian for E1 (dominant)

$$f(E,1) = K_{E1}\sigma_0\Gamma_0 \left[\frac{\varepsilon_\gamma\, \Gamma(\varepsilon_\gamma, T)}{\left(\varepsilon_\gamma^2 - E_0^2\right)^2 + \varepsilon_\gamma^2 \left[\Gamma(\varepsilon_\gamma, T)\right]^2} + 0.7\, \frac{\Gamma(\varepsilon_\gamma = 0, T)}{E_0^3} \right]$$

where $$\Gamma(\varepsilon_\gamma, T) = \frac{\Gamma_0}{E_0^2}\left(\varepsilon_\gamma^2 + 4\pi^2 T^2\right)$$

FISSION TRANSMISSION COEFFICIENTS

Gamma rays selection rules

γ **decay from initial CN level** $J_i^{\pi_i}$ **to final CN level** $J_f^{\pi_f}$

For gamma ray Eλ : $| J_i - \lambda | < J_f < J_i + \lambda$ **and** $\pi_f = (-1)^{\lambda}\, \pi_i$

For gamma ray Mλ : $| J_i - \lambda | < J_f < J_i + \lambda$ **and** $\pi_f = (-1)^{\lambda+1}\, \pi_i$

Renormalisation procedure

Gamma tr. coeff. renormalised using experimental measurements for excitation energy close to B_n

$$\langle \Gamma_{\gamma 0} \rangle_{exp} = \langle \Gamma_{\gamma 0} \rangle_{th} \quad \text{required}$$

$\Rightarrow$ **renormalisation factor = ratio of** $2\pi \dfrac{\langle \Gamma_{\gamma 0} \rangle_{exp}}{D_0}$ **with**

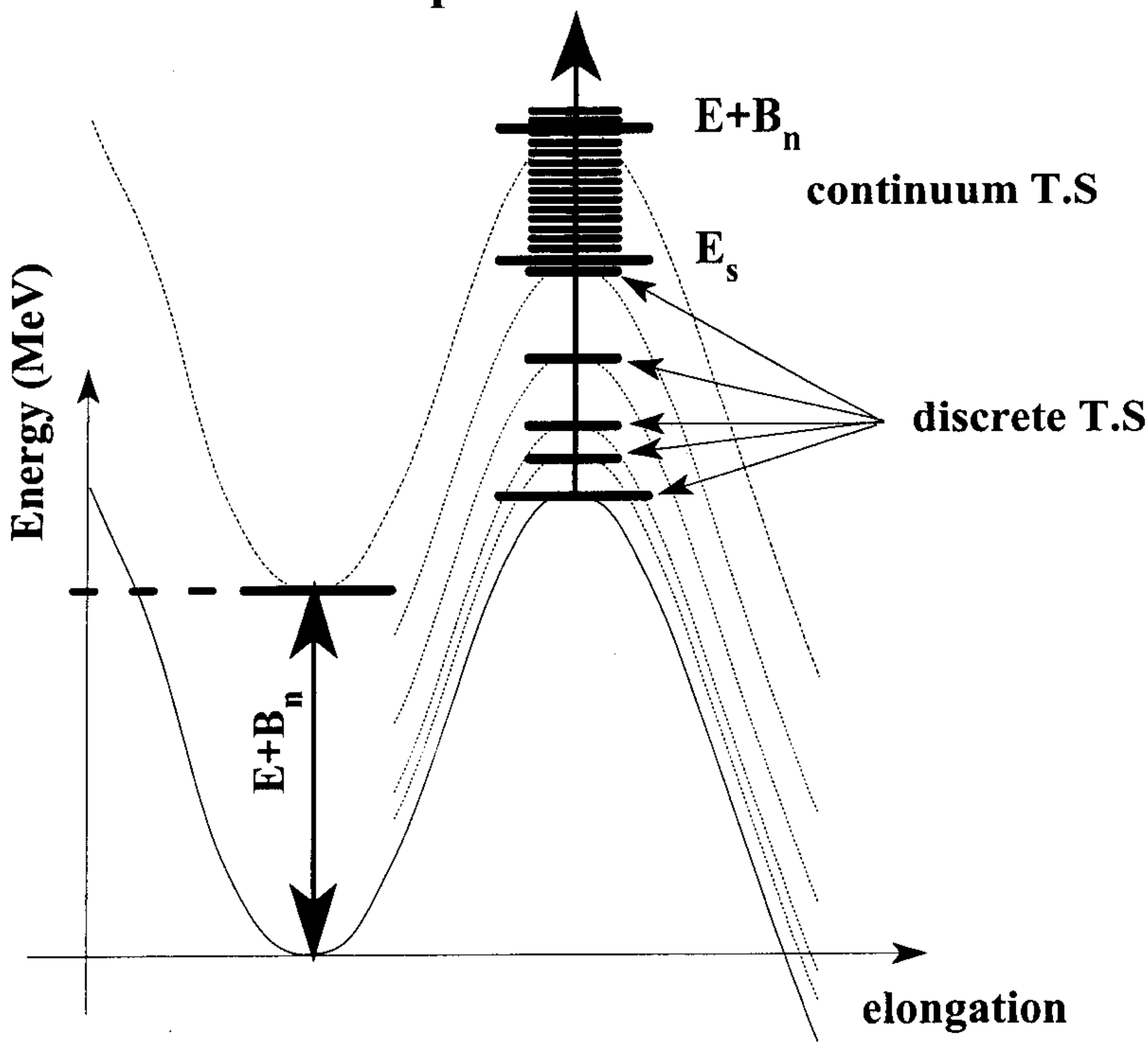
$$\sum_{J_i, \pi_i} \sum_{x,\lambda} \sum_{J_f = | J-\lambda |}^{J+\lambda} \sum_{\pi_f} \int_0^{B_n} T^{X\lambda}(\varepsilon)\, \rho(B_n - \varepsilon, J_f, \pi_f)\, f(J, J_f, \lambda, \pi, \pi_f)\, d\varepsilon$$

in which the function f is 0 or 1 to force compliance with selection rules, and the J_i **,** π_i **sum is over the CN levels that can be formed with s-wave incident neutrons. Also, D0 is the mean spacing of the s-wave resonances.**

FISSION TRANSMISSION COEFFICIENTS

See Michaudon, Nuclear Fission and Neutron-Induced Fission Cross Sections, Pergamon Press (1981)

Transition states concept



Fission Transmission coefficient

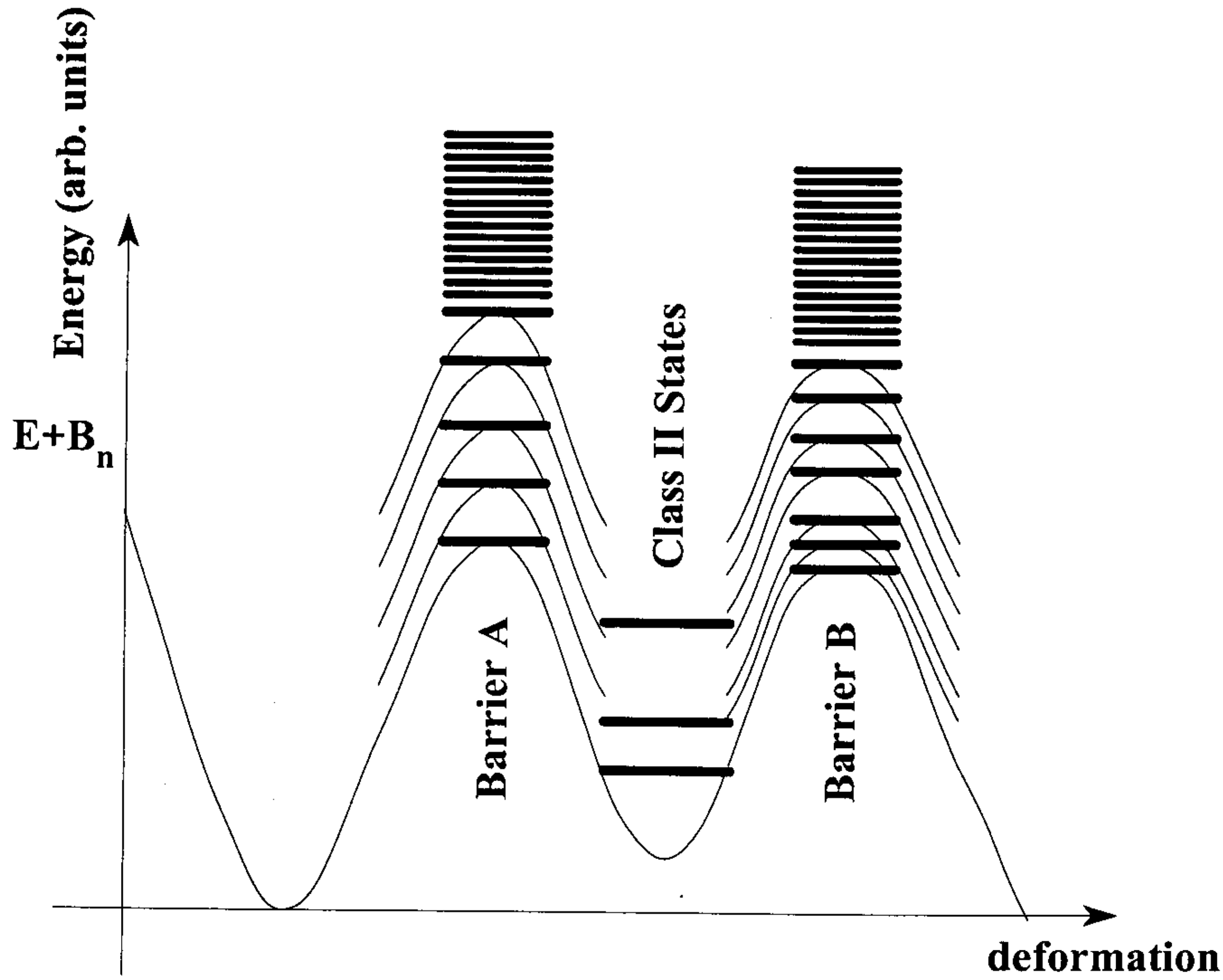
Fission tr. coeff. for a given state (E, J,π) of the CN

=

$$T_{fis}(E, J, \pi) = \sum_d T_{HW}(E-E_d) + \int_{E_s}^{E+B_n} \rho(E, J, \pi)\, T_{HW}(E-\varepsilon)\, d\varepsilon$$

FISSION TRANSMISSION COEFFICIENTS

Multiple humped barriers

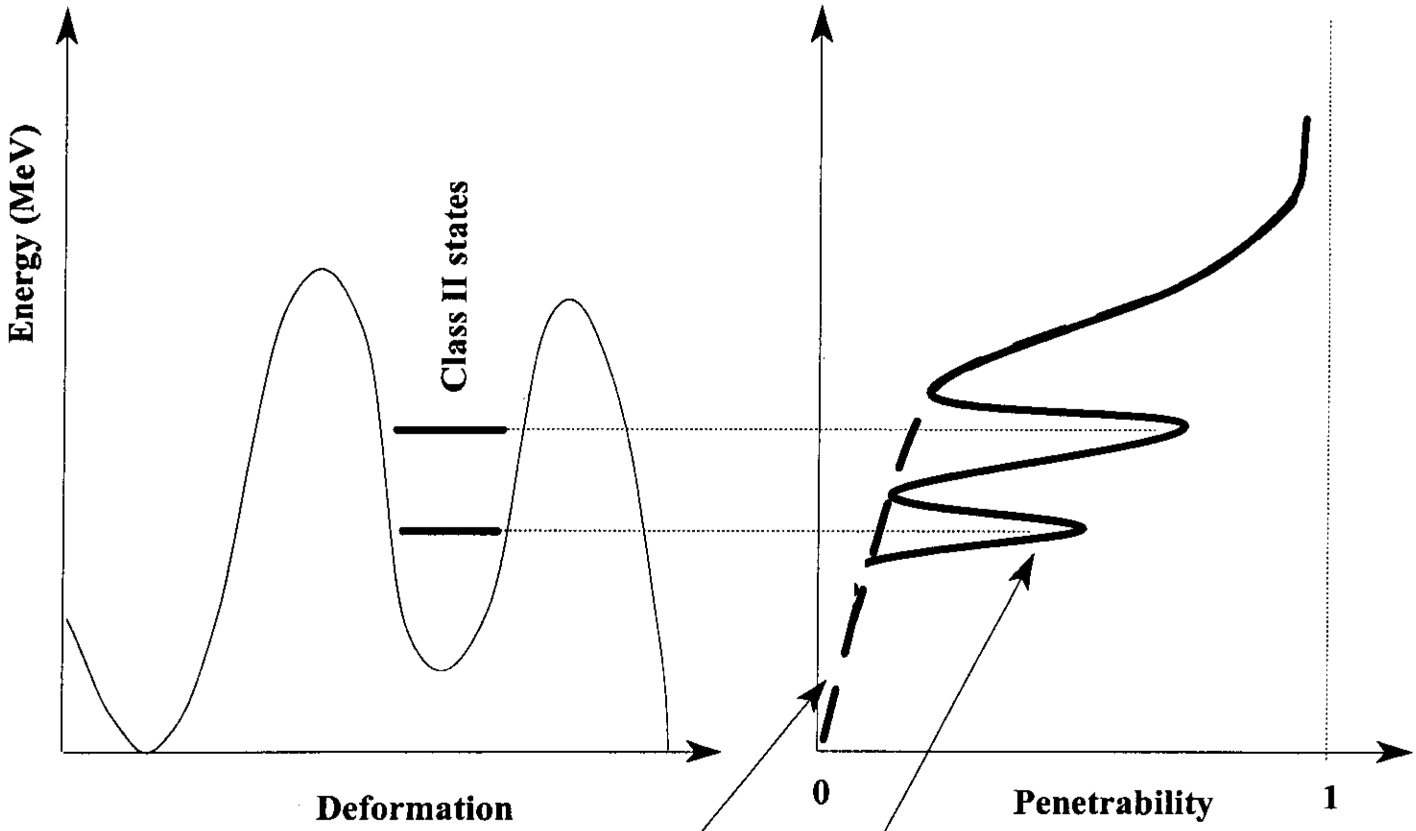


Fission transmission coefficient for two barriers

$$T_{fis}(E, J, \pi) = \frac{T_A \; T_B}{T_A + T_B}$$

FISSION TRANSMISSION COEFFICIENTS

Class II states



$$T_{fis} = \frac{T_A\,T_B}{T_A + T_B} \left(x\,\frac{4}{T_A + T_B} \right)$$

WIDTH FLUCTUATION CORRECTIONS

WIDTH FLUCTUATION CORRECTIONS

THE VARIOUS METHODS

HRTW

Dresner - Moldauer integral

GOE triple integral

PARTICULAR CHANNELS

Continuum channels

Gamma channels

Fission channels

GOE OR MOLDAUER ?

THE VARIOUS METHODS

HRTW (*see Hofmann et al., Z. Phys. A 297 (1980) 153 and references therein.*)

Based on independence hypothesis

$$\Rightarrow \sigma_{ab} = \frac{T_a \; T_b}{\sum_c T_c} \, W_{ab} = \frac{V_a \; V_b}{\sum_c V_c} \left[1 + \delta_{ab} \, (W_a - 1) \right]$$

From unitarity (i.e. $T_a = \sum_b \sigma_{ab}$) one obtains

$$T_a = V_a + (W_a - 1) \, \frac{V_a^{\,2}}{\sum_c V_c}$$

$\Rightarrow$ If W_a is known, the V_a's are deduced by iterations of the type

$$V_{ab}^{(i+1)} = \frac{T_a}{1 + (W_a - 1) \, \dfrac{V_a^{(i)}}{\sum_c V_c^{(i)}}} \quad \text{with } V_{ab}^{(0)} = \frac{T_a}{1 + (W_a - 1) \, \dfrac{T_a}{\sum_c T_c}}$$

Parametrisation of W_a from Random S-matrix analysis

$$W_a = 1 + \frac{2}{1 + T_a^{\,F}} + 87 \left(\frac{T_a - \tilde{T}}{\sum_c T_c} \right)^2 \left(\frac{T_a}{\sum_c T_c} \right)^5$$

$$\text{with } \tilde{T} = \frac{\sum_c T_c^{\,2}}{\sum_c T_c} \quad \text{and } F = \frac{\dfrac{4\tilde{T}}{\sum_c T_c} \left(1 + \dfrac{T_a}{\sum_c T_c} \right)}{1 + \dfrac{3\tilde{T}}{\sum_c T_c}}$$

THE VARIOUS METHODS

Dresner - Moldauer (*See Moldauer, Nucl. Phys. A344 (1980) 185.*)

Direct calculation of $W_{ab} = \left\langle \frac{\Gamma_a \Gamma_b}{\Gamma} \right\rangle \frac{\langle \Gamma \rangle}{\langle \Gamma_a \rangle \langle \Gamma_b \rangle}$ **assuming**

that the reduced widths $\Gamma / \langle \Gamma \rangle$ **follow a** χ^2 **distribution with distribution with** ν **degrees of freedom.**

$\Rightarrow$ W_{ab} **given by a simple integral**

$$W_{ab} = \left(1 + \frac{2\delta_{ab}}{\nu_a}\right) \int_0^{+\infty} \prod_c \left(1 + \frac{2T_c}{\nu_c \sum_i T_i} x\right)^{-\delta_{ac} - \delta_{bc} - \frac{\nu_c}{2}} dx$$

Parametrisation of ν **:**

- $\nu = 1$ **for Dresner**
- ν **deduced from Random S-matrix analysis for Moldauer**

$$\nu_a(T_a) = 1.78 + \left(T_a^{1.212} - 0.78\right) \exp\left(-0.228 \sum_c T_c\right)$$

W_{ab} **calculated using Gauss-Laguerre points and weights**

THE VARIOUS METHODS

GOE triple integral (*see Verbaarschot, Ann. Phys. 168 (1985) 368 and references therein.*)

W_{ab} calculated assuming that the Hamiltonian belongs to the GOE

$\Rightarrow$ W_{ab} given by a triple integral

$$\sigma_{ab} = \sum_i \alpha_i \frac{T_{a(i)} T_{b(i)}}{8} \int_0^{+\infty} d\lambda_1 \int_0^{+\infty} d\lambda_2 \int_0^{1} d\lambda \frac{\lambda(1-\lambda)|\lambda_1-\lambda_2|}{\sqrt{\lambda_1(1+\lambda_1)\lambda_2(1+\lambda_2)}(\lambda+\lambda_1)^2(\lambda+\lambda_2)^2}$$

$$\prod_c \frac{(1-T_{c(i)}\lambda)}{\sqrt{(1+T_{c(i)}\lambda_1)(1+T_{c(i)}\lambda_2)}} \left\{ \delta_{a(i)b(i)}(1-T_{a(i)}) \left[\frac{\lambda_1}{(1+T_{a(i)}\lambda_1)} + \frac{\lambda_2}{(1+T_{a(i)}\lambda_2)} + \frac{2\lambda}{(1-T_{a(i)}\lambda)} \right]^2 \right.$$

$$\left. + (1+\delta_{a(i)b(i)}) \left[\frac{\lambda_1(1+\lambda_1)}{(1+T_{a(i)}\lambda_1)(1+T_{b(i)}\lambda_1)} + \frac{\lambda_2(1+\lambda_2)}{(1+T_{a(i)}\lambda_2)(1+T_{b(i)}\lambda_2)} + \frac{2\lambda(1-\lambda)}{(1-T_{a(i)}\lambda)(1-T_{b(i)}\lambda)} \right] \right\}$$

PARTICULAR CHANNELS

Continuum channels

Problem : T_{eff} = integral $\Rightarrow$ >> 1

HRTW

Define R = number of levels (corresponding to T)

Define $T_{ave} = T_{eff} / R$

Use T_{ave} in HRTW method

Multiply results by R

Moldauer

Define $\nu_{ave} = R \ \nu(T_{ave})$

Use T_{ave} in Moldauer integral with ν_{ave} as exponent

GOE

Two situations (*see Verbaarschot, Ann. Phys. 168 (1986) 368.*)

- **ΣT weak $\Rightarrow$ use T_{ave} and multiply by R at the end**
- **ΣT >> 1 $\Rightarrow$ use asymptotic development in $1 / \Sigma T$**

$$\sigma_{ab} = \frac{T_a \, T_b}{\sum_c T_c} \left[1 + \frac{f(T_c)}{\sum_c T_c} + \frac{g(T_c)}{\left(\sum_c T_c\right)^2} + \ldots \right]$$

PARTICULAR CHANNELS

Gamma channels

Problem : great number of channels with very weak T_γ
$\Rightarrow$ individual treatment impossible

HRTW

No difficulty : use of global T_γ

Moldauer

Approximation for calculation of the product over $c \in \gamma$

$$\prod_{c\in\gamma}\left(1+\frac{2\,T_c}{\nu_c\sum_i T_i}\,x\right)^{-\frac{\nu_c}{2}} \approx \lim_{\nu_\gamma\to+\infty}\left(1+\frac{2\,T_\gamma}{\nu_\gamma\sum_c T_c}\,x\right)^{-\frac{\nu_\gamma}{2}}$$

$$= \exp\left(-\frac{T_\gamma^{eff}}{\sum_c T_c}\,x\right)$$

GOE

Same approximation as for Moldauer

$$\prod_{c\in\gamma}\frac{(1-T_c\,\lambda)}{\sqrt{(1+T_c\,\lambda_1)(1+T_c\,\lambda_2)}} = \exp\left[-(2\lambda+\lambda_1+\lambda_2)\frac{T_\gamma^{eff}}{2}\right]$$

PARTICULAR CHANNELS

Fission channels

Problem : great number of channels giving $T_{fis} >> 1$

$\Rightarrow$ individual treatment impossible

Same solution as for continuum channels

$\Rightarrow$ number of fission channels required

$\Rightarrow$ limit in integral over transition states

$$T_{fis}(E, J, \pi) = \sum_{d} T_{HW}(E-E_d) + \int_{E_s}^{\boxed{E+B_n}} \rho(E, J, \pi)\, T_{HW}(E-\varepsilon)\, d\varepsilon$$

$N_{fis}(E, J, \pi)$ = previous expression with $T_{HW} = 1$

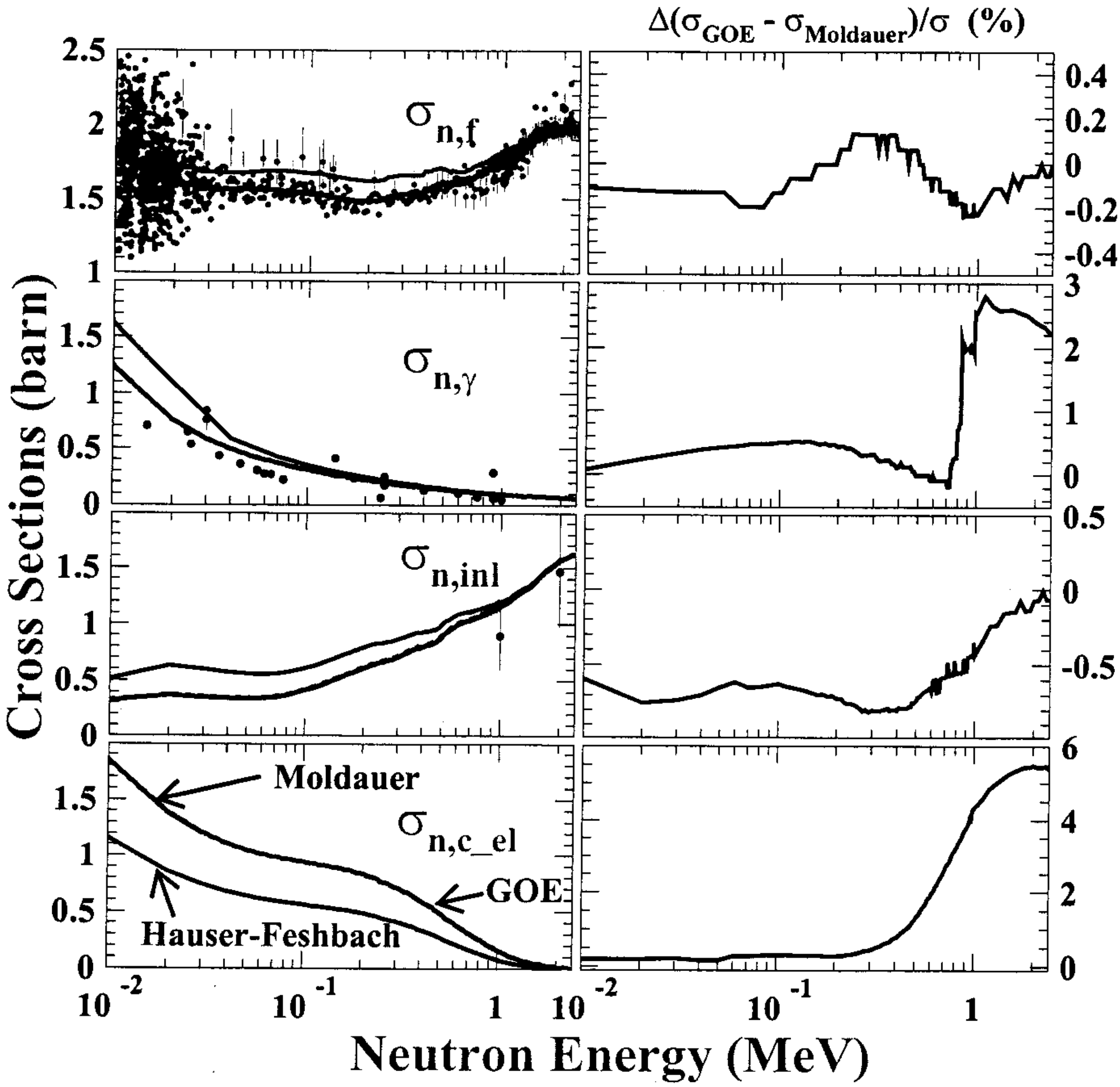
Generalisation for two barriers

- Calculate T_A (T_B) and N_A (N_B) $\Rightarrow \overline{T_A}$ and $\overline{T_B}$
- Calculate global $T_{fis} = \dfrac{T_A\, T_B}{T_A + T_B}$ and $\overline{T_{fis}} = \dfrac{\overline{T_A}\,\overline{T_B}}{\overline{T_A} + \overline{T_B}}$

$$\Rightarrow \text{global } N_{fis} = \frac{T_{fis}}{\overline{T_{fis}}} = \frac{T_A\, T_B}{T_A\, N_B + T_B\, N_A}$$

GOE OR MOLDAUER

Problem : GOE triple integral ⇒ important computation time
Moldauer simple integral ⇒ relatively fast

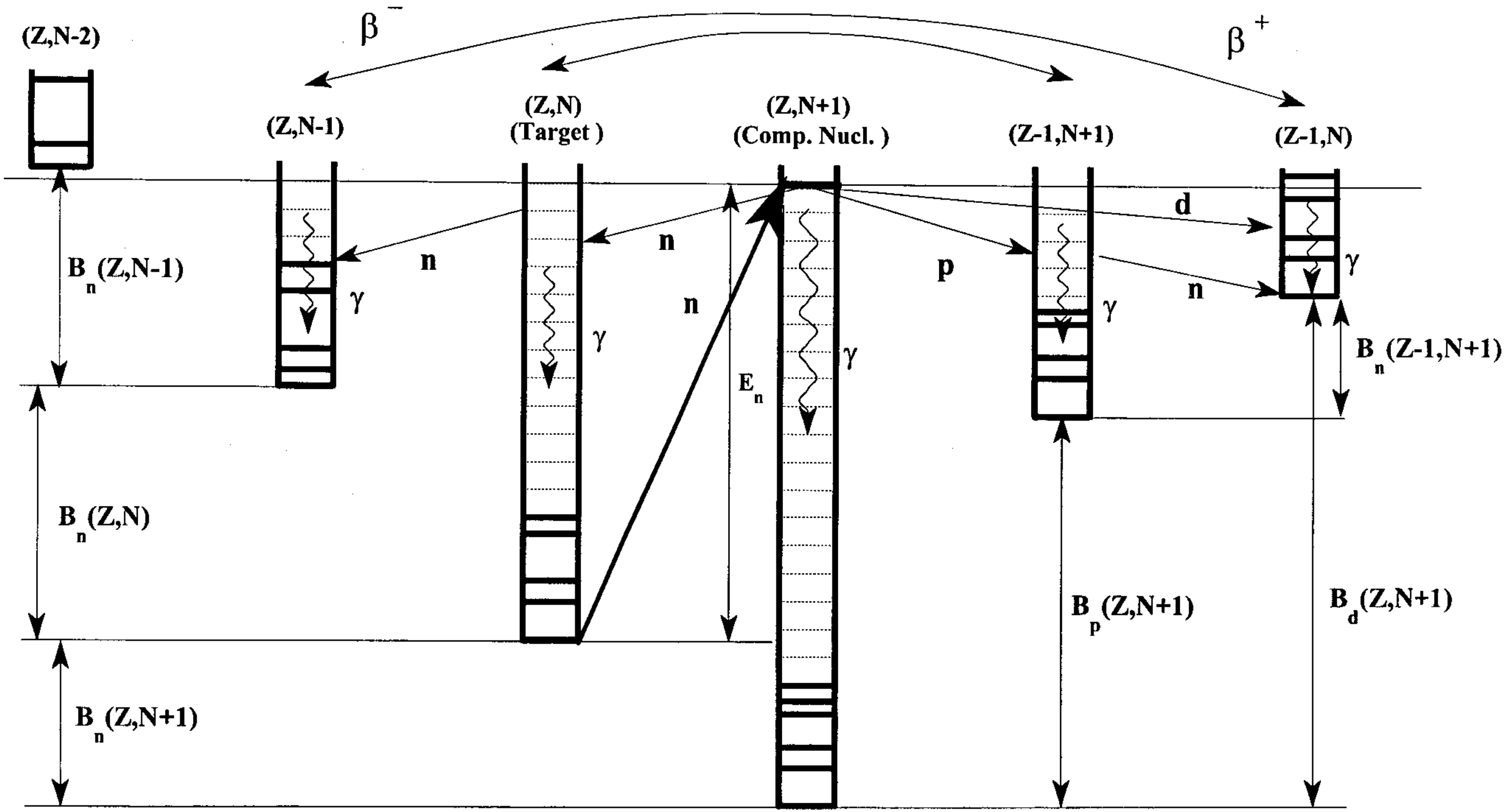


⇒ GOE ≈ Moldauer ⇒ Moldauer prefered

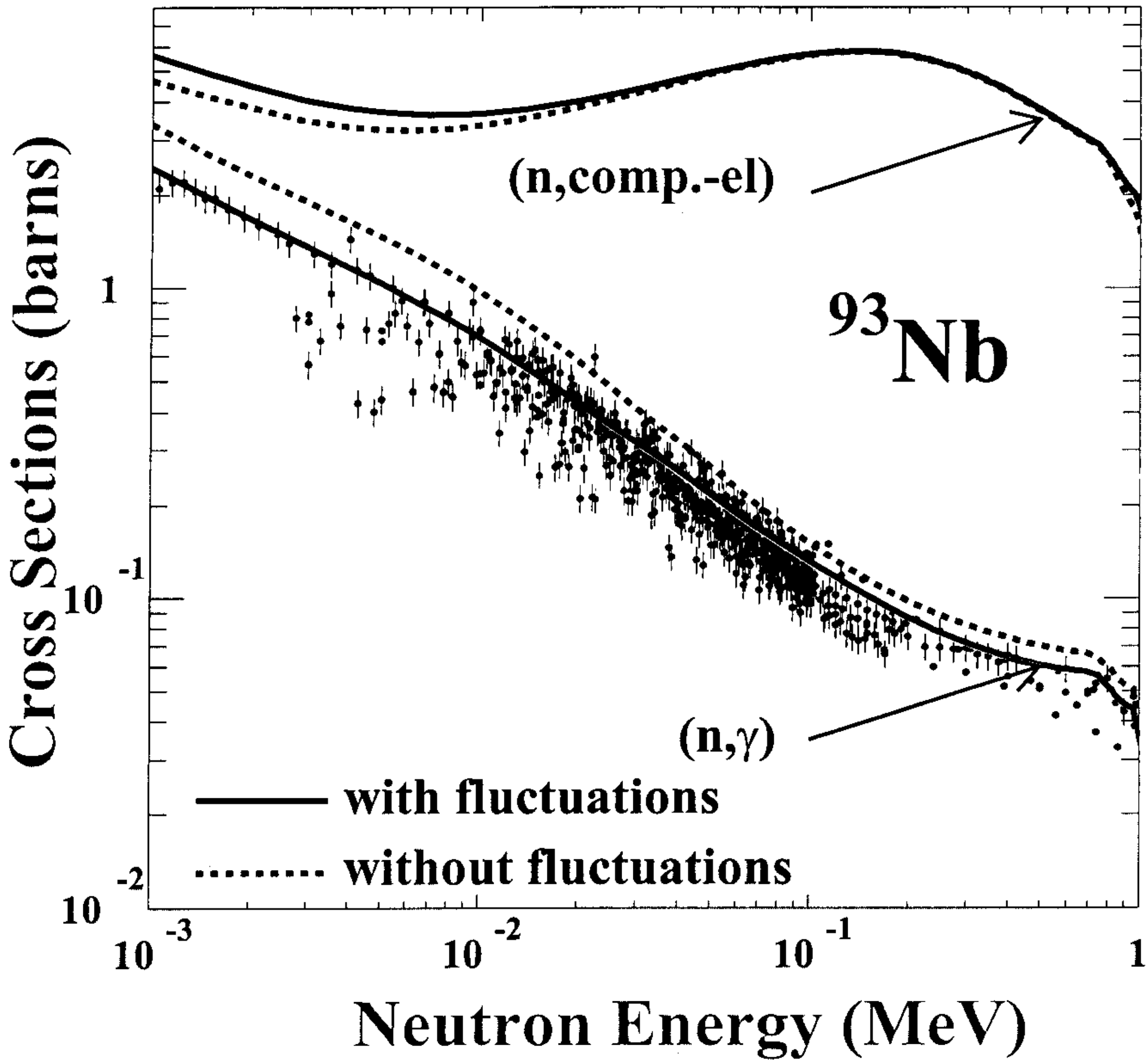
APPLICATIONS

APPLICATIONS

Illustration of a reaction chain



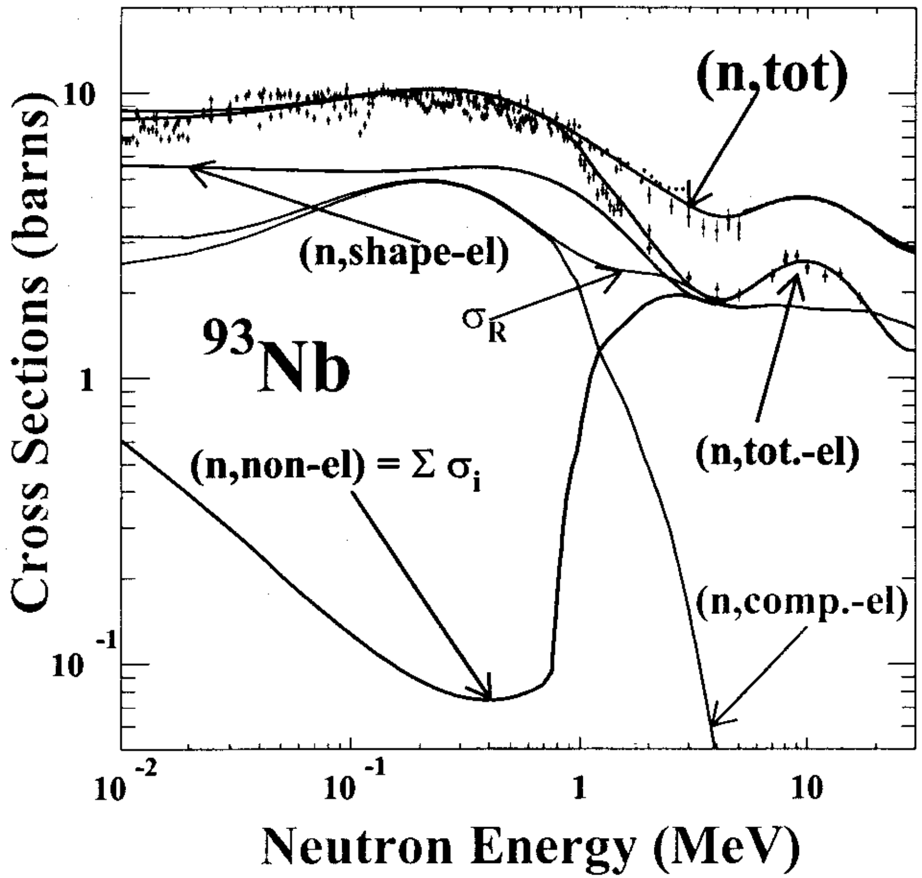
Width fluctuation correction effects

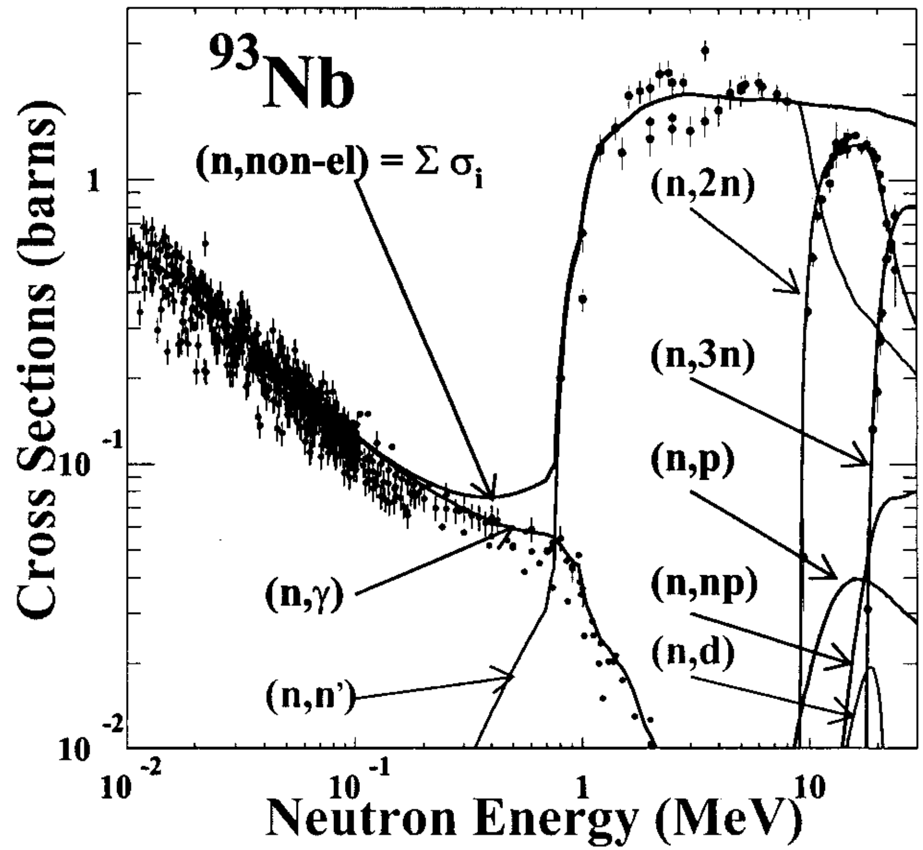


⇒ elastic enhancement

APPLICATIONS

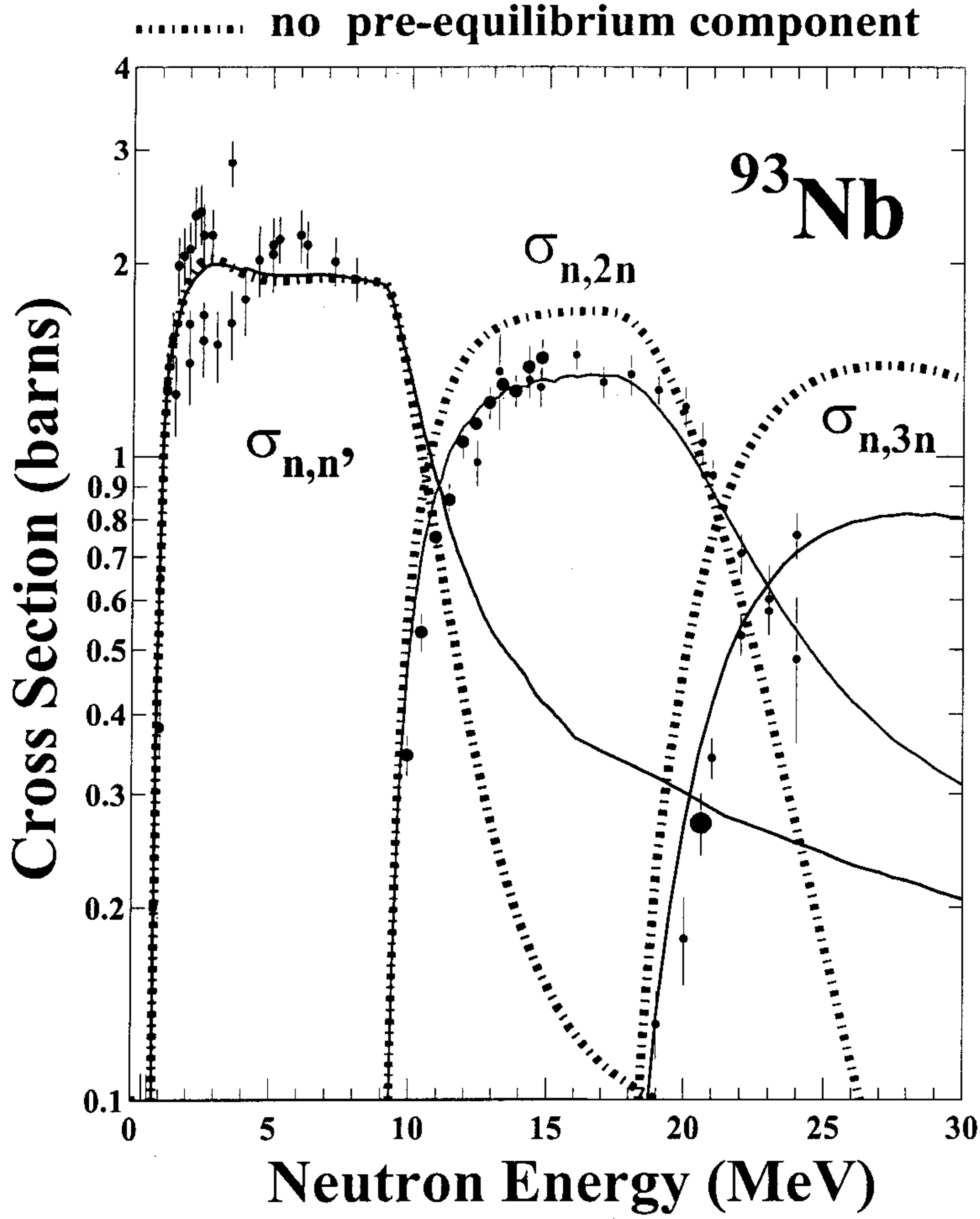
Cross sections splitting





APPLICATIONS

Pre-equilibrium model effects



⇒ key role above ≈ 10 MeV

CONCLUSION

The statistical nuclear reactions model is widely employed to analyse and predict nuclear data.

⇒ quite simple to implement

⇒ yields a lot of informations (observable or not)

Problem : good choice of all the ingredients ?

optical model potential ⇒ part. trans. coeff.
⇒ direct interaction component

level densities ⇒ govern channels competition

width fluctuation treatment ⇒ crucial for low energies

pre-equilibrium treatment ⇒ crucial for high energies

Remaining problems :

fission yields (generally systematics)

class two states treatment