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SMR.1227 - 21

SUMMER SCHOOL ON ASTROPARTICLE PHYSICS AND COSMOLOGY

12 - 30 June 2000

GAMMA RAY BURSTS

Part II

E. WAXMAN
Weizmann Institute
Rehovot
ISRAEL

strada costiera, 11 - 34014 trieste italy - tel. +39 0402240111 fax +39 040224163 - sci_info@ictp.trieste.it - www.ictp.trieste.it

V. High Energy Cosmic Rays: Introduction

[J.W. Cronin, "Some Unsolved Problems in Astrophysics", Princeton 1996 ;

Blandford & Eichler 1987, Phys. Rep. 154, 1]

The Flux Above 10^{17} Ev

1) SPECTRUM:

- $J(>E) \approx 1 \cdot (E/10^{19}\ \mathrm{eV})^{-2}\,\mathrm{Km}^{-2} \cdot \mathrm{yr}^{-1} \cdot \mathrm{sr}^{-1}$

- A Break $\sim 5 \cdot 10^{18}$ eV:

$$\begin{cases} J_G(E) \propto E^{-3.5} & (< 5 \cdot 10^{18}\mathrm{eV}) \\ J_{XG}(E) \propto E^{-2.6} & (> 5 \cdot 10^{18}\mathrm{eV}) \end{cases}$$

2) COMPOSITION: $\underset{\text{Heavy(Fe)}}{G} \longrightarrow \underset{\text{Light(p)}}{XG}$

The Flux Above 10^{17} eV

3) DIRECTIONAL:

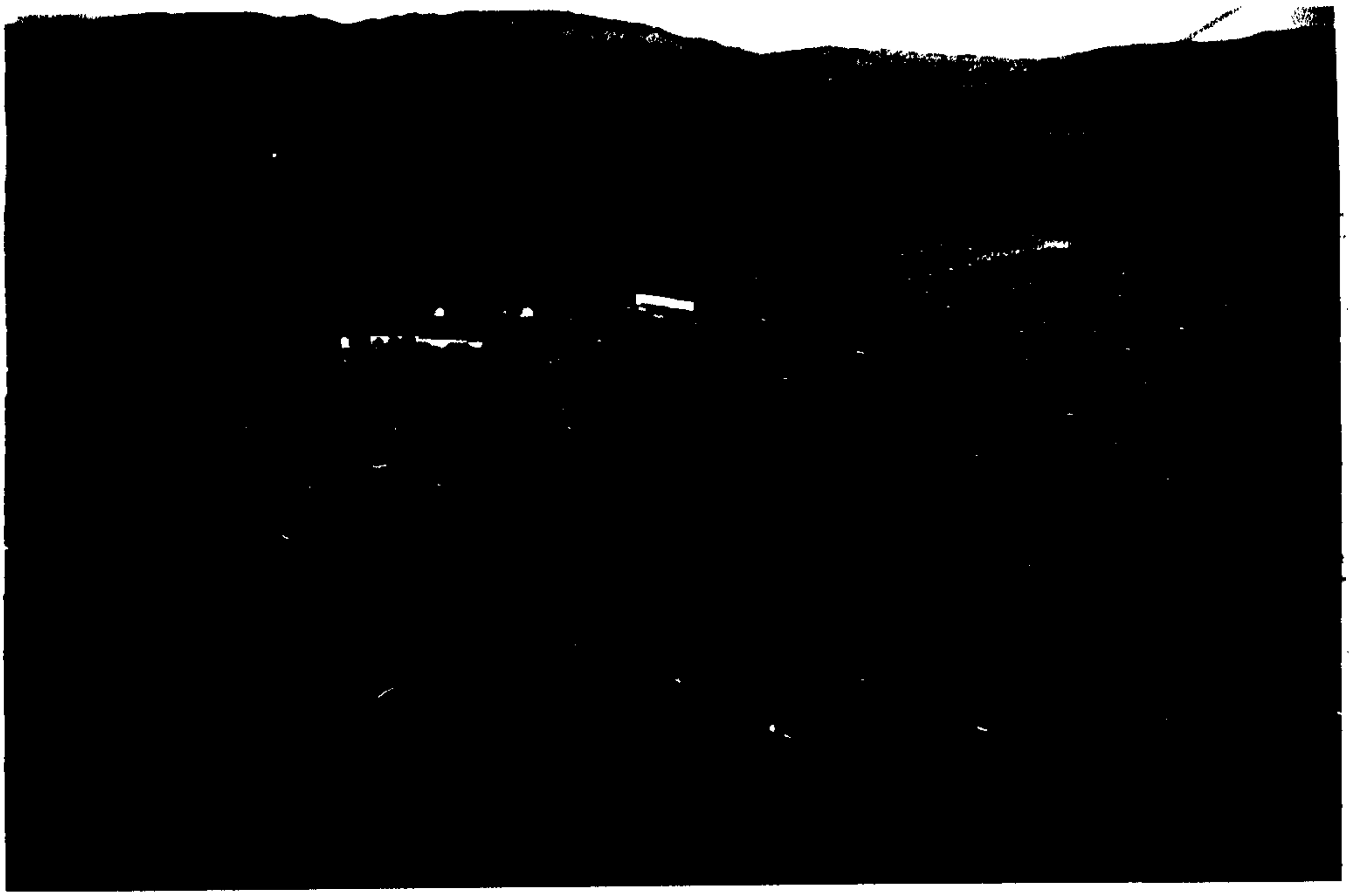
- No evidence for anisotropy (disk), or clustering

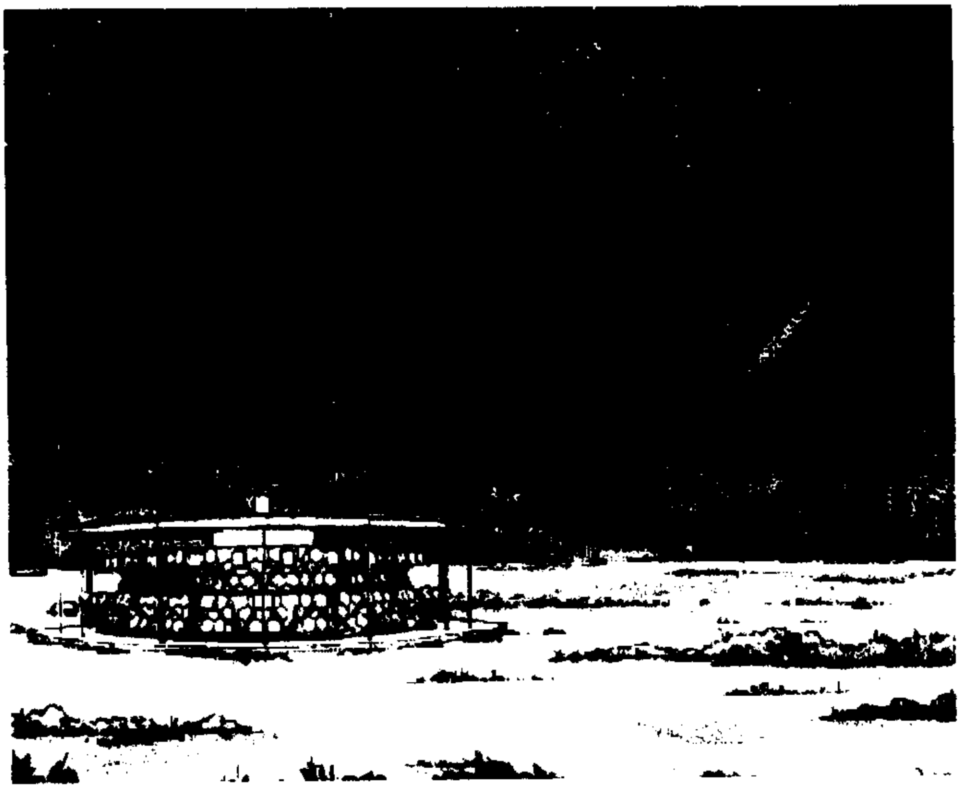
4) HIGHEST ENERGY EVENTS ($> 10^{20}$ eV):

Fly's Eye: $3 \pm 1 \cdot 10^{20}$ eV ; $l \cong 160°, b \cong 10°$

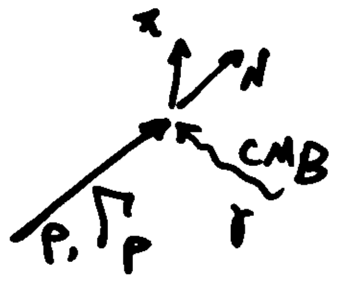
AGASA: $1.7 - 2.6 \cdot 10^{20}$ eV ; $l \simeq 130°, b \cong -40°$

[$p - Fe$, Not a γ , ν?]



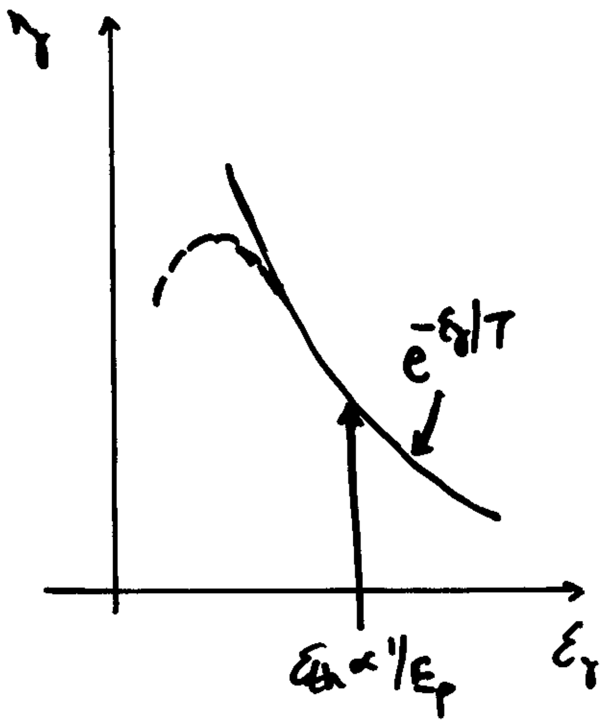


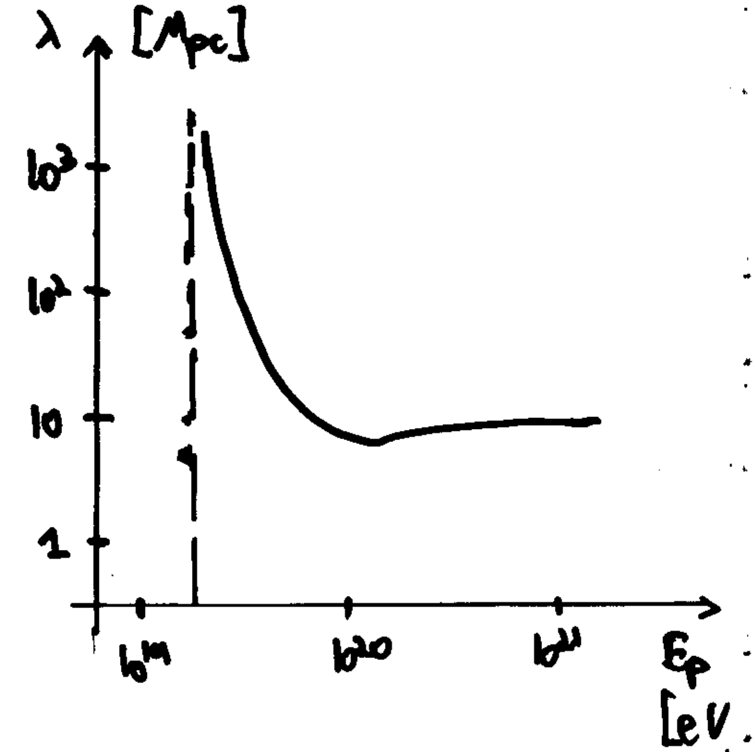
Greisen-Zatsepin-Kuzmin 'Cutoff'



$$\epsilon_\gamma \gtrsim \frac{0.2\,\mathrm{GeV}}{\Gamma_p} = 2\cdot 10^{-3} E_{20}^{-1}\ \mathrm{eV}$$

$$T_{CMB} = 2.4\cdot 10^{-4}\ \mathrm{eV}$$





$$n_{th} \propto \exp(-8E_{20}^{-1})$$

$$\Downarrow$$

$$\lambda_{\gamma p} \propto \exp(-8E_{20}^{-1})$$

$$\dot{n}_p(>E_p) \propto E_p^{-1}$$

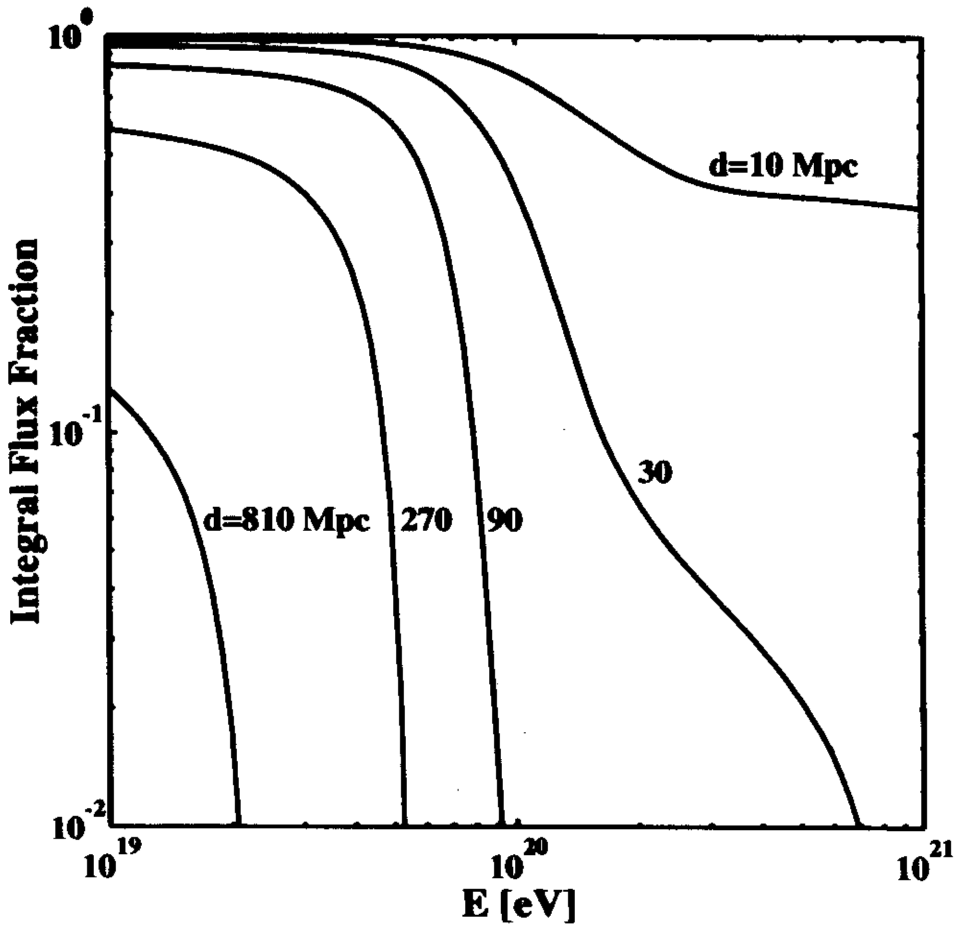
$$\Downarrow$$

$$J(>E_p) \propto \dot{n}_p(>E_p)\cdot D \approx$$

$$\approx \lambda(E_p)\cdot E_p^{-1}$$

Flux Variations $> 10^{20}$ eV

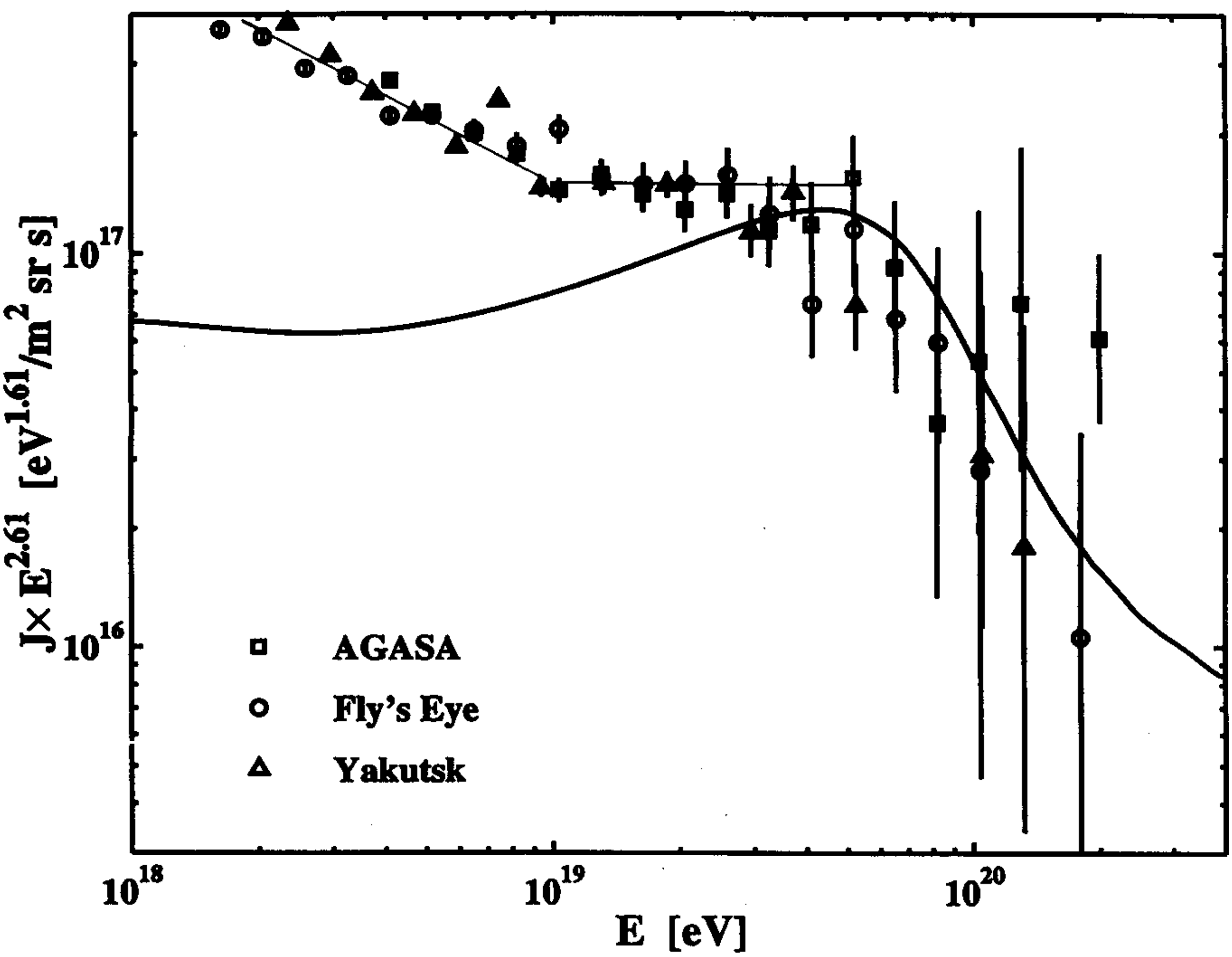
I. Inhomogeneities



(Waxman 1995, ApJ 452, L1)

Is the GZK "cutoff" seen?

$$E_p^2 \frac{dn_p}{dE_p} = 0.8 \cdot 10^{44} \ \mathrm{erg/Mpc^3\,yr}$$



Model: Waxman 95

Data: Fly's Eye 94

Yakutsk 92

AGASA 9?

GZK revisited

[Bahcall & Waxman 00]

$$N(E \geq E_c) =$$

$$\int_{E_c}^{\infty} dE \int d^3r \, \frac{1}{4\pi r^2} \cdot \underbrace{n(\underline{r})}_{\text{Source \# density}} \cdot \underbrace{\frac{\partial S}{\partial E}}_{\text{Source spectrum}} \cdot \underbrace{P(r, E; E_c)}_{\text{Survival probability}}$$

$$\Rightarrow \quad \sigma^2 = \iint dE \, dE' \iint \frac{d^3r \, d^3r'}{4\pi r^2 \cdot 4\pi r'^2} \, \bar{n}^2 \, \xi(\underline{r} - \underline{r}') \cdot \frac{\partial S}{\partial E} \frac{\partial S}{\partial E'} \cdot P \cdot P'$$

$$\langle n(\underline{r}) \, n(\underline{r}') \rangle = \bar{n}^2 \left[1 + \xi(\underline{r} - \underline{r}') \right]$$

$$\xi(\Delta) = \left(r_0/r\right)^{1.8}$$

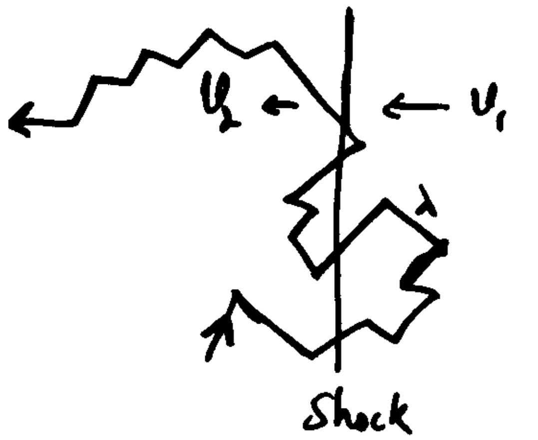
$$\Rightarrow \quad \frac{\sigma_{cluster}}{N_{av.}} = 0.9 \left(\frac{r_0}{10\,Mpc}\right)^{0.9} \quad @ \quad 10^{20}\,eV$$

$$= 2.6 \left(\frac{r_0}{10\,Mpc}\right)^{0.9} \quad @ \quad 10^{20.5}\,eV$$

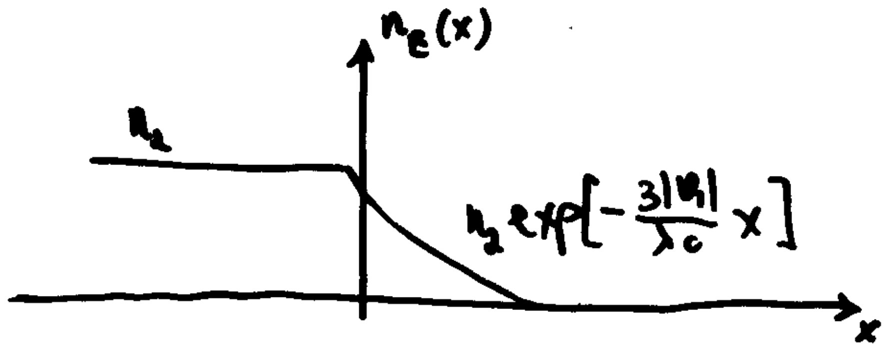
- Hopeless? No!

$$\theta_{anisot.} \approx \frac{r_0}{\lambda_p(E)} \approx 1 \left(\frac{r_0}{10\,Mpc}\right) \; @ \; 10^{20}\,eV$$

(Fermi) Shock Acceleration



$$\lambda \approx R_L \equiv \frac{E}{eB}$$



- Assume: Particle distribution ~ Isotropic

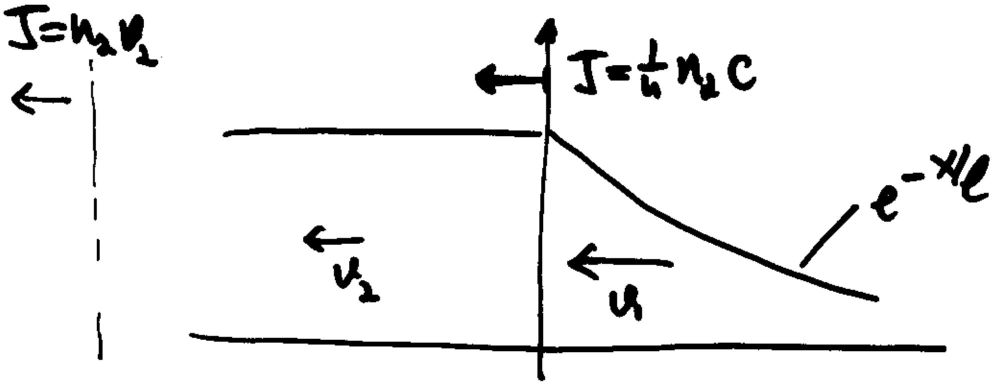
$\Rightarrow$ Diffusion eq.

$$\partial_t n + \underline{v} \cdot \nabla n - \nabla\left[D(\nabla n)\right] = 0 \,, \quad D = \frac{\lambda c}{3}$$

$$\rightarrow \quad v \partial_x n - \frac{\lambda c}{3} \partial_x^2 n = 0$$

$$n = \begin{cases} \text{Const.} \\ e^{-x/\ell} \end{cases} \,, \quad \ell^{-1} = \frac{3|v_1|}{\lambda c}$$

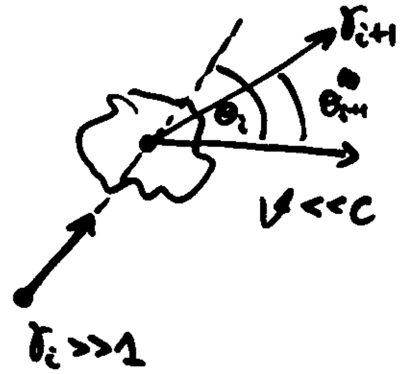
- Diffusion valid $\Leftrightarrow \ell \gg \lambda \Leftrightarrow \frac{v_1}{c} \ll 1$



- $$P_{escape} = \frac{n_2 v_2}{\frac{1}{4} n_2 c} = 4\beta_2$$

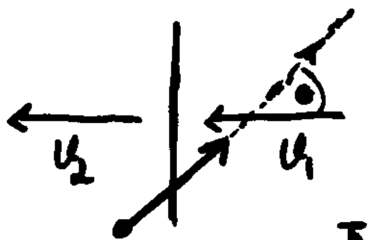
- Up-stream average residence time:

$$t_1 = \frac{\int_0^\infty dx\, n(x)}{\frac{1}{4} n_2 c} = \frac{\ell n_2}{\frac{1}{4} n_2 c} = 4\frac{\ell}{c} = \frac{4}{3}\frac{\lambda}{v_1}$$



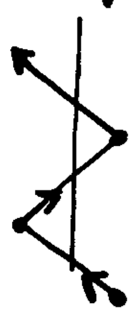
Scattering by Macro-scopic body: γ conserved in [cloud] rest-frame.

$$\Rightarrow \quad \frac{\gamma_{i+1}}{\gamma_i} - 1 = -\beta \left(\cos\theta_i - \cos\theta'_{i+1} \right) \qquad ; \quad \beta = v/c$$



$$\langle \cos\theta' \rangle = 0, \quad \langle \cos\theta \rangle = \frac{\int_{\pi/2}^{\pi} d\theta \sin\theta \cdot c\cos\theta \cdot \cos\theta}{\int_{\pi/2}^{\pi} d\theta \cdot \sin\theta \cdot c\cos\theta} = -\frac{2}{3}$$

cross-cycle



$$\left\langle \frac{\Delta\gamma}{\gamma} \right\rangle_{\text{cycle.}} = \frac{4}{3} \frac{\Delta v}{c}$$

$P_{esc.} = 4\beta_2 , \quad \left\langle \frac{E_{i+1}}{E_i} \right\rangle \equiv \langle 1 + \beta_i \rangle = 1 + \frac{4}{3}\Delta\beta$

$$\Rightarrow \quad \frac{dW}{dE} \propto E^{-\alpha} , \qquad \alpha = 1 - \frac{\ln(1-P_e)}{\langle \ln(1+\beta) \rangle} \xrightarrow[\beta \to 0]{} 1 + \frac{3\beta_2}{\Delta\beta}$$

- $\alpha = \frac{r+2}{r-1}$, $\quad r = \frac{\beta_1}{\beta_2}$ = Compression ratio

  Strong Shock , $\hat{\gamma} = 5/3$: $\quad r = 4$, $\quad \alpha = 2$

- $t_{acc.} \approx \frac{t_1}{\langle \Delta\gamma/\gamma \rangle} = \frac{\frac{4}{3}\frac{\lambda}{v_1}}{\frac{4}{3}\Delta\beta} = \frac{1}{\beta_1 \Delta\beta} \cdot \frac{\lambda}{c}$

Summary

- Non-relativistic, collisionless shocks:

  Power-law tail, $\frac{dN}{dE} \propto E^{-2}$

- Accelerator size: $R > l \approx \frac{\lambda}{\beta} \propto \frac{R_L}{\beta} = \frac{E}{eB\beta}$

$$E < \beta \cdot eBR$$

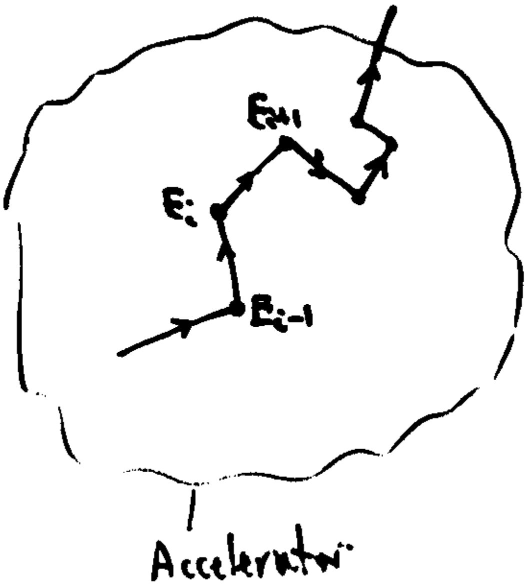
- Relativistic shocks - Not well understood

  See, However:

  Bednarz & Ostrowski 98, PRL 80, 3911

$$\frac{dN}{dE} \propto E^{-2.2}$$

VI. Fermi Acceleration



$E_{i+1} = (1+f_i)\, E_i$

P_e = escape probability between collisions

$$\ln \frac{E_n}{E_0} = \sum_i \ln(1+f_i)$$

- Assume: $\begin{cases} f_i \text{ independent of } i \\ f \lesssim 1 \end{cases}$

$\Rightarrow$ For $E_n \gg E_0$, $n \gg 1 \Rightarrow \ln \frac{E_n}{E_0}$ Gaussian

$$\left\langle \ln \frac{E_n}{E_0} \right\rangle = n \langle \ln(1+f) \rangle \; ; \quad \sigma^2 = n\,\sigma^2[\ln(1+f)]$$

$n \to \infty$: $\quad \ln \frac{E_n}{E_0} = n \langle \ln(1+f) \rangle$

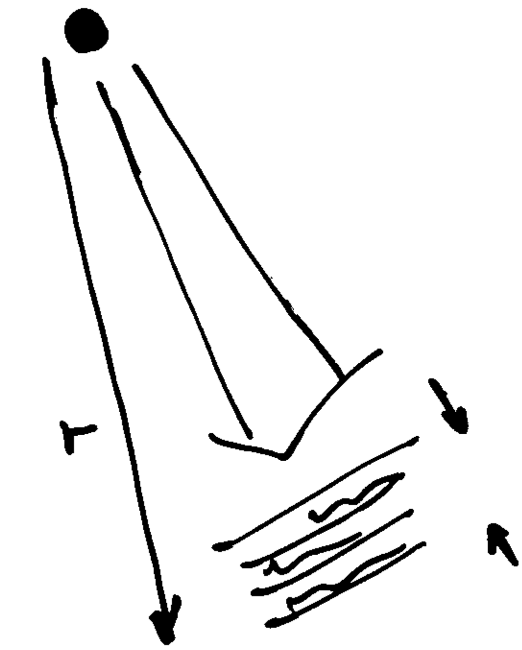
- $$P(>E) = (1-P_e)^{\ln(E/E_0)/\langle \ln(1+f)\rangle} = \left(\frac{E}{E_0}\right)^{\ln(1-P_e)/\langle \ln(1+f)\rangle}$$

$$\Rightarrow \quad \frac{dN}{dE} \propto E^{-\alpha}, \qquad \alpha = 1 - \frac{\ln(1-P_e)}{\langle \ln(1+f) \rangle}$$

VII. Cosmic-Rays & GRBs

[Waxman 00, Proc. Nobel Symposium, Physica Scripta T85, 117]

Acceleration:



$$\begin{cases} L_\gamma = 4\pi r^2 c \cdot \gamma^2 U_e \\ \dfrac{B^2}{8\pi} = \dfrac{\xi_B}{\xi_e} U_e \end{cases}$$

$$\Delta^{ob.} = \frac{r}{\gamma^2}, \quad \Delta^{th.} = \frac{r}{\gamma}$$

$$E^{ob.}/\gamma < eB\, {}^{r}/_{\gamma}$$

$$\Rightarrow \quad \frac{\xi_B}{\xi_e} > 0.02\, \gamma_{2.5}^{2}\, E_{p,20}^{2}\, L_{\gamma,52}^{-1}$$

[Independent of r]

Synch. loss:

- $\tau_{syn.} = \frac{6\pi m_p^4 c^3}{\sigma_T m_e^2} \frac{1}{EB^2} < \tau_{acc.} \approx \frac{E}{eBc}$

- $B < \cdots \quad \oplus \quad B \propto r^{-2}$

$\Rightarrow$

$$r > 10^{12} \gamma_{2.5}^{-2} E_{p,20}^{3} \text{ cm}$$

- $r \approx \gamma^2 c \Delta t$

$\Rightarrow$

$$\gamma > 130 \, E_{p,20}^{3/4} \Delta t_{-2}^{-1/4}$$

Cosmic-Ray Production Rate:

$$R_{GRB}(z\sim 1) \approx 4\cdot 10^{-9} \ /Mpc^3\,yr$$

$$\frac{R_{GRB}(0)}{R_{GRB}(1)} \approx \frac{1}{8} - 1$$

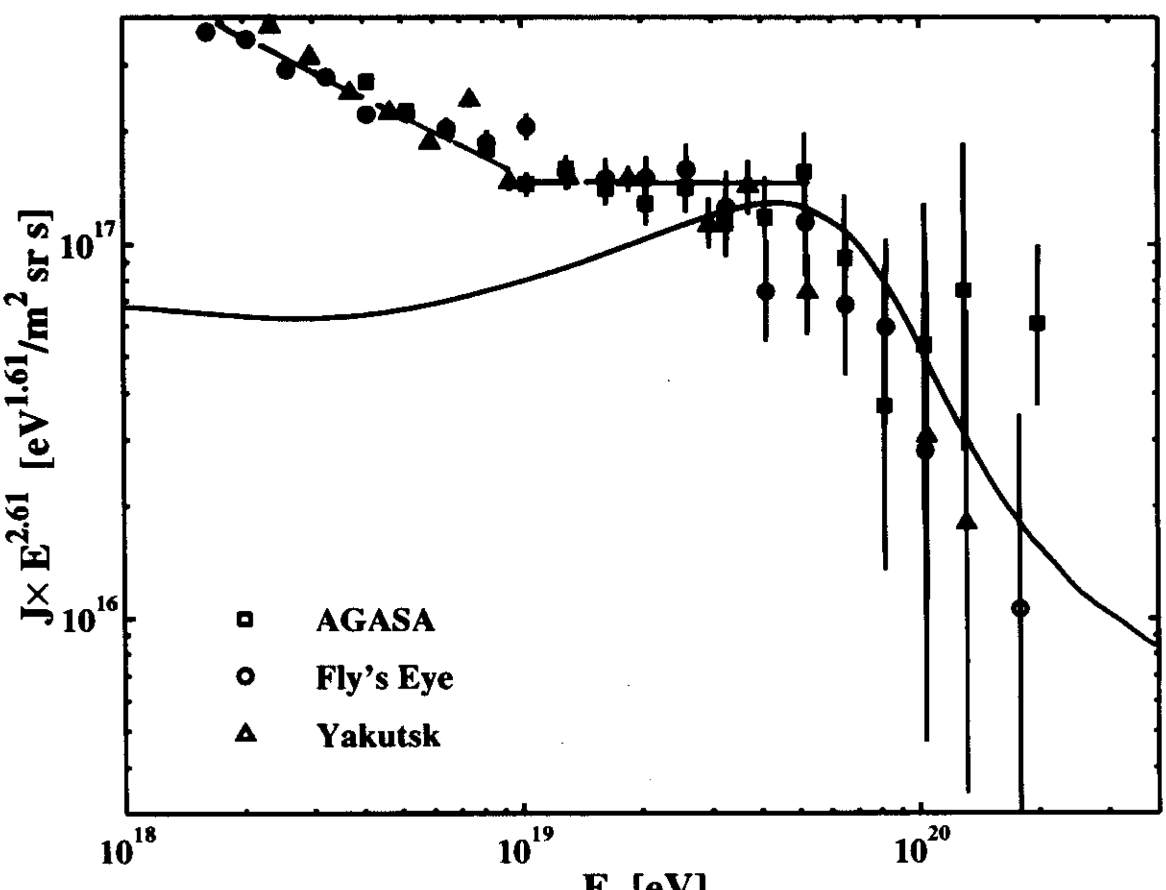
$$\Rightarrow \quad \dot{\mathcal{E}} \approx 10^{-9}\left(Mpc^{-3}\,yr^{-1}\right)\cdot 10^{53}\,(erg)$$

$$\approx 10^{44}\ \frac{erg}{Mpc^3\,yr}$$

Data versus Cosmological (GRB) Model

- Generation Rate & Spectrum:

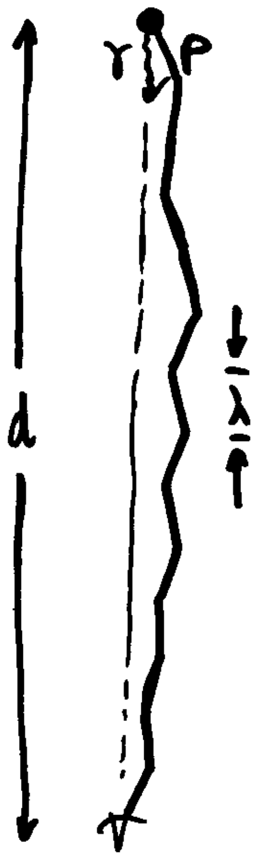
$$E^2 \mathrm{d}\dot{N}/\mathrm{d}E = 0.8 \times 10^{44} \mathrm{erg\ Mpc}^{-3}\ \mathrm{yr}^{-1}$$



[Waxman 1995]

Sources above 10^{20} eV:

- $R_{GRB}(d<100\,\mathrm{Mpc}) \approx \frac{1}{250\,\mathrm{yr}}$



- Inter-Galactic B, correlation length λ:

$$\theta_\lambda = \frac{\lambda}{R_L}$$

$$\langle\theta^2\rangle^{1/2} = \sqrt{\frac{2}{9}} \cdot \sqrt{\frac{d}{\lambda}} \cdot \frac{\lambda}{R_L}$$

$$t_{delay} = \theta^2 \frac{d}{4c}$$

$$\theta_s \approx 20^\circ \frac{\sqrt{d_{100}\lambda_1}}{E_{p,20}} B_{10nG}$$

$$t_{del} \approx 2\cdot10^7 \left(\frac{d_{100}}{E_{p,20}}\right)^2 \lambda_1 B_{10nG}^2 \;\mathrm{yr}$$

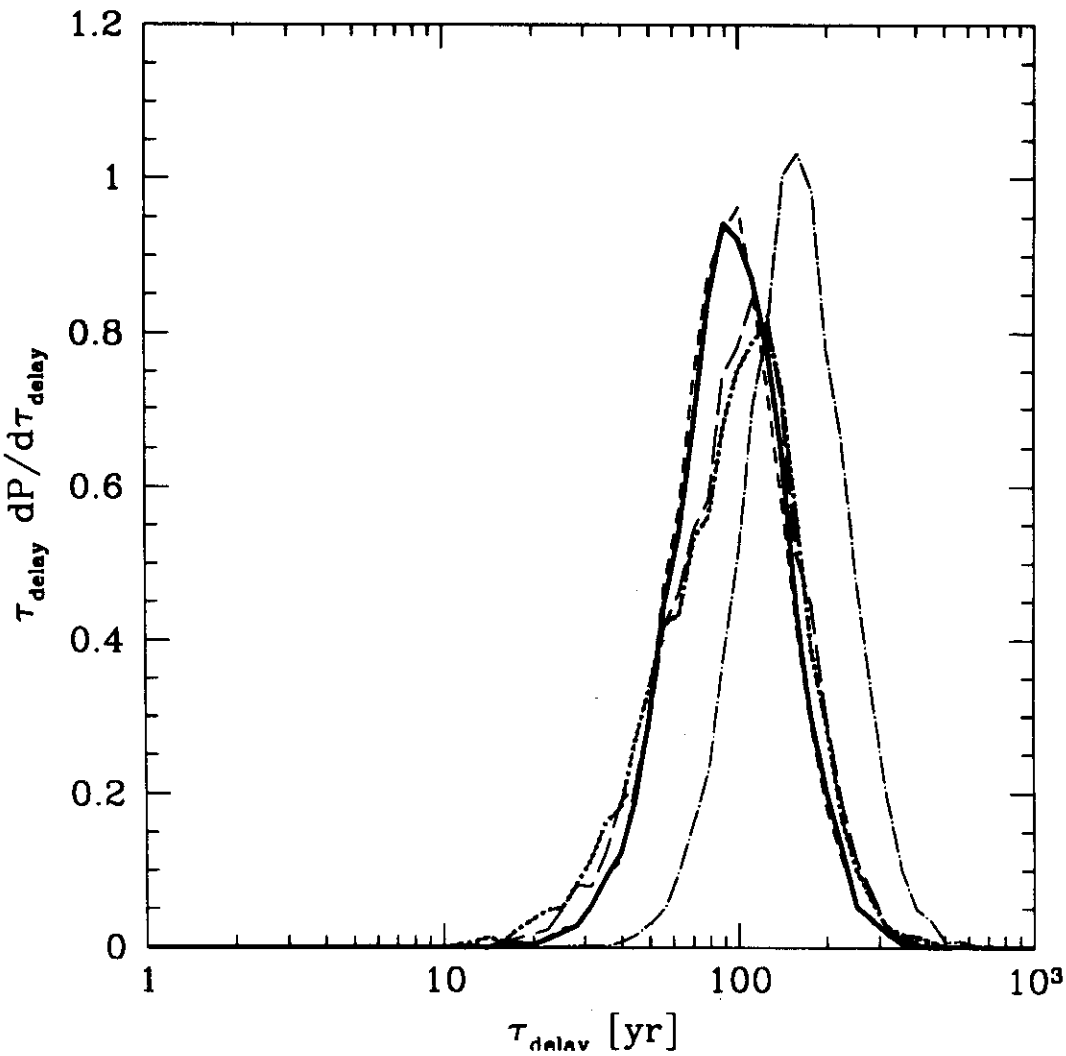
- π energy loss: $\left\langle \frac{E^{obs.} - E}{E} \right\rangle \sim 1$

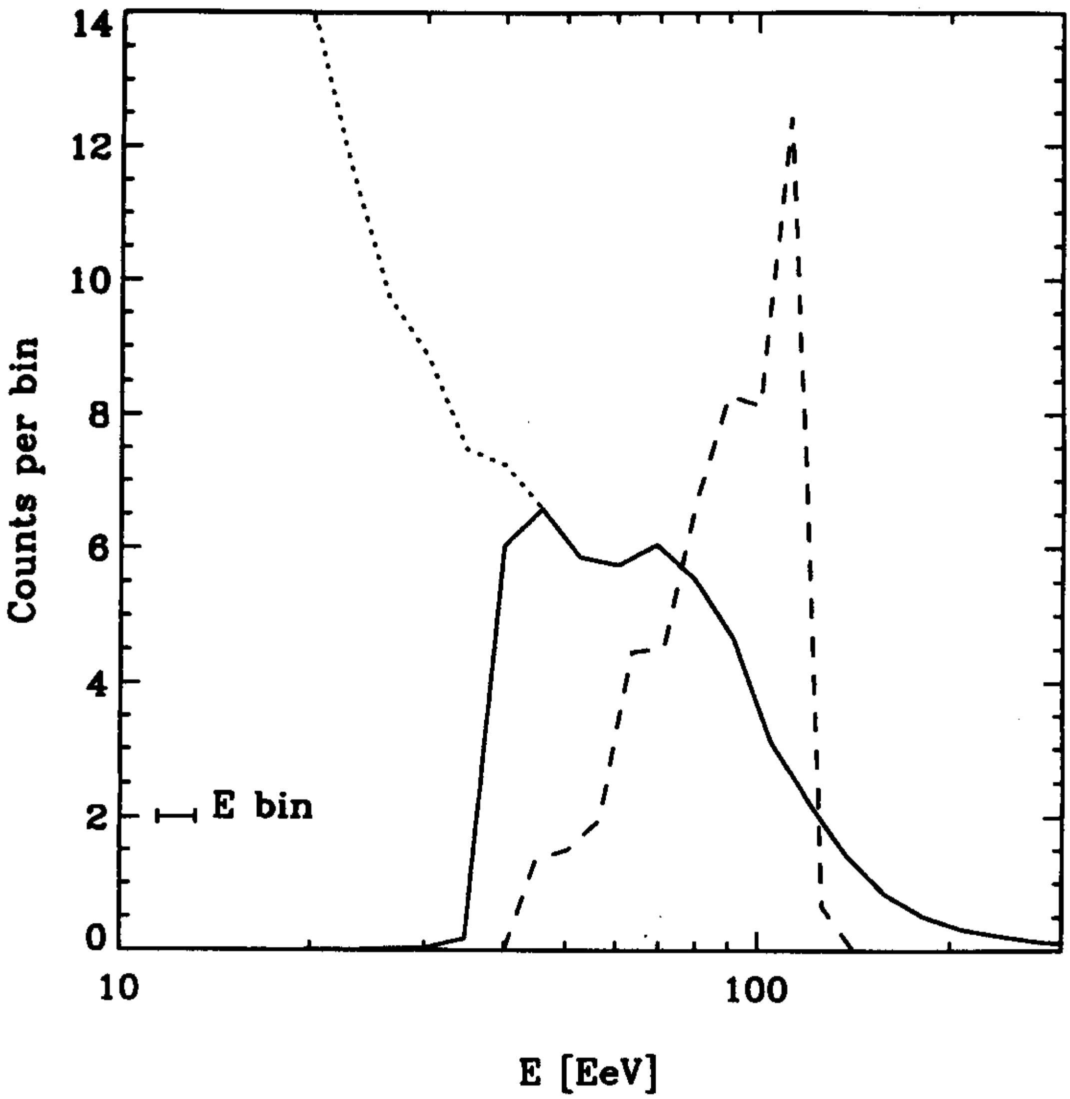
$$\Rightarrow \quad \frac{\Delta t^{obs.}}{t} \sim 1 \; : \quad t_{spread} \approx t_{delay}$$

Inhomogeneous IGM B Example

- IGM B formation $\leftrightarrow$ large scale structure: 3 Mpc structures of 10^{-11}G field and 0.2 filling factor; $< 10^{-15}$ G "void" field (Kulsrud et al. 96)

10^{20} eV Proton Delay



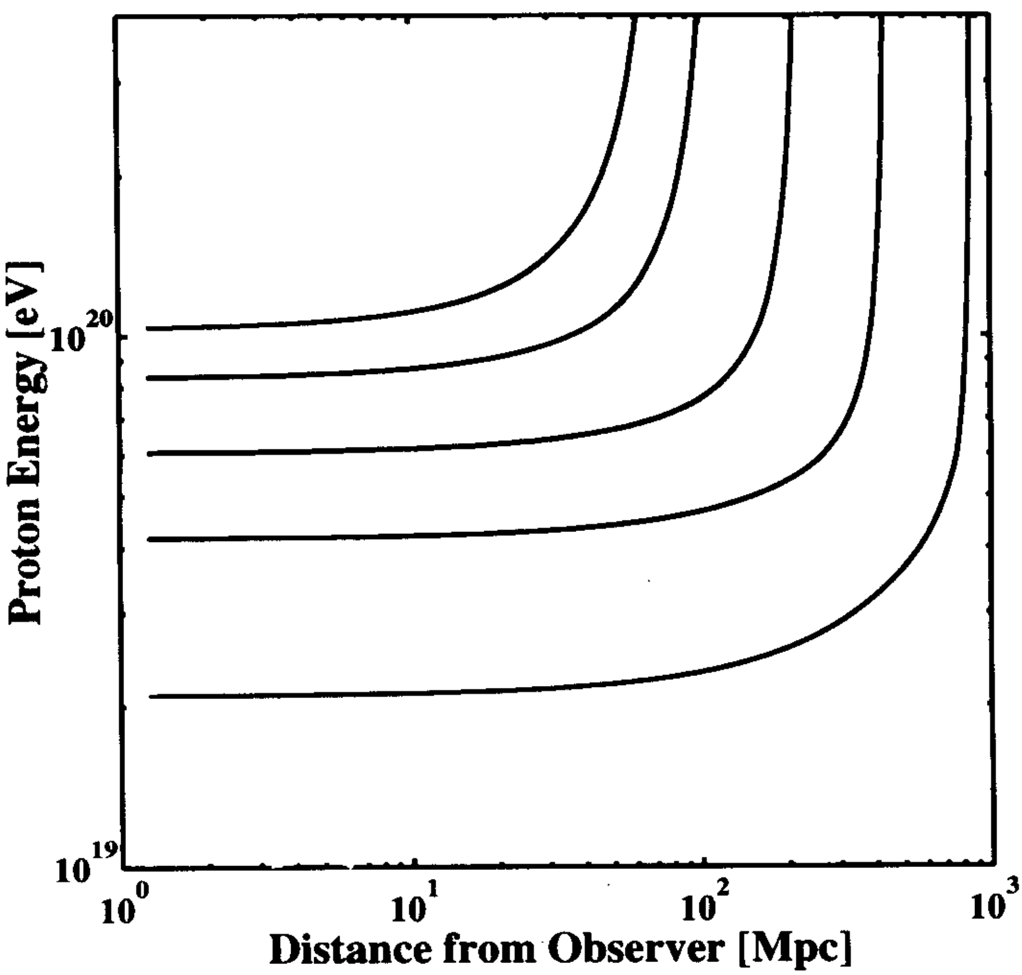


Lemoine, Sigl, Olinto & Schramm 97

Fig. 3.— Energy spectra for a continuous source (solid line), and for a burst (dashed line). Both spectra are normalized to a total of 50 particles detected. The parameters corresponding to the continuous source case are: $T_S = 10^4\,\mathrm{yr}$, $\tau_{100} = 1.3 \times 10^3\,\mathrm{yr}$, and the time of observation is $t = 9 \times 10^3\,\mathrm{yr}$, relative to propagation with the speed of light. A low energy cutoff results at the energy $E_S = 40\,\mathrm{EeV}$ where $\tau_{E_S} = t$ (see text). The dotted line shows how the spectrum would continue if $T_S \ll 10^4\,\mathrm{yr}$. The case of a bursting source corresponds to a slice of the image in the $\tau_E - E$ plane, as indicated in Figure 1 by dashed lines. For both spectra, $D = 30\,\mathrm{Mpc}$, and $\gamma = 2$.

SIGNATURES: I. BRIGHT SOURCES, E$\geq 5 \times 10^{19}$eV

- Energy Dependent Distance Cutoff



$d_c(E) \equiv$ max. distance of $>E$ sources

$\tau_c(E) \equiv$ arrival time spread of CRs of energy E, from distance $d_c(E)$

$$\Delta n_{sources}(d,E) = 4\pi R_{GRB} \tau_c(E) \cdot \frac{d^2}{d_c(E)^2} d^2 \Delta d$$

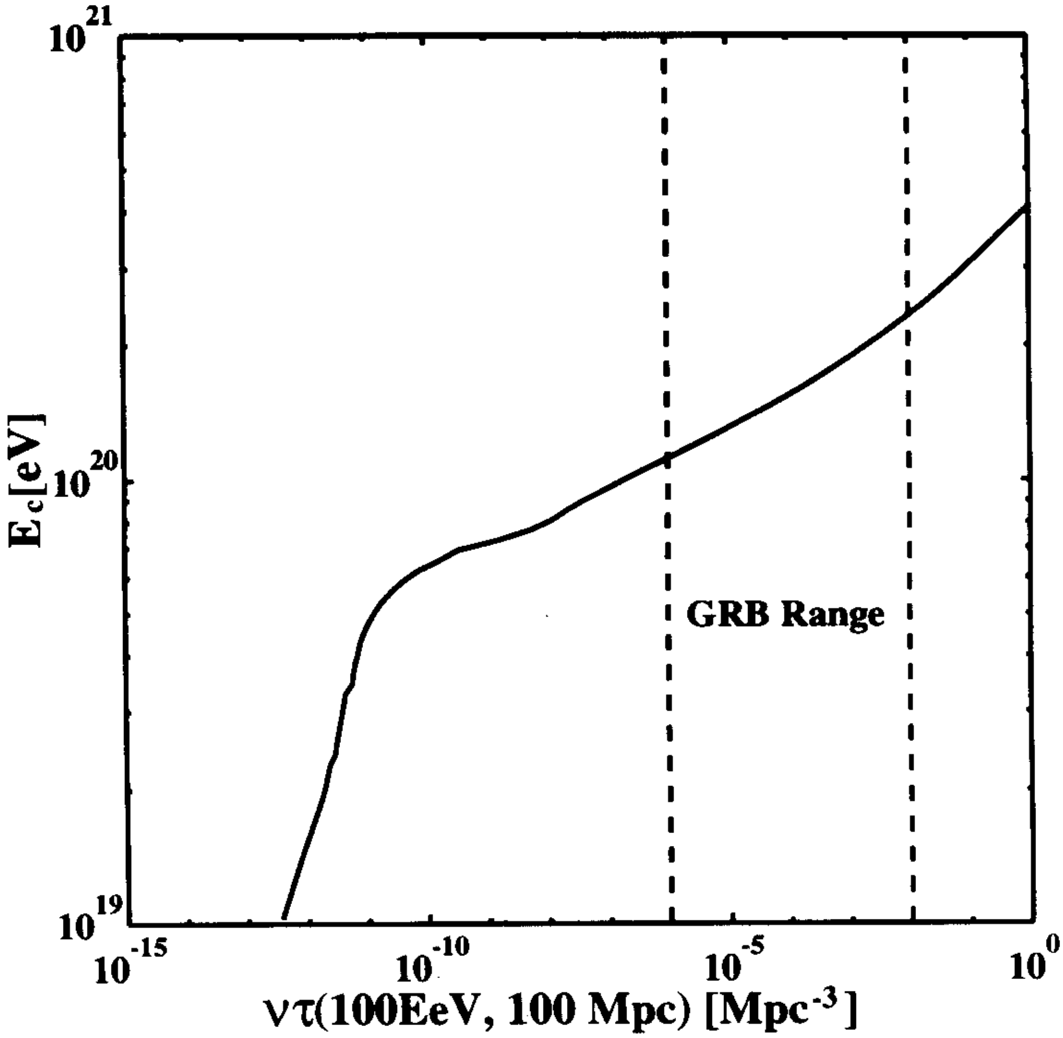
$$\Rightarrow \quad n_{sources}(E) = \frac{4\pi}{5} R_{GRB} d_c^3(E) \cdot \tau_c(E)$$

$E_c \equiv$ Energy above which one source is observed (on average)

$$\frac{4\pi}{5} R_{GRB} d_c^3(E_c) \tau_c(E_c) = 1$$

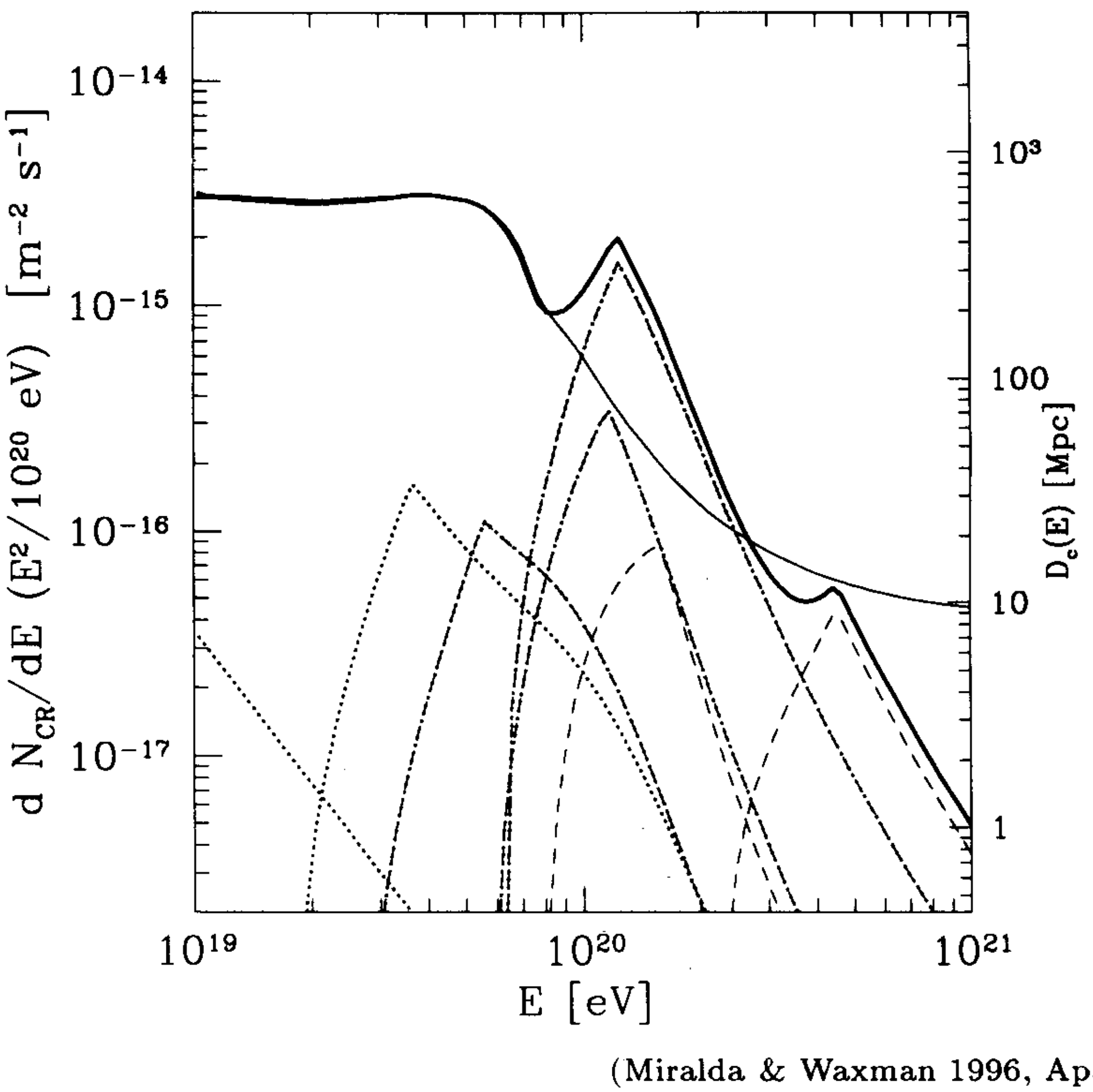
- CRITICAL ENERGY:

Above E_c — One Source (on average)



Monte-Carlo realization:

$$E_c = 1.4 \cdot 10^{20}\ \mathbf{eV}$$



(Miralda & Waxman 1996, ApJL in ~~press~~)

ApJ 462, L59

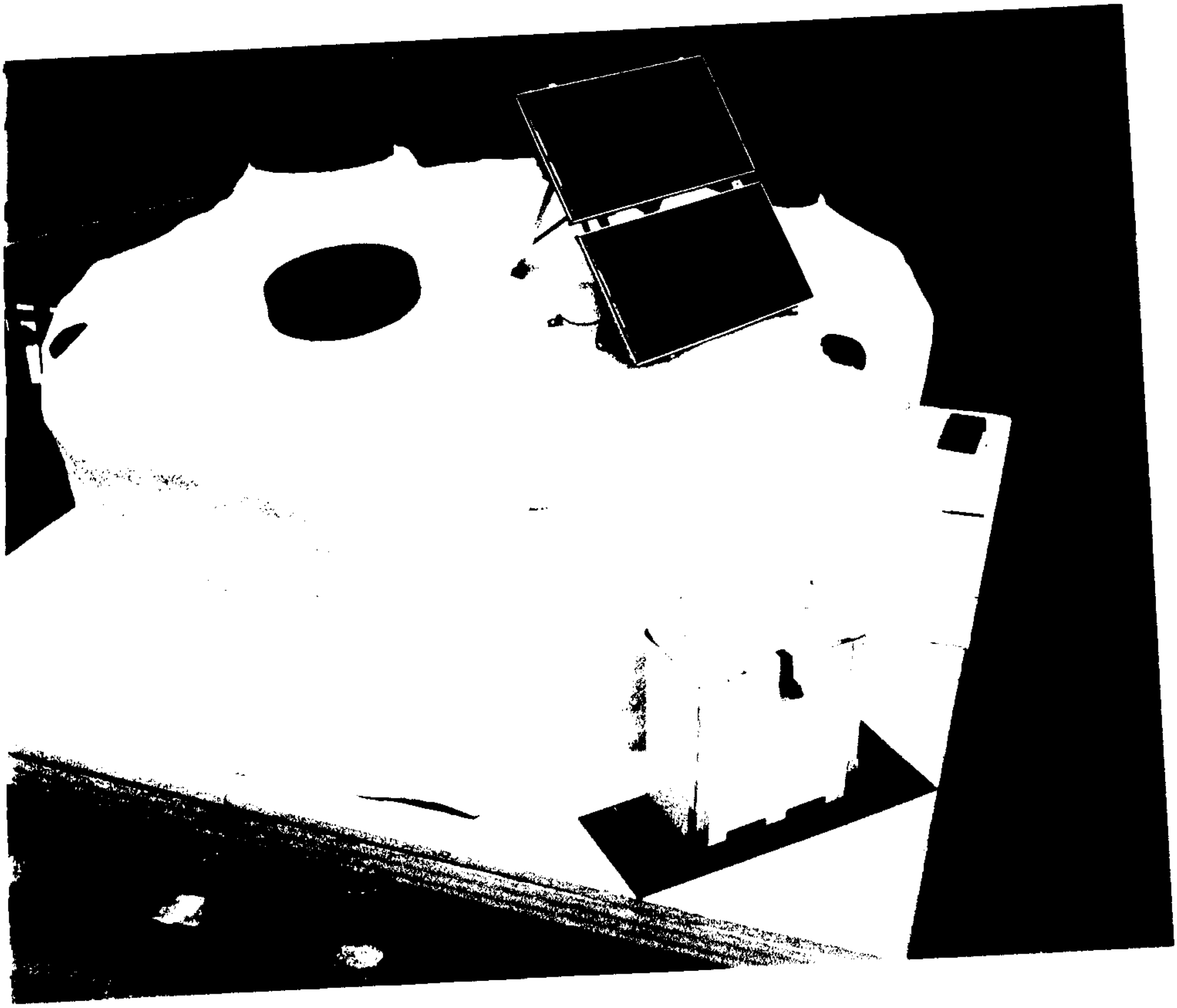
III. GRB-CR: summary

- GRB fireballs allow Fermi acceleration of protons to $>10^{20}$ eV
- Energy generation rate consistent with Observed CR flux at $E > 10^{19}$ eV
- Model predictions can be tested with planned Auger detectrons:

  Auger $\approx 5 \cdot 10^3$ km^2

  $\rightarrow$ $\times 100$ current exposure

  $\oplus$ Combined ground-array / fluorescence



VIII High Energy ν's

[Gaisser, Halzen & Stanev 1995

Phys. Rep. 258, 173 ;

Waxman & Bahcall

1997, PRL 78, 2292

1999, PRD 59, 023002

astro-ph/9909286 (ApJL)]

High Energy ν's: Motivation

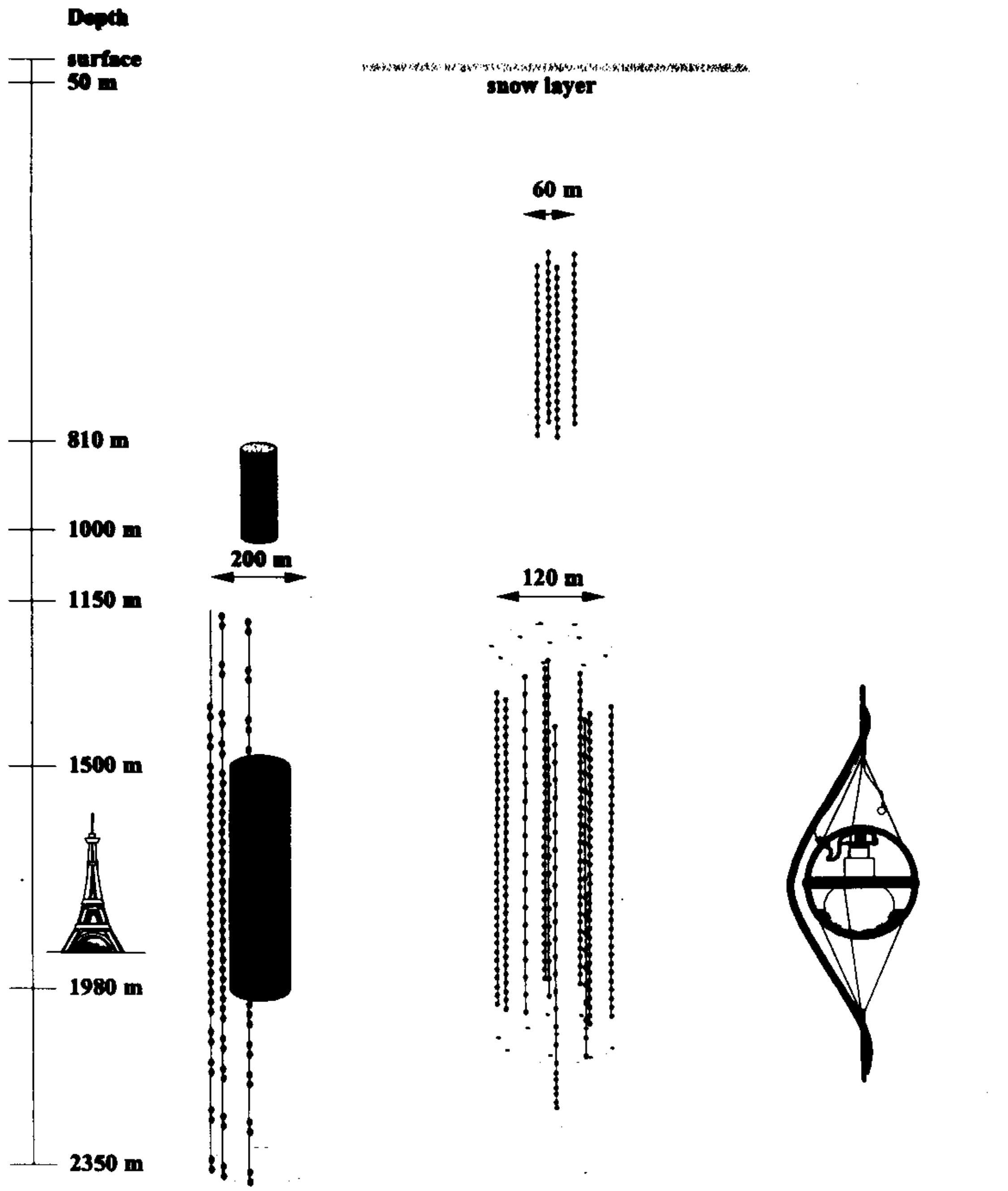
- $>10^{19}$ eV CRs exist
- Likely: Protons

- $\gamma + p \Rightarrow n + \pi^+$

$$\pi^+ \longrightarrow e^+ + \nu_e + \nu_\mu + \bar{\nu}_\mu$$

$\Rightarrow$ $\sim 1\,\text{km}^2$ ν detectors:

AMANDA $\longrightarrow$	IceCube	(South Pole)
ANTARES $\longrightarrow$		(France, Med. sea)
NuBE		(Hawaii)
NESTOR		(Greece, Med. sea)
NEMO		(Italy)



AMANDA as of 1998
Eiffel Tower as comparison
(true scaling)

zoomed in on
AMANDA-A (top)
AMANDA-B10 (bottom)

zoomed in on one
optical module (OM)

Flux Bound

[Waxman & Bahcall 98]
[" 99]

- J_{CR} ($>10^{19}$ eV) $\Rightarrow$

$$E^2 \frac{d\dot{N}_{CR}}{dE_{CR}} \approx 10^{44} \text{ erg}/\text{Mpc}^3 \cdot \text{yr}$$

- γ-p losses $\Rightarrow$ $z_{source} < 0.25$

  $\Rightarrow$ Present ($z=0$) Rate

- E calibration : ± 25 %

- $\tau_{\gamma p} < 1 \quad \Rightarrow$

$$E_\nu^2 \Phi_\nu < F_{max} = \frac{1}{4} \xi_z t_H \frac{c}{4\pi} E^2 \frac{d\dot{N}_{CR}}{dE_{CR}}$$

$$= 1.5 \cdot 10^{-8} \xi_z \quad \frac{\text{GeV}}{\text{cm}^2 \cdot \text{s} \cdot \text{sr}}$$

$\xi_z \equiv$ z (evolution, cosmology) Correction

- B-confinement ?

$\gamma + p \rightarrow \pi^+ + n$

$\lambda_n = 100 \left(E_n / 10^{19} \text{eV} \right)$ kpc

Redshift Correction:

$$n_\gamma(>E) = \int_0^{z_{max}} dz \, \frac{dt}{dz} \, \dot{n}_\gamma[(1+z)E, z]$$

$$\dot{n}[>E, z] = \dot{n}_0(>E) \cdot f(z) \quad ; \quad \dot{n}_0(>E) \propto E^{-1}$$

$$\Rightarrow \quad n_\gamma(>E) = \dot{n}_0(>E) \int_0^{z_{max}} dz \, \frac{dt}{dz} \, (1+z)^{-1} f(z)$$

$$\frac{dt}{dz} = -H_0^{-1} (1+z)^{-5/2} g(z)$$

$$g^2(z) = \Omega_m + \Omega_\Lambda (1+z)^{-3} + (1-\Omega_m-\Omega_\Lambda)(1+z)^{-1}$$

$$\xi_z \equiv \frac{\int_0^{z_{max}} dz \, g(z) \, (1+z)^{-7/2} f(z)}{\int_0^{z_{max}} dz \, g(z) \, (1+z)^{-5/2}}$$

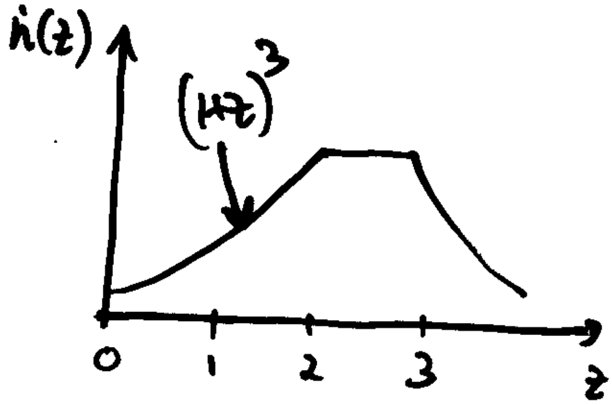
"Z" Corretions

- No source evolution:

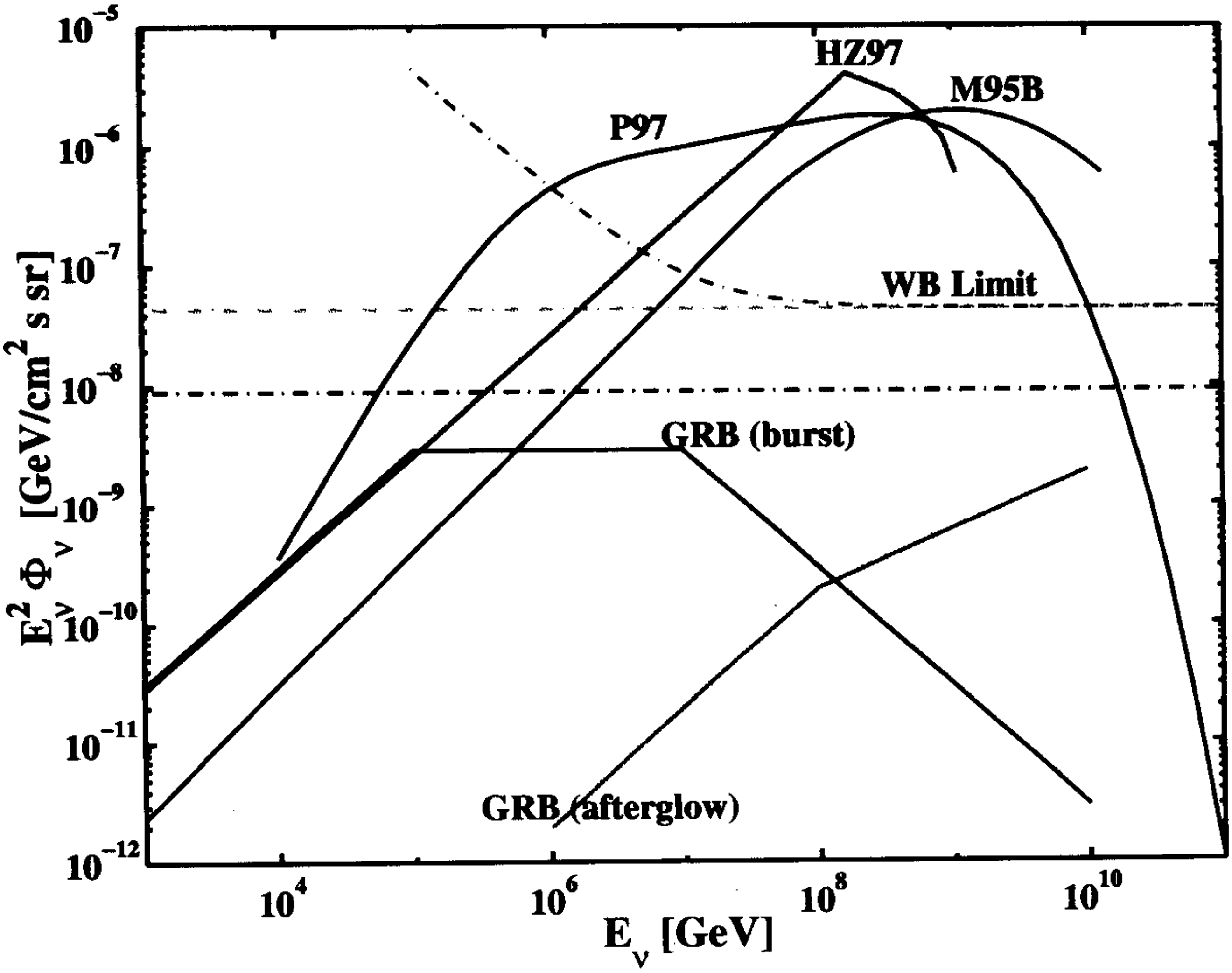
$$\xi_z \approx 0.6$$

- Strong Evolution -
  QSOs (Star-Formation ?):

$$\xi_z \approx 3$$



Bound vs. Models



AGNs: Protheroe 97 [P97], Halzen & Zas 97 [HZ97]

Mannheim 95 [M95B]

GRBs: Waxman & Bahcall [97, 99, 00]

Rise at $E < 10^{18}$ eV

(*) Direct (Balon) measurements :

$$j_p / j_{CR} \simeq 0.2 \quad \text{at} \quad 10^{14}\,\text{eV}$$

(*) Ground Array :

$$\frac{\partial (j_p / j_{CR})}{\partial E} < 0 \quad \text{at} \quad 10^{14}\,\text{eV} < E < 10^{16}\,\text{eV}$$

(*) Fly's Eye & AGASA:

$$j_p / j_{CR} < 0.1 \quad @ \quad 10^{17}\,\text{eV}$$

Violating the Bound

(1) "Hidden" Sources:

Only ν's get out.

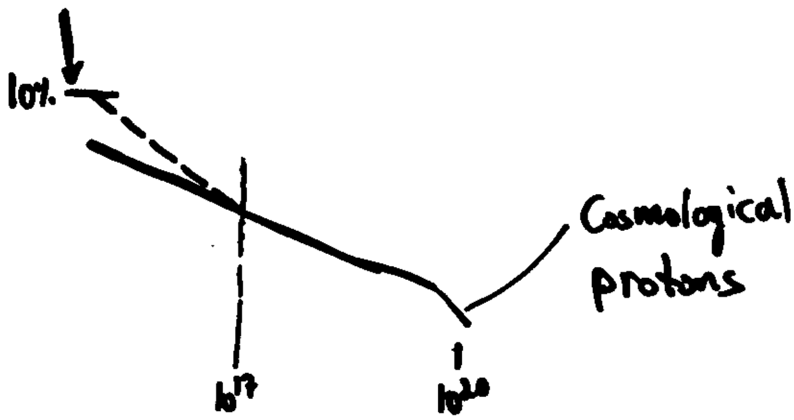
(2) (i) Proton fraction at $E \leq 10^{17}\,eV$ is $< 10\%$

(ii) $j_{CR}\,(E < 10^{17}) \propto E^{-3}$; $j_{p,\cos.} \propto E^{-2}$

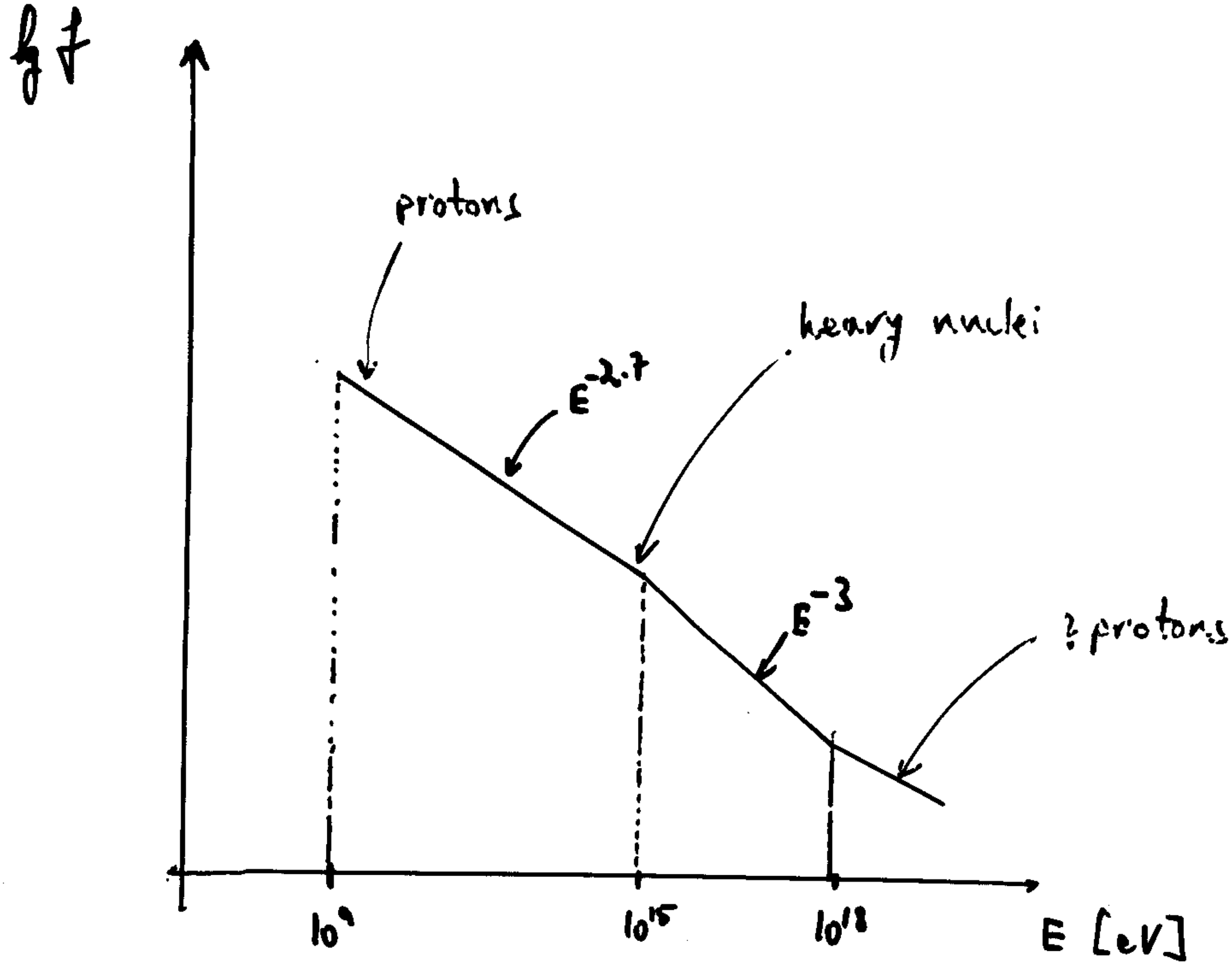
(iii) $j_{p,\cos.} / j_{CR} \Big|_{E=10^{17}} \lesssim 10\%$

Hide below j_{CR}:

$$F'_{max} \propto \left(\frac{10^{16}\,eV}{E_\nu}\right) F_{max} \quad @ \quad E_\nu < 10^{16}\,eV$$


10%
Cosmological protons
10^{17}
10^{20}

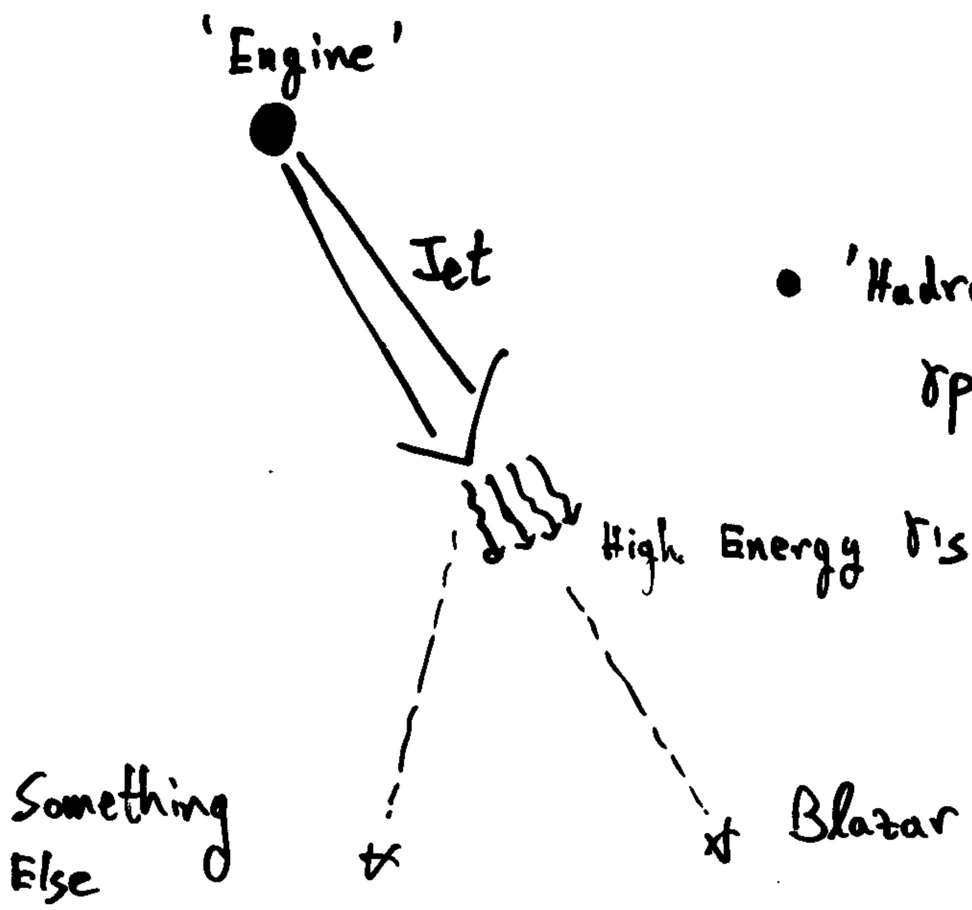
Flux and Composition



$$[f] = \frac{\#}{cm^2 \cdot s \cdot sr \cdot E}$$

$$u_{CR}[\sim 1\,GeV] = \frac{1\,eV}{cm^3}$$

Active Galactic Nuclei



- 'Hadronic Model'

$$\gamma p \rightarrow \begin{cases} \pi^0 \rightarrow \gamma\text{'s} \\ \pi^+ \rightarrow \nu\text{'s} \end{cases}$$

⊕ AGN Jets Produce γ-ray Background

AGN $\tau_{\gamma p}$

- $\varepsilon_p \varepsilon_\gamma \geq m_\pi m_p$ for γp

  $\varepsilon_{\gamma'} \varepsilon_\gamma \geq 2 m_e^2$ for $\gamma\gamma$

- $dN_\gamma / d\varepsilon_\gamma \propto \varepsilon_\gamma^{-2}$

  $\Rightarrow \quad \tau_{\gamma p} \propto \varepsilon_p \, , \quad \tau_{\gamma\gamma} \propto \varepsilon_\gamma$

$$\Rightarrow \quad \tau_{\gamma p}(\varepsilon_p) = 10^{-3} \tau_{\gamma\gamma}\left(\varepsilon_\gamma = \frac{2 m_e^2}{m_\pi m_p} \varepsilon_p\right)$$

$$\simeq \tau_{\gamma\gamma}(10\,\mathrm{GeV}) \cdot \left(\frac{\varepsilon_p}{10^{19}\,\mathrm{eV}}\right)$$

10GeV

$\uparrow \nu f_\nu$

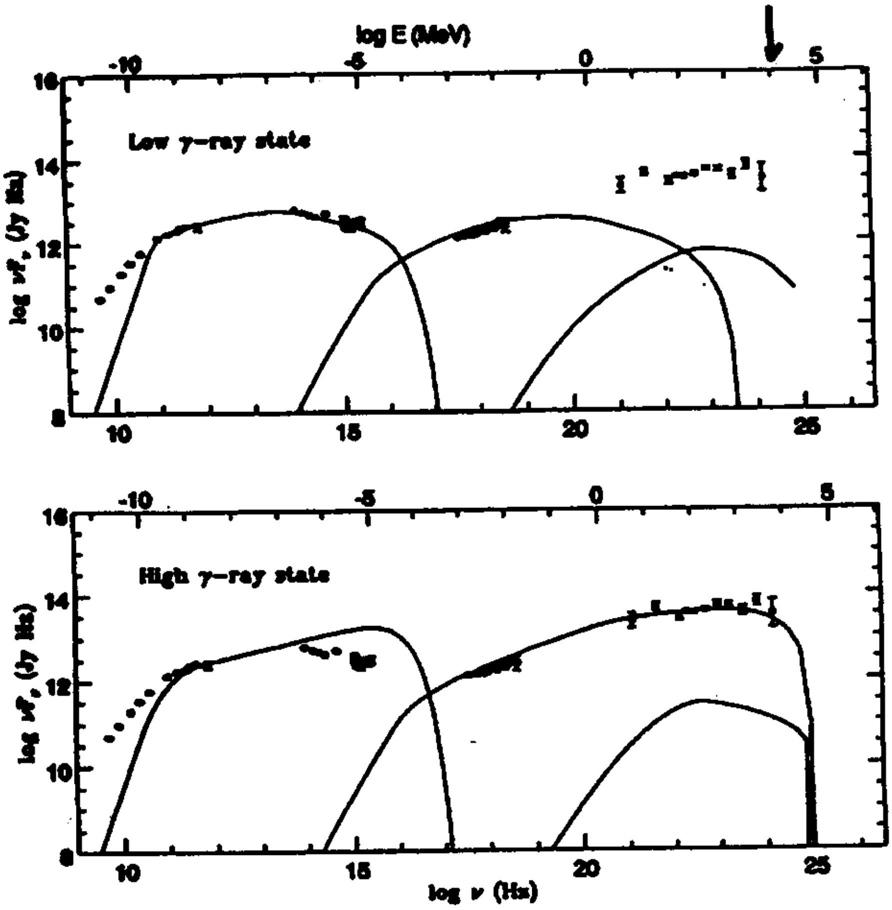


FIG. 6.—The best-fitting attempt to model numerically the 1991 June flare with SSC emission from a uniform, relativistic moving sphere.

3C279 Spectra

[Hartman et al. 96]

Mannheim et al. 96

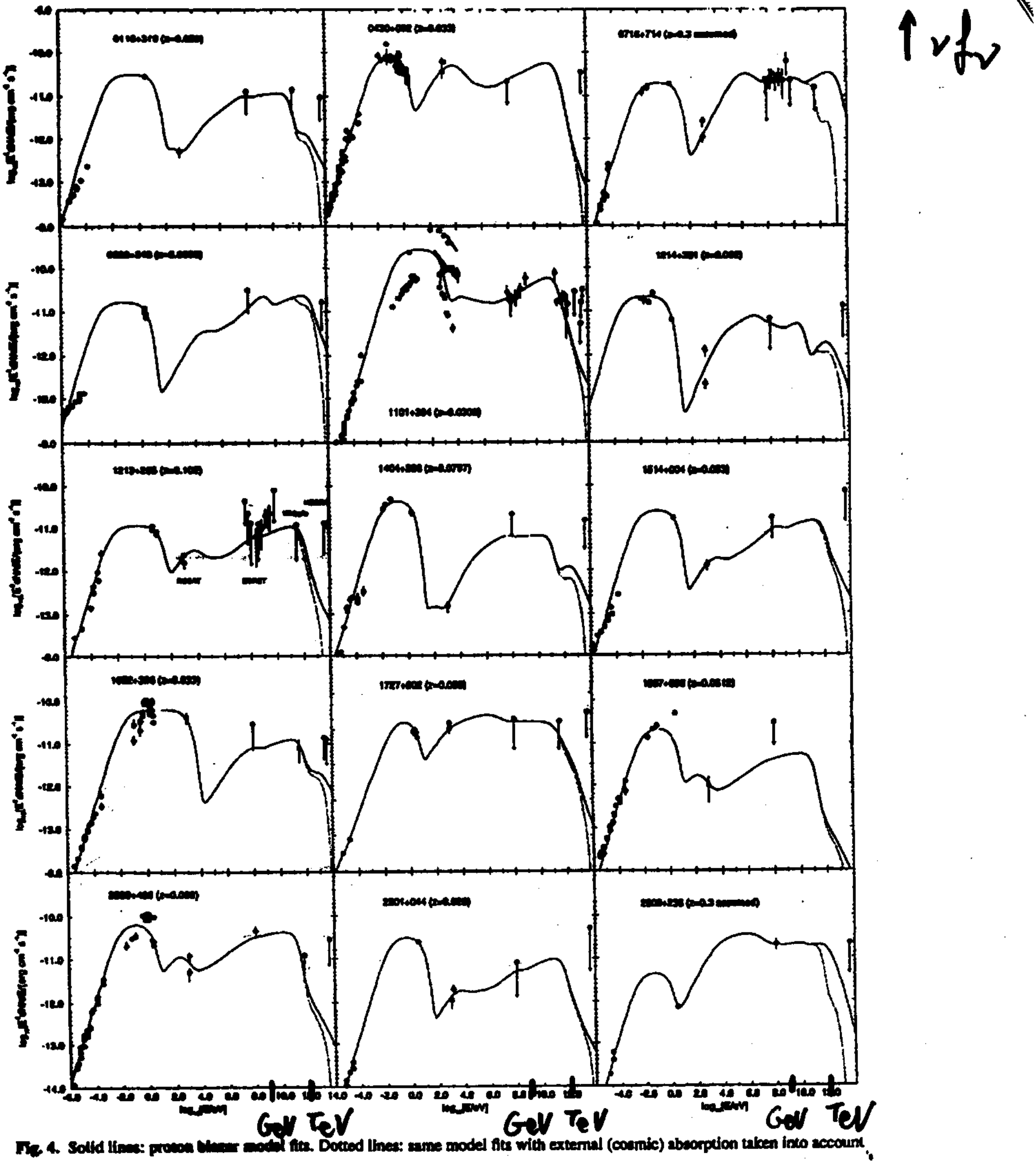


Fig. 4. Solid lines: proton blazar model fits. Dotted lines: same model fits with external (cosmic) absorption taken into account.

Mannheim et al. 00

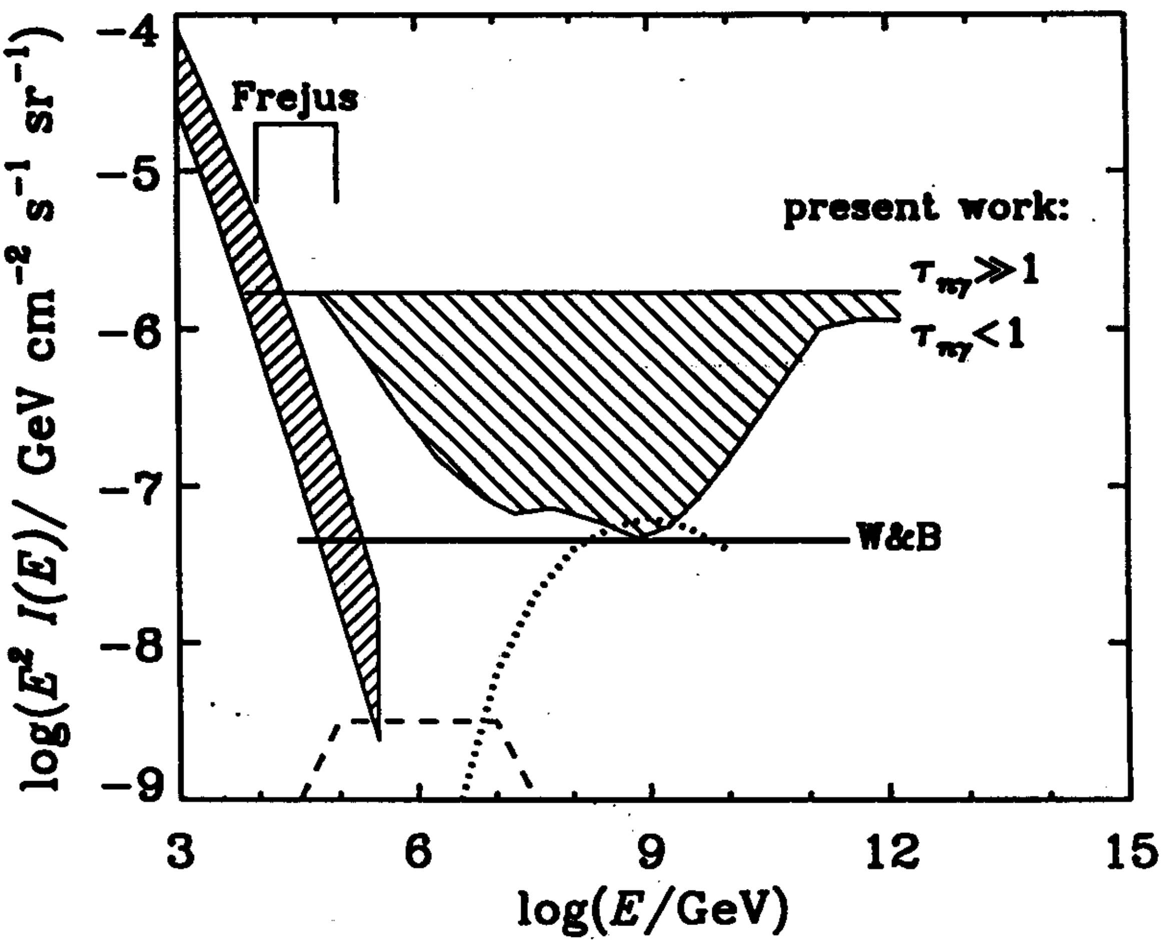
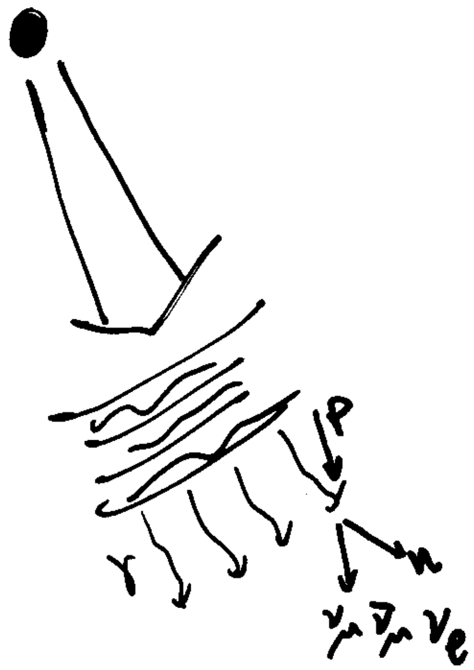


FIG. 3. Our neutrino upper bounds for optically thin pion photoproduction sources (curve labelled $\tau_{n\gamma} < 1$) and optically thick pion photoproduction sources (curve labelled $\tau_{n\gamma} \gg 1$); the hatched

GRB ν's



$$\frac{\epsilon_p}{\gamma} \cdot \frac{\epsilon_\gamma}{\gamma} > 0.2\,\mathrm{GeV}^2$$

$$\epsilon_\gamma = 1\,\mathrm{MeV} \longrightarrow \epsilon_p = 10^{16}\,\mathrm{eV} \rightarrow \epsilon_\nu \simeq 5 \cdot 10^{14}\,\mathrm{eV}$$

$n_\gamma(\epsilon_\gamma)\,d\epsilon_\gamma$ = # density of photons (in the fireball frame) at the energy range ϵ_γ to $\epsilon_\gamma + d\epsilon_\gamma$

$$t_\pi^{-1}(\epsilon_p) \equiv -\frac{1}{\epsilon_p}\frac{d\epsilon_p}{dt}$$

$$= \frac{1}{2\Gamma_p^2}\, c \int_{\epsilon_0}^{\infty} d\epsilon\, \sigma_\pi(\epsilon)\, \xi(\epsilon)\, \epsilon \int_{\epsilon/2\Gamma_p}^{\infty} dx\, x^{-2}\, n(x)$$

$\Gamma_p \equiv \epsilon_p / m_p c^2$

$\sigma_\pi(\epsilon)$ = Cross section for photon of energy ϵ in the proton frame

$\xi(\epsilon)$ = average fractional energy loss

$\epsilon_0 = 0.15\ \mathrm{GeV}$

Observed spectra:

$$n_\gamma(\epsilon_\gamma) \propto \begin{cases} \epsilon_\gamma^{-1} & \epsilon_\gamma^{ob.} < 1\,MeV \\ \epsilon_\gamma^{-2} & \epsilon_\gamma^{ob.} > 1\,MeV \end{cases}$$

$$\Rightarrow \quad \int_\epsilon^\infty dx\, x^{-2} n(x) \simeq \frac{1}{1+\beta} \frac{U_\gamma}{2\epsilon_{\gamma b}^3} \left(\frac{\epsilon}{\epsilon_{\gamma b}}\right)^{-(1+\beta)}$$

$$U_\gamma \equiv \int_{\frac{30\,keV}{\gamma}}^{3\,MeV/\gamma} d\epsilon\, \epsilon\, n(\epsilon)$$

$$\beta = \begin{cases} 1 & \epsilon_\gamma < \epsilon_{\gamma b} \\ 2 & \epsilon_\gamma > \epsilon_{\gamma b} \end{cases}$$

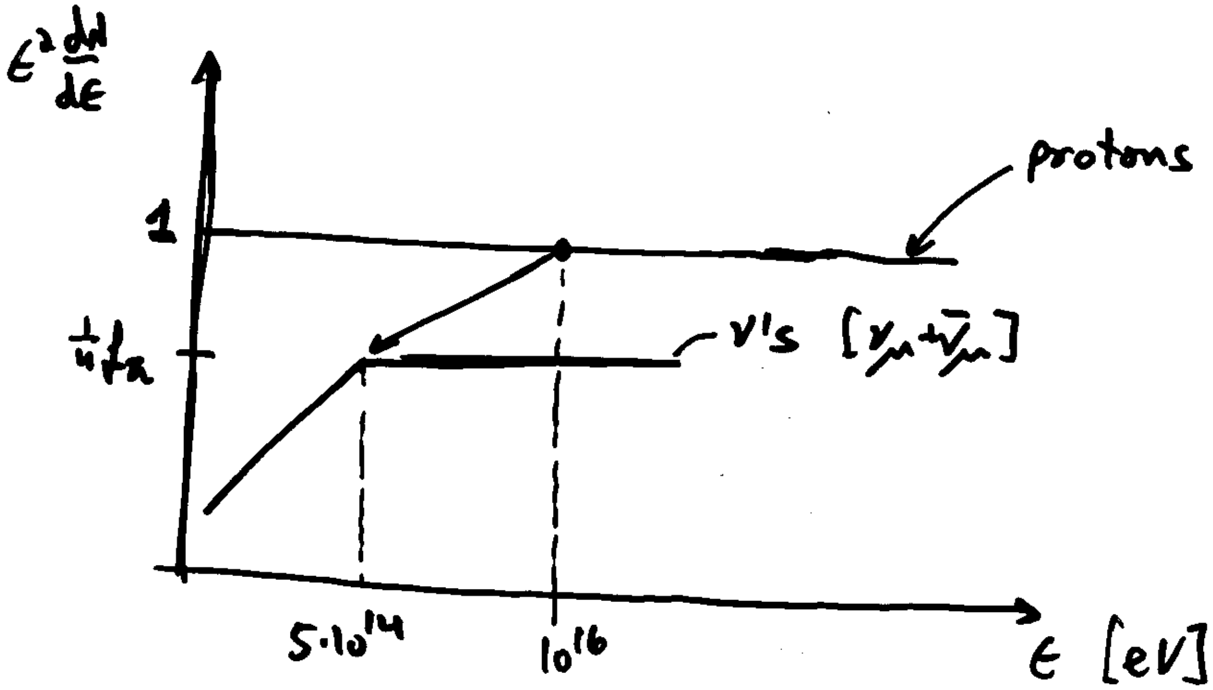
$$\epsilon_{\gamma b} = \epsilon_{\gamma b}^{ob.}/\gamma\,, \qquad \epsilon_{\gamma b}^{ob.} \approx 1\,MeV$$

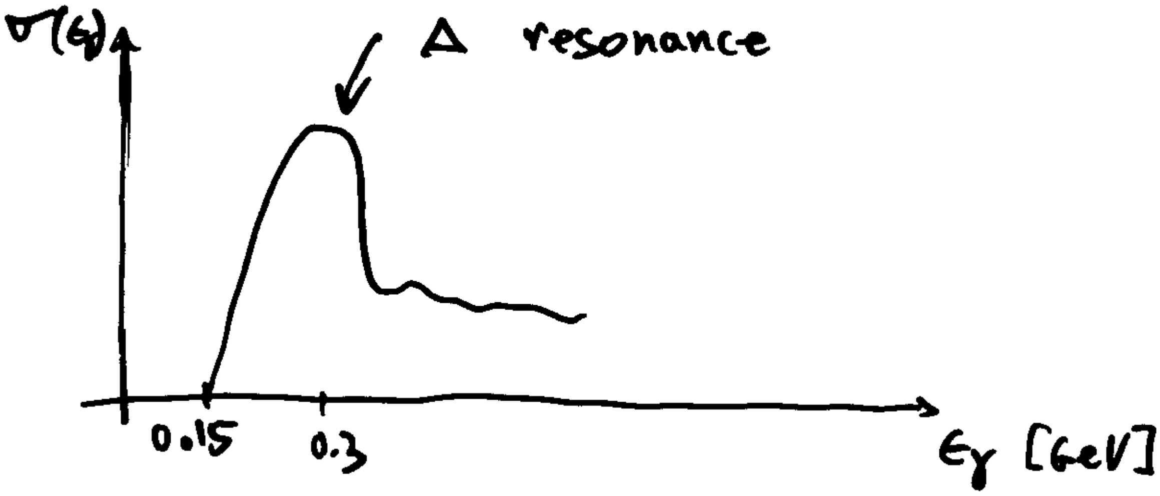
$$L_\gamma = 4\pi r^2 \cdot c \cdot \Gamma^2 U_\gamma$$

$$f_\pi \equiv \frac{r/\Gamma c}{t_\pi}$$

$$\Rightarrow \quad f_\pi = 0.20 \, \frac{L_{\gamma,52}}{\left(\epsilon^{ob}_{\gamma b}/1\,\mathrm{MeV}\right) \Gamma^4_{2.5} \, \Delta t_{ms}} \cdot \min\left[1, \frac{\epsilon_p^{ob.}}{\epsilon_{pb}^{ob}}\right]$$

$$\epsilon^{ob}_{pb} = 10^{16} \, \Gamma^2_{2.5} \left(\epsilon^{ob}_{\gamma b}/1\,\mathrm{MeV}\right)^{-1} \mathrm{eV}$$





Δ-resonance contributions

$$t_\pi^{-1} \simeq \frac{U_\gamma}{2\epsilon_{\gamma b}} c \sigma_{pe} \xi_{pe} \frac{\Delta\epsilon}{\epsilon_{pe}} \min\left[1, \frac{2\Gamma_p \epsilon_{\gamma b}}{\epsilon_{pe}}\right]$$

$$\begin{cases} \epsilon_{pe} = 0.3\,\mathrm{GeV}, \quad \Delta\epsilon = 0.2\,\mathrm{GeV} \\ \sigma_{pe} = 5 \cdot 10^{-28}\,\mathrm{cm}^2 \\ \xi_{pe} = 0.2 \end{cases}$$

ν flux

$$\varepsilon_\nu^2 \phi_{WB} = 1.5 \cdot 10^{-8} \frac{\mathrm{GeV}}{\mathrm{cm^2\,sec\,sr}}$$ [ν_μ + $\bar{\nu}_\mu$] [ν_e]

$$\Rightarrow \epsilon_\nu^2 \phi_\nu = f_\pi \cdot \frac{1}{2} \cdot \epsilon_\nu^2 \phi_{WB}$$

$$= 1.5 \cdot 10^{-9} \frac{f_\pi}{0.2} \min\left[1, \frac{\epsilon_\nu}{\epsilon_{\nu b}}\right] \frac{\mathrm{GeV}}{\mathrm{cm^2\,s\,sr}}$$

$$\epsilon_{\nu b} = 5 \cdot 10^{14}\, \Gamma_{2.5}^2 \left(\epsilon_{\gamma b}^{ob} / \cancel{1}\,\mathrm{MeV}\right)^{-1} \mathrm{eV}$$

Detection Rate

$$R = \int d\epsilon_\nu \cdot 4\pi \phi_\nu(\epsilon_\nu) \cdot P_{\nu\mu}$$

$$P_{\nu\mu} \simeq 10^{-6} \left(\frac{\epsilon_\nu}{1\,\mathrm{TeV}}\right)$$

$$\Rightarrow \quad J_\mu \cong 20 \, \frac{f_\pi}{0.2} \ \mathrm{km^{-2}\,yr^{-1}}$$

$$\phi_\nu P_{\nu\mu} \propto \begin{cases} \epsilon_\nu^{-1} & \epsilon_\nu > \epsilon_{\nu b} \\ \epsilon_\nu^{0} & \epsilon_\nu < \epsilon_{\nu b} \end{cases}$$

$\Rightarrow$ R dominated by $\epsilon_\nu \sim \epsilon_{\nu b}$

(π, μ) synchrotron losses:

- $t_{decay} = \frac{\epsilon}{mc^2} \cdot t_{decay, r.f.}$

$t_{decay} > t_{synchrotron} \rightarrow$ suppression of ν flux

$$\frac{\epsilon_\nu^s}{\epsilon_{\nu b}} = \left(\frac{\xi_B}{\xi_e} L_{\gamma,52}\right)^{-1/2} \gamma_{2.5}^2 \, \Delta t_{-2} \left(\frac{\epsilon_{\gamma b}}{1\,MeV}\right) \times$$

$$\times \begin{cases} 10 & \text{for } \bar{\nu}_\mu, \nu_e \\ 100 & \text{for } \nu_\mu \end{cases}$$

$\epsilon_\nu \gg \epsilon_\nu^s$: $P_{decay} \approx \frac{t_{syn.}}{t_{dec.}} \approx \left(\frac{\epsilon_\nu}{\epsilon_\nu^s}\right)^{-2}$

$\epsilon_\nu \gg \epsilon_\nu^s$: $\epsilon_\nu^2 \phi_\nu \approx 1.5 \cdot 10^{-9} \frac{f_\pi}{0.2} \left(\frac{\epsilon_\nu}{\epsilon_\nu^s}\right)^{-2}$ GeV/cm^{2}s sr

Implications:

- τ detection $\rightarrow$ Vacuum Oscillations ($\nu_\mu \rightarrow \nu_\tau$)

$$\Delta m^2 > 10^{-17} \frac{E_{\nu,14}}{D_{100}} \qquad \mathrm{eV}^2$$

- 1 sec $\nu - \gamma$ Arrival Time:

  (i) W.E.P. : $\Delta t \sim \frac{\phi}{c^2} \frac{L}{c} \sim 10^6$ sec (Galaxy)

  (ii) $1 - \frac{v}{c} = 10^{-16} \Delta t_{\mathrm{sec}} / D_{100}$