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SMR.1227 - 14

SUMMER SCHOOL ON ASTROPARTICLE PHYSICS AND COSMOLOGY

12 - 30 June 2000

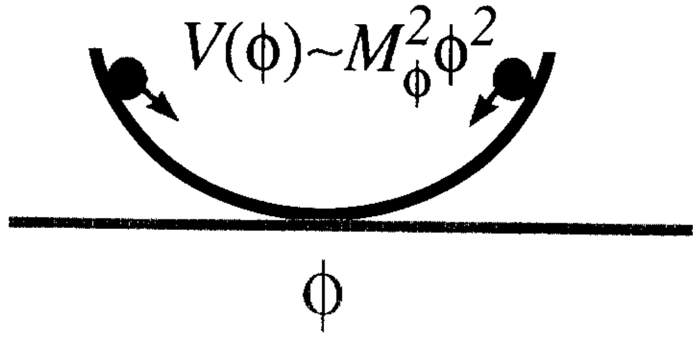
DYNAMICS OF INFLATION

Supplement to Lecture III

ROCKY KOLB
Fermi Lab. and Chicago
USA

The End of Inflation

- **After inflation universe frozen**



DEFROSTING:

- **Reheating (ca. early 1980s)**

  incoherent, nonresonant, linear decay of inflaton field

- **Preheating (ca. mid 1990s)**

  coherent, resonant, nonlinear particle production

PREHEATING/REHEATING ISSUES:

What is the "reheat" temperature?

Symmetry restoration?

Massive particle production?

baryogenesis/leptogenesis

dark matter

Light particle production?

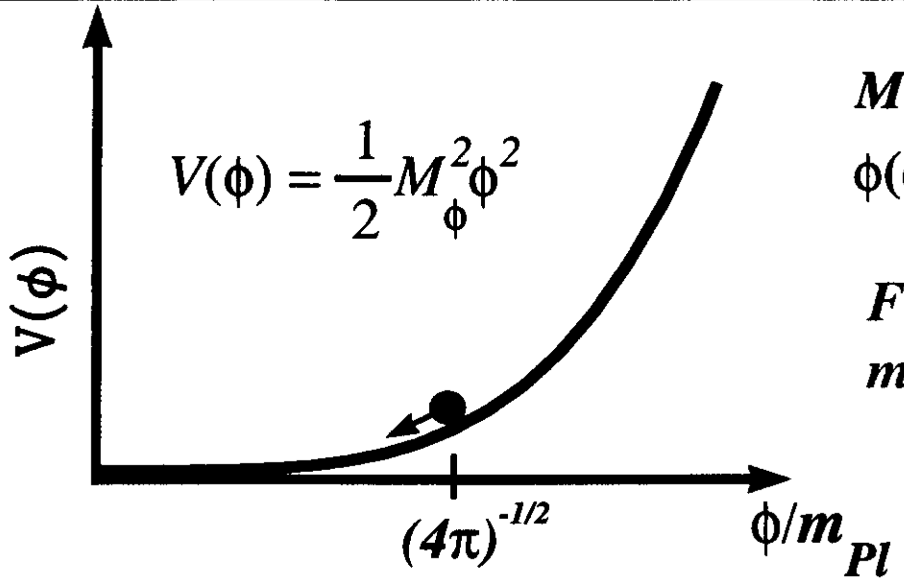
gravitinos

Perturbations?

curvature

isocurvature

Reheating



$M_\phi = 10^{13}$ GeV

$\phi(\text{end}) = (4\pi)^{-1/2} m_{Pl}$

Frozen Universe:
must defrost!!!!!!

- **coherent ϕ oscillations with decreasing amplitude**

  $\omega = M_\phi \; ; \; \rho_\phi \sim a^{-3}$

- **ϕ coupled to other fields**

  ρ_ϕ decays with width Γ_ϕ

- **decay produces massless degrees of freedom (γ)**

  thermalize to temperature T

- **when "all" energy extracted from ϕ, $T = T_{RH}$**

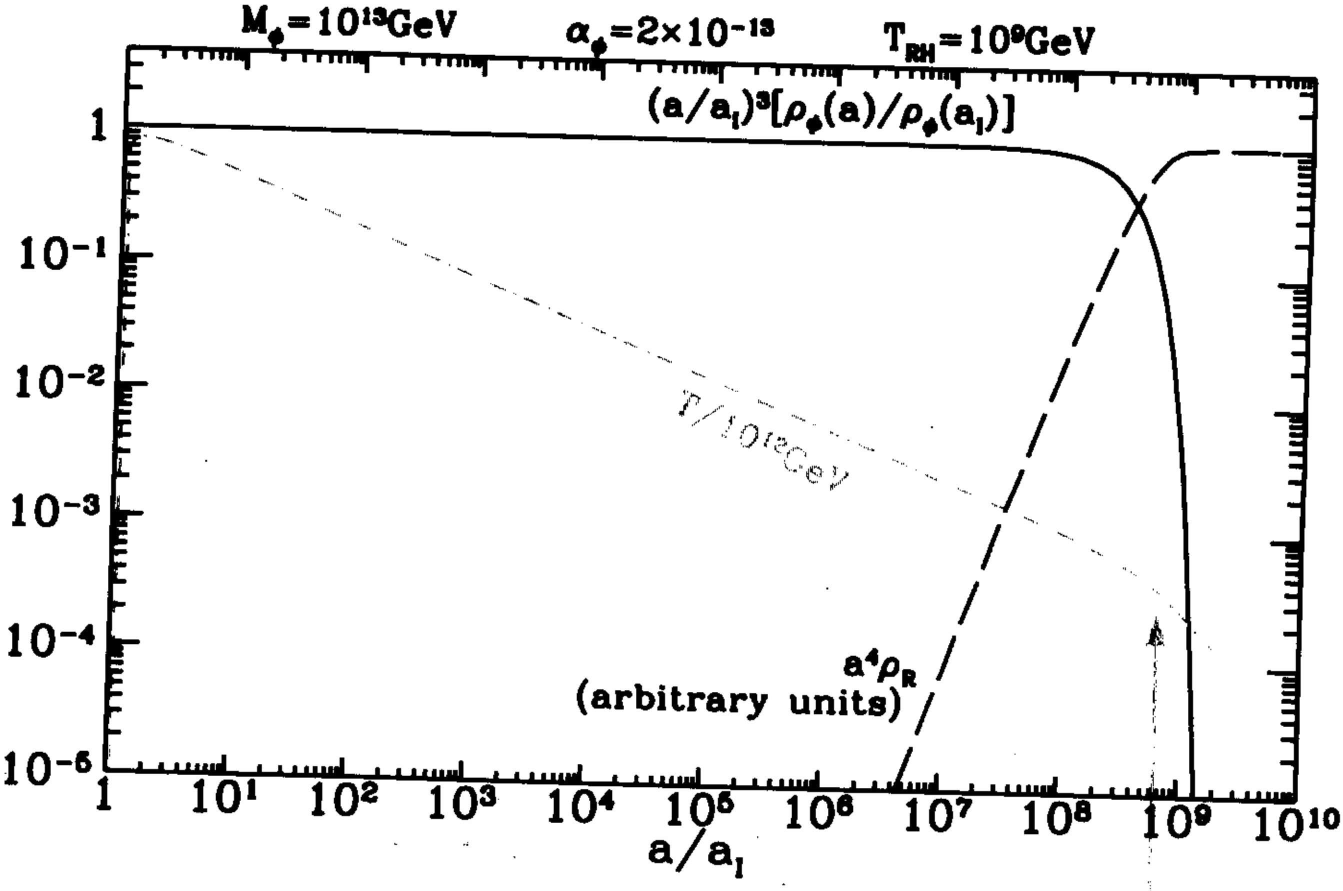
  gravitino limit: $T_{RH} < 10^9$ GeV

Reheating

$$\phi = \text{inflaton} \quad \Gamma_\phi = \alpha_\phi M_\phi$$

$$\dot{\rho}_\phi + 3H\rho_\phi + \Gamma_\phi \rho_\phi = 0$$

$$\dot{\rho}_R + 4H\rho_R - \Gamma_\phi \rho_\phi = 0$$



$T_{MAX} >> T_{RH}$ *(Scherrer & Turner; Chung, Kolb, Riotto)*

Preheating

Inflaton ϕ coupled to another scalar field χ: $V = \frac{1}{2} g^2 \phi^2 \chi^2$

- <u>Field eq.:</u> $\ddot{\chi}_k + 3\frac{\dot{a}}{a}\dot{\chi}_k + \left(\frac{\mathbf{k}^2}{a^2} + \cancelto{0}{m_\chi^2(0)} - \cancelto{0}{\xi R} + g^2\phi^2\right)\chi_k = 0$

  k - comoving momentum

  $\phi(t) = \Phi(t)\sin mt$ - inflaton field

- <u>Minkowski space:</u> $\ddot{\chi}_k + \left(k^2 + g^2\Phi^2 \sin^2(mt)\right)\chi_k = 0$

  Φ - constant

- <u>Mathieu* eq.:</u> $\chi_k'' + (A_k - 2q\cos 2z)\chi_k = 0$

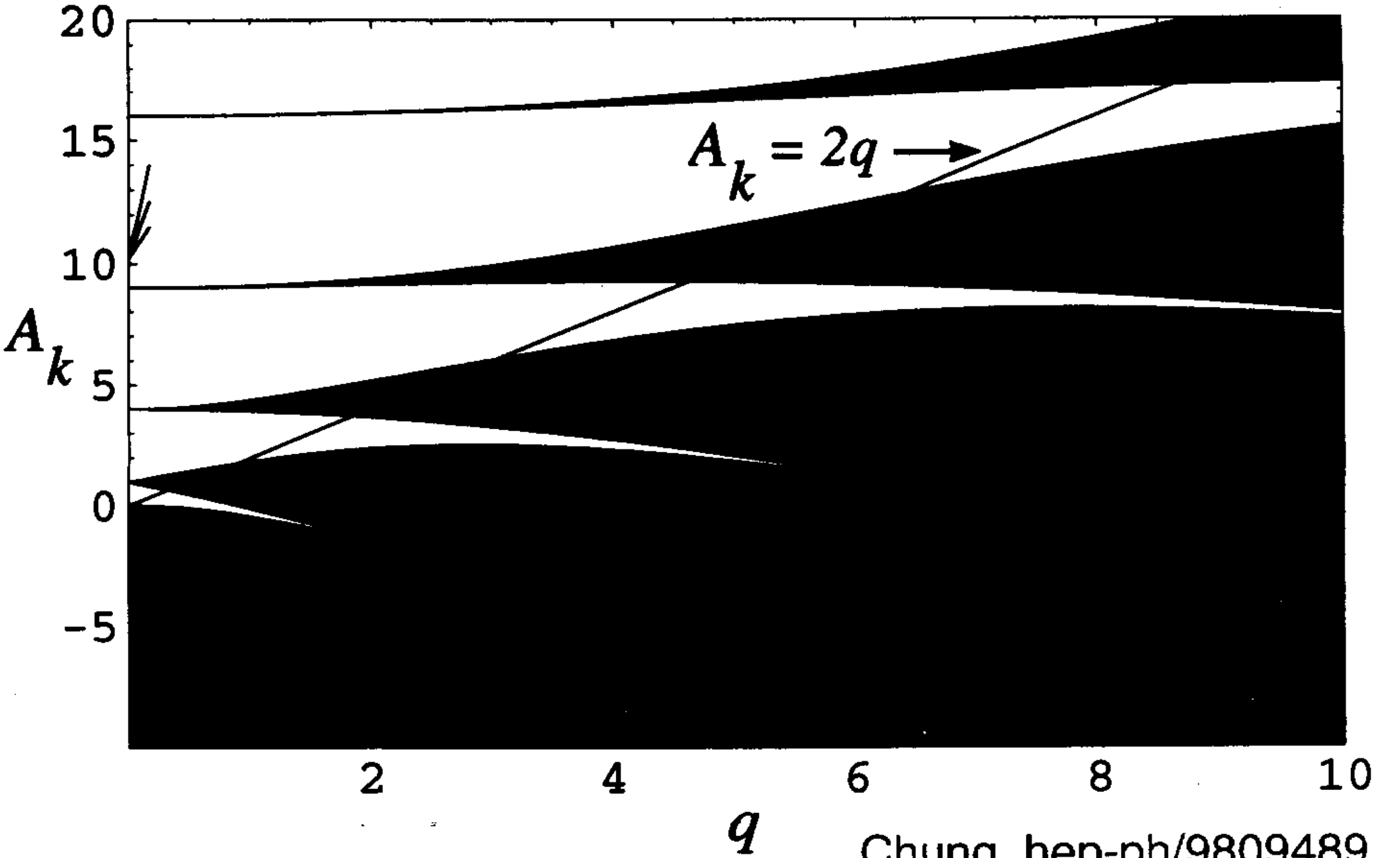
$$\left.\begin{aligned} z &= mt \\ q &= \frac{g^2\Phi^2}{4m^2} \\ A_k &= \frac{k^2}{m^2} + 2q \end{aligned}\right]$$ From Mathieu equation

---

* *E. Mathieu (1868): "Mémorie sur le movement vibratoire d'une membrane de form ellipitique."* Jour. de Math Pures et Appliquées

Instability regions

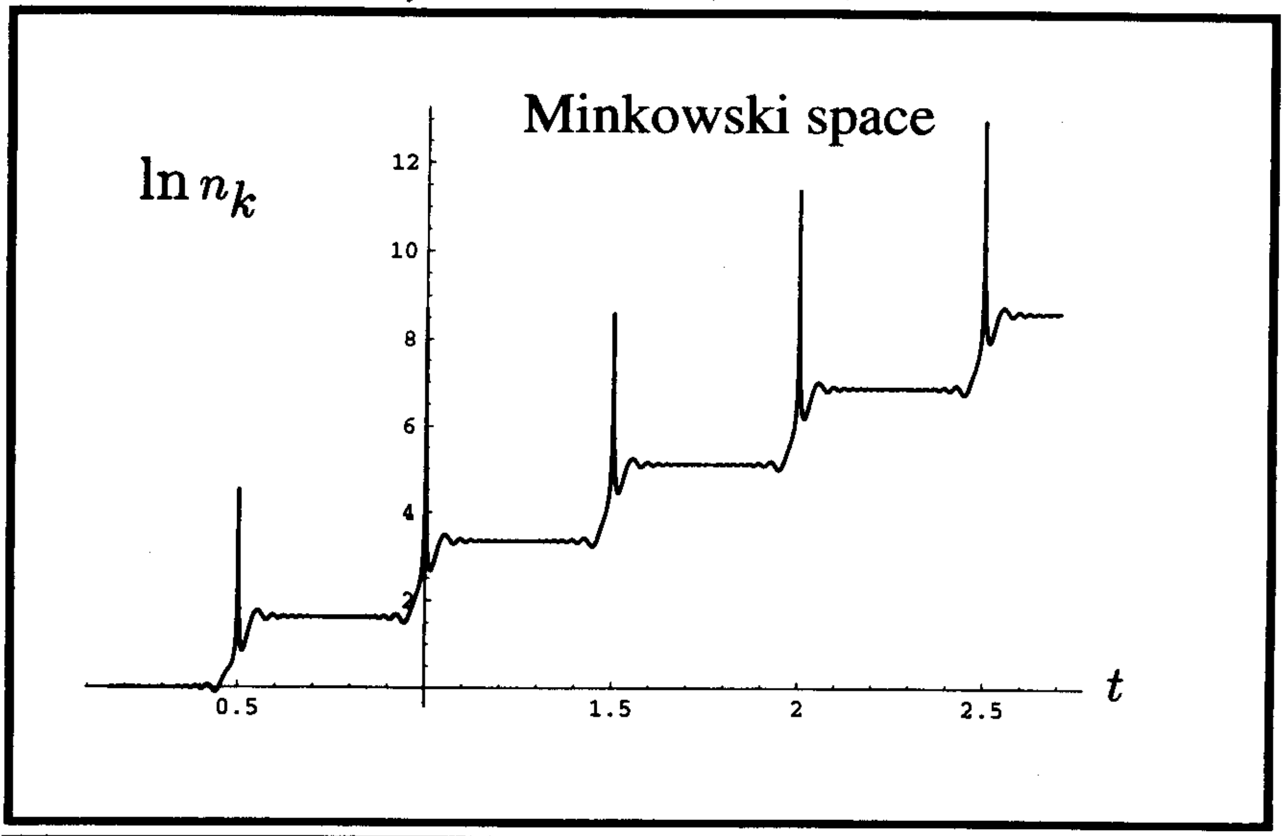
$$q = \frac{g^2\Phi^2}{4m^2} \qquad A_k = 2q + \frac{k^2}{m^2}$$

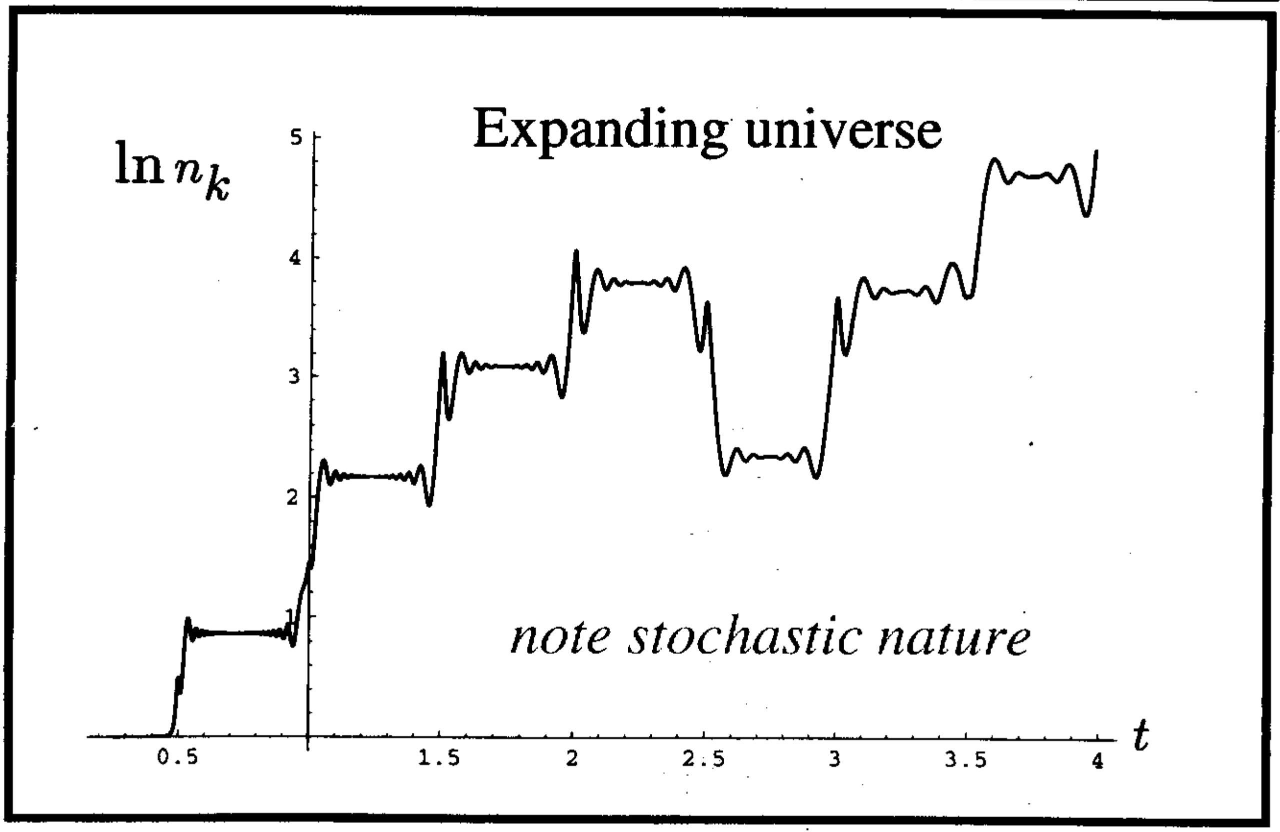


Chung, hep-ph/9809489

From Kofman, Linde, Starobinsky

Phys.Rev. D56 (1997) 3258





From Kofman, Linde, Starobinsky

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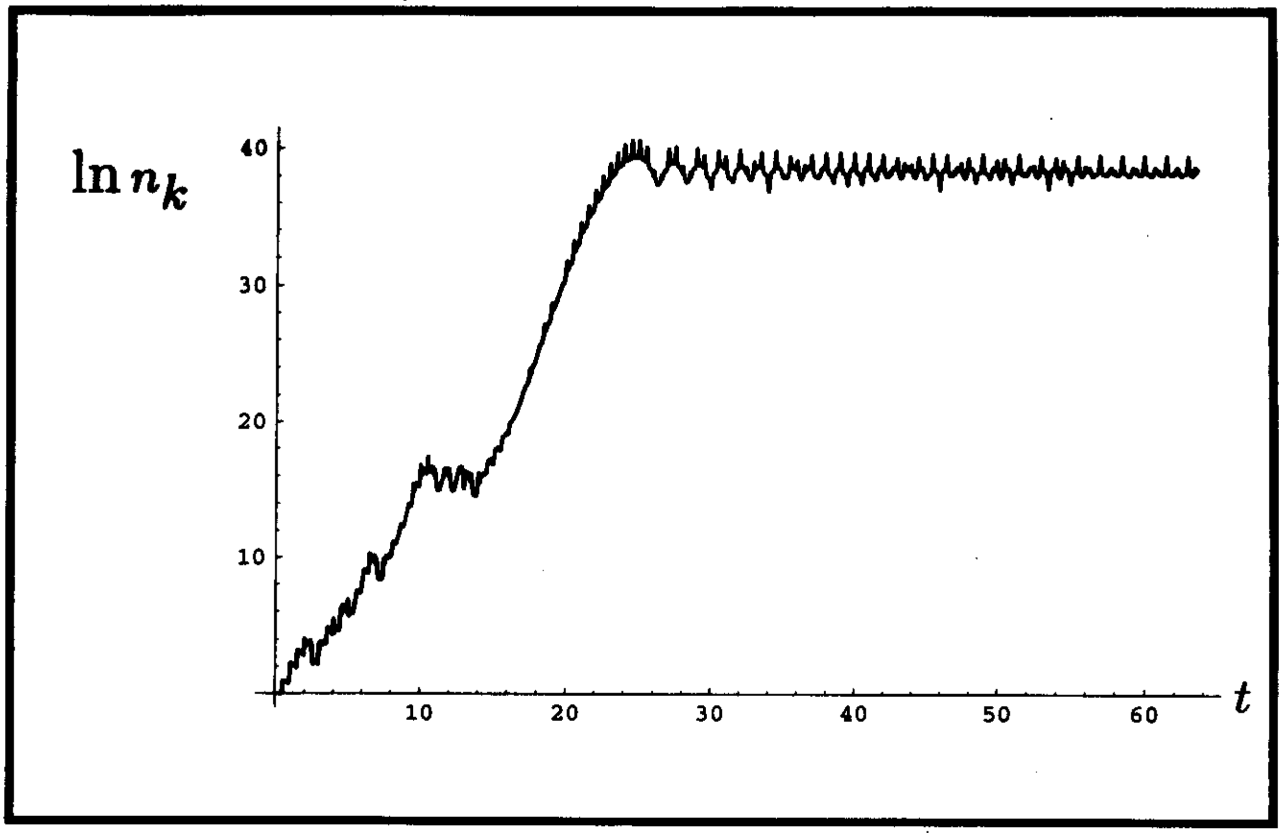


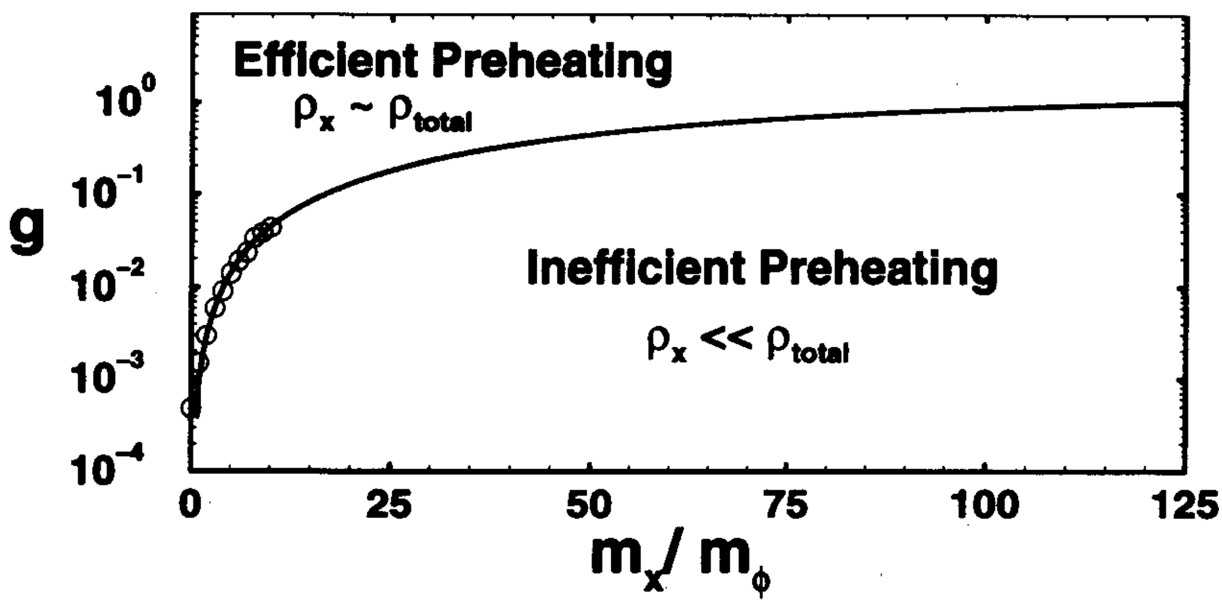
Complexity: rescattering; back reactions

Conclusion: in a few dozen oscillation periods of the inflaton field, a significant fraction of the inflaton field energy can be transferred into ***soft*** modes of another field χ ($<E_\chi^2> \sim g\, m_{Pl}\, m$).

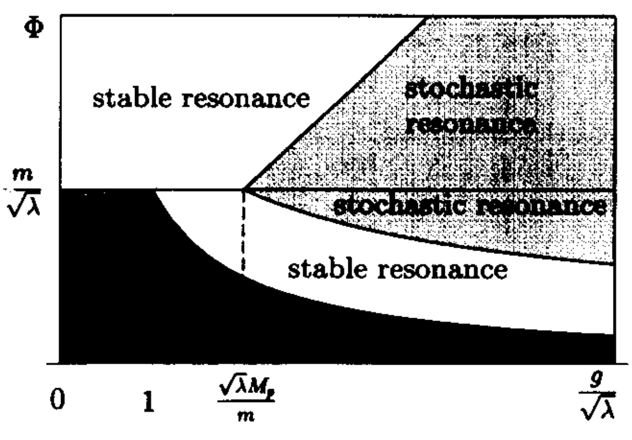
Note: it is inherently a coherent process---many inflaton quanta participate.

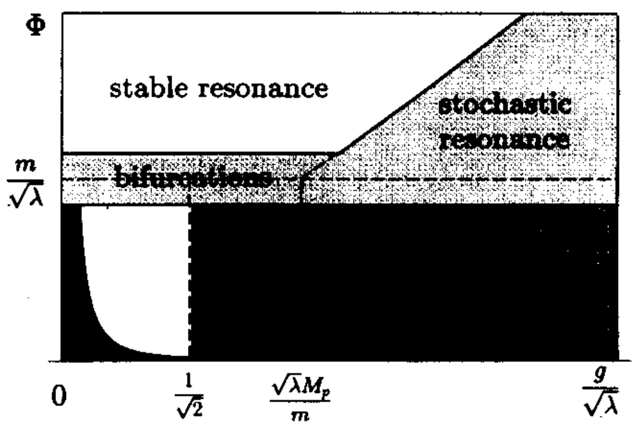
Preheating -- Parameter Space

Zlatev, Huey, Steinhardt $g^2X^2\phi^2$



Greene, Kofman, Linde, Starobinski $M^2\phi^2 + \lambda\,\phi^4$





Symmetry restoration

Tkachev; Kofman et al.

- suppose a fraction δ of ρ_ϕ in soft modes of another field χ.

- assume χ interacts with a field σ which undergoes SSB

$$V(\sigma,\chi) = -\frac{1}{2}\mu^2\sigma^2 + \frac{1}{4}\lambda\sigma^4 + h^2\sigma^2\chi^2$$

- lineup: $\phi \longleftarrow g \longrightarrow \chi \longleftarrow\!\!-\!\!-\!\!-\!\!-\; h \;-\!\!-\!\!-\!\!-\!\!\longrightarrow \sigma$

inflaton ↑ produces ↑ which restores symmetry of ↑

mass: m 0 μ

(can get by with fewer fields)

- at temperature $T >> T_C$ symmetry restored

$$T_C \sim \frac{\mu^2}{\lambda + h^2} \qquad \text{(recall } T_{RH} << T_C\text{)}$$

- soft background of χ can restore symmetry

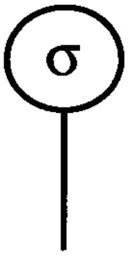
$$T_{EFF} >> T_C$$

Finite temperature symmetry restoration

$$V(\sigma) = -\frac{1}{2}\mu^2\sigma^2 + \frac{1}{4}\lambda\sigma^4$$

$$M^2(\sigma) = -\mu^2 + 3\lambda\sigma^2$$

- radiative corrections *(T=0)*

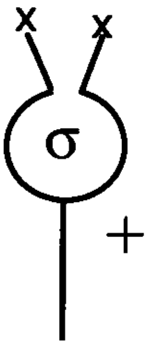


$$+\frac{1}{64\pi^2}M^4(\sigma)\ln M^2(\sigma)$$

- at finite temperature/density

$$D_\sigma(k) = i(k^2 - \mu^2 + i\epsilon)^{-1} + 2\pi f_\sigma \delta(k^2 - \mu^2)$$

- radiative corrections *(T≠0)*



$$+\frac{T^4}{2\pi^2}\int_0^\infty dx\, x^2 \ln\left[1 - \exp[-(x^2 + M^2/T^2)^{1/2}]\right]$$

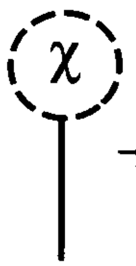
- at $T >> T_C$ $\qquad M^2(\sigma) = -\mu^2 + 3\lambda\sigma^2 \boxed{+\lambda T^2\sigma^2}$

$$\boxed{\lambda T^2 \sim \lambda\, n/\langle E\rangle}$$

Finite temperature symmetry restoration

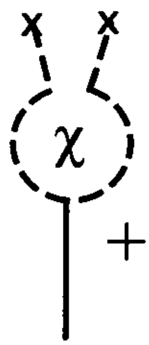
- now include $V(\sigma, \chi) = h^2\sigma^2\chi^2$

- radiative corrections *(T=0)*



$$+\frac{1}{64\pi^2}h^4\sigma^4\ln\sigma^2$$

- radiative corrections *(T≠0)*



$$+\frac{T^4}{2\pi^2}\int_0^\infty dx\ x^2\ln\left[1-\exp[-(x^2+h^2\sigma^2/T^2)^{1/2}]\right]$$

- at $T >> T_C$ $\quad M^2(\sigma) = -\mu^2 + 3\lambda\sigma^2 + \lambda T^2\sigma^2 \boxed{+\, h^2T^2\sigma^2}$

$$\boxed{h^2T^2 \sim h^2\, n/\langle E\rangle}$$

The fate of symmetry

- broken during inflation *($T \rightarrow 0$)*

- restored during preheating *(finite density effects)*

  $$h^2 n_\chi / \langle E_\chi \rangle \sim h^2 \rho_\chi / \langle E_\chi^2 \rangle$$

  $$\rho_\chi \sim \delta m^2 m_{PL}^2 \quad \text{and} \quad E_\chi^2 \sim g\, m\, m_{PL}$$

  $$h^2 n_\chi / \langle E_\chi \rangle \sim h^2 \delta m\, m_{PL} / g$$

  if express as a temperature $T_{EFF} \sim (\delta m\, m_{PL} / g)^{1/2}$

  could be as large as 10^{16}GeV

- broken after preheating

  — thermalization of χ

  — decay of χ

- inflation may not solve monopole problem

GUT *Baryogenesis*

Kolb, Linde, Riotto

- suppose a fraction δ of ρ_ϕ in soft modes of another field χ.

- identify χ as a B-L violating GUT boson *X*.

  $m_X \sim 10^{14\text{-}15}$GeV (probably Higgs)

- *X* produced ***out of equilibrium***

  inherently non-thermal spectrum

- produce asymmetry through "drift and decay"

  $X \longrightarrow bb$ $\bar{X} \longrightarrow \bar{b}\bar{b}$

  $X \longrightarrow \bar{b}\bar{b}$ $\bar{X} \longrightarrow bb$

- new twist here is produce particles (much) more massive than inflaton

  $m_\phi \sim 10^{13}$GeV

Baryogenesis issues

$$g^2\phi^2 X^2$$

inflaton ← ϕ X → ***GUT boson***

- can massive ($m_X > m$) bosons be made? **<u>YES!</u>**

  -analytic: Kolb, Linde, Riotto (PRL 96)
  -numerical: Kolb, Riotto, Tkachev (PRD 97)

  coherent process: $N\phi \longrightarrow \bar{X}X$

  $$\langle E_\chi^2 \rangle \sim g\, m_{Pl}\, m \sim g\, 10^{16}\text{GeV}$$

  m_{Pl} → 10^{19}GeV; m → 10^{13}GeV

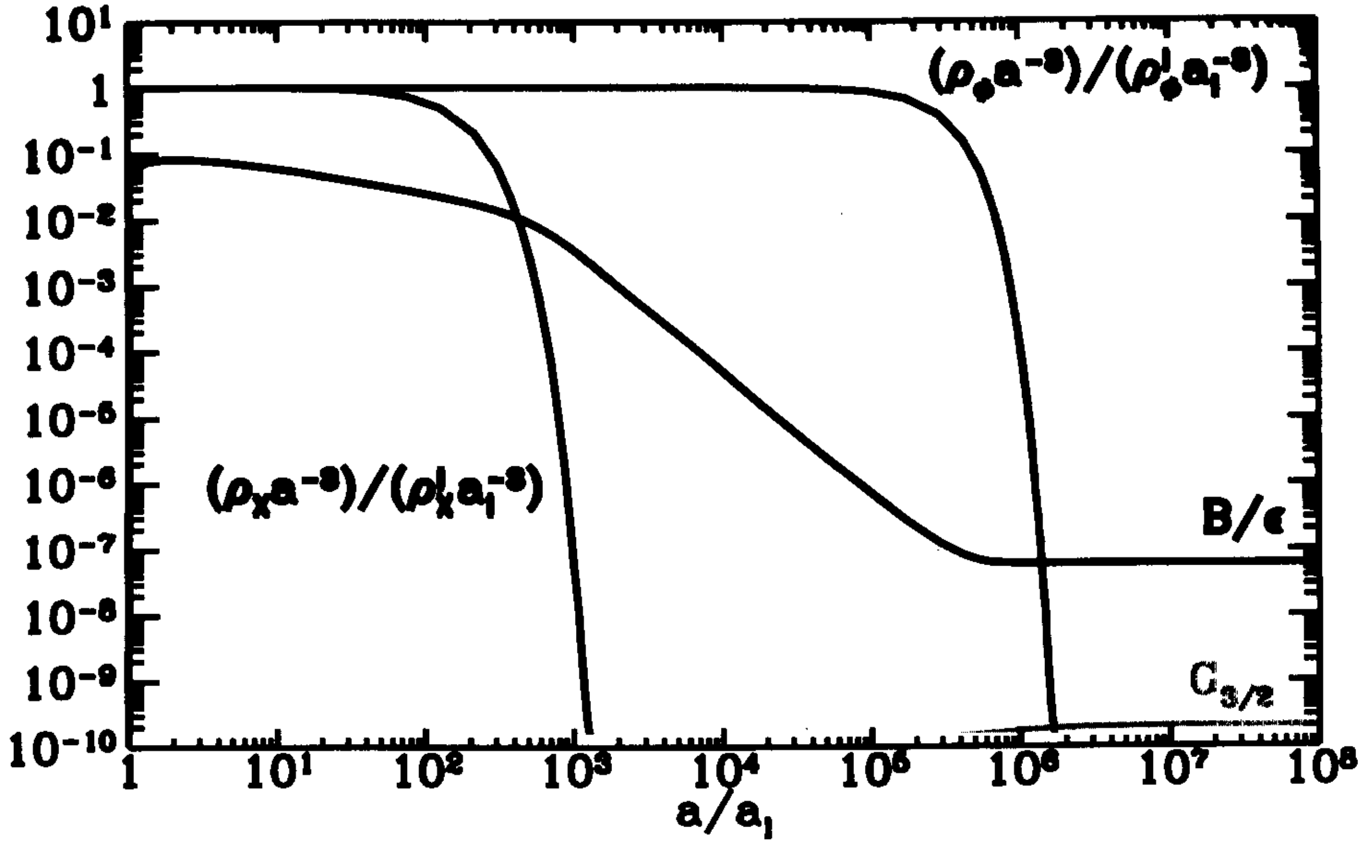
  GUT scale is the geometric mean of *inflaton* mass and *Planck* mass

  - g can't be too small
  - g can't be too large (unless SUSY)

- B-L violation necessary in X decay (points to SO(10)?)

$\not{CP}$: $\varepsilon \sim 10^{-3}$?

$\delta = 10^{-4}$



$m_\phi = 10^{13}$GeV $\quad m_X = 10^{14}$GeV

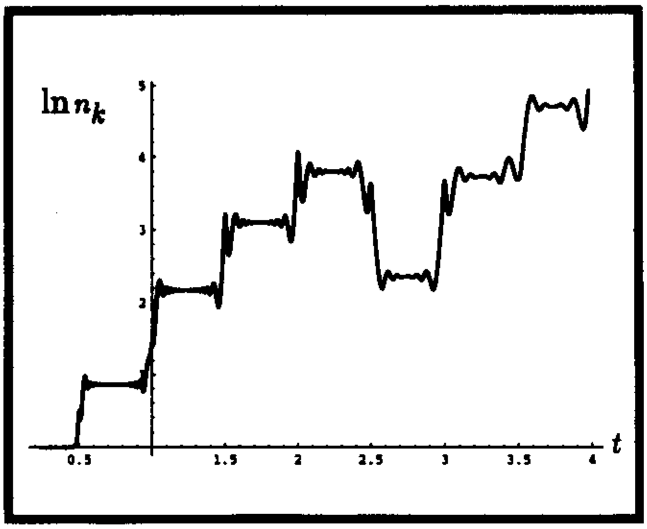
Baryo-conclusions

- Need large-field inflation model.
- Preheating gives new life to GUT baryogenesis
- Natural ***out of equilibrium state***

  (non-thermal origin of X)
- Restrictions on coupling constants and masses
- Numerical study of 'toy' models
  - preheating dynamics
  - reaction network
- 'Realistic' calculation still to be done

Isocurvature Fluctuations

Hui, Kolb, Stewart; Tkachev; Easther,...

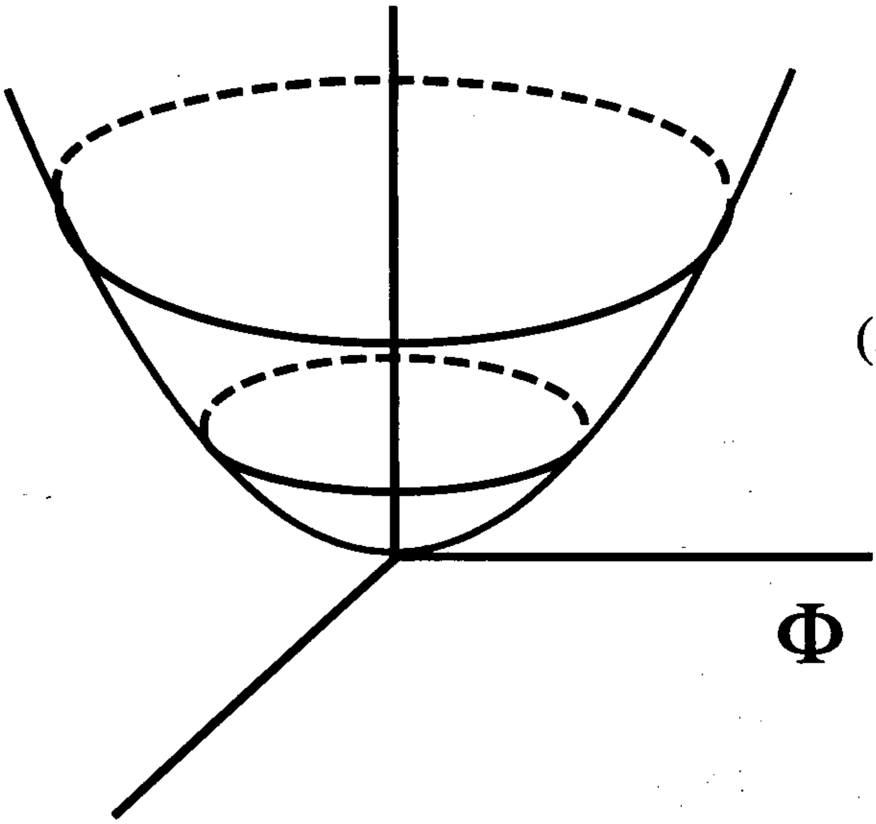


jumps when

ϕ

passes through origin

$V \quad M^2\phi^2 \rightarrow M^2|\Phi|^2$



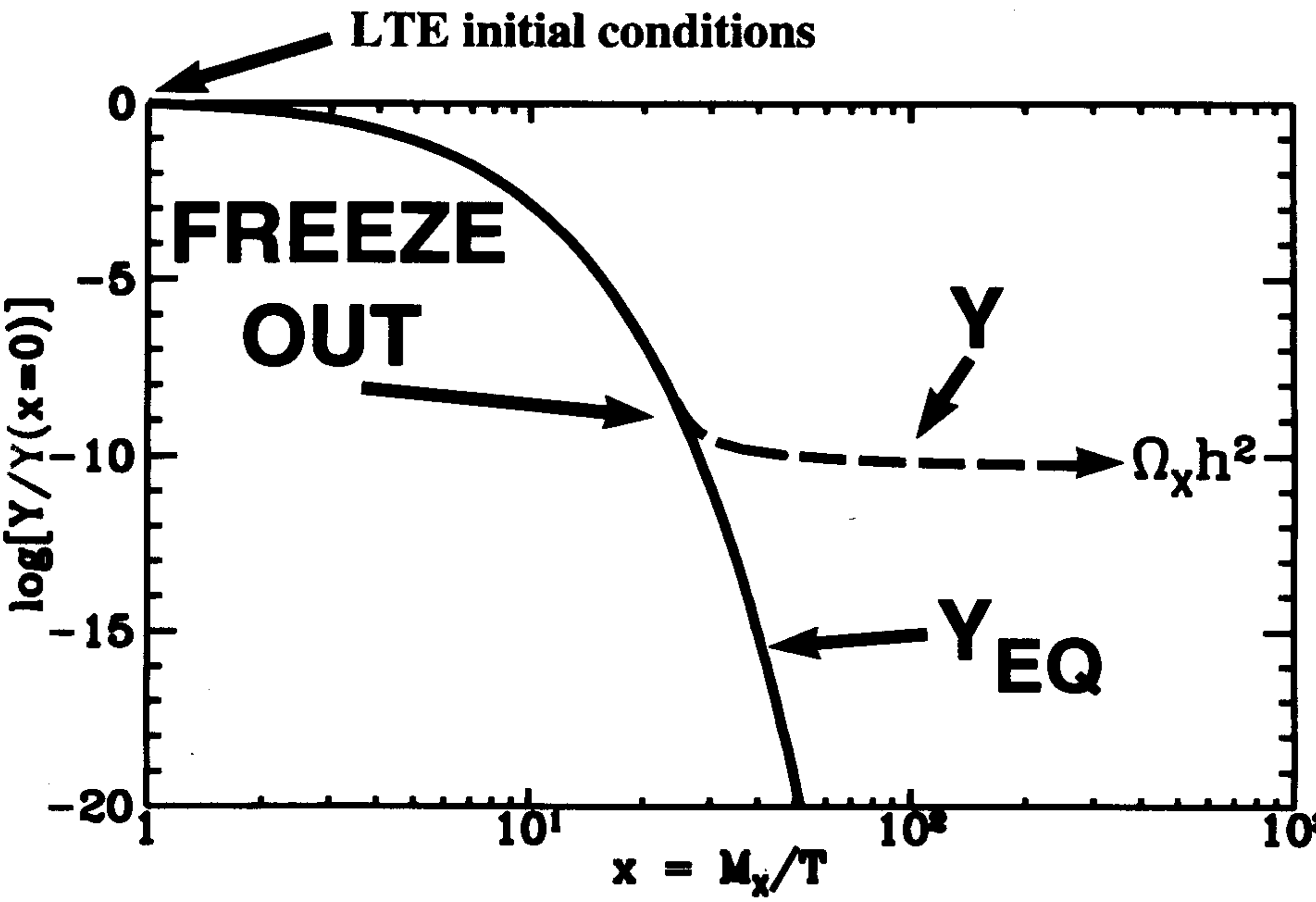
$\dot{\theta} \neq 0$

avoid origin

(angular momentum)


WIMPZILLAS!
SIZE
DOES
MATTER
$M \approx 10^{14}$ GeV

COLD THERMAL RELICS



$$Y \equiv \frac{n_X}{s} \approx Y_{EQ} \quad \text{at freeze out}$$

$$Y(x=\infty) \propto \frac{1}{M_X m_{Pl} \langle \sigma_A |v| \rangle}$$

Cold Thermal Relics

In some circumstances the annihilation cross section may be better approximated by $\langle\sigma_A|v|\rangle = \sigma_0 x^{-n}(1+bx^{-m})$, e.g., if both s-wave and p-wave annihilation processes are important. The modification to (5.45) is straightforward to compute: $Y_\infty \rightarrow Y_\infty/[1+(n+1)bx_f^{-m}/(m+n+1)]$, and $x_f \rightarrow x_f + \ln[1+b\{\ln(0.038(g/g_*^{1/2})m_{Pl}m\sigma_0)\}^{-m}]$.

As with a hot relic, the present number density and mass density of relic ψ's is easy to compute,

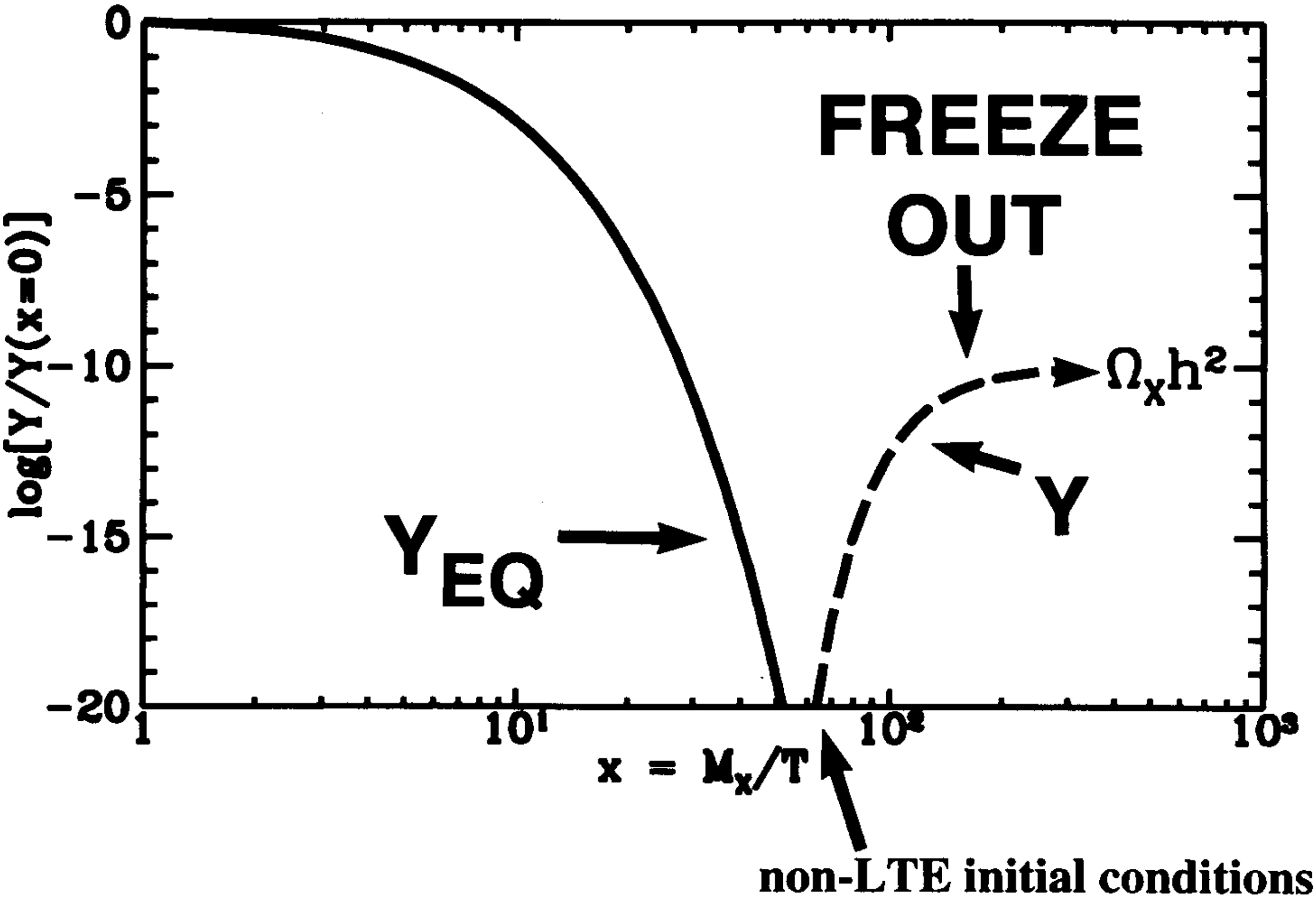
$$
\begin{aligned}
n_{\psi 0} &= s_0 Y_\infty = 2970 Y_\infty \ \mathrm{cm}^{-3} \\
&= 1.13\times 10^4 \frac{(n+1)x_f^{n+1}}{(g_{*S}/g_*^{1/2})m_{Pl}\, m\, \sigma_0}\ \mathrm{cm}^{-3} \qquad (5.46) \\
\Omega_\psi h^2 &= 1.07\times 10^9 \frac{(n+1)x_f^{n+1}\ \mathrm{GeV}^{-1}}{(g_{*S}/g_*^{1/2})m_{Pl}\sigma_0}. \qquad (5.47)
\end{aligned}
$$

It is very interesting to note that the relic density of ψ's is inversely

$$\Omega h^2 \propto \sigma_0^{-1}$$

σ_0	$\sim$	weak scale
M_X		undetermined

NONTHERMAL RELICS



$$\boxed{Y \equiv \frac{n_X}{s} \gg Y_{EQ} \quad \text{at freeze out}}$$

NONLTE:

radiation-dominated at freezeout: T_F

$$H^2(T_F) = \frac{\rho_\gamma(T_F)}{M_{Pl}^2} = \frac{T_F^4}{M_{Pl}^2}$$

$$\frac{\Omega_X}{\Omega_\gamma} = \frac{\rho_X(T_F)}{\rho_\gamma(T_F)}\,\frac{T_F}{T_0} = \frac{M_X n_X(T_F)}{T_F^4}\,\frac{T_F}{T_0}$$

interaction rate << expansion rate

$$\frac{n_X(T_F)\langle\sigma_A|v|\rangle}{H(T_F)} = \frac{\Omega_X}{\Omega_\gamma}\,\frac{T_0 M_{Pl} T_F}{M_X}\langle\sigma_A|v|\rangle \ll 1$$

safe limits: $\langle\sigma_A|v|\rangle < M_X^{-2} \qquad \Omega_X < 1$

$$\left(\frac{200\ \mathrm{TeV}}{M_X}\right)^2 \left(\frac{T_F}{M_X}\right) \ll 1$$

Cold Thermal Relics

Zel'dovich; Lee & Weinberg; Wolfram; Steigman, ...

- σ_0 weak scale for $\Omega \sim 1$
- limit to $\sigma_0(M)$: $M < 200$ TeV Griest & Kamionkowski

Inflation Relics

Chung, Kolb, Riotto; Kuzmin & Tkachev

- independent of σ_0
- $M < 10^{11}$ -- 10^{14} GeV (10^{8} -- 10^{11} TeV)

Re/Preheating Relics

Chung, Kolb, Riotto; Chung

- depends on σ_0
- $M \sim 10^{12}$ -- 10^{19} GeV

(Pseudo)solitons

Kolb & Tkachev; Frieman, Gelmini, Gleiser, Kolb; Griest & Kolb;
Frieman & Olinto; Kusenko, Kuzmin, Shaposhnikov, Tkachev, Tinaykov

- *axion miniclusters (Hogan & Rees)* $M \sim 10^{49}$ GeV
- *Q-balls, B-balls, superballs* ($M \gg M_{WEAK}$)

Inflation relics: Semiclassical quantum gravity

Changes in spacetime ➡ particle creation

(Arnowit, Birrell, Bunch, Davies, Deser, Ford, Fulling, Grib, Grischuck, Hartle, Hawking, Hu, Kofman, Linde, Mamaev, Misner, Mostepanenko, Page, Parker, Starobinski, Unruh, Vilenkin, Wald, Zel'dovich, ...)

- **vacuum quantum fluctuations**
- **mode functions change with time**
- **particle creation**

➡ { **Hawking radiation**
  density perturbations from inflation

Expanding universe ➡ particle creation
➡ dark matter

- **we require a particle** ***X*** {
  - ***massive (probably supermassive)***
  - ***stable (at least long lived)***
  - ***arbitrary interactions***
- **we assume**
  initial inflationary era
  transition to matter/radiation era
- **we extract**
  10^{-17} of energy into X

Consider scalar field X of mass M_X

$$X(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}a(\eta)} \left[a_k h_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + a_k^\dagger h_k^*(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$

scale factor → $a(\eta)$; annihilation & creation operators → a_k, $a_k^\dagger$

Mode equation (η = conformal time)

$$h_k''(\eta) + \left[k^2 + M_X^2 a^2 + (6\xi - 1)a''/a\right] h_k(\eta) = 0$$

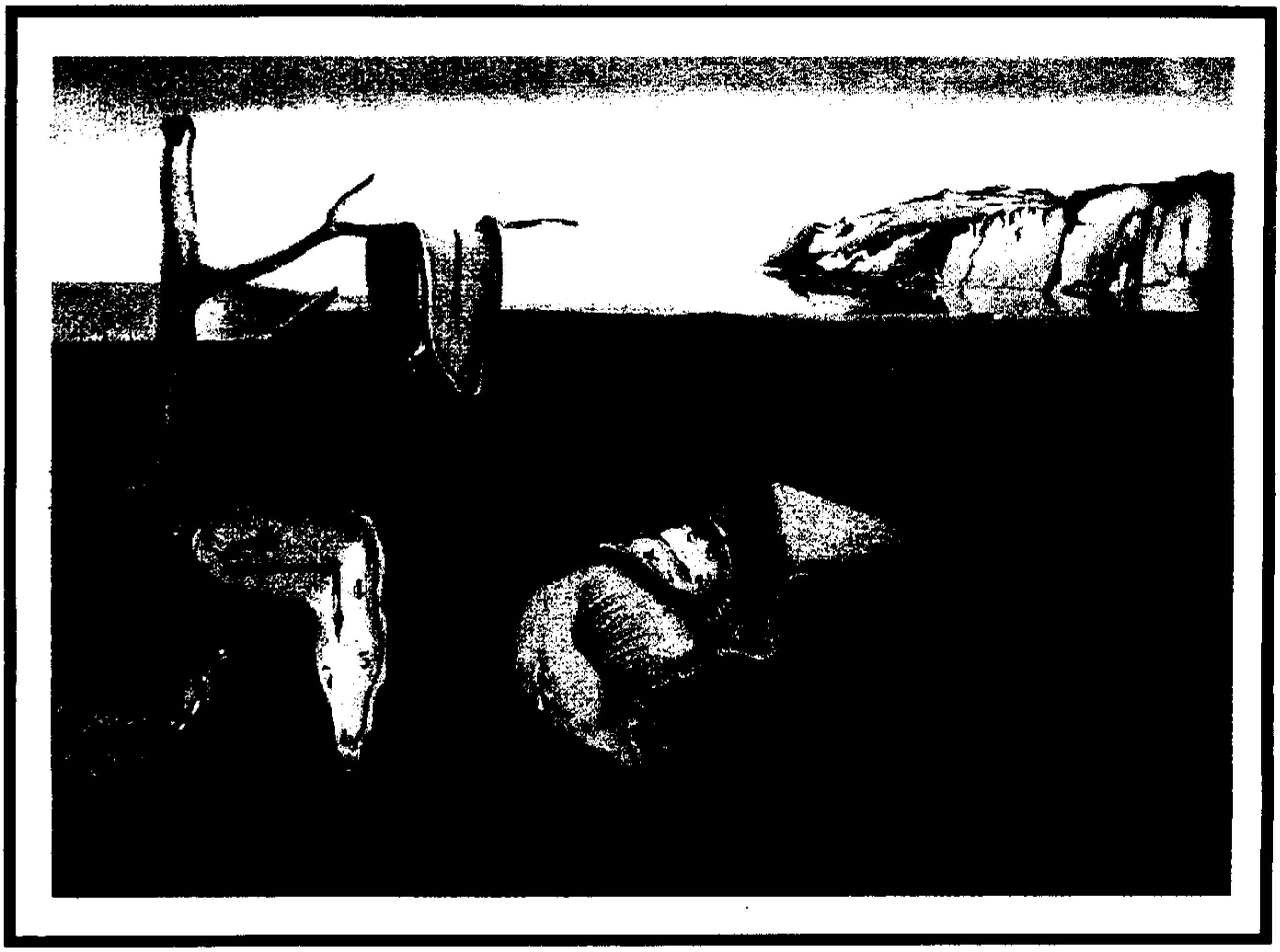
($6\xi - 1$) → *0*

$$h_k''(\eta) + \omega_k^2(\eta) h_k(\eta) = 0$$

Particle creation in nonadiabatic region

particle creation proportional to $\dfrac{\omega_k'}{\omega_k}$

Conformal Time



$$ds^2 = a^2(\eta)\left[d\eta^2 - d\vec{x}^2\right]$$

$$a^2(\eta)d\eta^2 = dt^2$$

No-particle state in past

$$a_k^0|0\rangle = 0 \qquad h_k^0 \longleftrightarrow a_k^0$$

$h_k(\eta)$ adulterated as $\omega_k(\eta)$ changes

Bogoliubov transformation:

$$h_k = \alpha_k h_k^0 + \beta_k h_k^{0*}$$

$$a_k = \alpha_k a_k^0 - \beta_k a_k^{0\dagger}$$

Particle creation

$$N_k = \langle 0|a_k^\dagger a_k|0\rangle \propto |\beta_k|^2$$

◆

Solve wave equation

$$h_k''(\eta) + \omega_k^2(\eta) h_k(\eta) = 0$$

$$\omega_k^2(\eta) = k^2 + M_X^2 a^2(\eta)$$

$$h_k^0 = 1/\sqrt{2\omega_k^0} \qquad h_k^{0\prime} = -i\sqrt{\omega_k^0/2}$$

Bogoliubov coefficient

$$|\beta_k|^2 = \frac{|h_k'|^2 + \omega_k^2 |h_k|^2 - \omega_k}{2\omega_k}$$

Number density proportional to

$$\int_0^\infty \frac{dk}{2\pi^2} k^2 |\beta_k(\infty, -\infty)|^2$$

Particle Creation

$$ds^2 = a^2(\eta)\left[d\eta^2 - d\vec{x}^2\right]$$

finite result so long as $\frac{d^n}{d\eta^n}\left[\frac{1}{a^2(\eta)}\frac{da^2(\eta)}{d\eta}\right]$ **finite!**

inflation *a~e^Ht*	$a^2(\eta) \propto \eta^{-2}$ $-\infty \le \eta \le 0$ $(0 \le t \le +\infty)$	**fails at** ***t=infinity***
matter/ radiation *a~t^2/3* *a~t^1/2*	$a^2(\eta) \propto \eta^2$ (MD) ; η (RD) $0 \le \eta \le \infty$ $(0 \le t \le +\infty)$	**fails at** ***t=0***

so ... start in inflation and end in matter/radiation

◆

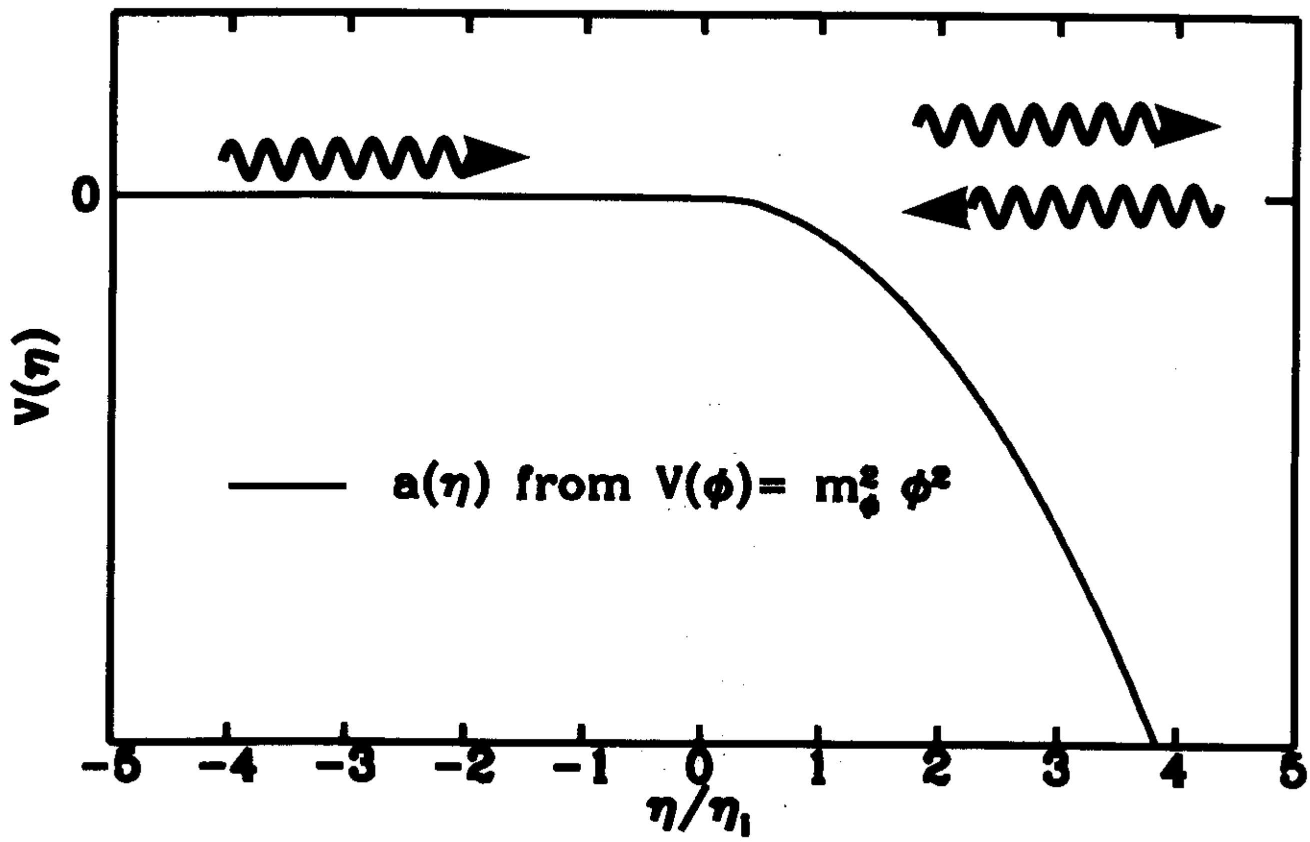
Mode Equation:

$$h''_{\tilde{k}}(\tilde{\eta}) + \left(\tilde{k}^2 + \frac{M_X^2}{H_i^2}\tilde{a}^2\right) h_{\tilde{k}}(\tilde{\eta}) = 0$$

$$\tilde{\eta} \to x \; ; \quad h_{\tilde{k}}(\tilde{\eta}) \to \psi(x) \; ; \quad \tilde{k}^2 \to E \; ; \quad \frac{M_X^2}{H_i^2}\tilde{a}^2 \to -V(x)$$

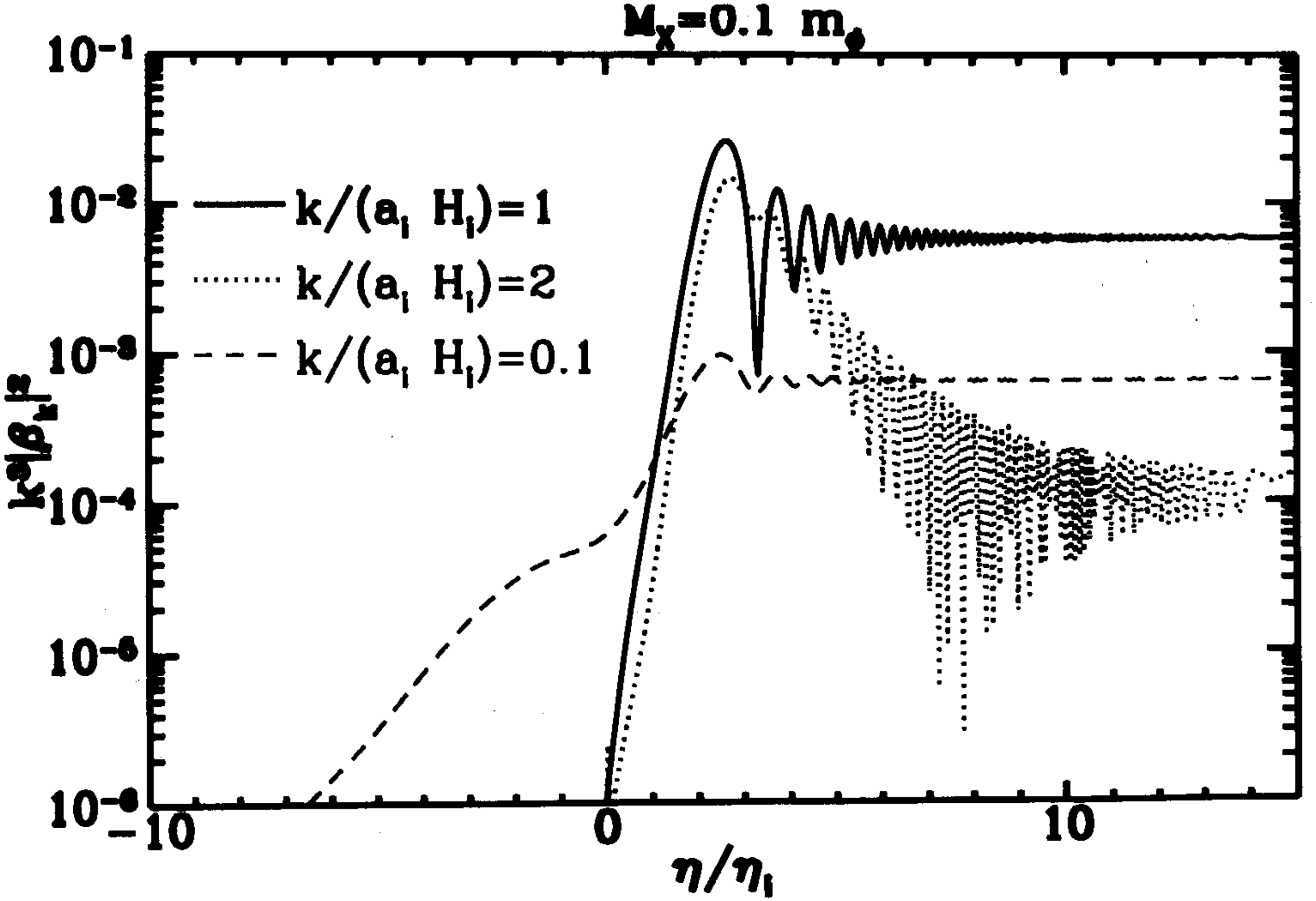
Wave Equation:

$$-\frac{\partial^2 \psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$$



Action at transition out of inflation ($\eta/\eta_i \sim 1$)

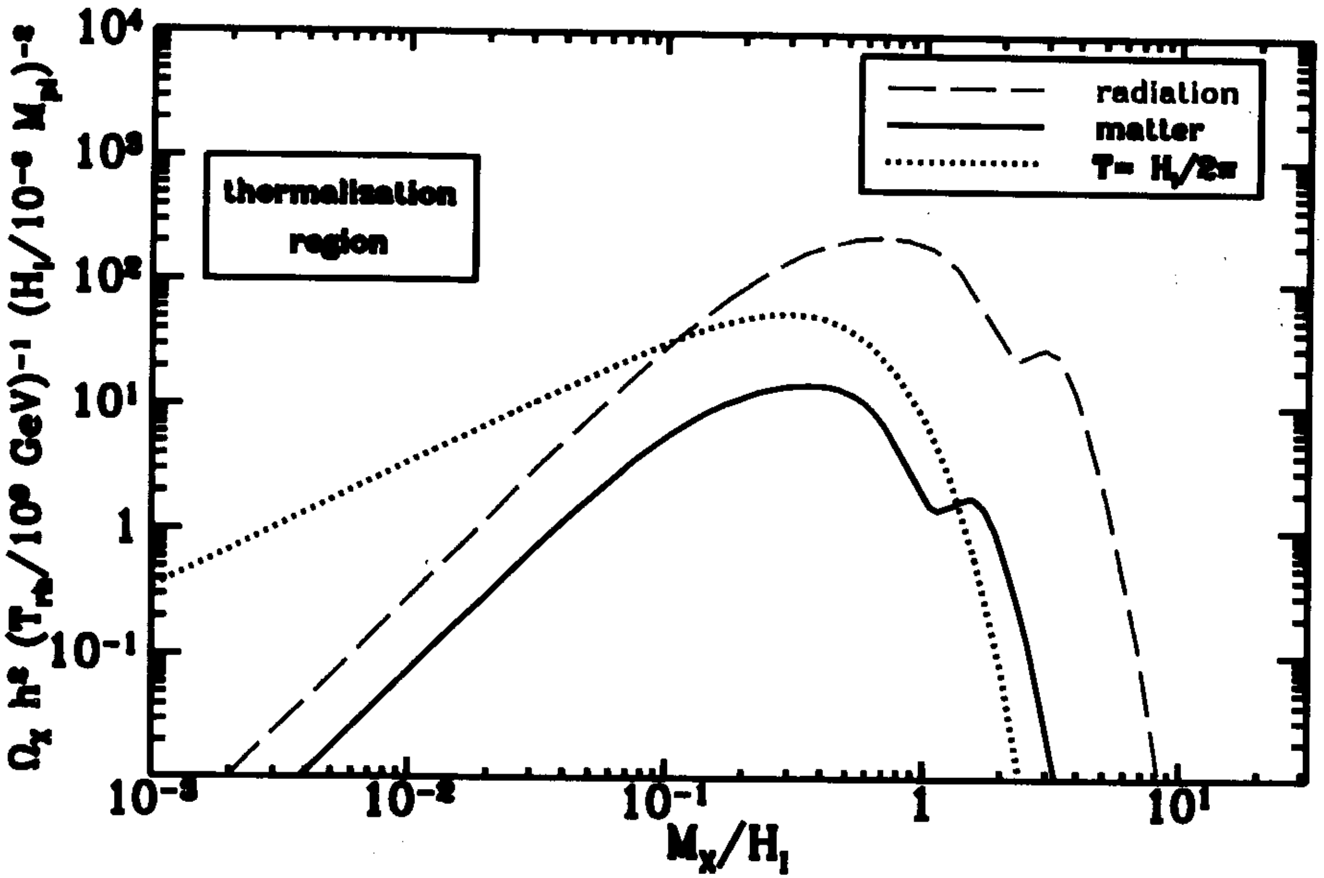
Particles produced with momentum $k \sim aH$



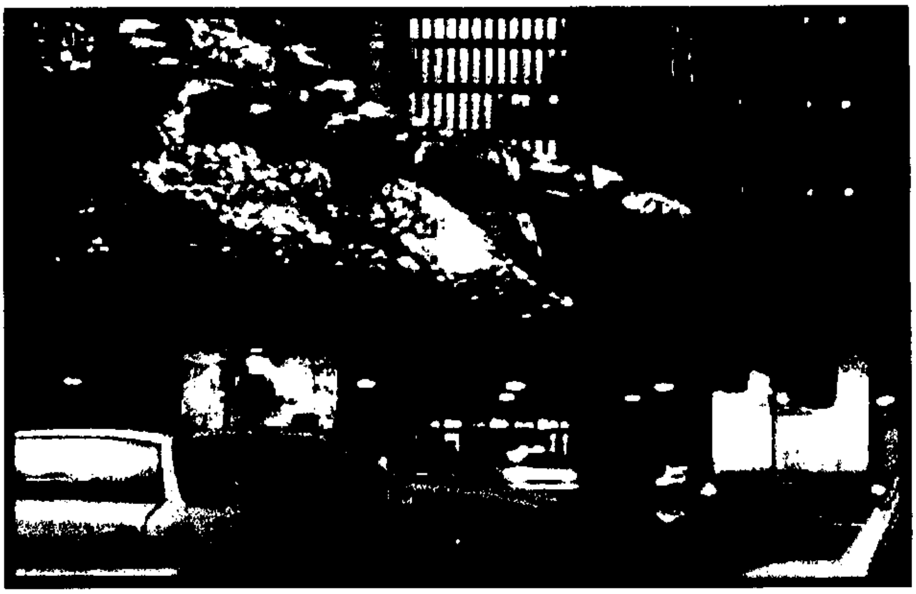
WIMPZILLA PRODUCTION:

Chung, Kolb, Riotto '98

$\Omega \sim 1$ for $M/H_{\text{INF}} \sim 1$ $\quad M \sim 10^{11}$--10^{14} GeV



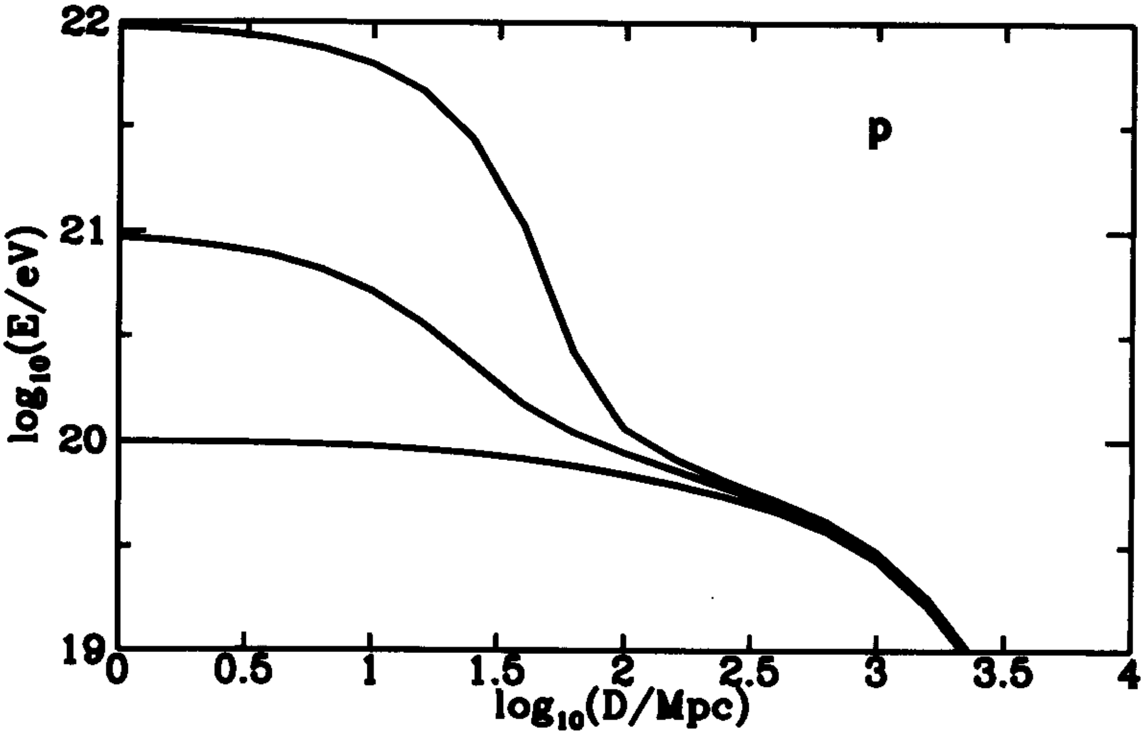
Wimpzilla Footprint:



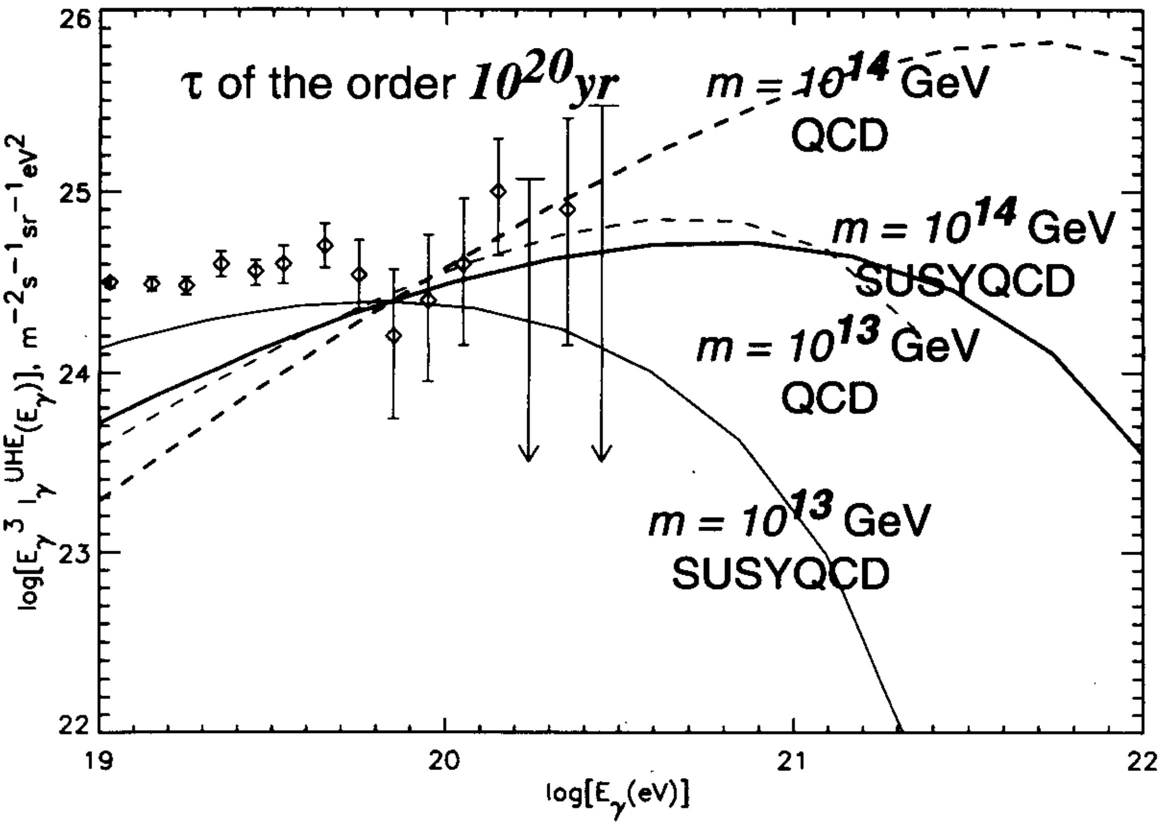
Interaction strength:
charged (heavyweight CHAMPS), hadronic, shadow, noninteracting?

Indirect detection:
neutrinos from the sun?

Stability:
stable, decay (cryptons) UHECR?



22
21
20
19
0
0.5
1
1.5
2
2.5
3
3.5
4
p
$\log_{10}(E/eV)$
$\log_{10}(D/Mpc)$



Blasi
PRD, 60 (99)

DARK MATTER

may be a

WIMP

thermal relic:

σ ~ ***weak scale***

M_X ***undetermined***

or may be a

WIMP ZILLA

nonthermal relic:

σ ***undetermined***

$M_X \sim H_{end\ inflation}$

To Be Explored

Fermions	Kuzmin & Tkachev
Non-Conformal	Kuzmin & Tkachev
Small-Field Models	
Hybrid Models	
Particle Models	Ellis et al.

Other Possibilities for Production

Reheating	Chung, Kolb, Riotto
Preheating	Chung
Bubble Collisions	Chung, Kolb, Riotto

Dark Matter:

WIMP

--- or ---

WIMPZILLA