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Embedding Gestalt Laws in Markov Random Fields

S.C. Zhu
Computer and Information Science Department
Ohio State University
Columbus, OH 43210-1277
United States of America

Embedding Gestalt Laws in Markov Random Fields

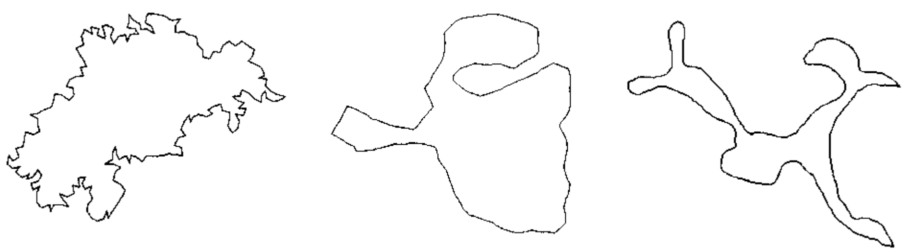
– a theory for shape modeling and perceptual organization

Song Chun Zhu

Computer and Information Science Department
Ohio State University

Generic Shape Modeling

Early human vision favors certain shapes and configurations without high level identification.



Random 2D shapes sampled from three models with decreasing entropy.

Existing Theories for Perceptual Organization

1, Gestalt psychology

–search for the best, simplest and most stable shape.

2, Likelihood principle (Helmholtz)

–assign a high probability for grouping two elements, if the placement of the two elements is less likely to be **accidental arrangement**

3, Minimum description length principle

–achieve the shortest coding length.

A key issue for perceptual organization is shape modeling.

A Unifying Theme

The three theories can be unified provided that

1. the likelihood probability accounts for the complexity and frequency of configurations occurring in nature, and thus yields the minimum coding length.

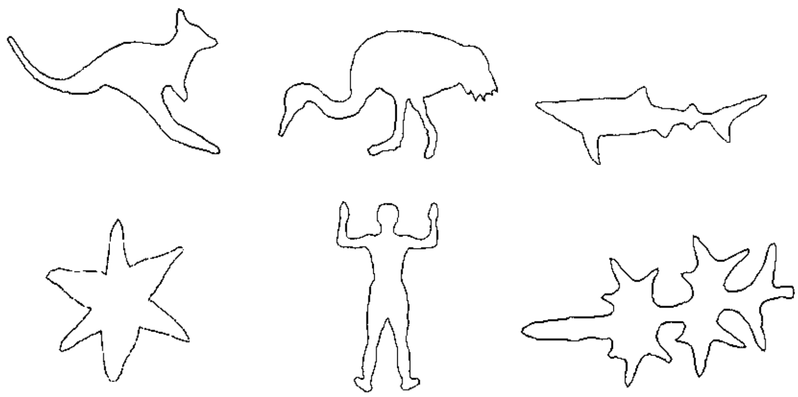
2. the structures of the likelihood probability reflects the Gestalt laws, and the Gestalt laws is effective only if they correspond to statistical regularities in real shapes.

According to the shape model $p(\Gamma)$, a higher probability means a better, simpler and more stable shape !!

Goal of Shape Modeling

Input: a set of shapes sampled from the true shape distribution $f(\Gamma)$.

$$\{\Gamma_1^{\text{obs}}, \Gamma_2^{\text{obs}}, \cdots \Gamma_M^{\text{obs}}\}$$



Output: a generic shape model $p(\Gamma)$.

What is a generic shape model?

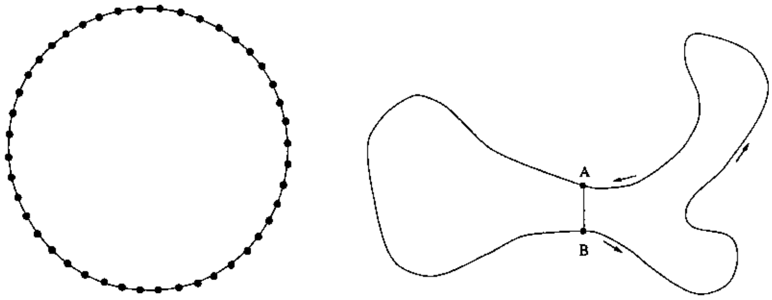
$p(\Gamma)$ is assigned to continuous curve $\Gamma(s)$, $s \in [0, 1]$.

$$\Gamma = ((x_0, y_0), (x_1, y_1), ..., (x_N, y_N))$$

$p(\Gamma)$ is defined on a manifold in $2N$ space, under constraints: being a simple closed curve, $ds = 1/N$.

1. $p(\Gamma)$ is invariant to translation, rotation, scaling.
2. $p(\Gamma)$ should capture the generic features of shape.

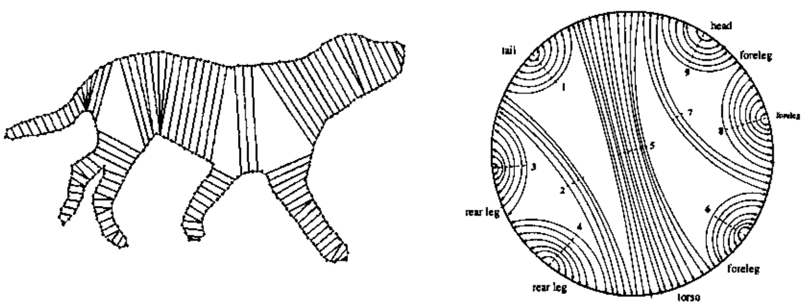
Region-based Features are Important



The Shape Features

The Gestalt laws:

co-linearity, co-circularity, proximity, parallelism, symmetry, closure, ...



$$\phi^{(1)}(s) = \kappa(s) = \frac{d\theta(s)}{ds}, \quad \text{and} \quad \phi^{(2)}(s) = \nabla\kappa(s) = \frac{d^2\theta(s)}{ds^2}.$$

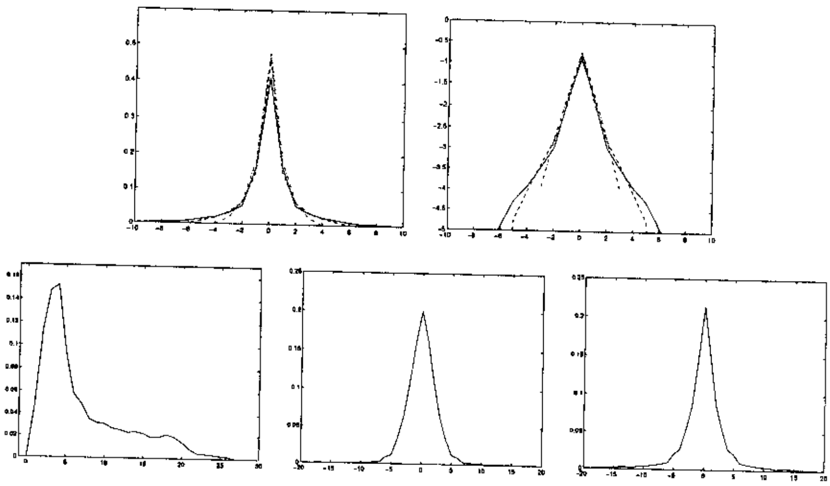
$$\phi^{(3)}(s) = r(s), \quad \phi^{(4)}(s) = \nabla r(s) = \frac{d\,r(s)}{ds}, \quad \phi^{(5)}(s) = \nabla^2 r(s) = \frac{d^2 r(s)}{ds^2}.$$

Statistics of Animate Object Shapes

Given a feature ϕ, we measure the empirical histogram - $\mu_{\text{obs}}(z)$,

- averaged over all observed shapes.

μ_{obs} is a one-dimensional marginal distribution of $f(\Gamma)$.



Shape Modeling by Maximum Entropy Principle

Given K features, a shape model $p(\Gamma)$ should duplicates the marginal distributions of the true model $f(\Gamma)$.

Of those models that satisfy the constraints, we pick up the one that has the maximum entropy.

$$p(\Gamma) = \frac{1}{Z}\exp\{-\int \lambda^{(1)}(\kappa(s)) + \lambda^{(2)}(\nabla\kappa(s)) + \lambda^{(3)}(r(s)) + \lambda^{(4)}(\nabla r(s)) + \lambda^{(5)}(\nabla^2 r(s))ds + \gamma\|B\|\}.$$

where $\lambda^{(\alpha)}(), \alpha = 1, 2, ...$ is the Lagrange multipliers, and α is the index of the feature.

Deformable templates are trivial solutions, which choose the entire object shape as features.

Feature Selection by the Minimum Entropy Principle

The marginal distribution of $p(\Gamma)$ with respect to a feature $\phi^{(\beta)}$ is denoted by $\mu_{\text{syn}}^{(\beta)}$.

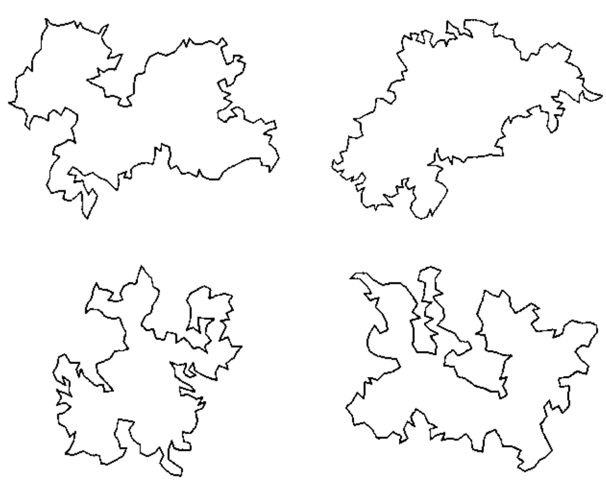
$$d(\mu_{\text{obs}}^{(\beta)}, \mu_{\text{syn}}^{(\beta)})$$

is a measure of the **non-accidental statistics** for $\phi^{(\beta)}$.

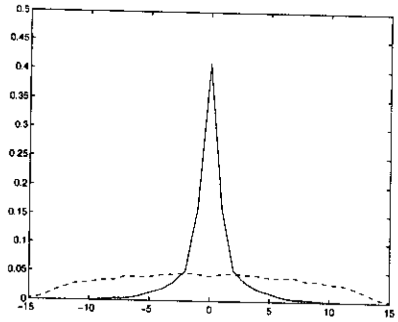
Choose the feature by maximizing the non-accidental statistics.

$$\begin{aligned}
\phi^{(\beta*)} &= \arg\max_{\beta} \ d(\mu_{\text{obs}}^{(\beta)}, \mu_{\text{syn}}^{(\beta)}) \\
&= \arg\max_{\beta} \ D(f \,||\, p) - D(f \,||\, p_{+}) \\
&= \arg\min_{\beta} \ \text{entropy}(p).
\end{aligned}$$

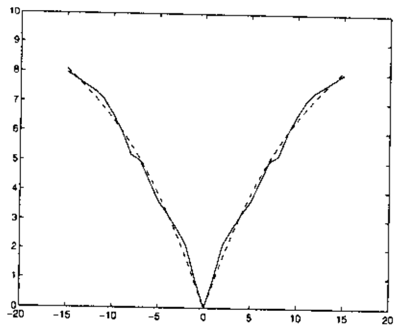
Random Samples of $p(\Gamma)$, $K = 0$



The Non-accidental Statistics for Curvature

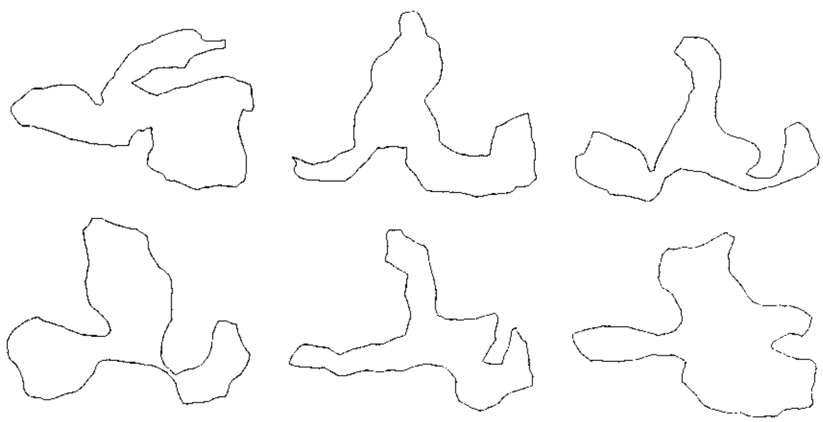


μ_{obs} vs. μ_{syn}

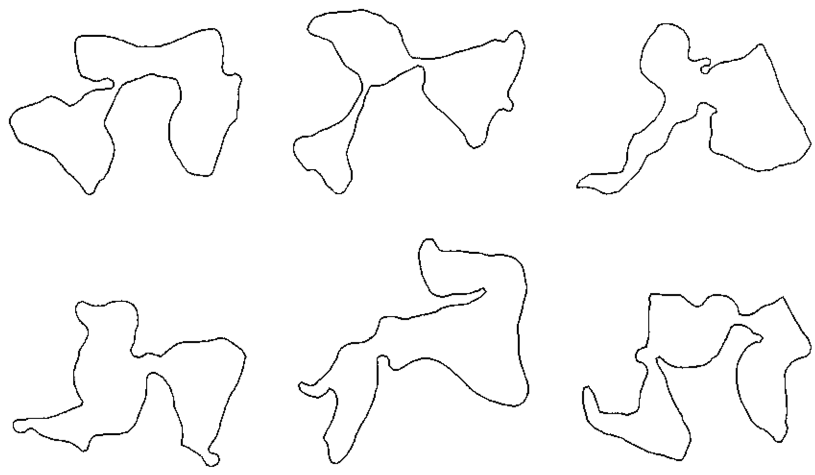


the learned $\lambda()$

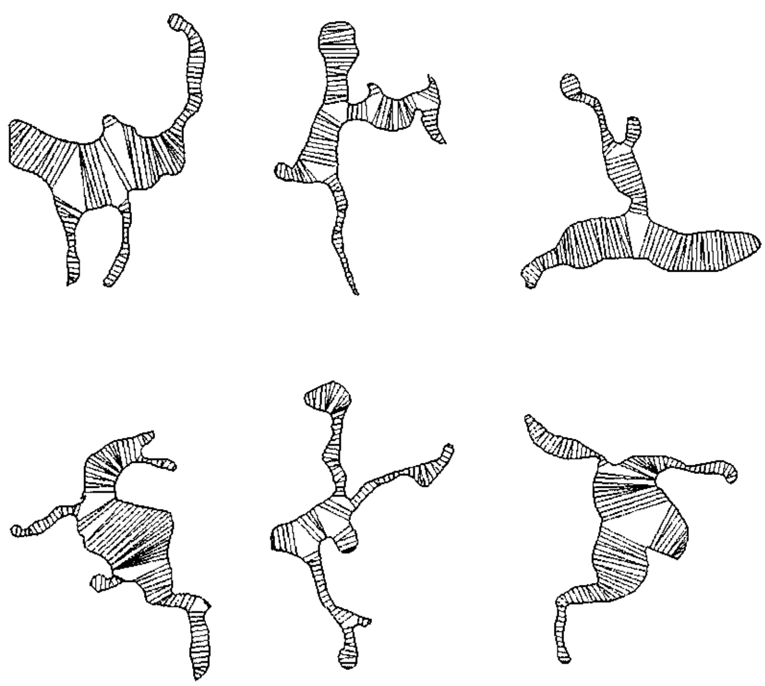
Random Samples of $p(\Gamma)$, $K = 1$



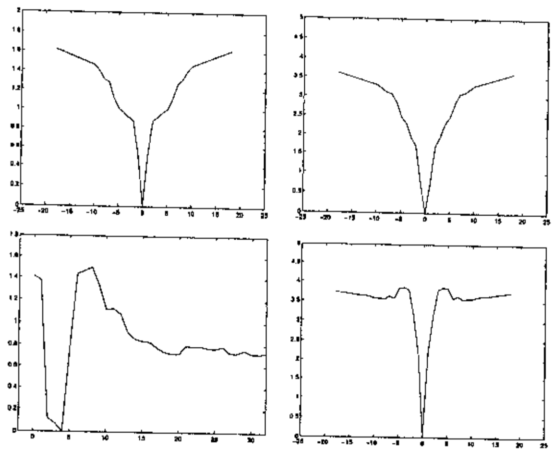
Random Samples of $p(\Gamma)$, $K = 2$



Random Samples of $p(\Gamma)$, $K = 4$

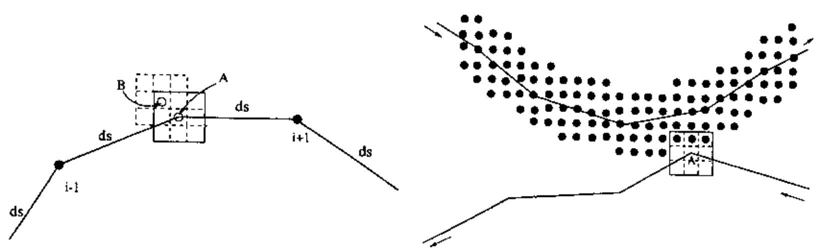


The Learned Potential Functions for 4 Features



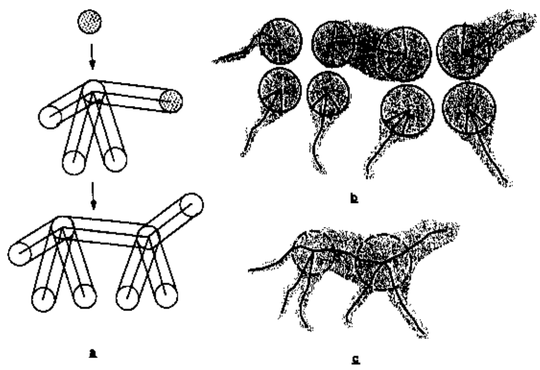
Stochastic Sampling by Markov Chain Monte Carlo

The sampling process:



The Role of Perceptual Organization

The essential goal of middle level vision and perceptual organization is to bridge the gap between lower level vision algorithms, such as edge detection, image segmentation, and the high level algorithms for object recognition. Therefore the models in the middle level should be **compatible** with both the low and high level representations.



Conclusion

A statistical framework for learning prob. models of shapes.

1. Study the statistics of natural shapes.
2. Measure the non-accidental statistics rigorously.
3. Shape modeling by the maximum entropy principle.
4. Feature selection by maximum non-accidental statistics.
5. Model learning and verification by stochastic sampling.

We argue that the learned models and the sampling process can be applied to shape inference.