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SMR.1221 - 8

SPRING WORKSHOP ON SUPERSTRINGS AND RELATED MATTERS

27 March - 4 April 2000

THE LARGE N LIMIT OF FIELD THEORIES AND GRAVITY

Lectures III, IV & V

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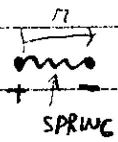
MYERS EFFECT

MYERS hep-th/9910053

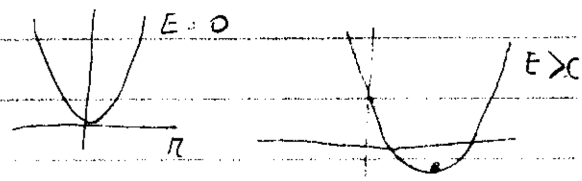
- BRANES IN TRANSVERSE RR OR NS-NS FIELDS EXPAND

- ELECTRIC DIPOLES



→ IN AN ELECTRIC FIELD THIS NEUTRAL OBJECT → EXPAND

$$V(r) = \tfrac{1}{2} k r^2 - qEr$$



- DO BRANE IN AN F_4 FIELD STRENGTH

D0 F_4 $\quad$ $F_4 = F_{0123}$

- FIRST $F_4 = 0$
- THINK OF D0 AS A COLLAPSED D2.

$$S_{D2} = \int dt\, d^2\sigma_2 \sqrt{\det(g_{ind} - F)} =$$

$$\int F = m = \text{D0 CHARGE}$$

$$S_{D2} = \int dt\, d\sigma_2 \sqrt{r^4 + m^2}$$

THIS IS MINIMIZED IF $r=0$

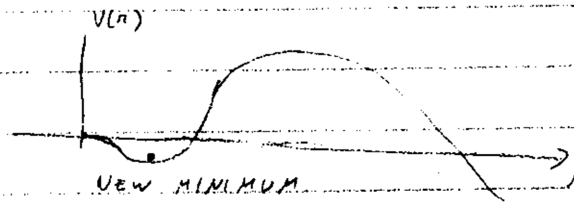
ENERGY $\sim m$ $\rightarrow$ D0 BRANE

. NOW $F_4 = f\, dt\, \varepsilon_{ijk}$

$$S_{D2} = \int dt \, \sqrt{r^4 + m^2} - \underbrace{\int A}_{\int_{B_3 \times \mathrm{TIME}} F_4}$$

$$S_{D2} = \int dt \, \sqrt{r^4 + m^2} - f r^3$$

$V_{min} \sim -f^4 m^3$



$\rightarrow r_{min} = fm$ $\Rightarrow$ THIS ANALYSIS VALID IF $r_{min} \gg \sqrt{\alpha'}$

THE CORRECT ANALYSIS FOR SMALL N

. D0 BRANES

. NEW COUPLINGS TO D-BRANES

$$\int d\sigma^{p+1} A_{0,\dots,p} \wedge F_{..} \xrightarrow[\text{T-DUALITY}]{} \int d\sigma^{p+1} \mathrm{Tr}\left[A_{0 \dots p IJ} \, [X_I, X_J] \right]$$

MORE PRECISELY

$$H_3^{RR} = f \varepsilon_{3\, ijk} \Rightarrow B^{RR}_{2\, ij} = \varepsilon_{ijk} X^k$$

NEW COUPLING ON D0

$$S_{int} = f \int dt \, \mathrm{Tr} \left(\varepsilon_{ijk} X^k [X^i, X^j] \right)$$

. WE CAN NOW MINIMIZE

THE POTENTIAL ON D0 BRANES

$$V = \mathrm{Tr}\Big[\sum_{I,J} [X^I, X^J]^2\Big] + f\, \varepsilon_{ijk}\, \mathrm{Tr}\big(X_i [X_j, X_k]\big)$$

USUAL

EQUATIONS OF MOTION

$$[X^k, [X^k, X^i]] + f \varepsilon^{ijk} [X^j, X^k] = 0$$

ASSUME $[X^i, X^j] = -f\, \varepsilon^{ijk} X^k$ $\quad X^i \to SU(2)$ REPRESENTATI

$X^i = f . J^i$

$\to$ SOLUTION

. NOTICE THAT IF WE CONSIDER COMMUTING MATRICES $\to$ WE ALWAYS HAVE $V > 0$, BUT WITH THE NON COMMUTING MATRICES WE CAN HAVE $V < 0$.

$\to$ THE POTENTIAL IS SMALLEST IF WE CHOOSE A REPRESENTATION WITH MAXIMAL DIMENSION:

$$2j + 1 = N$$

$$[X^i, X^k]^2 \sim f^4 (J^a)^2 \longrightarrow \mathrm{Tr}\,(\vec{J}^2) \sim j(j+1)(2j+1) \sim N^3$$

$$V_{min} \sim f^4 N^3$$

(AS WE OBTAINED ABOVE)

THEORY ON D2 $\to$ NON COMMUTATIVE GAUGE THEORY (THERE IS AN F FIELD) $\to$ THE MATRICES ARE A DESCRIPTION OF THE NON COMMUTATIVE SPHERE

BRANES MOVING:

GRAVITON MOVING ON CONSTANT F_{ijkl} IN M-THEORY
$\parallel$
$h\,\varepsilon_{ijkl}$ $\quad ijkl = 1,2,3,4$

$\rightarrow$ DLCQ IN DIRECTION 1.

$\Rightarrow$ GRAVITON $\rightarrow$ $D\phi$

· F $\rightarrow$ NS 3-FORM CHARGE $H_{jkl} = H_{234}$

$$\sqrt{\left(B_{IJ} + [X_I, X_K]\right)^2} \quad \text{in BI}$$

$\rightarrow$ COUPLING $\sim$ $B_{IJ0} \cdot [X^I X^J]$

IF $B_{IJ} = h\,\varepsilon_{IJK} X^K$ $\rightarrow$ SAME EFFECT

$\Rightarrow$ $D\phi$ $\rightarrow$ EXPAND TO SPHERICAL D2 BRANE IN DIRECTIONS 234

$\Rightarrow$ M2 BRANE IN M-THEORY

$\Rightarrow$ BACKGROUND IN 11-D IS NOT LORENTZ INVARIANT $\Rightarrow$

$\Rightarrow$ GET AN "ELECTRIC" FIELD WHEN WE BOOST

$\tilde{F}_{0234}$ $\rightarrow$ MAKES THE GRAVITON EXPAND INTO AN M2

· IF WE HAVE $AdS_7 \times S_4$ $\rightarrow$ KK PARTICLES WITH LARGE ANGULAR MOMENTUM $\rightarrow$ BECOME M2 BRANES.
$\hat{F}$

• ENHANÇON

hep-th/9911161
JOHNSON, PEET, POLCHINSKI

• D1-D5 system

$$dS^2 = Z_1^{-1/2} Z_5^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + Z_1^{1/2} Z_5^{1/2} dx^i dx^i + Z_1^{1/2} Z_5^{-1/2} dy^m dy^m$$

$$Z_1 = 1 + \frac{Q_1}{V r^2} \qquad \text{(I AM OMITTING } \alpha'\text{'s \& } \pi\text{'s)}$$

$$Z_2 = 1 + \frac{Q_5}{r^2}$$

$$\tau = \frac{1}{g} (Q_5 V + Q_1)$$

WE CAN HAVE $Q_1 < 0$ IN THESE FORMULAS

→ NOT THE SAME AS REPLACING BY ANTI BRANES

• E.g. K3 → INTERNAL SPACE

⇒ D5 → INDUCED (-1) D1 BRANE CHARGE

$$Q_1 \geq - Q_5$$

$Q_1 = - Q_5$ → NO EXPLICIT D1's

LOW ENERGY THEORY ON D5

→ $U(Q_5)$ $\mathcal{N}=2$ SYM WITHOUT ADJOINT HYPERS.

→ GO TO D2 - D6 TO AVOID PROBLEMS WITH IR. STABILITY (SO THAT WE HAVE A MODULI SPACE)

$$ds^2 = Z_2^{-1/2} Z_6^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + Z_2^{1/2} Z_6^{1/2} dx^i dx^i + V^{1/2} Z_2^{1/2} Z_6^{-1/2} ds^2_{K3}$$

$$Z_2 = 1 - \frac{gN}{2V}\frac{1}{r}$$

$$Z_6 = 1 + \frac{gN}{2r}$$

$$V(r) = V\frac{Z_2}{Z_6}$$

REPULSON SINGULARITY:

$$Z_2 \to 0 \Rightarrow G_{00} \to \infty$$

grav. POTENTIAL

CONSIDER D6 PROBE

$$S = \int e^{-\phi}\big(\underset{\substack{\uparrow \\ \text{VOLUME}}}{V(r)} - 1\big) + \int C_7 - \int C_3$$

EXPAND IN VELOCITIES

$$\mathcal{L} = \text{POSITION DEPENDENT TERMS} + v^2\left(V - 1 - \frac{gN}{r}\right)$$

$V = 1$ = SELF DUAL VOLUME

→ $V > 1$ AT ∞ SO THAT TENSION IS POSITIVE

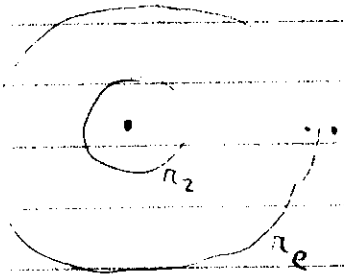
AT SOME r_e → TENSION BECOMES ZERO

$$r_e = \frac{2V}{V-1} r_2 \qquad r_e > r_2 = \frac{gN}{2V}$$

r_2: r OF REPULSON SINGULARITY

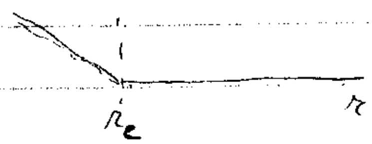
AT THIS POINT THE TENSION VANISHES AND IF WE

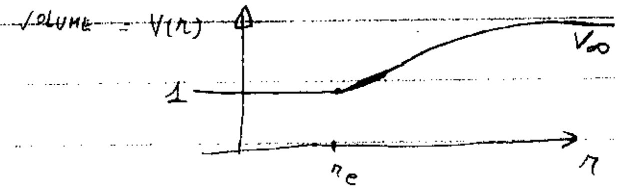
~~WANT TO CONTINUE~~



→ $\left| V Z_2 - \mu_2 Z_6 \right|$

⇒ NO MORE CANCELLATION OF FORCES ⇒ POTEN





⇒ WE CANNOT BRING D6 BRANES CLOSER THAN r_e

⇒ FLAT GEOMETRY INSIDE

→ INSIDE THIS FLAT GEOMETRY $V = V_* = 1$ → TENSIONLESS BRANE ?

→ WE SHOULD REMEMBER THAT AT THIS VOLUME WE HAVE AN ENHANCED SU(2) GAUGE SYMMETRY (COMING FROM D4 WRAPPING THE K3). THE D6 IS A MAGNETIC MONOPOLE IN THIS THEORY → HAS SOME SIZE → CORE OF THE MONOPOLE → RESTORED GAUGE SYMM

⇒ THE PROBE EXPANDS AS WE APPROACH r_e

→ RESOLUTION OF THE SINGULARITY

• MASS DEFORMATIONS OF $N=1$ SYM.

POLCHINSKI & ~~STRASSLER~~ hep-th/0003136

. ADD OPERATOR $\Delta_3 = \psi\psi + (\Phi[\phi,\phi])$

→ MASS FERMIONS → MASS FOR BOSONS

&

. SUSY DEFORMATION

$N=4$ SYM → $W = Tr\left([\phi_1, \phi_2]\phi_3\right)$

$\phi_1 = X_1 + iX_2$

$\phi_2 = X_3 + iX_4$

etc.

ADD +

$$\Delta W = m\, Tr\left(\phi_1^2 + \phi_2^2 + \phi_3^2\right)$$

(NOTICE THAT THIS ADD TERMS IN THE BOSONIC LAGRANGIAN OF THE FORM

$$Tr\Big[\underbrace{[X,X]^2}_{N=4\ SYM} + X[X,X] + m^2X^2\Big]\Big)$$

THE THEORY HAS ISOLATED VACUA
SUSY MINIMA .

$$\partial_i W = 0$$

$$\Rightarrow [\phi_i, \phi_j] = -m\,\varepsilon_{ijk}\,\phi_k$$

$\phi_i = 0$ → ONE VACUUM

$\phi_i =$ SU(2) REPRESENTATIONS

. EACH VACUUM $\rightarrow \sum_{d=1}^{N} d k_d = N$

$\downarrow$

~~NUMBER OF COPIES OF d-DIMENSIONAL SU(2) REPRESENTATION.~~

. HIGGS VACUUM $\rightarrow$ SU(N) TOTALLY BROKEN , $k_N = 1$

. MASSIVE VACUUM $\rightarrow$ NO UNBROKEN U(1)

. COULOMB VACUA $\rightarrow$ SOME UNBROKEN U(1)'s

. LARGE NUMBER OF VACUA $\rightarrow$ NUMBER $\sim e^{\text{const.}\sqrt{N}}$

. THE MASS DEFORMATION OF $\mathcal{N}=4$ SYM

$\rightarrow$ CORRESPONDS TO TURNING ON A COMBINATION OF H_{RR} & H'_{NS}

$$G_3 = i H_{RR} - \hat{\tau} H_{NS}.$$

. FERMION MASSES

$\lambda_\alpha \quad \alpha \in SU(4) \qquad m^{\alpha\beta} \lambda_\alpha \lambda_\beta$

$$m = \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \rightarrow SU(3)$$

$4 \times 4 = 10$ OF SO(6)

$\rightarrow m \rightarrow \overline{10} \rightarrow$ SELF DUAL TENSOR IN 6-d (COMPLEX)

$* T_{mnp} = i T$

$\frac{6 \cdot 5 \cdot 4}{3 \cdot 2} \cdot \frac{1}{2} = 10$ COMPLEX COMPONENTS

sf duality

SO WE SEE THAT THE MASS TERM TRANSFORM AS T_{mnp}.

A MORE PRECISE ANALYSIS SHOWS THAT IN OUR CASE WE HAVE

$$T_{\bar{i}\bar{j}k} = T_{i\bar{j}\bar{k}} = T_{\bar{i}j\bar{k}} = m\,\varepsilon_{ijk}$$

$$*_6 T = -iT$$

→ FERMION MASS TERM → ASSOCIATED WITH LOWEST KK HARMONIC OF T.

D3 T FIELD

. WE HAVE D3 BRANES IN A CONSTANT T FIELD.

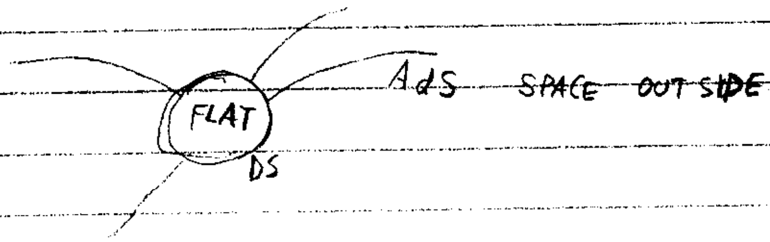
→ MYERS EFFECT ⇒ GROWS TO D5 BRANES, IN THIS SUSY CASE ALL D5 BRANES HAVE EQUAL ENERGIES; i.e. → SINGLE D5 OR MULTIPLE D5

→ IT CAN ALSO GROW INTO NS5 BRANES

PROPOSED SUGRA SOLUTION

. E.G. SPHERICAL DS in 789 S^2 $(H^{RR}_{456} = H^{NS}_{789})$
(OR AN NS-5 ALONG 456)
→ START FROM D3'S → ARRANGE THEM ON THE HIGGS BRANCH SO THAT THEY ARE ON THIS S^2 → PERTURB THE SOLUTION PUTTING $G_3 \neq 0$
→ NEAR THE BRANES → LOOK FOR DS ~~TYPE SOLUTION.~~

END RESULT:



. SINGULARITY GETS RESOLVED BY THE PRESENCE OF A BRANE.

. IT IS AN NS-5 IN THE CASE OF $\phi = 0$

. SL(2, R) WZW & strings in AdS_3

. AdS_3 x SOMETHING

~~WITH NS- BACKGROUND $H_3 = k\epsilon_3$ = VOLUME FORM IN AdS_3~~

. WHY?

. UNDERSTAND STRINGS ON CURVED SPACE-TIMES
$g_{\mu\nu}$ FACTOR IS NONTRIVIAL

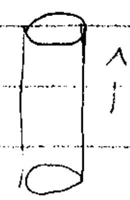
. "EXACTLY SOLVABLE"

. ~~BLACK HOLES → BTZ → $AdS_3/\mathbb{Z}$~~ QUOTIENTS

→ UNDERSTAND STRINGS & HORIZONS

. ~~HERE ALL WE WILL DO WILL BE TO DESCRIBE~~
THE SPECTRUM.

. AdS_3 → SL(2, R) GROUP MANIFOLD → MULTIPLE COVER



TIME = ~~NON-COMPACT~~

. WE WILL WORK IN GLOBAL COORDINATES
→ ~~UNDERSTAND THE CASE WITH~~
~~NO HORIZON~~ FIRST

. $$S = k \int \mathrm{Tr}\left[g^{-1}\partial_\alpha g \, g^{-1}\partial^\alpha g\right] + k S_{WZ}$$

- IN $AdS_3 \times S_3 \times K$: k = # of NS 5

WE TAKE Q_1 = # of FUND STRINGS

$g^2_{string} \sim k/Q_1 \to 0$

- LARGE Q_1 LIMIT

- SPECTRUM

IN GENERAL IN WZW MODELS

- CURRENT ALGEBRAS J^a_{Rm}, J^a_{Lm}

$$J^a_R(x^+) \sim Tr[T^a \partial_+ g g^{-1}]$$

$$J^a_L(x^-) \sim Tr[T^a g^{-1} \partial_- g]$$

- SIMMETRY

$$g \to h_L(x^+) \, g \, h_R(x^-)$$

- SPECTRUM IN THE QUANTUM THEORY

$$[J^a_n, J^b_m] = f^{abc} J^c_{n+m} + kn \, \eta^{ab} \delta_{n+m}$$

- HIGHEST WEIGHT REPRESENTATIONS

$J^a_n |\Psi\rangle = 0 \quad n > 0$

$J^a_0 |\Psi\rangle \Rightarrow$ REPRESENT OF THE J^a_0 ALGEBRA

- IN $SL(2,\mathbb{R})$:

• REPRESENTATIONS OF $SL(2,R)$

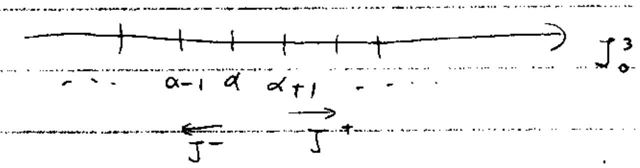
→ FUNCTIONS ON AdS_3 $\left(\mathcal{L}_2(AdS_3)\right)$

• CASIMIR $-J_3^2 + J_1^2 + J_2^2 = C_2$ $J^\pm = J_1 \pm i J_2$

• J_0^3 = ENERGY IN AdS_3 (ANG. MOM.) $J^3_{0R} = E + \ell$, $J^3_{0R} = E - \ell$

• CONTINUOUS

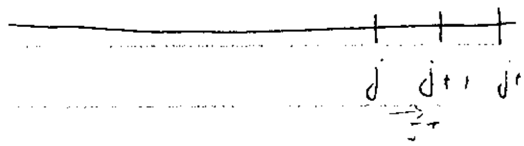
$$C_2 = \frac{1}{4} + s^2$$



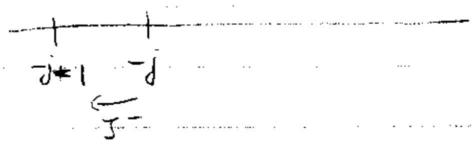
• DISCRETE

$$C_2 = -j(j-1)$$

(LOWEST ENERGY)



HIGHEST ENERGY



~~⊕~~ CASIMIR → LAPLACIAN ON WAVE FUNCTIONS

→ DISCRETE → OBEY BREITENLOHNER FREEDMAN BOUND $m^2 > -1$

CONTINUOUS → TACHYONS DO NOT OBEY ↑

. SPECTRUM → START WITH THE ABOVE AS HIGHEST WEIGHT REPRESENTATIONS

$J^3_{-m}|j\rangle$ → CREATES NEGATIVE NORM STATES
→ TEMPORAL OSCILLATIONS

. SHOULD BE REMOVED BY THE VIRASORO CONDITIONS

$L^T_m |\Psi\rangle = \delta_{m,0}|\Psi\rangle$

$L_m = L_m^{SL(2)} + L_m^{\text{INTERNAL CFT}}$

$c^T = 26$ (WE DO THE BOSONIC STRING FROM NOW ON

$L^{SL(2)} \sim \frac{:J^a J^a:}{k-2}$

. CONT

$L_0 = 1 \qquad \frac{\frac{1}{4} + s^3}{k-2} + N + h - 1 = 0$ → TACHYONS

. DISCR

$\frac{-j(j-1)}{k-2} + N + h - 1 = 0 \approx -\frac{j^2}{k} + N + h = 0$

UP TO FACTORS OF ORDER 1

$j \sim \underbrace{\sqrt{k(N+h)}}_{\text{MASS OF THE PARTICLE}}$

COMPARE TO

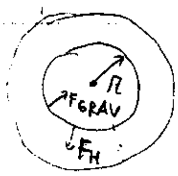
$-p_0^2 + \vec{p}^{\,2} + N + h \sim 0$

- AFTER IMPOSING THE VIRASORO CONDITIONS GHOSTS ARE REMOVED IF

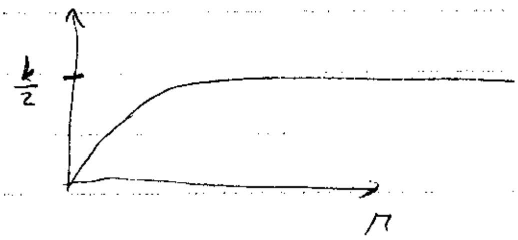
$$0 < j < \frac{k}{2}$$

- PUZZLE → WHAT IS THE ORIGIN OF THIS BOUND?
  - WHAT HAPPENS IF I HAVE A STATE WITH HIGHER MASS?

- RELATED PUZZLE: WHERE ARE THE LONG STRINGS



$E(r) \sim$



- FINITE ENERGY BARRIER FOR THE STRING TO ESCAPE TO INFINITY.

  → DUE TO THE ELECTRIC $H_{\text{or}\,\varphi}$ FIELD

- WHERE ARE THESE STATES IN THE SPECTRUM?

. SYMMETRY OF THE WZW

. USUAL $\quad h_L(x^+)\, g\, h_R(x^-) \qquad h_L \;\&\; h_R \to$ PERIODIC FUNCTIONS OF $\sigma \to \sigma + 2\pi$

. NEW: $\quad e^{w i \sigma_2 x^+}\, g\, e^{w i \sigma_2 x^-}$

$\uparrow$ TOPOLOGICALLY NONTRIVIAL IN THE GROUP.

. CALLED SPECTRAL FLOW

. ON THE CURRENTS

$$\tilde{J}^3_m = J^3_m - \frac{k}{2} w\, \delta_{m,0}$$

$$\tilde{J}^\pm_m = J^\pm_{m \pm w}$$

. VIRASORO GENERATORS

$$\tilde{L}_m = L_m + w J^3_m - \frac{k}{4} w^2 \delta_{m,0}$$

. HOW DOES THIS ACT?

. FIRST UNDERSTAND IT CLASSICALLY.

. START WITH A SIMPLE SOLUTION

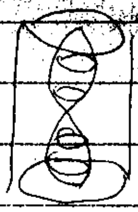
$$g \sim e^{\sigma_2 \underbrace{(x^+ + x^-)}_{c}\, \alpha/k}$$

$T^{SL(2)}_{++} = -\alpha^2/k$

$T^{TOT}_{++} = 0 = -\alpha^2/k + h = 0$ MASS SHELL COND.

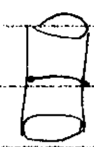
. ACT WITH GLOBAL ISOMETRIES

. ~~ACT WITH SPECTRAL~~ FLOW



$w=1$

. ~~SPACELIKE GEODESICS~~



$T^{SL(2)}_{++} \sim \alpha^2/k$ → NEVER OBEY MASS SHELL COND.

. SPECTRAL FLOW

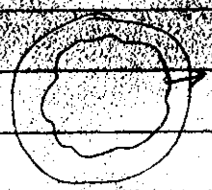
LONG STRINGS

$$T^{SL(2)}_{++} = \underbrace{\tilde{T}_{++}}^{\alpha^2/k} - w J^3_0 + \frac{k}{4} w^2$$

$$T^{SL(2)}_{++} + h = 0 \Rightarrow$$

$$\Rightarrow J^3_0 = \frac{k}{4} w + \frac{1}{w}\left(\alpha^2/k + h\right)$$

→ CORRECT ENERGY OF ~~A~~ A LONG STRING

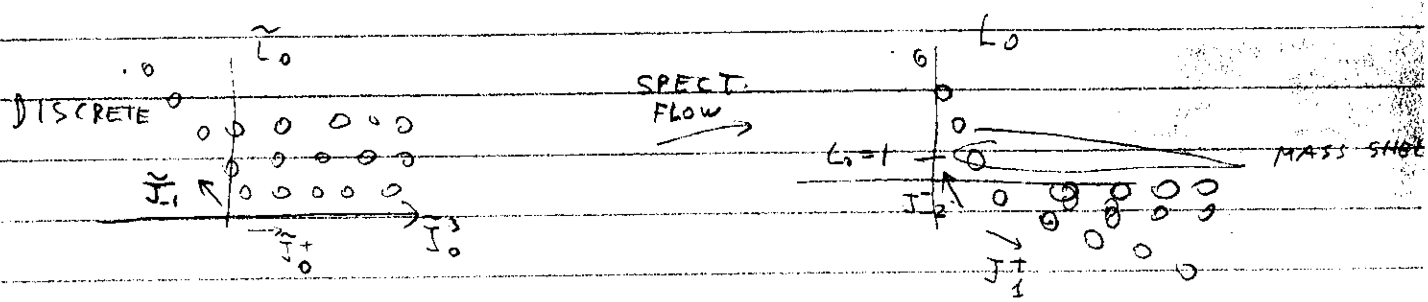


α → MOMENTUM IN RADIAL DIRECTION

• DO THIS IN THE QUANTUM THEORY

• START WITH USUAL REPRESENTATIONS OF $\tilde{J}^a_n$

→ VIEW THEM AS REPRESENTATIONS OF J^a_n



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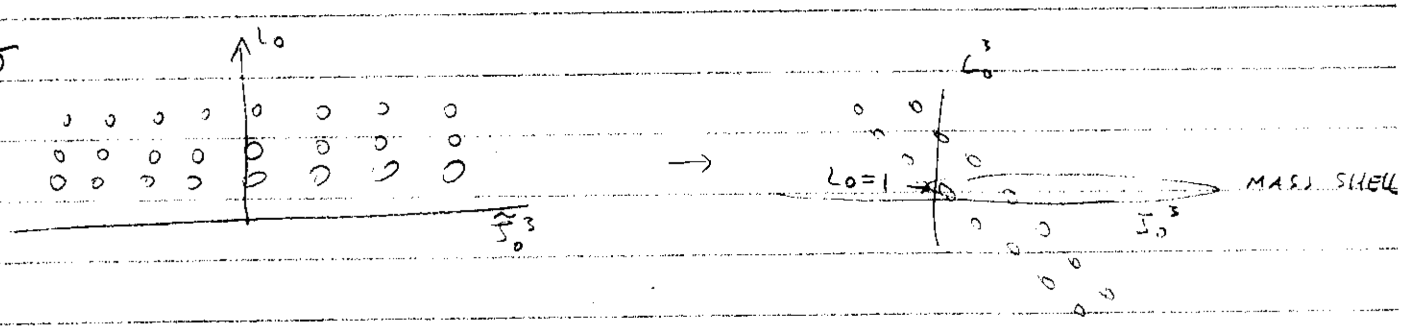


- FIND FOR CONTINUOUS

$$J_0^3 = \frac{k\omega}{4} + \frac{1}{\omega}\left(\frac{\frac{1}{4}+s^2}{k-2} + N + h - 1\right)$$

- DISCRETE

$$J_0^3 = \frac{k\omega}{4} + \frac{1}{\omega}\left(\frac{-\tilde{j}(\tilde{j}-1)}{k-2} + N - h - 1\right) = \underbrace{\frac{k}{2}\omega}_{\text{SPECT. FLOW}} + \underbrace{\tilde{j} + q}_{\text{ORIGINAL } \tilde{J}_0^3}$$

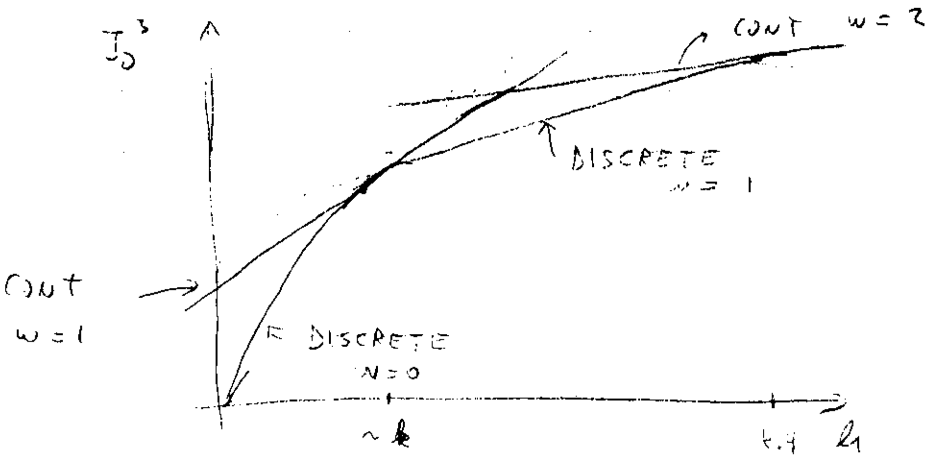
ORIGINAL $\tilde{J}_0^3$ ↓ INTEGER SHIFT FROM $\tilde{j}$

→ DETERMINE $\tilde{j}$

→ RESTRICT $\boxed{\frac{1}{2} < \tilde{j} < \frac{k}{2} - \frac{1}{2}}$

NEW CONDITION.

→ ALLOWS MASSIVE STATES ABOVE $J_0^3 \sim k$



- WE RESOLVED THE PUZZLES

- WE HAVE NOW CALCULATED INDEPENDENTLY THE ONE LOOP PARTITION FUNCTION AND WE FOUND PRECISE AGREEMENT WITH THIS SPECTRUM.

- WE COULD CALCULATE SCATTERING AMPLITUDES INVOLVING THESE LONG STRINGS.