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SMR.1227 - 20

SUMMER SCHOOL ON ASTROPARTICLE PHYSICS AND COSMOLOGY

12 - 30 June 2000

TOPOLOGICAL DEFECTS

Part II

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Please note: These are preliminary notes intended for internal distribution only.

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3. Strings:

(i) Topology : π_1.

Strings are formed if G/H has incontractable S^1's (circles).

Relevant Homotopy group is $\pi_1(G/H)$.

Theorem: If $\pi_n(G) = 1 = \pi_{n-1}(G)$ then

$$\pi_n(G/H) = \pi_{n-1}(H).$$

With $n=1 \Rightarrow \pi_1(G/H) = \pi_0(H)$

$\therefore$ if H has disconnected components, strings will be formed.

Alternately if $\pi_1(G) \neq 1$ ~~and~~ then strings may form. (Incontractable path in G must ~~not be contr~~ ~~be incontractable~~ ~~in H~~ not be in H.)

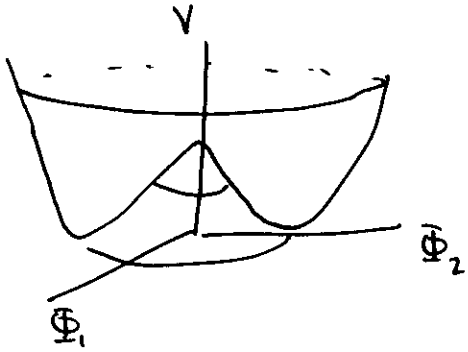
eg. $U(1) \to 1$.

$$\pi_0(1) = 1 \quad \text{but} \quad \pi_1(U(1)) = \mathbb{Z}$$

$\therefore$ look out for $U(1)$ factors in G and discrete factors in H if you are looking for strings.

(ii) Example: U(1) local

$$L = \frac{1}{2} \left| (\partial_\mu - ieA_\mu)\Phi \right|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{4} (\Phi^* \Phi - \eta^2)^2$$



$\Phi = (\Phi_1, \Phi_2)$ = complex.

$$U(1): \quad \Phi \longrightarrow \Phi' = e^{i\alpha(t,\vec{x})} \Phi$$

$$A_\mu \longrightarrow A_\mu' = A_\mu + \frac{1}{e} \partial_\mu \alpha .$$

$$\Sigma = \frac{U(1)}{\mathbb{1}} = S^1 .$$

$\pi_1(\Sigma) = \mathbb{Z} \Rightarrow$ strings are labeled by integers.

Bogomolnyi method for finding string solution:

$$E = \int d^2x \left[\frac{1}{2} |D_i \Phi|^2 + \frac{1}{2} B_z^2 + V(\Phi) \right].$$

Rescale:

$$\Phi_a = \eta Q_a \qquad (a = 1, 2).$$

$$A_m = \frac{\eta}{\sqrt{2}} v_m \qquad (m = 1, 2).$$

$$x^m = \frac{\sqrt{2}}{e\eta} \cdot y^m$$

Then:

$$E = \frac{\eta^2}{2} \int d^2y \cdot \Big[\frac{1}{4} \left(f_{mn} - \varepsilon_{mn} (1 - Q_a Q_a) \right)^2$$
$$+ \frac{1}{2} \left(\varepsilon_{mn} D_n Q_a + \varepsilon_{ab} D_m Q_b \right)^2$$
$$+ \frac{\beta - 1}{2} \left(Q_a Q_a - 1 \right)^2$$

$$+ \left\{ \frac{1}{2} f_{mn} \varepsilon_{mn} (1 - Q_a Q_a) - \varepsilon_{mn} \varepsilon_{ab} D_n Q_a D_m Q_b \right\} \Big]$$

with $\beta = \dfrac{2\lambda}{e^2}$, $D_m Q_a \equiv \partial_m Q_a + e\, \varepsilon_{ab} \cancel{A_m} v_m \cdot Q_b .$

But: $\frac{1}{2} f_{mn} \varepsilon_{mn} (1 - Q_a Q_a) - \varepsilon_{mn} \varepsilon_{ab} D_n Q_a D_m Q_b = \partial_p S_p$

where

$$S_p = \varepsilon_{pm} \left(Q_a D_m Q_b \cdot \varepsilon_{ab} + v_m \right).$$

Hence last term in E is a boundary term.

$\therefore$ for $\underline{\underline{\beta = 1}}$, E is minimized if

$$\left.\begin{array}{l} f_{mn} - \epsilon_{mn}\,(1 - Q_a Q_a) = 0 \\ \epsilon_{mn} D_n Q_a + \epsilon_{ab} D_m Q_b = 0. \end{array}\right\} \text{Bogomolnyi eq.}$$

and:

$$E_{min} = \frac{\eta^2}{2} \int d^2y \,.\, \partial_p S_p .$$

$$= \frac{\eta^2}{2} \int_{C_\infty} ds_p \,.\, \epsilon_{pm} \,.\, S_m .$$

$$= \frac{\eta^2}{2} \int_{C_\infty} ds_p \,.\, \epsilon_{pm} \,.\, v_m \qquad \text{since } D_m Q_b = 0 \text{ at } \infty .$$

Now: $\quad D_m Q_a = 0$ at $\infty \implies e\,\epsilon_{ab} v_m Q_b = -\partial_m Q_a .$

or, $Q_c v_m = +\frac{1}{e}\,\epsilon_{ca}\,\partial_m Q_a .$

or, $v_m = \frac{1}{e}\,\epsilon_{ca}\,Q_c\,\partial_m Q_a \qquad Q_c Q_c = 1$ at ∞.

$$\therefore \quad E_{min} = \frac{\eta^2}{2e} \int_{C_\infty} ds_p \,.\, \epsilon_{pm} \,.\, \epsilon_{ca}\, Q_c\, \partial_m Q_a .$$

$$= \frac{\eta^2}{2e} \int_0^{2\pi} d\theta \,\left(Q_1 \partial_\theta Q_2 - Q_2 \partial_\theta Q_1 \right).$$

$$= \frac{\eta^2}{2e} \,.\, 2\pi n = \pi \frac{\eta^2}{e} \,.\, n \qquad n = \text{winding \#}.$$

eg. $Q_1 = \cos(n\theta)$, $Q_2 = \sin(n\theta)$.

If $\Phi = f(r).\, e^{i\theta}$

$v_\theta = \frac{n}{r} v(r)$

then Bogomolnyi eqns. are:

$$\frac{df}{dr} = \frac{n}{r}(1-v)f$$

$$\frac{dv}{dr} = \frac{r}{n}(1-f^2).$$

– numerical estimate of $E_{min.}$ $\sim 10^{22}$ gms/cm for $\eta \sim 10^{16}$ GeV.

Note: $G\mu \sim \left(\frac{\eta}{m_{pl}}\right)^2$ $\therefore$ $G\mu \simeq 10^{-6}$ for GUT strings.

Eg. Other kinds of strings:

$\pi_1(G/H) = \mathbb{Z}_n \longrightarrow$ $\mathbb{Z}_n$ strings.

n $\mathbb{Z}_n$ strings are topologically trivial

$\therefore$ one can have a vertex of n strings.

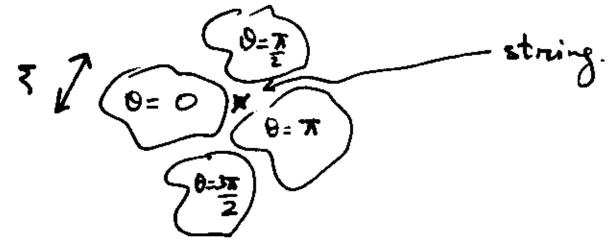
Z_5 strings.

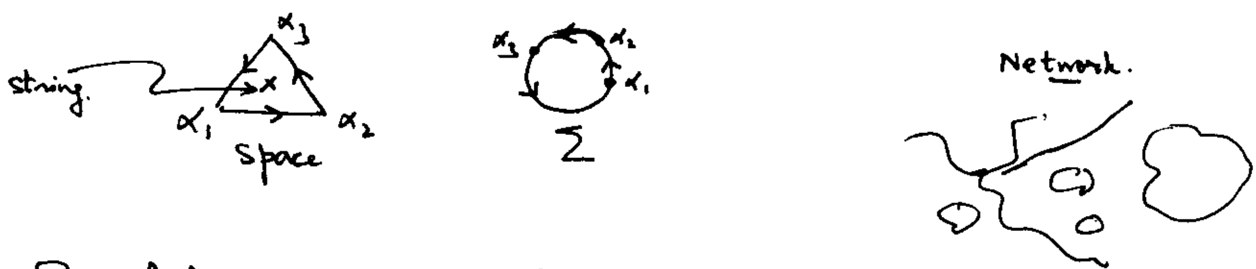
Non-Abelian strings – won't do here.

(iii) Semilocal and electroweak – will do later if time permits.

(iv) Formation and Evolution

U(1) strings.



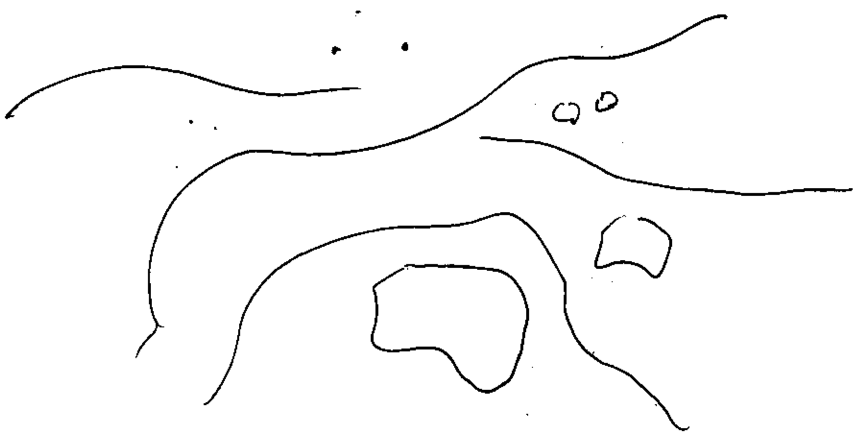


Simulations give the following results:

1) Loops: $dn = \alpha \frac{dR}{R^{4\pm}}$ scale invariant distribution.

2) Shape: $\ell = \beta R^{2\pm}$ Brownian

3) ∞ strings: $\rho_\infty \simeq 80\% \rho_{total}$ Percolation of strings



Evolution factors:

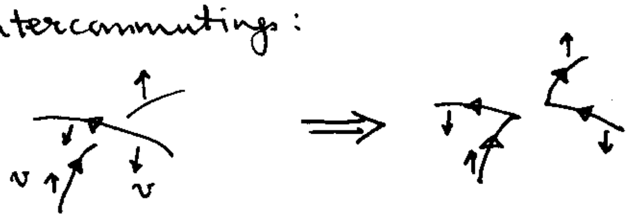
(a) tension:

$F \sim \mu/R$ R = curvature radius.

(b) friction: damps string motion when the ambient density is high. Unimportant at late times.

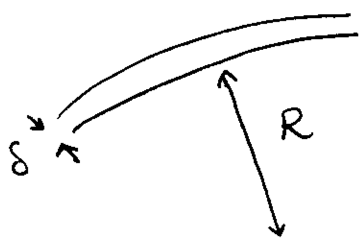
(c) Hubble expansion: stretches strings.

(d) Intercommuting:



(e) Gravitational radiation.

Effective action:



$\delta \simeq 10^{-30}$ cms   (GUT)

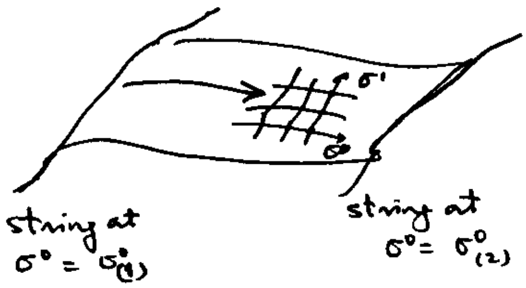
$R \simeq 10^{27}$ cms   (horizon size today)

$\frac{\delta}{R} << 1.$

$\Longrightarrow$  "zero thickness approximation"
"geometric limit".

Degrees of freedom: $x^\mu(\sigma^0, \sigma^1)$

"σ^1 is a parameter along the string"
σ^0 is a parameter in the time-like direction.



$$\underbrace{S_{FT}}_{\text{Field theory}} = \int d^4x\, L\sqrt{-g} = -\mu \int d^2\sigma \sqrt{-g^{(2)}} + \dots$$

$$= \underbrace{S_{NG}}_{\substack{\text{Nambu-Goto}\\ \text{action.}}} + \dots$$

$$g^{(2)}_{ab} = g_{\mu\nu}\, \partial_a x^\mu\, \partial_b x^\nu . \qquad a,b = 0,1 \;;\; \mu,\nu = 0,1,2,3.$$

In FRW, take $\sigma^0 = \tau$, $\sigma^1 = \zeta$ (gauge choice)

then:

$$\ddot{\vec{x}} + 2H(1 - \dot{\vec{x}}^2)\dot{\vec{x}} = \varepsilon^{-1}\left(\frac{\vec{x}'}{\varepsilon}\right)'$$

where:

$$\cdot = \frac{\partial}{\partial \tau} \qquad \tau = \text{conformal time.}$$

$$H = \frac{1}{a^2}\frac{da}{d\tau} = \frac{1}{a}\frac{da}{dt} \qquad (a\,d\tau = dt)$$

$$' = \frac{\partial}{\partial \zeta} \;; \qquad \varepsilon = \left(\frac{\vec{x}'^2}{1 - \dot{\vec{x}}^2}\right)^{1/2}$$

Approach 1: Solve eq. of motion for network on a computer. Take care of ~~collisions and~~ intersections and reconnection by hand. Very valuable but similar to studying a box of gas by following each particle.

Approach 2: "Statistical variables".

(v) Evolution: One scale model.

Network can have several different length scales. For example, the radius of curvature of long strings, the string separation, the size of the largest loops, etc. etc.

The "one scale model" is the assumption that all these length scales are roughly the same.

Define this single length scale L by:

$$\rho_\infty \equiv \frac{E_{strings}}{V} \equiv \frac{\mu}{L^2}.$$

(ρ_∞ = energy density in ∞ strings).

Then ρ_∞ changes due to two effects.

First : Hubble expansion dilutes the number density of strings and redshifts their velocities

Second: long strings chop up by forming small loops.

$$\therefore \quad \frac{d\rho_\infty}{dt} = -2H(1+v_{rms}^2)\rho_\infty - c\frac{\rho_\infty}{L}$$

$$v_{rms}^2 \equiv \frac{\int d\zeta \left(\frac{d\vec{x}}{dt}\right)^2 . \varepsilon}{\int d\zeta . \varepsilon}$$

c = parameter to account for chopping rate or "chopping efficiency". $\Big\} = \tilde{c}\, v_{rms}$

In terms of $L \left(= \sqrt{\frac{\mu}{\rho_\infty}}\right)$:

$$\frac{dL}{dt} = H.(1+v_{rms}^2)L + \frac{c}{2}$$

Scaling solution : $\rho_\infty \sim \frac{1}{t^2} \implies L \sim t$.

$\therefore$ set $L = \gamma(t).t$.

Then: $$\frac{\dot{\gamma}}{\gamma} = \frac{1}{t}\left[\frac{c}{2\gamma} - \left\{1 - Ht(1+v_{rms}^2)\right\}\right]$$

$\therefore$ scaling solution at: $$\boxed{\gamma = \frac{c}{2} \cdot \frac{1}{1-(Ht)(1+v_{rms}^2)}}$$

What is v_{rms}?

$$v_{rms}^2 = \frac{\int d\zeta\, \epsilon\, \dot{\vec{x}}^2}{\int d\zeta\, \epsilon} \qquad \dot{} = \frac{d}{dt}$$

This gives:

$$\dot{v}_{rms} = (1 - v_{rms}^2)\left[\frac{k}{L} - 2H v_{rms}\right]. \qquad k = \text{parameter called momentum parameter}$$

('k' is given by other average quantities of network.).

$\therefore$ one scale model eqns. are:

$$\frac{\dot{\gamma}}{\gamma} = \frac{1}{t}\left[\frac{\tilde{c}}{2\gamma} v_{rms} - \left\{1 - (Ht)(1 + v_{rms}^2)\right\}\right]$$

$$\dot{v}_{rms} = (1 - v_{rms}^2)\left[\frac{k}{L} - 2H v_{rms}\right].$$

Based on numerical simulations, Martins & Shellard give:

$$\tilde{c} = 0.23 \pm 0.04$$

$$k(v) = \frac{2\sqrt{2}}{\pi} \cdot \frac{1 - 8v^6}{1 + 8v^6}.$$

Is the one-scale assumption valid?

Numerical simulations $\longrightarrow$ No. Must include wiggliness.

Also, what sets scale of wiggliness $\longrightarrow$ probably grav./radn. back-reaction.

But one scale model might still provide a caricature of true network.

(vi) Cosmological constraints and signatures:

(a) CMB & P(k).

$$\frac{\delta T}{T} = \sum_{l,m} a_{lm} \cdot Y_{lm}(\theta,\varphi).$$

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{+l} \langle a^*_{lm} a_{lm} \rangle$$

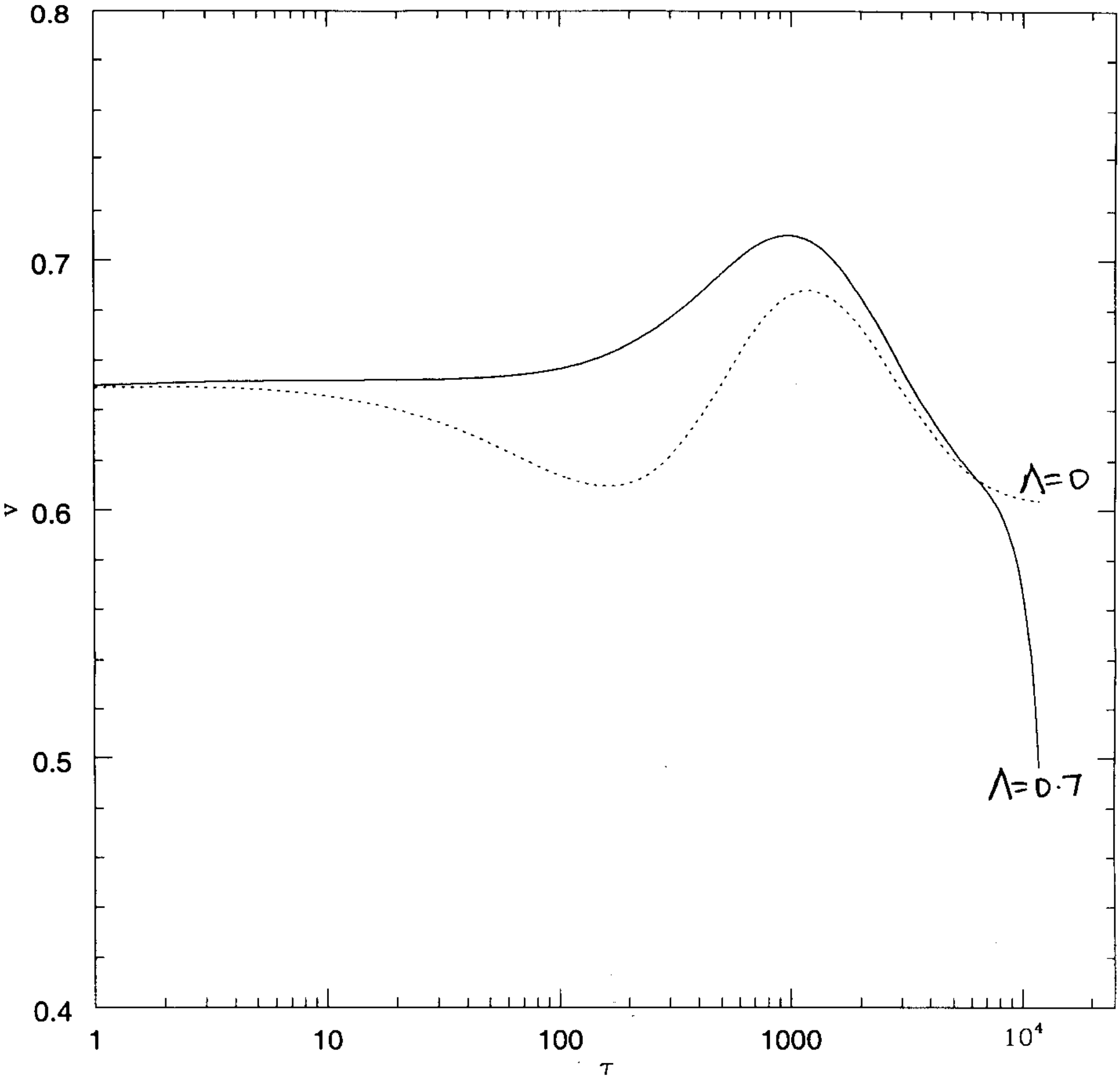
$\langle \ldots \rangle$ denote ensemble averaging.

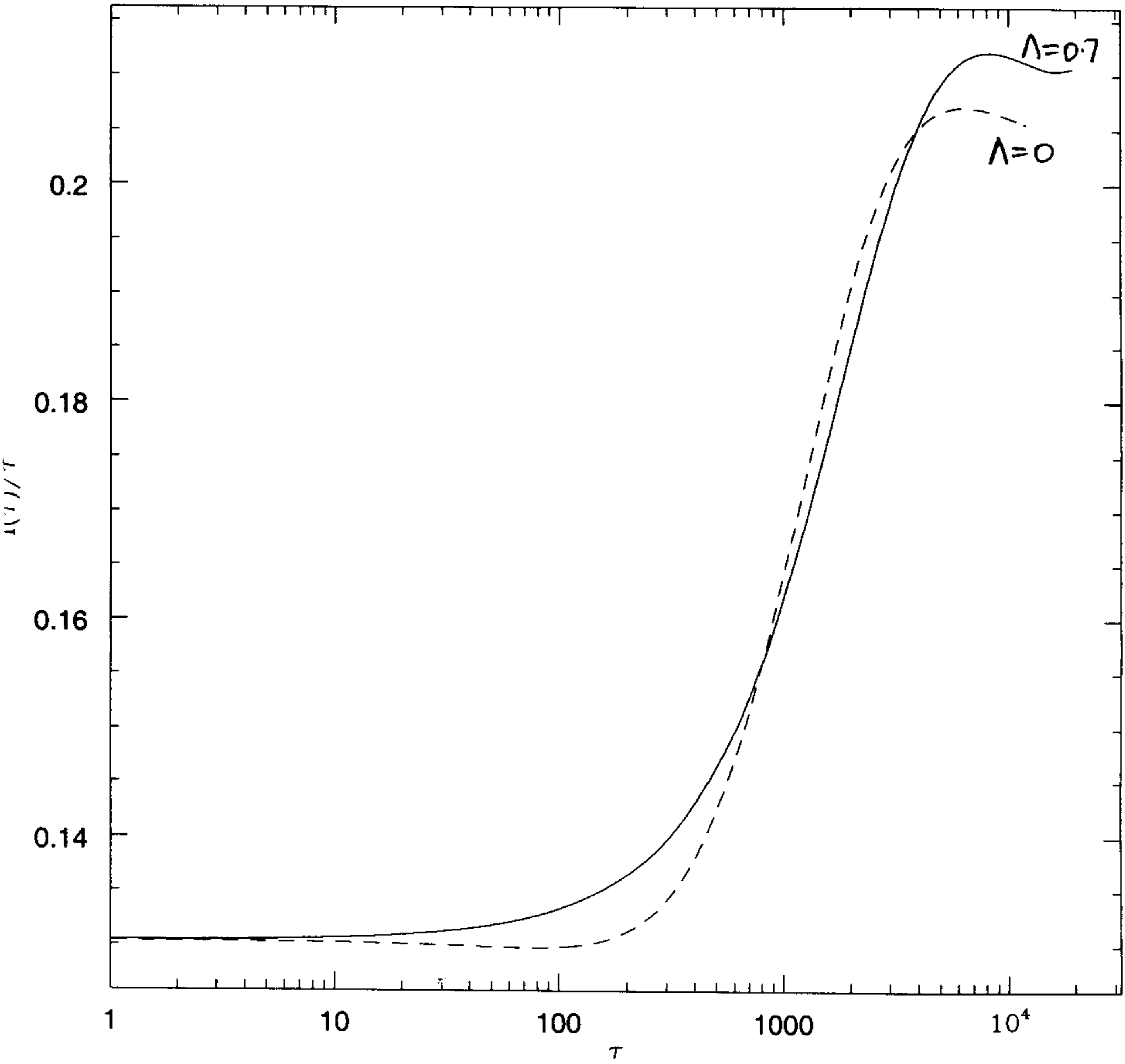
String model – straight segment gas

One scale model used to find length of segments and their velocities.

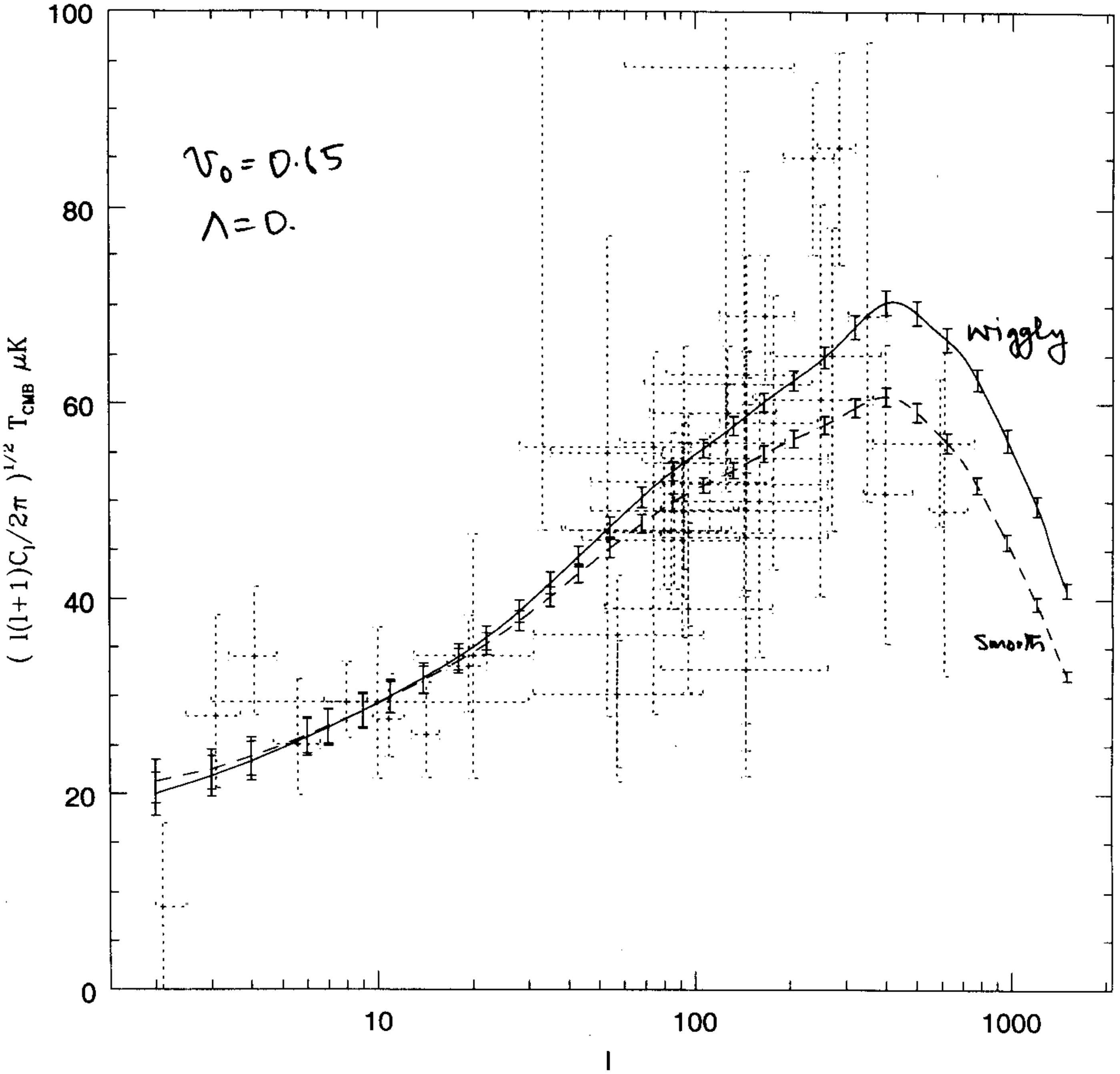
$T_{\mu\nu}$ of segment gas evaluated, plugged into CMBFAST.

Result: C_l graph, P(k) graph — for various parameters in one scale model, wiggliness and cosmology.

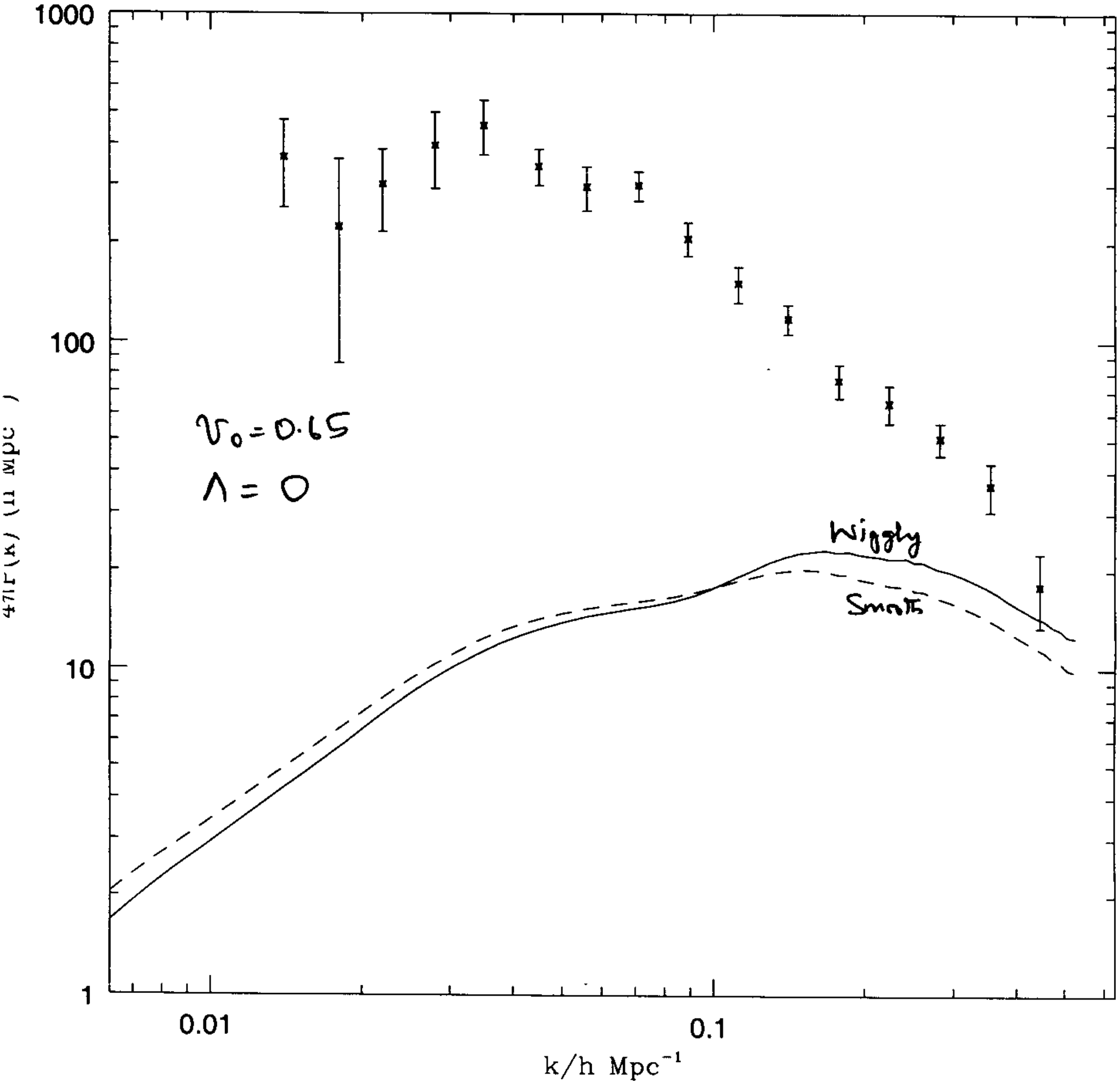

0.8
0.7
0.6
0.5
0.4
v
1
10
100
1000
10^4
τ
$\Lambda=0$
$\Lambda=0.7$

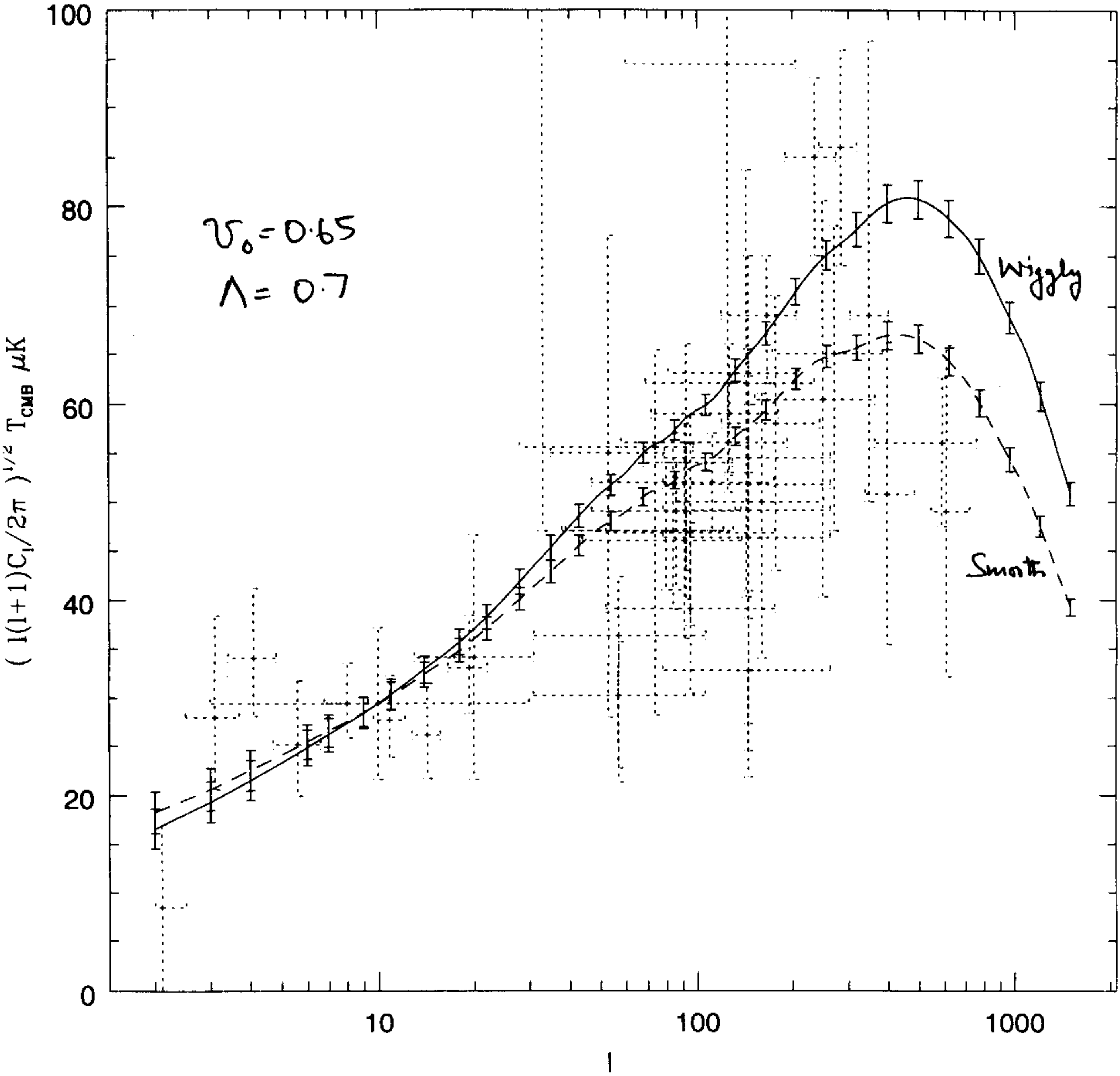


0.2
0.18
0.16
0.14
1
10
100
1000
10^4
τ
$\Lambda=0{\cdot}7$
$\Lambda=0$

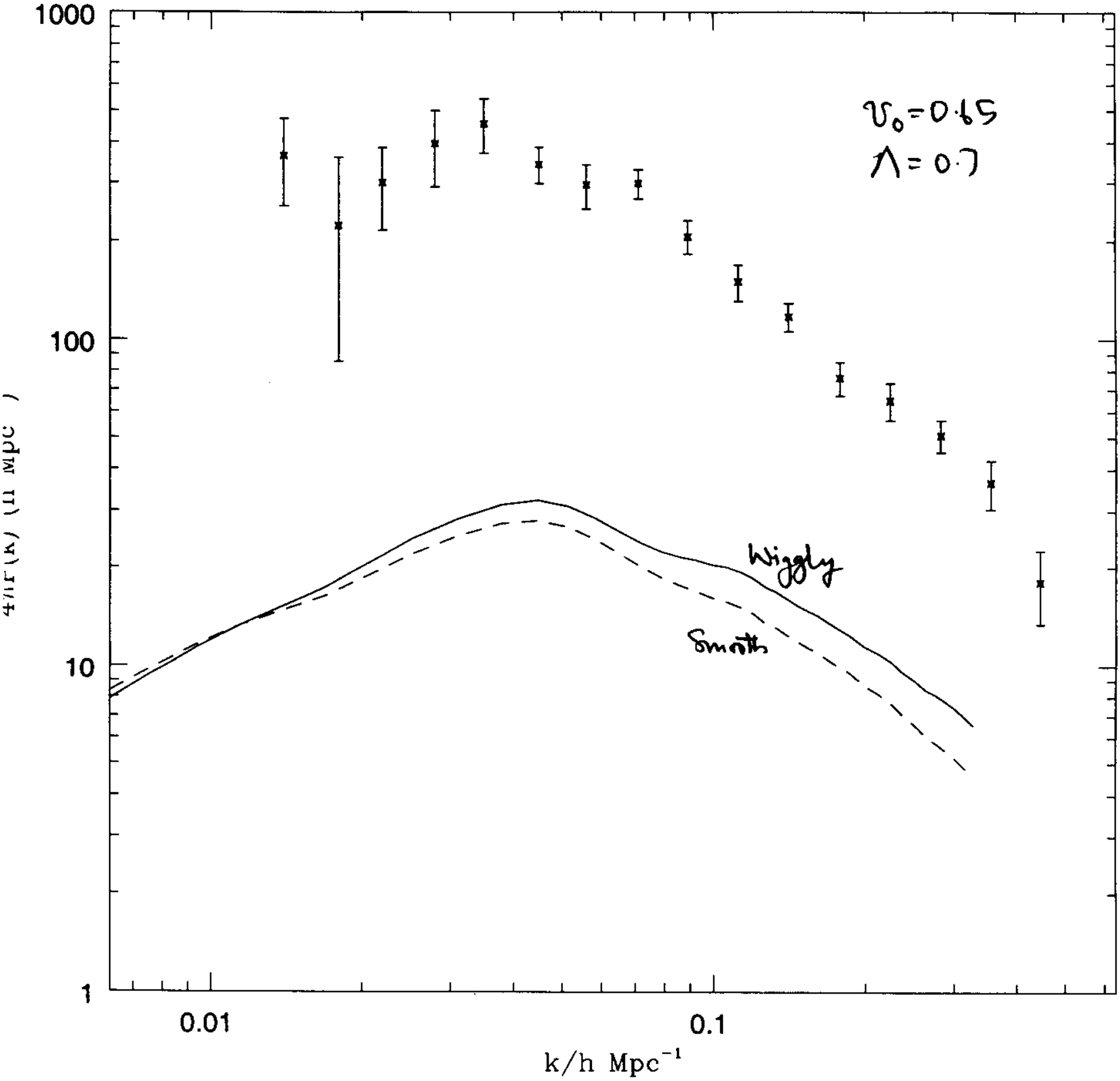


$\Omega_0 = 0.15$
$\Lambda = 0.$
wiggly
smooth
$(l(l+1)C_l/2\pi)^{1/2}\ T_{CMB}\ \mu K$
l
0
20
40
60
80
100
10
100
1000

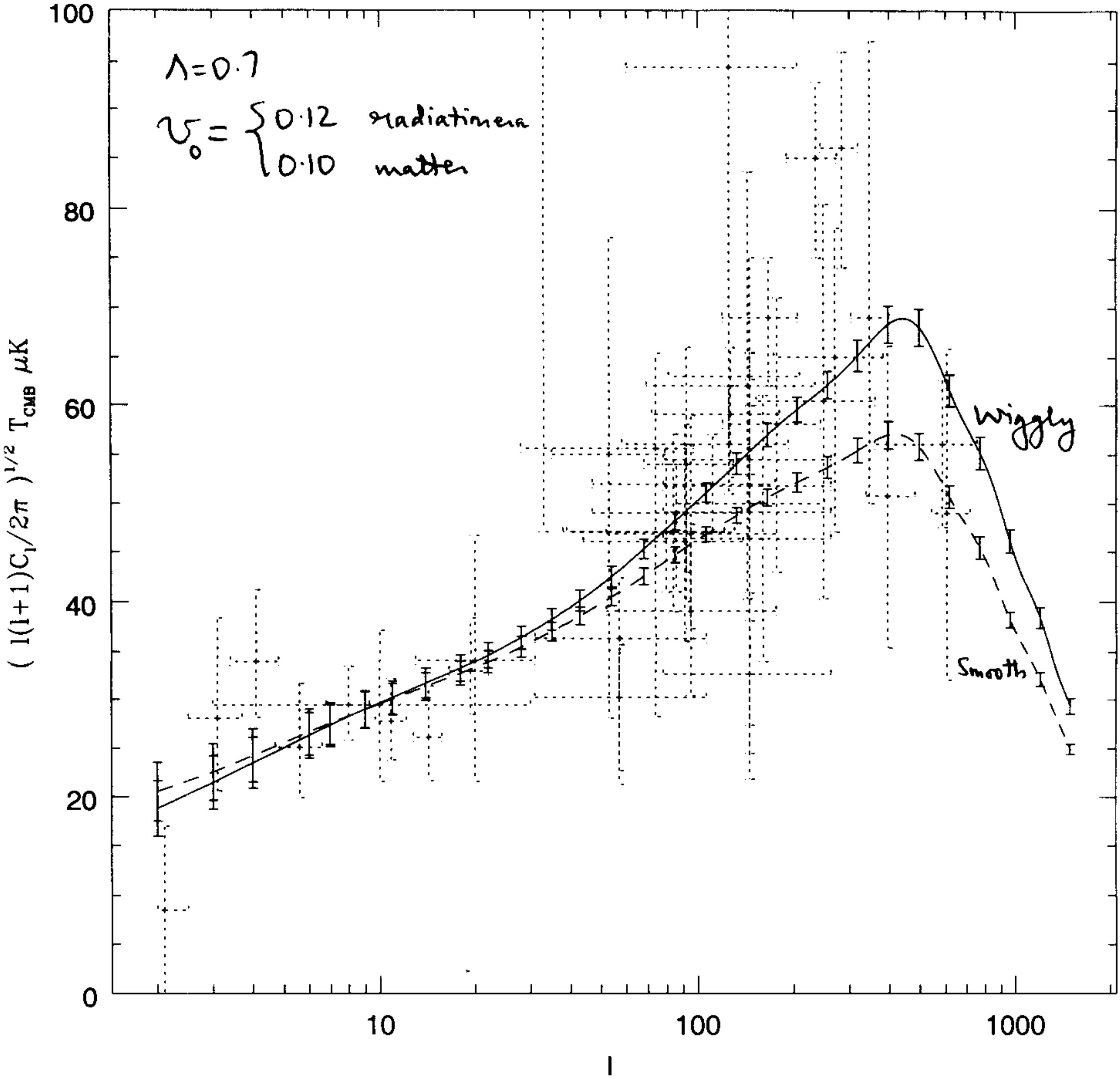

1000
100
10
1
0.01
0.1
k/h Mpc^{-1}
$4\pi P(k)$ (h Mpc)
$\Gamma_0 = 0.65$
$\Lambda = 0$
Wiggly
Smooth



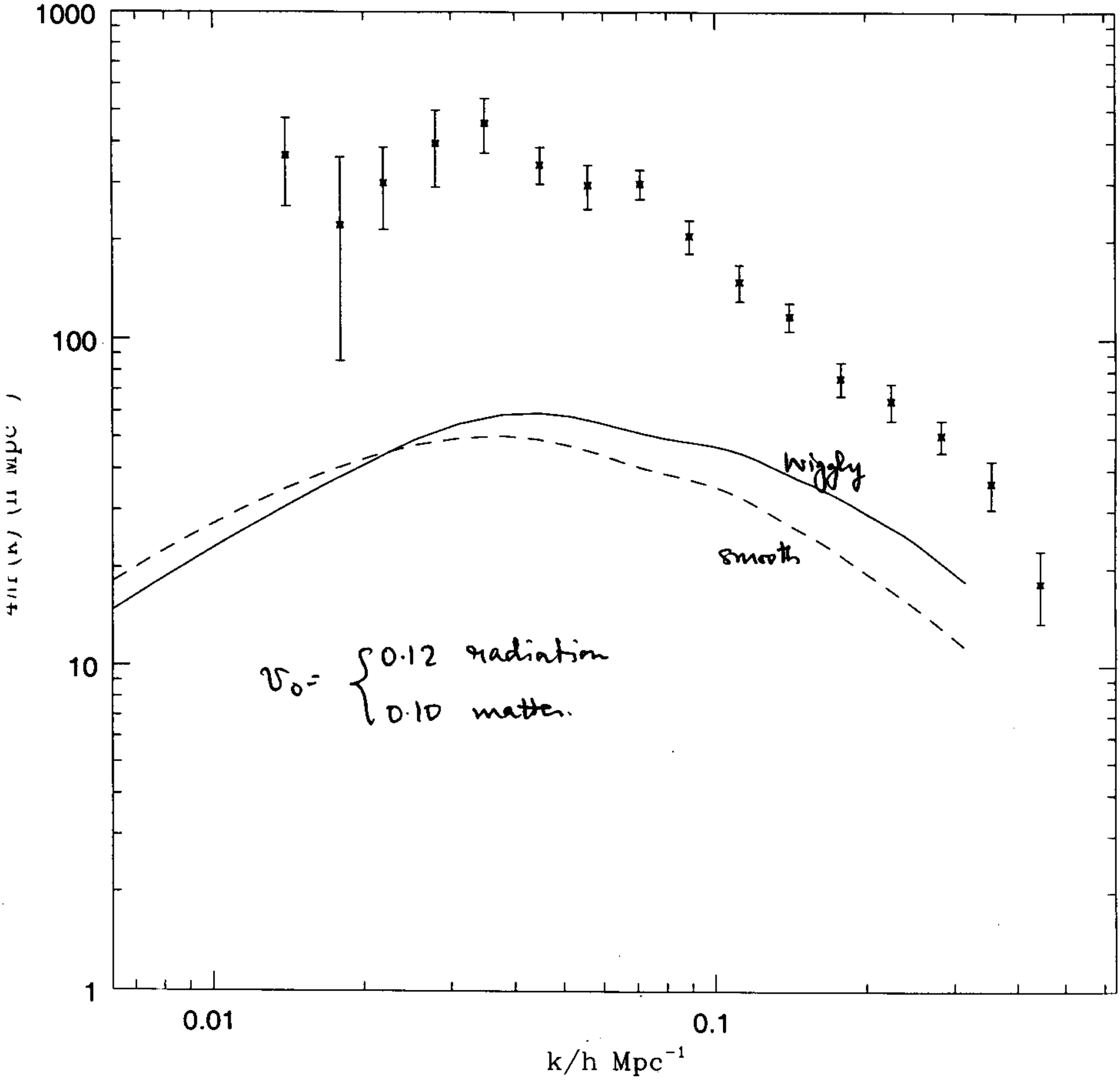
$\upsilon_0 = 0.65$
$\Lambda = 0.7$
Wiggly
Smooth
$(l(l+1)C_l/2\pi)^{1/2}\, T_{CMB}\ \mu K$
l
0
20
40
60
80
100
10
100
1000



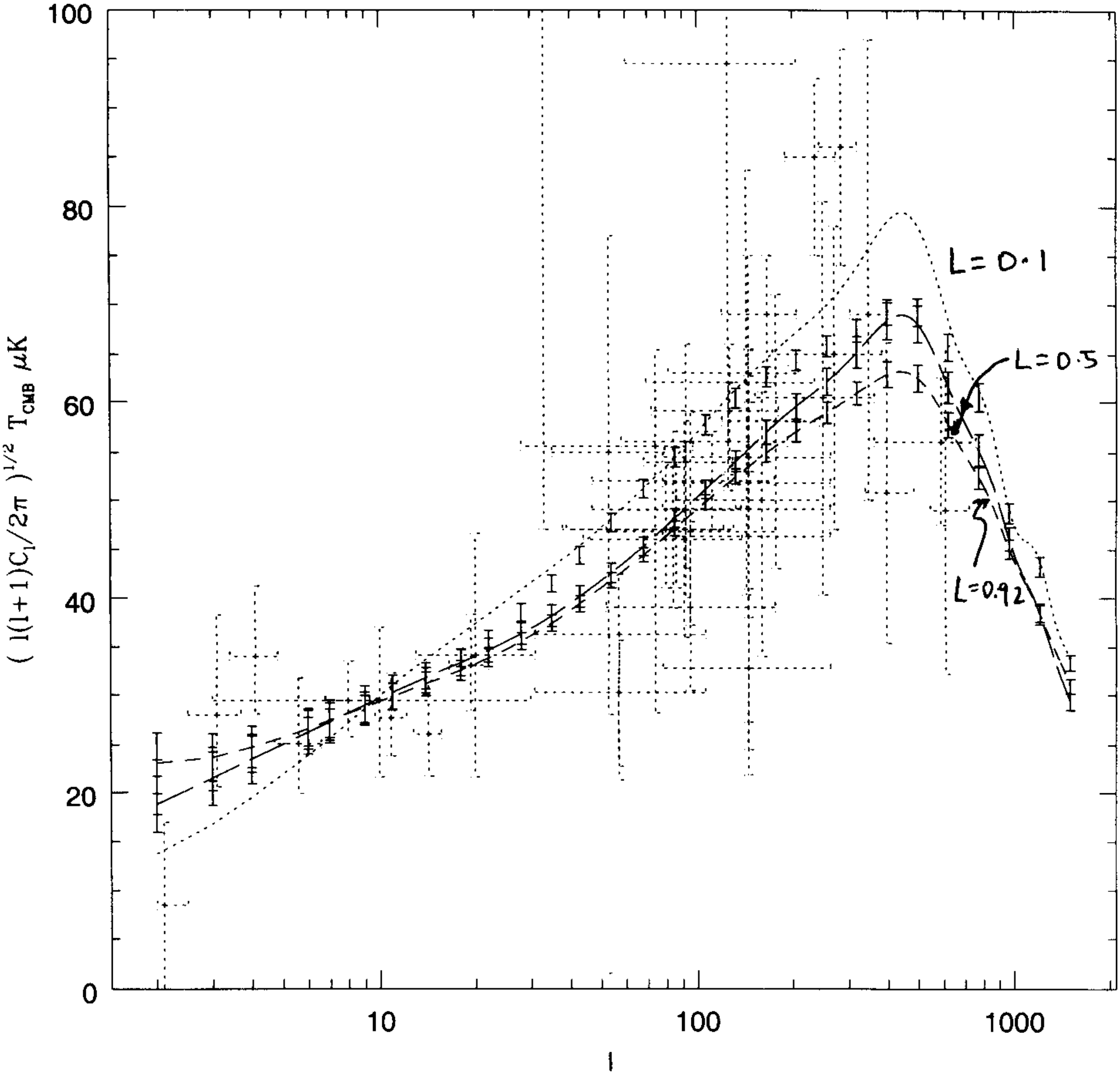
1000
100
10
1
0.01
0.1
k/h Mpc^{-1}
Ω_0=0.65
Λ=0.7
Wiggly
Smooth



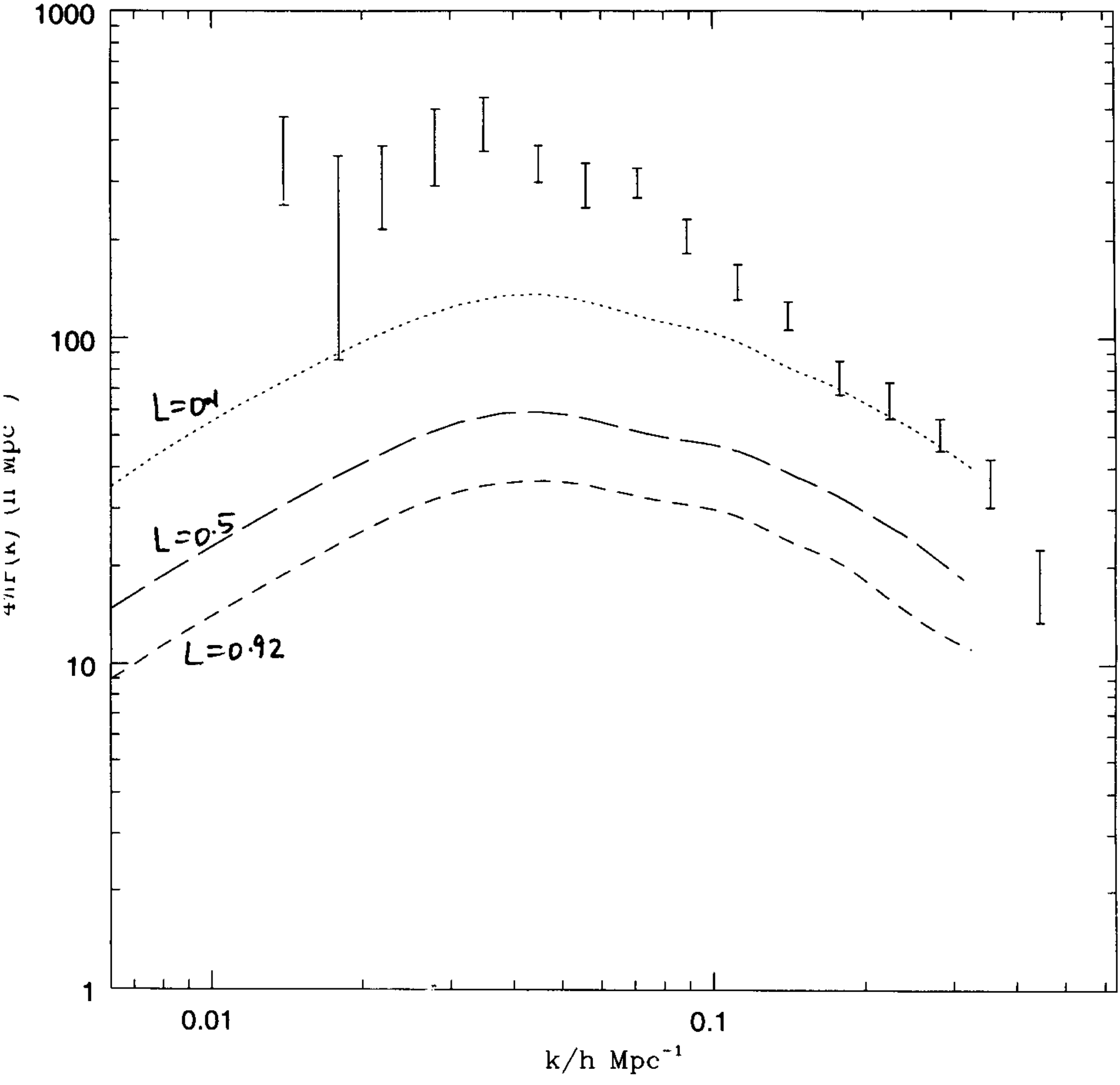
Λ=0·7
v_0 = 0·12 radiation era
0·10 matter
Wiggly
Smooth
$(l(l+1)C_l/2\pi)^{1/2}\ T_{CMB}\ \mu K$
l
0
20
40
60
80
100
10
100
1000

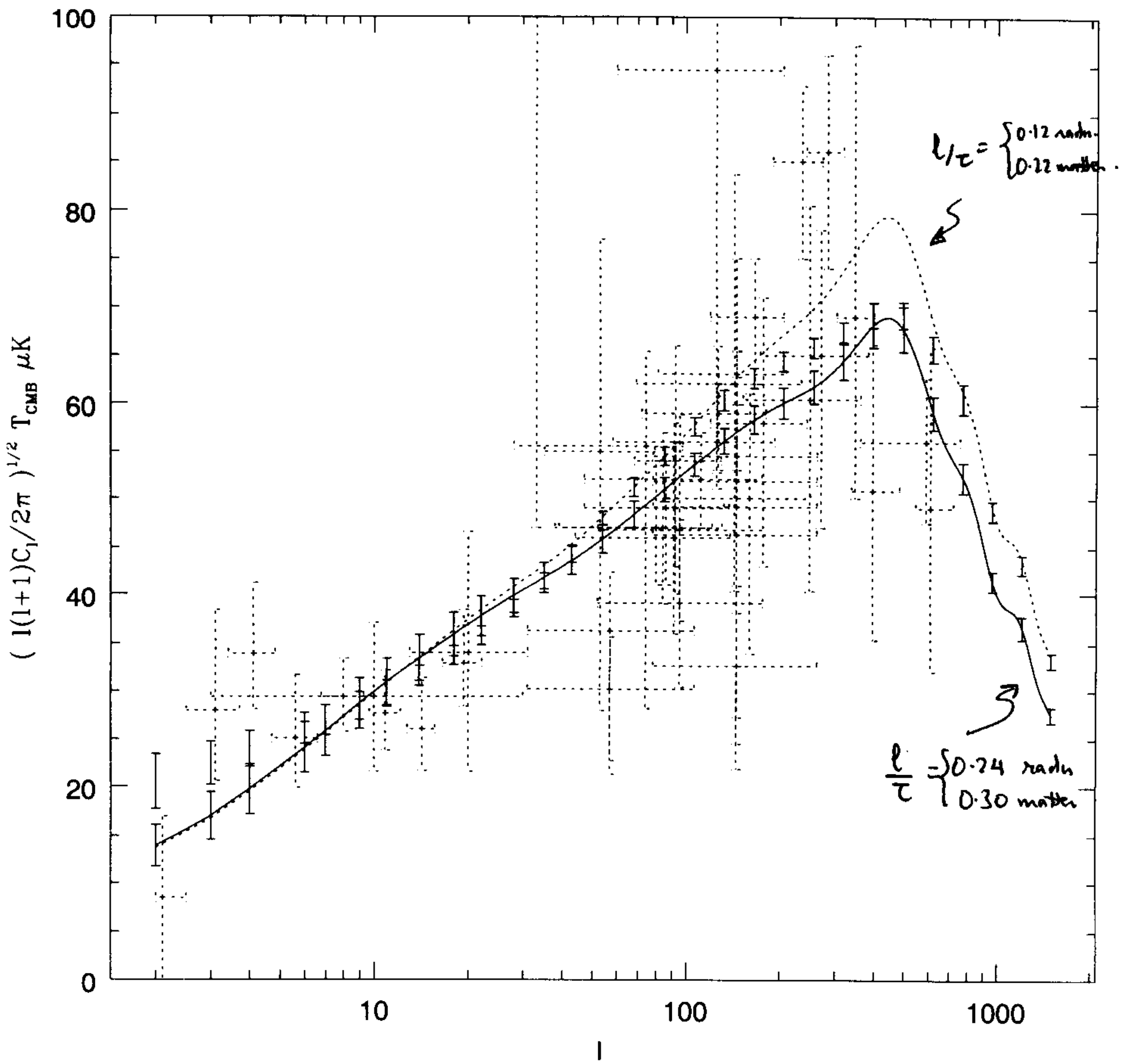


1000
100
10
1
0.01
0.1
k/h Mpc^{-1}
wiggly
smooth
$v_0 = \begin{cases} 0.12 & \text{radiation} \\ 0.10 & \text{matter} \end{cases}$

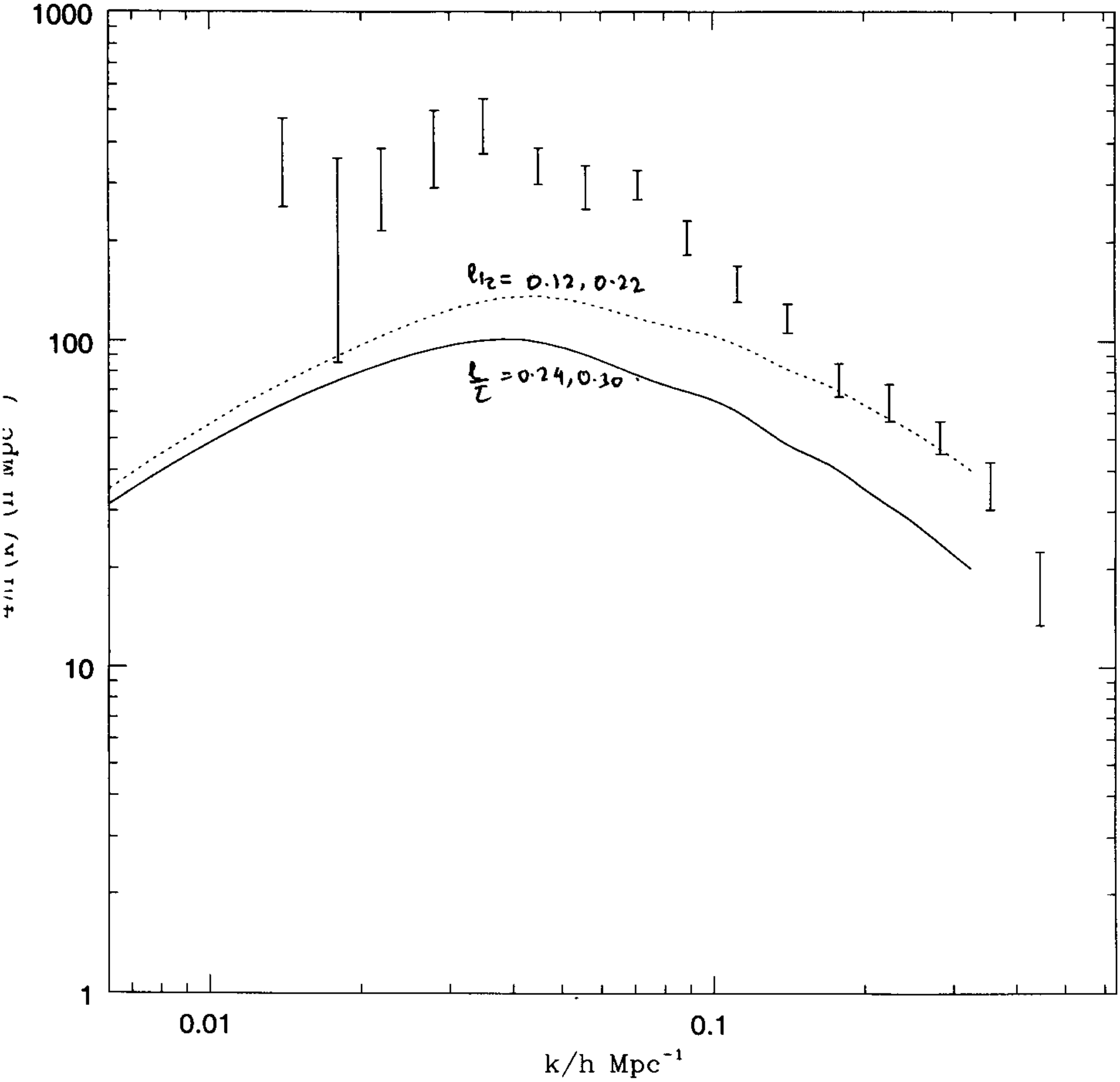


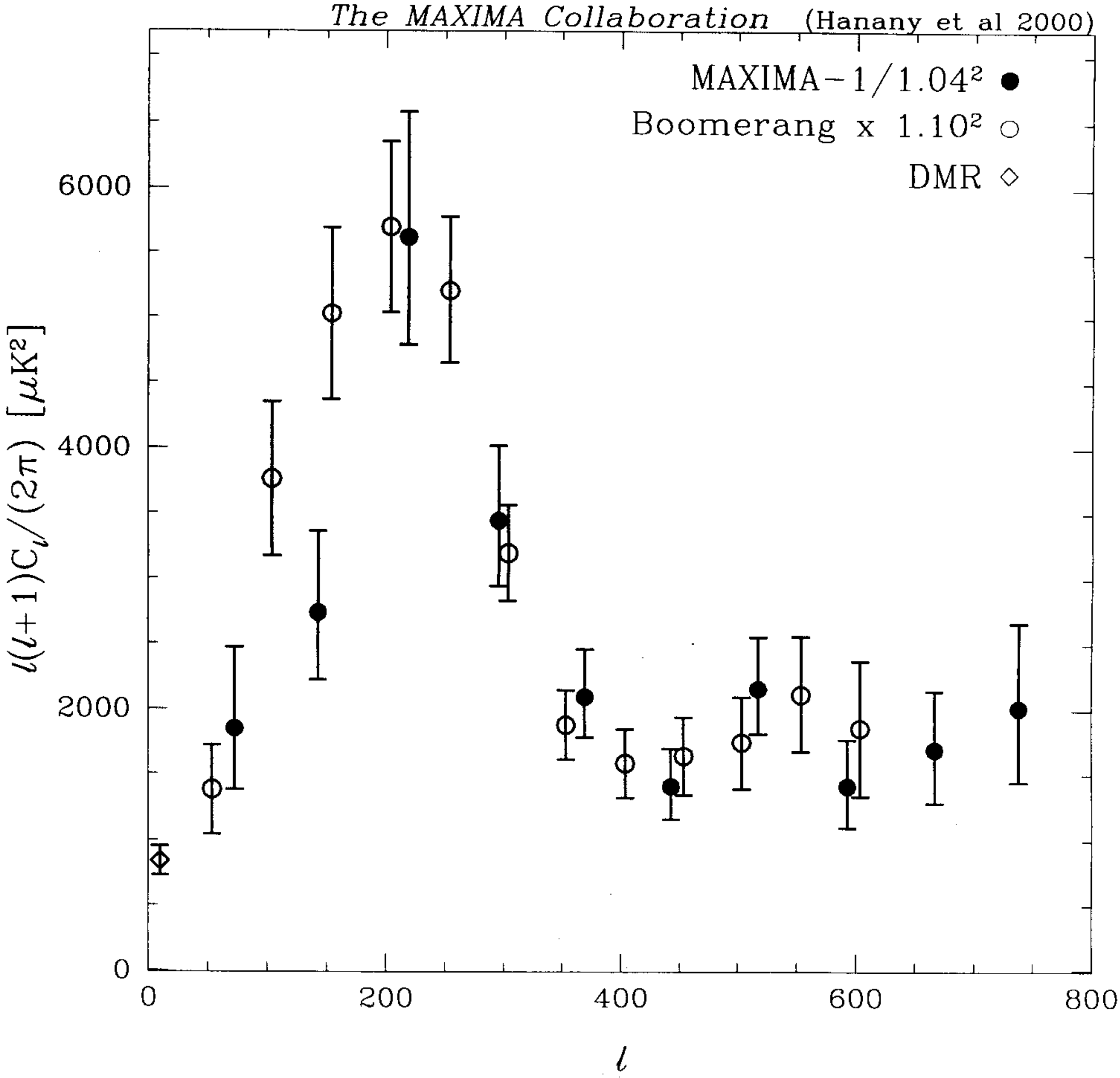
100
80
60
40
20
0
10
100
1000
l
$(\,l(l+1)C_l/2\pi\,)^{1/2}\ T_{CMB}\ \mu K$
L=0·1
L=0·5
L=0.92


1000
100
10
1
0.01
0.1
k/h Mpc^{-1}
$4\pi P(k)$ (h Mpc)
L=0.1
L=0.5
L=0.92

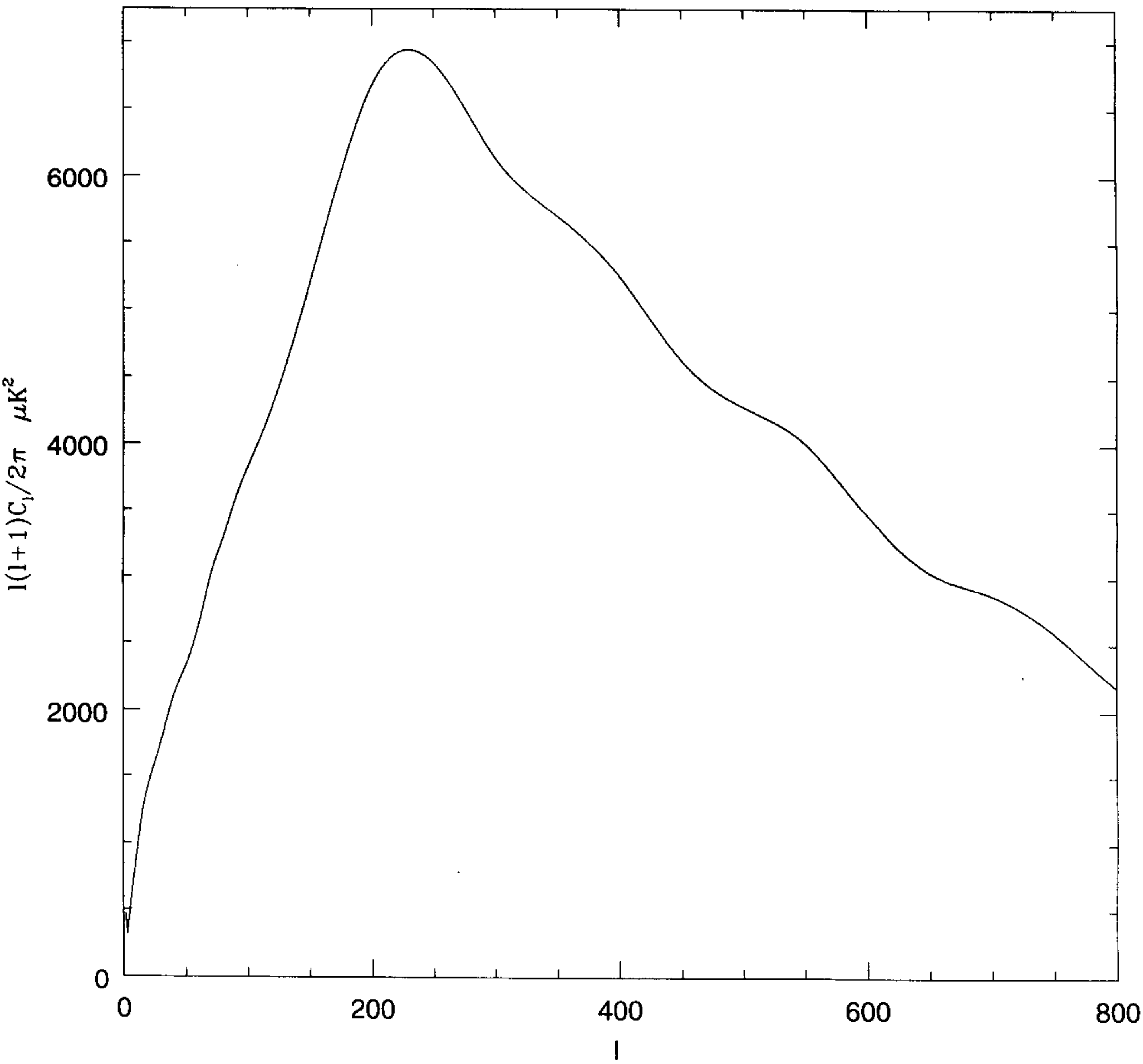


100
80
60
40
20
0
10
100
1000
l
$(\, l(l+1)C_l/2\pi\,)^{1/2}\, T_{CMB}\ \mu K$
$l/\tau = \begin{cases} 0{\cdot}12 \text{ radn.} \\ 0{\cdot}22 \text{ matter.} \end{cases}$
$\frac{l}{\tau} = \begin{cases} 0{\cdot}24 \text{ radn} \\ 0{\cdot}30 \text{ matter} \end{cases}$


1000
100
10
1
0.01
0.1
k/h Mpc^{-1}
$\ell_{1/2}$ = 0.12, 0.22
$\frac{L}{L}$ = 0.24, 0.30



The MAXIMA Collaboration (Hanany et al 2000)
MAXIMA−1/1.04²
Boomerang x 1.10²
DMR
6000
4000
2000
0
200
400
600
800
l(l+1)C_l/(2π) [μK²]
l



6000
4000
2000
0
0
200
400
600
800
l
l(l+1)C_l/2π μK²

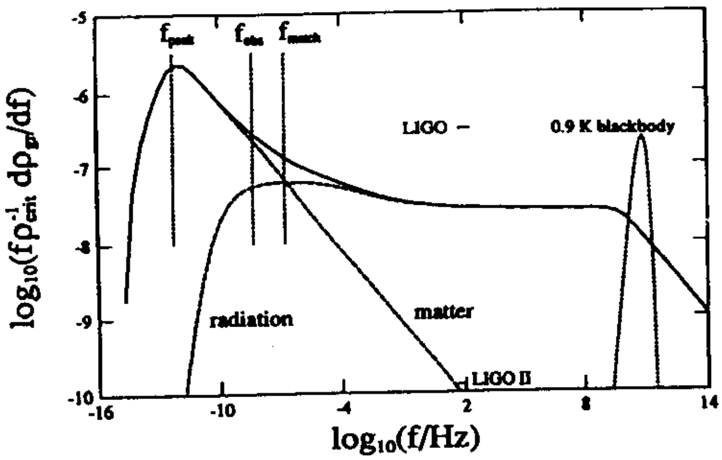


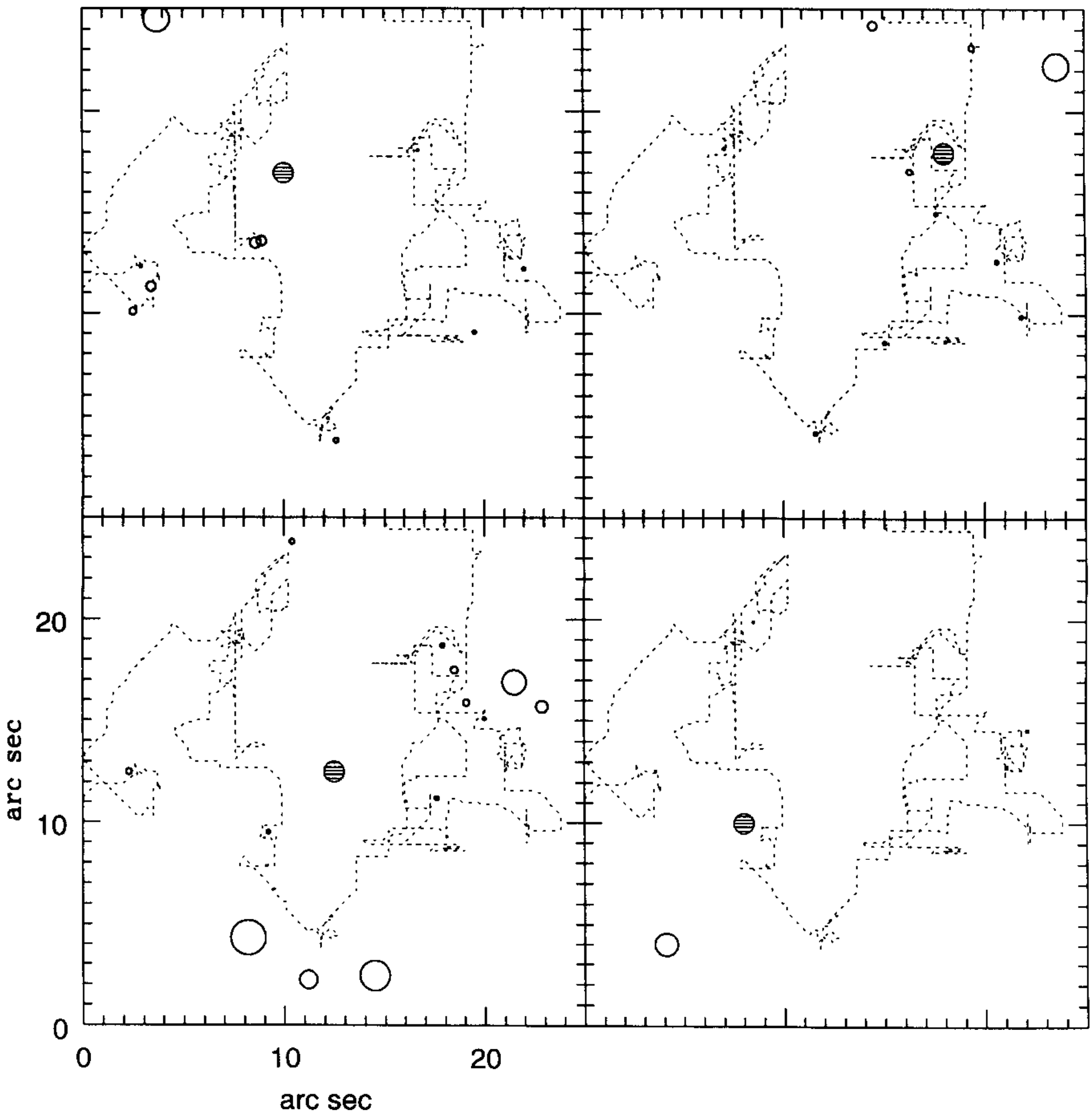
Fig. 10.5. Gravitational background spectrum for the parameters $G\mu = 10^{-}$ $\alpha = 10^{-3}$, $\beta = 4/3$ (from Caldwell & Allen [1992]). The contributions from t radiation and matter eras are indicated by dotted lines. The frequency f on t horizontal axis is defined as $f = \omega/2\pi$.

Here, $x = \ell/t_1$, $y = (t_1/t_0)^3$, t_0 is the present time and we have assum for simplicity that $\alpha >> \Gamma G\mu$, so that the upper limit of x-integrati can be changed to ∞. The θ-function in (10.4.16) is unimportant : $\omega > 4\pi/\Gamma G\mu t_0$ and β not very close to 2. The x- and y-integrations then easily performed, and we obtain

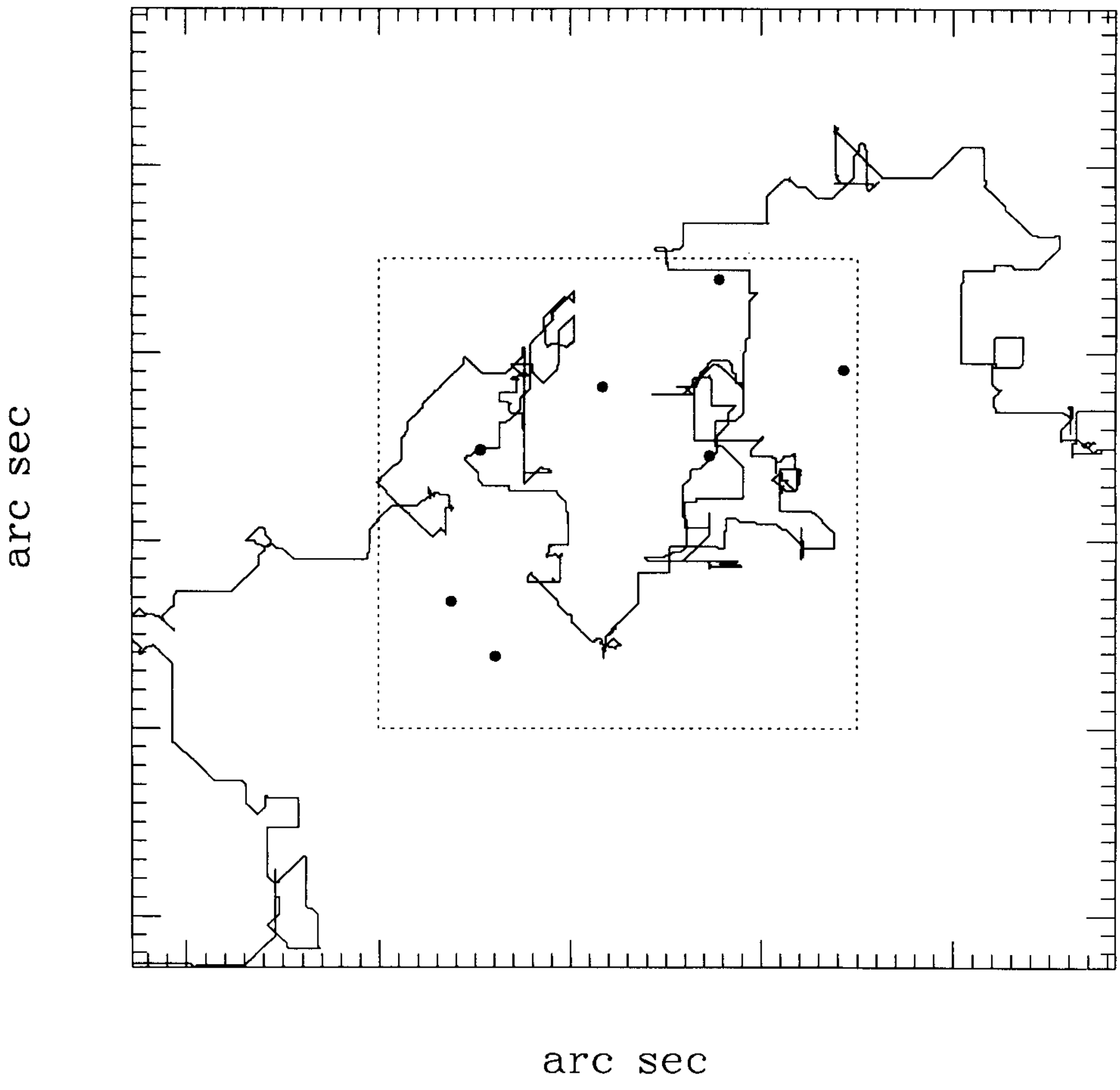
$$\Omega_g(\omega) = \frac{18\pi^2(\beta-1)^2\nu G\mu}{(3-\beta)\sin[(2-\beta)\pi]}\left(\frac{4\pi}{\Gamma\mu\omega t_0}\right)^{\beta-1} \qquad (10.4.$$

The peak of the spectrum is reached at $\omega_{max} \sim 4\pi/\Gamma G\mu t_0$ correspond to $\Omega_g(\omega_{max}) \sim G\mu$.

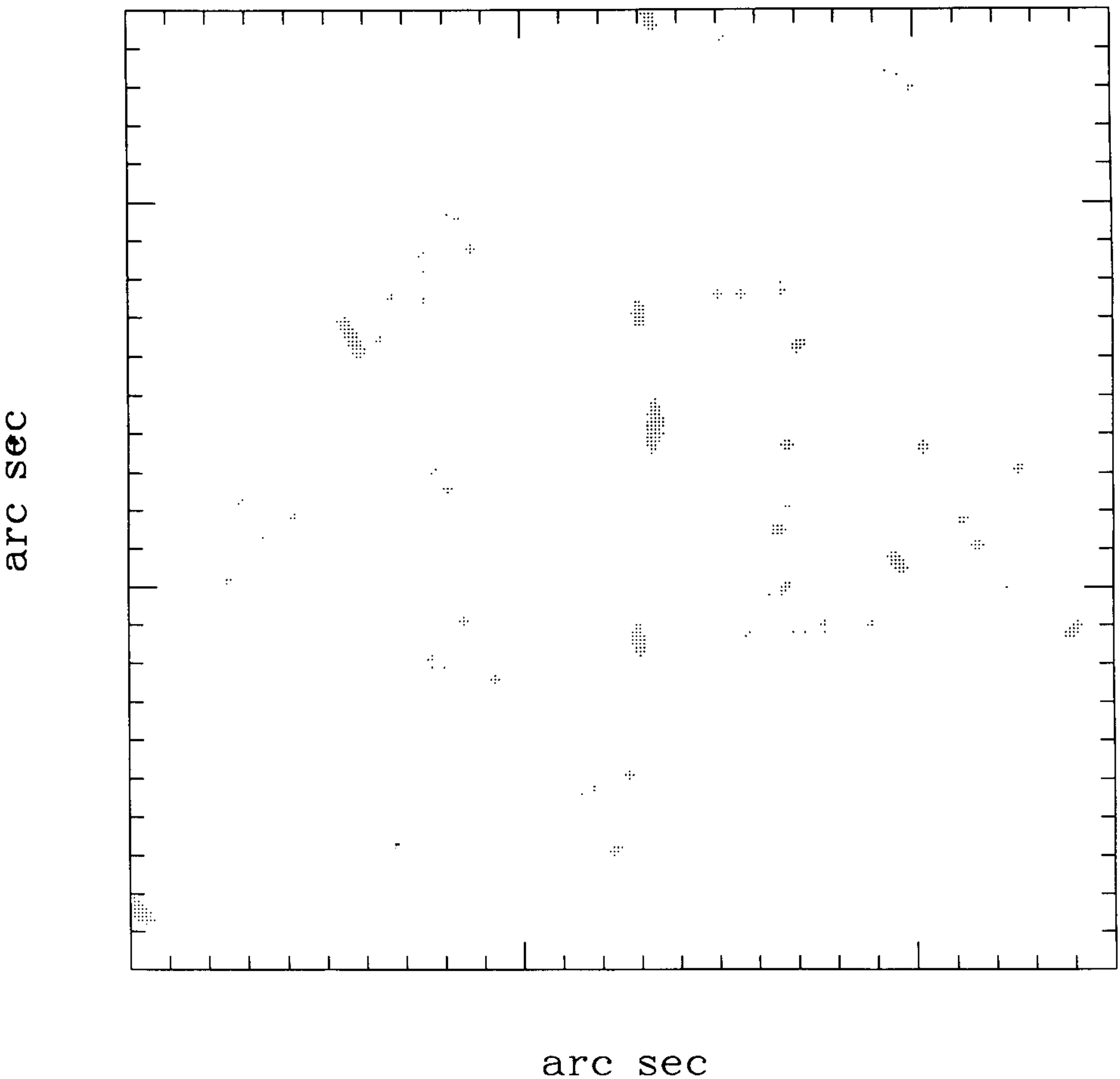
The full spectrum of the gravitational background for several value the parameters $G\mu$, α and β has been calculated numerically by Cald & Allen [1992]. They carefully treated the transition from the radia to the matter era and used a discrete spectral function with $P_n \propto$ instead of (10.4.16). A sample spectrum is shown in fig. 10.5. It sh be noted that the Caldwell & Allen estimates are based on the numer simulations of Allen & Shellard [1992] which do not include gravitati back-reaction. The additional smoothing from radiation damping act to reduce the high-frequency contributions to the radiation spect (refer to §7.7.1).



20
10
0
0
10
20
arc sec
arc sec



arc sec
arc sec


arc sec
arc sec

4. Magnetic Monopoles:

(i) Topology: π_2.

Using theorem:

$$\pi_2(G/H) = \pi_1(H) \quad \text{if } \pi_2(G) = \mathbb{1} = \pi_1(G).$$

For any Lie group $\pi_2(G) = \mathbb{1}$.

$\pi_1(G) = \mathbb{1} \Rightarrow$ eg. G has no ~~U(1) factors~~. incontractable paths (eg. U(1) factors).

Grand Unified Theories based on a simple group G. These satisfy $\pi_2(G) = \mathbb{1} = \pi_1(G)$.

Electromagnetism (and hypercharge) are based on a U(1) group. $\therefore$ H contains U(1).

$$\therefore \quad \pi_2\left(\frac{G_{GUT}}{H_{em}}\right) = \pi_1(H_{em}) = \mathbb{Z}$$

$\therefore$ Grand Unified Theories $\Rightarrow$ magnetic monopoles.

(ii) Examples:

(a) SU(2). (actually ~~S~~O(3)).

$$L = \frac{1}{2}(D_\mu \vec{\Phi})^2 - \frac{1}{4} W^a_{\mu\nu} W^{\mu\nu a} - V(\Phi).$$

$$V(\vec{\Phi}) = \frac{\lambda}{4}(\vec{\Phi}^2 - \eta^2)^2$$

$$D_\mu = \partial_\mu - i g W^a_\mu T^a.$$ Note: $D_\mu \vec{\Phi} \longrightarrow [D_\mu, \vec{\Phi}]$

Note: $$(D_\mu \vec{\Phi})^a = \partial_\mu \Phi^a - g f^{abc} W^b_\mu \Phi^c$$

Vacuum manifold: $\vec{\Phi}^2 = \eta^2 \Rightarrow S^2.$

$\langle \vec{\Phi} \rangle$: $SU(2) \to U(1)$ $(O(3) \to O(2))$.

$$\pi_2(G/H) = \pi_2\left(\frac{SU(2)}{U(1)}\right) = \pi_2(S^2) = \mathbb{Z}.$$

Bogomolnyi method:

$$E = \frac{1}{2}\int d^3x \left[\vec{B}^a \cdot \vec{B}^a + \vec{D}\Phi^a \cdot \vec{D}\Phi^a + \frac{1}{2}\lambda(\Phi^a\Phi^a - \eta^2)^2 \right]$$

$$\geq \frac{1}{2}\int d^3x \left[\vec{B}^a \cdot \vec{B}^a + \vec{D}\Phi^a \cdot \vec{D}\Phi^a \right]$$

$$= \frac{1}{2}\int d^3x \left[(\vec{B}^a \pm \vec{D}\Phi^a)^2 \right] \mp \int d^3x \, \vec{B}^a \cdot \vec{D}\Phi^a$$

$$\geq \int d^3x \, \vec{B}^a \cdot \vec{D}\Phi^a.$$

$$\int d^3x\ \vec{B}^a\cdot\vec{D}\Phi^a = \int d^3x\ \vec{B}^a\cdot\left[\vec{\nabla}\Phi^a - g f^{abc}\vec{W}^b\Phi^c\right].$$

$$= \int d^3x\left[\vec{\nabla}\cdot(\vec{B}^a\Phi^a) - \Phi^a\,\vec{\nabla}\cdot\vec{B}^a - g\vec{B}f^{abc}\vec{W}^b\Phi^c\right].$$

$$= \oint_{\substack{r\to\infty\\ \hat{n}=\hat{r}}} dS\,\hat{n}\cdot(\vec{B}^a\Phi^a). - \int d^3x.\ \Phi^a\left\{\vec{\nabla}\cdot\vec{B}^a + g f^{cba}\,\vec{W}^b\cdot\vec{B}^c\right\}$$

$$= \oint dS\,\hat{r}\cdot(\vec{B}^a\Phi^a) - \int d^3x.\ \Phi^a\underbrace{\left\{\vec{\nabla} - g f^{abc}\vec{W}^b\right\}}_{\vec{D}}\cdot\vec{B}^c$$

$$= \oint dS\ \hat{r}\cdot(\vec{B}^a\Phi^a)\quad -0 \qquad (\vec{D}\cdot\vec{B}^a=0).$$

since $D_\mu {}^*F^{\mu\nu} = 0$

$$= \frac{4\pi}{g^2}.\ m_V^2 \qquad \text{with}\quad m_V = g\eta = \text{mass of vector boson.}$$

$$\therefore\qquad E_{monopole} \geqslant \frac{4\pi}{g^2}m_V^2 = E_{BPS}$$

BPS (Bogomolnyi-Prasad-Sommerfield) monopole solution valid for $\lambda = 0$. $(V=0)$

BPS eqn: $\qquad \vec{B}^a \pm \vec{D}\Phi^a = 0.$

Exact solution: $\quad \Phi^a = \eta\, h(r).\dfrac{x^a}{r}\ ,\quad W_i^a = -[1-K(r)]\,\varepsilon^{aij}\dfrac{x^j}{gr^2}$

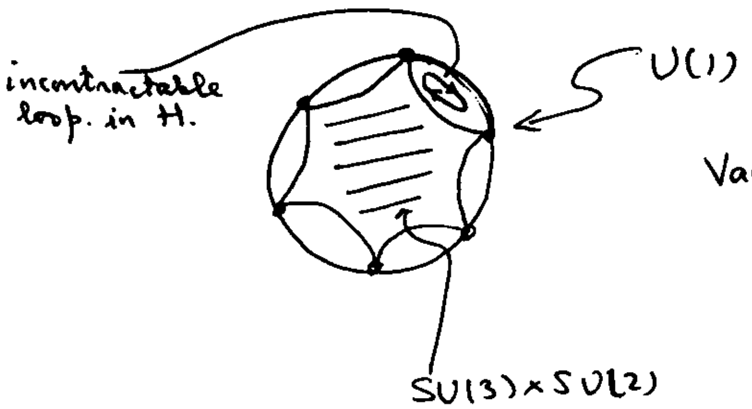
Then: $\quad h(r) = \text{Coth}(m_V r) - \dfrac{1}{m_V r}\ ,\quad K(r) = \dfrac{m_V r}{\text{Sinh}(m_V r)}$

(b) Electroweak monopoles: Skip.

(c) SU(5):

$$SU(5) \longrightarrow [SU(3) \times SU(2) \times U(1)] / \mathbb{Z}_6 .$$

The $\mathbb{Z}_6$ is the $\mathbb{Z}_3 \times \mathbb{Z}_2$ center of $SU(3)\times SU(2)$ and is also contained in the U(1) factor. These 6 elements have to be identified.



Vacuum manifold

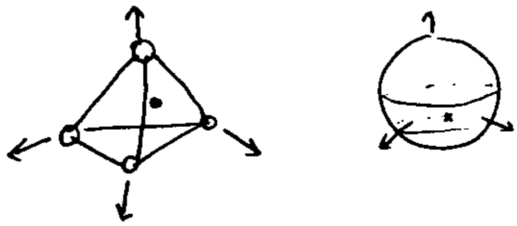
Fundamental monopole has SU(3), SU(2), U(1) magnetic charges.

	SU(3)	SU(2)	U(1)	
n=1	1/3	1/2	1/6	$(u,d)_L$
n=-2	1/3	0	-1/3	d_R
n=-3	0	1/2	-1/2	$(\nu,e)_L$
n=+4	1/3	0	2/3	u_R
n=5	—	—	—	
n=-6	0	0	-1	e_R
n=7	—	—	—	
n=8	—	—	—	

$\longrightarrow$ dual standard model.

(iii) Formation and Evolution:

Formation is exactly as in the case of domain walls, strings. Here S^2 is discretized as a tetrahedron.



Monopole density at formation $\sim \frac{1}{\xi^3}$.

Evolution:
(1) Coulomb attraction
(2) Friction
(3) Annihilation
(4) Hubble expansion.

-) Cosmological contraint:
Assume 1 monopole / horizon at formation (i.e. $\xi \sim$ max).

Then $\rho_m(t)$ = energy density in monopoles

$$\simeq \rho_m(t_{form}) \cdot \left[\frac{a(t_{form})}{a(t)}\right]^3 .$$ (pressure = 0 for monopole gas).

$$\therefore \quad \frac{\rho_m(t_{eq})}{\rho_{cr}(t_{eq})} = \frac{\rho_m(t_{form})}{\rho_{cr}(t_{form})} \cdot \left[\frac{a(t_{eq})}{a(t_{form})}\right]$$

$$\therefore \quad \Omega_m(t_{eq}) = \Omega_m(t_{form}) \left(\frac{t_{eq}}{t_{form}}\right)^{1/2} .$$

$$\Omega_m(t_{form}) \simeq \frac{m_M}{t_{form}^3} G t_{form}^2 = \frac{Gm}{t_{form}}.$$

$$\therefore \quad \Omega_m(t_{eq}) \simeq \frac{G m_M}{t_{form}} \left(\frac{t_{eq}}{t_{form}}\right)^{1/2}$$

$$= \left(\frac{m_M}{m_{pl}}\right) \cdot \left(\frac{t_{pl}}{t_{form}}\right) \cdot \left(\frac{t_{eq}}{t_{form}}\right)^{1/2} \qquad G = \frac{t_{pl}}{m_{pl}}.$$

GUT: $$\Omega_m(t_{eq}) \simeq \frac{10^{16}\,\mathrm{GeV}}{10^{19}\,\mathrm{GeV}} \frac{10^{-43}\,\mathrm{s}}{10^{-35}\,\mathrm{s}} \cdot \left(\frac{10^{12}\,\mathrm{s}}{10^{-35}\,\mathrm{s}}\right)^{1/2}$$

$$\simeq 10^{-46+35+23} = 10^{12}$$

$\therefore$ GUT monopoles would be a cosmological disaster.

Parker bound:

Magnetic monopoles would get accelerated by the galactic magnetic field causing it to decay. This gives "Parker bound" on the flux of magnetic monopoles.

Bound: Monopoles above $\sim 10^{11}$ GeV are not allowed.

Some model-dependent lighter monopoles are also ruled out by stellar physics.

(6) Solutions:

(i) Inflation

(ii) Langacker - Pi : monopoles get connected by strings.

(iii) Symmetry non-restoration

(iv) Sweeping by domain walls.

(7) Quantum effects:

Confinement of non-Abelian monopoles ?