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SMR.1221 - 2

SPRING WORKSHOP ON SUPERSTRINGS AND RELATED MATTERS

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WARPED BRANE WORLDS AND HIERARCHY PROBLEMS

Lecture I

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Basic Plan of my lectures :

Discovery of branes in string theory → many new ideas about fundamental approaches to various physical problems from string theory. "Brane world" scenarios are a striking example.

1ST 2 lectures : Branes, warp factors + their uses

I - RS scenarios : trapped gravity, hierarchy problem

II - Cosmological constant ?

Next 2 lectures : Discuss one of the promising ways of getting concrete brane worlds out of string theory -- Branes in CY compactifications.

III - Background about closed strings + mirror symmetry

IV - D-branes + open string mirror symmetry

Topic I: Gravity trapping + hierarchies

9906064, 9905221 } RS
9908076 } LR

- Its an old idea that we might live on a defect in higher dim'l space, e.g. domain wall

Problem: Why do we see 4d gravity? [gravity lives in the "bulk"]

Well, $S_{5d} \supset \int d^5x \sqrt{-G}\,[R \cdot M_5^3]$

Suppose the X_5 direction is a circle of radius r. Then matching to an "effective" $S_{4d} \rightarrow$

$$M_4^2 \sim r \cdot M_5^3$$

- r finite (+ small enough) $\Rightarrow$ M_4 finite, 4d gravity exists
- $r \rightarrow \infty$ $\Rightarrow$ no 4d gravity, of course.

<u>But</u> -- there is a more general possibility.

The metric can be <u>warped</u> to make nontrivial new scenarios.

<u>E.g.</u> Consider pure 5d gravity w/ cosm. const.

$$S = \int d^5x \sqrt{-G}\,[R - \Lambda] \quad + \int d^4x \sqrt{-g}\,(-V_{brane})$$

where $\quad g_{\mu\nu} = \delta^M_\mu \delta^N_\nu G_{MN}(X_5 = 0)$

$\mu, \nu = 1..4$

$M, N = 1,..,5$

Source term giving brane @ $X_5 = 0$

We want our world to look flat $\Rightarrow$ look for <u>solutions</u>:

$$ds^2 = e^{2A(X_5)} \eta_{\mu\nu}\, dX^\mu dX^\nu + dX_5^2$$

- This guarantees Poincaré inv't 4d slices

Now, Einstein's equations just become equations for A:

④

1) $6(A')^2 + \frac{1}{2}\Lambda = 0$

2) $3A'' + \frac{1}{2}V\,\delta(X_5) = 0$

For $\Lambda < 0$ (negative 5d c.c.)

1) $\Rightarrow$ $A' = \pm\sqrt{\frac{-\Lambda}{12}}$ $\Rightarrow$ $\boxed{A = \pm k X_5 \quad k \equiv \sqrt{\frac{-\Lambda}{12}}}$

2) $\Rightarrow$ integrating from $\int_{-\varepsilon}^{\varepsilon} dX_5$

$$3\Delta\left(\frac{dA}{dx}\right) = -\frac{1}{2}V$$

So to solve the equations, we must take

$$A = \begin{cases} -kX_5 & X_5 > 0 \\ kX_5 & X_5 < 0 \end{cases}$$

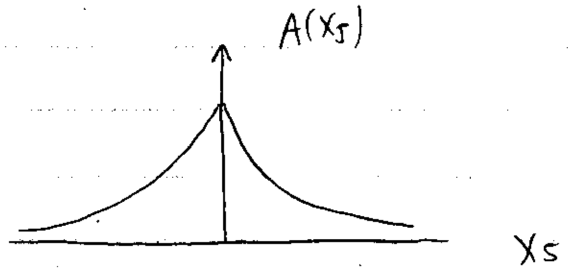
and further set $-6k = -\frac{1}{2}V$ $\Rightarrow$

$$\boxed{V = 12k = 12\sqrt{\frac{-\Lambda}{12}}}$$

This $\rightarrow$ a solution where

$$ds^2 = e^{-2k|X_5|} \eta_{\mu\nu}\, dX^\mu dX^\nu + dX_5^2$$

The warp factor is sharply peaked @ $X_5 = 0$, where the "Planck brane" is located :



This $\Rightarrow$ existence of "<u>localized gravity</u>" at the Planck brane. Namely, doing the naive $5d \rightarrow 4d$ reduction by integrating over X_5 :

$$M_4^2 = M_5^3 \int dX_5 \; e^{-2k|X_5|}$$

$\rightarrow$ <u>finite</u> despite ∞ 5^{th} dim !

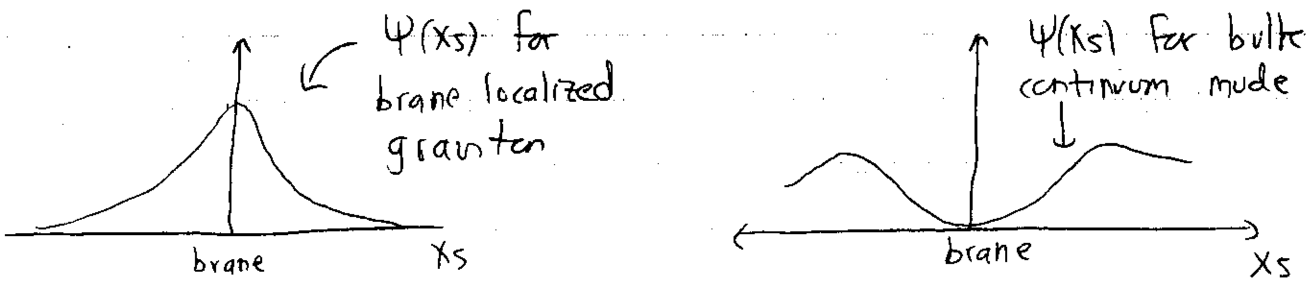
Now, there is a natural concern here :

- In unwarped transverse circle case discussed before, the lightest 5d KK states have mass $\sim \frac{1}{r}$.

r small enough → big gap, + the low energy theory is clearly 4d GR coupled to brane fields.

- In this case, ∞ X_5 ⇒ no gap in spectrum of bulk modes! Don't they ruin 4d EQFT?

No -- must account for the support of the wavefunctions of various modes @ the brane:

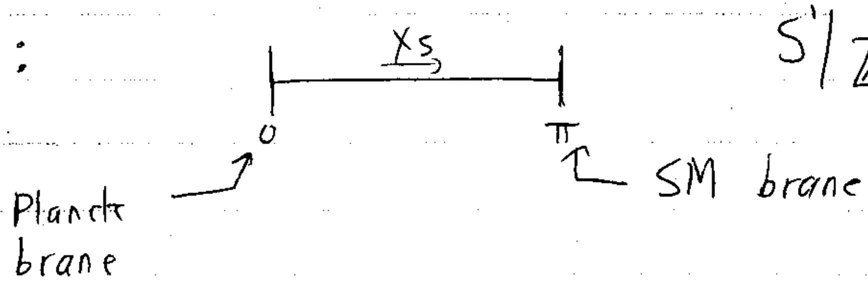


In fact the wavefunctions die fast enough so that Newton's $V(r) \sim \frac{1}{r}$ receives only $\frac{1}{r^3}$ corrections.

Warp factors + hierarchies

Consider now a case with two branes, located at $x_5 = 0, \pi$; and take x_5 to be compactified on $S^1/\mathbb{Z}_2$

an interval:



$$S = \int d^5x \sqrt{-G} \left\{ -\Lambda + R \right\}$$
$$+ \int d^4x \sqrt{-g_{SM}} \left\{ \mathcal{L}_{SM} - V_{SM} \right\} \leftarrow \text{brane at } x_5 = \pi$$
$$+ \int d^4x \sqrt{-g_{pl}} \left\{ \mathcal{L}_{pl} - V_{pl} \right\} \leftarrow \text{brane at } x_5 = 0$$

Again look for a solution:

$$ds^2 = e^{-2A(x_5)} \eta_{\mu\nu} dx^\mu dx^\nu + dx_5^2 \cdot r_c^2$$

$\pi r_c \equiv$ size of the interval

The Einstein equations are now:

(8)

1) $$\frac{6(A')^2}{r_c^2} + \frac{\Lambda}{2} = 0$$

2) $$\frac{3A''}{r_c^2} + \frac{1}{2}\frac{V_{pl}}{r_c}\delta(x_5) + \frac{1}{2}\frac{V_{SM}}{r_c}\delta(x_5-\pi) = 0$$

Like before, we can define $k \equiv \sqrt{\frac{-\Lambda}{12}}$ $(\Lambda < 0)$

and solve 1) with

$$A(x_5) = k\, r_c\, |x_5| \quad \Big\} \text{ consistent w/ } x_5 \to -x_5 \;\; \mathbb{Z}_2$$

[We'll let $-\pi \leq x_5 \leq \pi$ + find $\mathbb{Z}_2$ symmetric sol'n]

Now, this tells us that:

$$A'' = 2k\, r_c \left[\delta(x_5) - \delta(x_5-\pi)\right]$$

Comparing to 2) $\Rightarrow$

$$\left.\begin{array}{l} V_{pl} = -V_{SM} = 12k \\ (\text{recall } \Lambda \equiv -12k^2) \end{array}\right\} \text{fine tunes for flat sol'n}$$

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What is the 4d EQFT for this setup?

- 4d fields visible from metric ansatz:

$$ds^2 = e^{-2kT(x)} \underbrace{[\eta_{\mu\nu} + \bar{h}_{\mu\nu}(x)]}_{\text{Call this } \bar{g}_{\mu\nu}(x)} dx^\mu dx^\nu + T^2 dx_5^2$$

$T(x)$ = "radion field"; set $\langle T \rangle = r_c$, discuss later.

Then we compute the effective gravitational action

$$S_{eff} \supset \int d^4x \int dx_5 \; 2M_5^3 \; r_c \; e^{-2kr_c x_5} \sqrt{-\bar{g}}\, \bar{R}$$

$$\Rightarrow M_4^2 = 2M_5^3 r_c \int_0^\pi dx_5 \; e^{-2kr_c x_5}$$

$$= \frac{M_5^3}{k}\left[1 - e^{-2kr_c\pi}\right]$$

- So M_4 only weakly r_c dependent

- Planck brane sees $G_{MN}\,\delta^M_\mu \delta^N_\nu\,(x_5 = 0) \Rightarrow$

$$g^{Pl}_{\mu\nu} = \bar{g}_{\mu\nu}$$

- But SM brane sees $G_{MN}\,\delta^M_\mu \delta^N_\nu\,(x_5 = \pi) \Rightarrow$

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$$\boxed{g^{SM}_{\mu\nu} = e^{-2kr_c\pi}\,\bar{g}_{\mu\nu}}$$

So e.g. a scalar "Higgs" field sees a Lagrangian

$$\mathcal{L}_{SM} \sim \sqrt{-g^{SM}}\,\{ g^{\mu\nu}_{SM} D_\mu H^\dagger D_\nu H - \lambda(|H|^2 - V_0^2)^2 \}$$

Plugging in for g^{SM} from above $\Rightarrow$

$$\mathcal{L}_{SM} \sim \sqrt{-\bar{g}}\, e^{-4kr_c\pi} \{ \bar{g}^{\mu\nu} e^{2kr_c\pi} D_\mu H^\dagger D_\nu H -$$
$$\lambda (|H|^2 - V_0^2)^2 \}$$

Rescaling H to get a canonical kinetic term $\Rightarrow$

$$\mathcal{L}_{SM} \sim \sqrt{-\bar{g}}\, \{ \bar{g}^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda (|H|^2 - e^{-2kr_c\pi} V_0^2)$$

So now the physical mass scale is set by:

$$\boxed{V \equiv V_0\, e^{-2kr_c\pi}}$$

This will happen to all mass scales on the SM brane.

So to get a TeV weak scale out of $M_4^2 \cong V_0^2$, just need $e^{-kr_c\pi} \sim \frac{\text{TeV}}{M_4}$ $\Rightarrow$ no huge disparities required between the fundamental scales in the problem.

Is $\langle r_c \rangle$ of the required size attainable in a natural way as $\langle T(x) \rangle$?

Yes -- see Goldberger - Wise mechanism.

[Bulk massive scalar w/ interactions at two branes]