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SMR.1221 - 5

SPRING WORKSHOP ON SUPERSTRINGS AND RELATED MATTERS

27 March - 4 April 2000

WARPED BRANE WORLDS AND HIERARCHY PROBLEMS, AND BRANES IN CALABI-YAU SPACES

Lecture II, III and IV

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Please note: These are preliminary notes intended for internal distribution only.

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Lecture 2, Trieste 2000

Brane Worlds + The Cosmological Constant

0001197 ADKS

0001206 KSS

Basic heuristic idea (back to Rubakov + Shaposhnikov):

Perhaps if we live on a defect in a higher dim'l space (eg domain wall), it would be natural for the defect to be Flat ($\leftrightarrow$ cc problem).

How does this idea fare in "wall world" scenarios?

Recall RS from yesterday:

negative c.c. $\Lambda < 0$

$$S = \int d^5x \sqrt{-G}\,[R - \Lambda] + \int d^4x \sqrt{-g}\,(-V_{brane})$$

$$g_{\mu\nu} = G_{MN}(x_5 = 0)\,\delta^M_\mu\,\delta^N_\nu$$

Flat solution ansatz: $ds^2 = e^{2A(x_5)}\eta_{\mu\nu}\,dx^\mu dx^\nu + dx_5^2$

We saw yesterday that solving the Einstein eqns with this ansatz $\Rightarrow$ must have:

$$\boxed{V_{brane} = 12\sqrt{\frac{-\Lambda}{12}}}$$

This → Flat 4d, so $\Lambda_4 = 0$. <u>But</u>, one has to <u>tune</u> Λ_{bulk} in terms of V. If SM lives at $x_5 = 0$, small changes to SM parameters ⟹ V shifts to $V + \Delta V$ ⟹ no flat solution anymore! (w/o changing bulk thy)

This is the cosm const problem in wall world scenario, or at least a part of it.

In most microscopic theories, $\exists$ also bulk scalars. Do they change the situation? Take now:

$$S = \int d^5x \sqrt{-G}\,\left[R - \tfrac{4}{3}(\nabla\phi)^2\right] + \int d^4x \sqrt{-g}\,(-f(\phi))$$

no tree level bulk c.c., can say bulk is SUS

③

For simplicity, take

$$F(\phi) = V e^{b\phi} \qquad \left[\begin{array}{l}\text{similar result for}\\ \text{general } f \text{ easy}\end{array}\right]$$

and consider again the ansatz:

$$ds^2 = e^{2A(x_5)} \eta_{\mu\nu}\, dx^\mu dx^\nu + dx_5^2$$

The equations you get are:

1) $\frac{8}{3}\phi'' + \frac{32}{3} A'\phi' = bV e^{b\phi}\,\delta(x_5)$

2) $6(A')^2 - \frac{2}{3}(\phi')^2 = 0$

3) $3A'' + \frac{4}{3}(\phi')^2 = -\frac{1}{2} e^{b\phi} V\,\delta(x_5)$

<u>Important fact</u>: Finding flat solutions will <u>NOT</u> require a fine tune of $\{ V \xleftrightarrow[\text{of}]{\text{in terms}} \text{bulk parameters} \}$ in this case. How is this obvious?

- The <u>only</u> non-deriv. coupling of ϕ is to the brane tension term, $(-V)\, e^{b\phi}$

- Clearly, given a sol'n for one value of V, $\exists$ a sol'n for <u>any</u> value of V:

$$V \to V + \delta V \Rightarrow \text{can shift } \phi \text{ to hold } Ve^{b\phi} \text{ \underline{fixed}; then EOM unchanged}$$

<u>Upshot</u>: If $\exists$ flat sol'n for <u>any</u> V, $\exists$ flat sol'n for <u>arbitrary</u> V

Notice that the SM physics at $X_5 = 0$ is <u>purely</u> reflected through the wall source term.

- Suppose SM coupling constant $\neq \phi$ (eg if ϕ is dilaton, SM lives on NS brane)
- Then this means SM physics $\not\Rightarrow$ 4d c.c. !

What about 4d gravity?

To proceed, lets write down the explicit solutions.

Solving Bulk EOM :

$$\phi(X_5) = \frac{3}{4} \log \left| \frac{4}{3} X_5 + c \right| + d$$

$$A(X_5) = \frac{1}{4} \log \left| \frac{4}{3} X_5 + c \right| + \tilde{d}$$

for $X_5 = X_5^* = -\frac{3}{4} c$, $\exists$ singularity :

$$A \to -\infty, \quad |\text{curvature}| \to \infty$$

The warp factor $\in$ the metric vanishes (viewing X_5 as t, like a big bang/crunch).

Now, include wall source terms.

Lets specialize to a concrete interesting case :

$$b = -\frac{4}{3} \longrightarrow f(\phi) = V e^{-\frac{4}{3}\phi}$$

Then, satisfying the jump conditions at the wall can be done using :

$$\phi(X_5) = \begin{cases} \phi_1(X_5) = \frac{3}{4}\log\left|\frac{4}{3}X_5 + C_1\right| + d_1 & X_5 < 0 \\ \phi_2(X_5) = \frac{3}{4}\log\left|\frac{4}{3}X_5 + C_2\right| + d_2 & X_5 > 0 \end{cases}$$

and $A' = \frac{1}{3}\phi'$

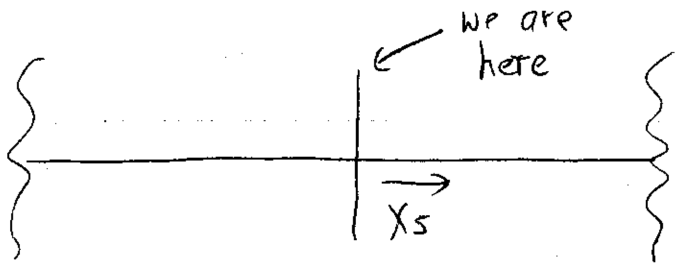
Matching conditions @ wall $\Rightarrow$

$$\left.\begin{aligned} C_1 &= -C_2 = C \\ d_1 &= d_2 = d \\ e^{-\frac{4}{3}d} &= \frac{4}{V}\cdot\frac{C}{|C|} \end{aligned}\right\}$$

Here for simplicity we are looking at a $\underline{\mathbb{Z}_2 \text{ symmetric}}$ sol'n

for <u>arbitrary</u> C.

- Suppose we choose $C > 0$. Then $\exists$ singularities at $X_5 = \pm\frac{3}{4}C$:



- Suppose the space <u>ends</u> @ the singularities.

Then:

$$M_4^2 \sim M_5^3 \int dx_5 \; e^{2A}$$

$$\sim 2M_5^3 \int_0^{3/4c} dx_5 \sqrt{\left|\tfrac{4}{3}x_5 - c\right|} \;=\; \underline{\underline{\text{finite}}}$$

$\therefore$ $\exists$ 4d gravity coupled to SM on $x_5 = 0$ brane

<u>Three Important Issues:</u>

1) What about bulk quantum corrections?

Expect them to exist. However, one can show that:

- SM brane scale $\sim$ TeV $\oplus$ $M_4 \sim 10^{19}$ GeV $\Rightarrow$!

$$M_5 \simeq 10^5 \text{ TeV} \gg \text{SM scale}$$

Now, "bulk" corrections to the 4d theory will be suppressed by powers of the bulk scale →

suppression factor $\left(\frac{\text{TeV}}{M_5}\right)$

- Too large for experiment
- <u>Much</u> smaller than SM contribution $(\text{TeV})^4$

We are only trying to explain the absence of leading SM contribution

2) What about <u>curved</u> 4d domain walls w/ 4d gravity

- For "fine tuned" $F(\phi)$ [eg $V e^{\pm\frac{4}{3}\phi}$] ∃ <u>no</u> <u>maximally</u> <u>symmetric</u> curved sdn's to EOM.

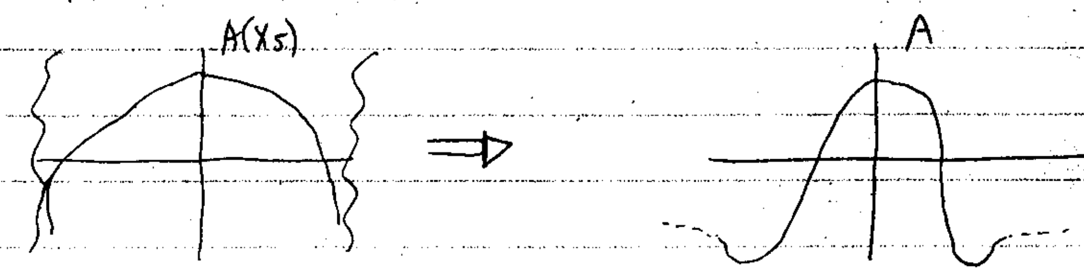
- Quantum corrections → hierarchically small curvatures (as in 1)).
- For generic $F(\phi)$, curved solns do exist with $\Lambda_{(4)} \lesssim (\text{TeV})^4$. However, they are in different superselection sectors: so it is interesting that flat ones exist, if they can be realized in a microscopic theory. For instance, today we know no pseudorealistic string model w/o SUSY w/ $\Lambda_{(4)}$ hierarchically smaller than expected.

3) What about the singularities?

It is important to find microscopic descriptions which "define" the physics of the singularities.

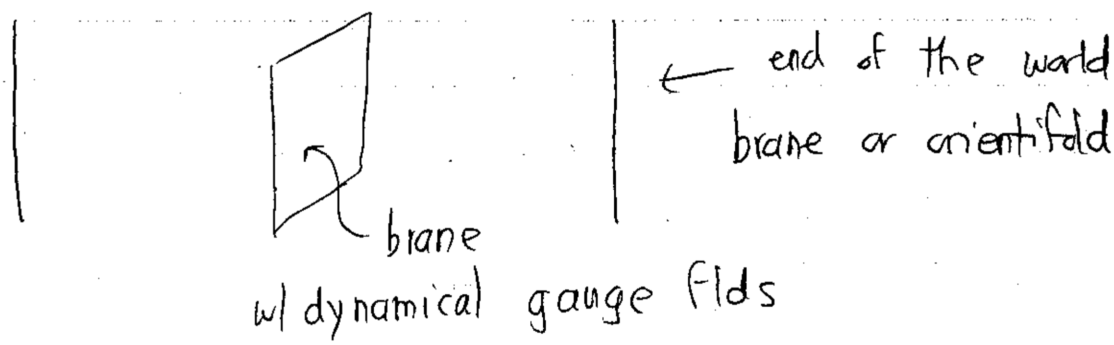
Some possibilities :

a)



Glue on a "throat" to another space ; $\exists$ approx. 4d gravity near $X_s = 0$ anyway.

b) 'Singularities' could be "orientifold-like" objects ; in type I' or Horava - Witten, $\exists$ natural scenarios :



c) Find realization in similar spirit which is smooth (see e.g. Luty et al) or where 'singularity' is hidden from physical probes (Youm ?).

Lecture #3, Trieste 2000

S. Kachru

In the next 2 lectures, we'll work up to the study of building blocks for microscopic "brane worlds" which clearly <u>are</u> realizable $\in$ string theory. They exhibit some interesting duality symmetries.

Type II Calabi-Yau Compactifications

To preserve SUSY in compactification of II A/B on a <u>smooth</u> <u>manifold</u> M of complex dimension d, one can show:

$$\text{Holonomy}(M) \subset SU(d)$$

} by requiring SUSY var'ns of fermions to vanish

is needed.

	d	theory
1 choice $\longrightarrow$	1	32 supercharges
2 choices $\longrightarrow$	2	16 supercharges
?? (> 1000s) choices $\longrightarrow$	3	8 supercharges

We'll focus on $d=3$ case $\Rightarrow$ 4d $\mathcal{N}=2$ SUSY.

These *Calabi-Yau* manifolds are:

- Ricci flat (solve Einstein's Eqn)
- Kähler (SUSY) $\rightarrow$ choose (1,1) Kähler form

The Ricci-Flat Kähler metrics come in a family of *real* dimension

$$\underset{\substack{\uparrow \\ \text{Kähler form}}}{h^{1,1}(M)} + 2\cdot \underset{\substack{\uparrow \\ \text{complex structure}}}{h^{2,1}(M)}$$

Example: Quintic in $\mathbb{P}^4$

$\mathbb{P}^4$: $(z_1, \ldots, z_5) \sim (\lambda z_1, \ldots, \lambda z_5)$ $\quad \lambda \in \mathbb{C}^*$

Quintic: $p = z_1^5 + \cdots + z_5^5 = 0$

- Complex structure deformations: representable as deformations of p modulo linear changes of variables

$$\Rightarrow \quad h^{2,1} = \frac{\cancel{5}\cdot\cancel{6}\cdot 7\cdot \overset{2}{\cancel{8}}\cdot 9}{\cancel{2}\cancel{3}\cdot\cancel{4}\cdot\cancel{5}} - 25 = \underline{\underline{101}}$$

- Kähler structure defs: just "size" of $\mathbb{P}^4$ $\Rightarrow$ $\underline{1}$ real variable

In fact, the Hodge diamond of a CY M is:

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{1,1} & & 0 & \\ 1 & & h^{2,1} & & h^{2,1} & & 1 \\ & 0 & & h^{1,1} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$

Spectrum of II A/B Theory on M:

4d $N=2$ theory has 2 kinds of light multiplets (other than gravity multiplet)

- Vector mult: Complex scalar $\oplus$ vector A^{μ} in adjoint rep of G (gauge group) + fermions
- Hyper mult: 2 complex scalars + fermions in <u>any</u> rep of G

The "moduli" of metric on M show up as scalars in such mults. in both II A/B.

- II A/B both have, in 10d, metric, $B^{NS}_{\mu\nu}$, ϕ ← dilaton
- II A also has 1-form + 3-form gauge fields C_μ, $C_{\mu\nu\rho}$
- II B has 0, 2 + 4-form C, $C_{\mu\nu}$, $C_{\mu\nu\rho\sigma}$

II A on M	II B on M
– kähler moduli, $B_{\mu\nu}$, $C_{\mu\nu\rho}$ →	– complex moduli, $C_{\mu\nu\rho\sigma}$ →
$h^{1,1}(M)$ vector mults	$h^{2,1}(M)$ vectors
– complex moduli, $C_{\mu\nu\rho}$ →	– kähler moduli, $B_{\mu\nu}$, $C_{\mu\nu}$,
$h^{2,1}(M)$ hyper mults	$C_{\mu\nu\rho\sigma}$ ⇒ $h^{1,1}(M)$ hypers
↑	↑
dilaton → $h^{2,1}+1$	dilaton → $h^{1,1}+1$

- By $N=2$ supersymmetry, several simplifications:

– no potential for scalars → "moduli spaces" of vacua

– $\mathcal{M} = \mathcal{M}_V \times \mathcal{M}_H$

metric on $\mathcal{M}_{V/H}$ indep of scalars in hyper/vector mults

	IIA	IIB
$\mathcal{M}_V$	$\dim_{\mathbb{C}} = h^{1,1}(M)$ "kähler mod space"	$\dim_{\mathbb{C}} = h^{2,1}(M)$ "C.S. moduli space"
$\mathcal{M}_H$	$\dim_{\mathbb{Q}} = h^{2,1}(M) + 1$	$\dim_{\mathbb{Q}} = h^{1,1}(M) + 1$

Structure of quantum corrections?

- String theory $\rightarrow$ 2 perturbative expansions
  - String loops, controlled by $g_s = e^{-\phi}$
  - Sigma model perturbation theory, controlled by $\frac{R^2}{\alpha'}$ (kähler moduli)

Lets consider e.g. $\mathcal{M}_V$ ($\mathcal{M}_H$ not well understood)

$\exists$ a holomorphic prepotential $F(\phi_i)$

$\rightsquigarrow$ gauge coupling functions, metric on $\mathcal{M}_V$

- In IIB, $\mathcal{M}_V$ indep of kähler moduli + g_s $\Rightarrow$ <u>exact</u> at σ-model tree level $\Rightarrow$ <u>computable</u> in <u>classical geometry</u>:

Ω = hol 3-form on CY

i, j, k index directions in Complex Str moduli space

$$\boxed{\frac{\partial^3 F}{\partial \phi_i \, \partial \phi_j \, \partial \phi_k} \sim \int_M \Omega \wedge (\partial_i \partial_j \partial_k \Omega)}$$

- In IIA, more complicated story.
  - kähler moduli $\in$ vector mults $\Rightarrow$ $\exists$ σ-model quantum corrections
  - dilaton still in hyper $\Rightarrow$ no g_s corrections

However, considerations of holomorphy (SUSY) $\Rightarrow$

$$\partial^3 F = \text{tree-level} + \text{non-perturbative in } \frac{R^2}{\alpha'}$$

Non-perturbative configs = "worldsheet instantons",

worldsheet $\mathbb{P}^1$ wraps curve in CY_3.

Formula: Vector mult scalars $\phi_i \leftrightarrow$ 4-cycles H_i
(2-forms b_i)

$$\frac{\partial^3 F}{\partial\phi_i\,\partial\phi_j\,\partial\phi_k} \underset{\substack{\text{large}\\ \text{radius}}}{\cong} H_i \cap H_j \cap H_k$$

$$\frac{\partial^3 F}{\partial\phi_i\,\partial\phi_j\,\partial\phi_k} = \int_M b_i \wedge b_j \wedge b_k + \sum_{\substack{\text{curves}\\ \text{through}\\ H_{i,j,k}}} C \int_C b_i \int_C b_j \int_C b_k \times e^{-\text{Area}(C)/\alpha'}$$

Mirror Symmetry: Says CY manifolds naturally come in pairs M, W with

$$\boxed{\text{IIA on } M \equiv \text{IIB on } W}$$

kähler / complex moduli of $M \leftrightarrow$ complex / kähler moduli of W

"Proof" (following SYZ) [really more of a construction]

Say IIA on M = IIB on W

Then full theories, including BPS branes + their properties, should match.

SUSY branes allowed by CY geometry:

- Can wrap "holomorphic" 0, 2, 4, 6 cycles
- Can wrap "special Lagrangian" 3-cycles

$$\Sigma \subset M \; : \; \omega|_\Sigma = 0,$$

$$\mathrm{Im}\, \Omega|_\Sigma = 0$$

Want particles in $\mathbb{R}^{3,1}$ $\to$

- Can use 0, 2, 4, 6 branes in IIA
- Can use 3 branes in IIB

Lets say we chose to study IIA 0-brane on M.

It yields a SUSY quantum mechanics with moduli spac

$$\mathcal{M}(D0 \text{ on } M) = M$$

So, if IIB on W = IIA on M, need a brane

whose moduli space $= M$!

- No choice - must be D3 on SUSY 3-cycle $\Sigma \subset W$

Properties of Σ? In particular, need

$$\dim_{\mathbb{C}} (\text{moduli of wrapped D3}) = 3$$

Now, McLean's theorem $\rightarrow$ Σ has $b_1(\Sigma)$ moduli

D-brane Wilson lines $\rightarrow$ $b_1(\Sigma)$ more from Wilson lines

$$\dim_{\mathbb{C}} (\text{D3 on } \Sigma \text{ moduli space}) = b_1(\Sigma)$$

Thus, must have $b_1(\Sigma) = 3$.

But, fixing pt in moduli of Σ, Wilson lines $\rightarrow$ a T^3.

Thus, M = a T^3 fibration!

- Obviously, switching role of M & W would $\Rightarrow$
  W = a T^3 fibration.

So, why not say Σ = the SUSY $T^3 \subset W$!

Then, "T-duality" on all 3-circles in Σ:

- Changes IIA $\leftrightarrow$ IIB ✓
- Takes wrapped D3 to D0 ✓

So: $\boxed{\text{CY manifolds w/ mirrors are } T^3 \text{ fibrations; "T-dualize" } T^3\text{s} \to \text{mirror geometry}}$ } hard to really see this in practice

<u>Example</u>: $T^2 = S^1_{R_1} \times S^1_{R_2}$ } "T^1" fibration over $S^1_{R_2}$

Complex structure modulus: $\tau \sim \frac{R_2}{R_1}$

Kähler structure ρ = volume = $R_1 R_2$

T-dualize along $S^1_{R_1}$ $\Rightarrow$

- define $R_1' = \dfrac{1}{R_1}$

Then $\tau_{new} = \dfrac{R_2}{R_1'} = R_2 R_1 = \rho_{old}$

$\rho_{new} = R_1' R_2 = \dfrac{R_2}{R_1} = \tau_{old}$

So we have taken

$$\begin{array}{cc} (\tau, \rho) & \text{IIA} \\ \downarrow & \\ (\rho, \tau) & \text{IIB} \end{array} \quad \checkmark$$

This is mirror symmetry for T^2.

- Can also do this successfully with K3 viewing it as a T^2 fibration.

Lecture #4, Trieste 2000

S. Kachr

Open Strings & Mirror Symmetry

KKLM 9912151
BDLR 9906200

Last time, we saw: IIA on M = IIB on W

This $\Rightarrow$ powerful tool for study 4d $N=2$ string vacua.

- Can also construct 4d $N=1$ "brane world" models out of Calabi-Yau spaces, by wrapping $(p+3)$ branes on SUSY p-cycles. What does mirror symmetry do for us here?

Type IIA CY brane worlds

- Only possibility: wrap D6 brane(s) on SUSY 3-cycle $\Sigma \subset$ Calabi-Yau M

SUSY 3-cycle Σ $\Rightarrow$

- $\omega|_\Sigma = 0$ ω = Kähler form on M
- $\mathrm{Im}\,\Omega|_\Sigma = 0$ Ω = hol (3,0) form on M

How to produce concrete examples of Σ ?

Take CY 3-fold w/ local complex coords $z_{1,2,3}$,

chosen so that : $\omega \sim \sum dz_i \wedge d\bar{z}_i$

$$\Omega \sim dz_1 \wedge dz_2 \wedge dz_3$$

Consider the fixed point locus of the real involution

$$z_i \to \bar{z}_i$$

$\to$ along this locus $z_i = \bar{z}_i$.

But under $z_i \to \bar{z}_i$ it is clear that :

- $\omega \to -\omega \quad \Rightarrow \quad \boxed{\omega|_{\text{fixed pts}} = 0}$
- $\Omega \to \bar{\Omega}$ so $\Omega - \bar{\Omega} = 0$ along fixed locus

$$\to \boxed{\mathrm{Im}\,\Omega|_{\text{fixed pts}} = 0}$$

So the fixed pt locus of real involution is SLAG

Lets give an example :

Work in $W\mathbb{P}^4[1,1,2,2,2]$

$$p = X_1^8 + X_2^8 + X_3^4 + X_4^4 + X_5^4 - 2(1+\varepsilon) X_1^4 X_2^4$$

defines a CY hypersurface

NOTE : $p = dp = 0$ has sol'ns when $\varepsilon = 0 \rightarrow$ a singular Calabi-Yau then)

Fixed point locus under $X_i \rightarrow \bar{X}_i$?

Let $u \equiv X_1^4$, work in $X_2 = 1$ patch WLOG.

$$p = 0 \Rightarrow u^2 - 2(1+\varepsilon)u + 1 + Q = 0$$

where $Q \equiv X_3^4 + X_4^4 + X_5^4$

Solutions ? $u_\pm = 1 + \varepsilon \pm \sqrt{\varepsilon^2 + 2\varepsilon - Q}$

So whats the point ? $Q < \varepsilon \Rightarrow$ 3-ball B_3

$Q = \varepsilon \Rightarrow$ "2 - sphere" S^2 (up to coverings)

The U-branches are glued together on the 2-sphere $\Rightarrow$ get two B_3s glued on a $S^2 = S^3$!

- Note that this S^3 shrinks to a point as $\varepsilon \to 0$

$\Rightarrow$ singularity is related to collapsing 3-cycle!

Field theory on wrapped D6 brane?

massless scalars?

McLean : "Geometrical" moduli space of Σ has (unobstructed) real dimension $b_1(N)$.

String theory complexifies this w/ Wilson lines.

Coordinates on moduli space?

* Choose a basis $\{\gamma_j\}$ for $H_1(\Sigma)$
* Choose discs D_j w/ $\partial D_j = \gamma_j$

⑤

* Define $W_j = \int_{D_j} \omega$ = "area" of holomorphic disc in D_js relative homology class

To complexify this:

b_1 Wilson lines $\int_{\gamma_j} A \Rightarrow$

$b_1(\Sigma)$ chiral flds $\phi_j = \frac{W_j}{\alpha'} + i a_j + \cdots$

For $\mathcal{N}=1$ SUSY QFT, natural "exactly computable" quantity is superpotential $W(\phi)$ [governs SUSY breaking, etc.]

What is $W(\phi_j)$?

- None to all orders in α'

Proof: a_j has a shift symmetry $\Rightarrow$ W indep of a_j to all orders in α' $\Rightarrow$ $W(\phi)$ has no poly. term:

<u>But</u>: $$e^{-(w_j + i a_j)} = e^{-\phi_j}$$

type terms are allowed to appear.

<u>Source</u>: "Disc Instantons"

Holomorphic maps $D \to M$, $\partial D = \gamma_j \subset \Sigma$

$\partial_n X(\sigma)$ at bdry in pullback of normal bundle to Σ

<u>So</u>: IIA D6 brane superpotentials are <u>purely</u> non-perturbative in α'!

Can compute e.g. $\phi \phi F$ couplings

$$\langle V_F{}^i \, V_\phi{}^j \, V_\phi{}^k \rangle$$

Denote by $d^{\{n_a\}}_{\{m_e\}}(i,j,k)$ = # maps $D \to M$ with

i) $[\partial D] = \sum m_e \gamma_e$

ii) $V^{i,j,k}$ mapped in cyclic order to intersection of ∂D with 2-cycles dual to $\gamma_{i,j,k}$

iii) $[D - \sum m_\ell D_\ell] =$ (closed 2 cycle in M)

$$= \sum_a n_a K_a \leftarrow \text{basis for } H_2(M)$$

Then:

harmonic 1-form associated to γ_i

$$\langle V_F^{\,i} \, V_\phi^{\,j} \, V_\phi^{\,k} \rangle \sim \sum_{m_\ell, n_a \geq 0} \int_{\partial D} \theta^i \int_{\partial D} \theta^j \int_{\partial D} \theta^k$$

$$d^{\{n_a\}}_{\{m_\ell\}}(i,j,k) \prod_{\ell=1}^{b_1(\Sigma)} e^{-m_\ell \phi_\ell} \prod_{a=1}^{h^{1,1}(M)} e^{-n_a t_a}$$

$$t^a \equiv \text{"Area" of } K_a = \int_{K_a} \omega$$

How do <u>closed string</u> moduli of M enter in brane theory?

- From above, see $W = W[t_a] \leftarrow$ depends on Kähler moduli

- BDLR argue that $\begin{cases} \text{Kähler} \\ \text{Complex} \end{cases}$ moduli of M only enter in $\begin{cases} \text{superpotential} \\ \text{D-terms} \end{cases}$ for branes wrapping Σ.

Type IIB on CY : Brane 'mirror' worlds

This side is, in some cases, much easier.

IIA	$\xleftrightarrow{\text{mirror}}$	IIB
6 brane on Σ		combination of D3, 5, 7, 9 on SUSY 0, 2, 4, 6 cycles

Focus for simplicity on Σ s.t. mirror is just

D5 on a curve $C = \mathbb{P}^1$.

- # massless fields? Given by small defs $\approx \dim H^0(N|_C)$

By classical deformation theory, deformations of $C \subset W$

can be obstructed. N^{th} order obstruction $\Rightarrow$

$$W = \Phi^{N+2} \quad \text{roughly.}$$

[assuming $\dim H^0(N|_C) = 1$ above]

Closed string moduli of mirror manifold W enter as

parameters in the wrapped D5 theory:

$$\begin{array}{ccc} \text{Complex Str} & & W(\Phi) \\ & \longrightarrow & \\ \text{Kähler Str} & & \text{D-terms} \end{array}$$

In examples, the moduli space of C can exhibit very intricate behavior as you vary complex structure parameters $\{\psi_i\}$ of W:

$$\mathcal{M}(C;\{\psi_i\}) \rightarrow W(\phi;\{\psi_i\})$$

<u>Fact</u>: $W(\phi;\{\psi_i\})$ is computable in the topological B-model $\Rightarrow$ it receives <u>no</u> instanton corrections & is <u>tree level</u> in α'.

Therefore, the IIA disc instanton sums should map over to <u>purely</u> <u>classical</u> geometrical data on IIB side in a mirror symmetric story:

$$W(\Phi_{IIA}\,;\,\{t_i\}) = W(\Phi_{IIB}\,;\,\{\psi_i\})$$

related by mirror map

Superpotential of D6 on Σ as function of kähler moduli of M

Superpotential of D5 on C as function of Complex moduli of W

How to match Σs with Cs in practice?

- Choose vanishing cycles Σ, mirror C = curve which collapses at mirror pt in moduli of W

Should 'eventually' be a powerful disc enumeration tool [just as closed string mirror symmetry enumerates rational curves].