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SMR/1220-17

Workshop on

Nuclear Reaction Data and Nuclear Reactors: Physics, Design and Safety

13 March - 14 April 2000

Miramare - Trieste, Italy

Level Densities

S. Hilaire
CEA - Bruyeres-le-Chatel
France

LEVEL DENSITIES

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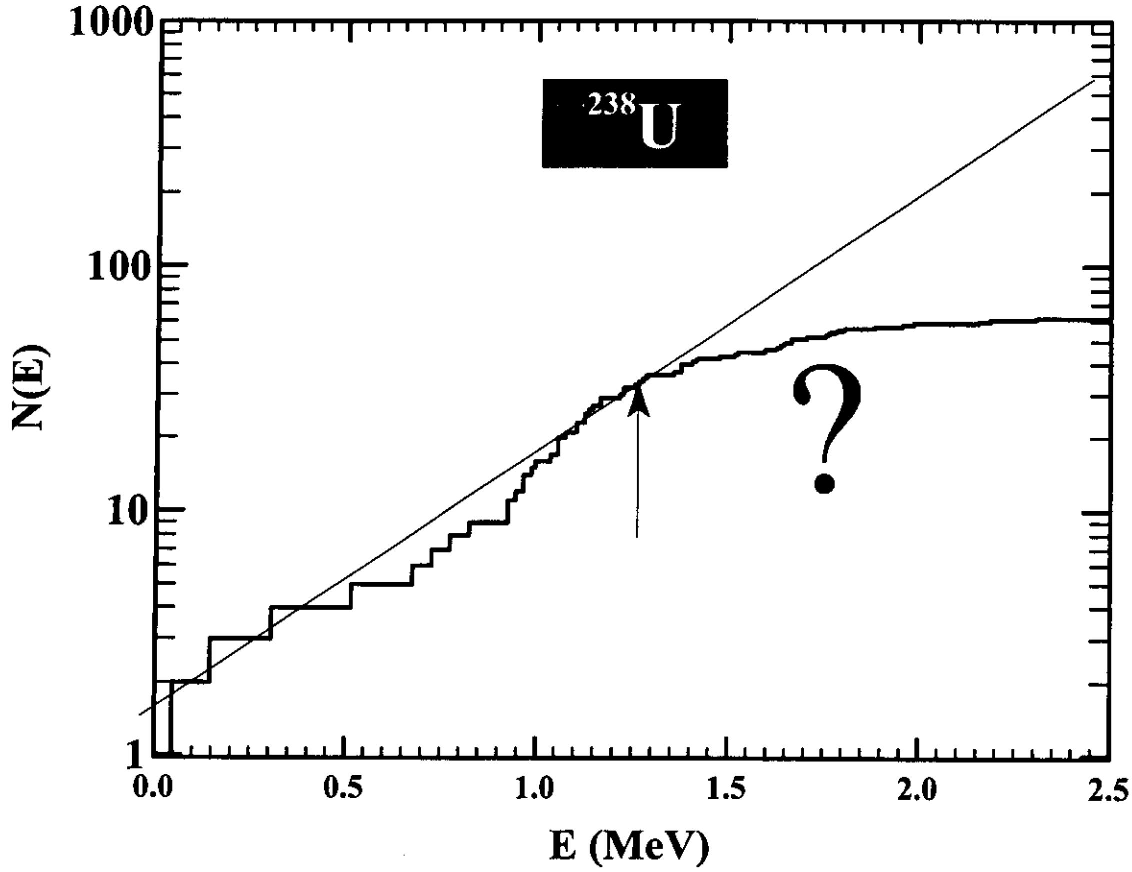
Miramare - Trieste
ICTP
22 March 2000

INTRODUCTION

- The cumulated number of nuclear discrete levels exhibits an exponential increase with increasing excitation energy



- Mean spacing of s-wave resonances around B_n close to a few eV

$\Rightarrow \approx 10^6$ levels / MeV

$\Downarrow$

Level density function $\rho(Z, N, U, J, \pi)$

- Crucial ingredient for the statistical and pre-equilibrium models

CONTENTS

EXPERIMENTAL INFORMATION

EXPERIMENTAL INFORMATION

See : Huizenga and Moretto, Ann. Rev. Nucl. Sci., 1972, Vol 22, p 427.
Melby et al., PRL 16, Vol 83, (1999) p 3150.

Model independent experimental data

- Low excitation energy E (<< B_n)
  - ⇒ Discrete excited nuclear levels (E, J, π)
- E close to a given value ($E = E_{proj} + B_{proj}$)
  - ⇒ Mean spacing of
    - resonances
    - neutron s-wave resonances

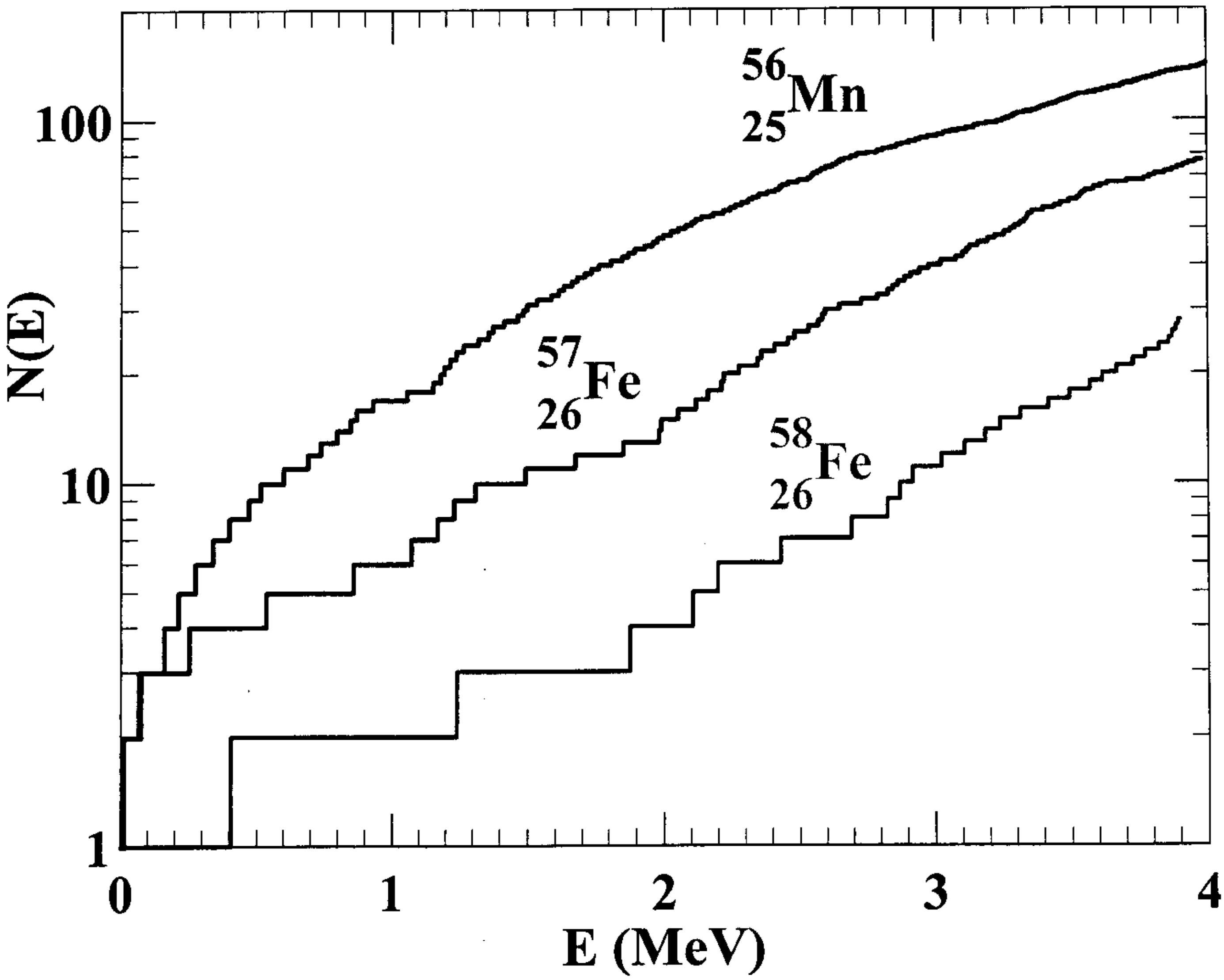
Model dependent experimental data

- E below B_n
  - ⇒ High resolution nuclear cross sections (inelastic & reaction)
  - ⇒ First generation γ-ray measurements
- E up to and above B_n
  - ⇒ Ericson fluctuations
  - ⇒ Evaporation spectra of particles
  - ⇒ Excitation functions analysis
  - ⇒ Mean compound nucleus lifetimes
  - ⇒ Heavy ions collisions

NUCLEAR DISCRETE EXCITED LEVELS

N(E) = cumulated number of levels up to excitation energy E

$$\rho(E) = \frac{dN(E)}{dE}$$



⇒ Exponential increase

Mass dependency

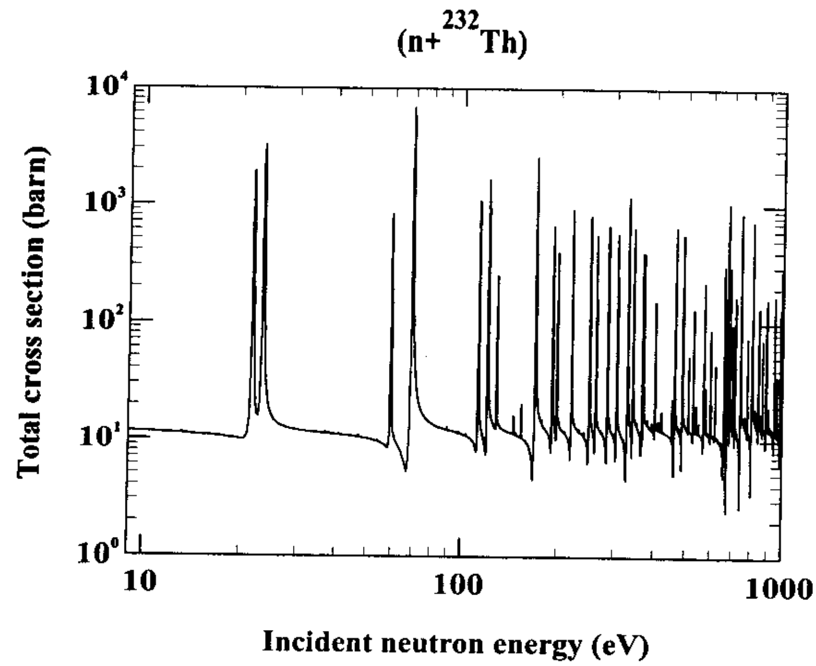
Odd-even effects

MEAN SPACING OF RESONANCES

General case

Nuclear Reaction $x + A \rightarrow C^* \rightarrow$ **decay**

$\Downarrow$



resonances (mean spacing D) $\Rightarrow$ levels of the composite nucleus C

$$\text{with} \begin{cases} E_x+B_x < E_C < E_x+B_x+W_x \quad (W_x = \text{energy spread of projectile beam}) \\ \vec{J_C} = \vec{I_A} + \vec{s_x} + \vec{l_x} \\ \pi_C = \pi_A (-1)^{l_x} \end{cases}$$

$$\frac{1}{D} = \sum_{J_C, \pi_C} \rho(E_x+B_x+W_x/2, J_C, \pi_C)$$

Experimental limitations

- The number of resonances must be high enough
- The width Wx must be weak enough
- The number of missed resonances must be estimated ($\Rightarrow$ experimental error)

MEAN SPACING OF RESONANCES

Special case for incident neutrons

E_n low enough $\Rightarrow$ s-waves only ($l = 0$)
$\Rightarrow$ mean spacing D_0

$$\frac{1}{D_0} = \rho(B_n, 1/2, \pi_A) \quad \text{for even-even target A}$$

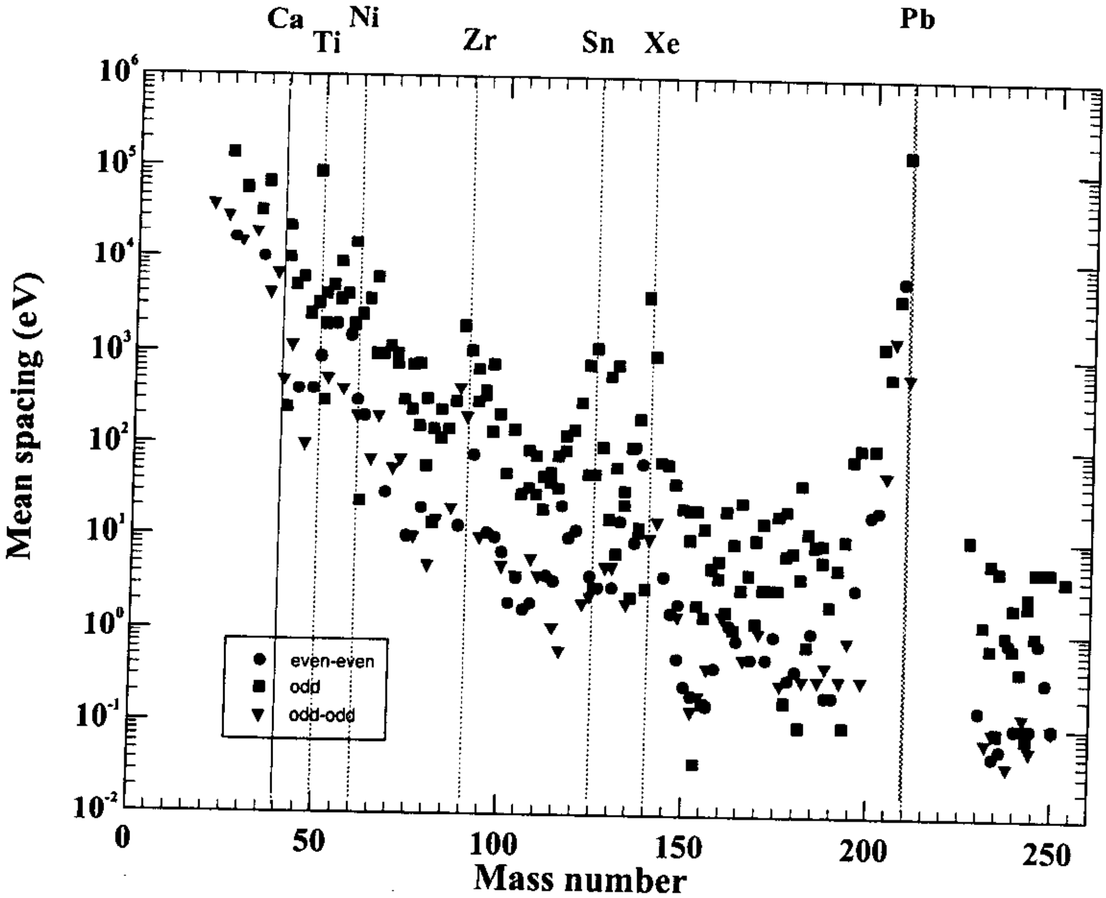
$$= \rho(B_n, I_A+1/2, \pi_A) + \rho(B_n, I_A-1/2, \pi_A) \text{ otherwise}$$

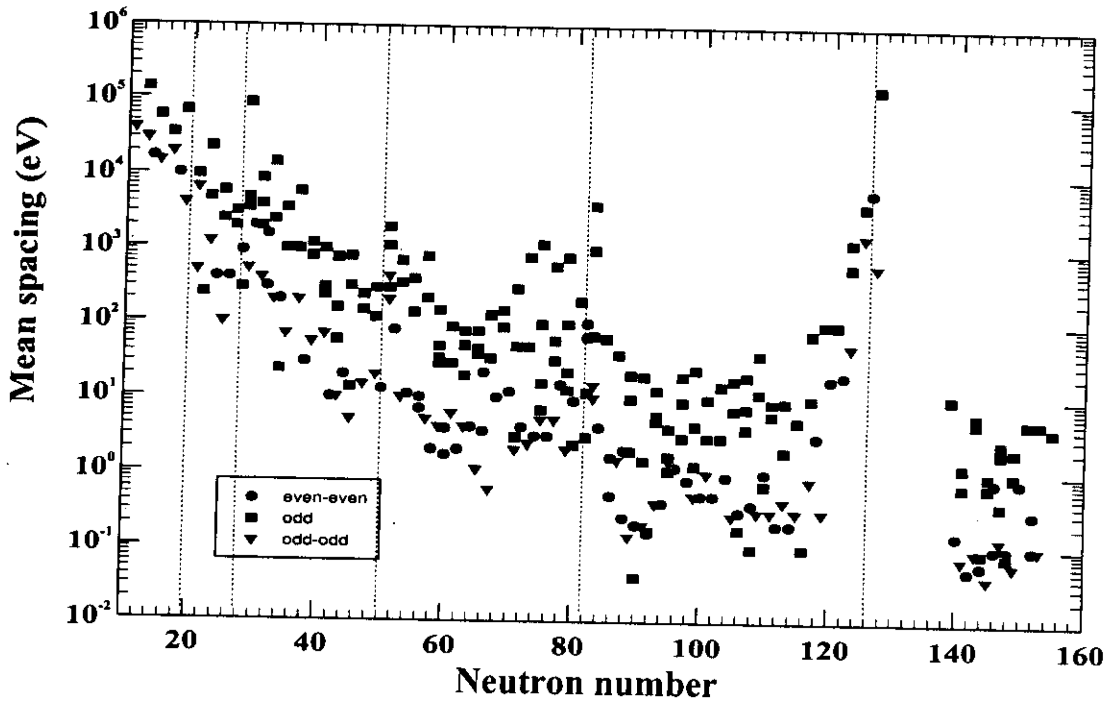
$\Longrightarrow$ **The level density experimental information is only model independent for even-even target and very low incident neutron energies**

$\Longrightarrow$ **Otherwise : assumption on parity or spin distributions necessary**

MEAN SPACING OF NEUTRON s-WAVE (l=0) RESONANCES

Data tabulated by Iljinov et al. NPA 543 (1992) p 517.





⇒ **Mass dependency**

Odd-even effects

Shell effects

THEORETICAL APPROACHES

THEORETICAL APPROACHES

General formulation

- Level density and partition function
- Level density and state densities

Level densities calculation

- Saddle point approximation
- Independent particle model
  - ⇒ Description
  - ⇒ Theoretical implications
  - ⇒ Application within the Fermi gas model
  - ⇒ Angular momentum and parity distributions
  - ⇒ Analysis of mean neutron resonances spacing
  - ⇒ Correction for pairing correlations
- Collective effects
  - ⇒ Vibrations
  - ⇒ Spherical or deformed nuclei
- Sophisticated microscopic approaches

Particle-hole level densities

- General principle
- Equidistant spacing model (ESM) expressions
- Refinements within ESM

GENERAL FORMULATION

Level density and partition function

Level density ρ = **Number of levels per MeV**

$$\rho(A, E) = \sum_{n, j} \delta(A-n)\ \delta(E-E_j(n))$$

$\Longrightarrow$ **Laplace transform of** ρ = **Partition function** $\mathcal{Z}$

$$\mathcal{Z}(\alpha, \beta) = \int_0^{\infty}\int_0^{\infty} \rho(A, E)\ \exp(\alpha A - \beta E)\, dA\ dE$$

$$= \sum_{n, j} \exp(\alpha n - \beta E_j(n))$$

$\Longrightarrow$ **Inverse Laplace transform of** $\mathcal{Z}$

$$\rho(A, E) = \left(\frac{1}{2i\pi}\right)^{-2} \int_{-i\infty}^{+i\infty}\int_{-i\infty}^{+i\infty} \mathcal{Z}(\alpha, \beta)\ \exp(\beta E - \alpha A)\, dA\ dE$$

If one knows the partition function $\mathcal{Z}$, one can theoretically get the level density ρ

Level densities and state densities

Level density $\rho(E, J)$ = **Number of levels per MeV**

State density $\rho(E, M)$ = **Number of states per MeV**

$$\rho(E, M) = \sum_{J \geq M} \rho(E, J) \quad \text{for } M \geq 0$$

$$\rho(E, -M) = \rho(E, M)$$

LEVEL DENSITIES CALCULATION
Saddle point approximation

See : Bethe, Rev. Mod. Phys.2, vol 9 (1937) 69.
Gilbert & Cameron, Can. J. Phys 43 (1965) 1446
Bohr & Mottelson, Nuclear Structure, vol 1. Benjamin NY 1969.

GOAL : calculate the inverse Laplace transform of ρ

$$\rho(A\,,E) = \left(\frac{1}{2i\pi}\right)^{-2} \int_{-i\infty}^{+i\infty}\int_{-i\infty}^{+i\infty} \mathcal{Z}(\alpha\,,\beta)\;\exp\left(\beta E - \alpha A\right) dA\;dE \qquad (1)$$

where $\mathcal{Z}(\alpha\,,\beta) = \sum_{n,\,j} \exp\left(\alpha n - \beta E_j(n)\right)$

METHOD : Steepest descent (saddle point approximation)

The integrand of (1) is expanded around its saddle point $(\alpha_0\,,\beta_0)$ defined by

$$-E = \left.\frac{\partial \ln \mathcal{Z}}{\partial \beta}\right|_{(\alpha_0,\,\beta_0)} \quad \text{and} \quad A = \left.\frac{\partial \ln \mathcal{Z}}{\partial \alpha}\right|_{(\alpha_0,\,\beta_0)}$$

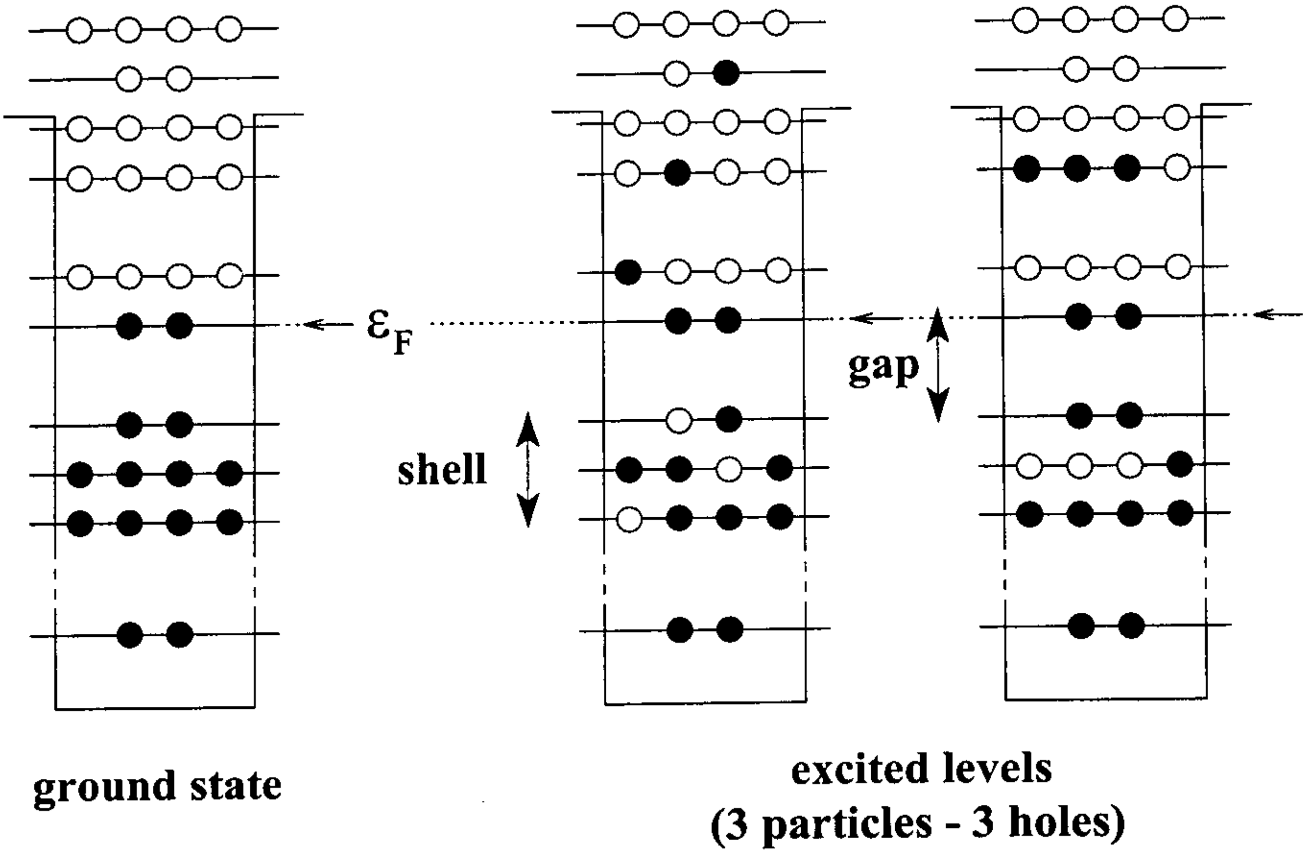
$$\rho(A\,,E) = \frac{\exp\,(S)}{2\pi\sqrt{\det}} \quad \text{with} \quad \begin{cases} S = \ln \mathcal{Z}(\alpha_0\,,\beta_0) + \beta_0 E - \alpha_0 A \\ \det = \begin{vmatrix} \dfrac{\partial^2 \ln \mathcal{Z}}{\partial \beta^2} & \dfrac{\partial^2 \ln \mathcal{Z}}{\partial \alpha\,\partial \beta} \\ \dfrac{\partial^2 \ln \mathcal{Z}}{\partial \alpha\,\partial \beta} & \dfrac{\partial^2 \ln \mathcal{Z}}{\partial \alpha^2} \end{vmatrix}_{\alpha_0,\,\beta_0} \end{cases}$$

This method can be generalized to account for other variables such as N, Z, M ...

LEVEL DENSITIES CALCULATION
Independent particle model

Description

Excited levels = independent excitations of the nucleus fermions



Level density problem = counting problem.

What is the number of ways , N(U) , of distributing the nucleons among the available levels for a fixed excitation energy U ?

$$\Rightarrow \quad \rho(U) = \frac{dN(U)}{dU}$$

LEVEL DENSITIES CALCULATION
Independent particle model

Theoretical implications

- Laplace transform of ρ = Partition function $\mathcal{Z}$

$$\mathcal{Z}(\alpha, \beta) = \sum_{n, j} \exp\left(\alpha n - \beta E_j(n)\right)$$

- IPM $\Rightarrow$ $E_j(n) = \sum_i n_i(j)\, \varepsilon_i(n)$ and $n = \sum_i n_i(j)$

$n_i(j)$ = occupation number of the s.p.s i in the independent particle state j of the nucleus with n fermions.

$$\Rightarrow \ln \mathcal{Z}(\alpha, \beta) = \ln \left[\sum_j \exp\left(-\beta \sum_i n_i(j)\, \varepsilon_i(n) + \alpha \sum_i n_i(j) \right) \right]$$

- Since $n_i(j) = 0$ or 1 for fermions

$$\mathcal{Z}(\alpha, \beta) = \prod_i \left(1 + e^{\alpha - \beta\, \varepsilon_i(n)} \right)$$

$\Downarrow$

$\varepsilon_i(n)$ values from model hamiltonian

$\Downarrow$

numerical or analytical calculation

LEVEL DENSITIES CALCULATION
Independent particle model

Application within the Fermi gas model (one type of particle)

See : Lynn, The theory of neutron resonance reactions, Clarendon Press Oxford, 1968, p 125.

- **FGM $\Rightarrow$ s.p.s continuously distributed with density** $g = \dfrac{3A}{2\,\varepsilon_0}$

  ε_0 = **Fermi energy**

  $$\varepsilon_0 = \left(\frac{9\pi}{8}\right)^{2/3} \frac{\hbar^2}{2\,m\,r_0^2} \qquad (\text{ m \& } r_0 = \text{nucleon mass \& radius})$$

- **Saddle point approximation**

  $$\Rightarrow \rho\,(U) = \frac{\exp\left(2\sqrt{aU}\right)}{4\sqrt{3}\,U} \qquad \text{and} \qquad a = \frac{\pi^2}{6}\,g$$

- **Protons and neutrons distinguished**

$$\rho\,(U) = \frac{\sqrt{\pi}}{12}\;\frac{\exp\left(2\sqrt{\;U}\right)}{{}^{1/4}\,U^{5/4}} \quad \text{with} \quad = \frac{\pi^2}{6}\,(g_\pi + g_\nu) = \frac{\pi^2}{4}\,\frac{A}{\varepsilon_0}$$

Using standard values for $\hbar$, m and r_0 yields $a \approx \dfrac{A}{13.5}$

$\Rightarrow$ **to be compared with experimental data analysis**

LEVEL DENSITIES CALCULATION
Independent particle model

Angular momentum and parity distributions

See : Lynn, The theory of neutron resonance reactions, Clarendon Press Oxford, 1968, p 142.

- **Angular momentum**

$$\text{IPM} \Rightarrow \rho(U,M) = \rho(U)\,\frac{1}{\sqrt{2\pi}\,\sigma_M}\exp\left[-\frac{M^2}{2\,\sigma_M^2}\right]$$

$$\text{with } \sigma_M^2 = K\sqrt{\frac{U}{a}}$$

This law can be shown using :

Saddle point approximation

See : Bohr & Mottelson, Nuclear Structure, vol 1, Benjamin NY (1965)

Central limit theorem

See : Ericson, Adv. Phys. 9 (1960) 425.

Physical argument on rotational energies

See : Lang & Le couteur, Proc. Phys. Soc. A67 (1953) 586.

$\Rightarrow$ demonstration

Spin dependency

Usually $\rho(U,J) = \rho(U,M=J) - \rho(U,M=J+1)$

Not valid for deformed nuclei (see later)

- **Parity**

Equipartition is assumed by default

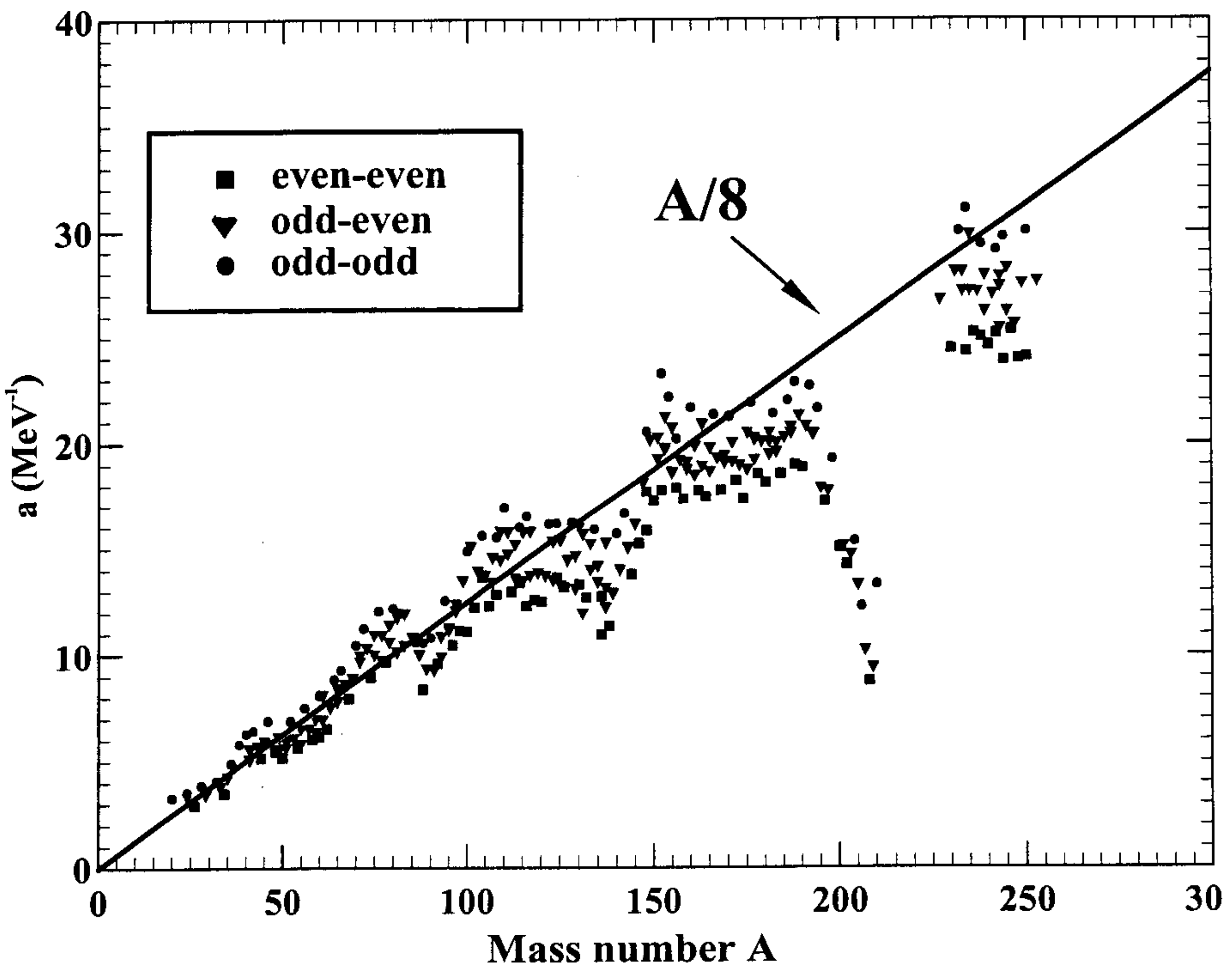
LEVEL DENSITIES CALCULATION
Independent particle model

Analysis of mean neutron resonances spacing

Given $\rho(U,J,\pi)$ expression one adjusts $a(B_n)$ to reproduce experimental D_0 values.

$$\rho(U,J,\pi) = \frac{1}{2}\frac{\sqrt{\pi}}{12}\frac{\exp\left(2\sqrt{aU}\right)}{a^{1/4}U^{5/4}}\frac{2J+1}{2\sqrt{2\pi}\,\sigma_M^3}\exp\left[-\frac{(J+1/2)^2}{2\sigma_M^2}\right]$$

with $\sigma_M^2 = I_{rig}\sqrt{\frac{U}{a}}$ ($I_{rig} = 9.65\ 10^{-3}\ r_0^2\ A^{5/3}$ in $\hbar^2\ MeV^{-1}$ units)



Odd-even effects $\Rightarrow$ dispersion

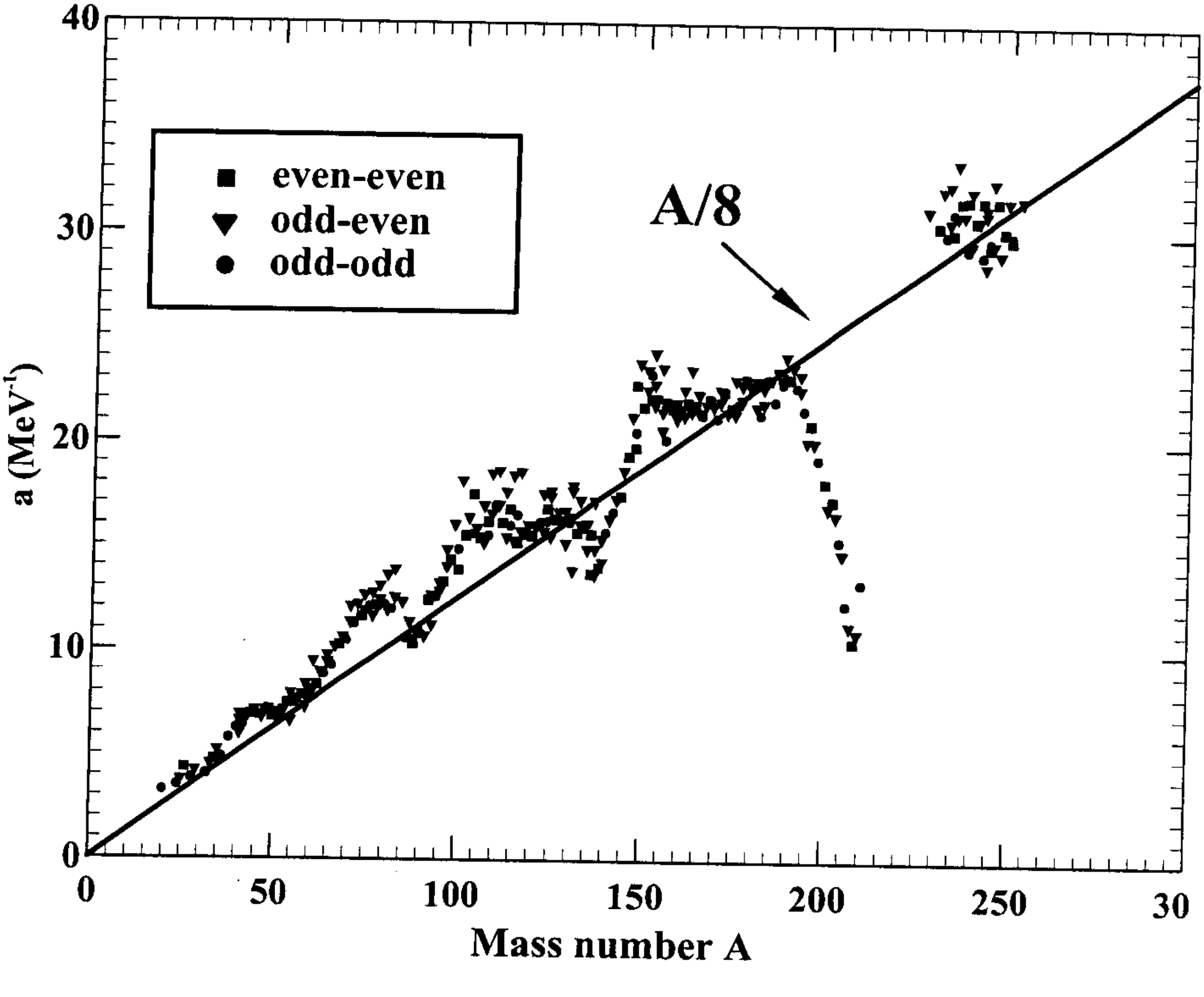
LEVEL DENSITIES CALCULATION
Independent particle model

Correction for pairing correlations

One replaces U by $U^{*}=U-\chi\,\Delta_0$ with $\Delta_0 = \frac{12}{\sqrt{A}}$

and $\chi = 0$, 1 or 2 for odd-odd, odd-even or even-even nuclei

(Δ_0 = average energy necessary to break a pair of nucleons)



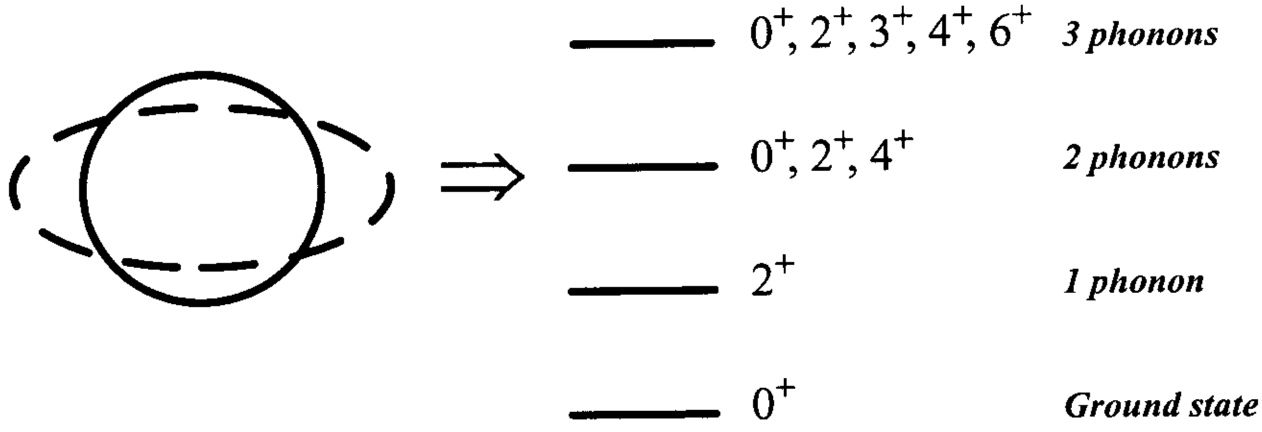
pairing correction $\Rightarrow$ reduction of the dispersion

$\Rightarrow$ still disagreement with A/13.5 of FGM

LEVEL DENSITIES CALCULATION
Collective effects

Harmonic Vibrations

Illustration



Partition function for phonons

Same derivation as for fermions (IPM model) but occupation numbers not restricted (0, 1, 2, ...).

$$\mathcal{Z}(\alpha, \beta) = \prod_{\text{phonons } i} \left(1 + e^{\alpha - \beta \varepsilon_i} + e^{2\alpha - 2\beta \varepsilon_i} + \dots \right)$$

Saddle point approximation

See : Lynn, The theory of neutron resonance reactions, Clarendon Press Oxford, 1968, p 151.

Vibrations ⇒ increase in the IPM level density

$$\rho_{\text{IPM+vib}} = \underbrace{\exp(\alpha A^{2/3} T^{4/3})}_{K_{\text{vib}}(U)} \rho_{\text{IPM}} \quad \text{with} \quad T = \sqrt{\frac{U}{a}}$$

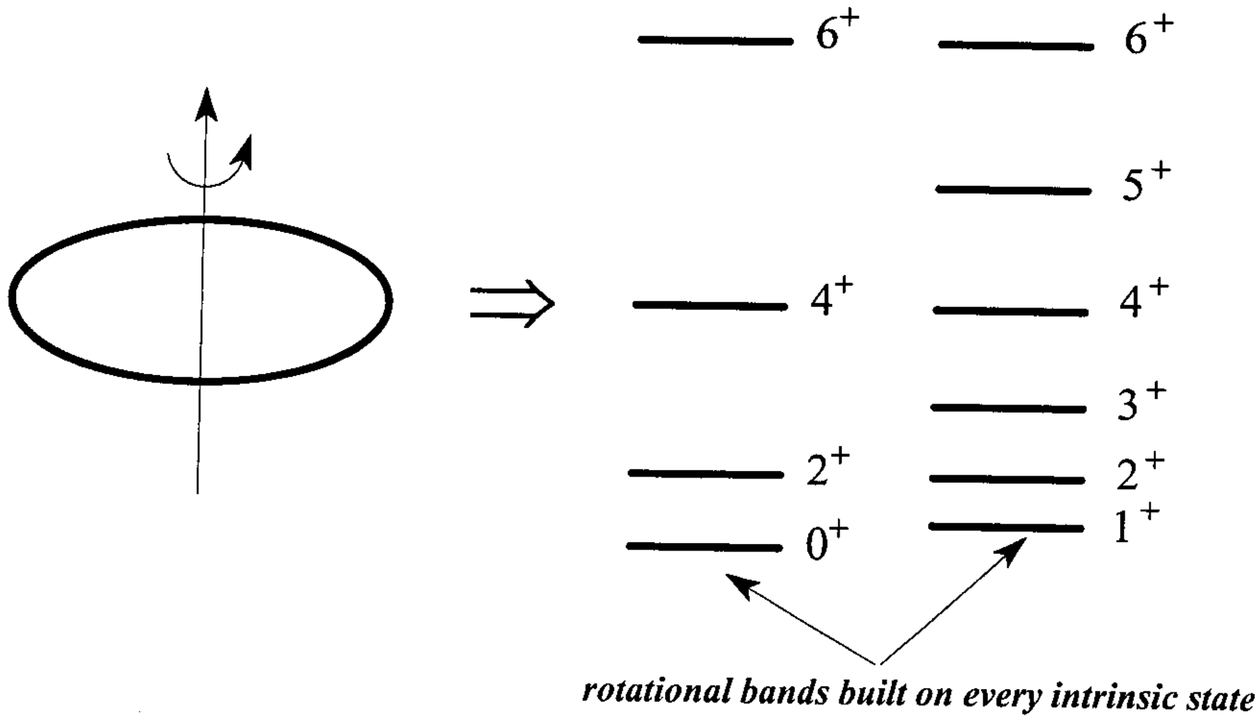
LEVEL DENSITIES CALCULATION
Collective effects

Spherical or deformed nuclei

Spherical $\Rightarrow$ $\rho(U, J, \pi) = \rho(U, M=J, \pi) - \rho(U, M=J+1, \pi)$

Deformed nuclei

See Bjornholm et al.,Physics and Chemistry of fission, IAEA, 1974 p 367.



The number of levels with a given spin J at excitation energy U is obtained by summing all the intrinsic states on which a rotational band can be constructed giving a level with spin J and an excitation energy U.

Deformation implies an enhancement such that

$$\rho_{def}(U) = K_{rot}(U)\, \rho_{sph}(U)$$

$K_{rot}(U)$ = **function of the type of deformation**

LEVEL DENSITIES CALCULATION
Sophisticated microscopic approaches

- **Combinatorial microscopic approach**

  See Berger et Martinot., Nuc. Phys. A226 (1974) 391.

  Direct couting approach without Saddle point approximation

  ⇒ Approximate treatment of pairing correlations

  ⇒ Access to every possible information (see later)

- **Superfluid Model approach**

  See Decowski et al., Nuc. Phys. A110 (1968) 129.

  Saddle point approximation with BCS-like equations

  ⇒ More rigourous treatment of pairing correlations

  ⇒ Two energetic regions : - Low energy : key role of pairing correlations
  - High energy : IPM behaviour (with $U^*=U-\Delta$)

- **SPA, SPA+RPA and Shell Model Monte Carlo methods**

  See Agrawal et al., Phys. Rev. C, vol 59,6 (1999) 3109 and references therein.

  Path integral approach with Saddle point approximation

  ⇒ More complex Hamiltonians than IPM & SM

  ⇒ Require time consuming computer calculations

- **Spectral distribution methods**

  See Chang et al., Ann. Phys. 66, (1971) 137.

  Approach based on random matrix theory

  ⇒ Pairing accounted for

  ⇒ Fast

- **Semiclassical methods (Thomas-Fermi approximation)**

  See : J.N.De et al., Phys. Rev. C, vol 57,3 (1998) 1398 and references therein.
  A.H.Blin et al., Nucl. Phys. A456 (1986) 109.

  Hamiltonian replaced by its classical counterpart and discrete partition function by an integral over phase space coordinates

  ⇒ Used to calculate level density parameter with model Hamiltonians

  ⇒ Provides analytical level density parameter expressions

PARTICLE-HOLE LEVEL DENSITIES
General principle

The problem

Total level densities = number of levels per MeV

Particle-hole level densities = number of levels per MeV that are based on excitations involving fixed numbers of p-h

The methods

Combinatorial approach with general single particle level schemes

BCS model
See Ignatyuk and Sokolov, Sov. J. Nucl. Phys. 17 (1973) 376.

Shell Model Monte Carlo approach
See Dean and Koonin, Phys. Rev. C, vol 60 (1999) 054306-1

Analytical solutions in the Equidistant Spacing Model

PARTICLE-HOLE LEVEL DENSITIES

Equidistant Spacing Model expressions

See Hilaire et al., Nucl. Phys. A632 (1998) 417 and references therein

1960 : Ericson

No Pauli exclusion principle

⇒ global overestimation

1971 : Williams

Pauli principle

⇒ overestimation above some tens of MeV because finite potential depth neglected

1976 : Běták and Dobeš

Finite depth accounted for

⇒ no more unphysical hole number but still overestimation because of mathematical approximations

1986 : Obložinský

Both restricted number of holes and particles

⇒ same mathematical problems as for Betak and Dobes

1995 : Anzaldo-Meneses

Correction terms to Williams expression

⇒ Improved mathematical method

1998 : Hilaire, Delaroche and Koning

Generalised expression

⇒ equivalent to exact calculations up to hundreds of MeV

PARTICLE-HOLE LEVEL DENSITIES

Refinements within ESM

1984 : C.Y. Fu (*Nuc. Sci. Eng 86 (1984) 344.*)

"Advanced" Pairing correction

⇒ Implementation in the usual Williams Formula of the Superfluid Model results in an approximate but easily tractable form.

1985 : Akkermans and Gruppelaar (*Z. Phys. A 321 (1985) 605.*)

Renormalisation function

⇒ Ensure consistency of p-h densitites with total densities

1985 : C.Y. Fu (*Nuc. Sci. Eng. 92 (1985) 440.*)

Advanced Spin cut-off Factor

⇒ Same principle as in 1984 but for p-h spin cut-off factor

1995 : C. Kalbach (*J. Phys. G. 21 (1995) 1499.*)

Shell Shifted Equidistant Spacing Model

⇒ Inclusion and treatment of a gap in single particle level schemes

1998 : Harangozo et al. (*Phys. Rev. C, vol 58,1 (1998) 295.*)

Energy dependent single particle states

⇒ Modification of single particle states density (g0 ⇒g(ε))
Single hole states density kept constant

SEMI-EMPIRICAL MODELS

SEMI-EMPIRICAL MODELS

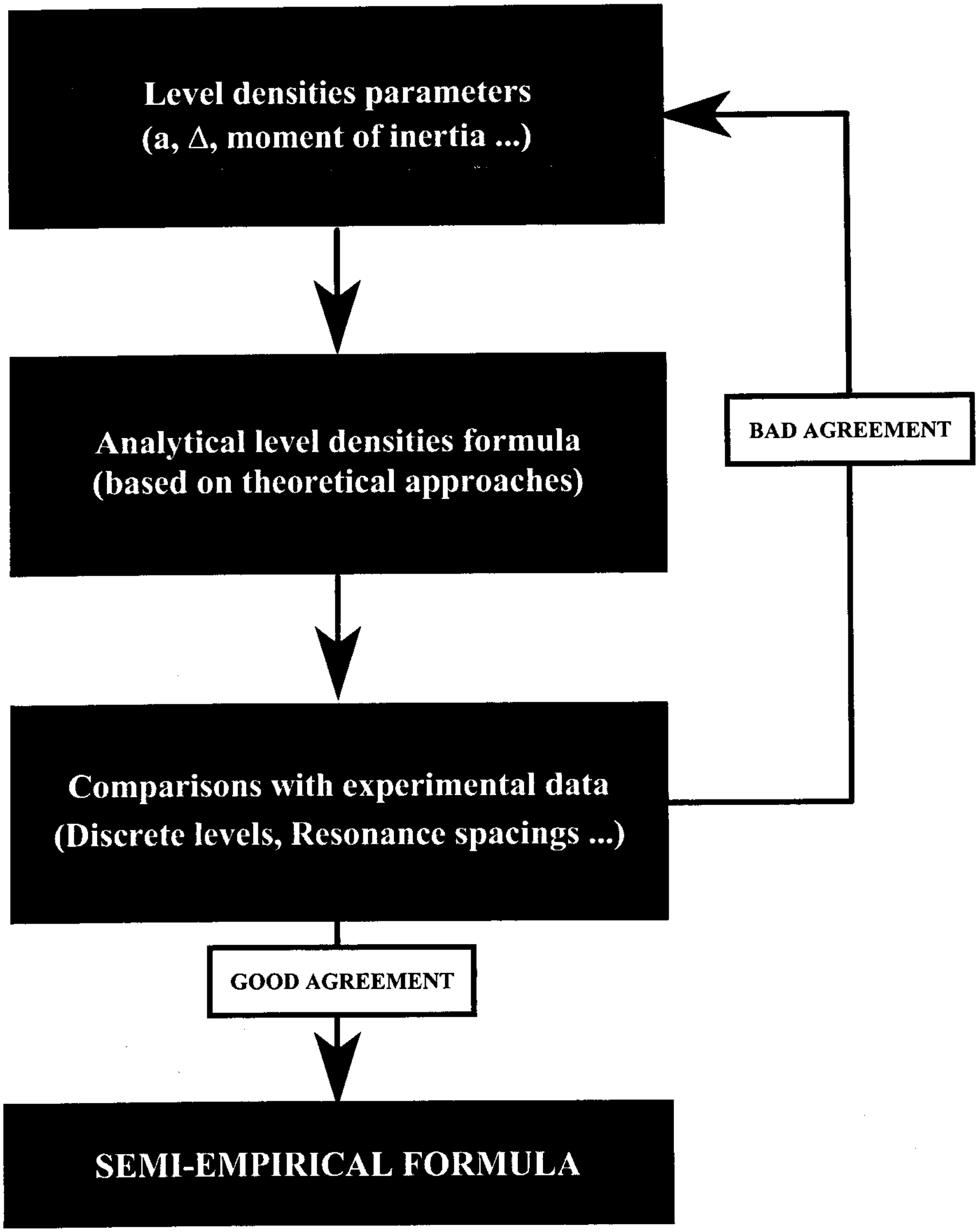
Semi-empirical approaches

- General principle
- Simplest applications
  - ⇒ Back-Shifted Fermi Gas model
  - ⇒ Gibert & Cameron model
  - ⇒ Shell effects on level density parameter
- More sophisticated expressions
  - ⇒ Baba expression
  - ⇒ Kataria-Ramamurthy-Kapoor formulation
  - ⇒ Ignatyuk formula
- Generalised Superfluid Model (GSM)

GSM and Combinatorial results analysis

- Why HFB+D1S single particle levels ?
- Results from the combinatorial method
- Spin and parity distributions
  - ⇒ Spin distribution
  - ⇒ Parity distribution
- Level density parameter (LDP) and Ignatyuk formula
  - ⇒ Intrinsic or effective level density parameter ?
  - ⇒ Results for Intrinsic LDP
- Collective effects
  - ⇒ Vibrational enhancement
  - ⇒ Rotational enhancement
- Towards a global level density expression ?

SEMI-EMPIRICAL APPROACHES
General principle



SEMI-EMPIRICAL APPROACHES
Simplest applications (1/2)

Back-Shifted Fermi Gas Model
See Dilg et al., Nucl. Phys. A217 (1973) 269.

$$\rho\,(U, J, \pi) = \frac{1}{2}\,\frac{\sqrt{\pi}}{12}\,\frac{\exp\left[2\sqrt{\ \ (U-\ )}\right]}{\ ^{1/4}\,(U-\ )^{5/4}}\,\frac{2J+1}{2\sqrt{2\pi}\,\sigma_M^3}\,\exp\left[-\frac{(J+1/2)^2}{2\sigma_M^2}\right]$$

$$\text{with } \sigma_M^2 = I_{rig}\sqrt{\frac{(U-\ )}{\ }}$$

a and Δ adjusted to reproduce both discrete levels and neutron resonances spacings

⇒ sets of tabulated values depending on the adopted spin cut-off parameter expression (α)

Problems

Nuclear discrete levels not very well described (pairing effects !)

Constant level density parameter

What's going on for nuclei without data ?

Gilbert and Cameron Model

See Gilbert et Cameron, Can. J. Phys, vol 43 (1965) 1447.

Cook et al., Aust. J. Phys 20 (1967) 477..

Mixing of a purely empirical expression for $U < U_{match}$ and a BSFG expression above U_{match}

Below U_{match} :

$$\rho(E) = \frac{1}{\ } \exp\left(\frac{E-}{\ }\right)$$

a , Δ, E_0 and T adjusted to reproduce
- **discrete levels**
- **D_0 values**

+ continuity for $U=U_{match}$ of the
- **two level densities**
- **two level density derivatives**

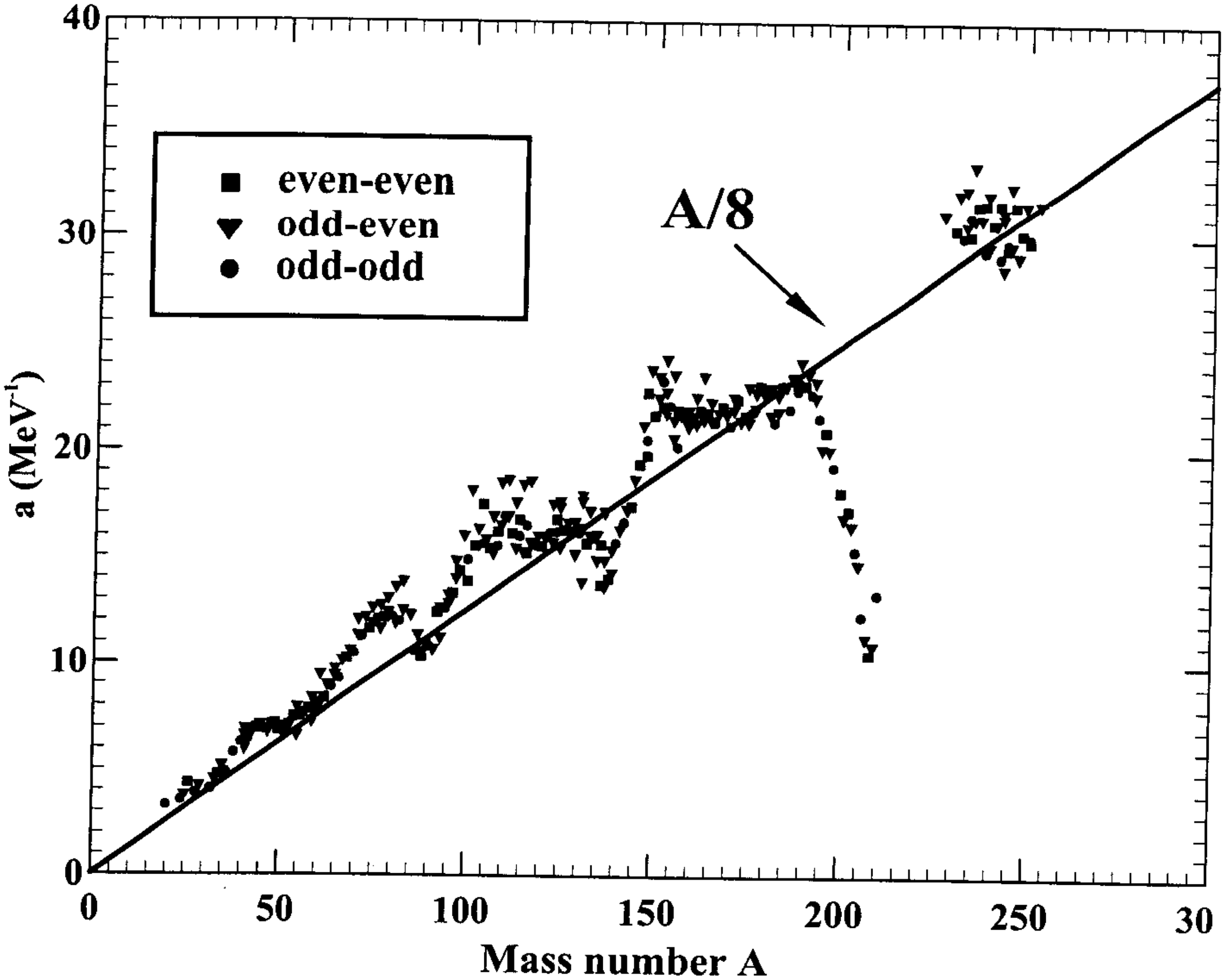
Problems

- **Constant level density parameter**
- **4 parameter expression**
- **Continuity not always possible**

But good discrete levels description

SEMI-EMPIRICAL APPROACHES
Simplest applications

Shell effects on level density parameter



⇒ decrease of level density parameter near shell closure

⇒ desagreement with Fermi Gas predicted LDP value (A/8 instead of A/13)

SEMI-EMPIRICAL APPROACHES
More sophisticated expressions

Baba expression
See Baba, Nucl. Phys. A159 (1970) 625.

Shell effects ⇒ stepwise function for the single particle state density

⇒ Fermi gas level density formula with level density parameter depending on excitation energy

Problem : choice of the stepwise function

Kataria-Ramamurthy-Kapoor formulation
See Kataria et al., Phys. Rev. C (1978) 549.

Shell effects ⇒ Fourier series expansion for the single particle state density

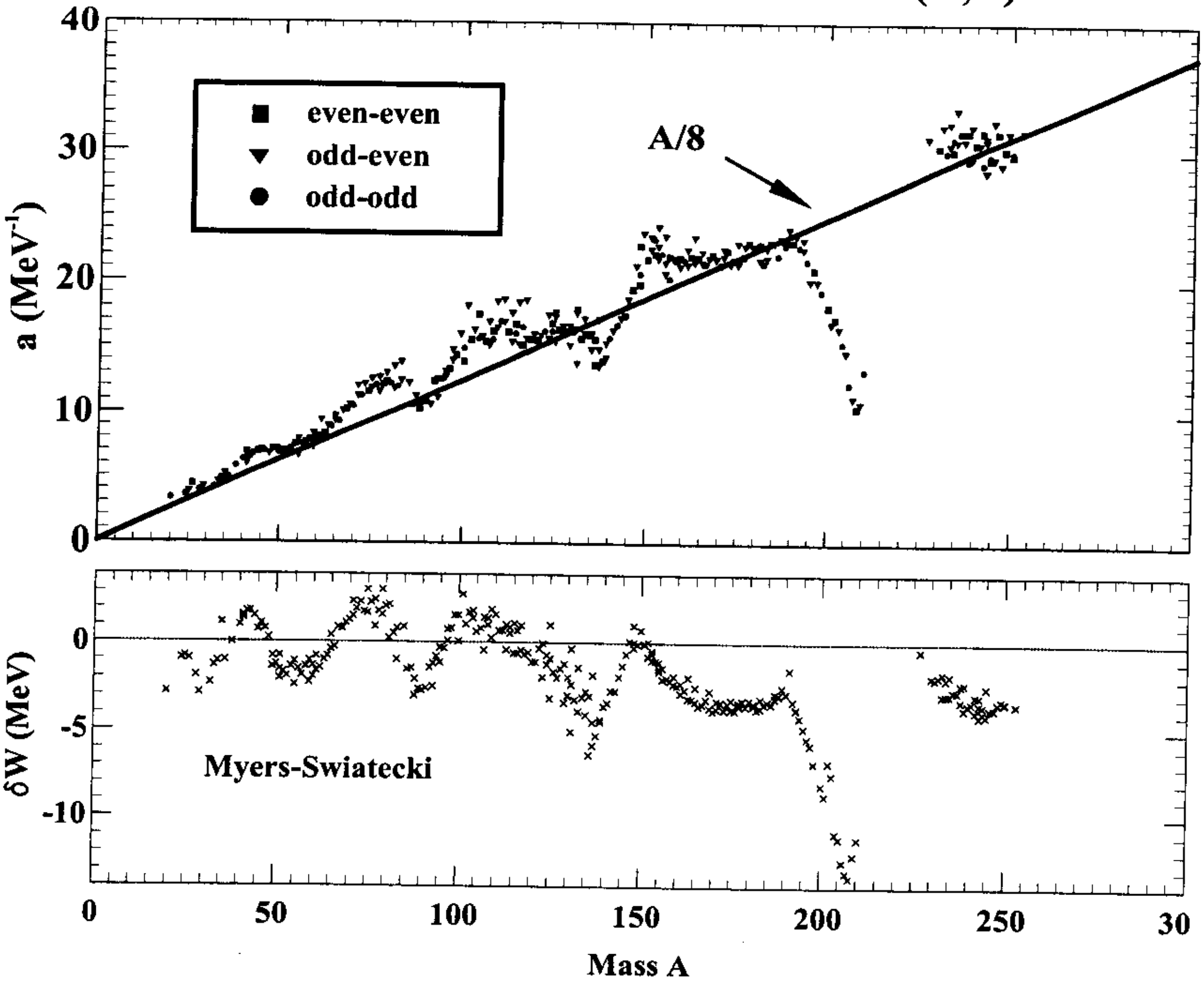
⇒ Saddle point expression with Entropie and Excitation energy function of Temperature T and oscillation frequency ω

Problem : choice of ω

SEMI-EMPIRICAL APPROACHES
More sophisticated expressions

Ignatyuk formula (*See Ignatyuk et al., Sov. J. Phys 21 (1975) 255.*)

Correlation between a and $\delta W(N,Z)$



⇒Fermi gas formula with LDP depending on excitation energy

$$a(N,Z,U) = \tilde{a}(A)\left[1 + \delta W(N,Z)\,\frac{1 - \exp(-\gamma U)}{U}\right]$$

with $\tilde{a}(A) = \alpha A + \beta A^2$ and $\delta W(N,Z) = M_{exp} - M_{ld}$

Problems : Choice of α, β, γ and $\delta W(N,Z)$ (i.e. liquid drop model)

SEMI-EMPIRICAL APPROACHES
Generalised Superfluid Model

Main characteristics (*See Svirin ans Smirenkin, Sov. J. Phys. 47 (1988) 54.*)

2 energy regions :

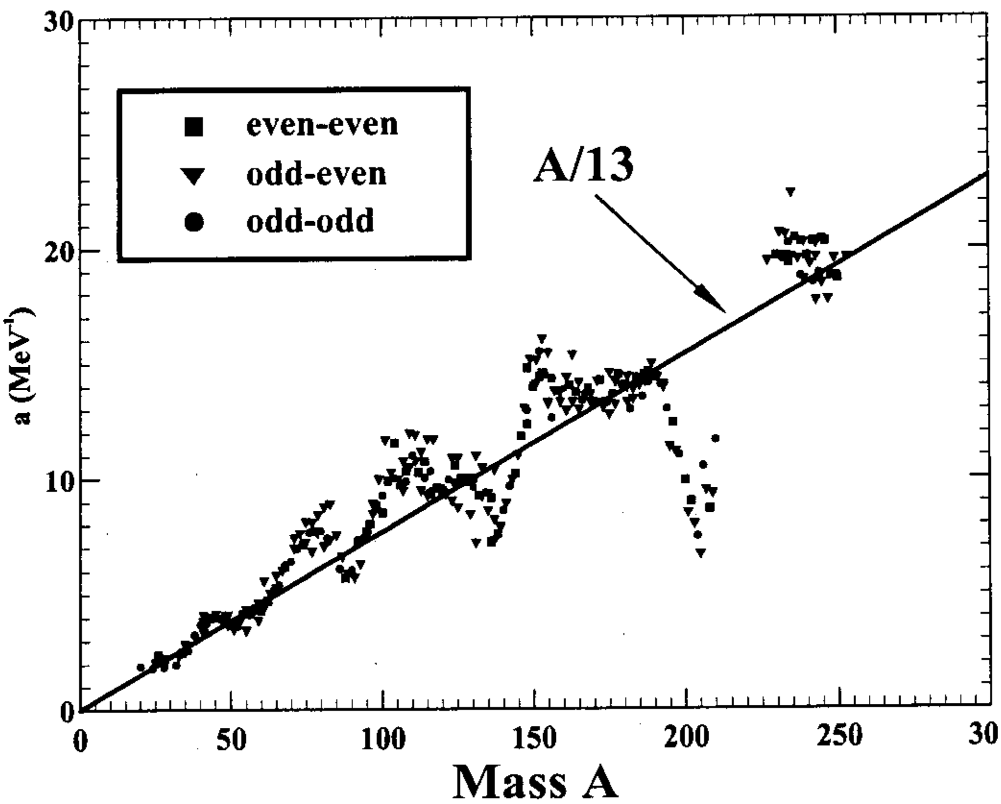
below U_c : Saddle Point Approximation with entropy, determinant and excitation energy = functions of temperature.

above U_c : Fermi Gas + Ignatyuk formula for a

In both regions explicit treatment of collective effects

$$\Rightarrow \rho(U, J, \pi) = K_{rot}(U)\, K_{vib}(U)\, \rho_{ph}(U, J, \pi)$$

Results



$\Rightarrow$ **A/8 behavior (instead of A/13) explanation**

GSM & COMBINATORIAL RESULTS ANALYSIS
Why HFB+D1S single particle states ?

HFB+D1S Method

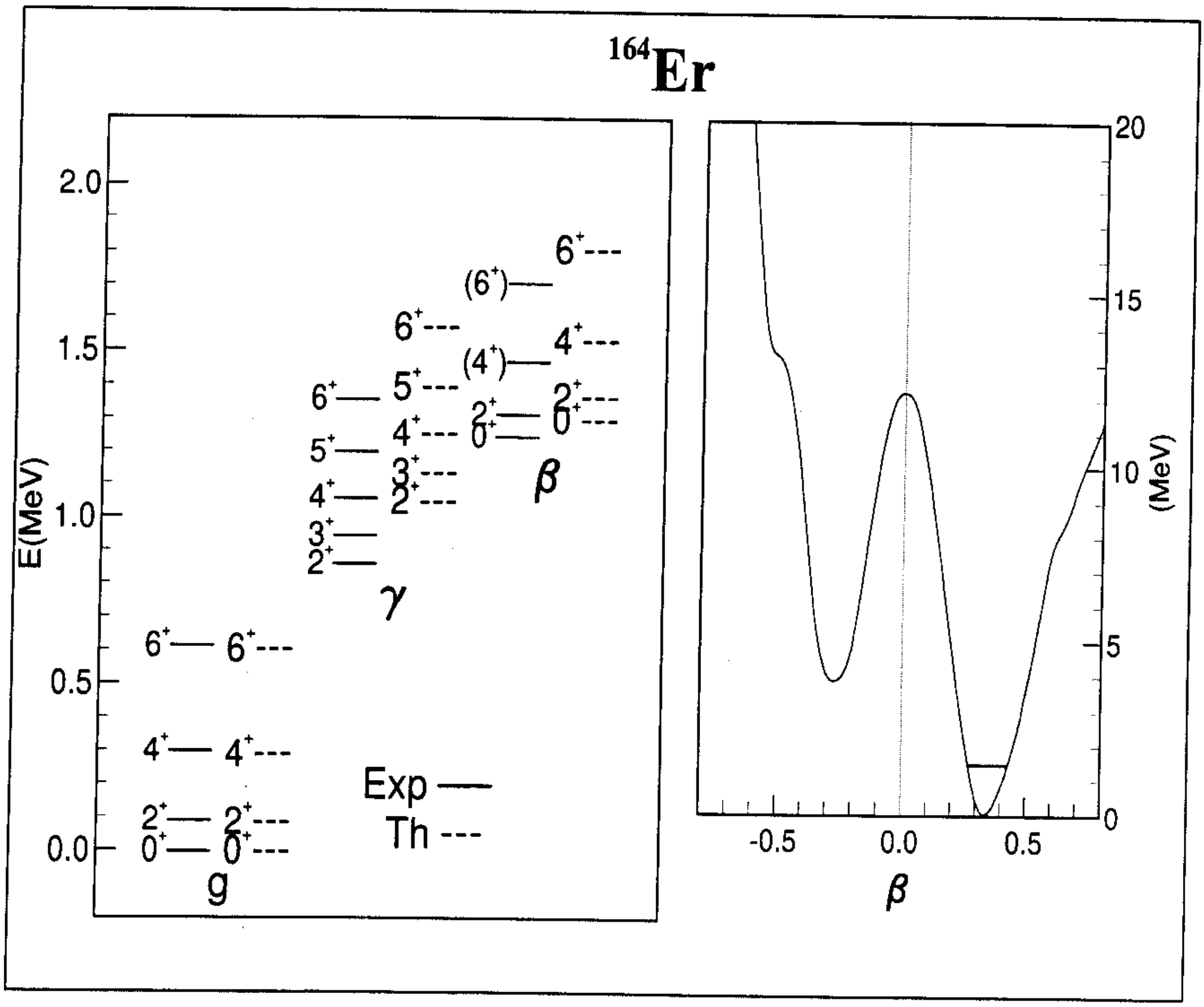
Microscopic self-consistent approach based on D1S effective nucleon-nucleon interaction (no adjustable parameters)

⇒ single particle levels (energy, spin, parity)

⇒ deformation and moment of inertia

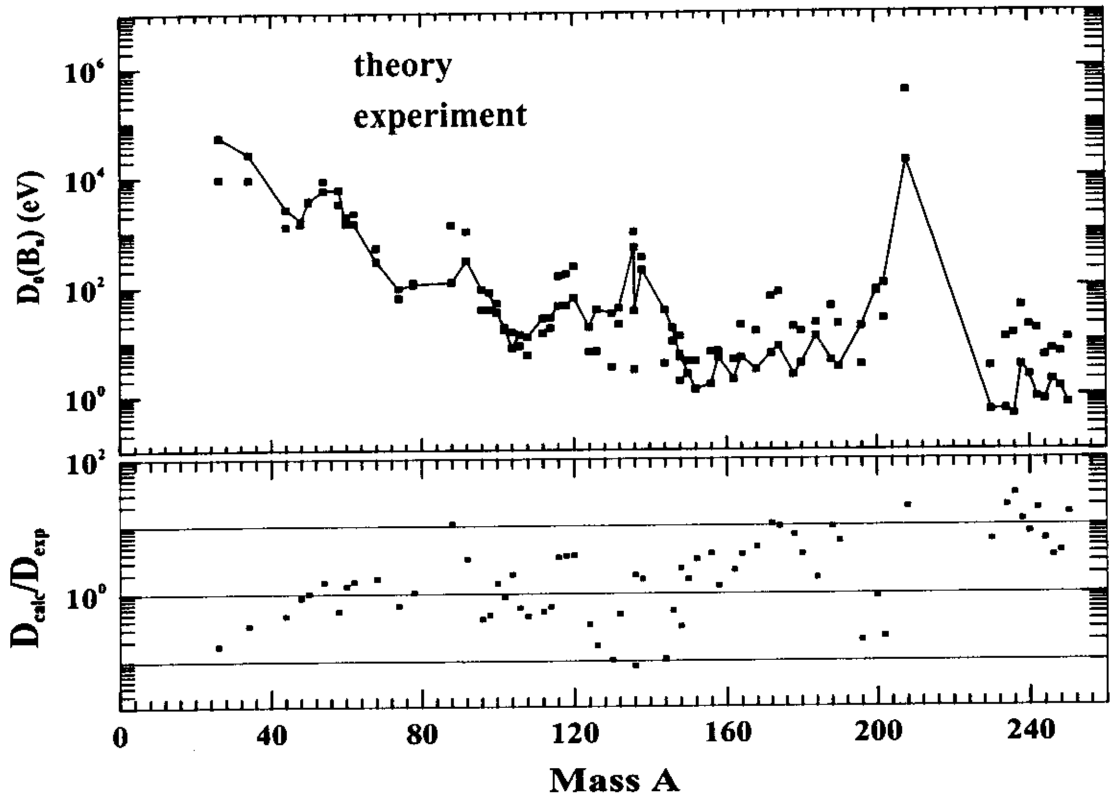
⇒ pairing correlations

HFB+D1S typical result

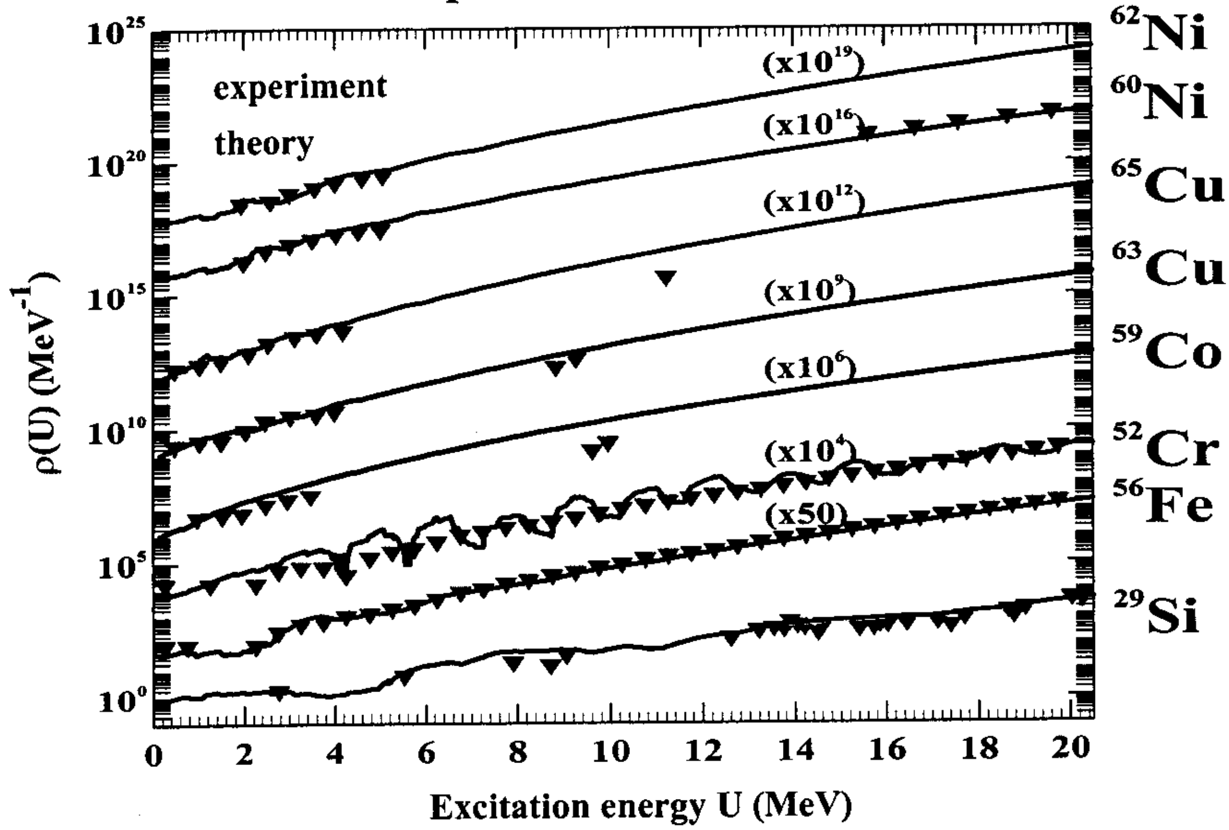


GSM & COMBINATORIAL RESULTS ANALYSIS
Why HFB+D1S single particle states ?

HFB+D1S s-wave resonances mean spacing



HFB+D1S level densities up to 20 MeV



GSM & COMBINATORIAL RESULTS ANALYSIS
Results from the combinatorial method

Each component of the Generalised Superfluid Model can be calculated

Level density parameter

With a fit of the combinatorial results using a BSFG expression where a is the adjustable parameter

Spin cut-off factor

With a fit of the distribution of $\rho(U,M)$ as a function of M using a Gaussian law

Parity distribution

Using the asymetry $[\rho(U,+)-\rho(U,-)] / \rho(U)$ as a function of U

Energy Shell correction δW

Using the Strutinsky method with HFB+D1S single particle levels

Damping factor

With a fit of the level density parameter with Ignatyuk formula given the previously determined δW

Rotational enhancement

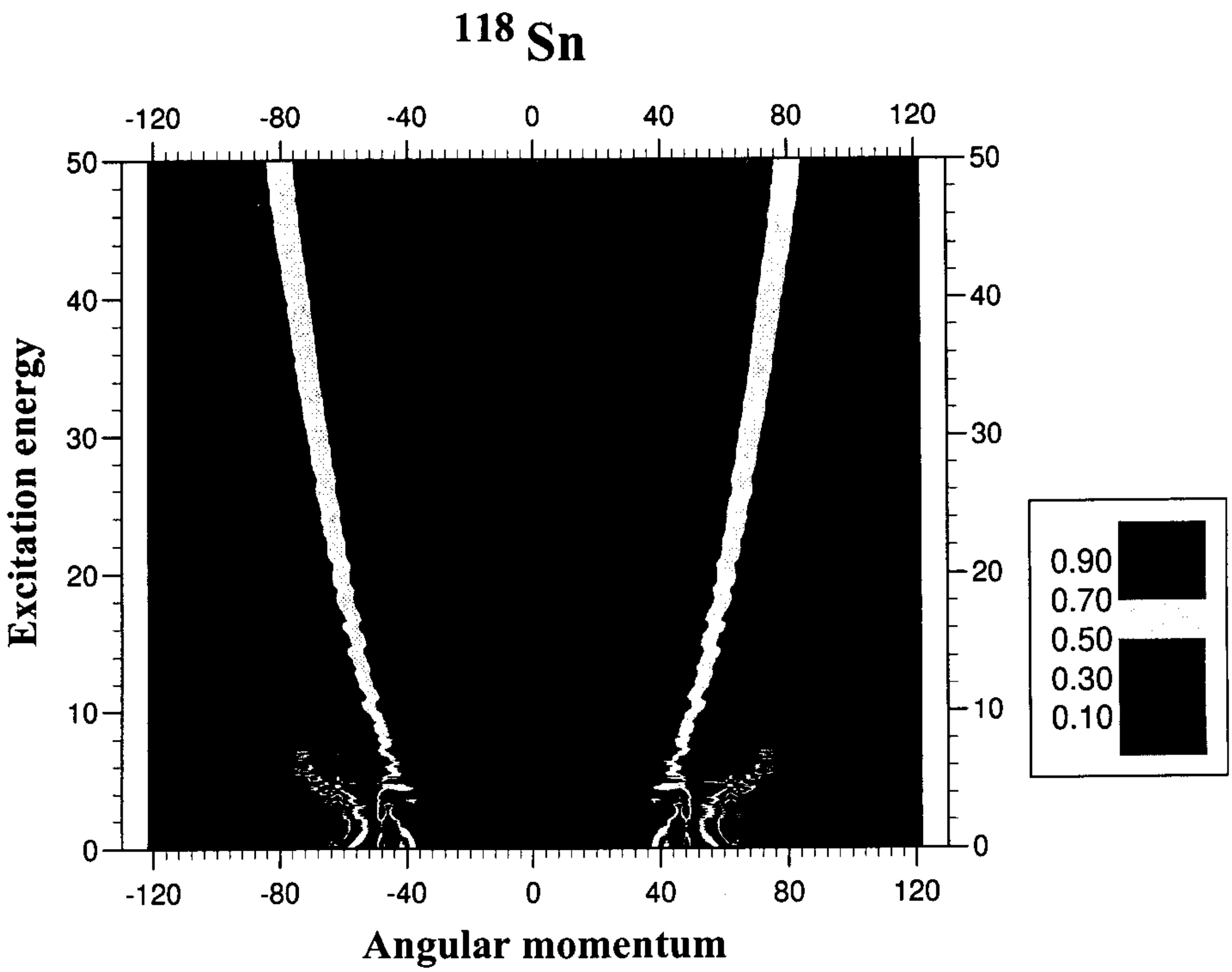
Ratio of the combinatorial level density with the constructed rotational bands and without

Vibrational enhancement

Ratio of the combinatorial level density with and without vibrational states included

Spin distribution (1/3)

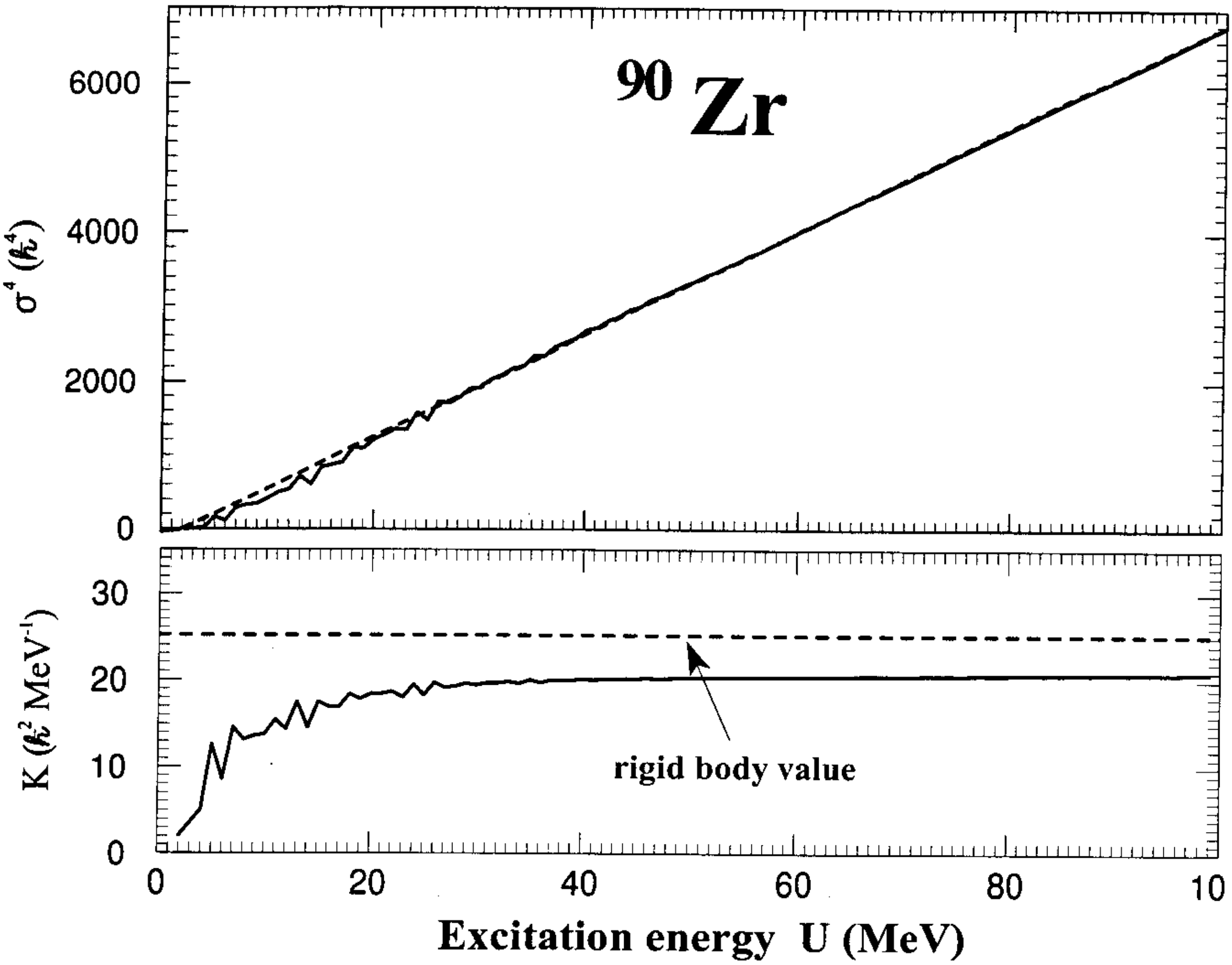
$$\text{Absolute error : } \left| \frac{\text{Gaussian - Combinatorial}}{\text{Combinatorial}} \right|$$



⇒ Large error for high angular momenta

Spin distribution (2/3)

Theoretically $\sigma^2 = K \sqrt{\frac{U-\Delta}{a}}$



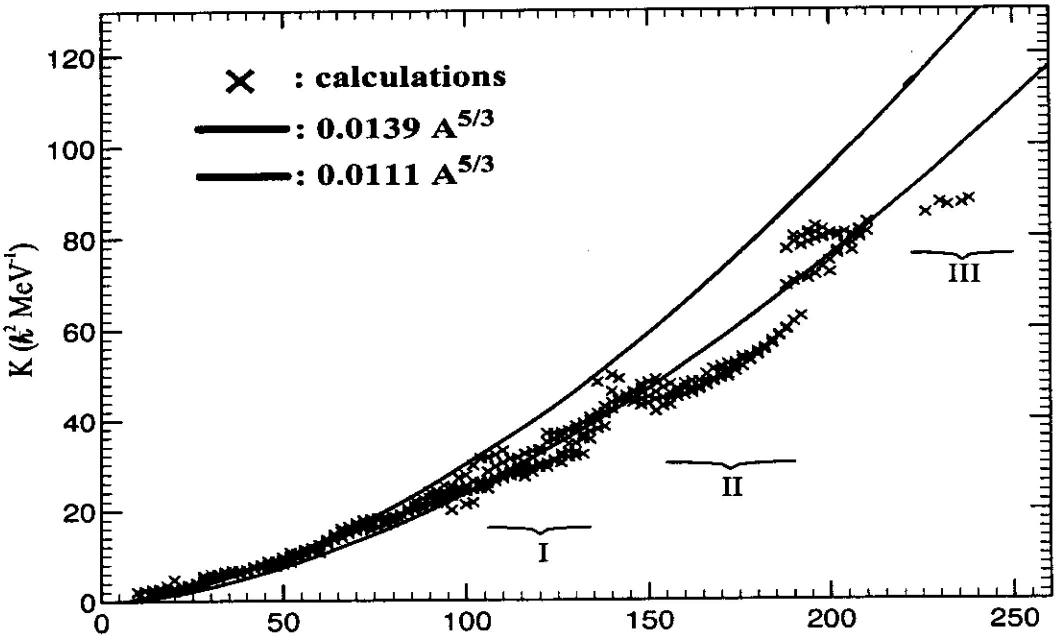
⇒ Overall linear behavior

Coefficient K different from rigid body value function of excitation energy

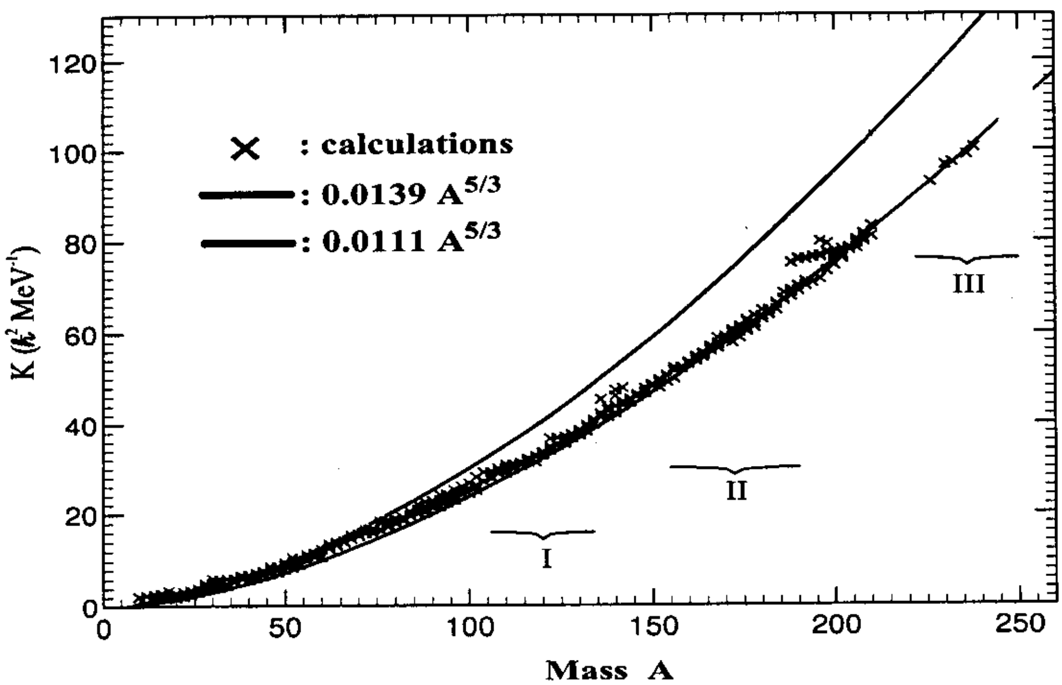
Spin distribution (3/3)

Calculation of $K=\sigma^2 \Big/ \sqrt{\frac{U-\Delta}{a}}$ for $U \rightarrow +\infty$

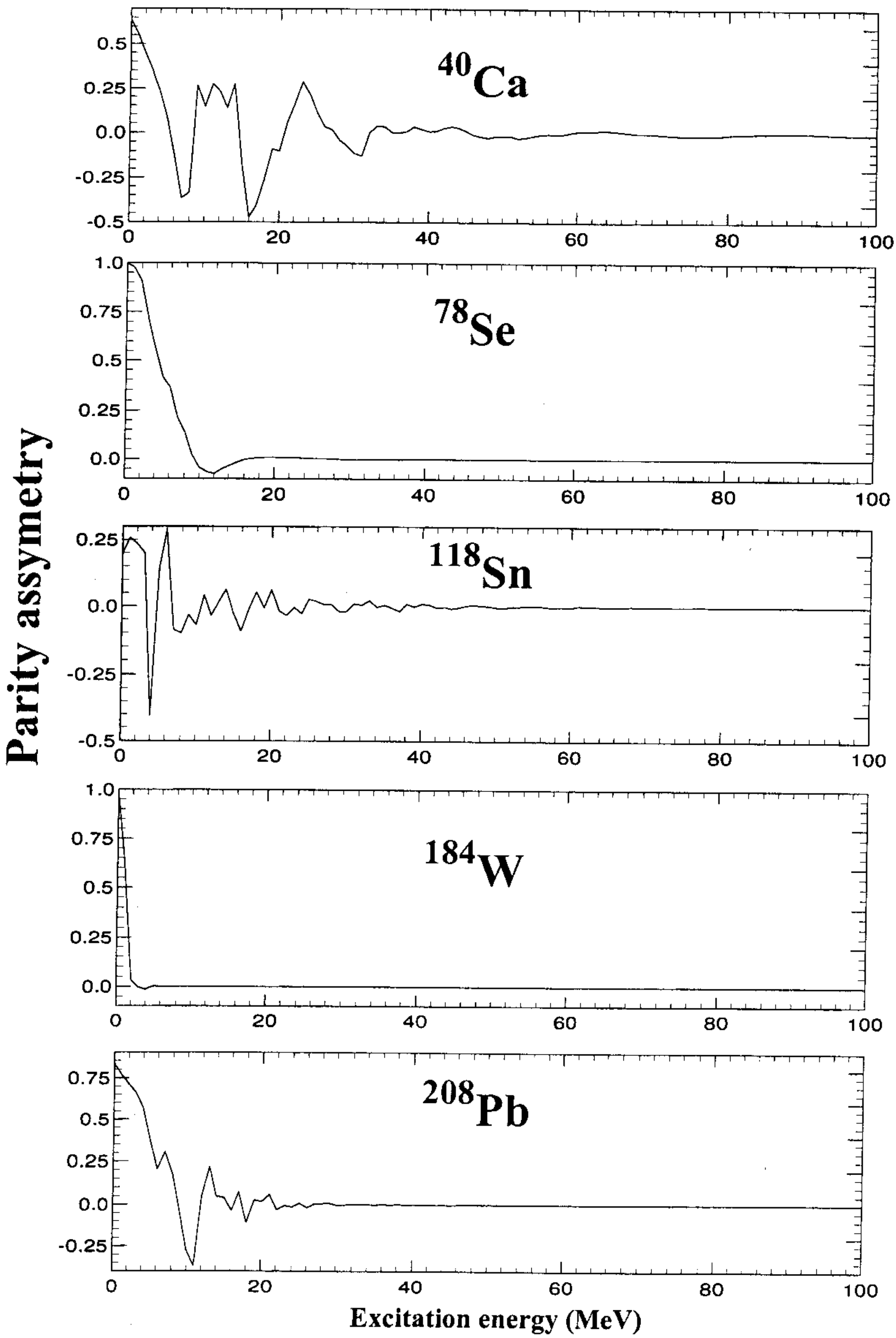
Intrinsic values



"Global values" (with rotational bands)



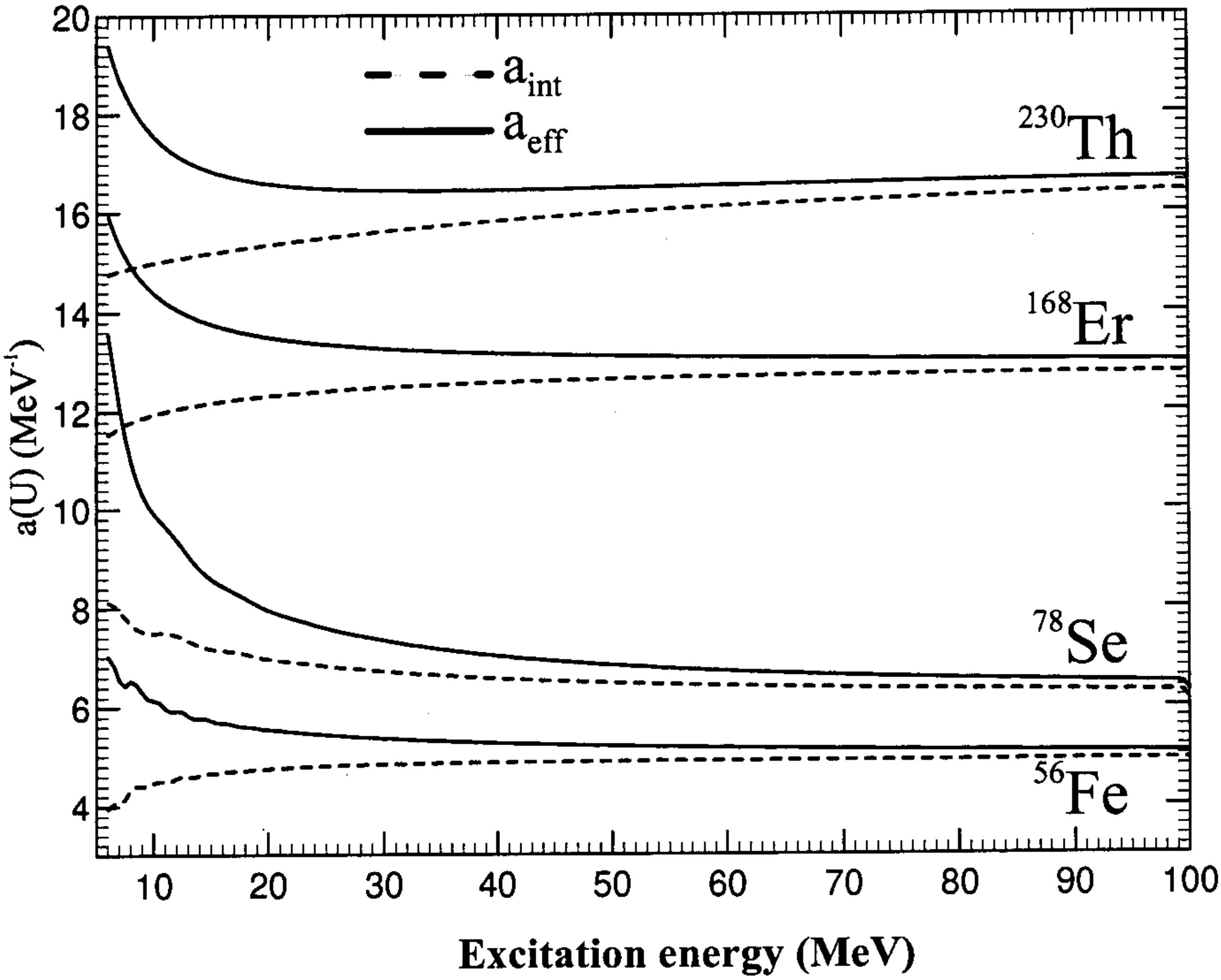
Parity distribution



GSM & COMBINATORIAL RESULTS ANALYSIS
Level density parameter and Ignatyuk Formula

Intrinsic of effective level density parameter

Comparison of ldp deduced from combinatorial calculations with (a_{eff}) and without (a_{int}) collective effects included



⇒ **Ignatyuk law only followed by a_{int}**

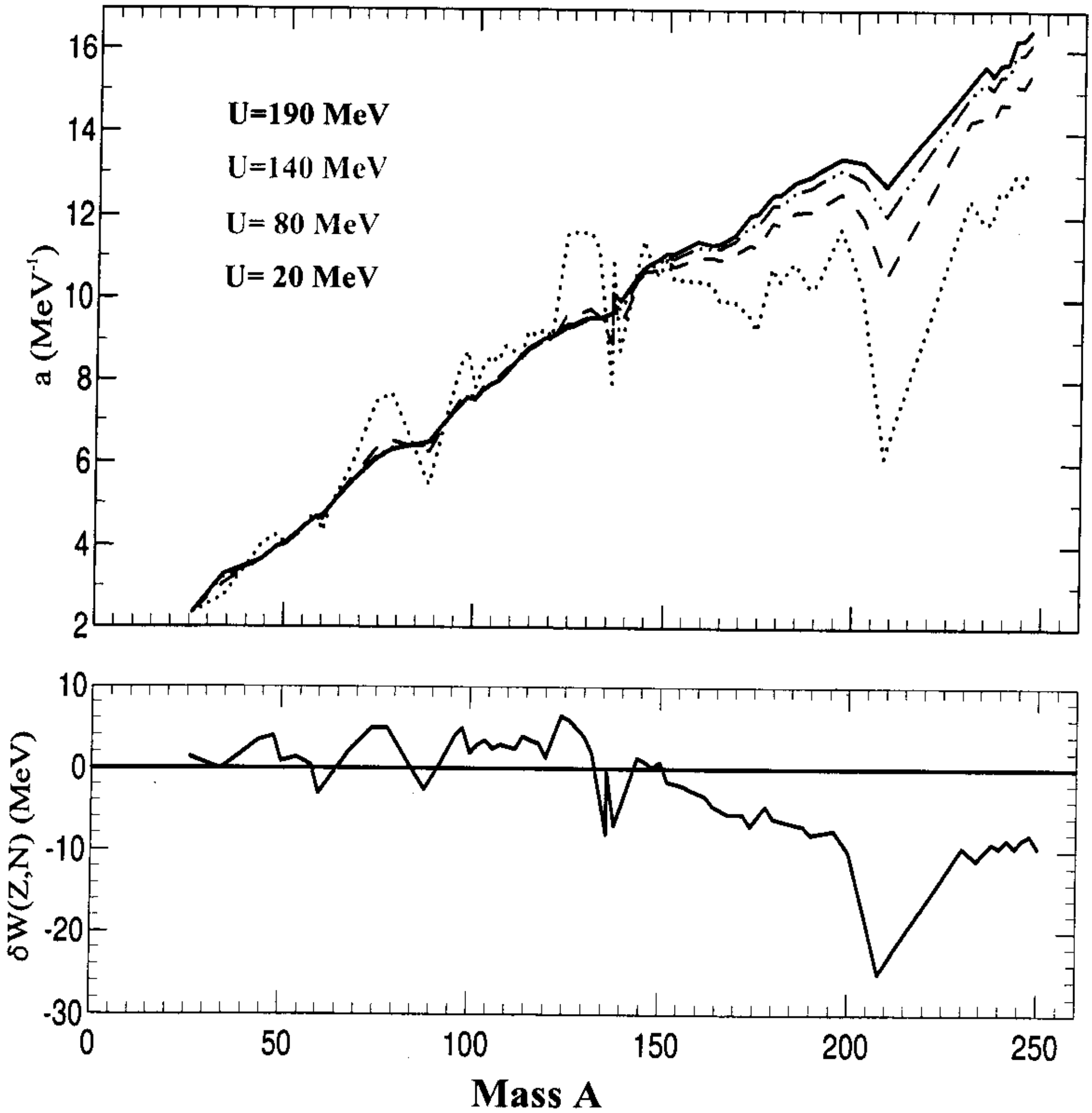
⇒ **Collective effects should be treated explicitly**

GSM & COMBINATORIAL RESULTS ANALYSIS
Level density parameter and Ignatyuk Formula

Results for intrinsic LDP (1/3)

$$a(N,Z,U) = \tilde{a}(A)\left[1 + \frac{\delta W(N,Z) f(U-\Delta)}{U-\Delta}\right]$$

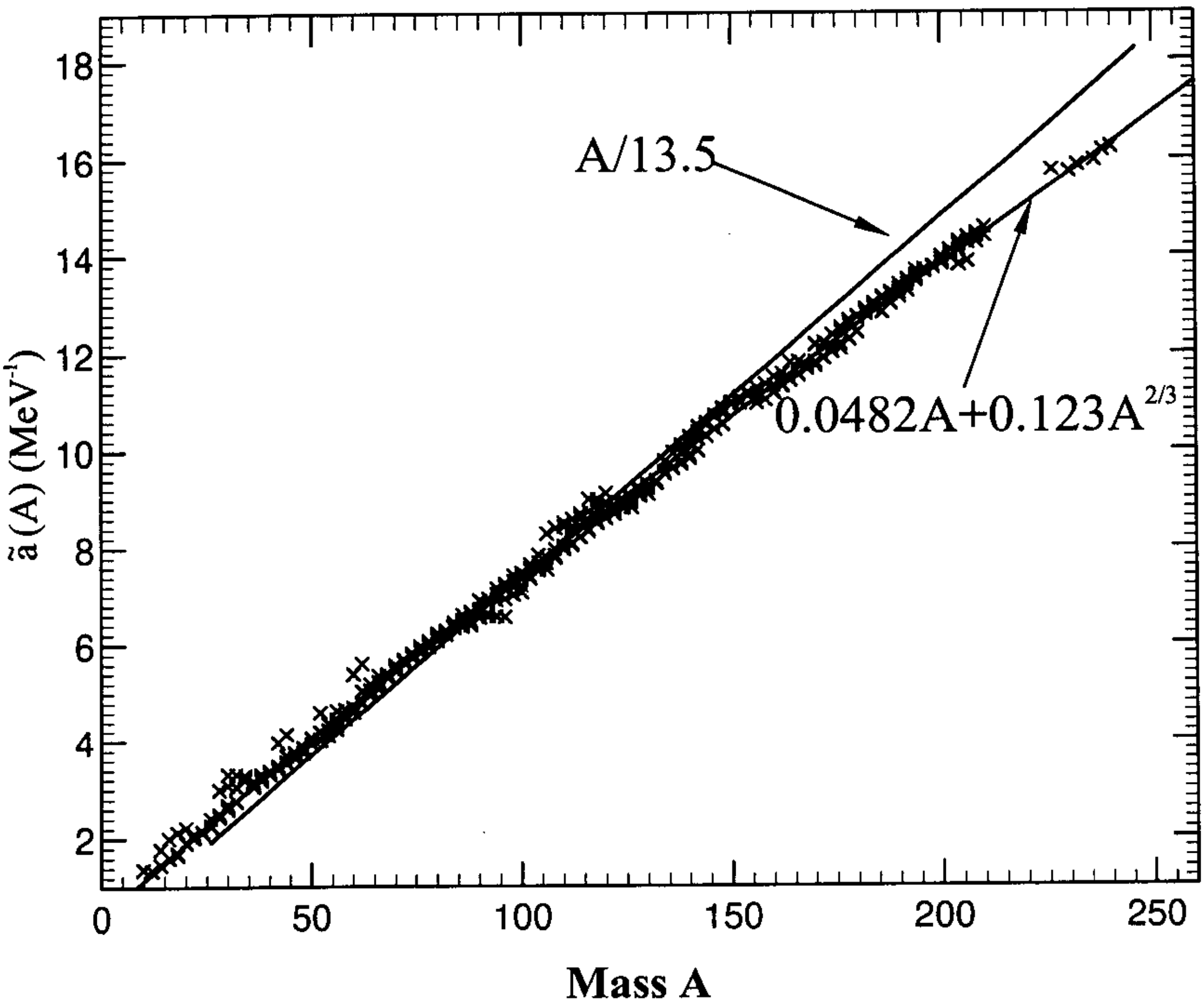
with $f(U) = 1 - \exp(-\gamma U)$



⇒ **Perfect correlation between Strutinsky Shell Correction and Combinatorial level density parameter**

Results for intrinsic LDP (2/3)

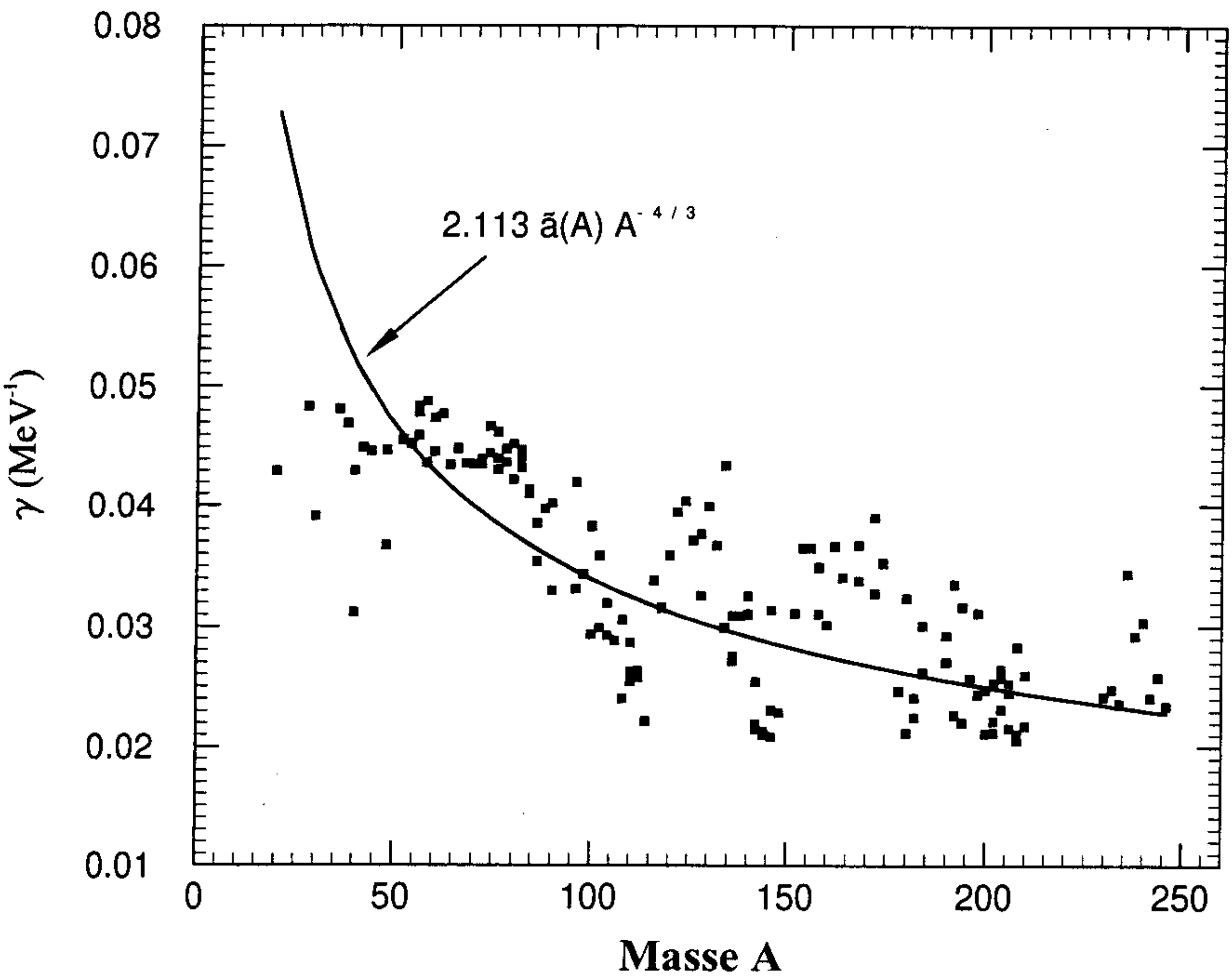
Calculation of $\tilde{a}(A)$ analysing 'à la' Ignatyuk the intrinsic combinatorial level density parameter



$\Rightarrow$ **A/13 behavior found again**

Results for intrinsic LDP (3/3)

Calculation of $\gamma(A)$ analysing 'à la' Ignatyuk the intrinsic combinatorial level density parameter



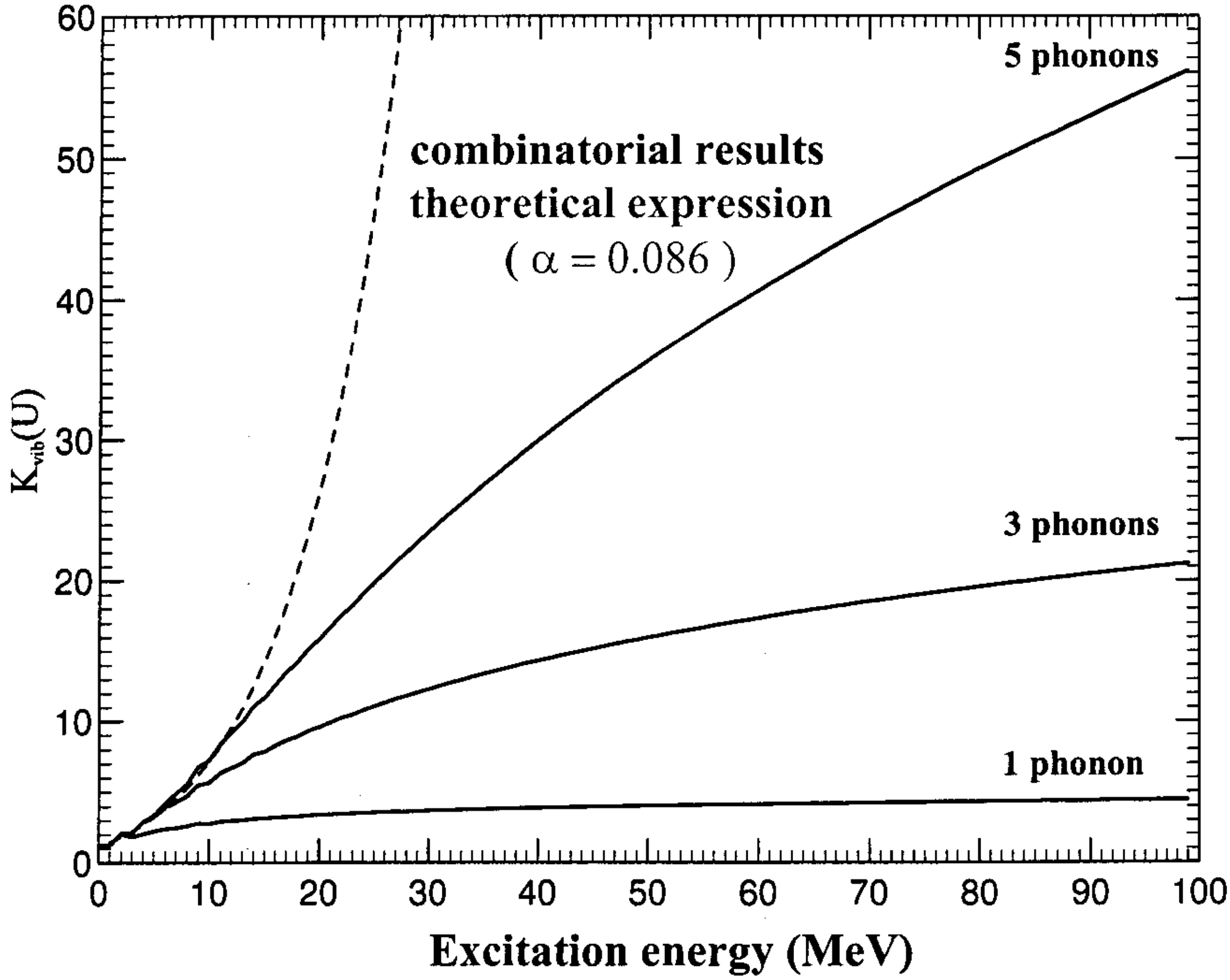
⇒ **Mass dependent γ**

Vibrational enhancement

Theoretical Expression

$$K_{vib}(U) = \exp\left[\alpha \, A^{2/3} \left(\frac{U-\Delta}{a} \right)^{2/3} \right]$$

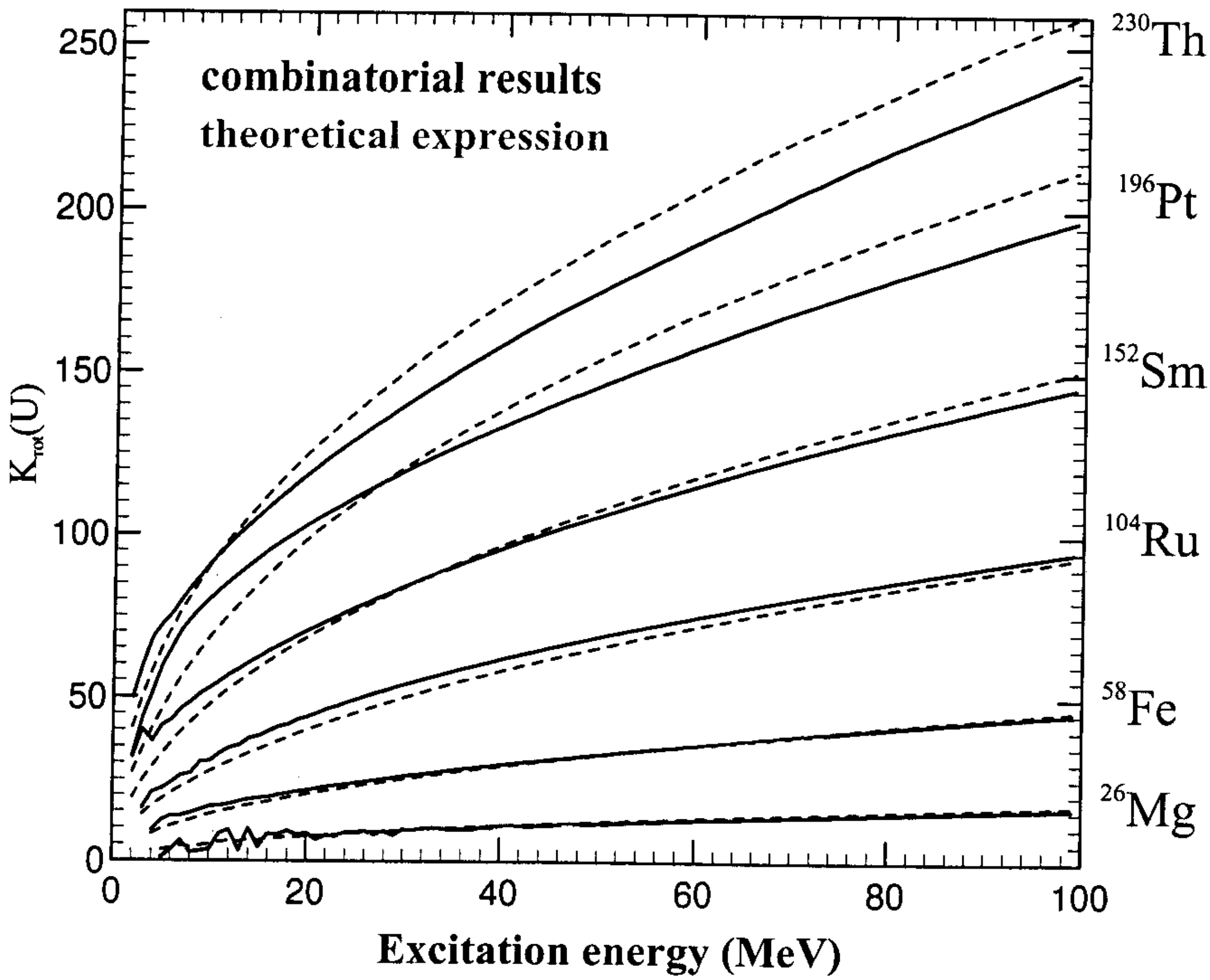
^{104}Ru



Rotational enhancement

Theoretical expression

$$\mathbf{K_{rot}(U)} = \sigma^2 = \mathcal{I}\sqrt{\frac{U-\Delta}{a}}$$



GSM & COMBINATORIAL RESULTS ANALYSIS

Towards a global level density expression (1/3)

1) Analytical expression for p-h level densities

$$\rho_{int}(U,J,\pi) = \frac{1}{2} \frac{2J+1}{2\sqrt{2\pi}\,\sigma_{eff}^3} \exp\left[-\frac{(J+1/2)^2}{2\sigma_{eff}^2}\right] \frac{\sqrt{\pi}}{12} \frac{\exp\left[2\sqrt{a_{int}(U-\Delta)}\right]}{a_{int}^{1/4}(U-\Delta)^{5/4}}$$

→ a_{int}: Ignatyuk A /13

→ $\sigma_{eff}^2 = I_{eff}\left(\frac{U-\Delta}{a_{int}}\right)^{1/2}$

→ $\frac{1}{2}$: parity distribution (to be improved)

2) Total level densities expression

$$\rho(U,J,\pi) = [1-q(U)]\; K_{vib}(U)\; K_{rot}(U)\; \rho_{int}(U,J,\pi) + q(U)\; \rho_{int}(U,J,\pi)$$

→ $K_{rot} = I_{rigid}\left(\frac{U-\Delta}{a_{int}}\right)^{1/2}$

→ K_{vib}: adjusted with D_0 values at B_n

→ $q(U)$: to be determined : $\lim_{U\to\infty} q(U) = 1$

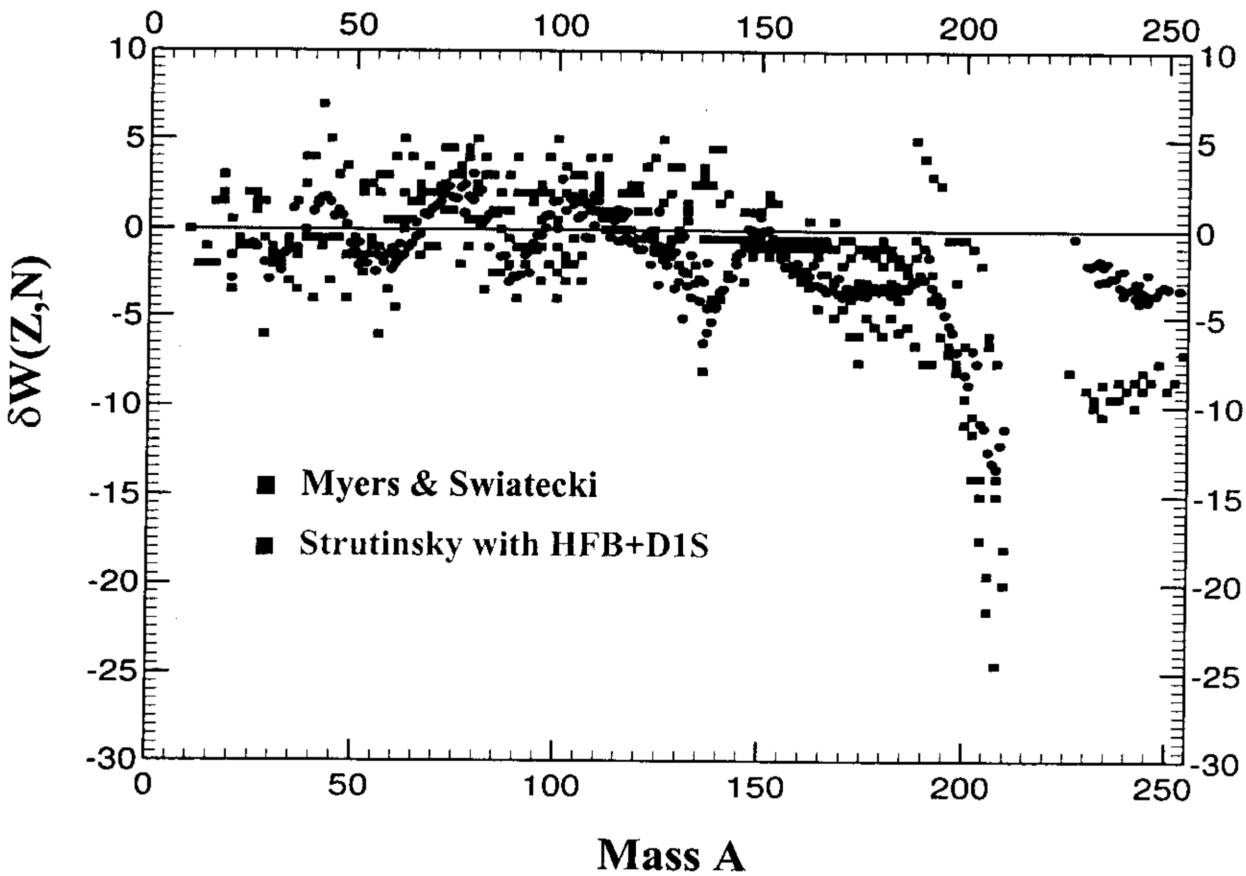
Towards a global level density expression (2/3)

Preliminary results

- **For 284 nuclei**

r_{ave} =2.54		r_{ave} =1.80
σ^2 = 2.30	⟹	σ^2 =0.96
factor 2 : 50%		factor 2 : 80%
factor 5 : 42%		factor 5 : 18%

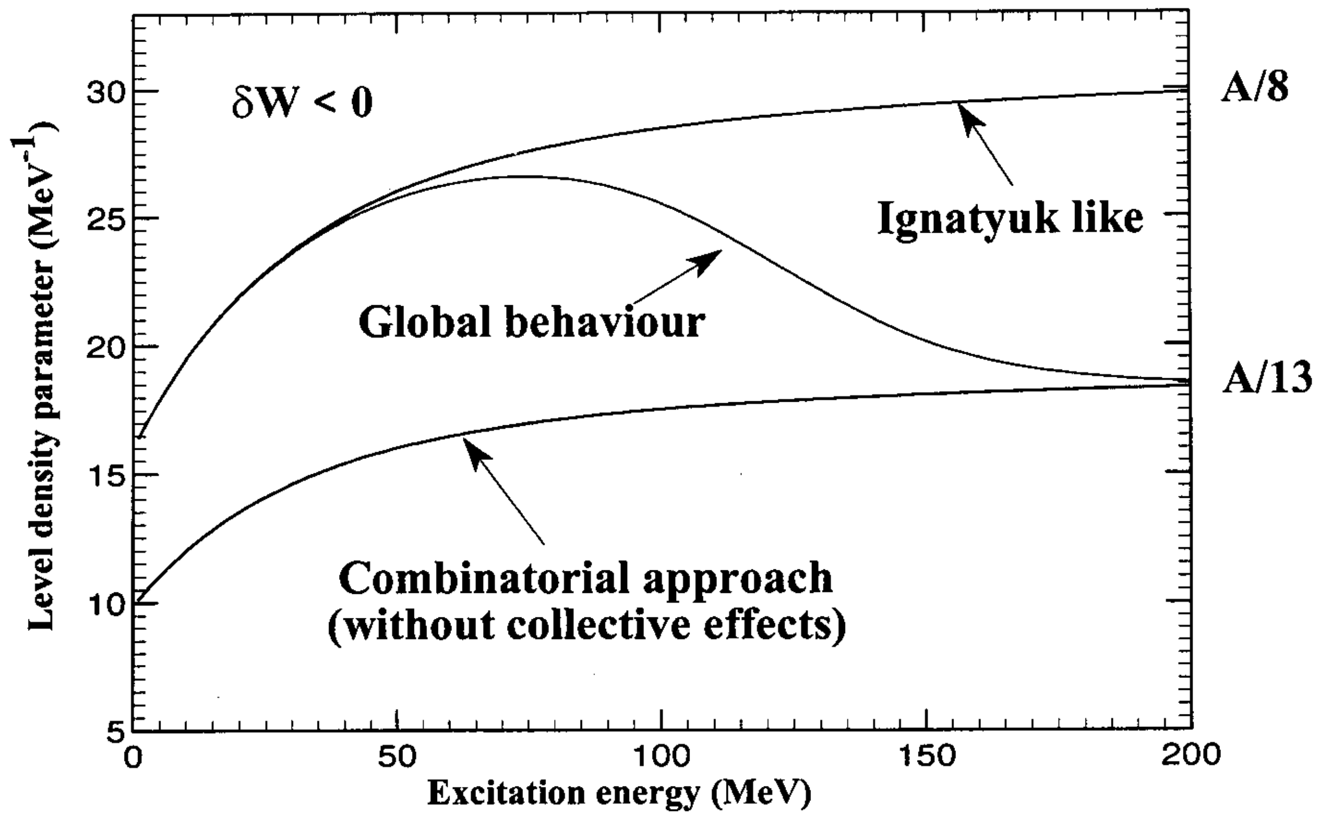
but

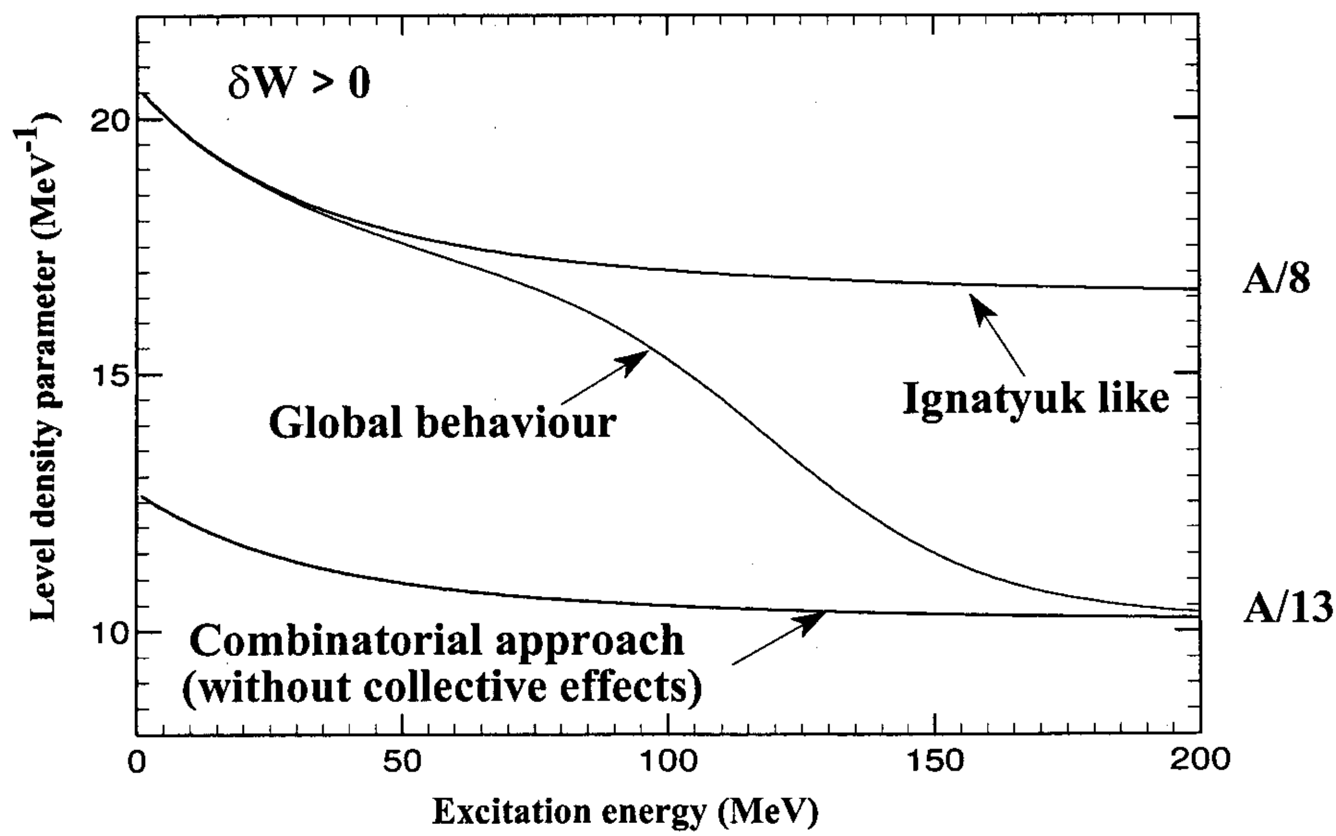


⟹ **Microscopic calculation of δW**
(Strutinsky + HFB-D1S single particle levels)

GSM & COMBINATORIAL RESULTS ANALYSIS
Towards a global level density expression (3/3)

Global level density parameter behaviour





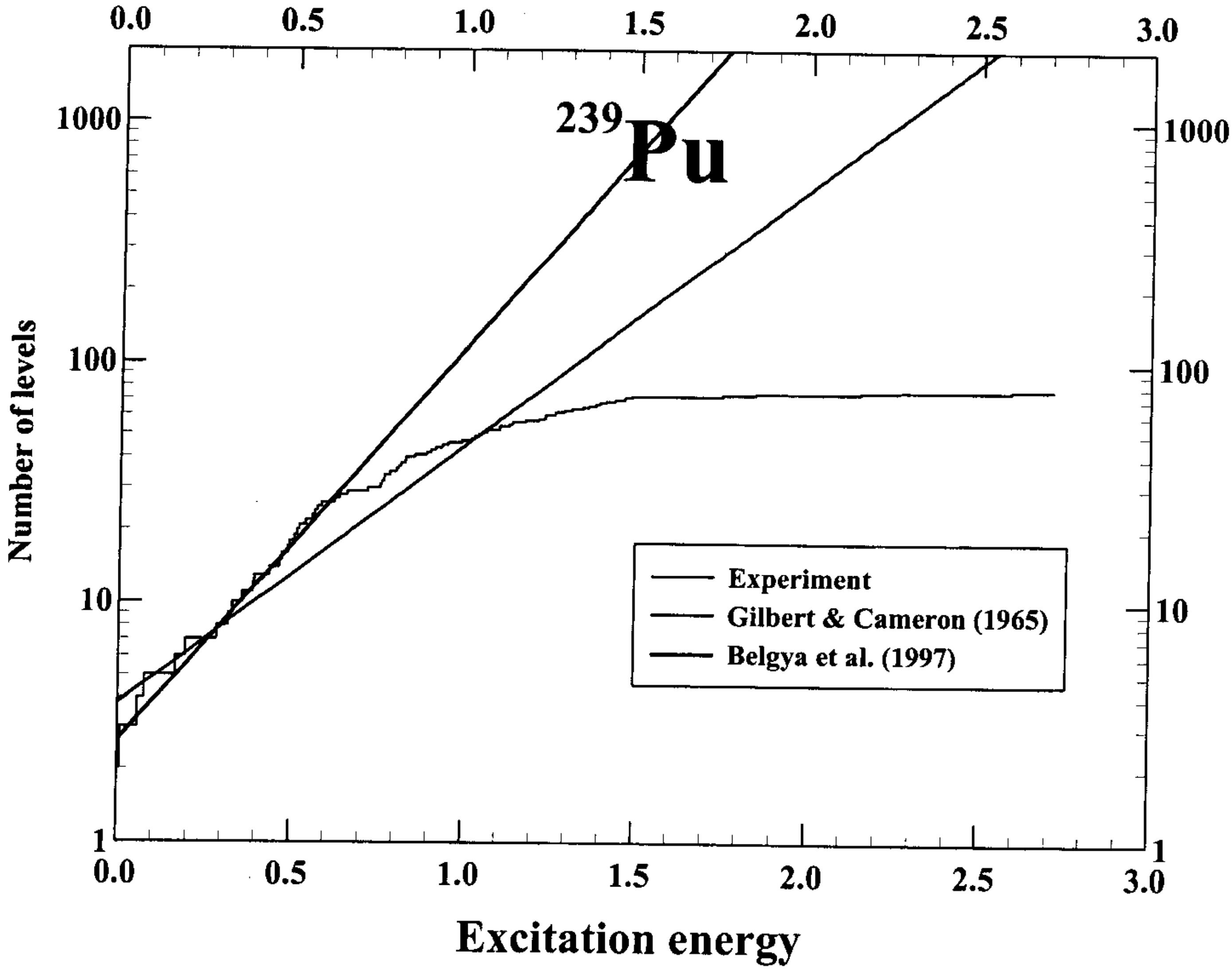
APPLICATIONS

APPLICATIONS

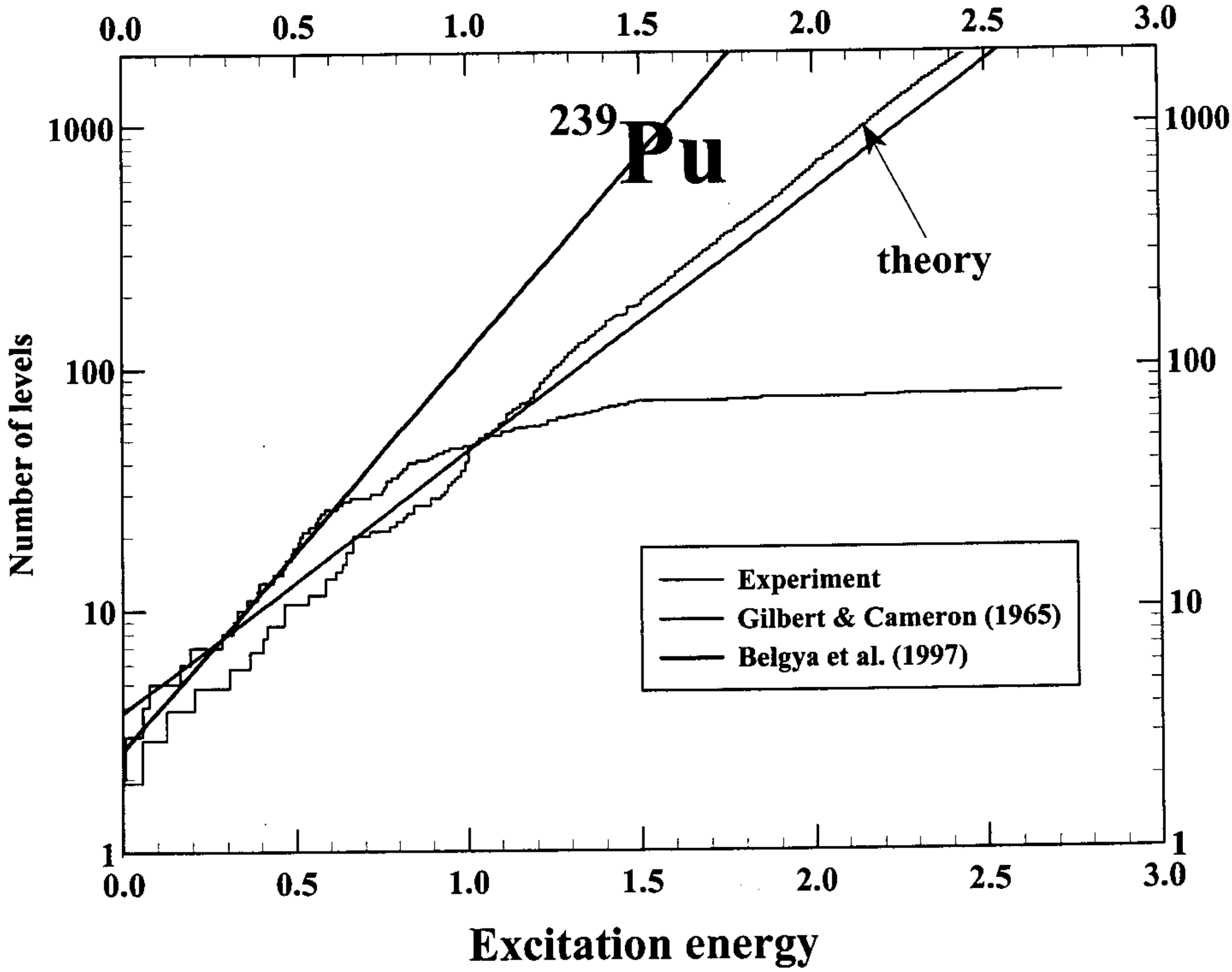
Temperature laws determination

Typical effect on cross section calculation

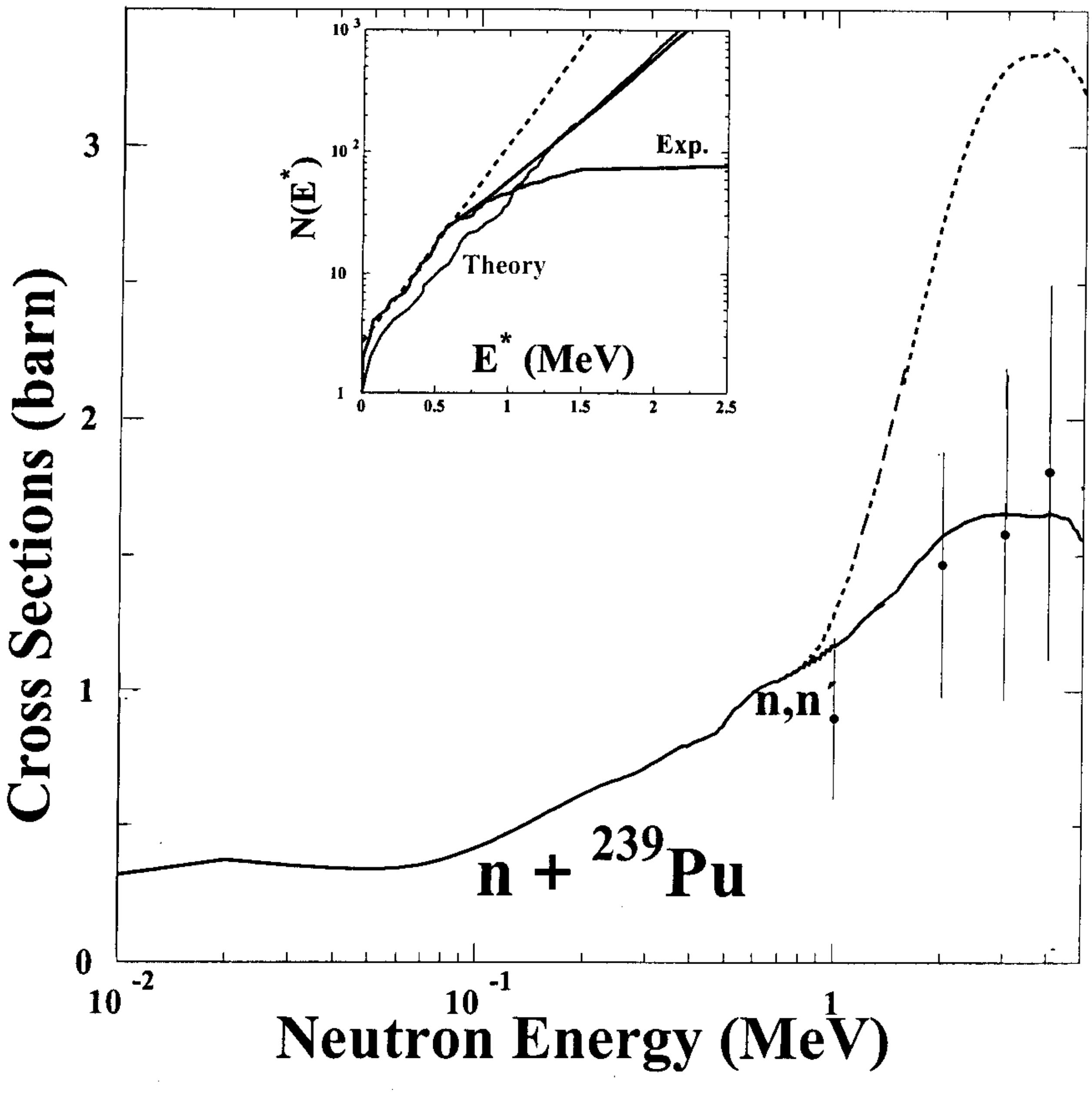
TEMPERATURE LAWS DETERMINATION



TEMPERATURE LAWS DETERMINATION



TYPICAL EFFECT ON XS CALCULATION



CONCLUSION

Many microscopic approaches are available to study level densities but these are difficult to use for pratical applications.

⇒ analytical expression generally prefered

Problem : adjustement of the parameters ?

because few experimental data available
when available ⇒ narrow energy range

⇒ adjustement reliing both on microscopic approaches and experimental data very promising.

microscopic approach ⇒ energetic behaviour
⇒ functional forms

experimental data ⇒ parameters values

Remaining problems :

vanishing of collective effects (A/8 → A/13 transition) ?

parity distribution ?

high spins ?

nuclei far from stability ?