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SMR.1221 - 11

SPRING WORKSHOP ON SUPERSTRINGS AND RELATED MATTERS

27 March - 4 April 2000

SOME ASPECTS OF LIFE ON A BRANE

Lectures III & IV

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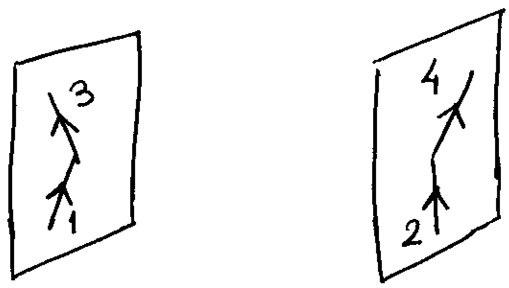
② Some aspects of high-E <u>string scattering</u>

based on hep-th/9909171
with B. Pioline

If $M_s \sim$ TeV question 'more urgent'

Of course 'touches upon' deep question of defn. of fundamental theory ...

But one particular context comes up:



how do strings on separate brane-worlds interact?

extra 'control parameter' (brane distance), can use it to learn something about this hard problem?

What is known:

① tree-level fixed-angle scattering exponentially soft   Veneziano

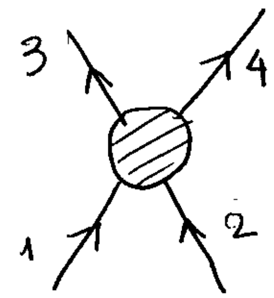
(2) Higher perturbative amplitudes also, but 'less soft'

$$A_N \sim e^{-\alpha' s f(\varphi)/N+1}$$

↑ # of loops

(Alessandrini, Amati, More
Gross-Mende
Gross-Manes

Kinematics



$$s = -(p_1+p_2)^2 = 4E^2 > 0$$

$$t = -(p_1+p_3)^2 = -s \sin^2 \varphi/2 < 0$$

$$u = -(p_1+p_4)^2 = -s \cos^2 \varphi/2 < 0$$

(φ: scattering angle in CM frame $2E$ total energy)

(3) pert. theory breaks down? (Ooguri-Mende)
power-behavior from D-instantons   Green
parton structure ?   (Shenker, matrix model ....)

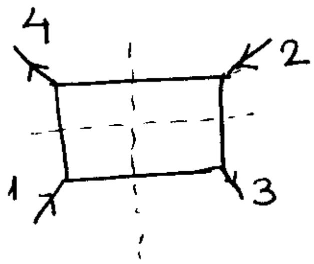
Here will mostly limit to comments in pert. theory, but extra parameter allows interesting extrapolation to 'interesting scaling limit'.

Comments on Veneziano amplitude

$$A_{disk} \sim \frac{\Gamma(-\alpha' t)\Gamma(-\alpha' u)}{\Gamma(\alpha' s)} + (s \leftrightarrow t) + (s \leftrightarrow u)$$

tr(1324) — no poles in intermediate channels

tr(1234), tr(1243) — have poles



t spacelike

u spacelike

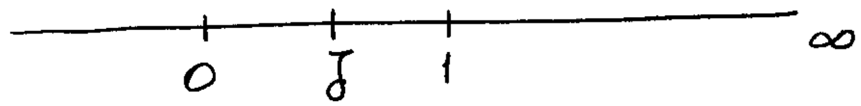
all three have asymptotic behavior:

$$A \sim e^{-s\log|s| - t\log|t| - u\log|u|}$$

$$\sim e^{-s\alpha'\left(\cos^2\frac{\varphi}{2}\log\left(\cos^2\frac{\varphi}{2}\right) + \sin^2\frac{\varphi}{2}\log\left(\sin^2\frac{\varphi}{2}\right)\right)}$$

↳ for ampls with poles interpret as res. at poles.

Saddle-point calculat



$$A \sim \int d\xi \, e^{-\alpha' \left(t \log|\xi| + u \log|1-\xi| \right)} \quad \left\{ \begin{array}{ll} \int_{-\infty}^{0} & \lambda_3\lambda_1\lambda_2\lambda_4 \\ \int_{0}^{1} & \lambda_1\lambda_3\lambda_2 \\ \int_{1}^{\infty} & \lambda_1\lambda_2\lambda_3 \end{array} \right.$$

where $x_1 = 0$, $x_2 = 1$, $x_4 = \infty$

$$x_3 = \xi = \frac{(x_1-x_3)(x_2-x_4)}{(x_1-x_2)(x_3-x_4)}$$

$SL(2,R)$-inv. cross ratio

unique saddle: $\tilde{\xi} = -\frac{t}{s} = \sin^2 \frac{\varphi}{2}$

$\rightarrow$ gives precisely desired expression

Remark 1 saddle ~~for~~ in integrnt region of (1324) amplitude $(0 < \tilde{\xi} < 1)$

no unitarity cuts $\rightarrow$ integral convergent (dominated by min.)

for other amplitudes deform contour to pass through same point

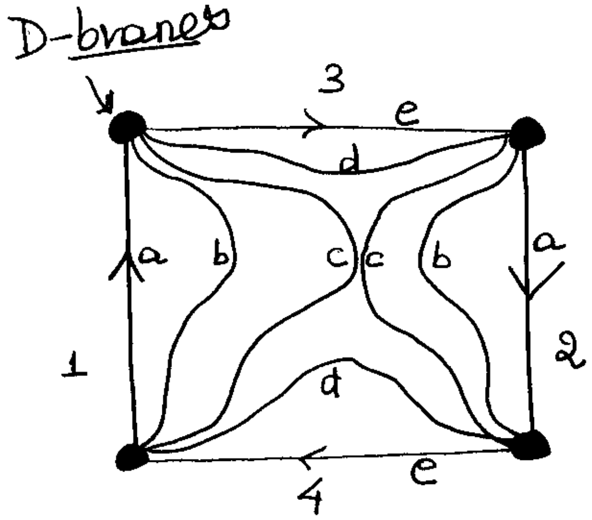
Remark 2

'T dualities' in collision plane

change $\vec{P}_i \longleftrightarrow \vec{W}_i$ "stretching"

$$\vec{X}_i = \vec{P}_i \tau + \frac{\vec{W}_i}{\pi} \sigma + \cdots\cdots$$

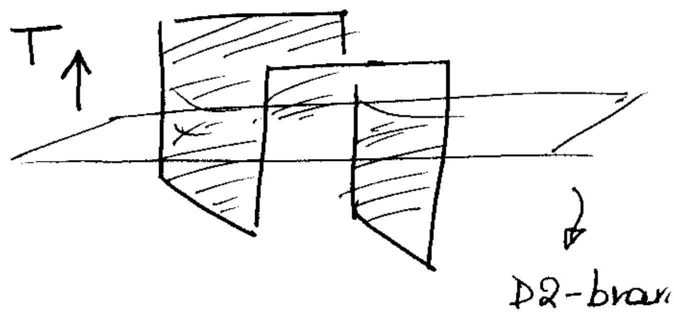
vertex ops same (up to $\bar{\partial} X \to -\bar{\partial} X$) $\Rightarrow$ Veneziano amplitude



'semiclassical tunneling'

explains exponential softness

soap-bubble problem



Remark 3

D-instanton $\rightsquigarrow$ Euclidean D2-brane at fixed time T

$\Rightarrow$ string endpoints can move freely

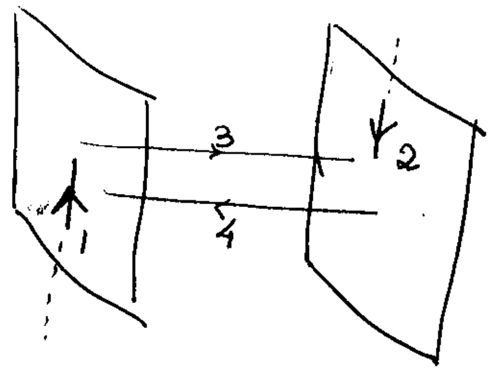
no tunneling $\longrightarrow$ no exponential suppression

Remark 4

in right-angle scattering, 'T-dualize' one direction only

$(\vec{P}_1, \vec{P}_2) \longrightarrow (\vec{W}_3, \vec{W}_4)$

in process



same as Veneziano amplitude

$$\Rightarrow \mathcal{A} \simeq e^{-\alpha' s \log 2}$$

(production of solitons in high-E collision)

Remark 5

Higher orders on same brane

$$\mathcal{A}_N \sim e^{-\alpha' s f(\varphi)/N+1}$$

(multiple cover of disk trajectory)

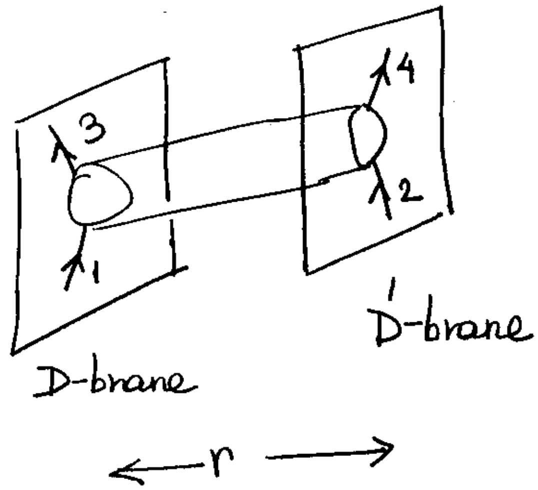
heuristically:

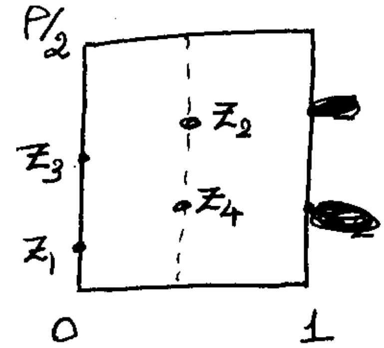
since scattering $\sim e^{-t\alpha'}$

preferable to do N+1 'less deep' scatterings

$\sqrt{t}/N+1$ each $\rightsquigarrow$ $\mathcal{A}_N \sim \left(e^{-\alpha' \frac{t}{(N+1)^2}}\right)^{N+1}$ ✓

Return to problem:





annulus diagram

$$\int dz_i\, d\rho\; e^{-\left\{r^2 \rho/4\pi\alpha' + P_i P_j\, G(z_i - z_j|\rho)\right\}} = \mathcal{E}$$

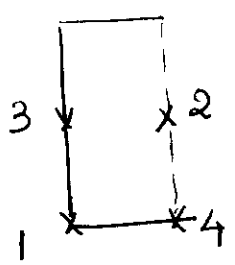
propagator on torus

$$G(z|\tau) = \log \chi$$

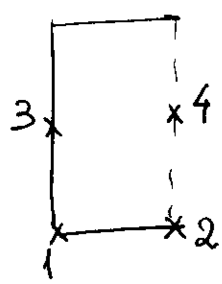
$$\chi = 2\pi e^{-\pi y^2/\tau_2} \left| \frac{\theta_1(z|\tau)}{\theta_1'(0|\tau)} \right|$$

$$\begin{cases} z = x + iy \\ \tau = \tau_1 + i\tau_2 \end{cases}$$

by Z_2 sym., equilibrium expected when charges 'half a cycle' apart



(A)



(B)

$$\mathcal{E}_{(A)} = \frac{r^2 \rho}{4\pi\alpha'^2} + 2s \log|\theta_3| + 2t \log|\theta_4| + 2u \log|\theta_2|$$

(B) $\quad s \leftrightarrow u$

θ's at zero argument & $\tau = i\rho/2$

Need still to solve ρ-eqn: (A)

$$\frac{r^2}{8\pi\alpha'^2} + s\frac{\theta_3'}{\theta_3} + t\frac{\theta_4'}{\theta_4} + u\frac{\theta_2'}{\theta_2} = 0$$

$$\left(\theta' = \partial_\rho \theta\right)$$

<u>with</u> $\quad s+t+u=0$

If $r=0 \rightsquigarrow$

$$\left.\begin{aligned} s &= a\theta_3^4 \\ t &= -a\theta_4^4 \\ u &= -a\theta_2^4 \end{aligned}\right\} + b\frac{\theta_3'\theta_3^3}{\eta'^2} \quad \text{with } b = \frac{r^2}{2\pi^3\alpha'^2}$$

solves <u>both</u> eqns

$$\therefore\ -u/s = \cos^2\varphi/2 = \left(\frac{\theta_2(\tilde\tau)}{\theta_3(\tilde\tau)}\right)^4$$

Picard map 1-1 from $\mathbb{C}$ to fund. domain for $\Gamma(2)$

$$\tilde{\mathcal{E}}_{ann} = \tfrac{1}{2}s\log|s| + \tfrac{1}{2}t\log|t| + \tfrac{1}{2}u\log|u|$$

$$= \tfrac{1}{2}\mathcal{E}_{\text{[illegible]}} \quad \text{as expected!}$$

for $r \neq 0$ need some more intricate identities, but problem still admits exact soln.

$$(A) \qquad -\frac{u}{s} = \cos^2 \varphi/2 = \frac{\theta_2^4(\tilde{\tau}) - u^2}{\theta_3^4(\tilde{\tau})}$$

$$(B) \qquad = \frac{\theta_3^4(\tilde{\tau}) - u^2}{\theta_2^4(\tilde{\tau})}$$

with $\boxed{v = \frac{r}{\pi \alpha \sqrt{s}}}$

∞ # of solutions in complex-τ plane. Which one to choose?

For $\underline{v=0}$: all saddles give same 'electrostatic energy' (though a priori different fluctuation det.)

Since $|\theta_3| > |\theta_2|$ on imaginary-τ integr. axis saddle A correct one?

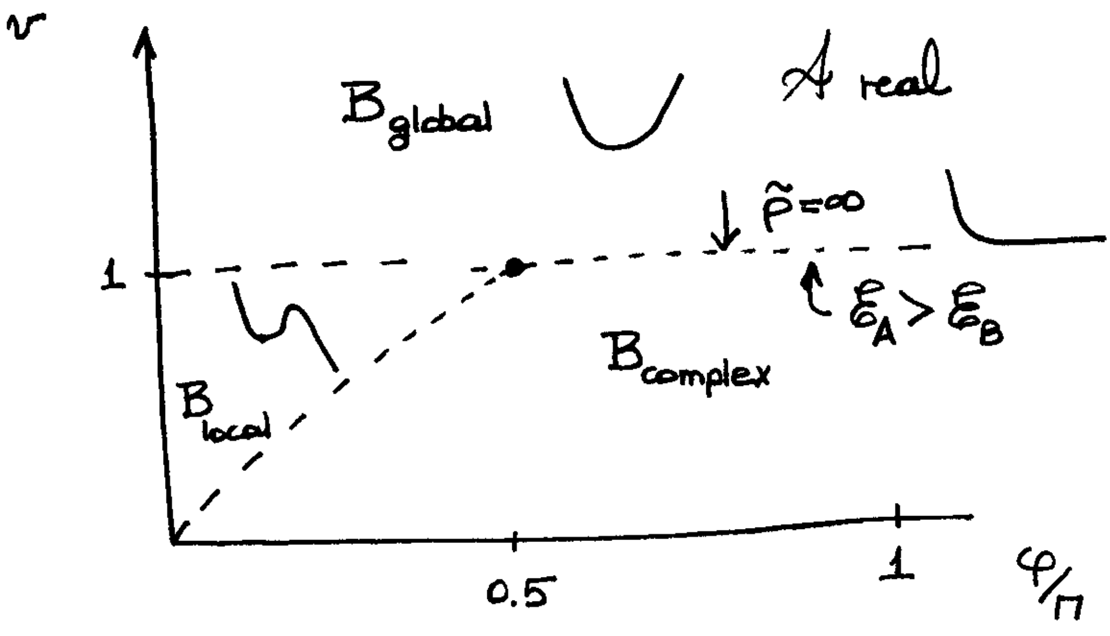
Will argue that <u>NOT</u>.

To find saddle, note first that problem well defined for $\underline{\nu > 1}$:

$\int$ is real and convergent because $\nexists$ unitarity cuts $\Rightarrow$ dominated by global minimum of 'energy' on integrnt. axis :

<u>saddle B</u> (since '1 2 2' attract must be closest to each other)

'Phase <u>diagram</u>'



for $\varphi > \pi/2$ check that:

$$A \sim e^{-2s\alpha' \log 2} \sim \left| A_{disk}(\pi/2) \right|^2$$

(Follows from

$$\mathcal{E}^{(B)} \overset{\rho\to\infty}{\sim} \frac{\pi s}{4}(u-1)\rho + 2s\log 2 \quad)$$

What happens as $u \to \infty$?

supergravity exchange ? (but in 'tensionless string' phase)

from $$\mathcal{E}^{(B)} \overset{\rho\to 0}{\sim} -\frac{\pi t}{\rho} + 2t\log 2 + \frac{\pi u s}{4}\rho + \cdots$$

find

$$\boxed{\mathcal{A} \underset{u\to\infty}{\sim} e^{-r\sqrt{|t|} - 2t\alpha' \log 2}}$$

cf with exchange of field-theory particles:

$$\mathcal{A} \sim \int \int dp_\perp \, \frac{e^{-ip_\perp r}}{p_\perp^2 + |t| + M^2} \, \rho(M) \, V(M) \, dM$$

vertex

density of states at mass M

$$\sim e^{-r\sqrt{|t|} + o\left(\frac{1}{r}\right)} \qquad \Big\{ \text{with } \rho \sim e^{M}$$

M .

$\therefore \quad e^{-2t\alpha' \log 2}$ 'stringy' enhancement

of off-shell vertex

('due to outward stretching
of energetic strings')

Questions: how to extend to higher orders?
Can we resum?
