



the
abdus salam
international centre for theoretical physics



SMR.1227 - 6

SUMMER SCHOOL ON ASTROPARTICLE PHYSICS AND COSMOLOGY

12 - 30 June 2000

**PARTICLE AND ASTROPHYSICS ASPECTS OF
ULTRAHIGH ENERGY COSMIC RAYS**

Lecture V

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Please note: These are preliminary notes intended for internal distribution only.



4.) Cosmic Rays as Tools to “Measure” Cosmic Magnetic Fields

a) Magnetic fields influence EM cascades via e^\pm synchrotron loss

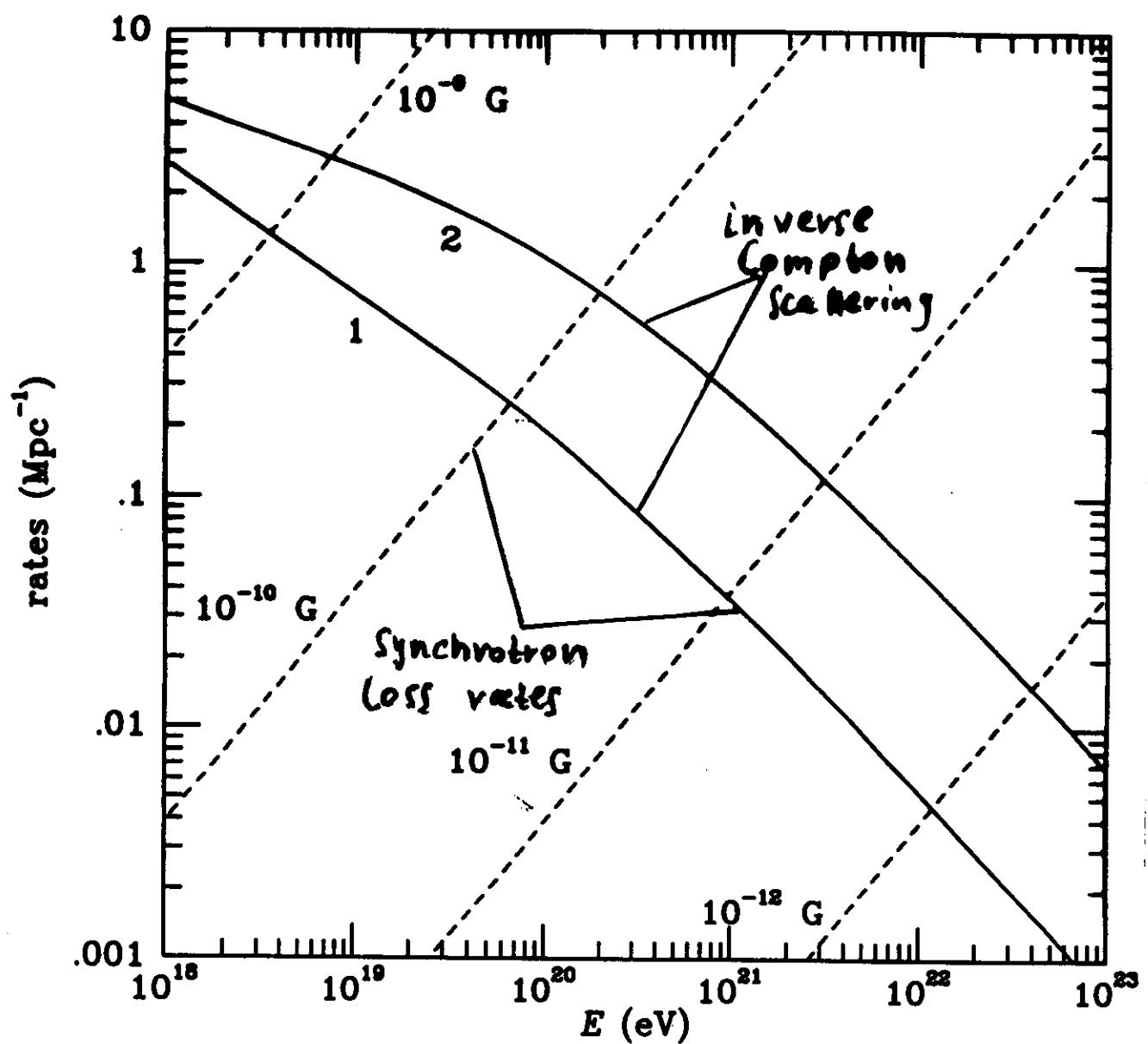
- In case of γ -rays produced as secondaries by nucleons undergoing pion production, there is a characteristic “cross-over” energy E_c between domination by synchrotron loss and by inverse Compton scattering. E_c only depends on the average extragalactic magnetic field (EGMF) strength.
- A solid angle averaged γ /charged cosmic ray flux ratio of $\simeq 10\%$ at ~ 10 EeV would hint to the operation of a TD scenario. At the same time, it would place an independent upper limit of $\simeq 10^{-11}$ G on the EGMF on scales of a few to tens of Mpc.

b) Magnetic fields deflect and delay charged particles

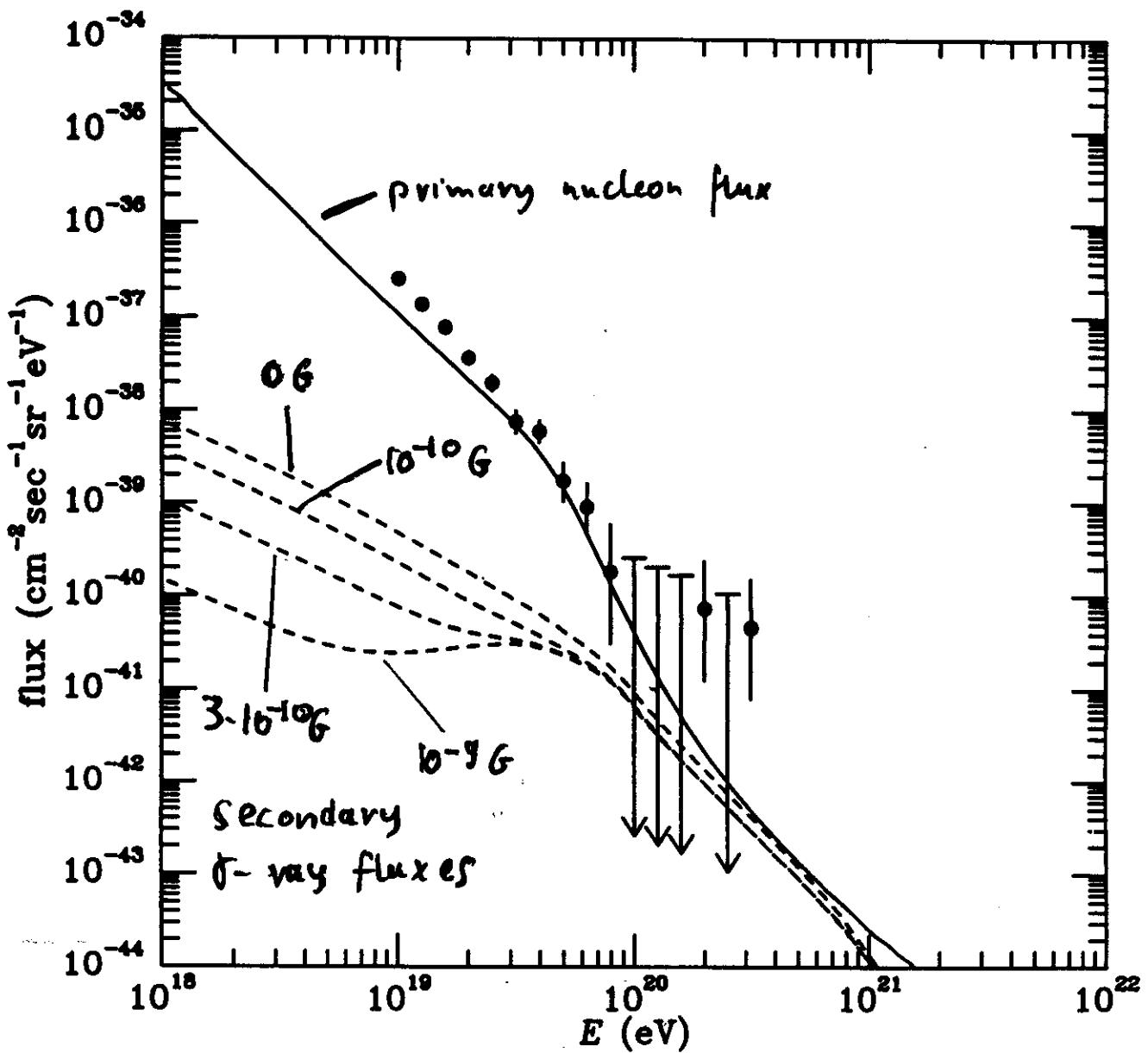
- AGASA observed pairs which might stem from bursting sources which cannot be much further away than ~ 100 Mpc (due to small attenuation length for the higher energy particle). Future observations with Auger could provide much more statistics. Comparison with Monte Carlo simulations might allow to construct the likelihood function for EGMF parameters and maybe even for distance and activity time scale of the sources.

Bursting TD sources might constitute especially attractive probes.

energy loss rates of electrons in the
EM cascade



uniform distribution of acceleration sources



S.Lee, A.V.Olinto, G.S., ApJ. 455 (1995) L21

Implications of AGASA pairs for B (assuming discrete sources)

relative angular deflection for two energies
 $\approx 2.5^\circ$ (resolution)

$$\Rightarrow B_{\text{rms}} \approx 5 \cdot 10^{-10} \left(\frac{d}{30 \text{ Mpc}} \right)^{-1/2} \left(\frac{l_c}{1 \text{ Mpc}} \right)^{-1/2} \text{ G}$$

compare with best limit on large-scale magnetic fields from Faraday rotation of radio wave polarization from high- z sources:

$$B_{\text{rms}} \leq 10^{-9} \left(\frac{l_c}{1 \text{ Mpc}} \right)^{-1/2} \text{ G}$$

\implies potentially constrains origin of Galactic field: $B_{\text{rms}} \sim 10^{-12} - 10^{-9}$ G
if no dynamo, much smaller otherwise

\longrightarrow probes primordial magnetic fields

Angle-Time-Energy Images of Ultrahigh Energy Cosmic Ray Sources

assume:

- particle of charge z , energy E
- (homogeneous) statistical description of magnetic field: strength B_{rms} , coherence scale l_c

=> average time delay

$$\bar{\tau}_E \approx \frac{\alpha_{\text{rms}}^2 d}{4} \approx 2 \left(\frac{d}{30 \text{ Mpc}} \right)^2 \left(\frac{E}{10^{20} \text{ eV}} \right)^{-2} \left(\frac{B_{\text{rms}}}{10^{-11} \text{ G}} \right)^2 \left(\frac{l_c}{1 \text{ Mpc}} \right) \text{ yr}$$

in general 3 time scales involved:

delay time $\bar{\tau}_E$, emission time scale T_s ,
experimental lifetime T_{obs}

Distributions sensitive to RELATIVE SIZE

Next step: include localized regions of strong magnetic fields

M. Lemoine, G.-S., P. Biermann

Assume

- particle of charge z and energy E
- magnetic field of strength B_{rms} and coherence length l_c

Then average time delay over distance $d \gtrsim l_c$ is

$$\tau_E \approx \frac{\alpha_{\text{rms}}^2 d}{4} \approx 2 \left(\frac{d}{30 \text{ Mpc}} \right)^2 \left(\frac{E}{100 \text{ GeV}} \right)^{-2} \left(\frac{B_{\text{rms}}}{10^{-6}} \right)^2 \left(\frac{l_c}{1 \text{ Mpc}} \right) \text{ yr}$$

Assume observed clusters of events stem from a discrete source

Define likelihood function

$$L(T_{100}, T_s, d, r, N_0, l_c, n_B) = \left\langle \prod_{j=1}^N e^{-\beta_j} \frac{\beta_j^{n_j}}{n_j!} \right\rangle$$

j : energy-time bin (no angular binning here)

β_j : predicted II events in bin j

n_j : observed " " " "

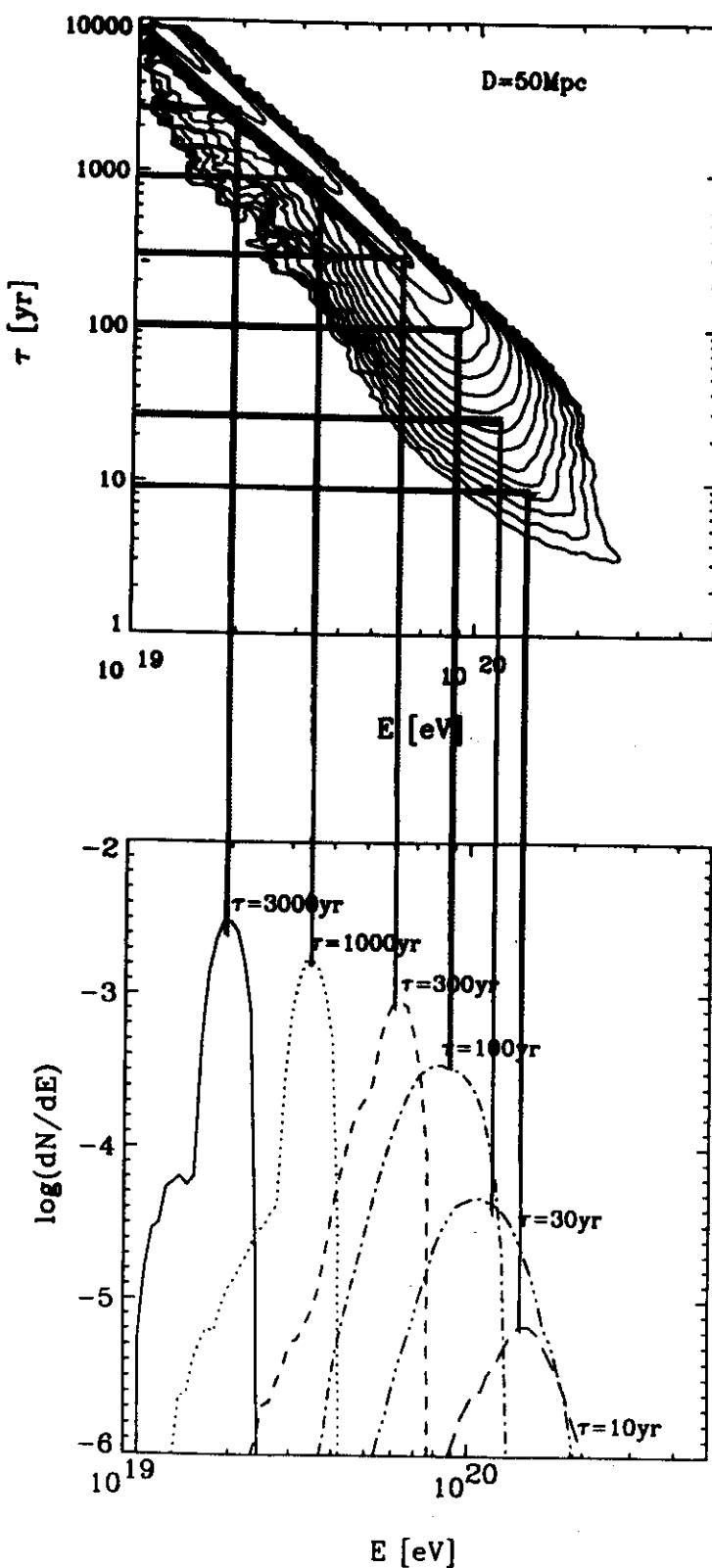
T_s : source emission time scale

γ : injection (power law) spectral index

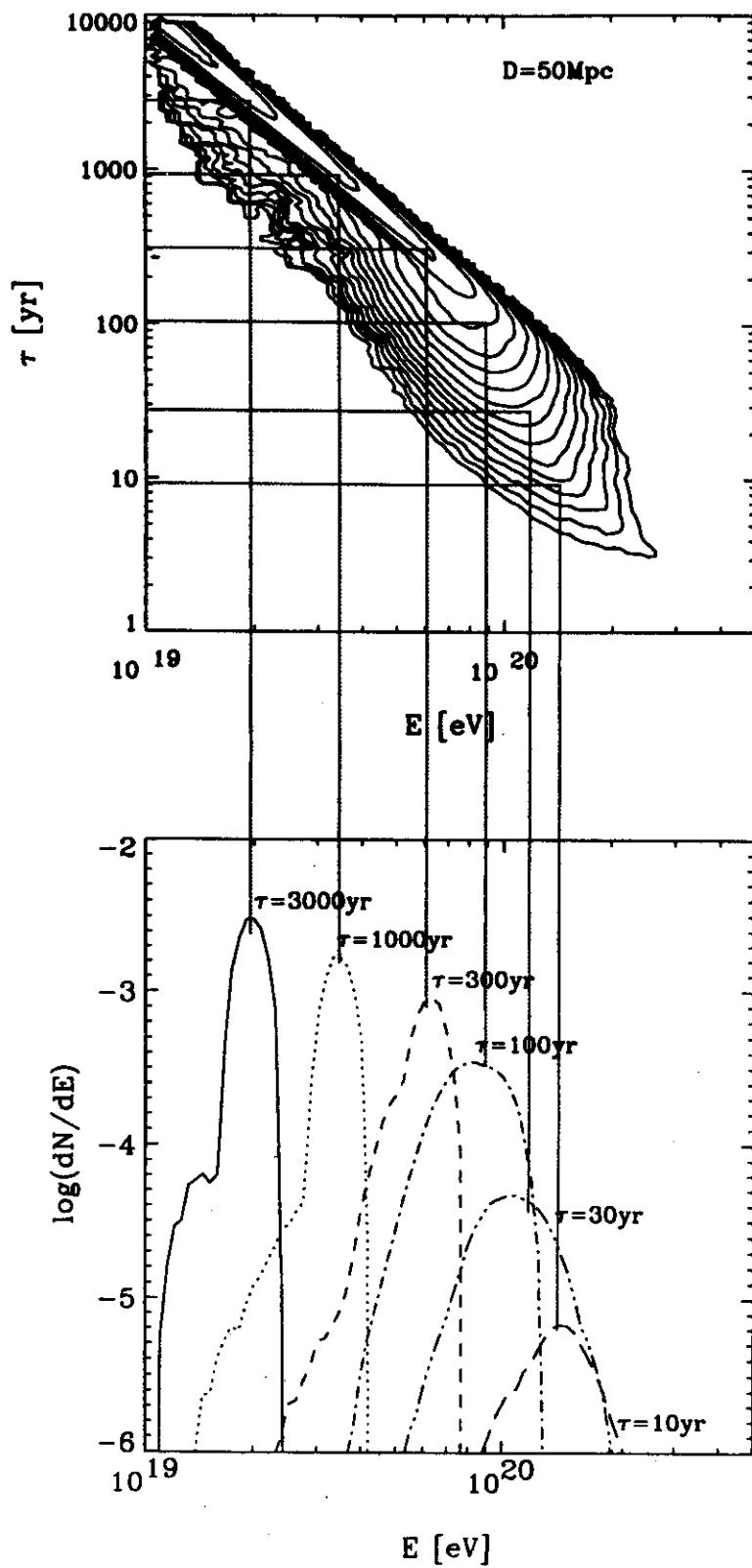
N_0 : total source fluence at detector over infinite time

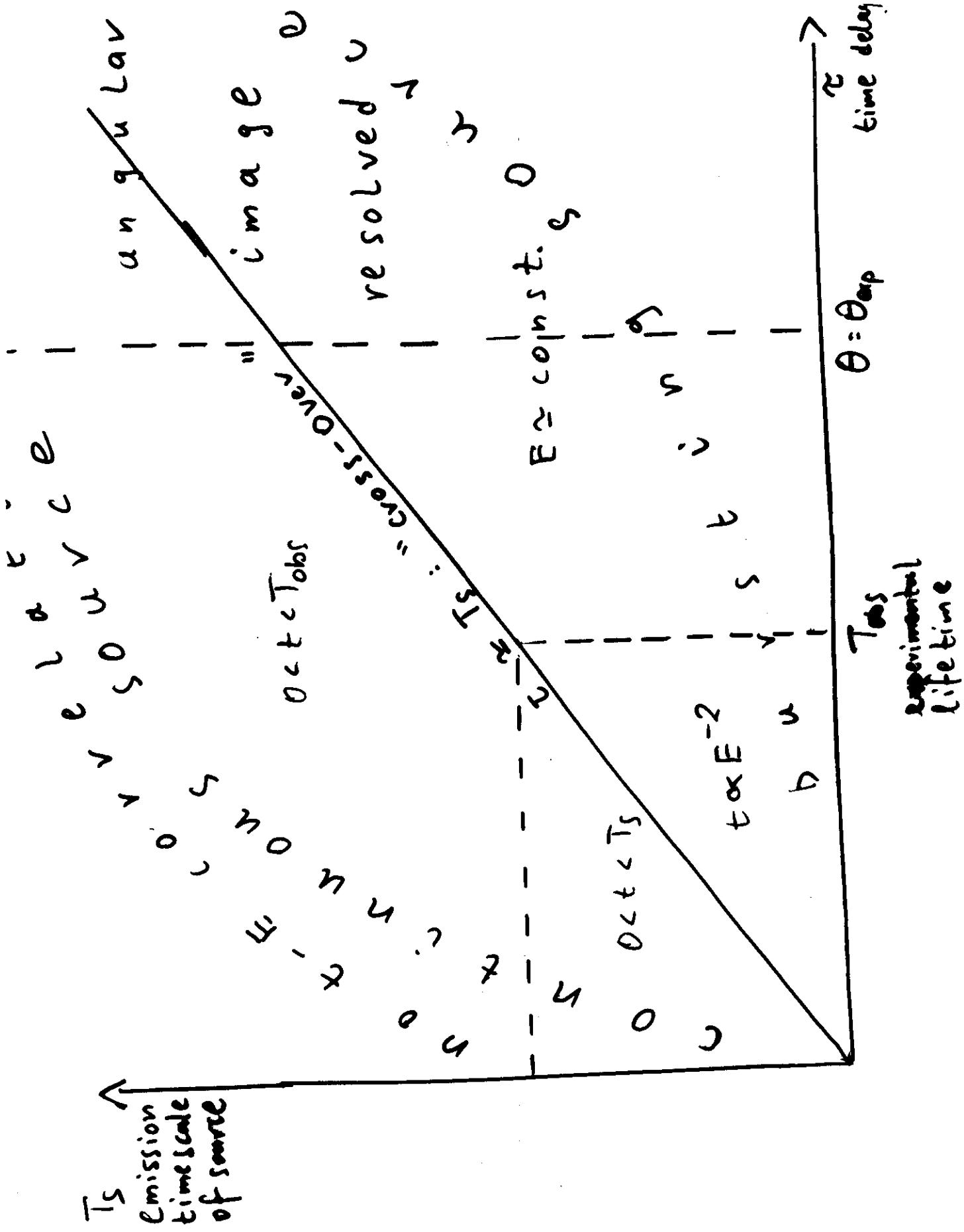
n_B : magnetic field spectral index

Emission time scale << delay time



Emission time scale << delay time





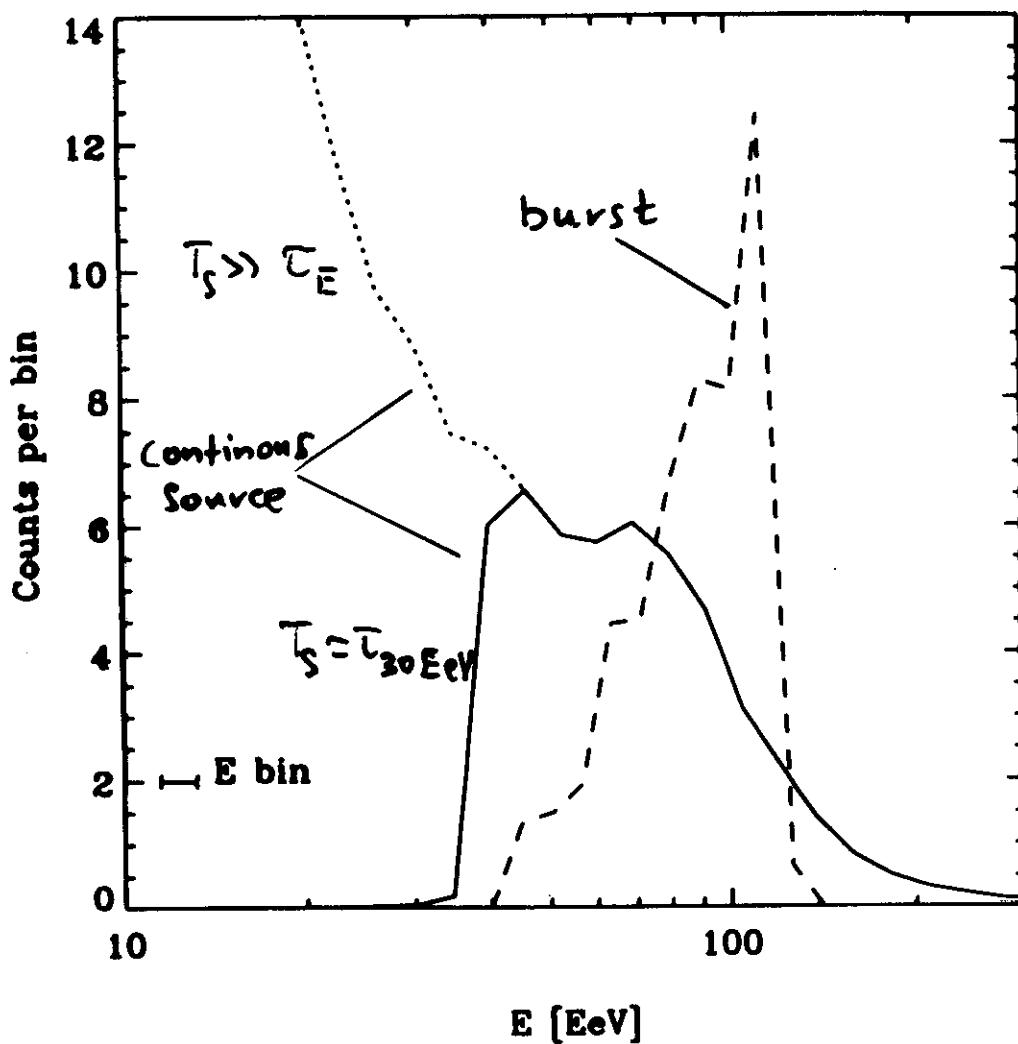


Fig. 3.— Energy spectra for a continuous source (solid line), and for a burst (dashed line). Both spectra are normalized to a total of 50 particles detected. The parameters corresponding to the continuous source case are: $T_S = 10^4$ yr, $\tau_{100} = 1.3 \times 10^3$ yr, and the time of observation is $t = 9 \times 10^3$ yr, relative to propagation with the speed of light. A low energy cutoff results at the energy $E_S = 40$ EeV where $\tau_{E_S} = t$ (see text). The dotted line shows how the spectrum would continue if $T_S \gg 10^4$ yr. The case of a bursting source corresponds to a slice of the image in the $\tau_E - E$ plane, as indicated in Figure 1 by dashed lines. For both spectra, $D = 30$ Mpc, and $\gamma = 2$.

$\tau_E \gg T_{\text{orb}}, T_E \gg T_S$; e.g. GRB, topological defect

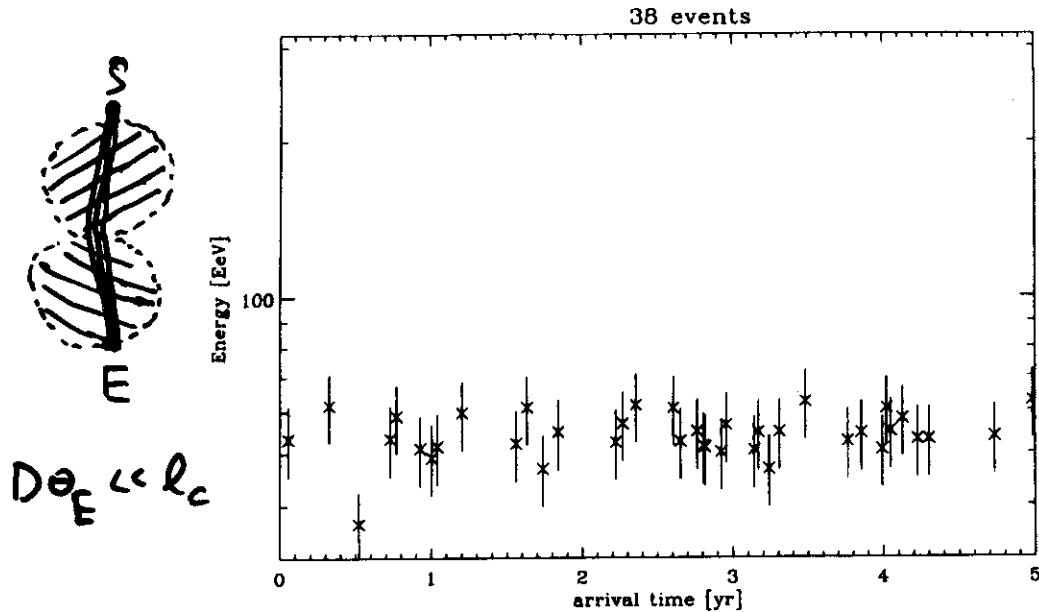


Figure 2: Same as for Fig. 1, except for the parameters $\tau_{100} = 50$ yr, and $N_0 = 4 \times 10^3$. This serves as an example for a burst with a long time delay, but still in the limit of small deflection, $D\theta_E/l_c \ll 1$, leading to a small detected energy dispersion.

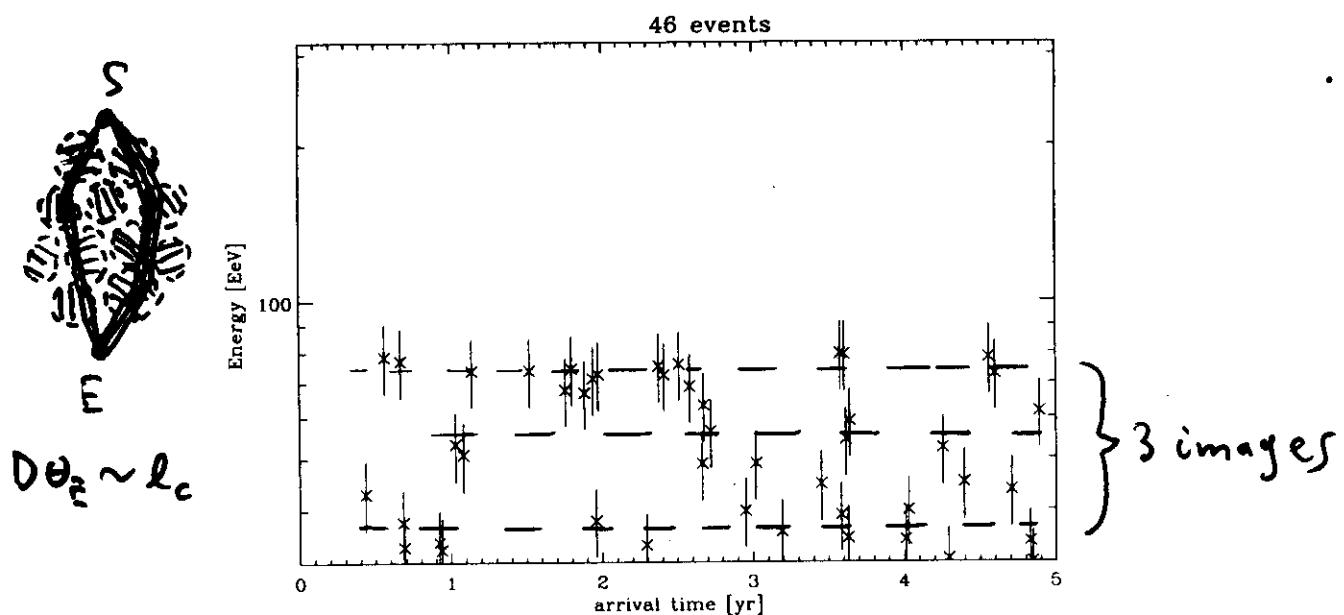
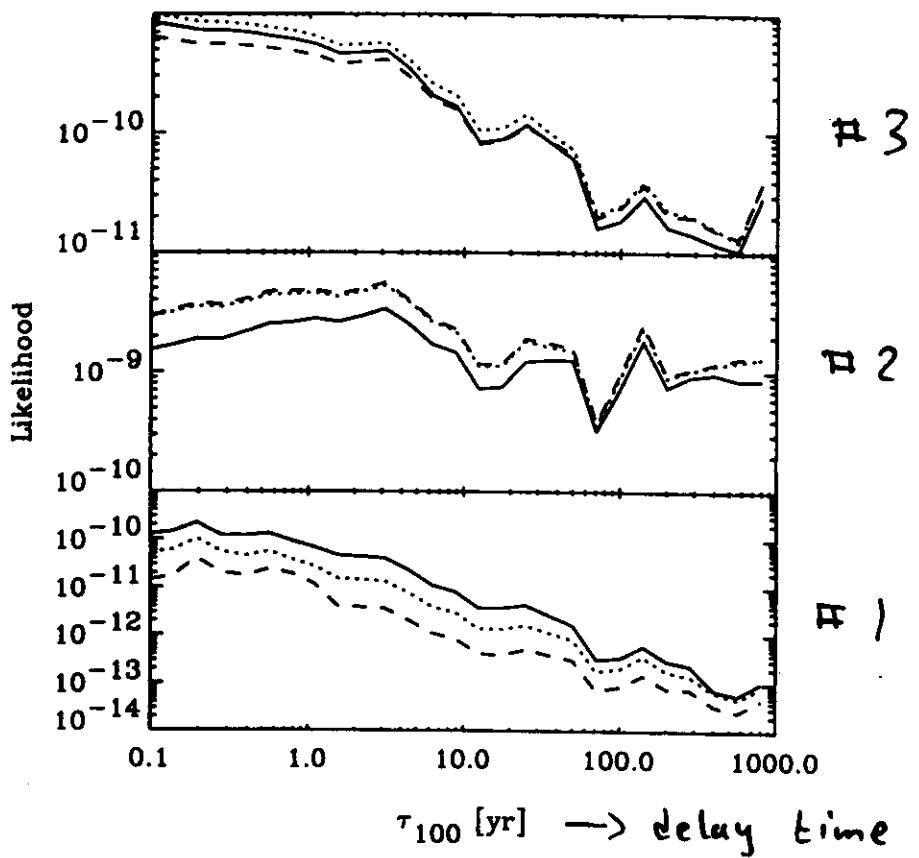


Figure 3: Same as Fig. 1, except for the parameters $\tau_{100} = 250$ yr, $N_0 = 2 \times 10^4$, and $l_c \simeq 0.25$ Mpc. This serves as an example for a burst with a long time delay for intermediate deflection, $D\theta_E/l_c \sim 1$. The distinct sub-bands are due to multiple source images.



Likelihood marginalized with respect
to emission time scale and source fluence.

$D = 30 \text{ Mpc}$

$\overline{\cdots \cdots} \quad \gamma = 1.5$
 $\cdots \cdots \quad \gamma = 2.0$
 $\cdots \cdots \quad \gamma = 2.5$

Likelihood Function for a cluster with $D \approx 50$ \rightarrow small coherence length

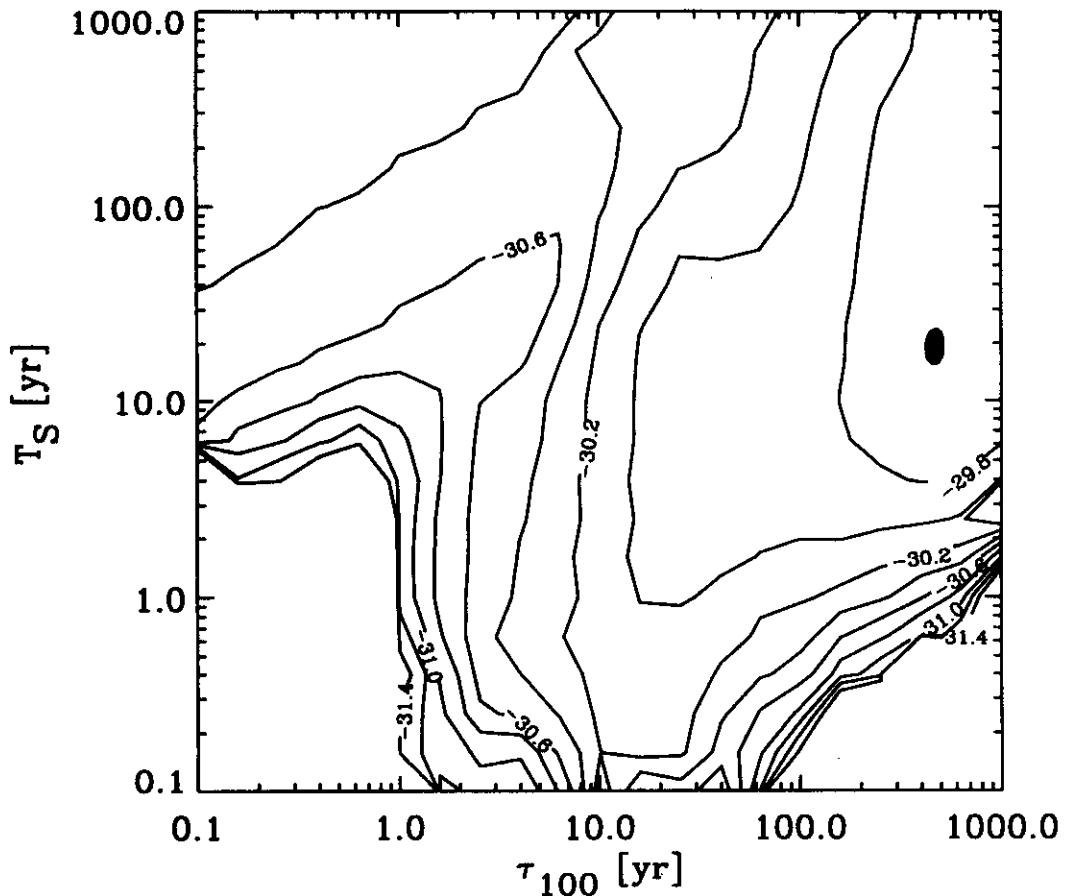


Fig. 7. The likelihood for the cluster shown in Fig. 3, marginalized over N_0 and γ , plotted in the $\tau_{100} - T_S$ plane, for $D = 50$ Mpc and $l_c \simeq 0.25$ Mpc (the true values). The maximum of the likelihood occurs for $\tau_{100} \simeq$ a few hundred years, $T_S \simeq$ a few years which is a good reconstruction of the true values. The contours shown go from the maximum down to values of about 0.01 of the maximum in steps of a factor $10^{0.2} = 1.58$. Note that values in the range $\tau_E = T_S$ with $E \gtrsim 80$ EeV and $T_S \gtrsim 10$ yr are not significantly excluded, as expected (see text). The fall-off at $\tau_{100} \gtrsim 50$ yr and $T_S \lesssim 3$ yr is a numerical artifact due to the limited number of propagated particles (4×10^4 per parameter set) which causes too patchy histograms in arrival time.

Likelihood Function for a cluster with $D\theta_E \sim l_c$
 \rightarrow large coherence length

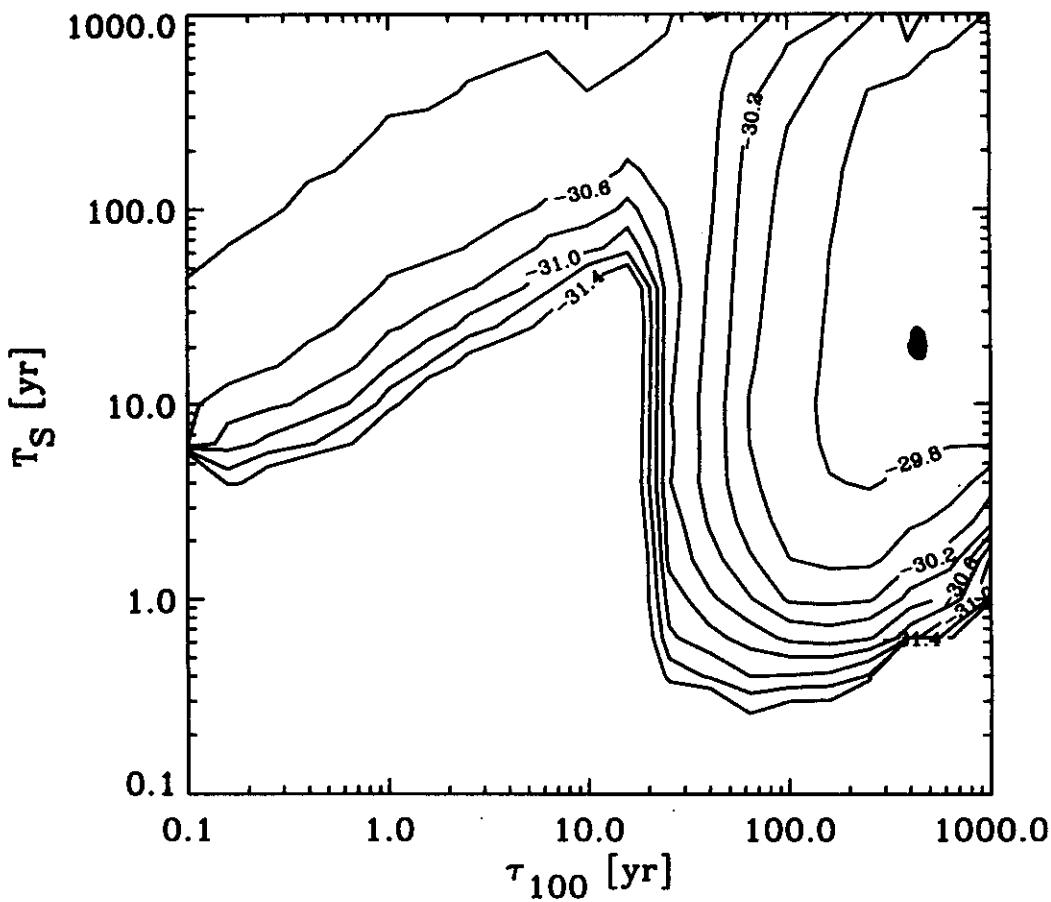


Fig. 11. Same as Fig. 7, but assuming a magnetic field coherence length $l_c \simeq 1$ Mpc. The maximum of the likelihood is near $\tau_{100} = 400$ yr, $T_S = 20$ yr which again is a reasonable reconstruction of the true values. Note that if τ_{100} were known to be smaller than $\simeq 50$ yr, a coherence length as large as $l_c \simeq 1$ Mpc could be ruled out, but $l_c \simeq 0.25$ Mpc would be allowed (see Fig. 7 and discussion in the text).

have to increase τ_{100} to satisfy $D\theta_E \sim l_c$
 \Rightarrow likelihood function has no power at small τ_{100}

$\tau_E \lesssim T_S$ for all events, e.g. AGN

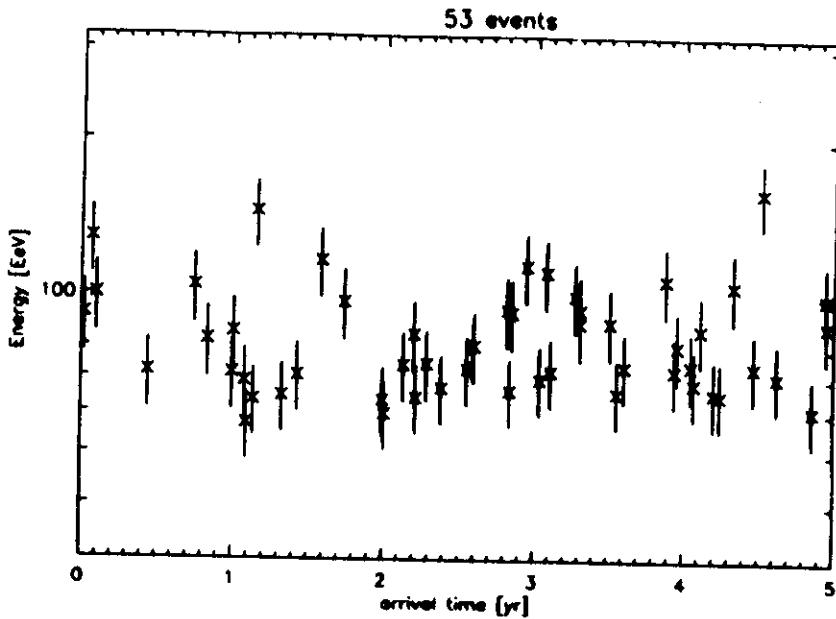


Figure 4: Same as Fig. 1, except for the parameters $\tau_{100} = 50$ yr, $T_S = 200$ yr, and $N_0 = 1.4 \times 10^4$. A lower cut-off in energy occurs at $E_C \approx 60$ EeV where $\tau_{E_C} \approx T_S$.

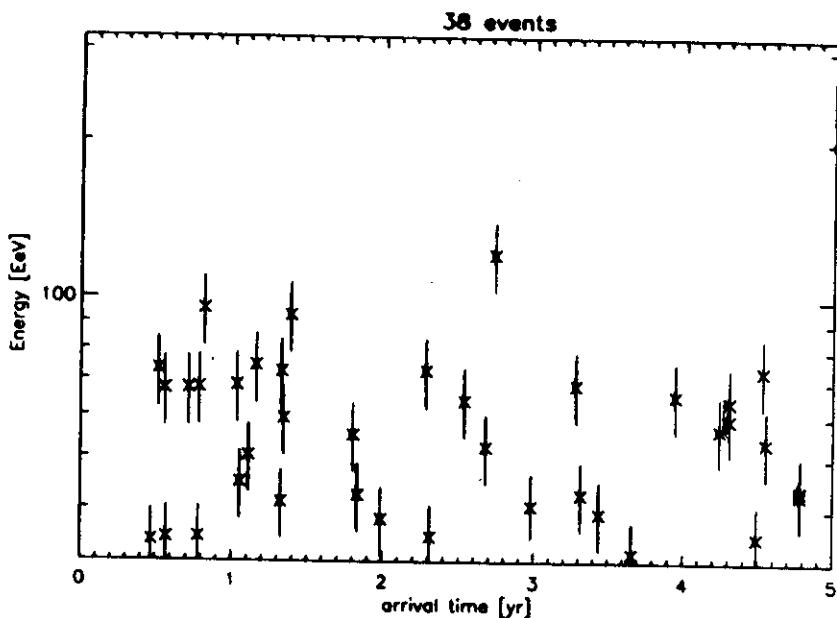
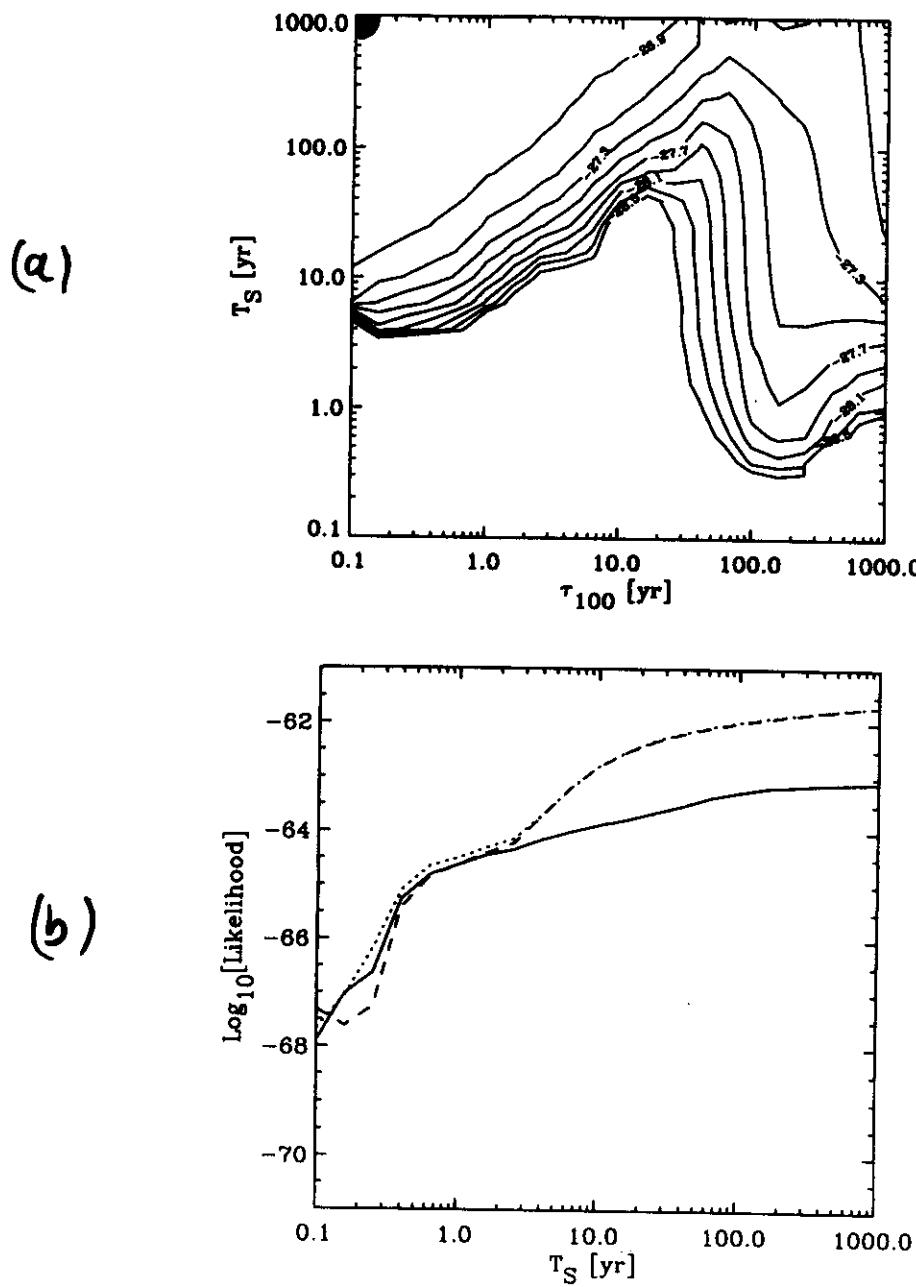
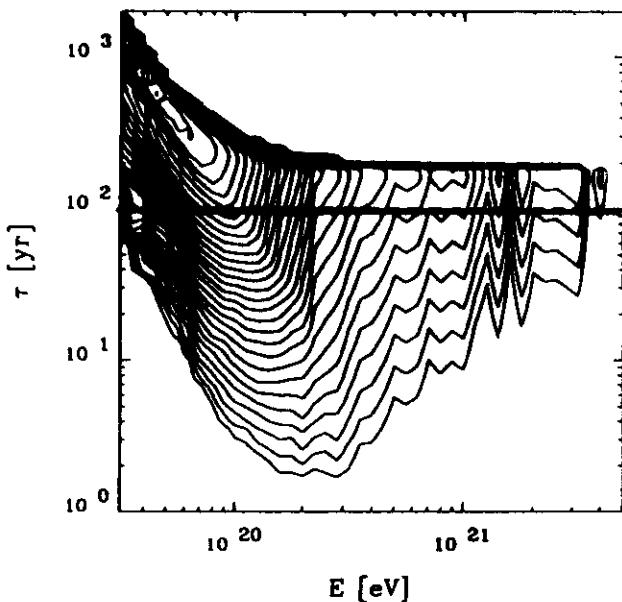


Figure 5: Same as Fig. 1, except for the parameters $\tau_{100} = 0.1$ yr, $T_S = 500$ yr, and $N_0 = 10^4$. This serves as an example of a source that is continuously emitting at all relevant energies.

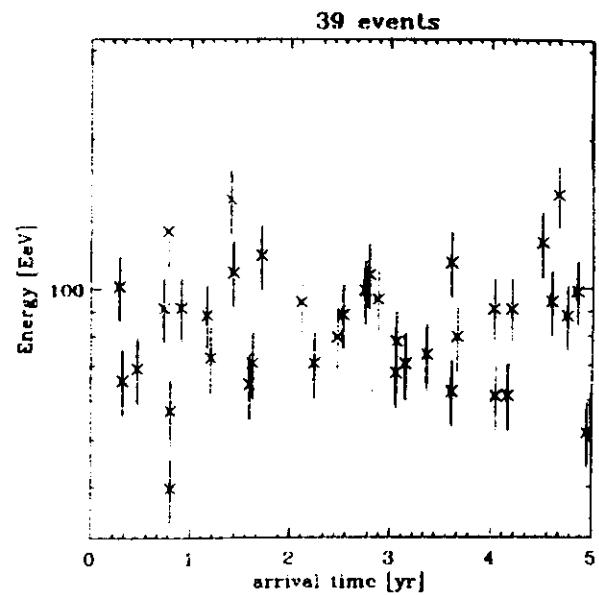


"Cross-over", $\tau_E \approx \tau_S \gg \tau_{\text{obs}}$

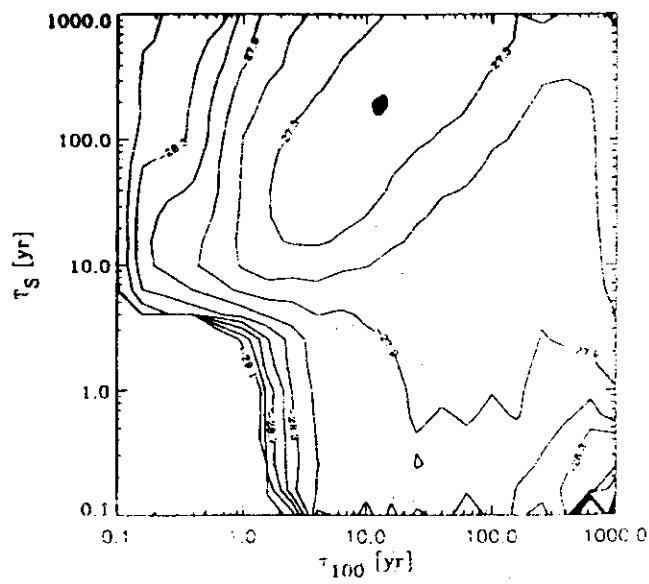
e.g. hot spots in radio galaxies



(a)



(b)



(c)

Low-energy cut-off is a signature for a "cross-over" where

$$\tau_{E_c} \approx T_s$$

If τ_E is large enough such that deflection angle

$$\theta_E \approx 2^\circ \left(\frac{d}{10 \text{ Mpc}} \right)^{-1/2} \left(\frac{\tau_E}{10^4 \text{ yr}} \right)^{1/2}$$

can be resolved, $T_s \gg T_{\text{obs}}$ can be measured

$$1 \cdot 10^3 \left(\frac{\theta}{2^\circ} \right)^2 \left(\frac{d}{10 \text{ Mpc}} \right) \text{ yr} \lesssim T_s \approx \tau_E \lesssim 10^4 \dots 10^7 \left(\frac{E}{100 \text{ EeV}} \right)^{-2} \text{ yr}$$

\uparrow
angular image
resolvable

\uparrow
constraints on magnetic
fields, e.g. Faraday rotation

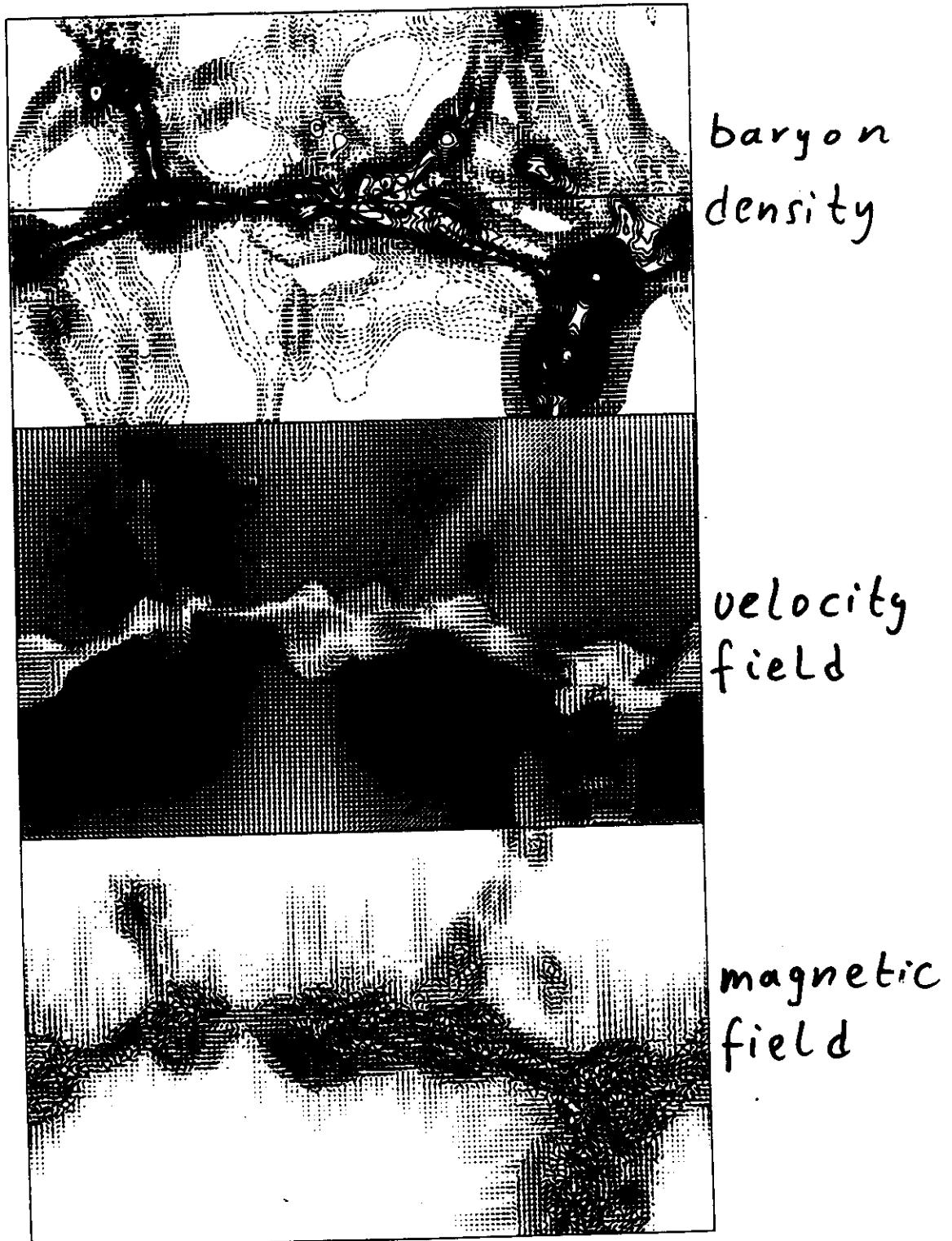
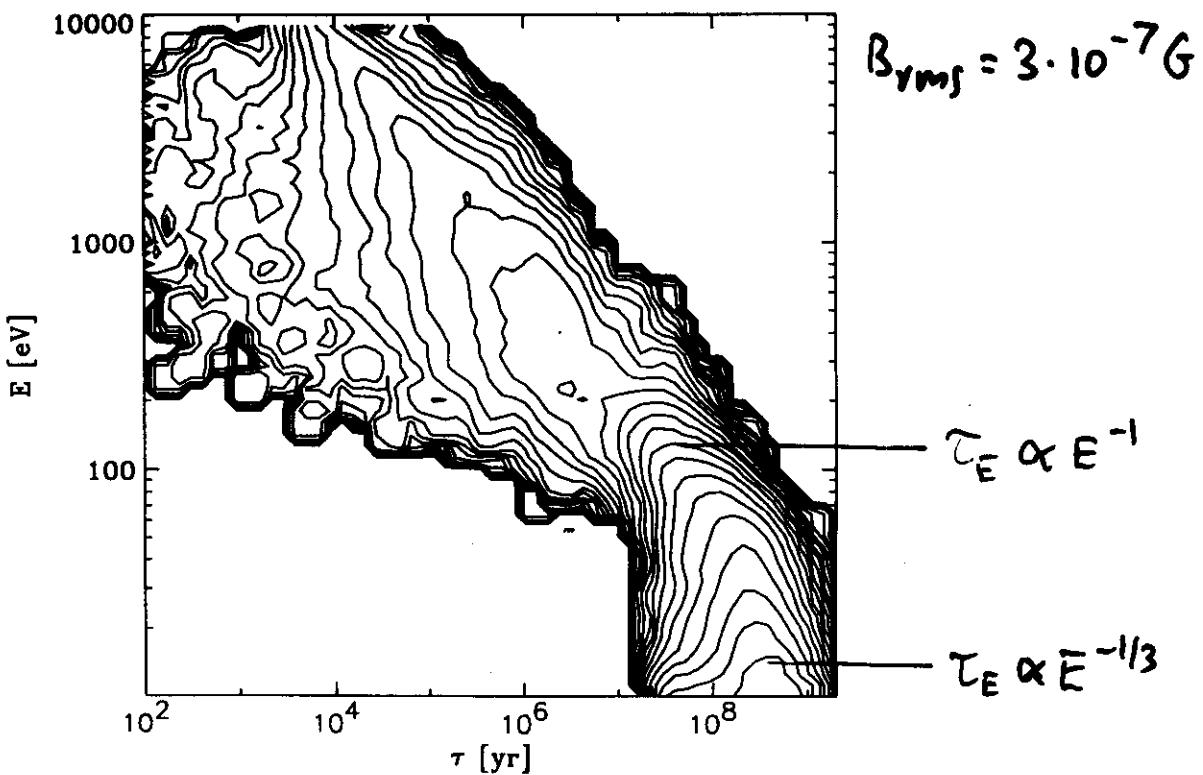
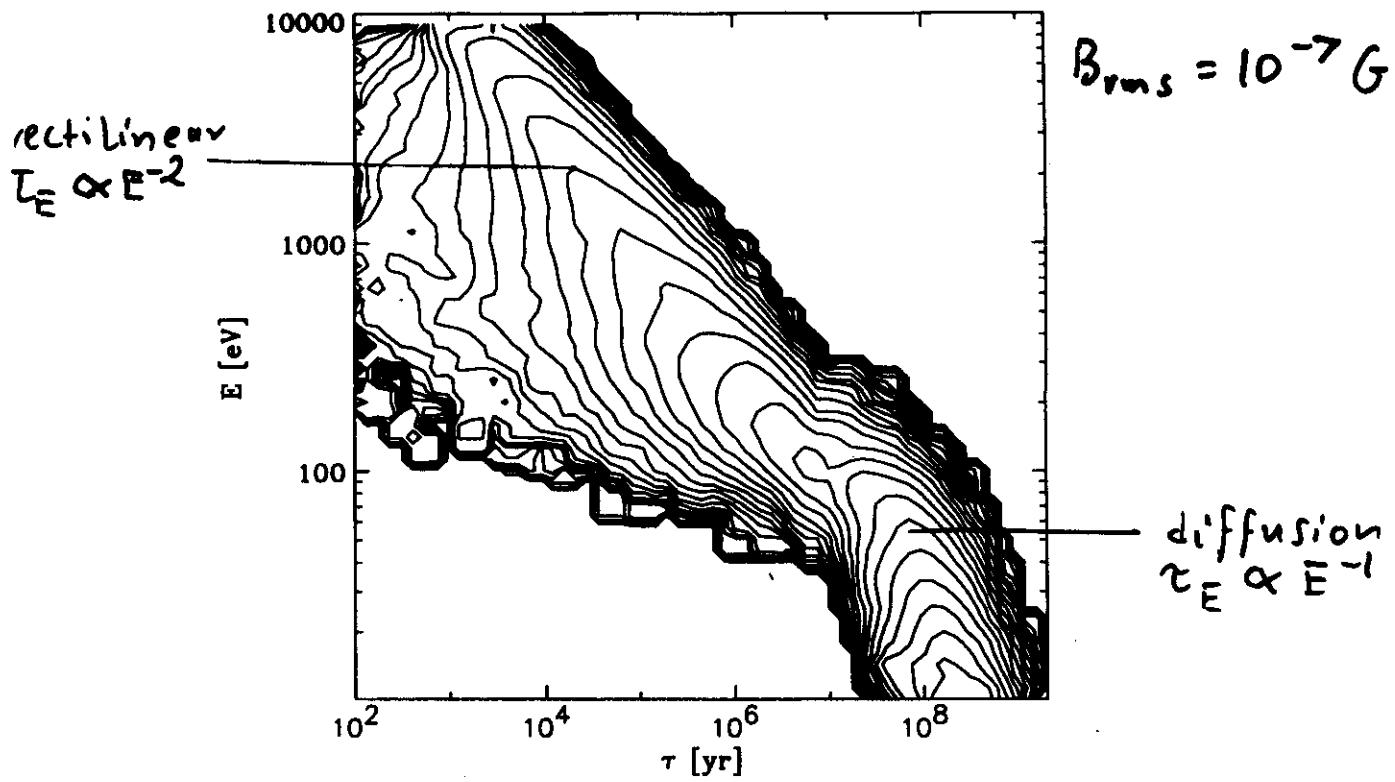


Fig. 1. Two-dimensional cut of the simulated universe at $z = 0$. The plot shows a region of $32h^{-1} \times 20h^{-1} \text{Mpc}^2$ with a thickness of $0.25h^{-1} \text{Mpc}$, although the simulation was done in a box of $(32h^{-1} \text{Mpc})^3$ volume. The first panel shows baryonic density of $\rho_b \geq \bar{\rho}_b$ and the dotted lines contour those of $10^{-1.2} \bar{\rho}_b \leq \rho_b < \bar{\rho}_b$. In the third panel, the vector length is proportional to the log of magnetic field strength.

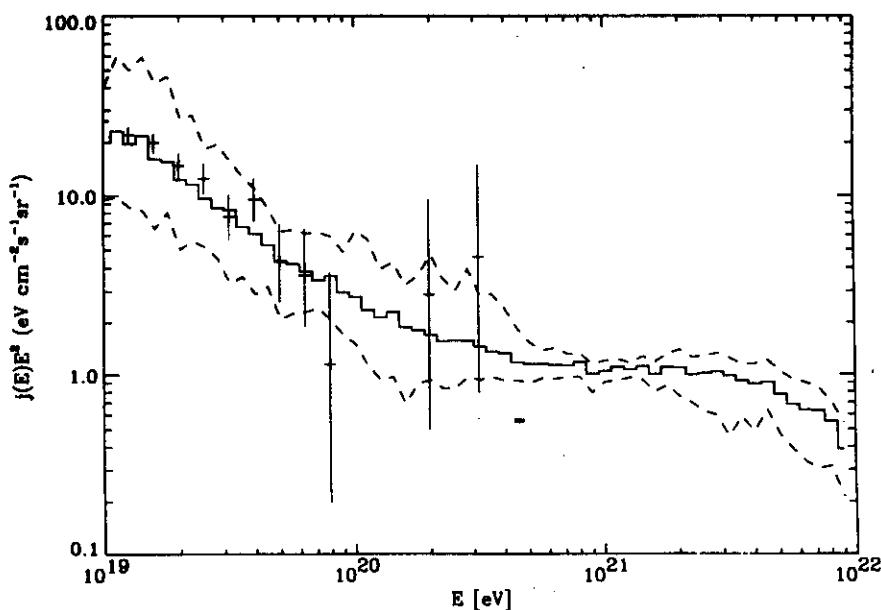
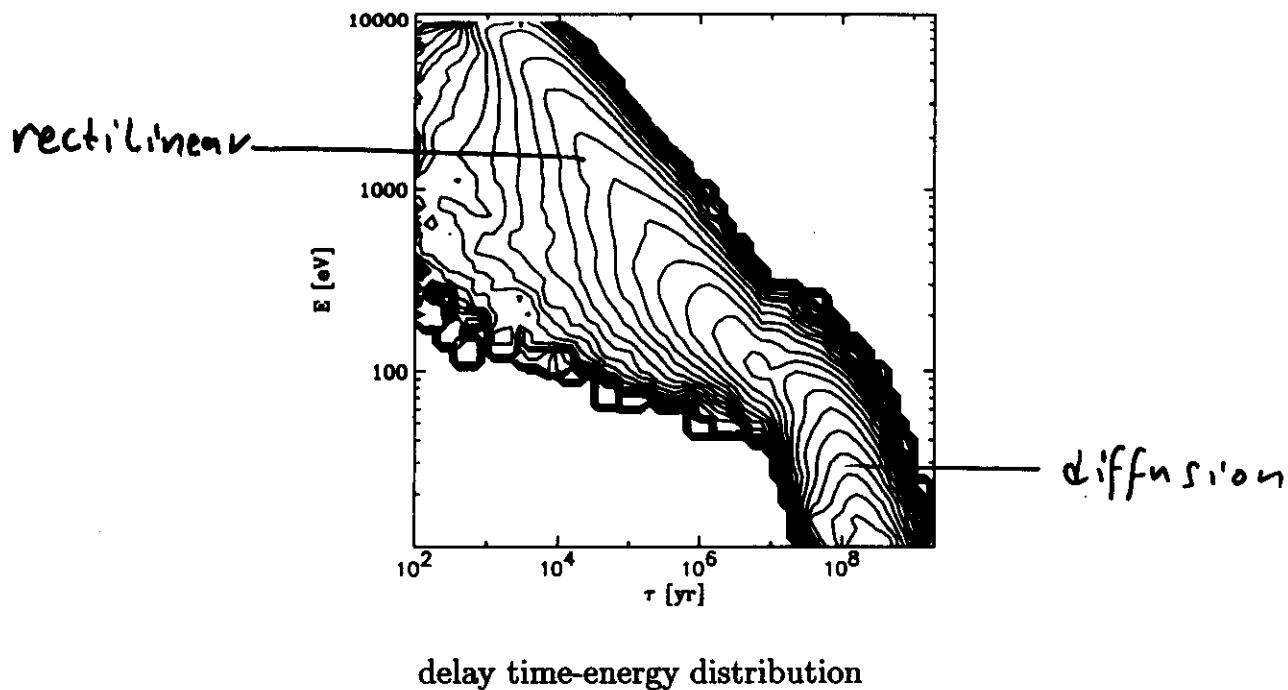
$D = 10 \text{ Mpc}$



Supergalactic plane as a sheet with Gaussian profile, width=10 Mpc

Kolmogorov field with $B_{\text{rms},\text{max}} = 10^{-7}$ G, $L_{\text{max}} \simeq 10$ Mpc, $l_c \simeq 1$ Mpc

continuous source with $T_S \gg \tau_{10} \sim 2 \times 10^8$ yr, $j(E) \propto E^{-2.4}$ up to 10^{22} eV, $D = 10$ Mpc

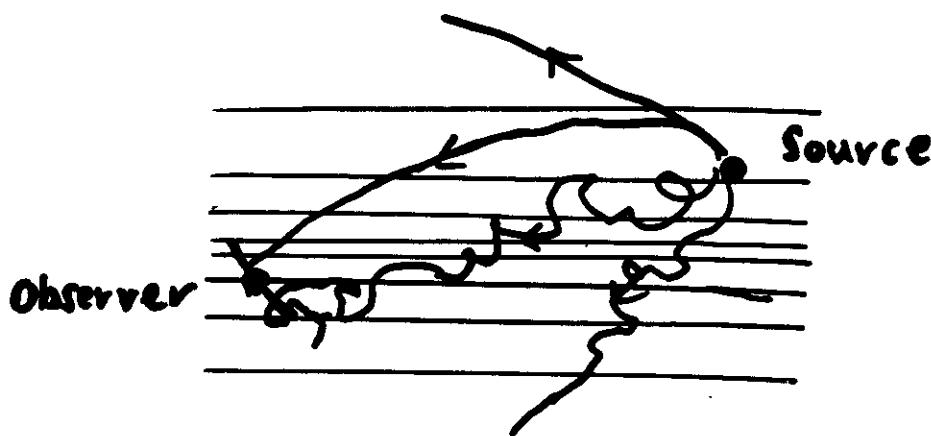


average spectrum and variance: Higher and lower fields produce much poorer fits

Simple Model:

Supergalactic plane as a sheet with thickness $\approx 10 \text{ Mpc}$, typically Gaussian profile
B-field has a Kolmogorov spectrum with $B_{\text{rms,max}}$ varied, $L_{\text{max}} \approx 10 \text{ Mpc}$, $L_c \approx 1 \text{ Mpc}$, weighted by density profile

3-field realizations represented on a spatial grid via Fourier transformation



propagate many particles for different realizations
record energies E , delay times τ_E , angles θ, φ

typically continuous source, injection spectrum
 $\Phi(E) \propto E^{-2.4}$ up to $\approx 10^{21} - 10^{22} \text{ eV}$, then
roll off

Important Influences / Effects :

- almost rectilinear propagation

$$\tau_{\text{rec}} \ll D$$

$$E \gtrsim 10^{20} \left(\frac{D}{10 \text{ Mpc}} \right)^{1/2} \left(\frac{B_{\text{rms}}}{10^{-7} \text{ G}} \right) \left(\frac{l_c}{1 \text{ Mpc}} \right) \text{ eV}$$

then $j(E) \propto \frac{\Phi(E)}{4\pi D^2} \lambda(E)$

- Opposite limit : diffusive regime

$$j(E) \propto \Phi(E) \tau_E \propto \Phi(E) / D(E)$$

in equilibrium and up to energy losses

- energy losses, especially pion production (GZK effect)

tends to increase low energy flux,
especially if diffusive regime reaches
beyond $E_{\text{GZK}} \simeq 7 \cdot 10^{19} \text{ eV}$

- inhomogeneity suppresses low energy flux within strong field regions (escape?)
mostly dependent on D , inhomogeneity length scale and contrast, less on profiles

Approximation in diffusive regime

Solve diffusion-continuous energy loss equation

$$\partial_t n(\tilde{v}, E, t) = + \vec{\nabla} \cdot [D(\tilde{v}, E) \vec{\nabla} n(\tilde{v}, E, t)] - \frac{d}{dt} \Phi(E) n(\tilde{v}, E, t)$$
$$+ \Phi(\tilde{v}, E, t)$$

partial differential equation

integrate over age of Universe $\simeq 0(10^{10} \text{ yr})$

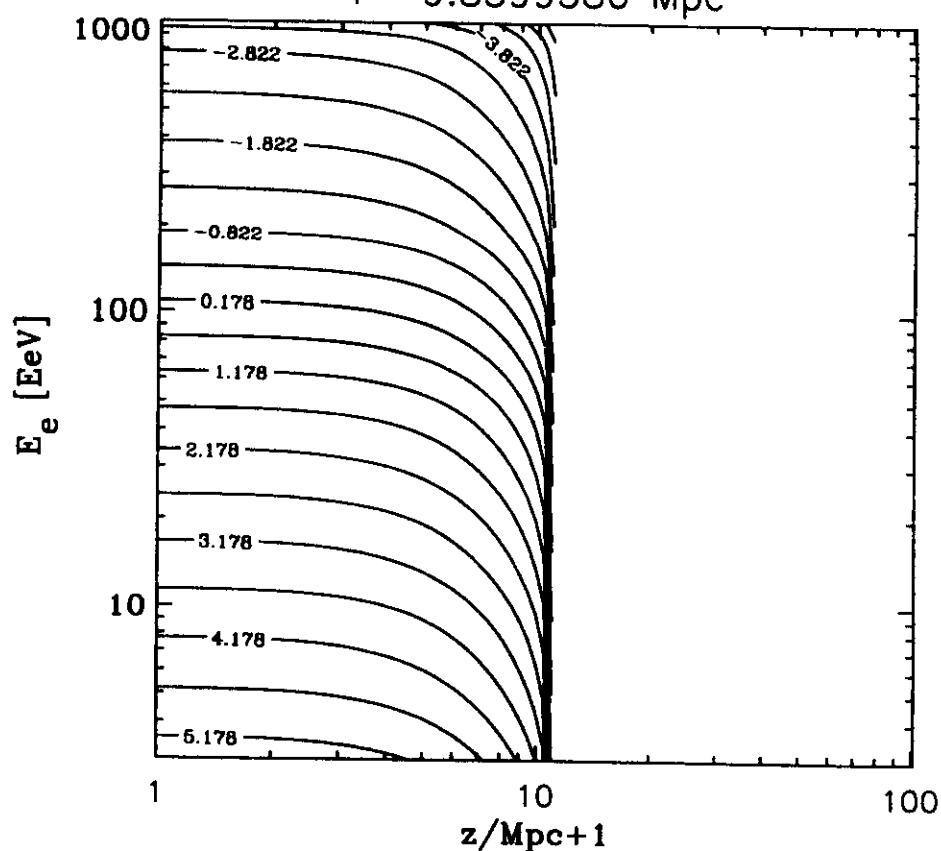
→ Usually reach equilibrium if $\Phi(t)$ and
 $T_E \lesssim 10^{10} \text{ yr}$

- analytical (Syrovatsky) solution if homogeneous $\rightarrow P.B.$
- otherwise numerical

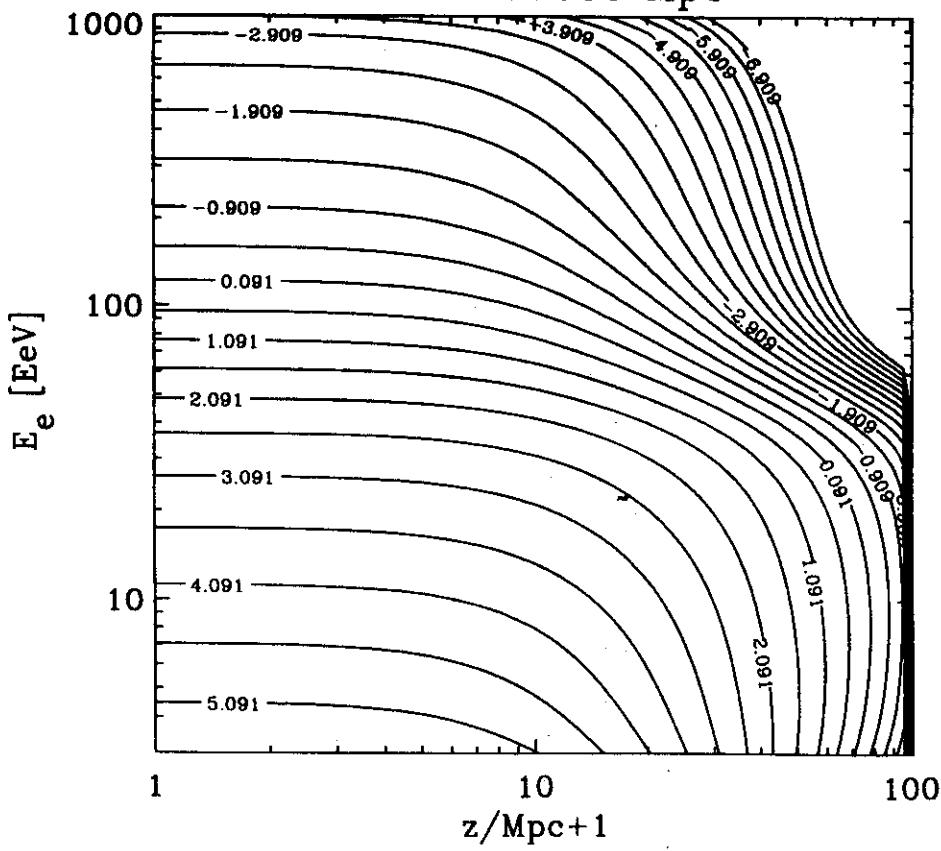
Warning: pion production is stochastic, not continuous, especially for $D \lesssim 10 \text{ Mpc}$

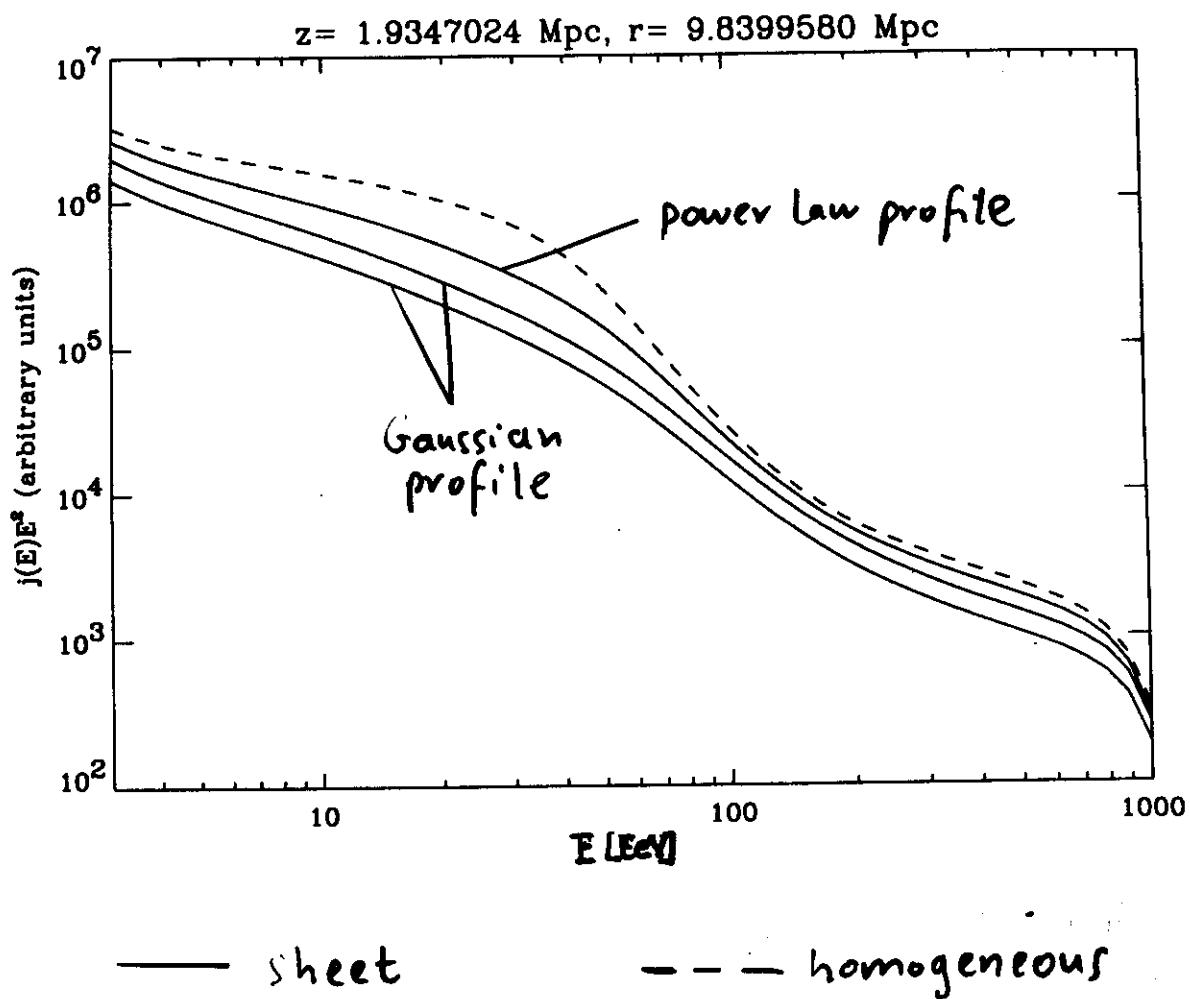
$$B_{\text{rms}} = 5 \cdot 10^{-8} \text{ G}$$

$$r = 9.8399580 \text{ Mpc}$$



$$r = 9.8399580 \text{ Mpc}$$



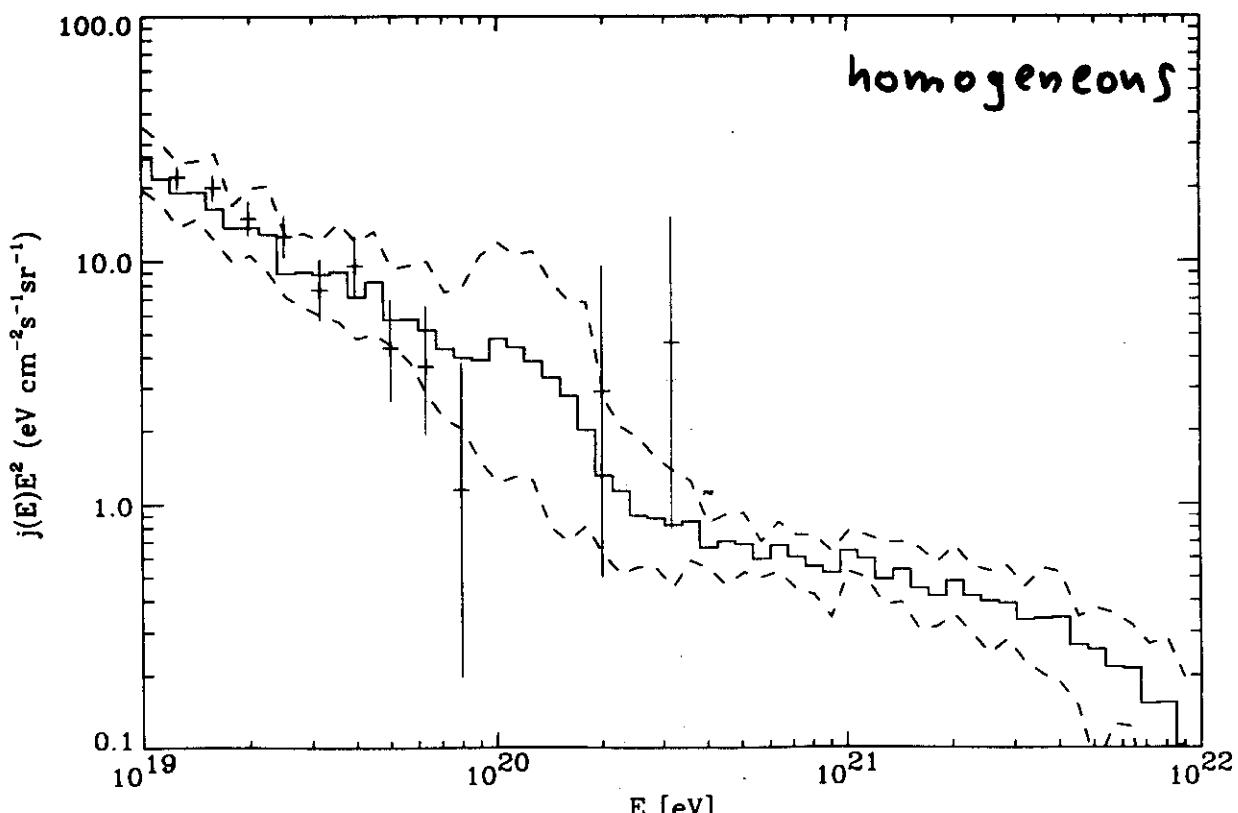
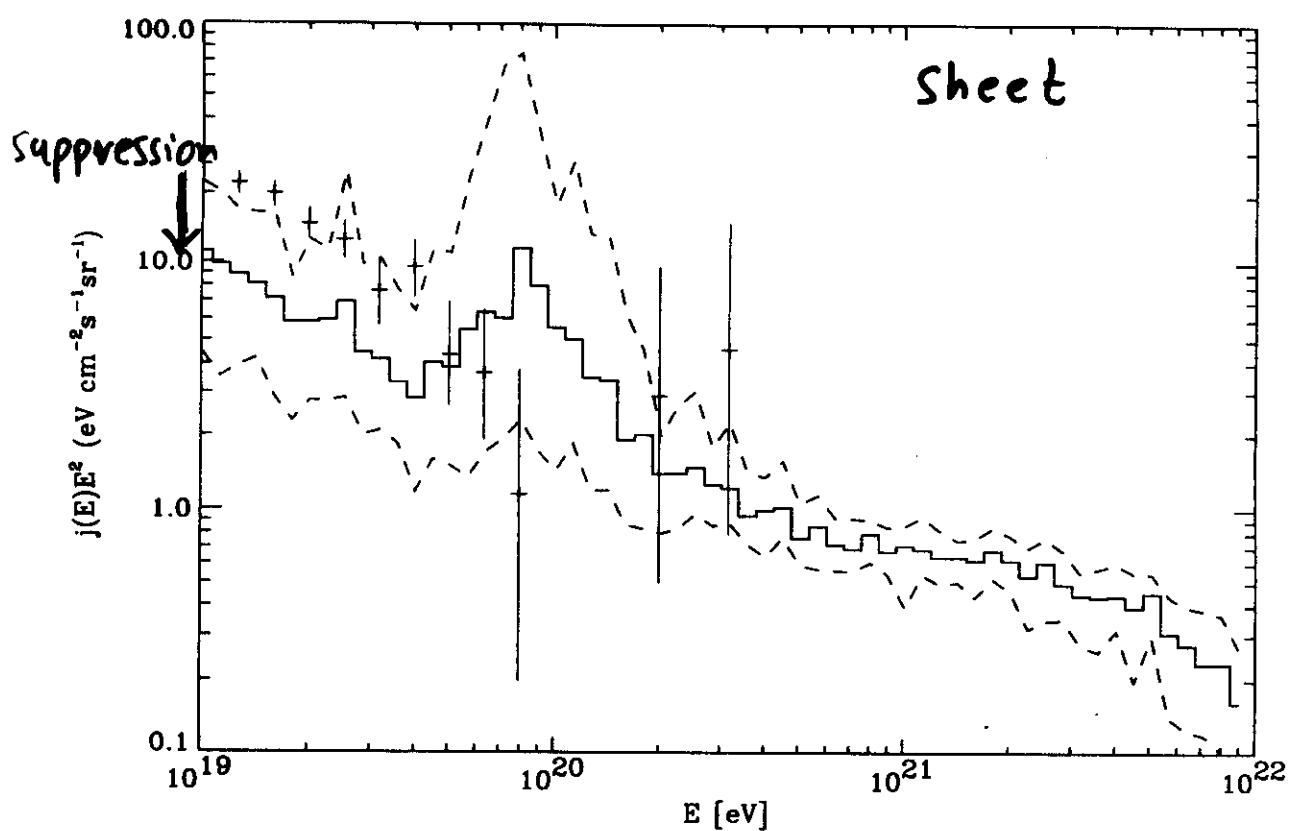


Solution of diffusion - continuous energy loss equation

$$B_{\text{rms}} = 5 \cdot 10^{-8} \text{ G}, D \approx 10 \text{ Mpc}$$

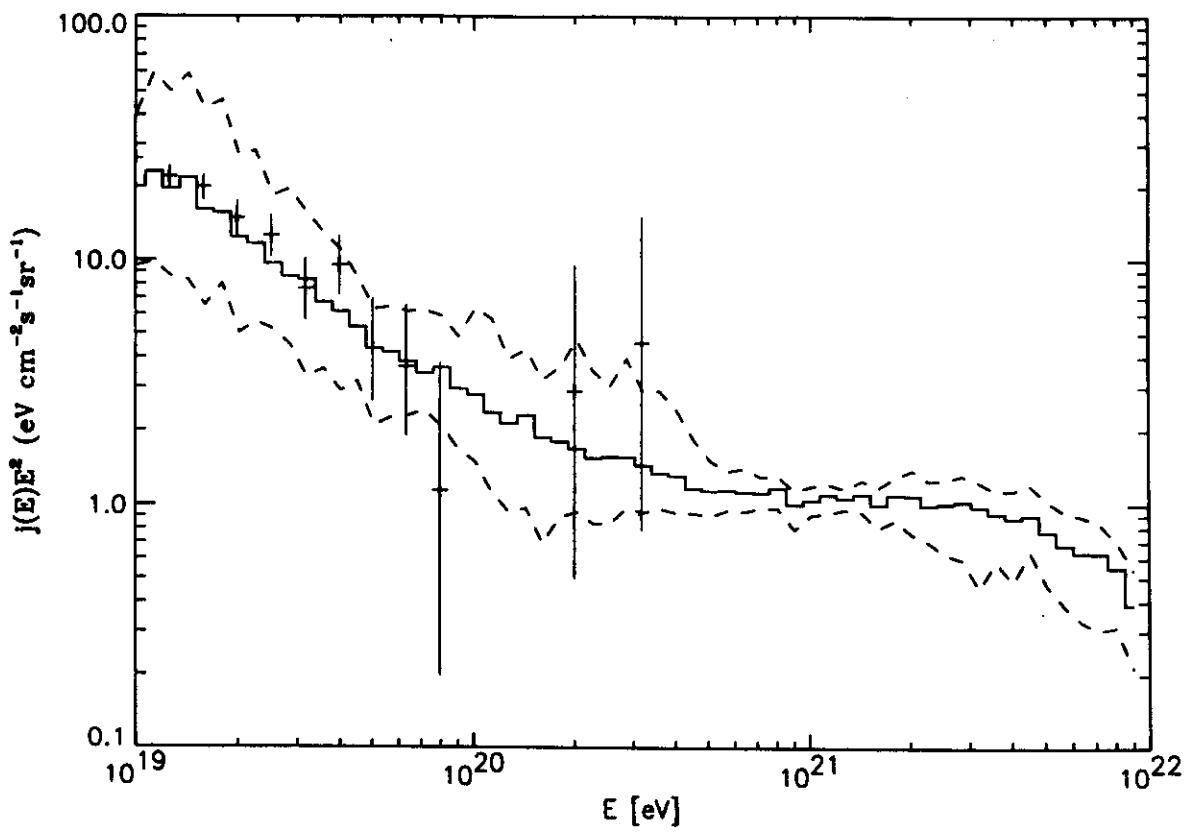
confirms suppression

$$D = 10 \text{ Mpc} \quad B_{\text{rms}} = 5 \cdot 10^{-8} \text{ G}$$



— average - - - standard deviation

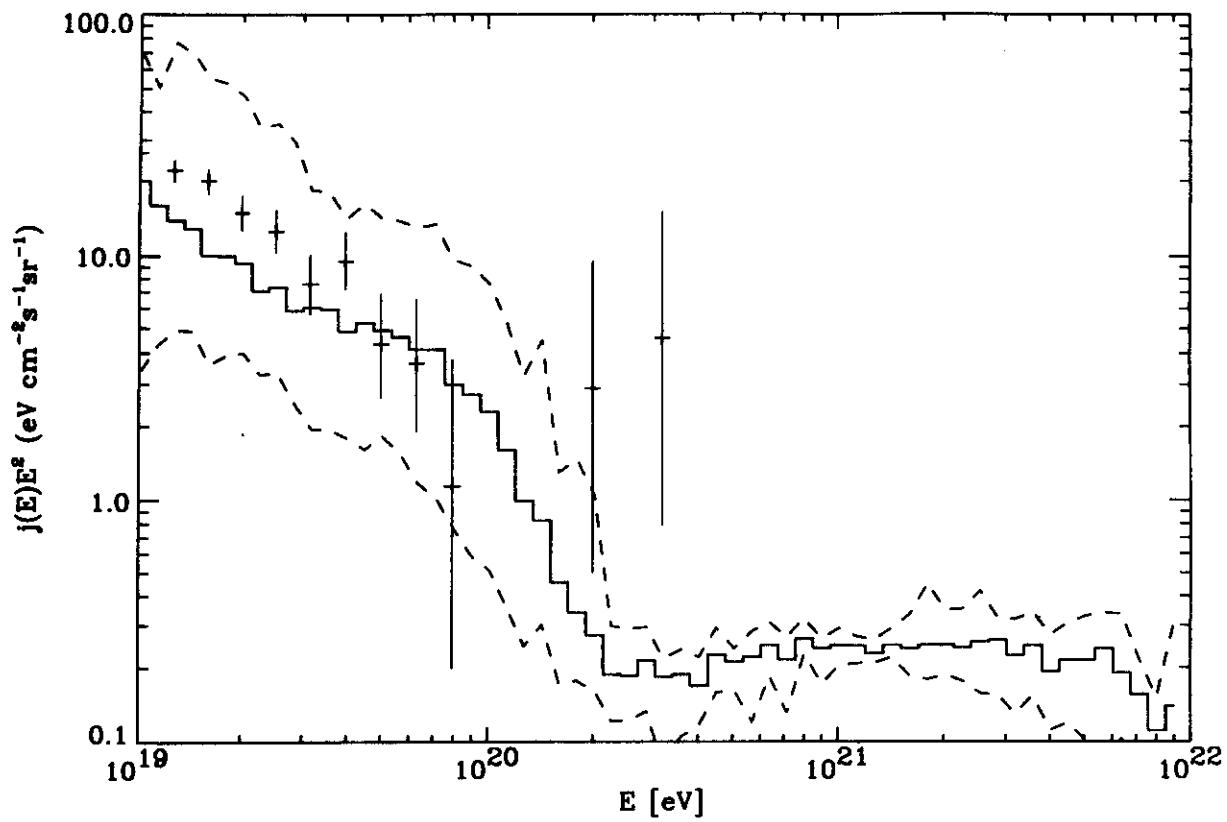
$$D = 10 \text{ Mpc} \quad B_{\text{rms}} = 10^{-7} \text{ G}$$



low energy depletion from sheet compensated
by diffusively enhanced pion production

\implies best fit on average

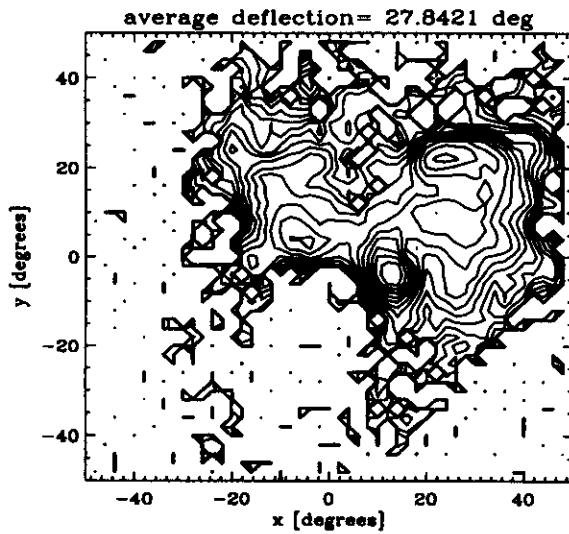
$$D = 10 \text{ Mpc} \quad B_{rms} = 3 \cdot 10^{-7} \text{ G}$$



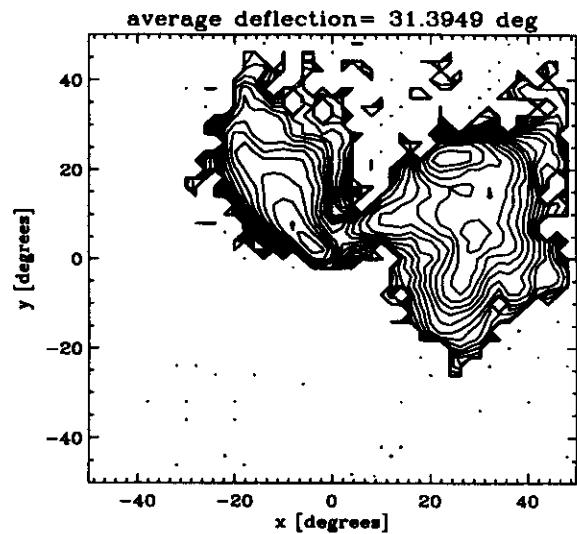
Low energy depletion from sheet overcompensated
by diffusively enhanced pion production

$$B_{\text{rms},\text{max}} = 5 \times 10^{-8} \text{ G}$$

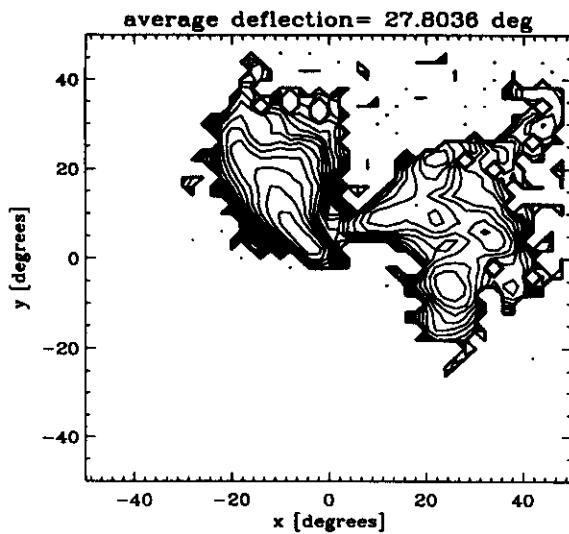
continuous source with $T_S \gg \tau_{10} \sim 10^8 \text{ yr}$, $j(E) \propto E^{-2.4}$ up to 10^{22} eV , $D = 10 \text{ Mpc}$



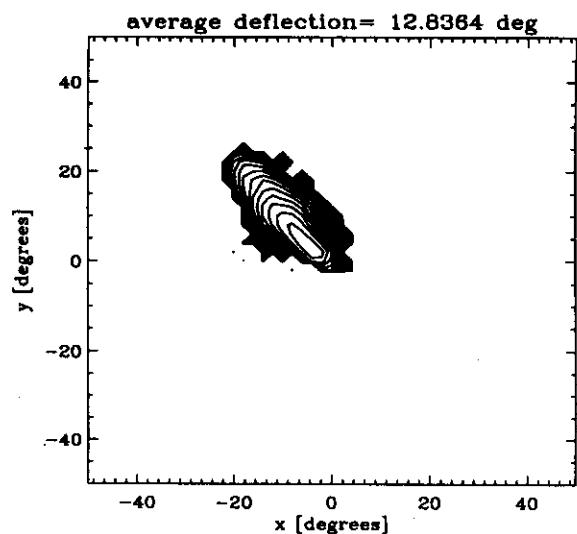
$$E > 30 \text{ EeV}$$



$$E > 60 \text{ EeV}$$

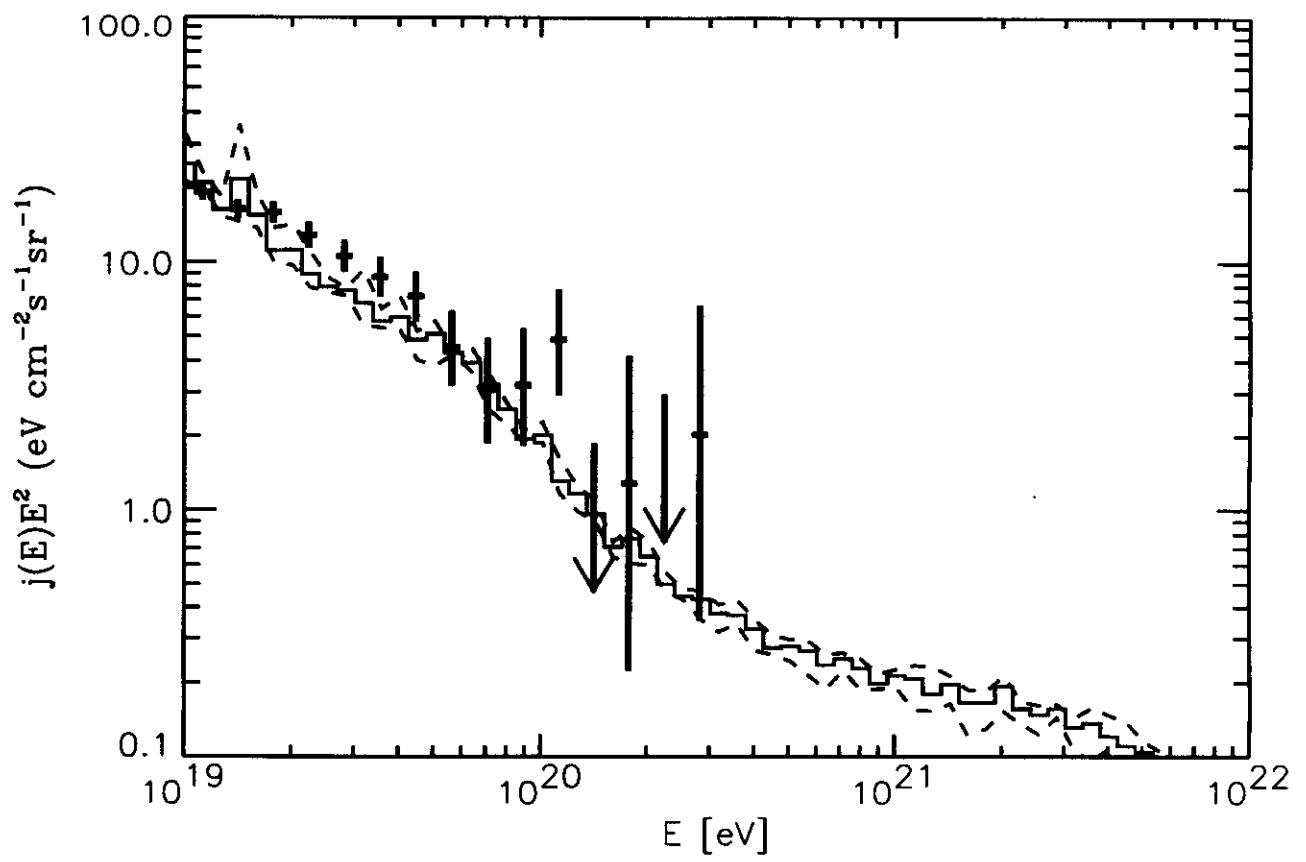


$$E > 100 \text{ EeV}$$



$$E > 200 \text{ EeV}$$

$$D\theta_E/l_c \simeq 1 \text{ at } E \simeq 150 \text{ EeV} \implies l_c \text{ can be estimated}$$



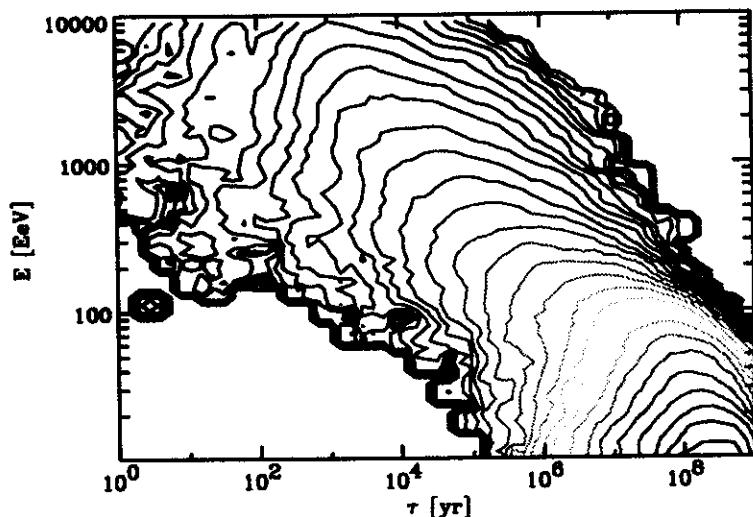
Supergalactic plane as a sheet with Gaussian profile, width=10 Mpc

Kolmogorov field with $B_{\text{rms,max}} = 0.05 - 0.5 \mu\text{G}$, $L_{\text{max}} \simeq 10 \text{ Mpc}$, $l_c \simeq 1 \text{ Mpc}$

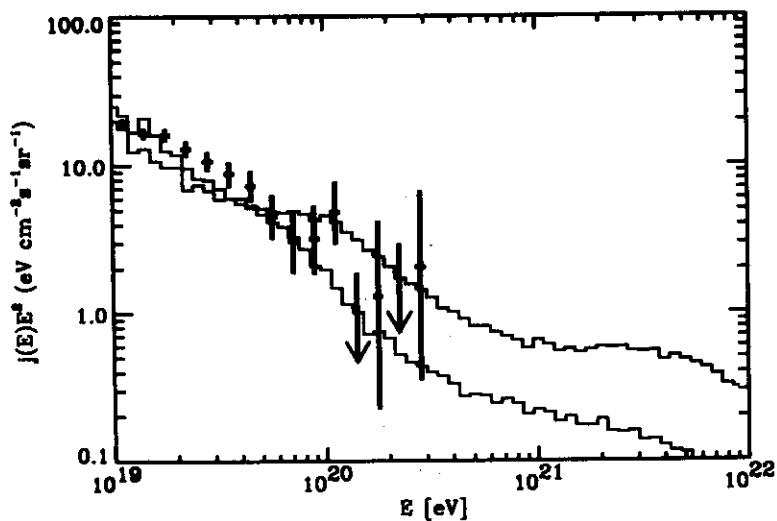
continuous source distribution following plane profile

with $T_S \gg \tau_{10} \sim 2 \times 10^8 \text{ yr}$, $j(E) \propto E^{-2.4}$ up to 10^{22} eV

distance to center (Virgo) = 20 Mpc



delay time-energy distribution

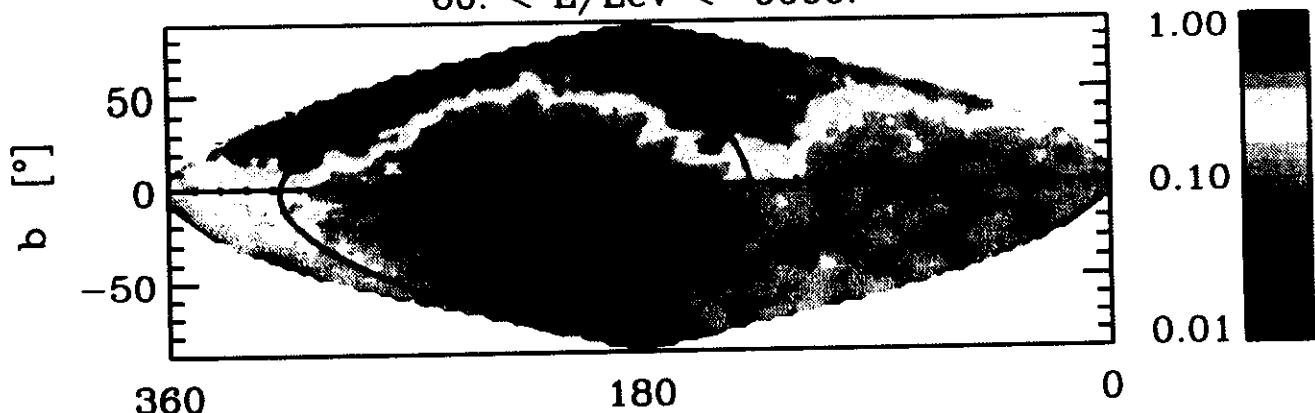


average spectrum: $B_{\text{rms,max}} = 0.05/0.5 \mu\text{G}$ (blue/red)

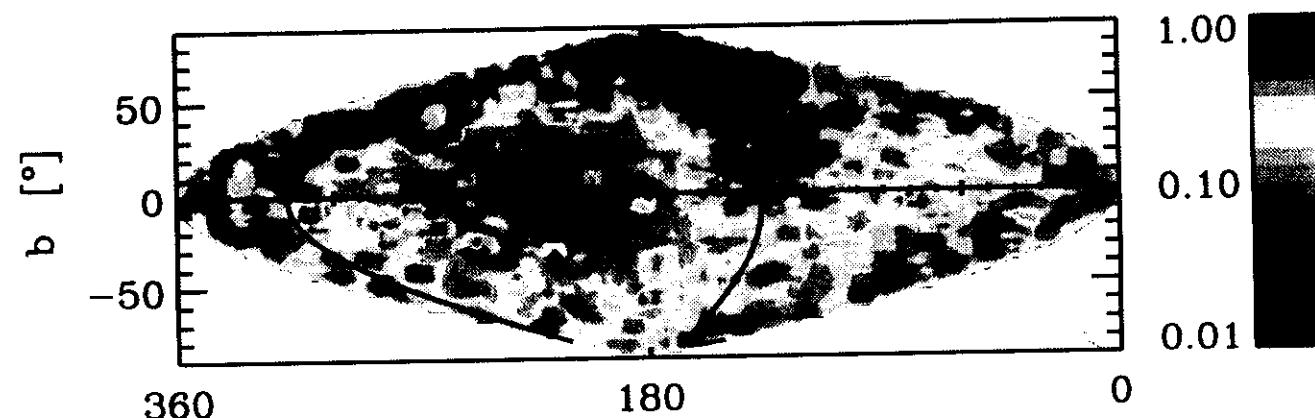
cosmic variance negligible

Realization averaged angular distributions

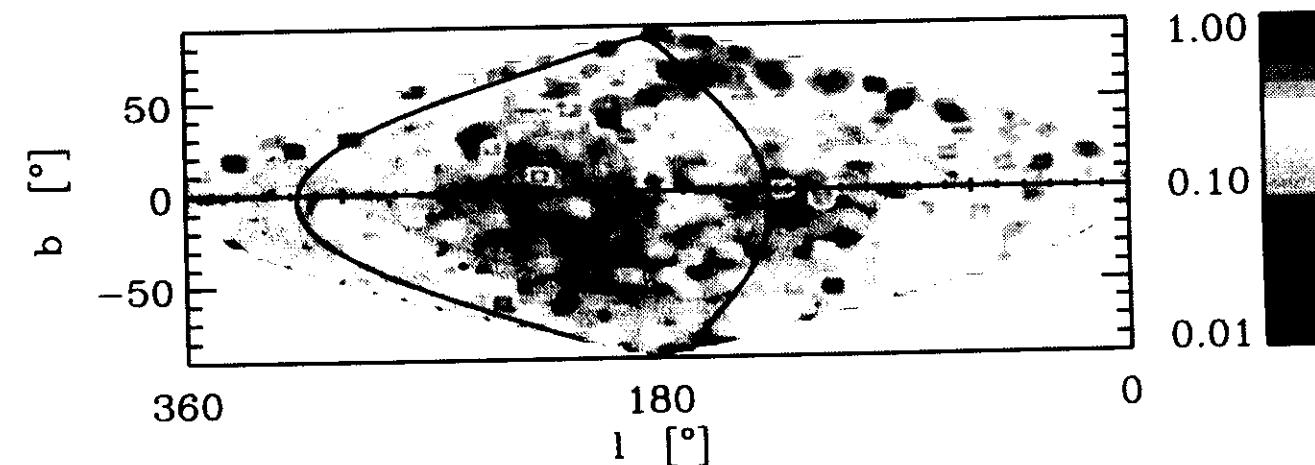
$60. < E/\text{EeV} < 9000.$



$B_{\text{rms,max}} = 0.05 \mu \text{G}$, 20 Mpc source distribution radius

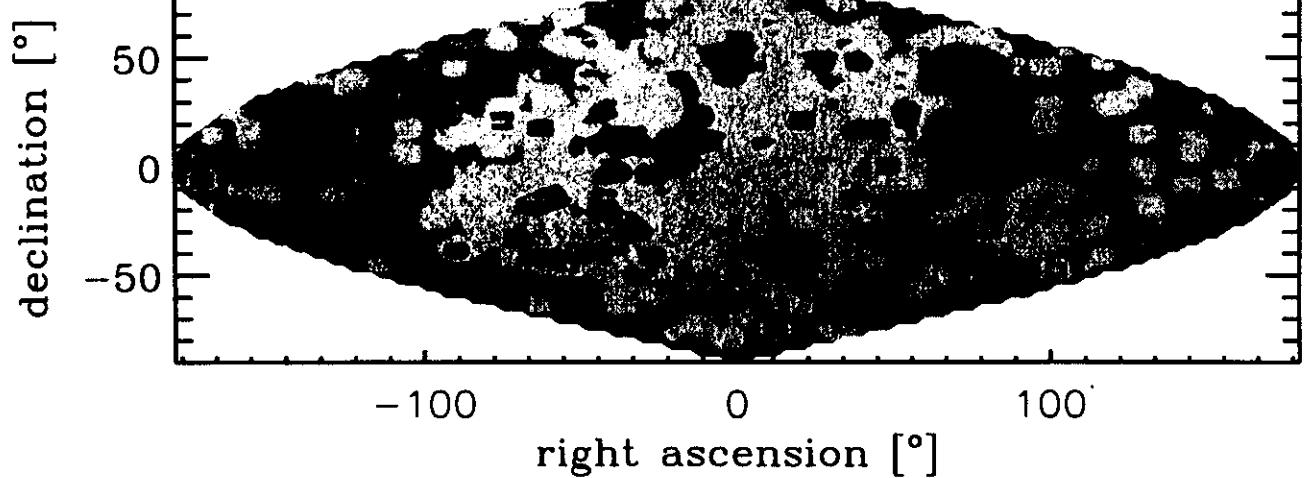


$B_{\text{rms,max}} = 0.05 \mu \text{G}$, ∞ source distribution radius

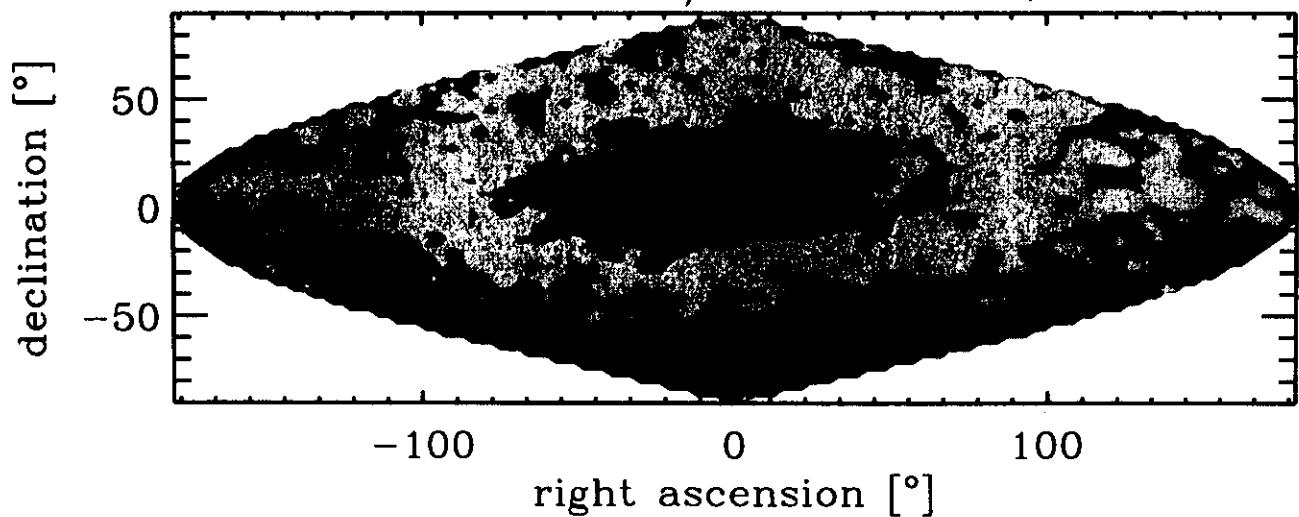


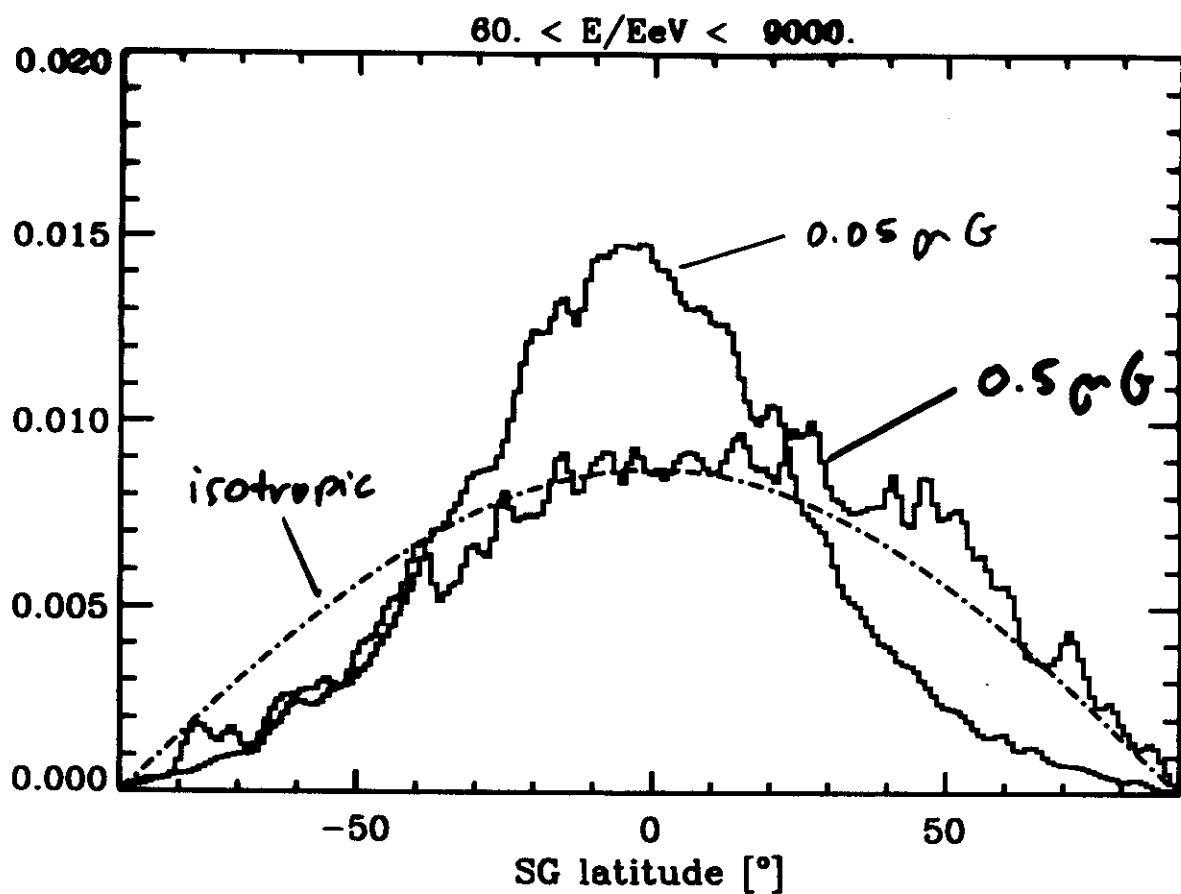
$B_{\text{rms,max}} = 0.5 \mu \text{G}$, 20 Mpc source distribution radius

$50. < E/E\text{eV} < 9000.$



$500. < E/E\text{eV} < 9000.$





Realization averaged Supergalactic latitude distribution

for $B_{\text{rms,max}} = 0.05/0.5 \mu \text{G}$ (blue/red)

Consequences:

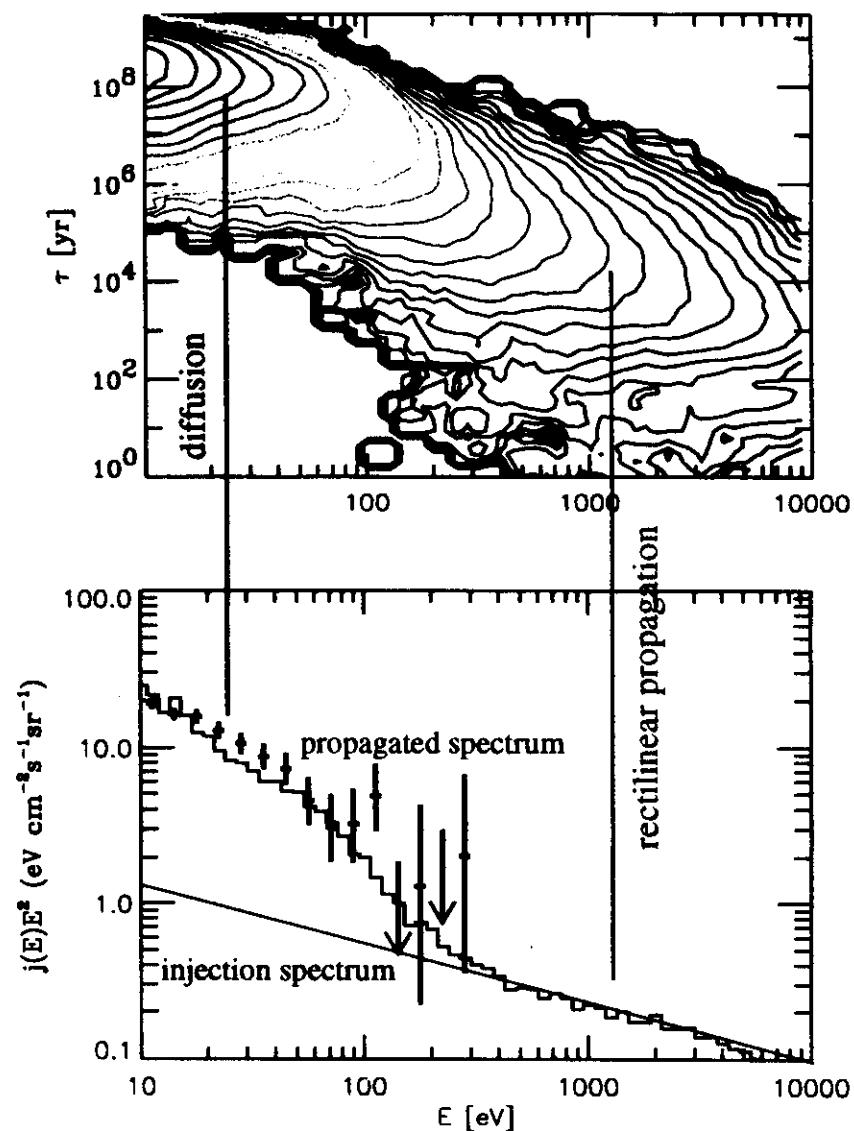
Anisotropy decreases with increasing magnetic field strength due to diffusion
and with increasing source distribution radius

Clustering increases with coherence and strength of magnetic field
due to magnetic lensing and with increasing source distribution radius

Confidence levels strongly increase with exposure

Monte Carlo simulations of sources in a magnetized Supergalactic Plane

Kolmogorov spectrum with maximal field strength of 0.5 micro Gauss in plane center



Angular distribution explains large scale isotropy (by diffusion) and small scale clustering (by magnetic lensing):

