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SPRING WORKSHOP ON SUPERSTRINGS AND RELATED MATTERS

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TACHYON POTENTIAL AND STRING FIELD THEORY

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Lectures at Trieste

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* Framework for open string field theory

* Computation of the tachyon potential

* Computation of the D-brane mass

* Closed String Field Theory. Open-closed theory

Algebraic Framework

$|\Phi_i\rangle$ CFT in states

$|\Phi\rangle = \phi^i |\Phi_i\rangle$

String field

target space

CFT

dual basis "out states" $\langle \Phi^i|$

$\langle \Phi^i | \Phi_j \rangle = \delta^i_j$

* $\langle \, , \, \rangle$ Symplectic odd bilinear form

recall $\langle c_{-1} c_0 c_1 \rangle \neq 0$

$$\langle A, B \rangle = -(-)^{AB} \langle B, A \rangle$$

* Q BRST operator $Q^2 = 0$

$$\langle QA, B \rangle = -(-)^A \langle A, QB \rangle$$

* BPZ - conjugation $|A\rangle \longmapsto \langle bpz(A)|$

$$\langle A, B \rangle = \langle bpz(A) | B \rangle$$

* Hermitian conjugation $|A\rangle \longrightarrow \langle hc(A)|$

$$\overline{\langle hc(A) | B \rangle} = \langle hc(B) | A \rangle$$

$$hc\,(Q|A\rangle) = \langle hc(A) | Q$$

* Star conjugation $\quad * = hc^{-1} \circ bpz \quad (= hc \circ bpz^{-1})$

$$\overline{\langle A, B \rangle} = \langle B^*, A^* \rangle$$

Reality condition on string field $\Phi^* = \Phi$ will make

$$\langle \Phi, Q\Phi \rangle \text{ real}$$

<u>Sequences of products</u>

$$b_1(A) = QA$$

$$b_2(A, B) = A * B \equiv AB$$

$$b_3(A, B, C) = (ABC)$$

$$\vdots$$

(i) $b_1 \circ b_1 = 0$

(ii) $b_1 \circ b_2 + b_2 \circ (b_1 \otimes \mathbb{1} + \mathbb{1} \otimes b_1) = 0$

(iii) $b_1 \circ b_3 + b_2 \circ (b_2 \otimes \mathbb{1} + \mathbb{1} \otimes b_2)$
$\quad + b_3 \circ (b_1 \otimes \mathbb{1} \otimes \mathbb{1} + \mathbb{1} \otimes b_1 \otimes \mathbb{1} + \mathbb{1} \otimes \mathbb{1} \otimes b_1) = 0$

$\rightarrow$ (i) $Q^2 A = 0$

derivation

(ii) $Q(AB) + (QA)B + (-)^A A(QB) = 0$

(iii) $Q(ABC) + (AB)C + (-)^A A(BC)$
$\quad + ((QA)BC) + (-)^A (A(QB)C) + (-)^{A+B}(A, B(QC)) = 0$

Cyclicity:

$$\langle A_1, b_n(A_2, \ldots A_{n+1})\rangle = \underline{\text{sign}} \langle A_2, b_n(A_3, A_4 \ldots A_1)\rangle$$

<u>String action</u>:

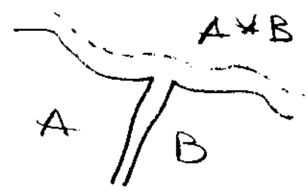
$$S = \sum_{n=1}^{\infty} \frac{1}{n+1} \langle \Phi, b_n(\Phi, \ldots \Phi)\rangle$$

$$= \frac{1}{2}\langle \Phi, Q\Phi\rangle + \frac{1}{3}\langle \Phi, \Phi * \Phi\rangle + \ldots$$

if <u>$*$ is associative</u>

$$\boxed{S = \frac{1}{2}\langle \Phi, Q\Phi\rangle + \frac{1}{3}\langle \Phi, \Phi * \Phi\rangle}$$

Witten's open SFT



Tachyon - potential

$$S(\Phi) = -\frac{1}{g_0^2}\left(\frac{1}{2}\langle\Phi, Q\Phi\rangle + \frac{1}{3}\langle\Phi, \Phi * \Phi\rangle\right)$$

$M = \dfrac{1}{2\pi^2 g_0^2}$ = Mass of brane represented by the OSFT

$|T\rangle$ tachyon string field $\quad |T\rangle = t\, c_1|0\rangle$

$\uparrow$ $Sl(2,R)$

$$\langle 0|c_{-1}c_0c_1|0\rangle = 1$$

$$BPZ(\Phi_n) = (-)^{n+d}\,\Phi_{-n}$$

$$Q = c_0L_0 + b_0M + \tilde{Q}$$

$$V(T) = -S(T) = M(2\pi^2)\left(\frac{1}{2}\langle T, QT\rangle + \frac{1}{3}\langle T, T*T\rangle\right)$$

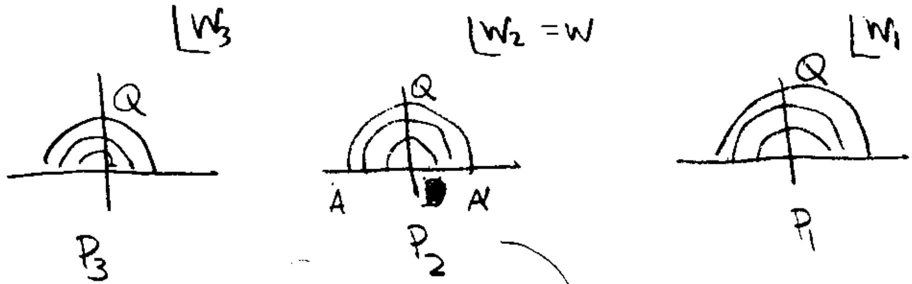
$$\frac{V(T)}{M} = (2\pi^2)\left(\frac{1}{2}\langle T, QT\rangle + \frac{1}{3}\langle T, T*T\rangle\right)$$

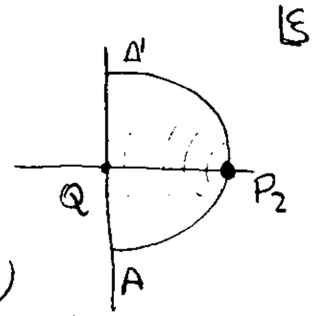
hope for $\quad \dfrac{V(T_c)}{M} = -1$

$\langle T| = t\,\langle 0|c_{-1} \quad \rightarrow \quad \langle T, QT\rangle = t^2\langle 0|c_1 c_0 L_0 c_1|0\rangle$
$= -t^2$ //

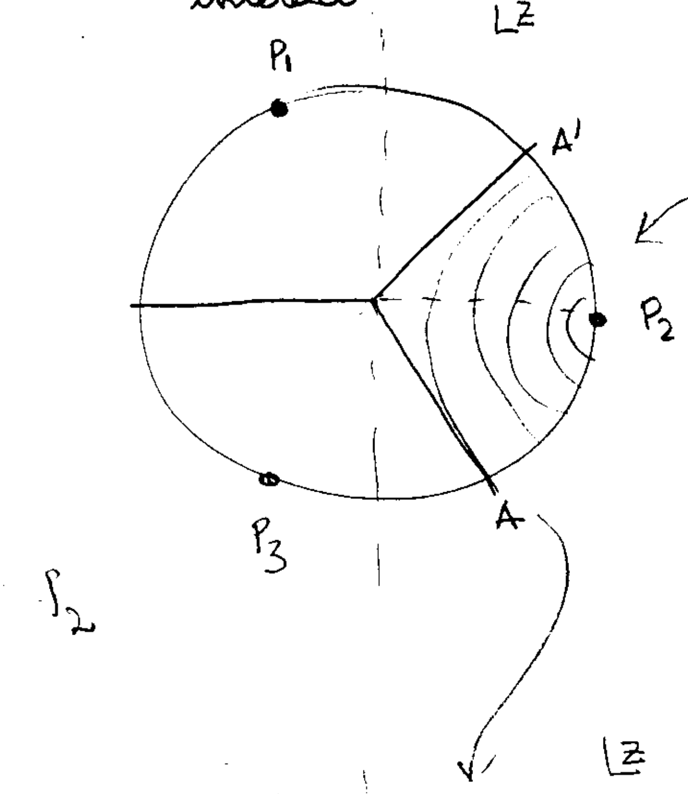
Define vertex by description of pictures:

3 - free strings - w_1, w_2, w_3





come together to interact



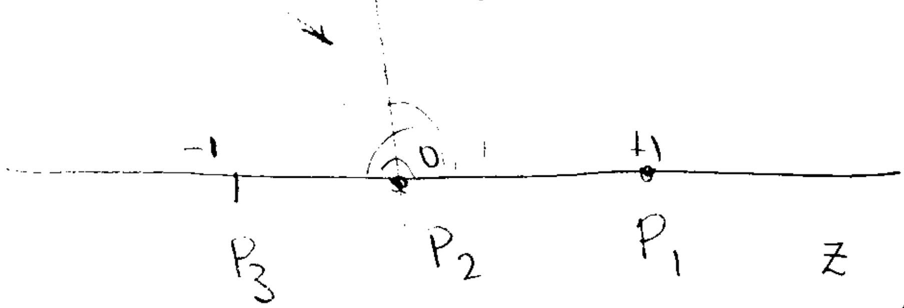
$$\xi = \frac{1+iw}{1-iw}$$

$$\left.\begin{array}{l} w=0 \to \xi = 1 \\ w = i \to \xi = 0 \\ w = 1 \to \xi = i \end{array}\right\} \checkmark$$

$$\tilde{z} = \xi^{2/3} = \left(\frac{1+iw}{1-iw}\right)^{2/3}$$

⌊z

$$\tilde{z} = 1 + \frac{4i}{3} w + O(w^2)$$



$$\left\{\begin{array}{l} \tilde{z} = 1 \to z = 0 \\ \tilde{z} = -1 \to z = \infty \\ \tilde{z} = e^{2\pi i/3} \to z = 1 \end{array}\right.$$

$$z = \left(\frac{\tilde{z}-1}{\tilde{z}+1}\right)\left(\frac{e^{\frac{2\pi i}{3}}+1}{e^{\frac{2\pi i}{3}}-1}\right)$$

Find the $\Theta(w)$

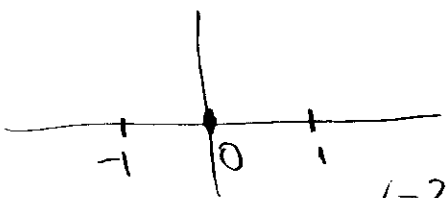
$$z = \frac{\frac{4i}{3}w + \Theta(w^2)}{2 + \frac{4iw}{3}} \left(\frac{-\frac{1}{2} + i\frac{\sqrt{3}}{2} + 1}{-\frac{1}{2} + i\frac{\sqrt{3}}{2} - 1} \right)$$

$$z = \frac{2iw}{3} \left(\frac{1 + i\sqrt{3}}{-3 + i\sqrt{3}} \right) = \frac{2iw}{3} \left(\frac{(1+i\sqrt{3})(-3-i\sqrt{3})}{9+3} \right)$$

$$= \frac{2iw}{3} \left(\frac{-3+3-4i\sqrt{3}}{12} \right) = \frac{2iw}{3} \left(-\frac{i\sqrt{3}}{3} \right) w$$

$$z = \frac{2\sqrt{3}}{9} w + \Theta(w^2)$$

How about the other coordinates?



$T(z) = ?$ $\qquad -1 \to 0 \to 1$

$$\left(\frac{-2}{-1} \right) \cdot \frac{\overset{0}{T(z)}}{\underset{+1}{T(z) - 1}} = \frac{\overset{-1}{z+1}}{\underset{0}{z}} \cdot \left(\frac{1}{2} \right)$$

$$\frac{2T}{T-1} = \frac{z+1}{z} \frac{1}{2}$$

$$4Tz = Tz + T - (z+1)$$

$$3Tz - T = -(z+1) \qquad \boxed{T(z) = \frac{z+1}{1-3z}}$$

$$T(3z-1) = -(z+1)$$

z small $\qquad T(z) = \dfrac{1+z}{1-3z} \approx 1+4z$

$$z_{P_1} = 1 + 4\left(\frac{2\sqrt{3}}{9}\right)w$$

thus

$$\boxed{\begin{aligned} z_{P_1} &= 1 + \frac{8\sqrt{3}}{9}w + \Theta(w^2) \\ z_{P_2} &= \frac{2\sqrt{3}}{9}w + \Theta(w^2) \\ z_{P_3} &= -1 + \frac{8\sqrt{3}}{9}w + \Theta(w^2) \end{aligned}}$$

Local coordinates for insertions

$$\langle \Phi, \Phi * \Phi \rangle = \langle f_1 \circ \Phi(0),\ f_2 \circ \Phi(0)\ \ f_3 \circ \Phi(0) \rangle$$

$$= \langle \Phi(w_1 = 0)\ \ \phi(w_2 = 0)\ \ \phi(w_3 = 0) \rangle$$

concrete meaning of vertex

expressed in z_1 coord

Thus $\quad C(w) = \dfrac{c(z)}{\left(\frac{dz}{dw}\right)}$

$$\langle f_1 \circ C(0),\ f_2 \circ C(0),\ f_3 \circ C(0) \rangle$$

$$= \left\langle \frac{C(+1)}{\frac{8\sqrt{3}}{9}}\ \frac{C(0)}{\frac{2\sqrt{3}}{9}}\ \frac{C(-1)}{\frac{8\sqrt{3}}{9}} \right\rangle = \frac{3^4\sqrt{3}}{2^7} \underbrace{\langle C(1)\, C(0)\, C(-1) \rangle}_{\substack{(1-0)(1-(-1))(0-(-1)) \\ = +2}}$$

$$\langle T, T * T \rangle = \frac{3^4\sqrt{3}}{2^6}$$

Thus: $\quad \dfrac{V(t)}{M} = (2\pi^2)\left(-\frac{1}{2}t^2 + \frac{1}{3}\underbrace{\left(\frac{3^4\sqrt{3}}{2^6}\right)}_{\alpha} t^3\right)$

EOM: $\quad -t + \alpha t^2 = 0 \qquad t_c = -\frac{1}{\alpha}$

$$\frac{V(t_c)}{M} = (2\pi^2)\left(-\frac{1}{2}\frac{1}{\alpha^2} + \frac{1}{3\alpha^2}\right) = -\frac{2\pi^2}{6\alpha^2}$$

$$= -\frac{\pi^2}{3}\frac{1}{\alpha^2} = -\frac{\pi^2}{3}\frac{2^{12}}{3^9}$$

$$= -\pi^2 \frac{2^{12}}{3^{10}} = -\frac{4096}{59049}\pi^2 \cong -0.684$$

$\sim 70\%$

of expected vacuum energy

Where is the tachyon string field

$\langle \Phi, Q\Phi \rangle$ and $\langle c_1 c_0 c_1 \rangle \neq 0$ Φ must be of gh # 1

Classify states from the viewpoint of matter CFT (recall $k^2 = -k_0^2 + k_1^2 + \cdots$)

Allow all ghost oscillators that add up to ghost # +1 $\quad c_1 \; c_0 \; c_{-1} \cdots$ $\quad b_{-2} \; b_{-3} \cdots$ $\quad$ set $\mathcal{G}$

Matter classify by primaries and descendents

set M_1 $\quad |0\rangle$ m"primary" + descendents

set M_2 $\quad |$m-primaries with $k_0 \neq 0\rangle$ + descendents

set M_3 $\quad |$ m-primaries with $k_0 = 0\rangle$ + descendent

$\Downarrow$

must have $\underline{\text{dim} > 0}$

only k_0 gives trouble

These are all states from matter ...

Thus set

$$\left.\begin{aligned} \mathcal{H}_1 &\equiv M_1 \otimes \mathcal{G} \\ \mathcal{H}_2 &\equiv M_2 \otimes \mathcal{G} \\ \mathcal{H}_3 &\equiv M_3 \otimes \mathcal{G} \end{aligned}\right\} \text{disjoint}$$

H_1 = union of $\mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \mathcal{H}$

<u>Claim</u>: - The string field is in $\mathcal{H}_1$ ($\mathcal{H}_1^{com} = \mathcal{H}_2 \oplus \mathcal{H}_3$)

Must have that the kinetic term does not couple $\mathcal{H}_1$ to the others $\langle \mathcal{H}_1, Q\,\mathcal{H}_1^{comp} \rangle =$

$\langle \mathcal{H}_1, Q\mathcal{H}_2 \rangle = 0$ by momentum conservation

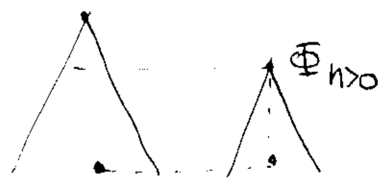
$\langle \mathcal{H}_1, Q\mathcal{H}_3 \rangle =$ $\quad Q = c_n L_n^{matt} + ccb$

$Q : \mathcal{H}_3 \to \mathcal{H}_3$ it either adds L_{-n}'s or ghost oscill

$\langle \mathcal{H}_1, \mathcal{H}_3 \rangle$ is zero due to the matter sector

$\langle L_{-I} |0\rangle, L_{-J}\, \Phi_{h>0} \rangle$

to be nonzero have to have the same dimension



thus $I > J$ but then flip the L_{-I}'s to the right and get zero

For the interactions need to have

$$\langle \mathcal{H}_1, \mathcal{H}_1, \mathcal{H}_1^{comp} \rangle = 0$$

No one point functions for fields in $\mathcal{H}_1^{comp}$.

Manifest for $\mathcal{H}_2$ because of momentum
For $\mathcal{H}_3$ factorize on the matter part

$$\langle L_{-I} |0\rangle , \; L_{-J} |0\rangle , \; L_{-K} \Phi_{h>0} \rangle$$

move all Virasoro's around until they dissappear

$$\sim \langle |0\rangle, |0\rangle, \Phi_{h>0} \rangle = \langle \Phi_{h>0} \rangle = 0$$

1-point function

So how does the spectrum look?

Note: Cannot use L_{-1}^{m}

$$t\, c_1|0\rangle \qquad L_0 = -1 \quad \ell = 0$$

$$r\, c_0|0\rangle \qquad L_0 = 0 \quad \ell = 1$$

$$\frac{v}{\sqrt{13}}\, L_{-2} c_1|0\rangle\,, \quad u\, c_{-1}|0\rangle\,, \quad w\, b_{-2} c_0 c_1$$

Siegel Gauge

String field can be made to satisfy

$b_0|\Phi\rangle = 0$ by virtue of the gauge

invariance $\qquad \Phi \approx \Phi + Q\epsilon$

This is a major simplification ① much less fields
② kinetic terms simplify

$b_0|\Phi\rangle = 0$ means $|\Phi\rangle = \ldots$ no c_0 $\ldots$

$$\langle \Phi, Q\Phi \rangle = \langle \Phi, c_0 L_0 \Phi \rangle \;\checkmark$$

Suppose $b_0|\Phi\rangle \neq 0$ and $L_0|\Phi\rangle \neq 0$.

$$|\Phi\rangle \approx \underbrace{|\Phi\rangle - Q\frac{b_0|\Phi\rangle}{L_0}}$$

$$b_0|\Phi\rangle - b_0 Q\frac{b_0|\Phi\rangle}{L_0} = b_0|\Phi\rangle - \overbrace{\{b_0, Q\}}^{L_0}\frac{b_0|\Phi\rangle}{L_0}$$

$$= 0 \, !$$

Thus can get rid of W, but <u>not quite</u> of r.

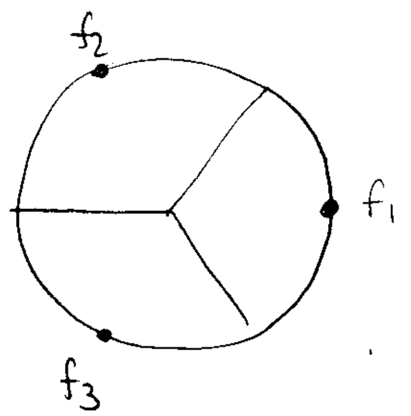
The gauge fixes things completely: there are no residual gauge transformations that act nontrivially on the gauge slice ($L_0 \neq 0$) If there were, there would be some nonzero string field that is pure gauge: let $|\Phi_0\rangle$ be such

$$|\Phi_0\rangle \neq 0 \, , \quad b_0|\Phi_0\rangle = 0 \, , \quad |\Phi_0\rangle = Q|\Lambda_0\rangle$$

$$\{Q, b_0\}|\Phi_0\rangle = 0 \rightarrow L_0|\Phi_0\rangle = 0$$

$$\rightarrow |\Phi_0\rangle = 0$$

Twist invariance



$$f_1(w) = \left(\frac{1+iw}{1-iw}\right)^{2/3}$$

$$f_2(w) = e^{\frac{2\pi i}{3}} f_1(w)$$

$$f_3(w) = e^{\frac{2\pi i}{3}} f_2(w) = e^{\frac{-2\pi i}{3}}\left(\frac{1+iw}{1-iw}\right)^{2/3}$$

Let $\tilde{I}(z) = \frac{1}{z}$ $\qquad M(z) = -z$

$$f_1 \circ M = \tilde{I} \circ f_1$$

$$f_2 \circ M = e^{2\pi i/3}\left(\frac{1-iw}{1+iw}\right)^{2/3} = \tilde{I} \circ f_3$$

$$= \tilde{I} \circ f_3$$

$$f_3 \circ M = e^{-2\frac{\pi i}{3}}\left(\frac{1-iw}{1+iw}\right) = \tilde{I} \circ f_3$$

So, suppose we have a correlator

$$\langle f_1 \circ A ,\ f_2 \circ B ,\ f_3 \circ C \rangle$$

A, B, C grassman odd operators

$$= (-)^{h_A + h_B + h_C} \langle f_1 \circ M \circ A ,\ f_2 \circ M \circ B ,\ f_3 \circ M \circ C \rangle$$

$$= (-)^{h_A + h_B + h_C} \langle \tilde{I} \circ f_1 \circ A\ \ \tilde{I} \circ f_3 \circ B ,\ \tilde{I} \circ f_2 \circ C \rangle$$

$$= (-)^{\sum h_A + 1} \langle f_1 \circ A ,\ f_2 \circ C ,\ f_3 \circ B \rangle$$

<u>Cyclicity of amplitudes</u>

$$\langle f_1\circ A,\ f_2\circ B\ \ f_3\circ C\rangle = \langle f_1\circ B,\ f_2\circ C,\ f_3\circ A\rangle$$

cost is 2 minus signs

<u>Therefore</u>

$$\langle f_1\circ A\ \ f_2\circ B\ \ f_3\circ C\rangle = \Omega_A\Omega_B\Omega_C\,\langle f_1\circ C\ \ f_2\circ B\ \ f_3\circ A\rangle$$

$$\langle A.BC\rangle = \Omega_A\Omega_B\Omega_C\,\langle CBA\rangle$$

$\boxed{\Omega = (-)^{n+1}}$ $\qquad c_1|0\rangle$ is <u>twist even</u>

Claim that a twist odd field cannot couple to two twist even fields:

$$\langle E_1 E_2 O\rangle + \langle E_1 O E_2\rangle \qquad = \langle O E_2 E_1\rangle = -\langle E_1 E_2 O\rangle$$

$$\langle E_2 E_1 O\rangle + \langle E_2 O E_1\rangle$$
$$\langle O E_1 E_2\rangle + \langle O E_2 E_1\rangle$$

cyclic the same as the above

thus get zero!

So keep only twist <u>even fields in $\mathcal{H}_1$</u>

Thus work with

$$|T\rangle = t c_1|0\rangle + u c_{-1}|0\rangle + \frac{v}{\sqrt{13}} L_{-2} c_1|0\rangle$$

$$\frac{V}{M} = 2\pi^2 \Big(-\tfrac{1}{2}t^2 + \frac{3^3\sqrt{3}}{2^6} t^3$$

$$- \tfrac{1}{2}u^2 + \tfrac{1}{2}v^2$$

$$+ \frac{11.3\sqrt{3}}{2^6} t^2 u - \frac{5.3\sqrt{39}}{2^6} t^2 v$$

$$+ \frac{19}{2^6\sqrt{3}} t u^2 + \frac{7.83}{2^6\ 3.\sqrt{3}} t v^2 - \frac{11.5\sqrt{13}}{2^5.\ 3\sqrt{3}} t u v$$

$t_c = 0.542 \qquad u_c \simeq 0.173 \qquad v_c = 0.187$

$$\frac{V(t_c)}{M} = -0.949 \quad //$$

Calculation of the D-brane mass

$$S = -\frac{1}{g_o^2}\left(\frac{1}{2}\langle\Phi|Q_B|\Phi\rangle + \frac{1}{3}\langle\Phi, \Phi * \Phi\rangle\right)$$

Work with $\alpha' = 1$
String tension $T = \frac{1}{2\pi}$

Compact time x^0 of length 1

Coordinates $x^0 \overbrace{x^1 \ldots x^n}^{x^i}$ $(25-n)$ compact x^α

World sheet fields $\underset{\downarrow \text{ Neuman}}{X^0}, \underbrace{X^1, \ldots X^n}_{\text{Dirichlet}}$ $\quad BCFT(X^\alpha)$

Work with $\alpha' = 1$ $\quad$ Mink. $= (-1, 1, 1 \ldots)$
T = string tension $= \frac{1}{2\pi}$

This D-brane looks like a particle in Minkowski $(1, n)$ and it will have finite mass M (closed string metric)

M would seems independent of the BCFT as it will only depend on the open string coupling g_o. But g_o is related to g_c via the BCFT

Let $\quad |k_0\rangle \equiv e^{ik_0 X^0(0)}|0\rangle$

and normalize with

$$\langle k_0 | c_{-1} c_0 c_1 | k_0' \rangle = \delta_{k_0, k_0'} \quad \text{--- (1)}$$

Step 1 Consider time dependent displacements of the brane

What is the string field ?

$$\hat{\Phi} = \sum_{k_0} \phi^i(k_0)\, c_1 \alpha_{-1}^i |k_0\rangle \quad (2)$$

note $k_0 = \frac{2\pi}{n}$

$e^{ik_0 t}$ is periodic with $t \to t+1$

Evaluate the action

$$S_2(\hat{\Phi}) = -\frac{1}{g_0^2} \sum_{k_0, k_0'} \frac{1}{2} \phi^i(k_0)\, \phi^{i'}(k_0') \langle -k_0' | c_{-1} \alpha_1^i c_0 L_0 c_1 \alpha_{-1} | k_0 \rangle$$

$$= -\frac{1}{2} \frac{1}{g_0^2} \sum_{k_0} \phi^i(k_0)\, \phi^i(-k_0) \left(-(k_0)^2\right)$$

here $L_0 = k.k = -(k_0)^2 + (\vec{k})^2$

$$S_2(\hat{\Phi}) = \frac{1}{2} \frac{1}{g_0^2} \sum_{k_0} \phi^i(-k_0)\, k_0^2\, \phi^i(k_0) \quad (3)$$

Now define a "hoped for displacement" $\phi^i(t)$

$$\phi^i(t) = \sum_{k_0} e^{\imath k_0 t} \phi^i(k_0) \qquad \cdots\cdots \quad (4)$$

<u>try</u> $\int_0^1 dt\, \partial_t\phi^i \partial_t \phi^i = \sum_{k_0,k_0'} \int_0^1 dt\, (\imath k_0)(\imath k_0')\, e^{\imath(k_0+k_0')t} \phi^i(k_0)\, \phi^i(k_0')$

$$\int_0^1 dt\, e^{\imath(k_0+k_0')t} = \delta_{k_0,-k_0'}$$

$$= \sum_{k_0} (\imath k_0)(-\imath k_0)\, \phi^i(k_0)\, \phi^i(-k_0)$$

$$= \sum_{k_0} \phi^i(-k_0)\, k_0^2\, \phi^i(k_0)$$

So indeed

$$\boxed{S_2(\Phi) = \frac{1}{2g_0^2} \int dt\, \partial_t\phi^i\, \partial_t\phi^i \overset{?}{\sim} \frac{1}{2} M v^2} \quad (5)$$

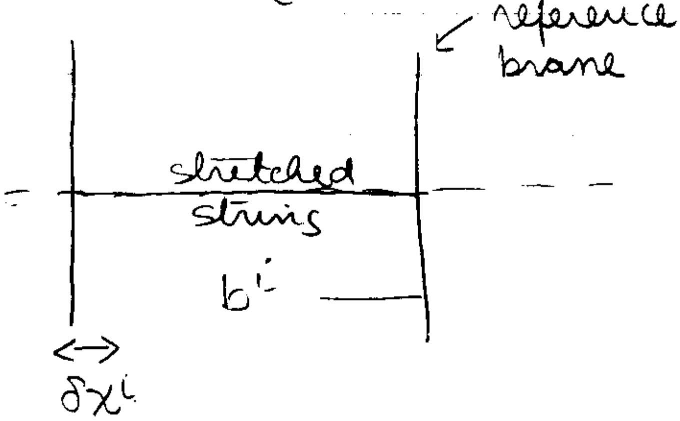
The only problem is that ϕ^i may only represent displacement up to normalization!

<u>Step 2</u> - Relate distance between branes to mass shifts of stretched strings

Mass streched string $=$ $T\,L$ $=$ (Tens)(length)

$$m = \frac{L}{2\pi}$$

$$m^2 = \frac{L^2}{(2\pi)^2}$$



$$\Delta m^2 = \frac{1}{4\pi^2}\left[(b^i + \delta x^i)^2 - (b^i)^2\right]$$

$$\boxed{\Delta m^2 = \frac{1}{2\pi^2}\, b^i\,\delta x^i} \qquad (6)$$

String field time independent displacement

From (4) set $\quad \delta\phi^i \equiv \delta\phi^i(k_0 = 0)\,; \qquad \delta\phi^i(k_0 \neq 0) = 0$

back in (2)

$$\boxed{\hat{\Phi} = \delta\phi^i\, c_1\, \alpha^i_{-1}|0\rangle} \qquad (7)$$

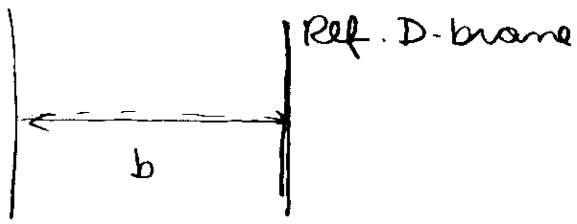
Want to find $\frac{\delta\phi^i}{\delta x^i} \sim$ constant

Then back in (5) $S = \frac{1}{2g_0^2}\left(\frac{\delta\phi^i}{\delta x^i}\right)^2 \int dt\, \partial_t x^i \partial_t x^i$

$$\boxed{M_{\text{brane}} = \frac{1}{g_0^2}\left(\frac{\delta\phi^i}{\delta x^i}\right)^2} \quad (8)$$

Step 3 SFT computation of the mass shift

Introduce fields η, η^* to represent masses of stretched strings (distance b)



$$\eta^* c_1 |k_0, b\rangle \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \eta\, c_1 |-k_0, -b\rangle \otimes \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Action includes a tr factor

What is the CFT operator associated to $|k_0, b\rangle$??

$$|k_0, b\rangle = e^{ik_0 X^0}\, e^{i\frac{b}{\pi} X_L^i}$$

because it has the right $L_0 \cdots : -k_0^2 + \ldots$

X^i has Dirichlet BC $\quad X^i_L = -X^i_R = x_B/2$

boundary of the world sheet

$$\partial X^i_L(x)\,\partial X^i_L(y) \sim -\frac{1}{2}\frac{1}{(x-y)^2}$$

$$T_{X^i} = -\partial X^i_L \partial X^i_L$$

$$T_{X^i}\; e^{i\alpha X^i_L} = \frac{\alpha^2}{4} e^{i\alpha X^i_L}$$

$$L_0\left(e^{i\frac{b}{\pi}X^i_L}\right) = \left(\frac{b}{2\pi}\right)^2 \quad \checkmark \text{ the right } m^2$$

$$g_0^2 S_2(\eta^*, \eta) = -\frac{1}{2}\cdot \underset{\eta,\,\eta^*}{\underset{\uparrow}{2}} \quad \eta^* \eta \left(-k_0^2 + \frac{b^2}{(2\pi)^2}\right)$$

$$= \eta^*\left(k_0^2 - \frac{b^2}{(2\pi)^2}\right)\eta$$

How about the fluctuation? From ⑦ add CP

$$\delta\phi^i\, c_i\, \alpha^i_{-1}|0\rangle \otimes \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} \qquad \partial x\,\partial x = -\ln$$

$$\partial X_L \partial X_L \sim -\frac{1}{2}$$

$$\leftrightarrow \boxed{\delta\phi^i\, c\,\sqrt{2}\, i\,\partial X_L \otimes \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}}$$

$$g_0^2 S_3 = -\frac{1}{3}\langle \Phi, \Phi, \Phi\rangle$$

put the η^* and η fields on the mass shell (ops are dim = 0) while $\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}1&0\\0&0\end{pmatrix} = \begin{pmatrix}1&0\\0&0\end{pmatrix}$!

$$\begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}1&0\\0&0\end{pmatrix} = \begin{pmatrix}0&0\\0&1\end{pmatrix}\begin{pmatrix}1&0\\0&0\end{pmatrix} = \begin{pmatrix}0&0\\0&0\end{pmatrix} !$$

6 ways to insert the operators. 3 vanish and 3 are the same $\cdot\ tr(\cdot) = 1$

$$g_0^2 S_3 = -\frac{1}{3}\cdot 3 \cdot \Big\langle e^{-i\frac{b}{\pi}X_L^i - ik_0X_0} c(\infty)$$

$$\eta^*\eta\,\delta\phi^i \quad \sqrt{2}\,i\,\underline{\underline{\partial X_L}}\, c(-1) \quad e^{i\frac{b}{\pi}X_L^i + ik_0X_0} c(0)\Big\rangle$$

$$= -\sqrt{2}\,i\ \frac{ib}{\pi}\left(-\frac{1}{2}\right)\frac{1}{(1-0)}\ \eta^*\eta\,\delta\phi^i$$

$$= \sqrt{2}\,\frac{b}{\pi}\,\frac{1}{2} = \frac{b}{\sqrt{2}\pi}\ \eta^*\eta\,\delta\phi^i$$

$$g_0^2(S_2+S_3) = \eta^*\left(k_0^2 - \frac{b^2}{(2\pi)^2} + \underbrace{\frac{b\,\delta\phi^i}{\sqrt{2}\,\pi}}_{\Delta m^2}\right)\eta$$

Recall from (6)

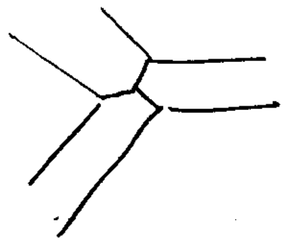
$$\frac{1}{2\pi^2}\,b\,\delta x^i = \frac{b}{\sqrt{2}\,\pi}\,\delta\phi^i$$

$$\frac{\delta\phi^i}{\delta x^i} = \frac{1}{\sqrt{2}\,\pi}$$

back in (8) $\quad \boxed{M_{brane} = \frac{1}{2\pi^2 g_0^2}}$

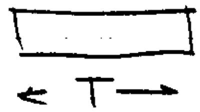
Open, closed and Open/closed

Open string Feynman rules



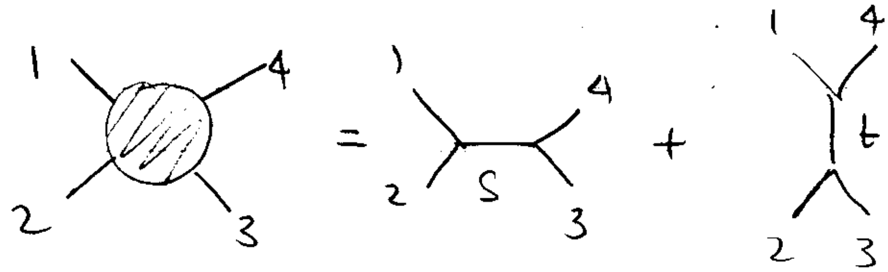
vertices

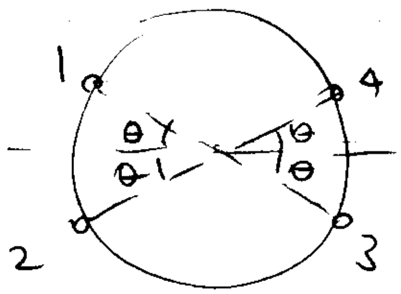
propagator



strips

$T \in [0, \infty]$





full moduli space

$\theta \in [0, \frac{\pi}{2}]$

s-channel $\theta \in [0, \frac{\pi}{4}]$

t-channel $\theta \in [\frac{\pi}{4}, \frac{\pi}{2}]$

cover of moduli space $\sim$ gauge invariance $\sim$ associativity

Closed String

$$Q = c_0^+ L_0^+ + c_0^- L_0^- + \cdots$$

$$c_0^\pm = c_0 \pm \bar{c}_0$$

$$\langle 0 | c_{-1} \bar{c}_{-1} \, c_0 \bar{c}_0 \, c_1 \bar{c}_1 | 0 \rangle \neq 0$$

$\langle \psi Q \psi \rangle$? no good

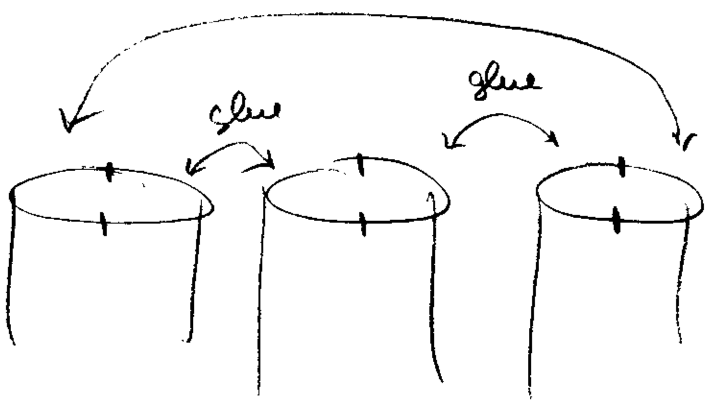
$S_2 \sim \langle \psi | c_0^- Q | \psi \rangle$ good if $\left.\begin{matrix} (L_0 - \bar{L}_0)|\psi\rangle = 0 \\ (b_0 - \bar{b}_0)|\psi\rangle = 0 \end{matrix}\right\}$ off-shell conditions

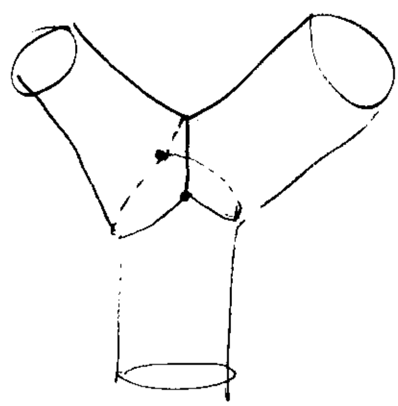
$$\delta|\psi\rangle = Q|\epsilon\rangle, \quad b_0^-|\epsilon\rangle = 0$$

$$\delta S_2 \sim \langle \epsilon | Q c_0^- Q | \psi \rangle$$

$$= \langle \epsilon | c_0^- b_0^- \, Q c_0^- Q | \psi \rangle = 0$$

= get L_0^-'s that vanish and $b_0^- |\psi\rangle = 0$ ✓





Strebel polyhedron

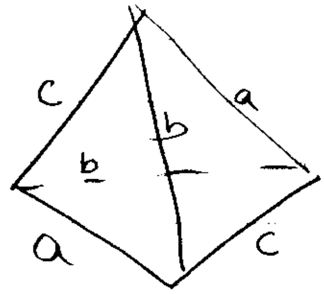
Do not cover

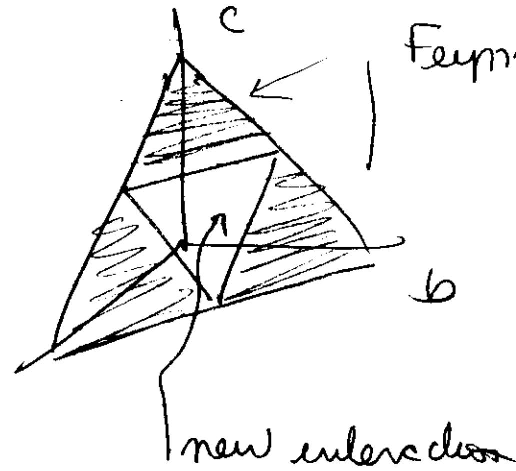
$M_{0,4}$

need 4-string

elementary vertex.

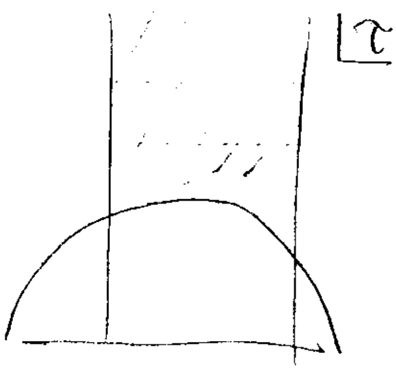
$a+b+c=2\pi$

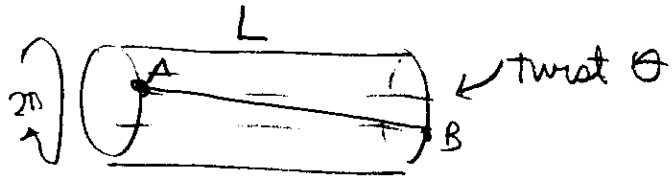




n-point interaction

* n-faced polyhedron
* perimeter of each face $= 2\pi$
* All closed curves are longer than 2π.

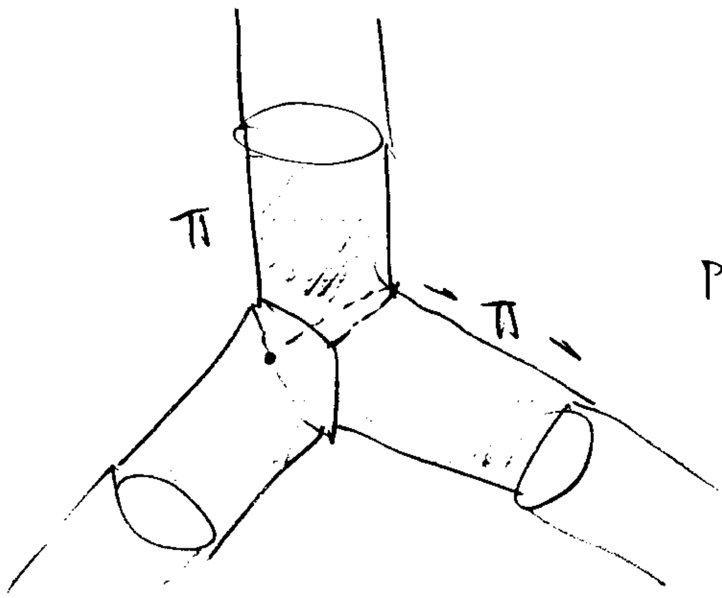




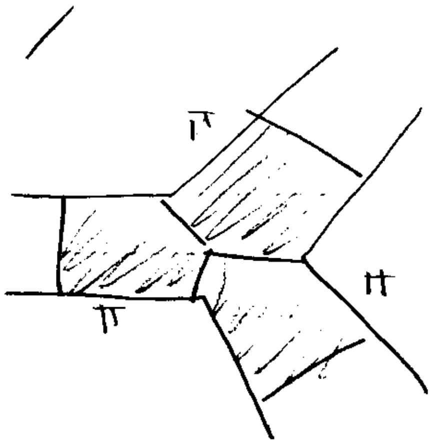
$$\tau = \frac{iL+\theta}{2\pi}$$

$$l_{AB} = \sqrt{L^2+\theta^2} \geq 2\pi$$

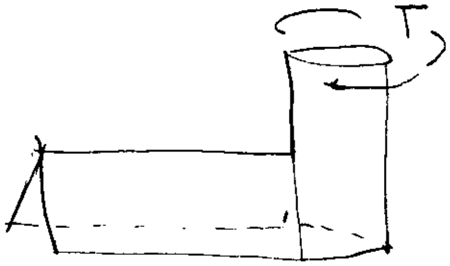
$$|\tau| = \frac{|iL+\theta|}{2\pi} > 1 \checkmark$$



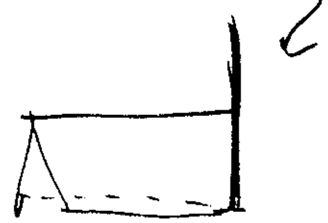
prevent <u>short closed</u> $(< 2\pi)$ <u>curves</u>



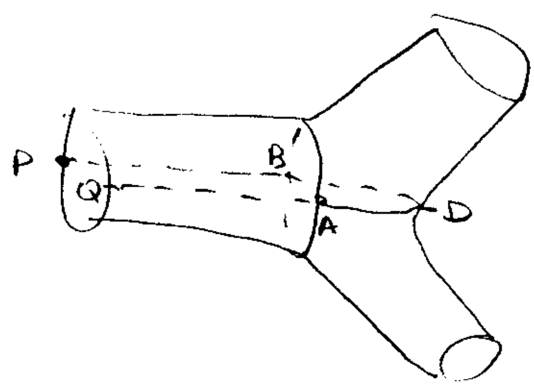
Add stubs to <u>open string</u>
- lose associativity
- can now incorporate closed strings off-shell



stop at $T = 2\pi$
otherwise



open-closed vertex



cut open