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SUMMER SCHOOL ON ASTROPARTICLE PHYSICS AND COSMOLOGY

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FINITE TEMPERATURE FIELD THEORY AND PHASE TRANSITIONS

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Please note: These are preliminary notes intended for internal distribution only.

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FINITE TEMPERATURE FIELD THEORY

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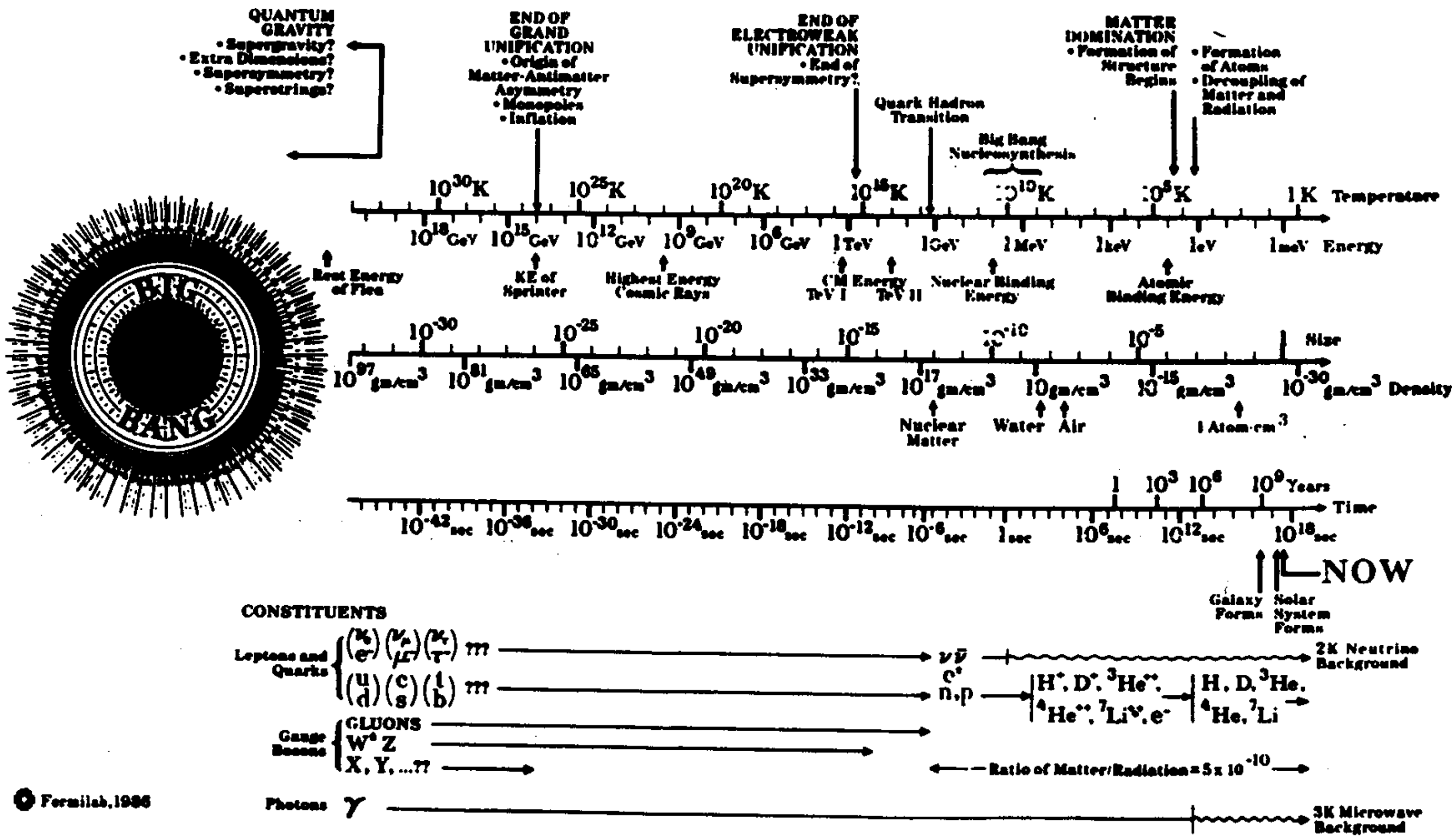
Thermal History of the Universe



Transitions we shall be concerned with

Energy/T_c scale	Symmetry	order parameter	"Toy" models
EW phase Transition ~ 100 GeV	$SU(2)\times U(1)_Y \to U(1)_{em}$	$\langle\phi\rangle$	"$\lambda\phi^4$", simple scalar gauge models
QCD chiral ~ 100 MeV	$SU(N_f)\times SU(N_f) \to SU(N_f)$	$\langle\bar\psi\psi\rangle$	$(\bar\psi\psi - \bar\psi\gamma_5\psi)^2$, O(4) sigma model
QCD confining ~ 100 MeV	No symm. change	No order param	Pure Yang Mills

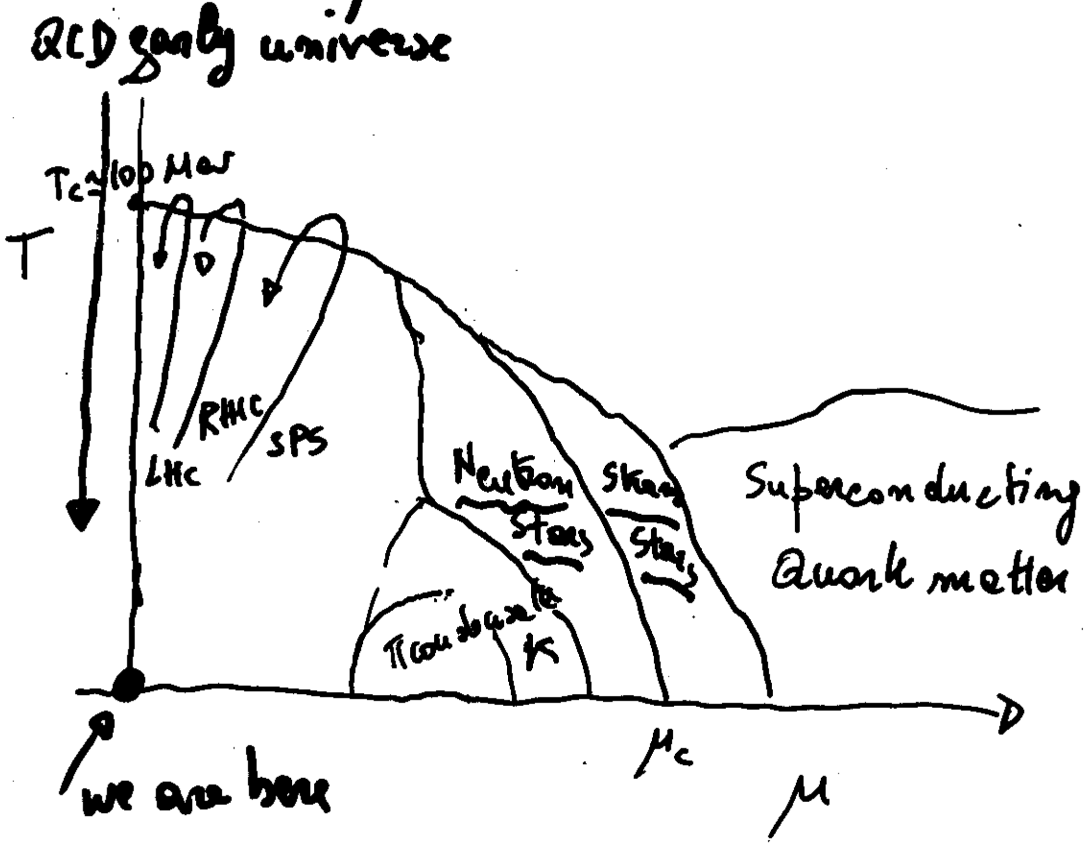
- **Main points:**

I. Universe "almost" at equilibrium while crossing T_c

II. Density very high, but
particles $\simeq$ # antiparticles

We mostly study $\mu = 0$, $T > 0$
equilibrium field theory

However, $\mu \neq 0$ important too:



Sketchy QCD phase diagram

Plan

0. Basics on $T>0$, $\mu\geq 0$ field theory

I. Critical Phenomena: a brief survey

II. Toy Electroweak : $\lambda\phi^4$

- Symmetry restoration at high T

III. Toy QCD : $\times$: 4-fermion $(\bar{\psi}\psi)^2$

- Phase diagram in the T,μ plane

IV. QCD :

- The lattice
- The phase diagram in the T,μ plane $g=\infty$
- Universality and deconfinement [Yang Mills]
- Universality and chiral transition

V. EW

- 4d perturbative analysis
- 3d reduced theory, and lattice
- 4d on the lattice

- Basic facts on

$T>0, \mu \geq 0$ Field Theory at equilibrium

- Most important function

$$\mathcal{Z} = \mathcal{Z}(V, T, \mu):$$

Grand Canonical Partition function

- $\mathcal{Z}$ fixes the system's state:

$$P = T \frac{\partial \ln \mathcal{Z}}{\partial V} \qquad N = T \frac{\partial \ln \mathcal{Z}}{\partial \mu}$$

$$S = \frac{\partial (T \ln \mathcal{Z})}{\partial T} \qquad E = -PV + TS + \mu N$$

- $\mathcal{Z} = \mathrm{Tr}\, \hat{\rho}$

$$\hat{\rho} = e^{-(H - \mu \hat{N})/T}$$

Functional Integral Representation of Z

- Consider first the transition amplitude for returning to the original state $|\phi_a\rangle$ after time t:

$$\langle \phi_a | e^{-iHt} | \phi_a \rangle = \int [d\pi] \int_{\phi(x,0)=\phi_a(x)}^{\phi(x,t)=\phi_a(x)} [d\phi] \times$$

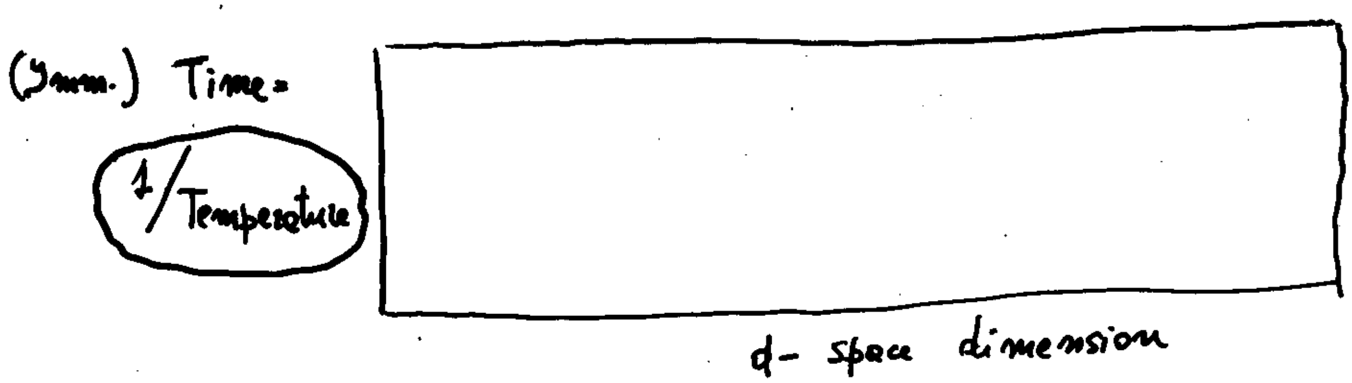
$$\exp\left[i \int_0^t dt \int d^3x \left(\pi(\vec{x},t) \frac{\partial \phi(\vec{x},t)}{\partial t} - \mathcal{H}(\pi,\phi) \right) \right]$$

- Compare with

$$Z = \mathrm{Tr}\, e^{-\beta(H-\mu N)} = \int d\phi_a \langle \phi_a | e^{-\beta(H-\mu N)} | \phi_a \rangle$$

$$\boxed{\beta \equiv \frac{1}{T} \Rightarrow i t}$$

- The partition function $\mathcal{Z}$ has the interpretation of the partition function of statistical field theory in $d+1$ dimensions:



- Introduce the integral of the lagrangian density:

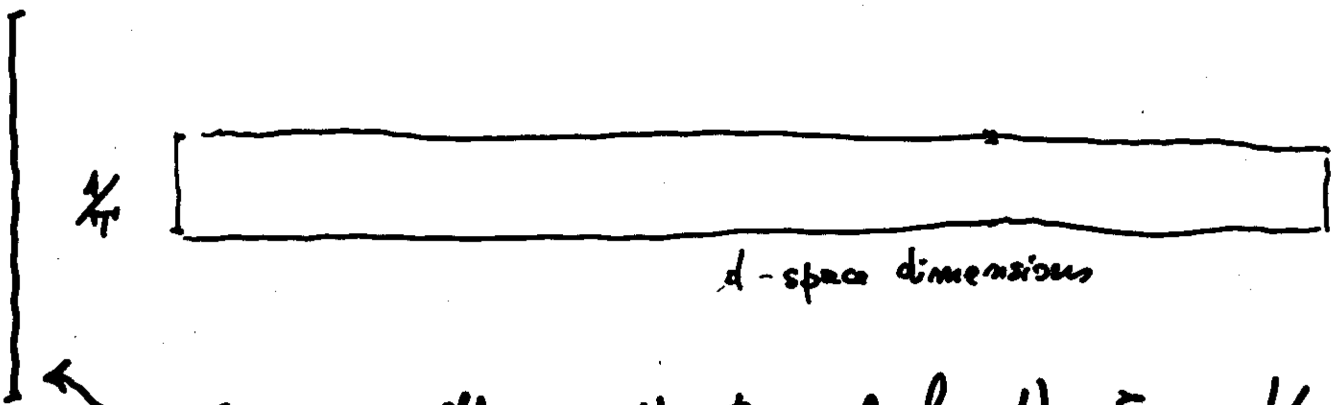
$$S(\phi,\psi) = \int_0^{1/T} d\tau \int d^d x \, \mathcal{L}(\overset{\text{bosons}}{\phi},\underset{\text{fermions}}{\psi})$$

$$\boxed{\mathcal{Z} = \int [d\phi][d\psi] \exp\left[-S(\phi,\psi)\right]}$$

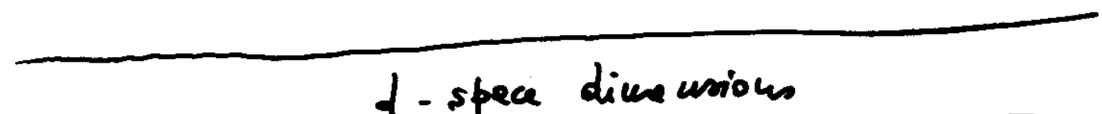
$$\underbrace{\phi(\tau=0,x) = \phi(\tau=1/T,x)}_{\text{periodic b.c for bosons}} \; ; \; \underbrace{\psi(\tau=0,x) = -\psi(\tau=1/T,x)}_{\text{antiperiodic b.c. for fermions}}$$

The idea of

Dimensional Reduction



Suppose this is the typical length $\xi \gg 1/T$:
the system is "effectively" d-dimensional:



- When:
  1. T "very" high $\gg$ any mass
  2. Close to a 2^{nd} (or higher order transition):
     $\xi \to \infty$! Short distance physics/heavy modes are not important.

POWER OF SYMMETRIES !!! [Peter on

- Note: Bosons and Fermions <u>ARE</u> different! [next slide]

Consider thermal Green functions describing propagation from $(\vec{y}, t=0)$ to $(\vec{x}, t=\tau)$

BOSONS:

$$G_B(\vec{x},\vec{y};\tau,0) = \mathrm{Tr}\left\{\hat{\rho}\,\underline{T_\tau}\left[\hat{\phi}(\vec{x},\tau)\hat{\phi}(\vec{y},0)\right]\right\}/Z$$

$\underline{T_\tau}$: imaginary time ordering operator

$$T_\tau\left[\hat{\phi}(\tau_1)\hat{\phi}(\tau_2)\right] = \hat{\phi}(\tau_1)\hat{\phi}(\tau_2)\theta(\tau_1-\tau_2) + \hat{\phi}(\tau_2)\hat{\phi}(\tau_1)\theta(\tau_2-\tau_1)$$

Use: $[T_\tau, e^{-\beta H}] = 0$; $e^{\beta H}\phi(y,0)e^{-\beta H} = \phi(y,\beta)$

Heisenberg time evolution

To get:

$$G_B(\vec{x},\vec{y};\tau,0) = G_B(\vec{x},\vec{y};\tau,\beta) \Rightarrow \boxed{\hat{\phi}(\vec{x},0) = \hat{\phi}(\vec{x},\beta)}$$

FERMIONS:

$$T_\tau\left[\hat{\psi}(\tau_1)\hat{\psi}(\tau_2)\right] = \hat{\psi}(\tau_1)\hat{\psi}(\tau_2)\theta(\tau_1-\tau_2) - \hat{\psi}(\tau_2)\hat{\psi}(\tau_1)\theta(\tau_2-\tau_1)$$

$$G_F(\vec{x},\vec{y},\tau,0) = -G_F(\vec{x},\vec{y};\tau,\beta) \Rightarrow \boxed{\hat{\psi}(\vec{x},0) = -\hat{\psi}(\vec{x},\beta)}$$

Mode Expansion and Decoupling

- **Bosons** : Periodic

$$\phi(x,t) = \sum' e^{i\omega_n t}\, \phi_n(x)$$
$$\omega_n = 2n\pi T$$

- **Fermions** : Antiperiodic

$$\psi(x,t) = \sum' e^{i\omega_n t}\, \psi_n(x)$$
$$\omega_n = (2n+1)\pi T$$

Write again $S(\phi,\psi) = \int_0^{1/T} dt \int d^d x\, \mathcal{L}(\phi,\psi)$

$\int dt \to \sum$:

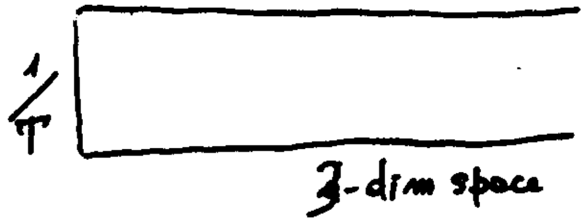
> A $d+1$ statistical field theory at $T>0$ is equivalent to a d - theory with an infinite number of fields

Q: can infinite become just 1 ?

A: YES for bosons: only mode "0" survives when dimensional reduction "works"

Finite T at a Glance

- 4-d theory at $T > 0$ is equivalent to the Euclidian (imaginary time) field theory defined on a finite time interval.



Fermion: a.p. b.c
Bosons: p. b.c

- "Dimensional reduction", when 'true' means that

  - $1/T \to 0 \Rightarrow$ System becomes 3-dim.

  - Only Fourier components of each bose field with vanishing Matsubara frequency will contribute to the dynamics.

  - Fermions 'decouple'.

- N.B.: Decoupling Scenario extremely likely, but not a theorem! Also: Dim reduction window can be very small

In practice:

Q: Which Calculational Scheme?

4-d Theory $T > 0, \mu \geq 0$

Try to go to 3-d via dim. red.

either

Pert. theory

Smart guess

$\boxed{\text{3-d theory}}$

Use Standard field theory methods

Use Standard field Theory methods

- Lattice + MC
- Perturbation Theory
- Lattice + Strong Coupling exp
- Other non perturbative ($\frac{1}{N}$ methods

A: No Unique Answer

- General Idea: Dimensional Reduction, Critical Phenomena, Universality
- Model dependent strategy

II. Critical Phenomena -
~ brief survey ~

- The 'macroscopic' view:
  - Equation of State
  - Critical Exponents

- The 'intermediate' description
  - The Effective Potential

- Amusing Things:

I. Equation of State

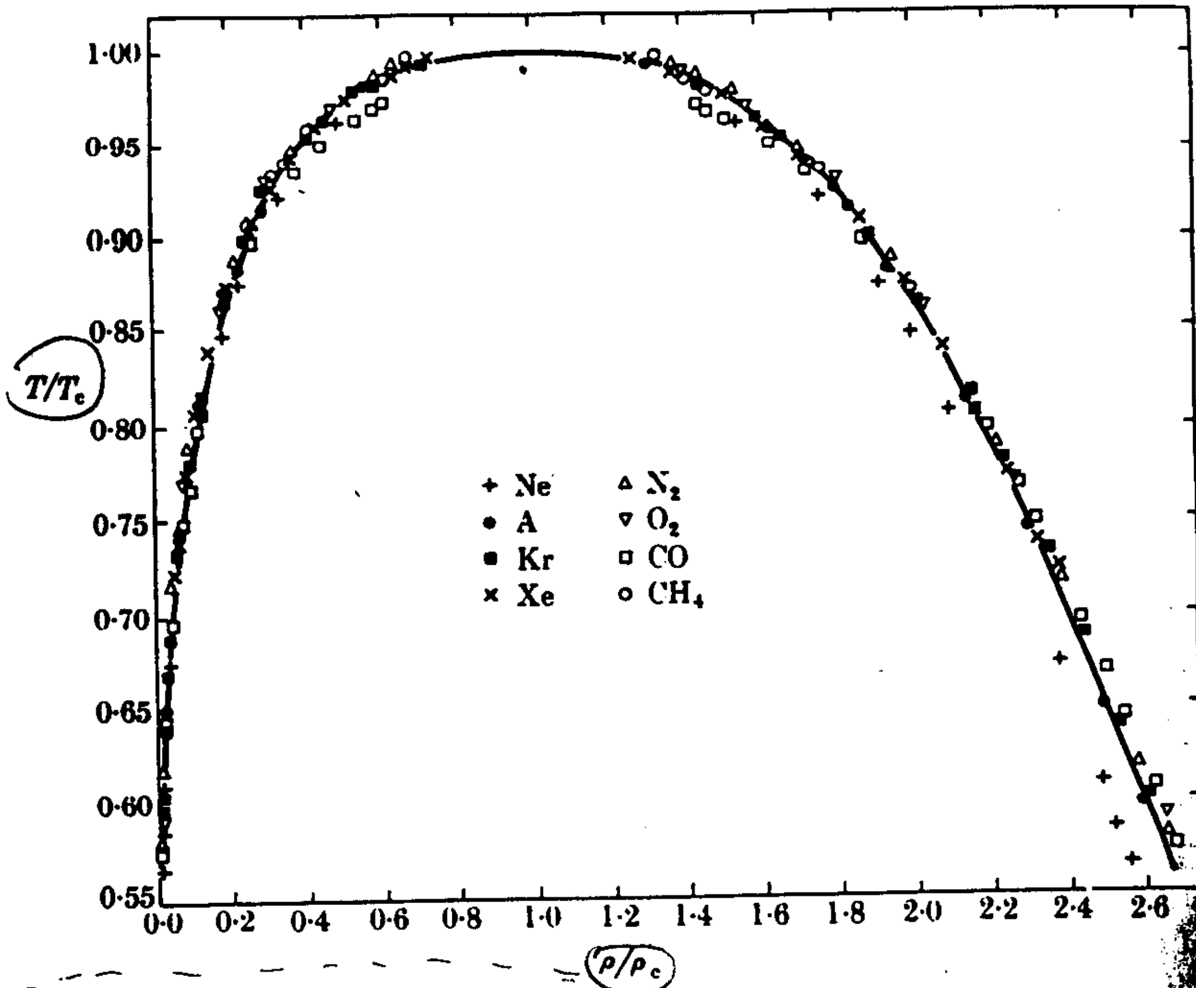


FIG. 1.8. Measurements on eight fluids of the coexistence curve (a reflection of the $P\rho T$ surface in the ρT plane analogous to Fig. 1.3). The solid curve corresponds to a fit to a cubic equation, i.e. to the choice $\beta = \frac{1}{3}$, where $\rho - \rho_c \sim (-\epsilon)^\beta$. From Guggenheim (1945).

50 YEARS AGO: MODERN ERA OF CRITICAL PHENOMENA

REDUCED VARIABLES ⟺ UNIVERSALITY

- After many experiments:

$$\left(\rho - \rho_c\right) = A\left(\frac{T-T_c}{T_c}\right)^{\beta}$$

A GENERAL RELATIONSHIP: THE

EQUATION OF STATE

for continuous transitions [2^{nd} order or higher]

- The EOS applies to magnetic systems

$$M = A\left(\frac{T-T_c}{T_c}\right)^{\beta}$$

- And can be generalized to include an external magnetic field h

$$\frac{h}{M^{\delta}} = f\left(\frac{T-T_c}{M^{1/\beta}}\right) \xrightarrow{T=T_c} M = h^{1/\delta}$$

$h=0$

- Remarkable properties:

  - The EOS contains the "usual" definition of critical exponents [set $T=T_c$ or $h=0$]
  - Gives information on the behaviour at T_c working at $T \neq T_c$

A "dictionary"

	Magnets	Fermions
external S.B. field	h	m
response function	M	$\langle \bar\psi \psi \rangle (m)$
order parameter [= response function → external S.B. → 0]	$M(h=0)$ ↓ spont. magnetization	$\langle \bar\psi \psi \rangle$ ↓ chiral condensate

~ Different Systems ~
~ Same Description ~

Moreover : [Continuous Transitions Only]

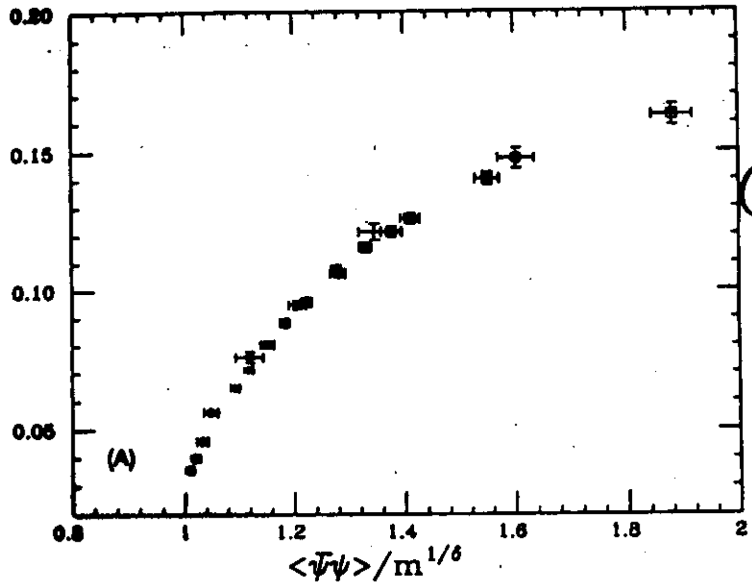
Systems with the ! same critical exponents/behaviour !
are said to be in the same

UNIVERSALITY class

→ Even very different one from another

Fermionic QED : Kocić Kogut Wang MPL

Chiral Equation of State



$(\beta_c - \beta)/<\bar{\psi}\psi>^{1/\beta_{mag}}$

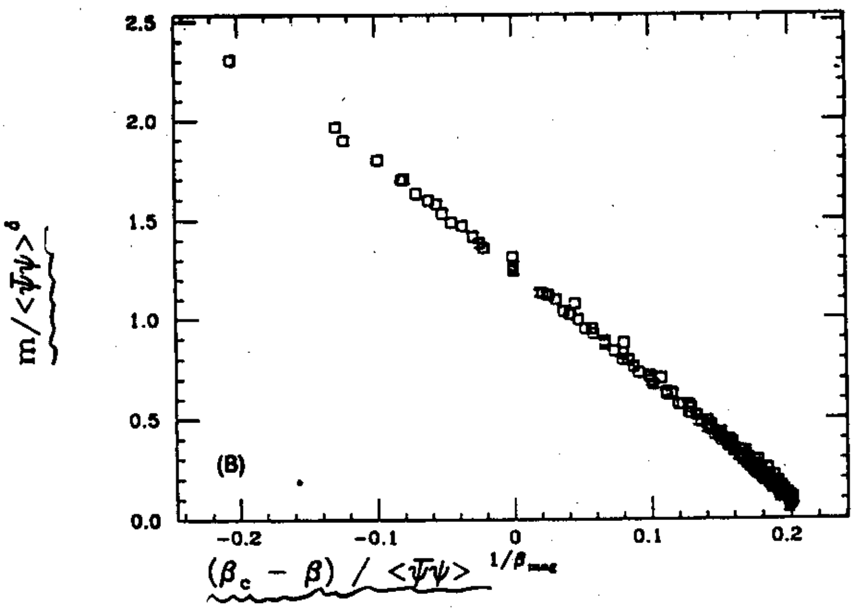


Fig. 6. (a) Chiral equation of state for $\beta_c = 0.257$, $\delta = 2.2$ and $\beta_{mag} = 0.833$ on the 16^4 lattice. $(\beta_c - \beta)/\langle\bar{\psi}\psi\rangle^{1/\beta_{mag}}$ is plotted versus $\langle\bar{\psi}\psi\rangle/m^{1/\delta}$. We plot only points in the strong coupling region. (b) Chiral equation of state on the 24^4 lattice for the same β_c, δ and β_{mag}.$m/\langle\bar{\psi}\psi\rangle^{\delta}$ is plotted versus $(\beta_c - \beta)/\langle\bar{\psi}\psi\rangle^{1/\beta_{mag}}$. All the points are shown.

• The "concept" of EOS "determines" the critical exponents

but

at the price of a rather subjective (= prone to errors) procedure

In slightly more detail:

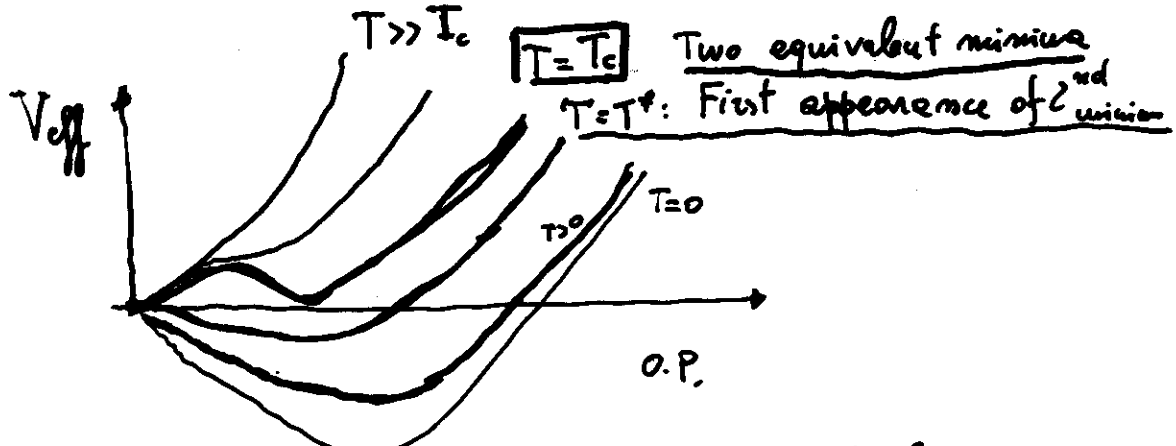
- Behaviour of the order parameter can be inferred by its "probability distribution"

Works also for discontinuous [1^{st} order] P.T.

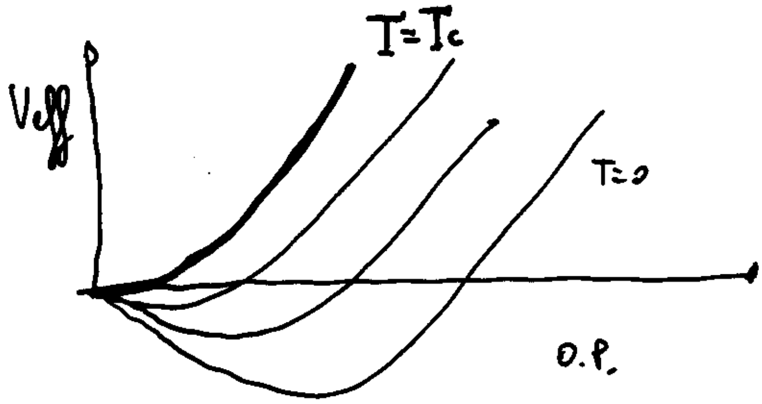
- Idea [Landau]

Guess/Derive a function V_{eff} of the order parameter and external fields which describes the state of the system

1st order



2nd order



} Critical behaviour, Critical exponents

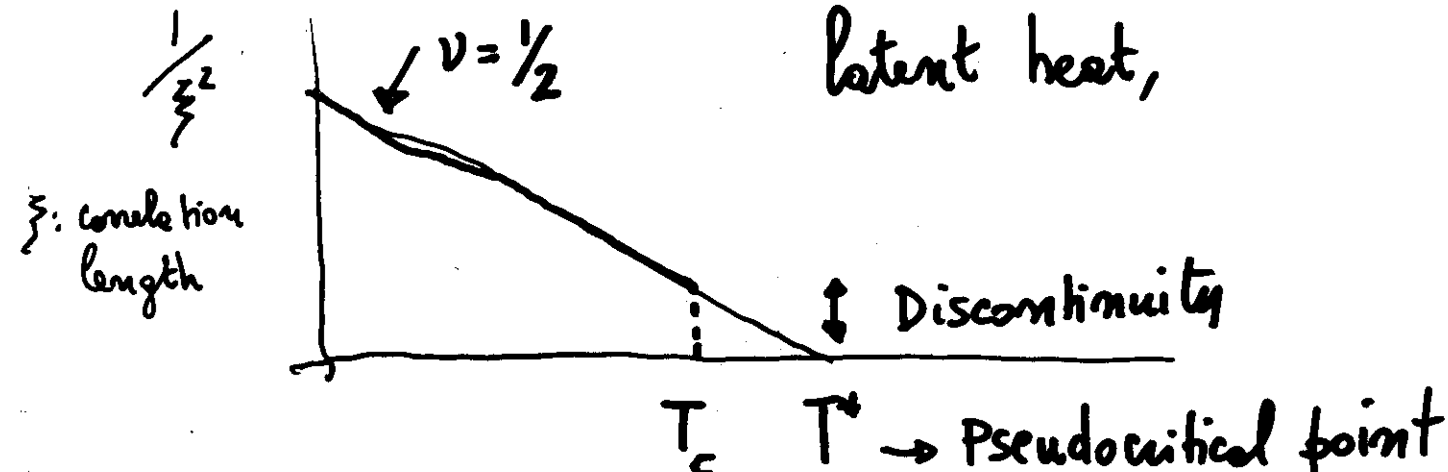
• Ist order Transitions •

• 1st order transitions can lead to cohexisting phases:

→ Domain walls,
Topological defects

• However, consequences are experimentally observable only when the transition is 'strong enough'

• Strength of a 1st order transitions:



latent heat,

When $T^* \to T_c$ Transition is 2nd order

I^{st} order transitions are characterized by their

- "Strength" given by latent heat + relative position of critical & pseudocritical point

[Topological Defects require Strong 1^{st} Transition]

II^{nd} order transitions [and higher] are characterized by

Critical exponents associated with

Diverging correlation length

↓

DIMENSIONAL REDUCTION

UNIVERSALITY

III.

"Toy" Electroweak.

- Symmetry spontaneosly broken $T = 0$
- Symmetry restored high T

- Build
  $V_{eff}(\phi) \longrightarrow \phi = \langle \hat{\Phi} \rangle$

from E.W. Action

$$S = \underbrace{\int dt}_{\text{Temp. dependence}} \int d^3x \left[S_{gauge} + \left| D_\mu \hat{\phi} \right|^2 + V(\hat{\phi}) \right]$$

V Time $V_{eff}(\phi) = \int dt\, d^3x \left[S_{gauge} + \left| D_\mu \hat{\phi} \right|^2 + V(\hat{\phi}) \right] \delta(\phi - \hat{\phi})$

constant background value

Higgs field

$$\hat{\phi} = \langle \hat{\phi} \rangle + \cancel{\text{fluctuactions}} = \phi$$

$T = 0$ [tree level]

$$V_{eff}(\phi, T=0) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

$\phi \to -\phi$ Symmetry: $V(\phi, T=0) = V(-\phi, T=0)$

N.B. discrete symmetry is 'enough' for O.P.

$T > 0$ one loop contribution

Effects of a non-interacting bose-fermi gas with frequencies depending on ϕ

$$V_{eff\ 1\text{-loop}} = T \sum_i{}' \mp \int \frac{d^3p}{(2p)^3} \ln \left| 1 \pm e^{-\sqrt{p^2 + \omega^2(\phi)}/T} \right.$$

$$\simeq \underbrace{\phi^2 T^2}\ [\text{bosons + fermions}]$$

$$\simeq \underbrace{-\phi^3 T}\ [\text{bosons alone}] + \cdots$$

All in all: [see e.g. Rubakov - Shaposhnikov 95 review]

$$V_{eff}(\phi, T) = -\frac{m^2}{2}\phi^2 + \frac{1}{2}\gamma T^2 - \frac{1}{3}\alpha T\phi^3 + \frac{1}{4}\lambda\phi^4$$

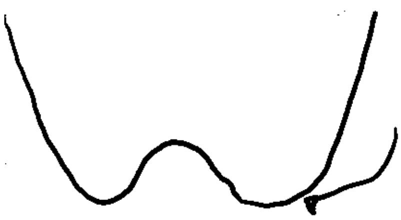
$$\boxed{= \frac{1}{2}\gamma\left(T^2 - m^2/\gamma\right)\phi^2 - \frac{1}{3}\alpha T\phi^3 + \frac{1}{4}\lambda\phi^4}$$

$$V_{eff} = \frac{1}{2}\gamma\underbrace{\left(T^2 - T^{*2}\right)}_{\downarrow\ \frac{m^2}{\gamma}}\phi^2 - \frac{1}{3}\alpha T\phi^3 + \frac{1}{4}\lambda\phi^4$$

- γ, α "inherit" all of the "exact" dynamics!
- further corrections ignored

V_{eff} 'dramatic' simplification! [far from exact]

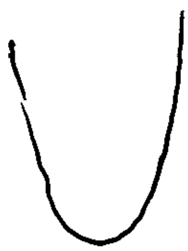
- $T=0$



SSB $\quad V_{eff} = -\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4$

$\phi^2 = 2m^2/\lambda$

- $T \gg T_c$



$V_{eff} \simeq \frac{1}{2}\gamma T^2 + \frac{1}{4}\lambda\phi^4$

$\phi = 0$

Symmetry restoration at high $T \simeq \gamma T^2$

Issues

- Order of the Transition

  1st, because of cubic term $[\alpha \neq 0]$ ✓

- Onset for metastability

$$\begin{cases} \dfrac{\partial^2 V}{\partial \phi^2} = 0 \\ \dfrac{\partial V}{\partial \phi} = 0 \end{cases} \rightarrow T = T^*$$

- Critical temperature

$$V(\phi_1) = V(\phi_2) \Rightarrow T_c = T_* \Big/ \sqrt{1 - \frac{2\alpha^2}{9\lambda\gamma}} > T_*$$

- Strength

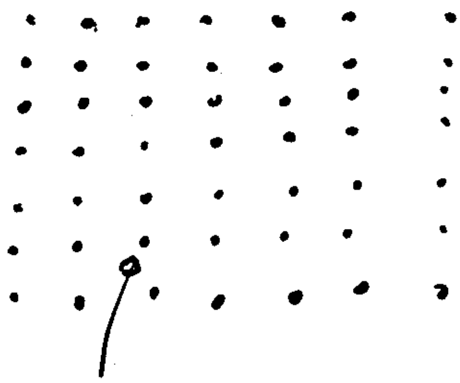
$$\frac{\phi(T_c)}{T_c} = \left[2\alpha/3\lambda \ ; \ 0\right] \quad \text{Jump} = 2\alpha/3\lambda$$

$$T_c - T_* = T_* \left[1 - 1 \Big/ \sqrt{1 - \tfrac{2}{9}\alpha^2/\lambda\gamma} \right]$$

- Transition weekens when

$$\left.\begin{array}{l} \alpha \rightarrow 0 \quad \Rightarrow 2^{nd} \text{ order} \\ \gamma \rightarrow 0 \\ \lambda \rightarrow \infty \end{array}\right\} \begin{array}{l} \text{Strength} \rightarrow 0 \\ T^* \rightarrow T_c \end{array}$$

- To 'monitor' the strength of the transition study 2-dimensional Potts model with q states



$\sigma_i, \; i=1,q$

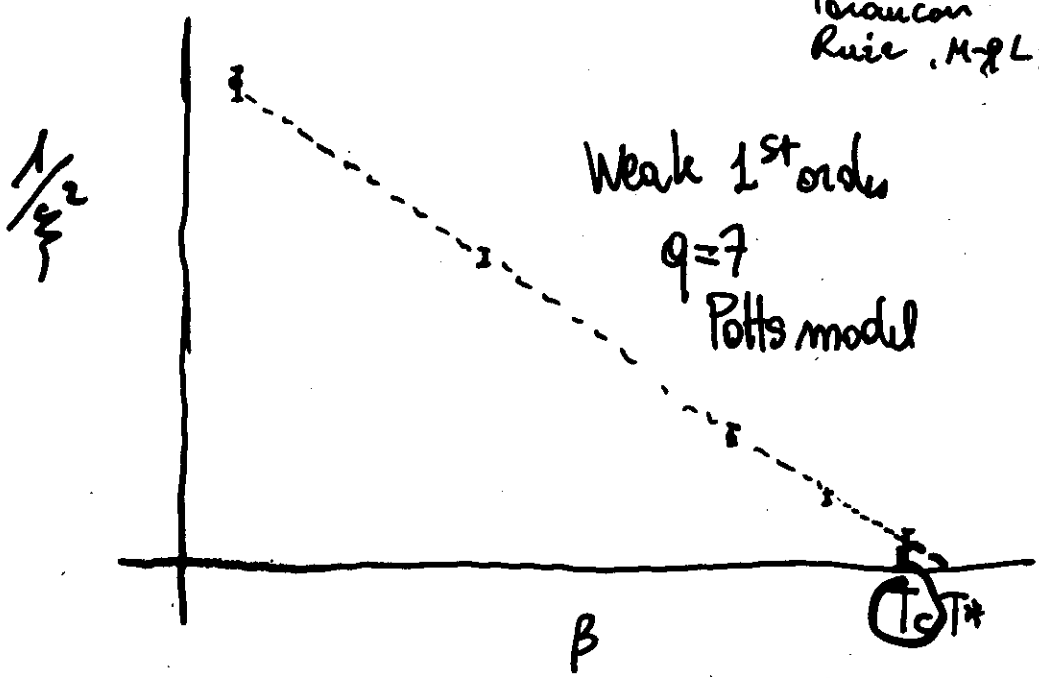
$$S = -\beta \sum{}' \delta_{\sigma_i \sigma_j}$$

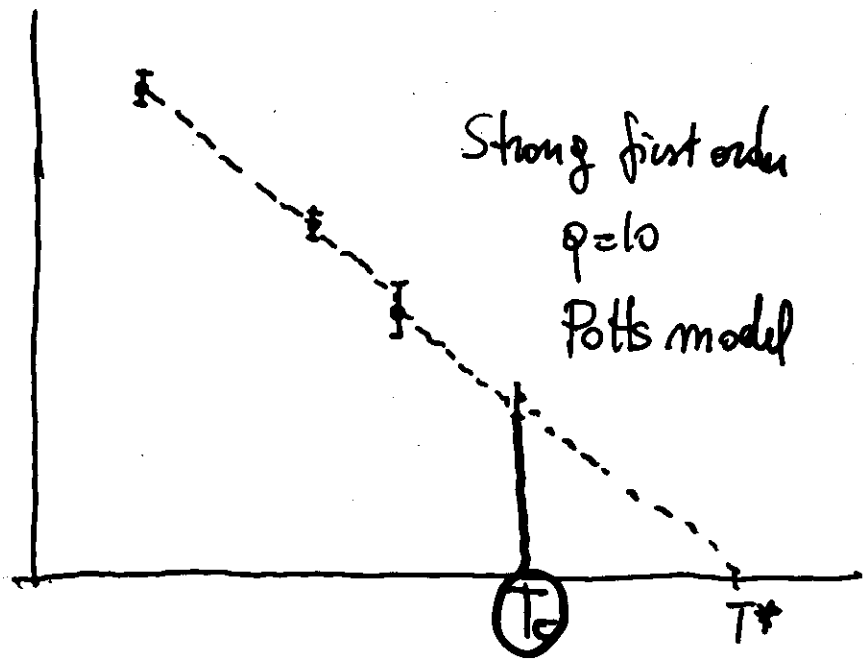
- spins like to be aligned
- There is a transition to disordered state at β_c

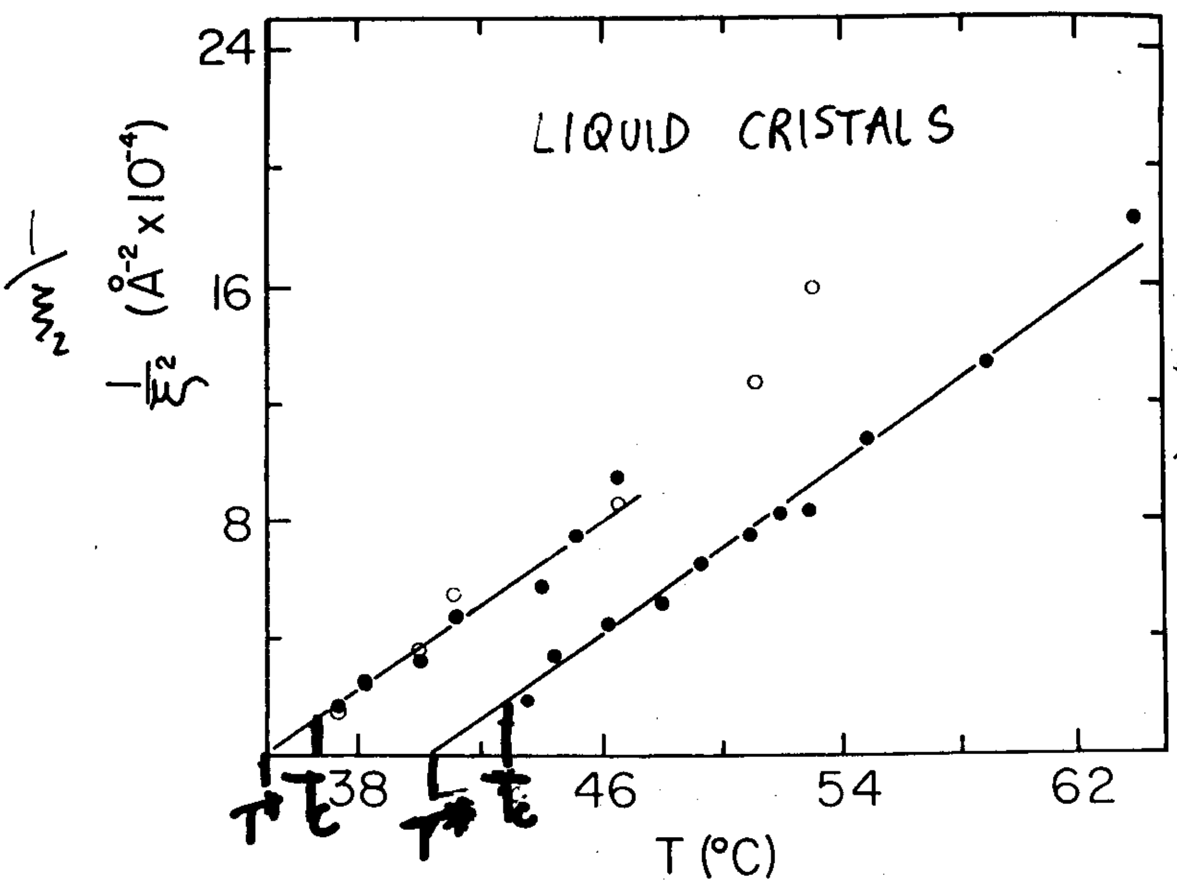
- Model is analitically "solved": β_c and latent heat are known for any number of states [Potts, Wu, Baxter]

Latent heat,
$$\begin{cases} 0 & q \leq 4 \quad 2^{nd} \text{ order transition} \\ >0 & q>4, \text{ increasing with } q, \; 1^{st} \text{ order transition with increasing strength} \end{cases}$$

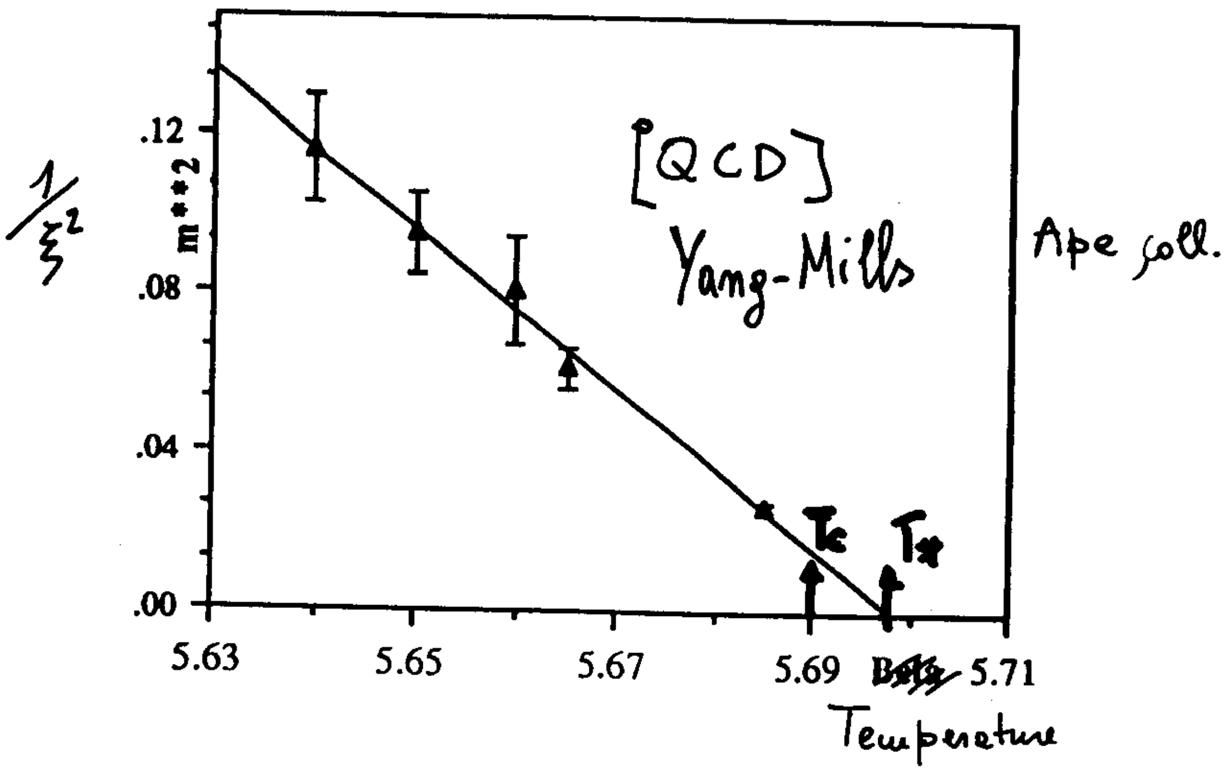
Fernandez
Tarancon
Ruiz , M-gL. 92







Stinson, Listner ~1970



1/ξ²
m**2
.12
.08
.04
.00
[QCD]
Yang-Mills
Ape coll.
Tc
T*
5.63
5.65
5.67
5.69
Beta
5.71
Temperature

- Insight from Toy studies
- High T Restoration observed
- Effective Potential for EW Transitions

can accomodate al possible behaviours from

$\gtrsim$ 2nd; very weak first order, strong first order
- 'Exact' calculations a must!
- Weak firts order transitions do exhist

in nature!

- Real thing
- State-of-art calculations will indicate

that EW is at most weak first

→ disappointing!

- SUSY can make it strong

IV . Fermions. [Toy QCD]

- Fermionic system are not amenable to direct dimensional reduction, as fermions decouple when conditions for dimensional reductions are fulfilled.
  [no zero modes]
- Nice insight into the thermodynamics comes from mean field analys of the 'bosonized' model: [but lattice is very important]
- We consider 3-d Gross Neveu model:
- For large g^2 it has χ sym. breaking at $T, \mu = 0$
- Rich meson spectrum, and 'baryon' (fermion)
- Interacting continuum limit
- Also amenable to 'exact' lattice calculations

- Model is described by the Lagrangian density:

$$\mathcal{L} = \bar\psi\left(\partial\!\!\!/ + m\right)\psi - \frac{g^2}{N_f}\left[(\bar\psi\psi)^2 - (\bar\psi\gamma_5\psi)^2\right]$$

- Symmetry is $U(1)$

$$\psi_i \to e^{i\alpha\gamma_5}\psi_i$$
$$\bar\psi_i \to \bar\psi_i\, e^{i\alpha\gamma_5}$$

- ~~Continuous~~ symmetry $\to$ Goldstone modes

- Introduce bosonic auxiliary field $\sigma, \vec\pi$ by adding the irrelevant term: $\left(\bar\psi\psi + \frac{\sigma}{g^2}\right)^2 + \left(\bar\psi\gamma_5\psi + \frac{i\pi\gamma_5}{g^2}\right)^2$

$$\mathcal{L} = \bar\psi\left(\partial\!\!\!/ + m + \sigma + i\pi\gamma_5\right)\psi + \frac{N_f}{2g^2}\left(\sigma^2 + \pi^2\right)$$

- Continuous χ symm is also reflected by rotations in the $\binom{\sigma}{\vec\pi}$ chiral sphere.

Mean Field Analysis

Hands
Kim
Kogut
1995-8

- $T, \mu = 0$

$$\langle \sigma \rangle = -g^2 \langle \bar{\psi}\psi \rangle = g^2/V \; \mathrm{tr}\, S_F$$

- Self consistent solution $[m=0]$

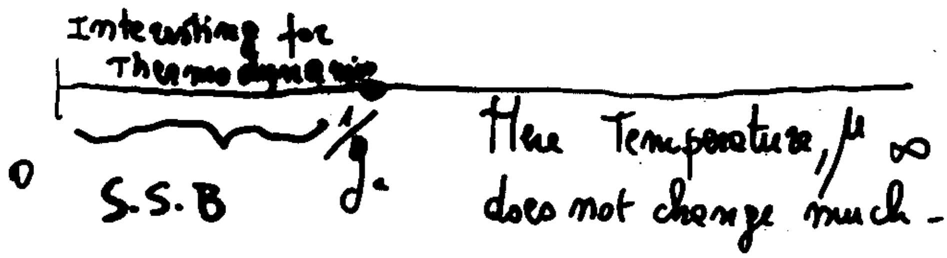
$$\langle \sigma \rangle = g^2 \int \mathrm{tr} \frac{1}{i\not{p} + \underbrace{[\langle \sigma \rangle]}_{\text{Dynamical Fermion Mass}}}$$

$$1/g^2 \sim \int d^3p \Big/ p^2 + (\langle \sigma \rangle + m)^2$$

- Solution $\underbrace{\langle \sigma \rangle \neq 0 \text{ exhists when } m=0}_{\text{S.S.B}}$ if:

$$1/g^2 < 1/g_c^2 = 2\Lambda/\pi^2$$

- " Phase diagram" as a function of coupling g



• Introducing a chemical potential for Baryon Number -

$$\mu \underset{\substack{\downarrow \\ n^+ - n^-}}{N} \to \mu \underset{\substack{\downarrow}}{J_0} = \mu \bar{\Psi} \gamma_0 \Psi$$

$n^+ - n^-$

0th component of a conserved current $\bar{\Psi} \gamma_\mu \Psi$

$$p_0 \to p_0 - i\mu$$

• $T=0$, $\mu \neq 0$ -

$$1/g^2 = \int d^3 p \Big/ (p_0 - i\mu)^2 + p^2 + \Sigma^2(\mu, T)$$

• $T \neq 0$, $\mu \neq 0$

$$\int d^3 p \to \sum_{\text{Matsubara}} \int d^2 p$$

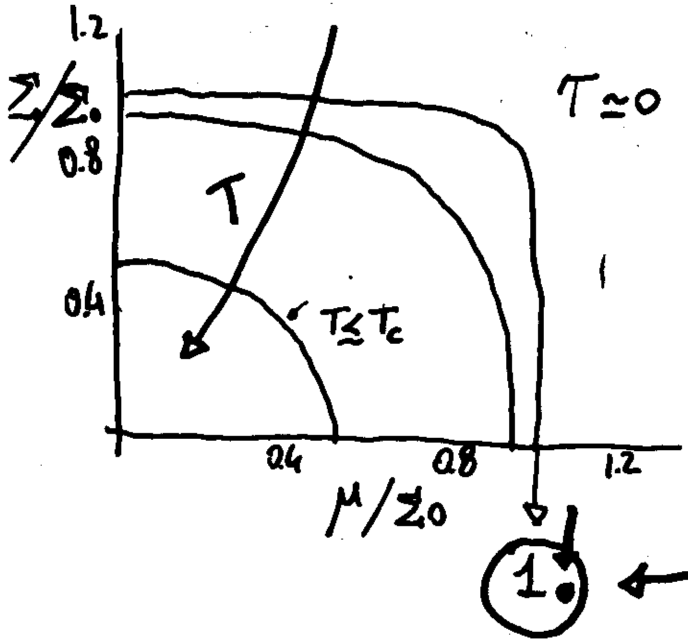
$$\boxed{\frac{1}{g^2} = 4T \sum_{n=-\infty}^{\infty}{}' \int \frac{d^2p}{(2\pi)^2} \frac{1}{((2n-1)\pi T - i\mu)^2 + p^2 + \Sigma^2(\mu, T)}}$$

$\Sigma = \langle \sigma \rangle$

- Eliminate coupling, cutoff in favour of $\Sigma_0 = \Sigma(0,0)$

$$\Sigma - \Sigma_0 = -T \left[\ln\left(1 + e^{-(\Sigma-\mu)/T}\right) + \ln\left(1 + e^{-(\Sigma+\mu)/T}\right)\right].$$

- Behaviour of the O.P. at fixed temperature as a function of μ



Hands Kim Kogut 97

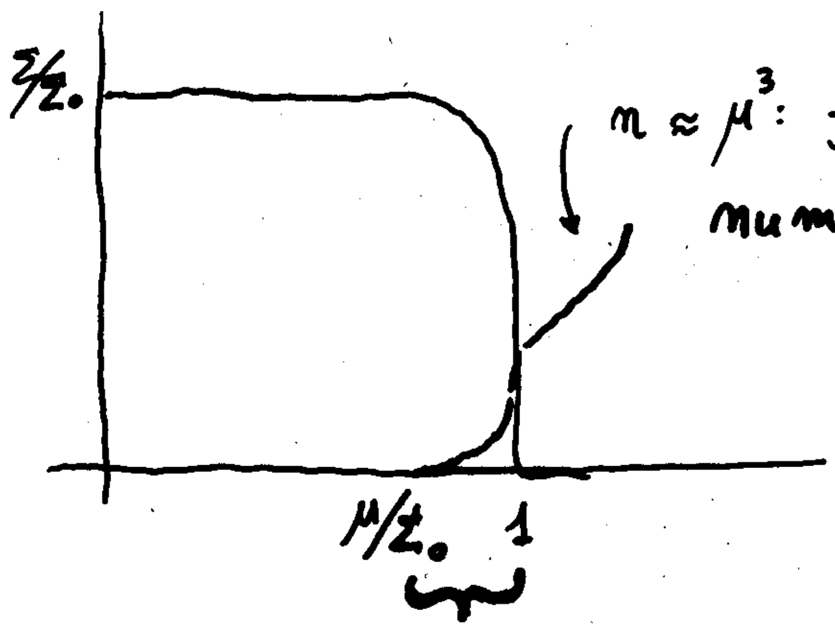
$$\boxed{\mu_c = m_F}$$

- Transition is everywhere continuous but at

$T = 0$

The equation of state:

$T \simeq 0$

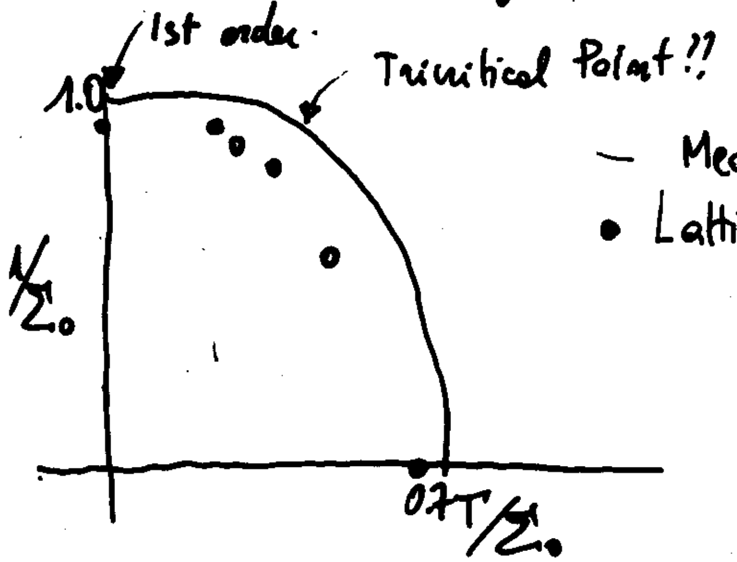


$n \approx \mu^3$: free fermions

number density $= \langle J_0 \rangle \times 10$

Nuclear matter phase: for $T = 0$ should disappear?

The phase diagram: Mean Field vs. Lattice



— Mean field

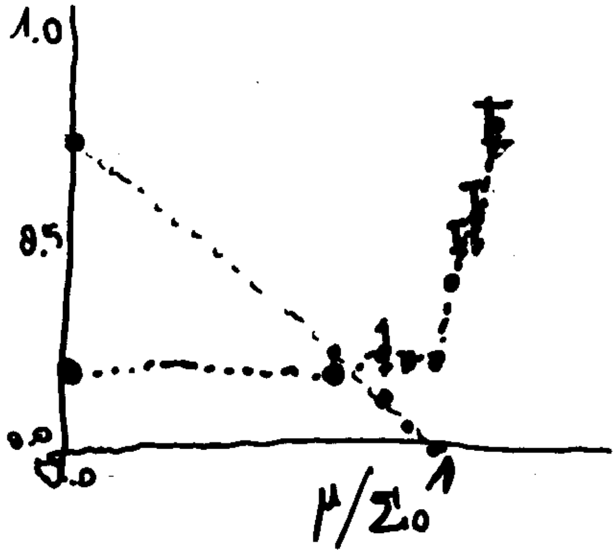
• Lattice

$N_f = 4$

Note: mean field $\equiv$ leading order $1/N_f$

For detailed dynamics: LATTICE!

• Mass spectrum



$\bullet$: $m_f = m_f - \mu$: Typical "mean field"

I : m_π = loss of Goldstone behaviour beyond $\mu_c \simeq m_f$

Masses are very important:

→ For experiments [eg. RHIC, LHC]

→ For studying patterns of χSB

4. Fermion summary:

• Nice results for the phase diagram obtained in a mean field approximation for auxiliary field σ

• Results can be cross checked with exact lattice calculations: They are important since

• Agreement qualitatively ok - But not 'perfect'!

V QCD

· Theory of Quarks and Gluons ·

Main features

· Symmetries $\quad SU(3)_c \times \underbrace{SU(N_f) \times SU(N_f)}_{\text{approx. chiral}} \times Z_A(N_f)$

$$\downarrow$$

$$SU(N_f)$$

· Goldstone Bosons [π] [PCAC]

· Mass Gap [massive baryons]

· Dynamics : Confinement

Quarks hidden inside hadrons
$\Longrightarrow$ all 'objects' colorless

Non Abelian $SU(3)_c$

· Asymptotic Freedom
Coupling Decreases at Short Distance

· Gluons carry Color Charge

When increasing T

- Symmetry Restored : $\langle \bar{\psi}\psi \rangle \to 0$

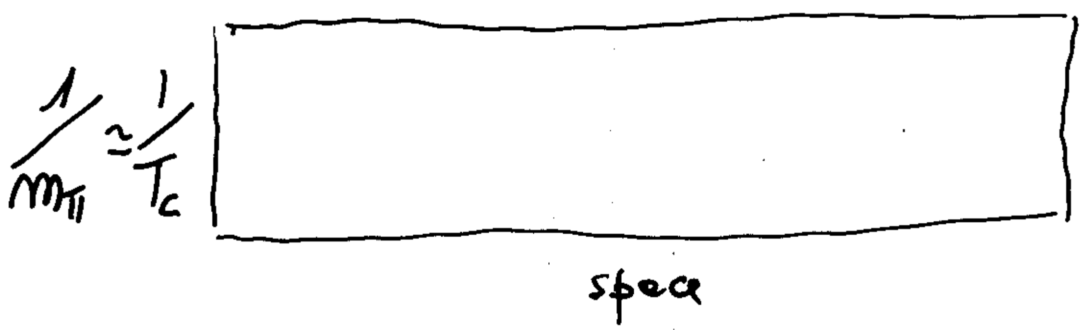
  [high T favors disorder]

- Recombination with thermally excited pairs 'breaks' the confining string

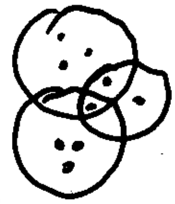
Deconfinement

- Bound States might survive giving complicated dynamics (non-perturbative) for $T \gtrsim T_c$

Physical Scale : m_π



When increasing density:

• Overlapping Hadrons - 

Asymt. Freedom, Short distance behaviour nearly 'free'

· Long distance behaviour screened

$\Rightarrow$ nearly free quarks.

Deconfinement * χ Symm. Restoration

Recall However

• χSymm restoration in GN at high μ

• no asymptotic freedom

Actual mechanism: Dynamical Question

• Physical Scale m_B : ✓ Fermi Statistics

$$n_B = \Theta(\mu - m_B)\,\mu^3$$

- Status of the art:
    - Lattice Studies Satisfactory at high T
    - Difficult/Impossible at high μ
    - Interesting predictions from
        - Four Fermion models with appropriate symmetry
    - However dynamics is important

- From the lattice
    - Strong coupling expansion
    - Yang Mills
    - Real things -

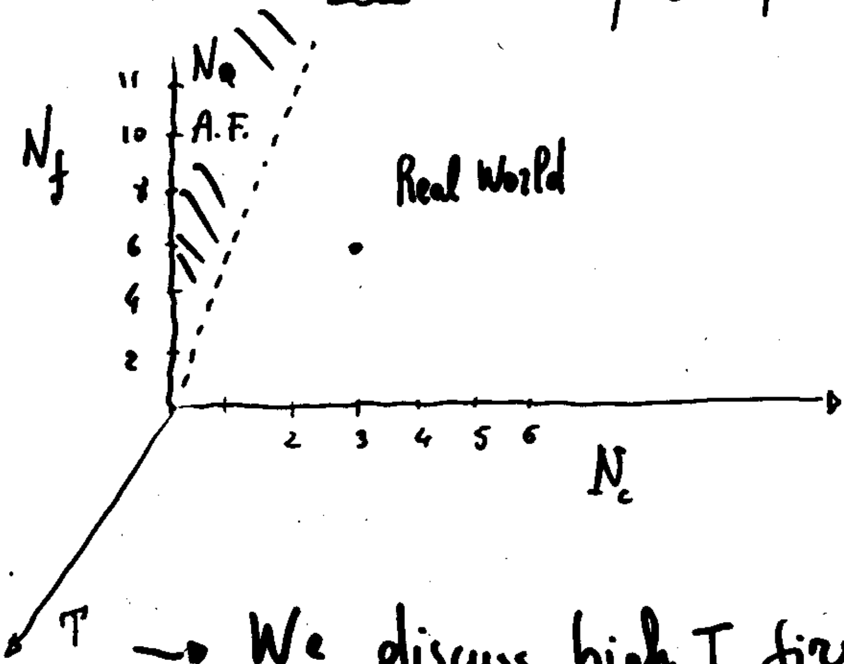
Phases of QCD

- Features of the normal phase
  - Asympotic freedom
  - Confinement
  - Spontaneous Chiral Symmetry Breaking

- Both confinement and SSB can disappear at high T, high μ

- Asymptotic freedom does not depend on T, μ:
  holds true till $N_f < 11/2 \, N_c$

• Consider QCD in N_{color} and N_{flavor} space



T → We discuss high T first

• There are two important limits which are amenable to a symmetry analysis

- $m_q = \infty$

  Static quarks do not contribute to the dynamics

  Pure Yang Mills $\Rightarrow Z(N_c)$

- $m_q = 0$

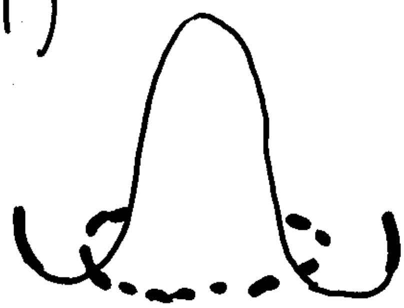
  Chiral symmetry

  $SU(N_f) \times SU(N_f)$

  "Reasonable" for $N_f = 2$ as $m_u \simeq m_d \simeq 0$

  $SU(2) \times SU(2) \to O(4) : \begin{pmatrix} \sigma \\ \vec{\pi} \end{pmatrix}$

• $V(\sigma^2+|\Pi|^2)$



• When "we" select one direction in the chiral sphere "we" spontaneously break chiral symmetry in that direction:

$\langle\sigma\rangle \neq 0$ $\Rightarrow$ Three massless pions

$O(4) \xRightarrow{\langle\sigma\rangle} O(3)$ [Eq. $SU(2)\times SU(2) \to SU(2)$]

• At high T

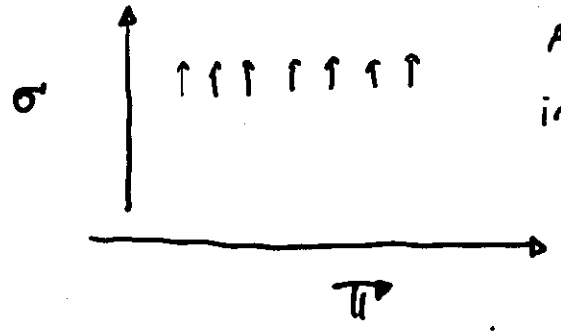
$\langle\Pi\rangle, \langle\sigma\rangle = 0$

No Goldstone particles

• Mechanism for χ restoration can be for instance understood by analogy with magnetic systems

$\langle\hat{\sigma}\rangle$ [$\langle\bar{\Psi}\Psi\rangle$] is 'like' $\underline{M}$

Broken Symmetry



All 'spins' aligned in $\langle\sigma\rangle$ directions in chiral space

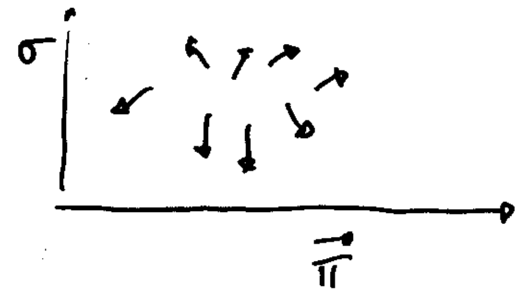
T

Massless pions

σ, $\langle\bar{\psi}\psi\rangle \neq 0$

$\langle\vec{\pi}\rangle$, $\langle\bar{\psi}\gamma_5\psi\rangle = 0$

Restored symmetry



"Spins" randomly oriented in chiral sphere

$\langle\bar{\psi}\psi\rangle \neq 0$

$\langle\bar{\psi}\gamma_5\psi\rangle = 0$

No massless particles

- Mechanism analogous to that of Gross Neveu model [?]

- If dim. reduction 'works' 3-d $O(4)$ magnet should characterize the transition

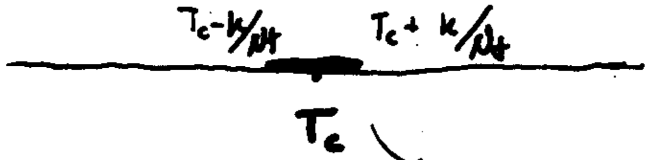
$$S = \frac{(\partial_\mu\sigma)^2}{2} + \frac{(\partial_\mu\vec{\pi})^2}{2} + \frac{\mu^2}{2}\left(\sigma^2 + \vec{\pi}^2\right) + \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2\right)^2$$

Possible sources of violations of dimensional reduction/universality scenario

- Scenario holds true but only in a very narrow window:

  $Z(N) \times Z(N)$

  cfr ↔ GN at large T: 2d Ising exponents only in $O(1/N_f)$ window



shrinks → 0 in large N_f

- Strong Deconfining?
- Decoupling of fermions - or lack thereof?

  Theories with χSB produced by $\langle\bar\Psi\Psi\rangle$ have fermionic zero modes

$$\langle\bar\Psi\Psi\rangle = \lim_{V\to 0} \pi \rho(0)/V$$

Density of eigenvalue near zero for the Dirac operator

- Role of the anomaly? or lack thereof

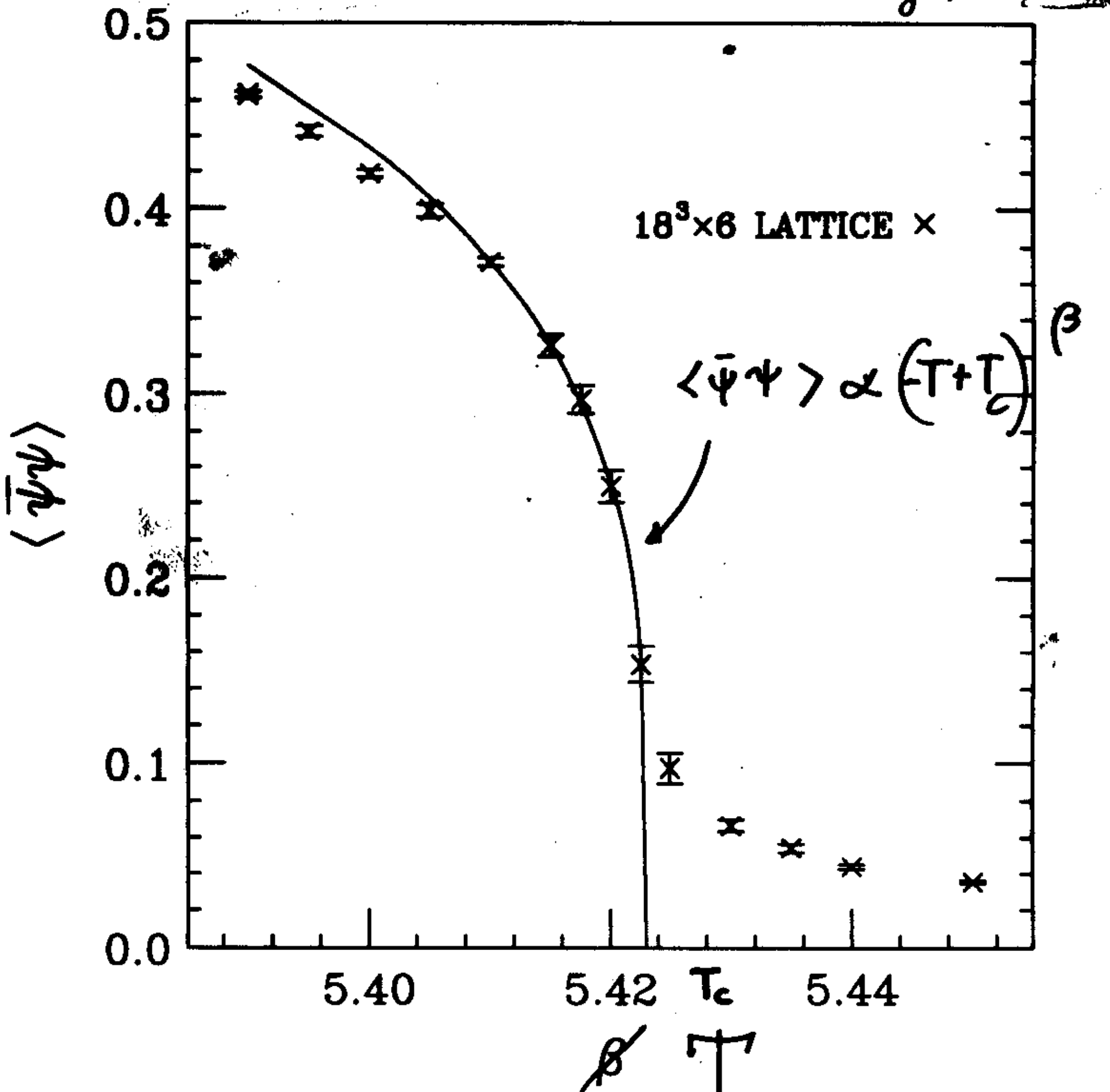
- σ model picture assumes that axial $U_A(1)$ remains broken across the transition: $U_A(1) \to Z_A(N_f)$

- High T kills instanton, and $U_A(1)$ might be effectively restored ⟹ more light modes contributing to effective V

//

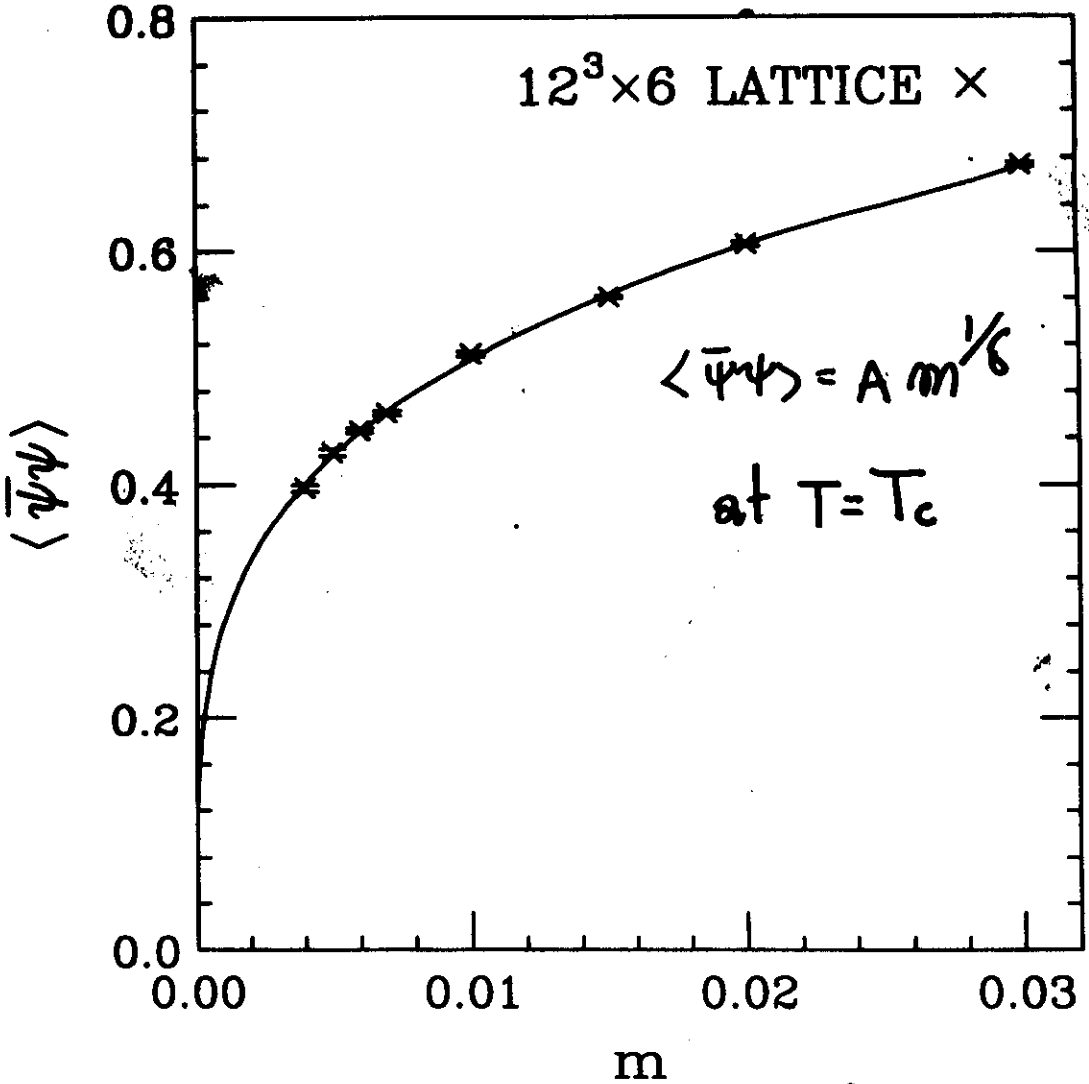
Critical Exponents of 2fl. QCD high T chiral Transition

J. Kogut and D. [redacted]
2000



$\beta_{QCD} = .27(3)$ $\cancel{\beta_{mf} = .5}$

$\beta\,\sigma^{O(4)}_{model} = .38(1)$ $\beta\,\sigma\,model\ O(2) = .35(1)$



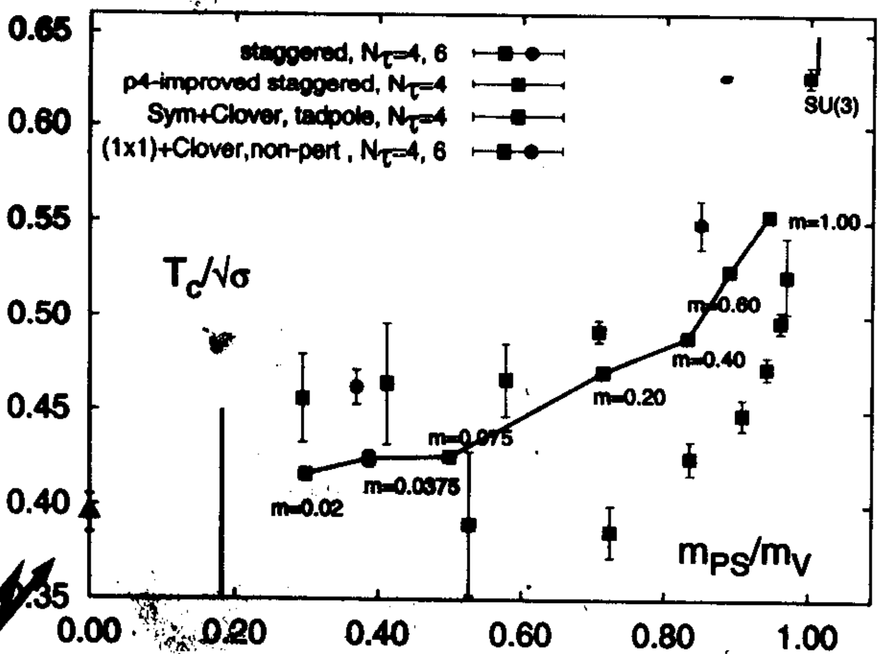
$\delta_{QCD} = 3.89(3)$ $\delta_{m\text{-field}} = 3$

$\delta_{O(4)\ \sigma\,model} = 4.8(2)$ $\delta_{O(2)\ \sigma_{model}} = 4.8(2)$

Courtesy F. Karsch - 2000

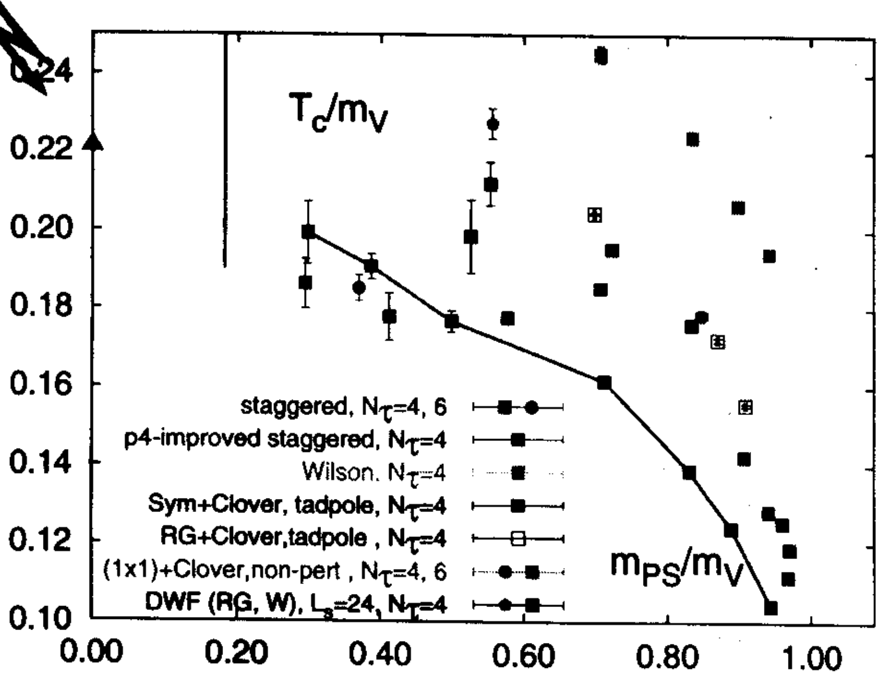
2 flavour QCD: $T_c(m_{PS})$

$m_{PS} \sim 0.3$ GeV ⇓ ⇓ $m_{PS} \sim 1.2$ GeV



rapid drop of $T_c/\sqrt{\sigma}$

[redacted] ⇕ O(4) scaling: [redacted]



note: O(2) will not change the [illegible]

improvement mandatory

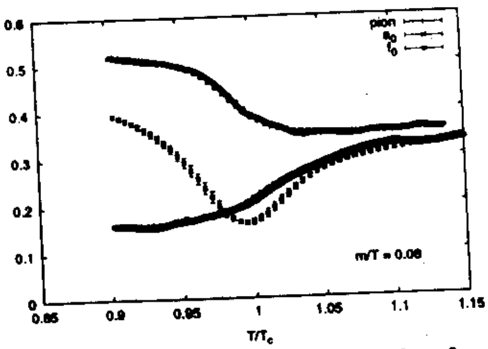
discretisation errors not yet under control

Clover: R.G. Edwards, U.M. Heller, PL B462 (1999) 132
S. Ejiri et al. (CP-PACS) hep-lat/9909075
improved staggered: Bielefeld data
domain wall fermions: P.M. Vranas et al. (Columbia group), hep-lat/9911002

Status of QCD χ high T transition with $N_f = 2$

- 2nd order transition favoured / confirmed $T_c \simeq 170$ MeV
- Exponents $\approx O(4)$ [continuum prediction] or possibly $O(2)$ [lattice prediction]
- Possible non-restoration of $U_A(1)$ symmetry:
    - Also suggested by meson behaviour;

$$\begin{array}{ccc} \pi & & \\ \Big\updownarrow SU(2) & & \\ f_0 & \overset{U_A(1)}{\longleftrightarrow} & a_0 \quad \text{scalar} \end{array}$$



From E. Laermann, 1998

Figure 3. Masses of π, a_0 and f_0 taken from the generalized susceptibilities, eq. (4), for two flavors of staggered quarks as a function of the temperature.

Analytic arguments in favor of non-restoration }
- Discussed also in:
  Goméz Nicola - Alvarez Estrada, 1994
  [illegible] 1998

• Chiral Symmetry of two color QCD

• Enlarged chiral symmetry!

$$\underbrace{SU(N_f)\times SU(N_f)}_{\text{QCD}} \to \underbrace{SU(2N_f)}_{\text{bicolor QCD}}$$

Pauli-Gürsey symmetry

• Only discernable condensate $\langle\bar\psi\psi\rangle^2+\langle\psi\psi\rangle^2$

• $S = (\psi, \bar\psi)\begin{pmatrix} J\tau_2 & \not{D}+m \\ \not{D}^{+}+m & J\tau_2 \end{pmatrix}\begin{pmatrix}\psi \\ \bar\psi\end{pmatrix}$ Twice as N_f

$$\left\{\boxed{\tau_2 D \tau_2 = D^*}\right\}$$

• Recently rediscovered at high $\underline{\mu}$

• Even more recently, re-discussed at high $\underline{T}$.

→ Universality class of 2 flavor-2color

high T χ trans.: $O(6)$ if 2^{nd} order

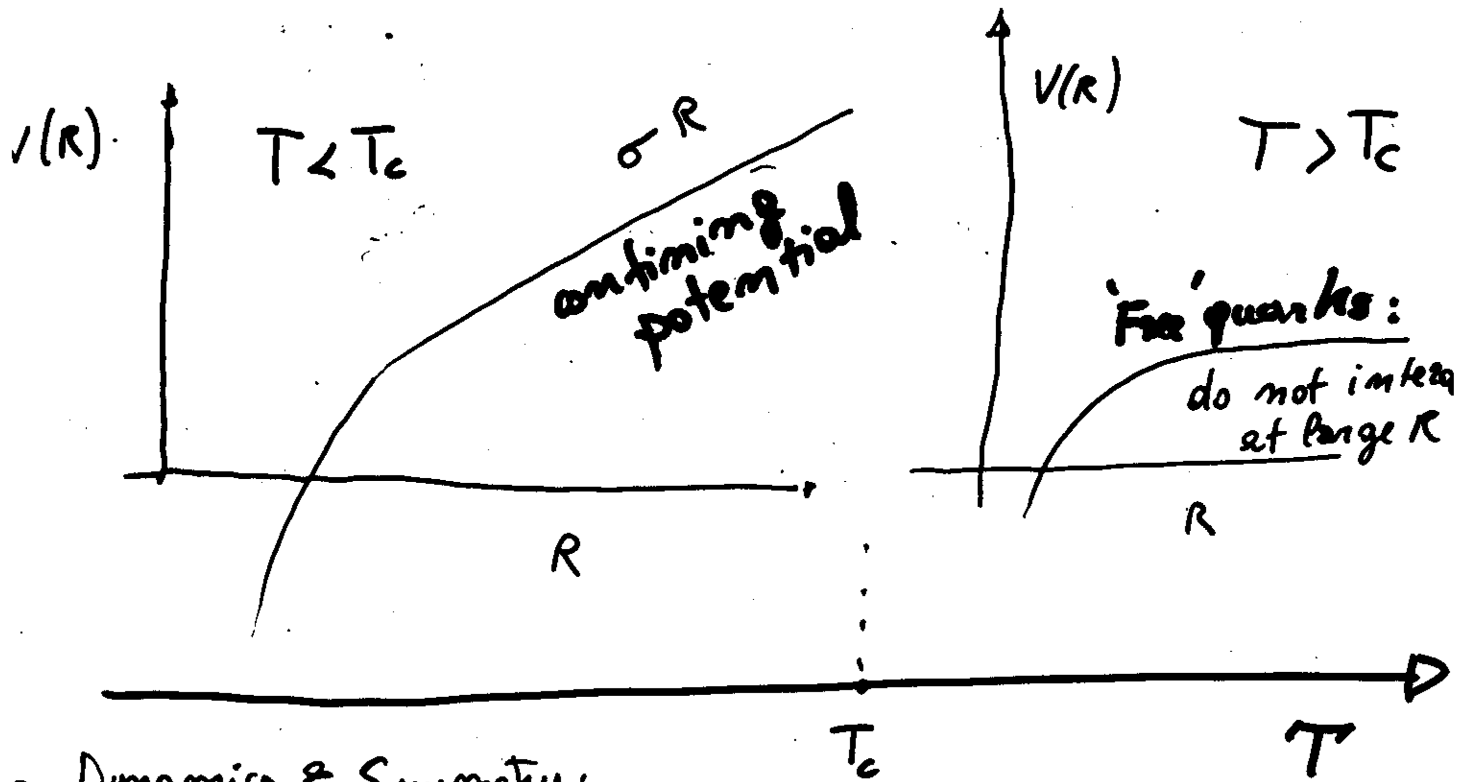
Wirstam, 2000

• Universality of χ P.T. depends on color too!

2^{nd} case relevent to symmetry analysis:

$m_q \to \infty$: Pure Yang Mills $\quad S = -F_{\mu\nu}F^{\mu\nu}$

- Deconfinement Transition:



- Dynamics & Symmetry:

$$e^{-V(R,T)/T} \propto \langle P(\vec{0})\, P^{+}(\vec{R}) \rangle \xrightarrow[R\to\infty]{} \underbrace{|\langle P \rangle|^2}_{\text{order par. for } Z(N_c) \text{ symmetry}} = \begin{cases} 0 & \text{confined} \\ \neq 0 & \text{deconf.} \end{cases}$$

P : $\exp i \int_0^{1/T} A_0\, dt$: 'cost' of a static source violating $Z(N_c)$ sym.

order par. for $Z(N_c)$ symmetry

• Universality Class of the Deconfinement Transition in Yang-Mills [Pure Gauge]
in 4-dim

• 3 colours: 3 state, 3-d Potts model:
[Z_3] 1st order

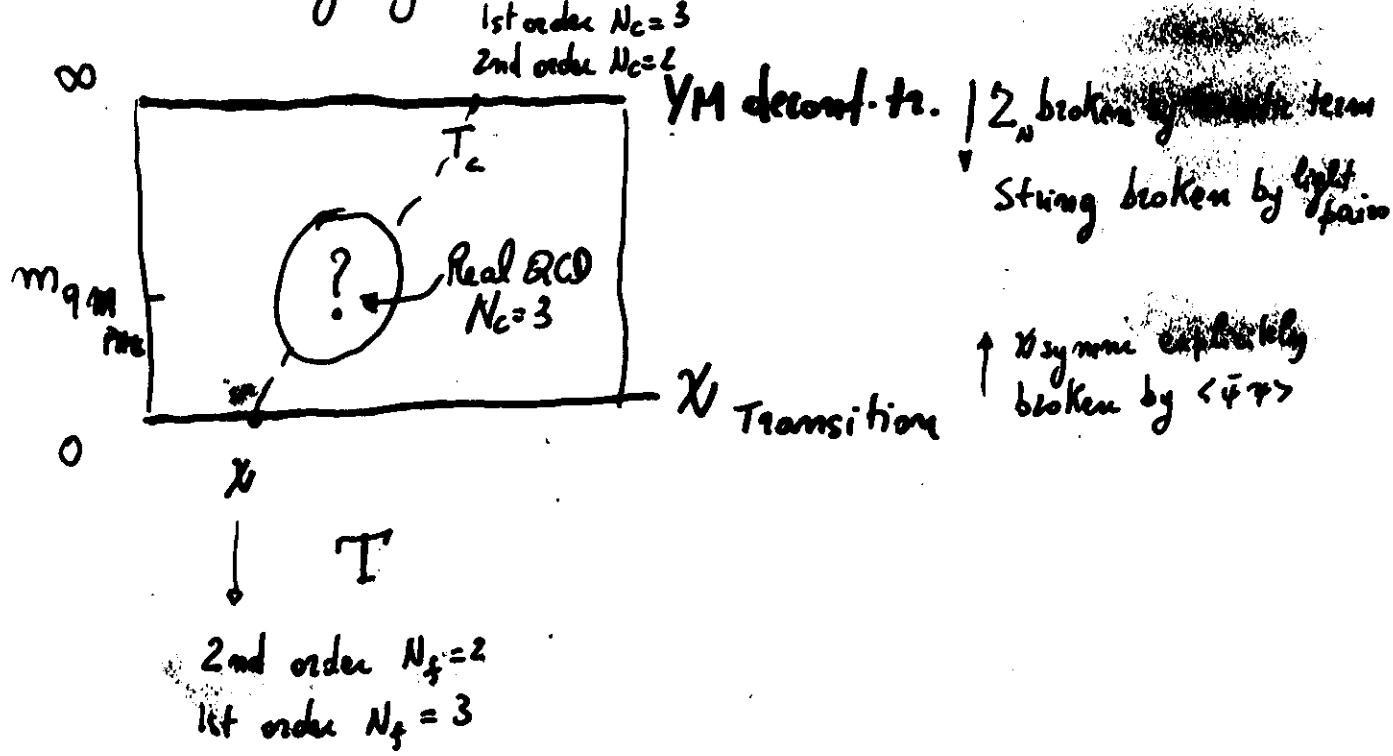
• 2 colours: 3-d Ising model
[Z_2] 2nd order

Beautiful example of universality + dimens. reduction

Source		$SU(2)$	Ising [4]
$\langle \lvert L \rvert \rangle$	β/ν	0.525(8)	0.518(7)
$D\langle \lvert L \rvert \rangle$	$(1-\beta)/\nu$	1.085(14)	1.072(7)
	$1/\nu$	1.610(16)	1.590(2)
	ν	0.621(6)	0.6289(8)
	β	0.326(8)	0.3258(44)
χ_v	γ/ν	1.944(13)	1.970(11)
$D\chi_v$	$(1+\gamma)/\nu$	3.555(15)	3.560(11)
	$1/\nu$	1.611(20)	1.590(2)
	ν	0.621(8)	0.6289(8)
	γ	1.207(24)	1.239(7)
	$\gamma/\nu + 2\beta/\nu$	2.994(21)	3.006(18)
g_r	$-g_r^\infty$	1.403(16)	1.41
Dg_r	$1/\nu$	1.587(27)	1.590(2)
$(\omega = 1)$	ν	0.630(11)	0.6289(8)

SU(2):
Engels, Mashkevic, Scheidl
Zinovjev '95
Ising
Ferrenberg, Landau '91

- Summary of high T QCD phase transition



- T_c deconfinement + Universality ok
- T_c chiral + Universality $\simeq$ ok

Open questions:

- Interrelation deconfinement/chiral ?
- Why $T_{deconf/chiral}$ $m_q=\infty$ $\gg$ T_χ $m_q=0$?
- How will the $N_f=2$ scenario meet with the $N_f=3$?

· Non-zero Baryon Density -

· Why it's so difficult
· What do we know
· What can we do

· For susceful lattice calculations, one does not need to explain lattice details.

· But now we are facing problems --- back to the Z --

$$Z(\mu, T) = \int_0^{1/T} dt \int e^{-S(\bar{\psi}, \psi, U)} d\bar{\psi}\, d\psi\, dU$$

Formulation

· Introducing baryon chemical potential

$$\mu \bar{\psi} \gamma_0 \psi \to p \to p_0 - i\mu$$

Search for effective Action.

$$S_{QCD} = F_{\mu\nu} F_{\mu\nu} + \bar{\psi} \underbrace{(\not{D} + m + \mu \gamma_0)}_{M} \psi$$

$$S_F = \bar{\psi} M \psi$$

• $\mathcal{Z}(T,\mu) = \int d\bar{\psi}\, d\psi\, dU\, e^{-S_g - \bar{\psi} M \psi}$

$$= \int dU \det M\, e^{-S_g} \quad !$$

$$= \int dU\, e^{-S_g - \log[\det M]}$$

• $S_{eff}(U) = S_{gauge} + \log[\det M(U)]$

$\boxed{\text{• Exact } S_{eff}(U) \text{ only depending on gauge fields}}$

• $\mathcal{Z}(T,\mu) = \int dU\, e^{-S_{eff}(U)}$

usually computed with

importance sampling . . but

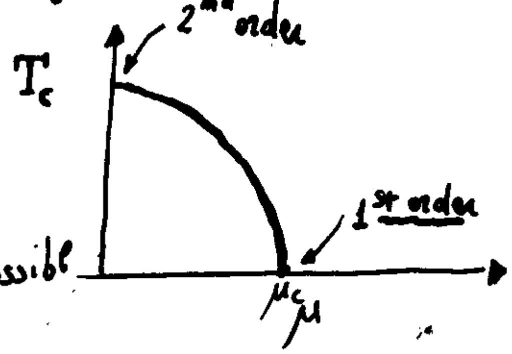
$S_{eff}(U)$ complex when $\mu \neq 0$

for $N_c = 3$ QCD.

What do we know

Comes from

- Strong coupling lattice analysis
  - $V_{eff}(\langle\bar\psi\psi\rangle, \mu, T)$
    - Similar to Gross Neveu
    - Systematic improvment possible
    - Incorporates confinement



- Model Studies at $T \simeq 0$
  - New phases at high μ: [Alford, Rajagopal, Wilczek; Schuryak, Schaefer, Rapp, Velkovsky]
    colored diquark condensate
    color superconductivity
  - Role of confinement?
- Lattice Studies of models with $S(\mu) = S^*(\mu)$
  - Gross Neveu: Toy model for QCD symm restoration
  - $N_c = 2$ QCD: diquark condensation
    deconfinement at high μ

Construction of Effective Potential for QCD at $g = \infty$

Petersson et al. $\simeq$ 85
Damgaard et al.

$V_{eff}(\langle \bar\Psi \Psi \rangle, T, \mu)$ from the exact

QCD Action

$$S = -\frac{1}{2} \sum_x' \sum_{j=1}^{3}{}' \eta_j(x) \left[\bar\chi(x) U_j(x) \chi(x+j) - \bar\chi(x+j) U_j^+(x) \chi(x) \right] \text{ space}$$

$$-\frac{1}{2} \sum_x \eta_0(x) \quad \left[\bar\chi(x) U_0(x) e^{\mu} \chi(x+\hat{0}) - \bar\chi(x+\hat{0}) U_0^+(x) \chi(x) e^{-\mu} \right] \text{time}$$

$$-\frac{1}{3} \sum_x' 6/g^2 \sum_{j,\kappa=0}^{4}{}' \left[1 - \mathrm{Re}\,\mathrm{Tr}\, U_{\nu\mu}(x) \right] + \sum_x' m \bar\chi(x) \chi(x)$$

Note • inclusion of chemical potential μ:

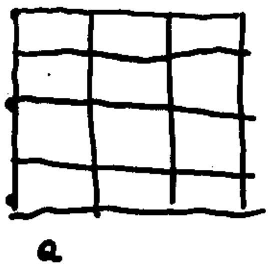
$\xrightarrow[U]{e^{\mu}}$: Forward propagation enhanced

$\xleftarrow[U^+]{e^{-\mu}}$: Backward propagation suppressed

• Temperature straightforward

The lattice

Scalar Fields on the lattice



$$\varphi(x) \to \varphi(x_n)$$

$$\partial_\mu(\phi(x)) = \frac{\phi(x+a_\mu) - \phi(x)}{a}$$

$$\int \partial_\mu \varphi \, \partial_\mu \varphi + \frac{1}{2} m^2 \varphi^2 - V(\phi) = \sum_n' \left\{ \left(\frac{\varphi_n - \varphi_n}{2a} \right)^2 + \right.$$

$$\frac{1}{2} m^2 \varphi_n^2 + \sum_n' V(\varphi_n)$$

$L \to \infty$; a : fixed

$$\tilde{\varphi}_n = \frac{a^4}{8\pi^4} \int_{-\pi/2}^{\pi/2} \hat{\varphi}(p) \, e^{-i p(n) a} \, d^4 p$$

- a : ultraviolet cutoff

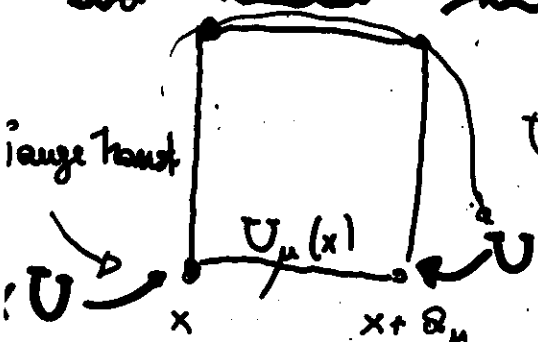
Continuum limit $a \to 0$

Gauge Fields on the Lattice

- Gauge Invariance Particularly simple on the lattice!

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F_{\mu\nu} \qquad F^{\alpha}_{\mu\nu} = \partial_\nu A^{\alpha}_{\mu} - \partial_\mu A^{\alpha}_{\nu} - g f^{\alpha\beta c} A^{\beta}_{\mu} A^{\gamma}_{\nu}$$

- Gauge Invariant Object on the lattice



$$U_\mu(x) = U(x, x+a_\mu) = \exp\left[ig \int_x^{x+a_\mu} A_\mu \, dX_\mu\right] \in SU(3)$$

Plaquette: $\mathrm{Tr}\, U_\mu(x) U_\nu(x+\hat{\mu}) U^{+}_\mu(x+\nu) U^{+}_\nu(x) = U_{\mu\nu}(x)$

- Build Action

$S = S$ (gauge invariant objects) such that

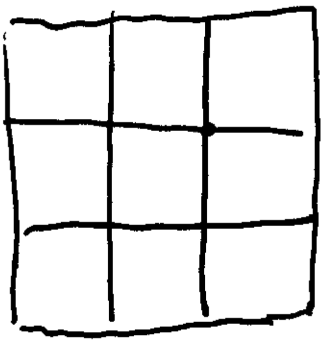
$$\lim_{a \to 0} S = S_{QCD}$$

$$\boxed{\text{N.B.} \quad a \to 0 \;\Downarrow\; g \to 0 \text{ Asympt Freedom}}$$

$$S_g = \sum_x{}' \, 6/g^2 \left[1 - \mathrm{Re}\,\mathrm{Tr}\, U_{\nu\mu}(x)\right] / 3$$

$$\lim_{a \to \phi} S_g \Rightarrow S_{\text{yang mills}}$$

Fermions on the lattice

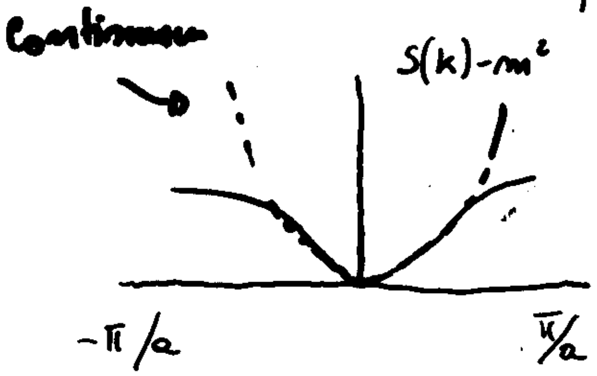


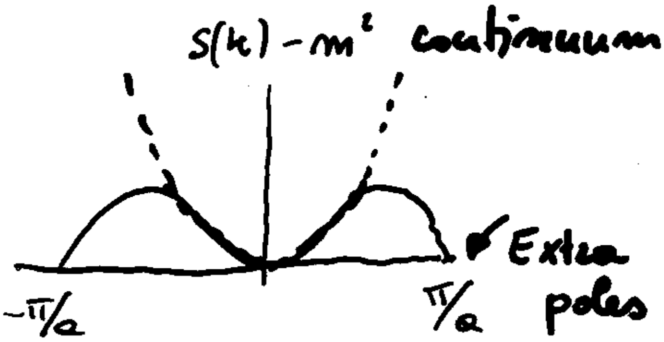
As bosons

$\psi_n = \psi(x_n)$

but:

Extra complications due to different dispersions relations:





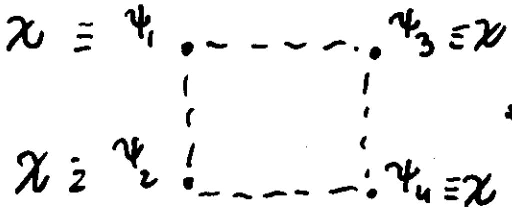
BOSONS : ~OK

FERMIONS :

EXTRA SPECIES

Removed by

e.g. Staggered Fermion Trick



Dirac Spinor 'distributed' on different sites

$g = \infty$:

'Main tricks'

- Drop the gauge Action: $\cancel{\frac{1}{g^2} \mathrm{Tr}\, \Box}$
- Perform independent link integration

$$Z = \int dU\, d\bar{\chi}\, d\chi\; e^{-S(\bar{\chi}, \chi, U)}$$

Special links are integrated first

$$Z = \int \prod_{t\,\text{links}} dU_t\; d\bar{\chi}\, d\chi\; e^{-\frac{1}{4N} \sum'_{\langle x,y \rangle} \underbrace{\bar{\chi}(x)\chi(x)\bar{\chi}(y)\chi(y)}_{4\text{-Fermion}}} \cdot e^{-S_t}$$

· Assume homogeneous χ and linearize by

$$e^{-\frac{1}{4N} \bar{\chi}(x)\chi(x)\;\bar{\chi}(y)\chi(y)} = \int d\lambda\; e^{-\frac{N}{d}\lambda^2 - \bar{\chi}(x)\lambda\chi(x)}$$

c.f. Gross-Neveu

$$\langle \bar{\chi}\chi \rangle = \frac{2N}{d}\lambda \implies \boxed{S = \bar{\chi} M \chi}$$

• Integrate over $d\bar{\chi}\, d\chi$

$$\mathcal{Z} = \int d\bar{\chi}\, d\chi\, dU_t\, e^{-\bar{\chi} M \chi}\, d\lambda \longrightarrow \bar{\Psi} \not{\partial} \Psi \; : \; m_{ass} = m + \lambda$$

$$\mathcal{Z} = \int \underbrace{\left[1 - d \;\; QCD\right]^{V}}_{Z_0} d\lambda$$

gives Effective Potential for λ, or, equivalently

for $\langle \bar{\chi}\chi \rangle = \frac{2 \cdot N}{d} \lambda = 2\lambda \quad [N=3, d=3]$

Temperature^{-1}

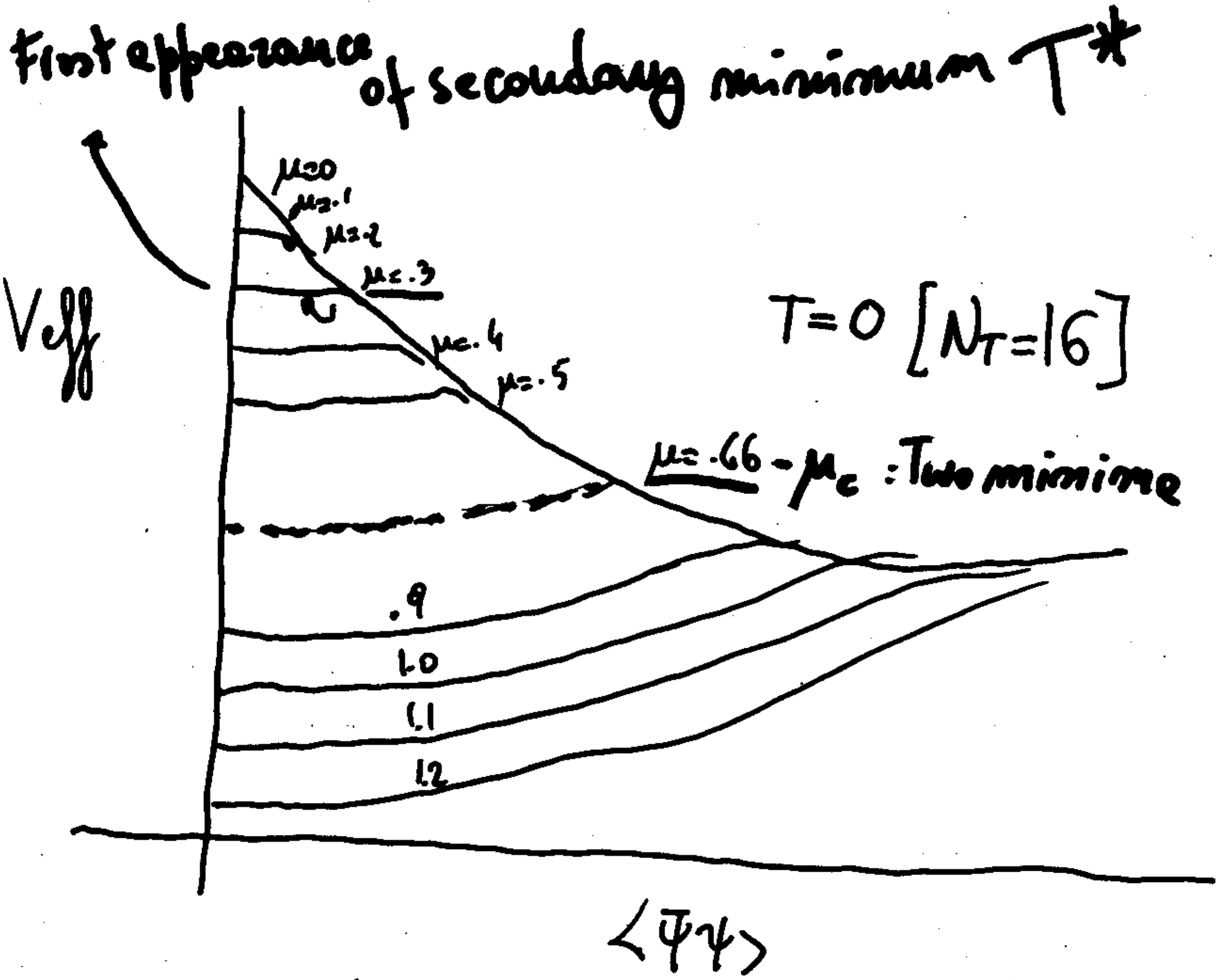
$$\mathcal{Z}_0 = 2\cosh\left(3 N_t \mu\right) + \frac{\sinh\left[(3+1) N_t \lambda'\right]}{\sinh N_t \cdot \lambda'}$$

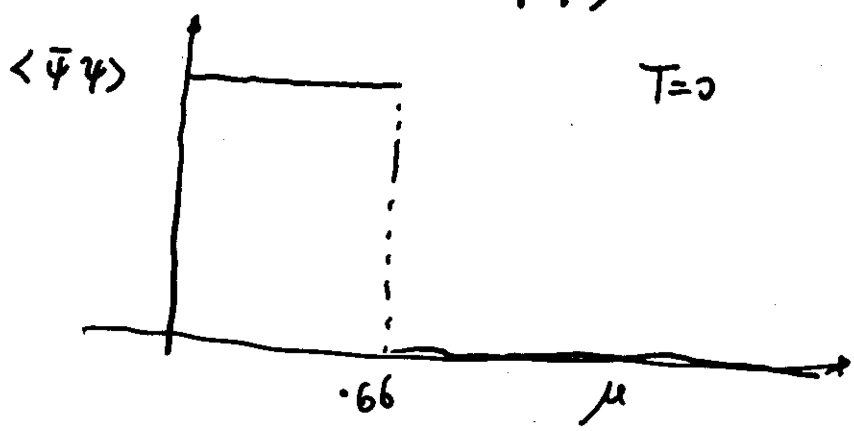
chemical potential

canonical part. function

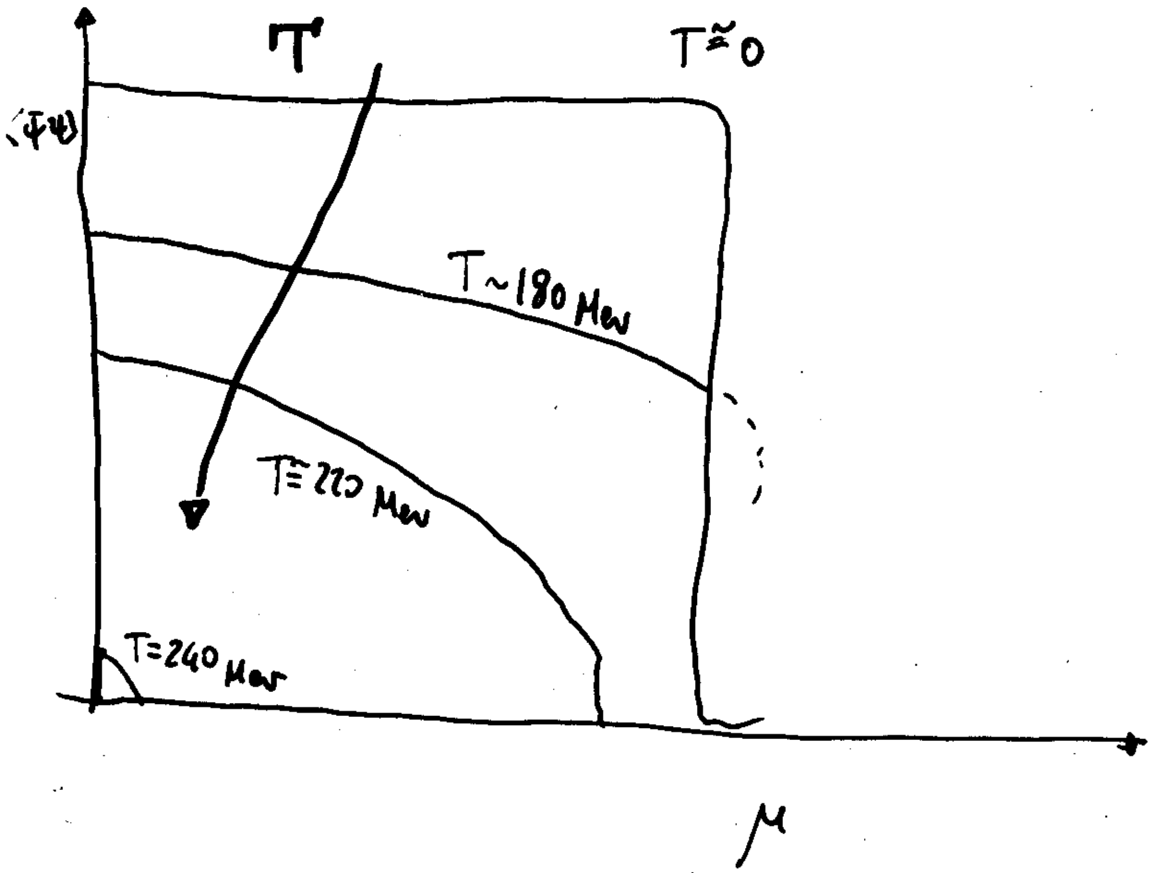
• Fugacity expansion: $\mathcal{Z}_0 = \sum_{-V}^{V}{}' C_n\, e^{3 N_t \mu n}$ $\hookrightarrow$ Colorsinvariance: only multiple of 3 allowed!

• $V_{eff} = -\ln \mathcal{Z}_0 = V_{eff}\left(\langle \bar{\Psi}\Psi \rangle, \mu, T\right)$

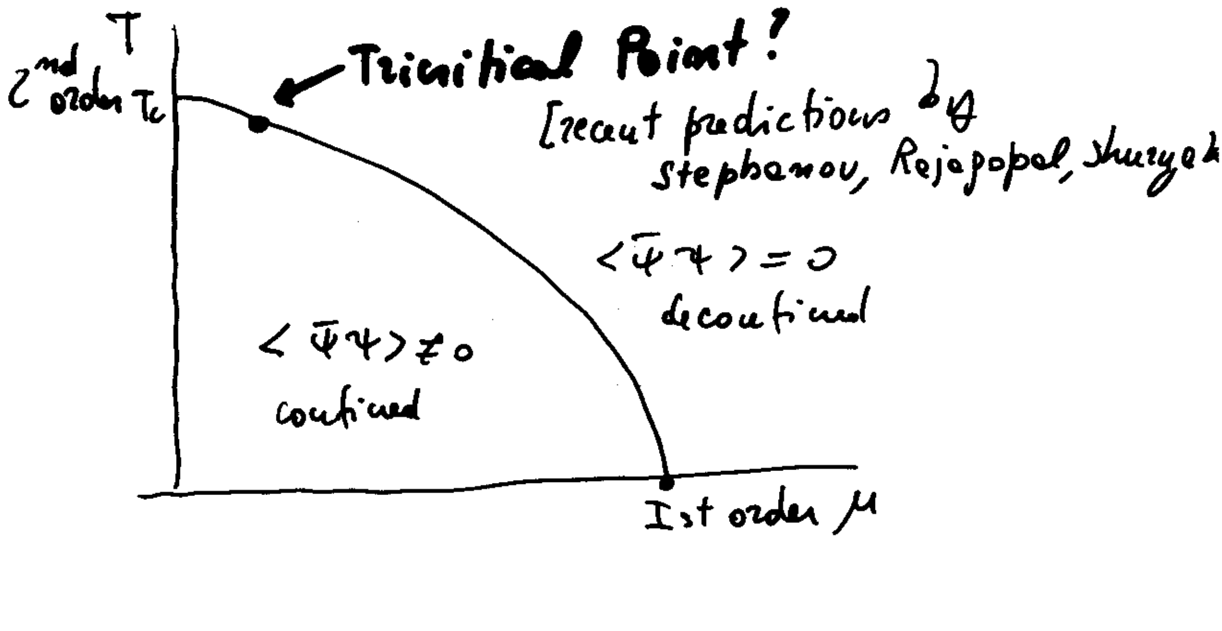

First appearance of secondary minimum T^*
V_{eff}
$\mu=0$
$\mu=.1$
$\mu=.2$
$\mu=.3$
$\mu=.4$
$\mu=.5$
$T=0$ [$N_T=16$]
$\mu=.66 - \mu_c$: Two minima
.9
1.0
1.1
1.2
$\langle\bar\Psi\psi\rangle$


$\langle\bar\Psi\psi\rangle$
$T=0$
.66
μ

Results from $g = \infty$ analysis of QCD



- Strong 1st order chiral P.T. at $T=0$ $\mu \geq \mu_c$
- Temperature smoothens the transition
- Continuous transition at $\mu = 0$; $T = T_c$



- Towards the continuum limit?

$$\mu_c(g=\infty) < m_B$$

- $1/g^2$ corrections can be included:

$\beta = 1/g^2$	$\frac{1}{3} M_N$	μ_{critical} (quark)
0	.972	.571
1	.936	.586
3	.850	.617
5	.735	.648

Lattice Units

get closer

continuum

- Improved s.c. Techiques an alternative to numerical simulations?

Two color QCD - high μ (and T)

- Remember:

$$S = (\bar{\Psi}\ \Psi)\begin{pmatrix} j & M \\ M^+ & j \end{pmatrix}\begin{pmatrix} \bar{\Psi} \\ \Psi \end{pmatrix}$$

$SU(2N_f)$: Enlarged Symmetry $\rightarrow$ broken to $Sp(2N_f)$ at $\mu=T=0$

- By increasing T
  Restore $SU(2N_f)$
- By increasing μ
  - Excite baryons = pions when $\mu = \frac{m_B}{2} = \frac{m_\pi}{2}$!
  - Baryons are bosons! [qq]
  - They can create qq condensate which still breaks χ symmetry!

- Symmetries VERY different from $SU(3)$!
  Colorless diquark condensate [$SU(2)$]
  vs
  Colored diquark condensate [$SU(3)$]
- Dynamics might be more similar?

SU(2)$_c$ Lattice Gauge Theory

Symmetries and Spectrum (99)

Simon Hands
John Kogut
M-P. L
Susan E. Morrison

2. Action and symmetries

We will outline plausible patterns of symmetry breaking for SU(2) lattice gauge theory with fermions in the fundamental representation of the gauge group. We emphasize the distinction between the model with continuum fermions in a pseudoreal representation (such as the 2 of SU(2)) which has the symmetry breaking pattern $SU(2N_f) \to Sp(2N_f)$ in contrast to the lattice model with staggered fermions which, as we shall outline below, has the unorthodox pattern $\underline{SU(2N_f) \to O(2N_f)}$. We also consider the consequences of introducing a chemical potential corresponding to non-zero baryon density.

Peskin-Verbaarschot

Let us start with the kinetic term in the lattice action for a gauged isospinor doublet (2) of staggered fermions $\chi, \bar{\chi}$. For clarity we will consider $N = 1$ flavours, although this will be generalised when the numerical simulations are discussed,

$$S_{\text{kin}} = \sum_{x,\nu=1,3} \frac{\eta_\nu(x)}{2} \left[\bar{\chi}(x)U_\nu(x)\chi(x+\hat{\nu}) - \bar{\chi}(x)U_\nu^\dagger(x-\hat{\nu})\chi(x-\hat{\nu})\right]$$
$$+\sum_x \frac{\eta_t(x)}{2} \left[\bar{\chi}(x)e^{\mu}U_t(x)\chi(x+\hat{t}) - \bar{\chi}(x)e^{-\mu}U_t^\dagger(x-\hat{t})\chi(x-\hat{t})\right]. \quad (2.1)$$

Symmetry: SU(2) at $\mu=0$ $N_F=1$

The U's are 2×2 complex matrices acting on colour indices. The chemical potential enters via the timelike links in the standard way. It is straightforward to rearrange (2.1), using the Grassmann nature of $\chi, \bar{\chi}$, the fact that $\eta_\mu(x) \equiv (-1)^{x_0+\ldots+x_{\mu-1}} = \eta_\mu(x \pm \hat{\mu})$, and the property $\tau_2 U_\mu \tau_2 = U_\mu^*$, where τ_2 is a Pauli matrix:

$$S_{\text{kin}} = \sum_{x\,\text{even},\,\nu=1,3} \frac{\eta_\nu(x)}{2} \left[\bar{X}_e(x)U_\nu(x)X_o(x+\hat{\nu}) - \bar{X}_e(x)U_\nu^\dagger(x-\hat{\nu})X_o(x-\hat{\nu})\right]$$
$$+\sum_{x\,\text{even}} \frac{\eta_t(x)}{2} \left[\bar{X}_e(x)\begin{pmatrix} e^{\mu} & 0 \\ 0 & e^{-\mu}\end{pmatrix} U_t(x)X_o(x+\hat{t})\right.$$
$$\left.-\bar{X}_e(x)\begin{pmatrix} e^{-\mu} & 0 \\ 0 & e^{\mu}\end{pmatrix} U_t^\dagger(x-\hat{t})X_o(x-\hat{t})\right] \quad (2.2)$$

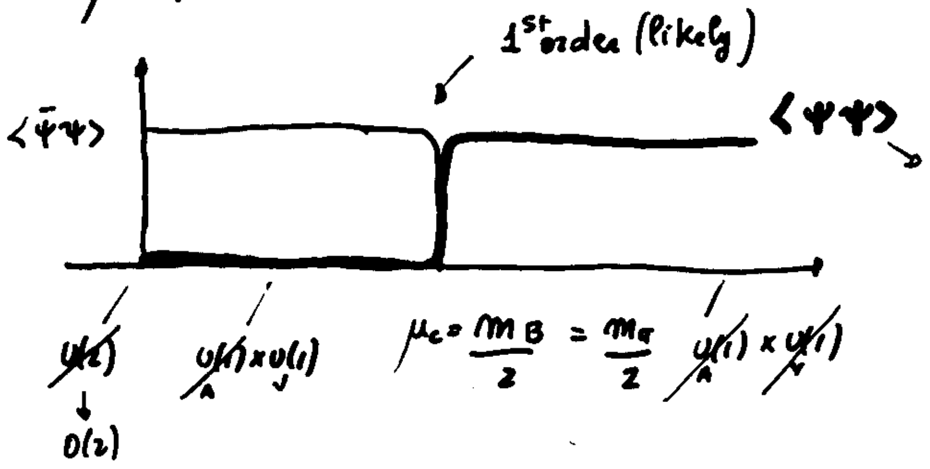
with the definitions

$$X = \begin{pmatrix} \chi_o \\ -\tau_2\bar{\chi}_o^{tr} \end{pmatrix} \begin{matrix} \to \text{quark} \\ \to \overline{\text{quark}} \end{matrix}$$

– Two Color QCD –

Lattice Symmetries → $\mu, T \neq 0$

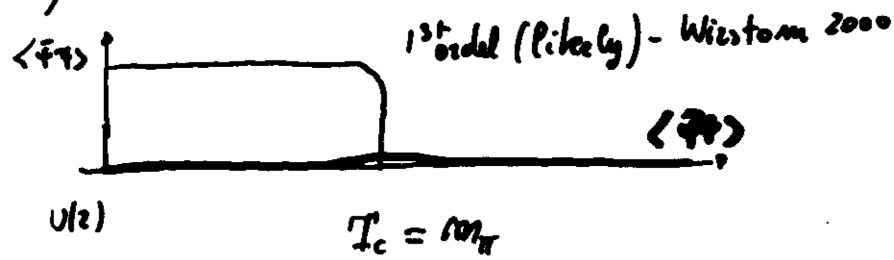
I $\mu \neq 0;\ T = 0$



→ χsymm still broken

NB $U_A(1) \times U_V(1)$ same as $SU(3);\ \mu = 0$

I $\mu = 0;\ T \geq 0$



. Differences with SU(3)

- Massless Baryons.
- Diquark condensate colorless (colored) in SU(2) (SU(3))
- High T and High μ Transitions look 'less different' in SU(2): both could be 1st order, both happen $\simeq m_\pi$???? ????
- $$QCD_2\left(\mu_B, \mu_I = 0\right) = QCD_2\left(\mu_B = 0, \mu_I\right)$$

SU(2) symmetry, and symmetry breaking

- Implications on the propagators

$\mu = T = 0$

$G_\pi = G_{\text{scalar diquark}}$

$\updownarrow \quad \langle\bar{\psi}\psi\rangle \quad \updownarrow$

$G_\delta = G_{\text{pscalar diquark}}$

$\xrightarrow{T > T_c}$ $G_\pi = G_{sd} = G_\delta = G_{psd}$

$\downarrow \mu > \mu_c$

$G_\pi = G_\delta$

$G_{\text{scalar diquark}}$

$\updownarrow \langle\psi\psi\rangle$

$G_{\text{pscalar diquark}}$

masses look however degenerate

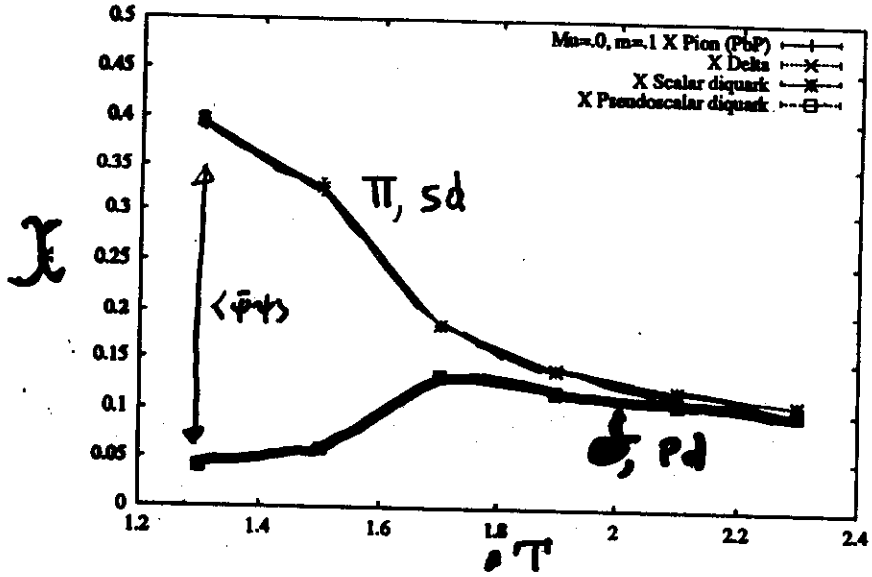


FIG. 7. Scalar and pseudoscalar susceptibilieties as a function of β demonstrating chiral symmetry restoration at high temperature

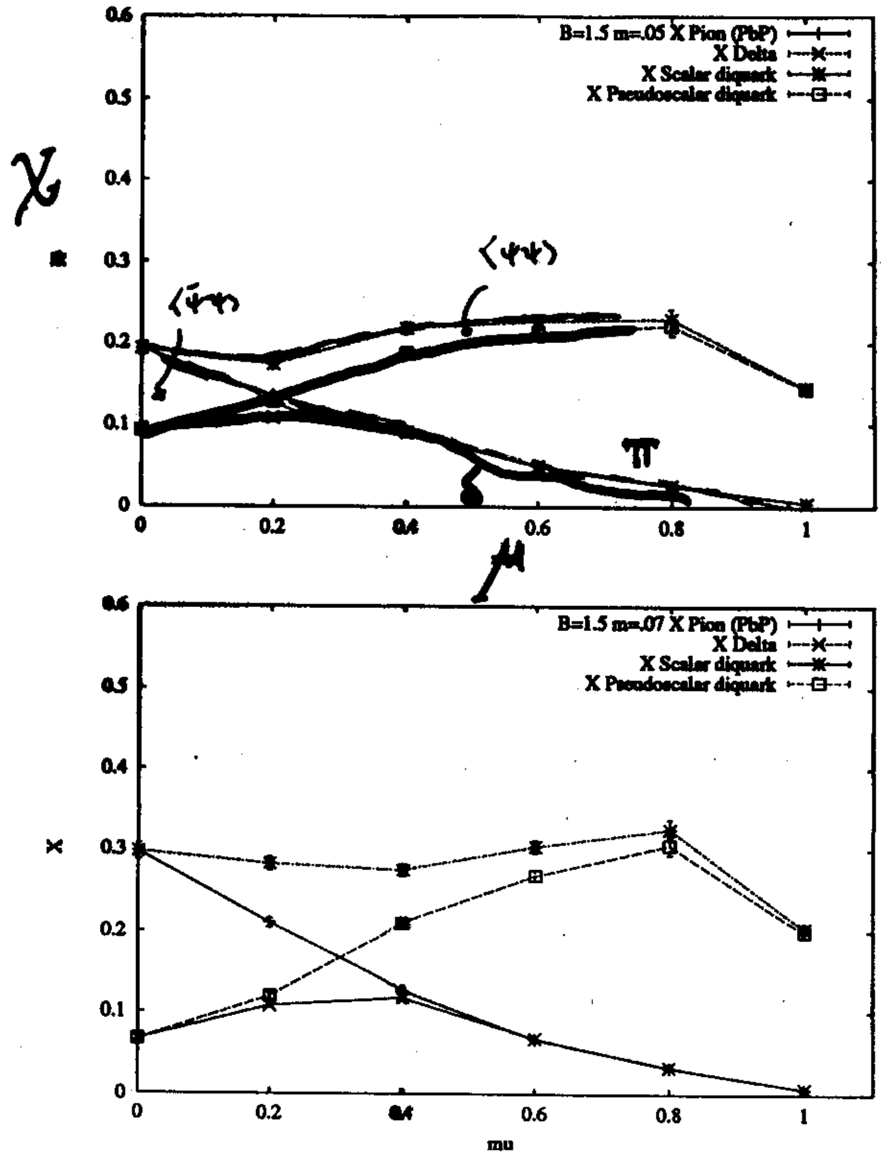


FIG. 8. Scalar and pseudoscalar susceptibilieties for mesons and diquarks a function of μ at $\beta = 1.5$ and mass $= .05$ (upper) and mass $= .07$. See text for discussions

Deconfinement with Matter Fields

INTERQUARK POTENTIAL

M.P.L

from Polyakov loops

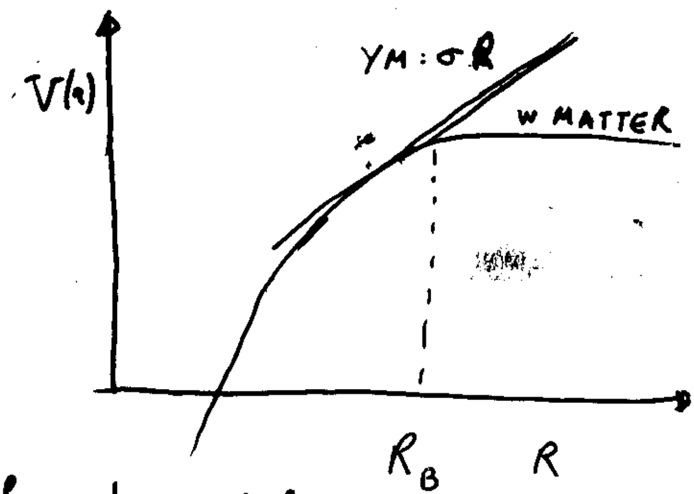
$$\cdot\ \langle P(\vec{0})\, P^{+}(\vec{R})\rangle \propto e^{-V(R,T)/T} \qquad \left[= \langle P(\vec{0})\, P(\vec{R})\rangle\right]$$

- Pure Yang Mills [$Z(N_c)$ symmetry] ← O.P.

$$\langle P(\vec{0}) \cdot P^{+}(\vec{R})\rangle \underset{R\to\infty}{\propto} e^{-V(\infty,T)/T} \propto \langle |P|\rangle^2 = \begin{cases} 0 & \text{confined} \\ \neq 0 & \text{deconfin.} \end{cases}$$

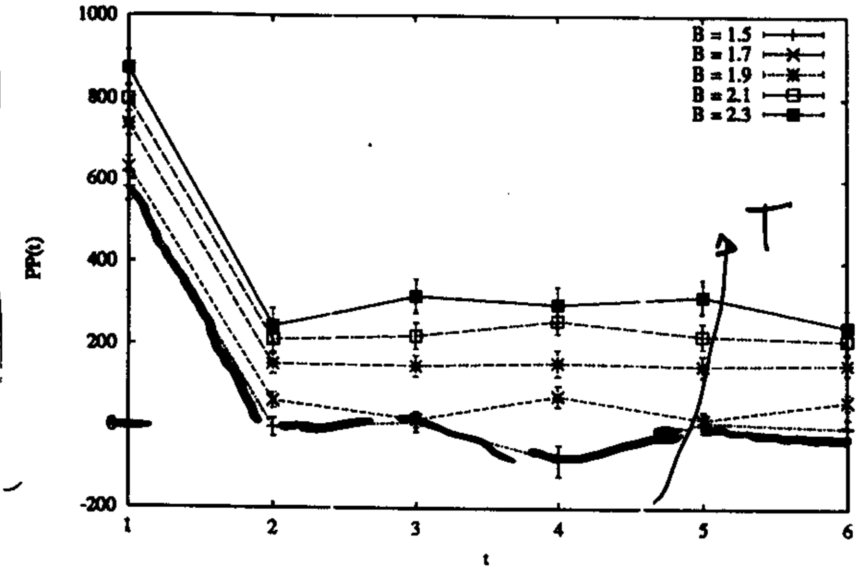
- With matter [$\cancel{Z(N_c)}$]

$$Q\bar{Q} \to (Q\bar{q})(q\bar{Q})$$



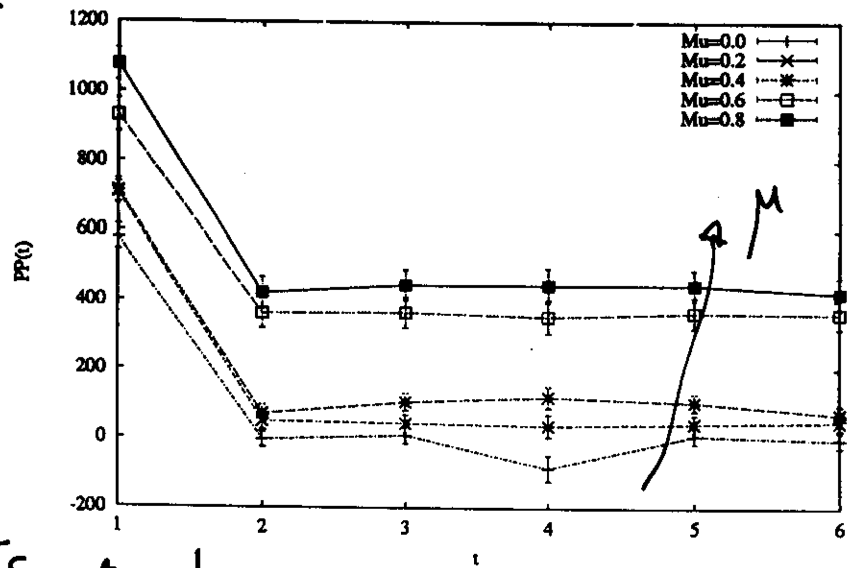
- Scale: $V(R_B) = 2M_{Q\bar{q}}$ Alexandrou et al.
- "Easier" with temperature — de Tar, Kaczmarek, Karsch, Laermann
- Even easier with finite density? Engels, Kaczmarek, Karsch, Laermann
- Strategy
  - → Look for sudden changes of screening properties
  - → Contrast finite T, finite μ behaviour.

Standard long range Screening in a Hot System



String Breaking at high T

First Direct Observation of long Range Screening in a Dense System!

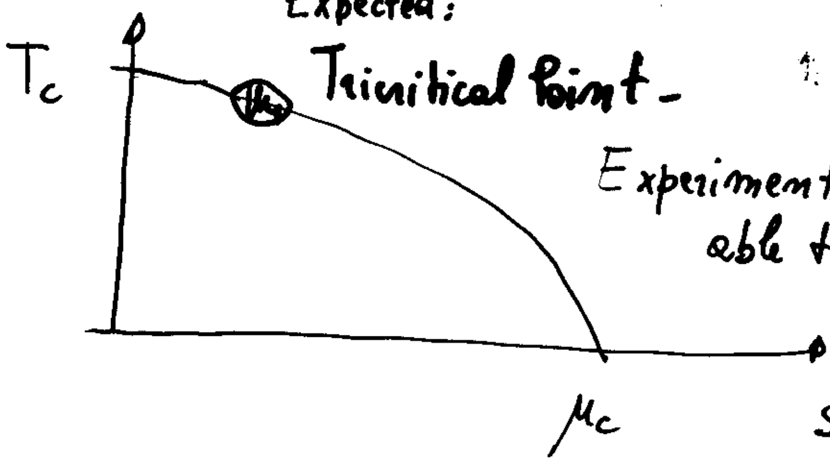


String Breaking at high μ

FIG. 4. Correlations of the zero momentum Polyakov loop as a function of the space separation. The upper diagram is for $\mu = 0$, and β as indicated. The lower part is for $\beta = 1.5$ and μ as indicated. In both cases we observe long range ordering possibly associated with deconfinement

Summary finite density QCD

- Expected:



Experiments might be able to 'see' it at RHIC or LEP II

Stephanov, Rajagopal, Shuryak ; pp

- Calculational tools ?

  - → • pushing strong coupling expansion might be worthwhile

  - → • When μ is purely imaginary, action is real in $SU(3)$! Alford, Kapustin, Wilczek ; pp
    - Might work at large $[T \simeq T_c]$ –
    - Seems to be the case at least at strong coupling M.-P. L

VI High T electroweak phase transition

Perturbation theory

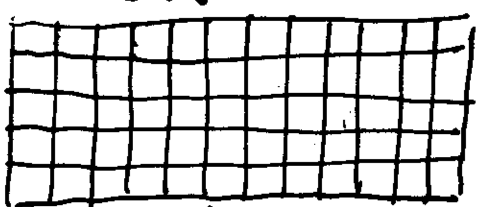
From exact $S \to V_{eff} = \frac{1}{2}\gamma\left(T^2 - T_*^2\right)\phi^2 - \frac{1}{3}\alpha T \phi^3 + \frac{1}{4}\lambda\phi^4$

$$T_*^2 \downarrow m_H^2/4\gamma^2$$

$$T_c = \frac{m_H}{\sqrt{2\gamma - 4\alpha^2/9\lambda}} \to \text{• Linear dependence } T_c \text{ on } m_H$$

4-d Lattice analysis

$t = 1/T$



$1/m_{PHYS} > a$: ultraviolet cutoff

$d=3$ space, ideally ∞

$$\mathcal{L} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + (D_\mu \phi)^\dagger (D_\mu \phi) - m^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

3-d Effective analysis

$$\mathcal{L} = \frac{1}{4} F^a_{ij} F^a_{ij} + (D_i \phi)^\dagger (D_i \phi)^\dagger + m_3^2 \phi^\dagger \phi + \lambda_3 \left(\phi^\dagger \phi\right)^2$$

$m_3^2 \downarrow T$; $\lambda_3 \to m_{Higgs}$

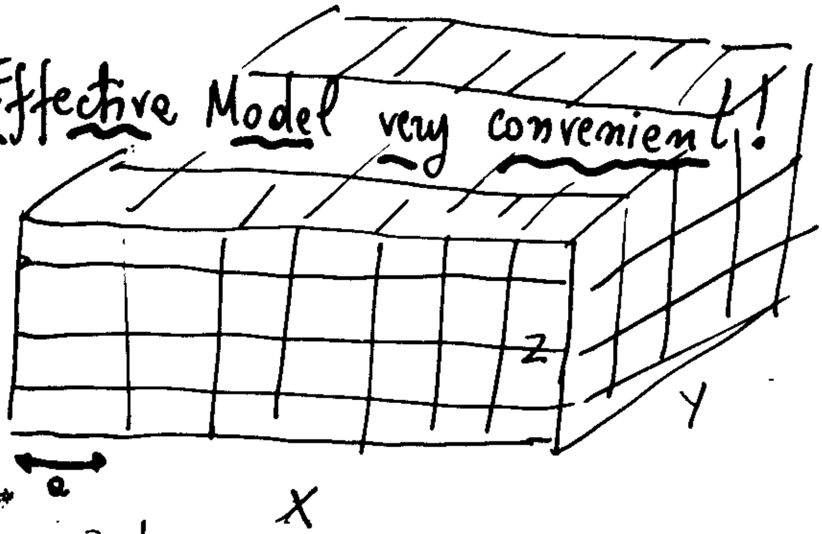
N.B.: could be general!

$$\left(g_3, m_3, \lambda_3\right) \longleftrightarrow \left(\text{4-d model parameters}\right)$$

S.M. Kajantie, Laine, Rummukainen, Shaposhnikov

M.S.S.M. Laine, Rummukainen

- <u>3-d</u> <u>Effective</u> <u>Model</u> <u>very</u> <u>convenient</u>!



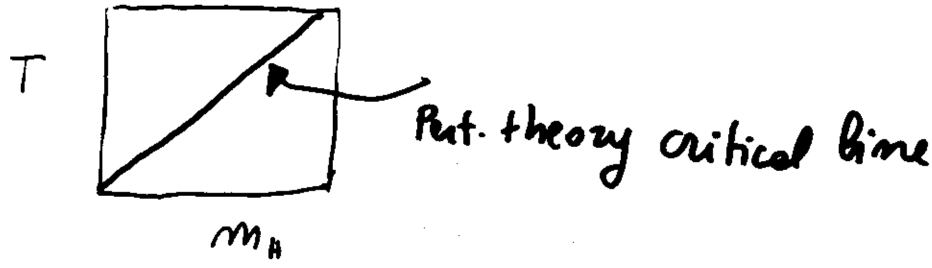
- Of course, 3d is cheaper than 4d
- Heavy modes, requiring small a, are gone!

  <u>Better</u> <u>than</u> <u>4-d</u>!
- light modes are still there

  <u>Much better</u> <u>than</u> <u>Pert. Theory</u>:

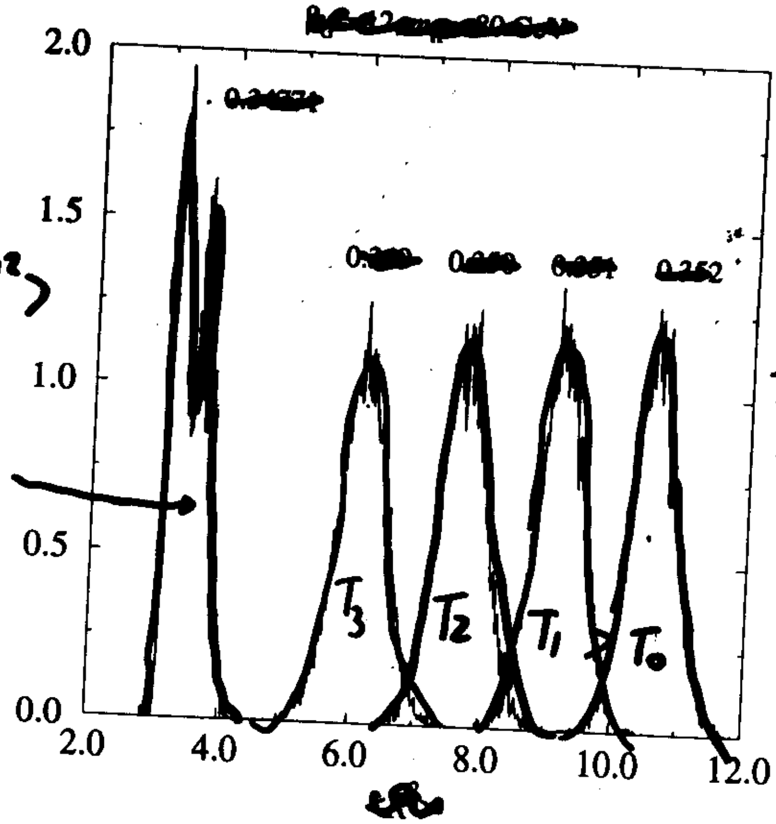
  For instance, can handle a light W at the P.T.

- Either 4-d and 3-d:
  - scan T, m_H space searching for phase transitions:



$m_H \simeq 80$ GeV

$-V_{eff}(\langle|\phi|^2\rangle)$



Two state signal: proximity of a p.T.

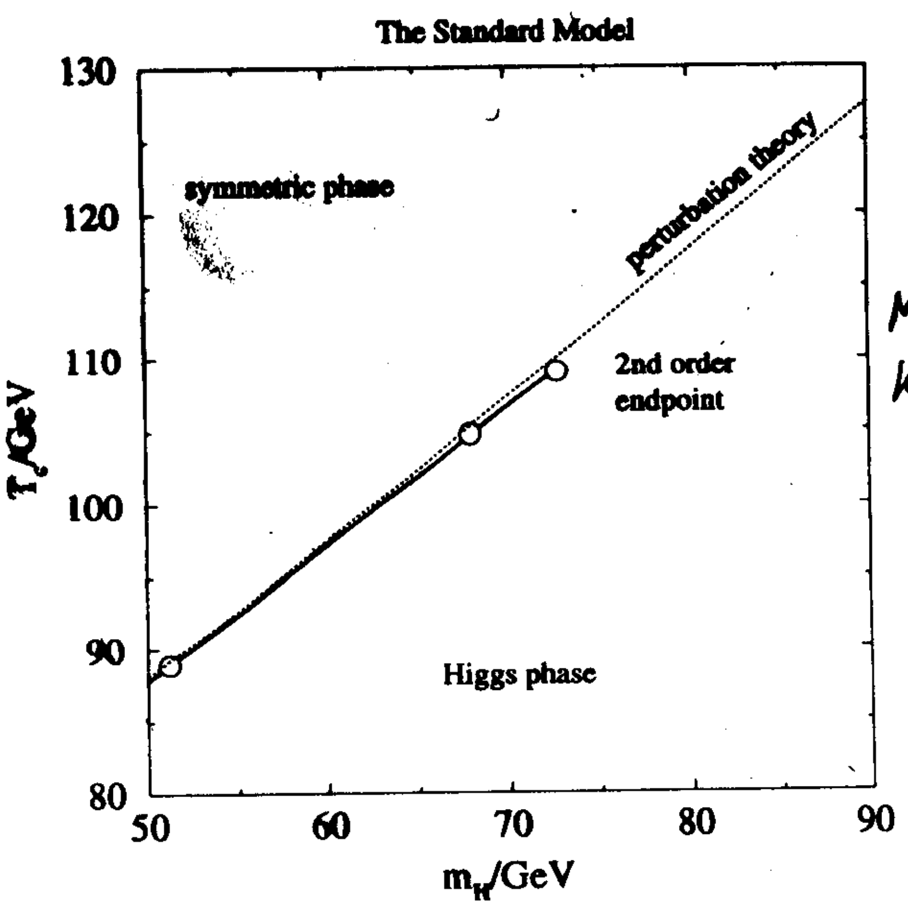
$T_0 < T_1 < T_2 < T_3 < T_4 \simeq T_c$

$\langle|\phi|^2\rangle$

Kajantie, Laine, Rummukainen, Shaposhnikov

High T EW Transition – Summary

- The phase diagram in the T, m_H plane.



Mikko Laine &
Kari Rummukainen, 98
[still ok]

Figure 1. The phase diagram of the Standard Model. The non-perturbative endpoint location has been studied with 3d simulations in [11–14] and with 4d simulations in [15–18]. In perturbation theory (dotted line), the transition is always of the first order.

COMMENTS

ELEGANCE AND POWER OF DIMENSIONAL REDUCTION AND UNIVERSALITY

LATTICE CALCULATIONS MANDATORY TO SETTLE QUALITATIVE ISSUES