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*Excited Electronic States  
of Surfaces and Molecules*

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# Excited Electronic States of Surfaces and Molecules

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- Introduction
- Surface Excitons
- Excited Molecules
- Interrelation with the Geometry

**In cooperation with**

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## Outline:

- Theoretical Description of Optical Spectra

$\Longleftrightarrow$  Hierarchy of *ab-initio* Techniques:

1. Density-functional Theory DFT (LDA + Pseudopotentials)

$\Rightarrow$  Electronic Ground State  $|N, 0\rangle$

[ + geometric structure]

2. Electronic Self Energy  $\Sigma$  (within *GW* Approx.)

Dyson's equation for  $G_1$

$\Rightarrow$  Single-particle Excitations  $N \rightarrow N \pm 1$

Electronic Spectrum; Band Structure

M. S. Hybertsen and S. G. Louie, PRL 55, 1418 (1985);

R. W. Godby et al., PRL 56, 2415 (1986).

3. Electron-Hole Interaction  $K^{\text{eh}}$

Bethe-Salpeter equation for  $G_2$

$\Rightarrow$  Coupled Electron-Hole Excitations  $N \rightarrow N^*$

Optical Spectrum, Excitons, etc.

G. Strinati, PRB 29, 5718 (1984);

S. Albrecht et al., PRL 80, 4510 (1998);

L. X. Benedict et al., PRL 80, 4514 (1998);

M. Rohlfing and S. G. Louie, PRL 80, 3320 (1998),

PRL 82, 1959 (1999).

## Quasiparticle (QP) band structure calculations

- Density-functional theory:

$$\left\{-\nabla^2 + V_{\text{ext}} + V_{\text{Coul}} + V_{\text{xc}}\right\} \psi_{\mathbf{n}\mathbf{k}}^{\text{DFT}} = \varepsilon_{\mathbf{n}\mathbf{k}}^{\text{DFT}} \psi_{\mathbf{n}\mathbf{k}}^{\text{DFT}}$$

Hohenberg, Kohn, and Sham 1965

- Green-function approach + QP approximation:

$$\left\{-\nabla^2 + V_{\text{ext}} + V_{\text{Coul}} + \Sigma(\varepsilon_{\mathbf{n}\mathbf{k}}^{\text{QP}})\right\} \psi_{\mathbf{n}\mathbf{k}}^{\text{QP}} = \varepsilon_{\mathbf{n}\mathbf{k}}^{\text{QP}} \psi_{\mathbf{n}\mathbf{k}}^{\text{QP}}$$

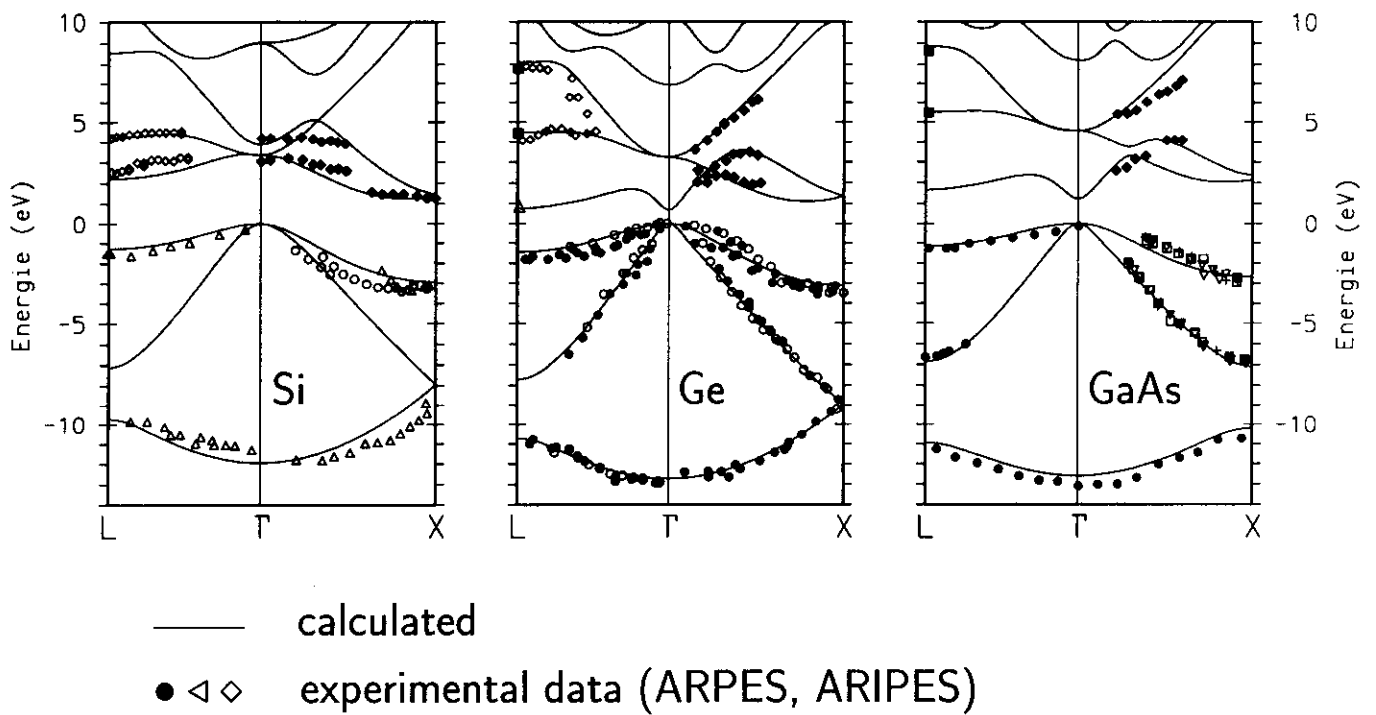
$\Sigma = iG_1W$  GW approximation for the self energy

$G_1$  one-particle Green function

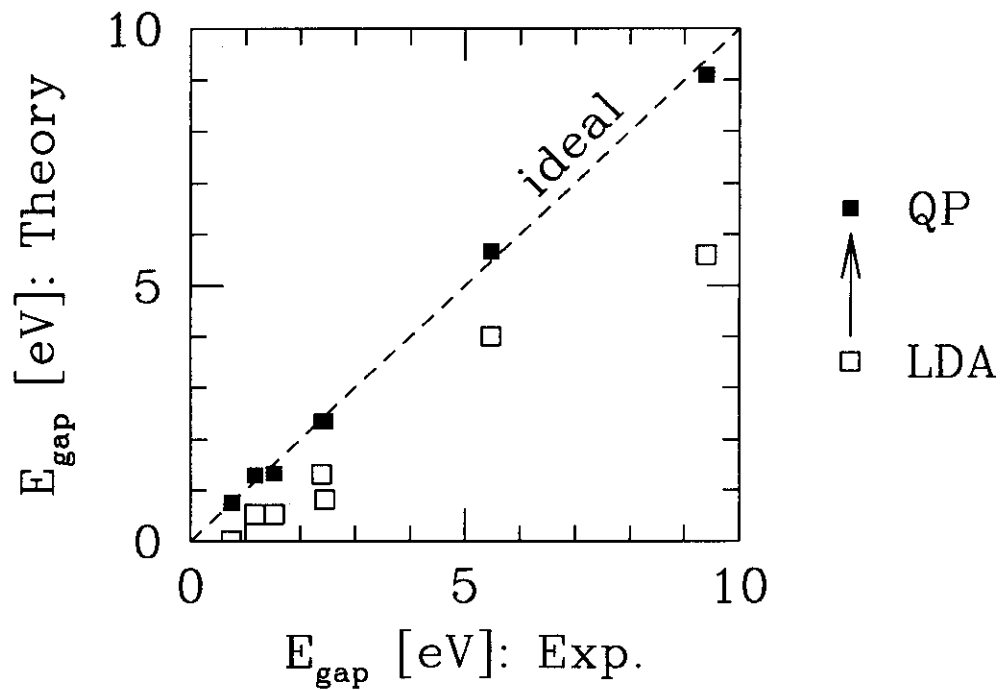
$W = \epsilon^{-1}v$  screened Coulomb interaction

Hedin 1965, Hybertsen and Louie 1985

## QP band structures of Si, Ge, and GaAs:



## QP gap energies of various semiconductors:



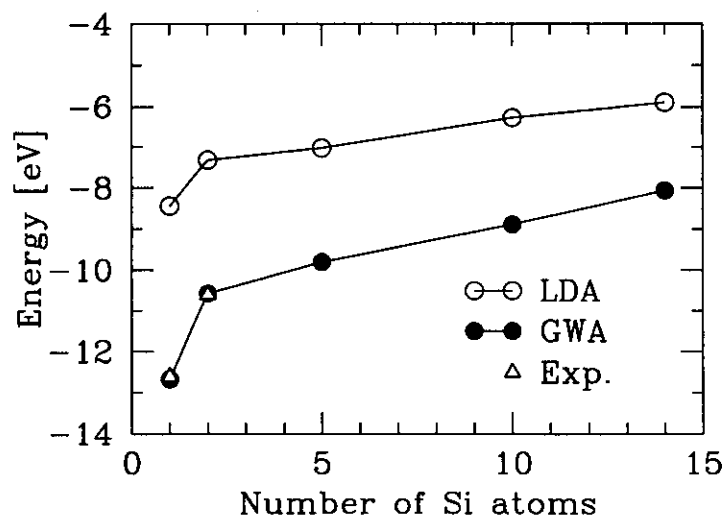
## Ionisation Energies of Atoms and Molecules:

| [eV]                          | LDA <sup>a</sup> | GWA   | Exp. <sup>b</sup> |
|-------------------------------|------------------|-------|-------------------|
| He                            | 15.48            | 24.68 | 24.586            |
| Ne                            | 13.06            | 21.47 | 21.564            |
| Ar                            | 10.35            | 15.94 | 15.759            |
| CO                            | 9.1              | 14.3  | 14.01             |
| HCl                           | 8.0              | 12.6  | 12.75             |
| CH <sub>4</sub>               | 9.3              | 12.5  | 12.99             |
| C <sub>2</sub> H <sub>4</sub> | 6.9              | 10.4  | 10.5              |

<sup>a</sup>LDA:  $-E_{\text{HOMO}}$

<sup>b</sup>Exp.: Landolt-Börnstein, Vol. I-1 (Springer, Berlin 1950);  
G. Herzberg, *Molecular Spectra* (Van Nostrand, N.Y. 1966).

## Si<sub>m</sub>H<sub>n</sub>: Quasiparticle HOMO Energies

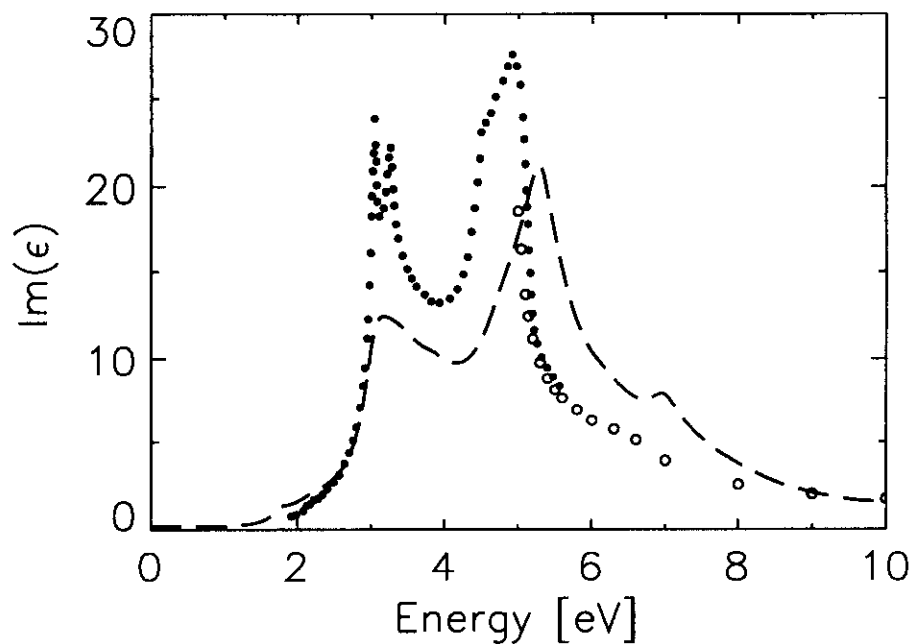


Exp.: U. Itoh et al., J. Chem. Phys. 85, 4867 (1986).

## GaAs: Absorption Spectrum ?

- Usual evaluation of the optical spectrum:

$$\epsilon_2(\omega) = \frac{4\pi e^2}{\omega^2} \sum_{v\mathbf{ck}} |M_{v\mathbf{ck}}|^2 \delta(\omega - (\epsilon_{c\mathbf{k}}^{\text{QP}} - \epsilon_{v\mathbf{k}}^{\text{QP}}))$$



• • • Exp.

- The "form" of the spectrum is not correct.
- Bound excitons are not described.

- Quasiparticle band structure:

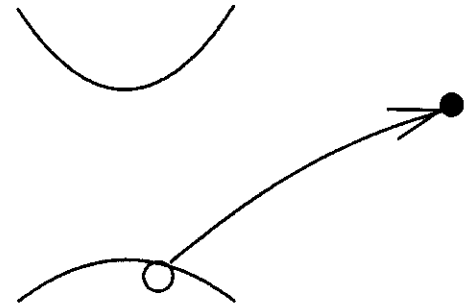
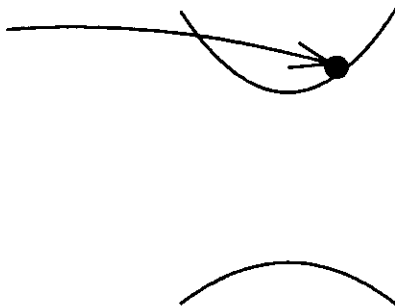
describes individually excited holes and electrons  
 ( $\leftrightarrow$  One-particle Green function  $G_1$ )

Quasi-electron:  $N \rightarrow N + 1$

(inverse photoemission; tunneling)

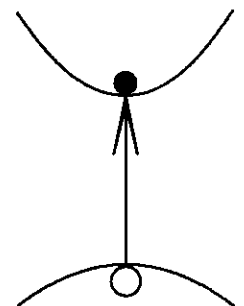
Quasi-hole:  $N \rightarrow N - 1$

(photoemission; tunneling)



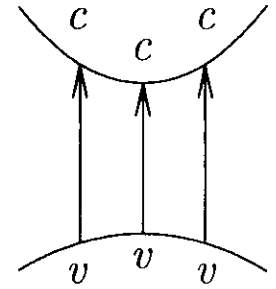
- Optical experiment (e.g., absorption):

- Creation of electron-hole pair:  $N \rightarrow N^*$
- Electron-hole interaction
- Requires a two-particle approach on top of the QP band structure



## Coupled Electron-hole excitations:

$$|S\rangle = \sum_v^{\text{hole}} \sum_c^{\text{elec}} \sum_{\mathbf{k}} A_{v\mathbf{c}\mathbf{k}}^S \hat{a}_{v\mathbf{k}}^\dagger \hat{b}_{c\mathbf{k}+\mathbf{Q}}^\dagger |0\rangle$$



$|0\rangle$  ground state of the many-electron system

$\hat{a}_{v\mathbf{k}}^\dagger, \hat{b}_{c\mathbf{k}+\mathbf{Q}}^\dagger$  creates quasi-hole, -electron

$A_{v\mathbf{c}\mathbf{k}}^S$  coupling coefficients

$|S\rangle$  corresponds to the two-particle Green function:

$$G_2(\omega) \hat{=} \sum_S \frac{|S\rangle\langle S|}{\omega - \Omega_S}, \quad \Omega_S \text{ excitation energy}$$

The Bethe-Salpeter Equation for  $G_2$  leads to

$$(\varepsilon_{c\mathbf{k}+\mathbf{Q}}^{\text{QP}} - \varepsilon_{v\mathbf{k}}^{\text{QP}}) A_{v\mathbf{c}\mathbf{k}}^S + \sum_{v'\mathbf{c}'\mathbf{k}'} \langle v\mathbf{c}\mathbf{k} | K^{eh} | v'\mathbf{c}'\mathbf{k}' \rangle A_{v'\mathbf{c}'\mathbf{k}'}^S = \Omega^S A_{v\mathbf{c}\mathbf{k}}^S$$

$\varepsilon_{v\mathbf{k}}^{\text{QP}}, \varepsilon_{c\mathbf{k}+\mathbf{Q}}^{\text{QP}}$  single-QP energies

Strinati 1984

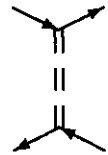
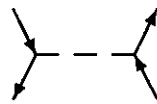
$K^{eh}$  electron-hole interaction

The electron-hole interaction  $K^{eh}$ :

$$K^{eh}(12, 34) = \frac{\delta[V_{\text{Coul}}(1)\delta(13) + \Sigma(13)]}{\delta G_1(42)} \quad \mathbf{1}=(\mathbf{r}_1, \sigma_1, t_1)$$

assume that  $\Sigma = iG_1W$  and  $G_1 \frac{\delta W}{\delta G_1} \approx 0$

$$\Rightarrow K^{eh}(12, 34) = -i\delta(13)\delta(2^-4)v(14) + i\delta(14)\delta(23)W(1^+3)$$



$$\langle v\mathbf{c}\mathbf{k}|K^{eh}|v'\mathbf{c}'\mathbf{k}'\rangle =$$

$$\int d\mathbf{x}d\mathbf{x}' \psi_{\mathbf{c}\mathbf{k}+\mathbf{Q}}^*(\mathbf{x})\psi_{v\mathbf{k}}(\mathbf{x}) v(\mathbf{r}, \mathbf{r}') \psi_{\mathbf{c}'\mathbf{k}'+\mathbf{Q}}(\mathbf{x}')\psi_{v'\mathbf{k}'}^*(\mathbf{x}') \quad (\text{I})$$

$$- \int d\mathbf{x}d\mathbf{x}' \psi_{\mathbf{c}\mathbf{k}+\mathbf{Q}}^*(\mathbf{x})\psi_{\mathbf{c}'\mathbf{k}'+\mathbf{Q}}(\mathbf{x})\psi_{v\mathbf{k}}(\mathbf{x}')\psi_{v'\mathbf{k}'}^*(\mathbf{x}') \quad \times \quad (\text{II})$$

$$\frac{i}{2\pi} \int d\omega e^{-i\omega 0^+} W(\mathbf{r}, \mathbf{r}', \omega) \quad \times$$

$$\left[ \frac{1}{\Omega_S - \omega - (\varepsilon_{\mathbf{c}'\mathbf{k}'+\mathbf{Q}}^{\text{QP}} - \varepsilon_{v\mathbf{k}}^{\text{QP}}) + i0^+} + \frac{1}{\Omega_S + \omega - (\varepsilon_{\mathbf{c}\mathbf{k}+\mathbf{Q}}^{\text{QP}} - \varepsilon_{v'\mathbf{k}'}^{\text{QP}}) + i0^+} \right]$$

$$=: K^{eh,x} + K^{eh,d}$$

(I) repulsive exchange term  $K^{eh,x}$

(II) attractive direct term  $K^{eh,d}$

## Optical spectrum:

- Free transitions  $v\mathbf{k} \longrightarrow c\mathbf{k}$ :

$$\epsilon_2(\omega) = \frac{4\pi e^2}{\omega^2} \sum_{v\mathbf{k}} |M_{v\mathbf{k}}|^2 \delta(\omega - (\epsilon_{c\mathbf{k}}^{\text{QP}} - \epsilon_{v\mathbf{k}}^{\text{QP}}))$$

$$M_{v\mathbf{k}} = \vec{\lambda} \cdot \langle v\mathbf{k} | \vec{V} | c\mathbf{k} \rangle$$

$\vec{\lambda}$  polarization vector of the light

$\vec{V}$  velocity operator

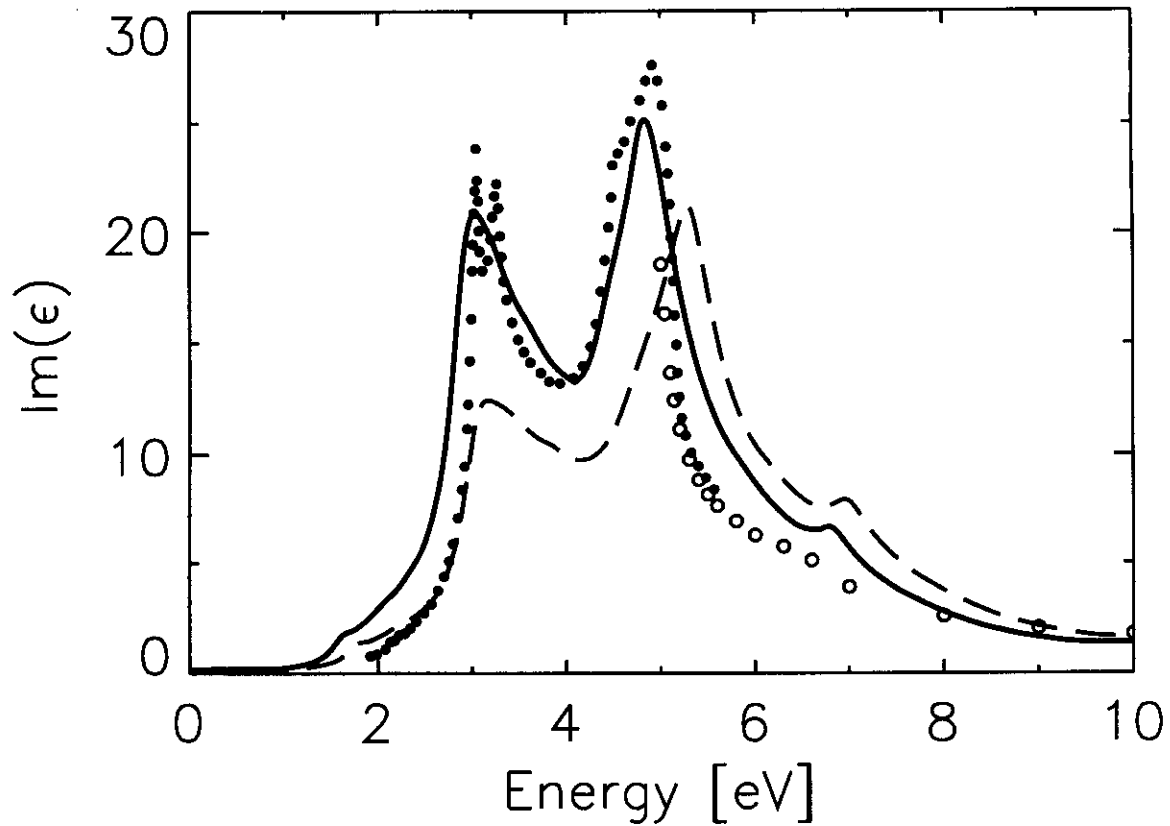
- Coupled transitions  $|S\rangle$ :

$$\epsilon_2(\omega) = \frac{4\pi e^2}{\omega^2} \sum_S |M_S|^2 \delta(\omega - \Omega_S)$$

$$M_S = \vec{\lambda} \cdot \langle 0 | \vec{V} | S \rangle = \sum_{v\mathbf{k}} A_{v\mathbf{k}}^S M_{v\mathbf{k}}$$

- $\epsilon_2 \longrightarrow \epsilon_1, n, k, R, T, A, \dots$

## GaAs: Absorption Spectrum



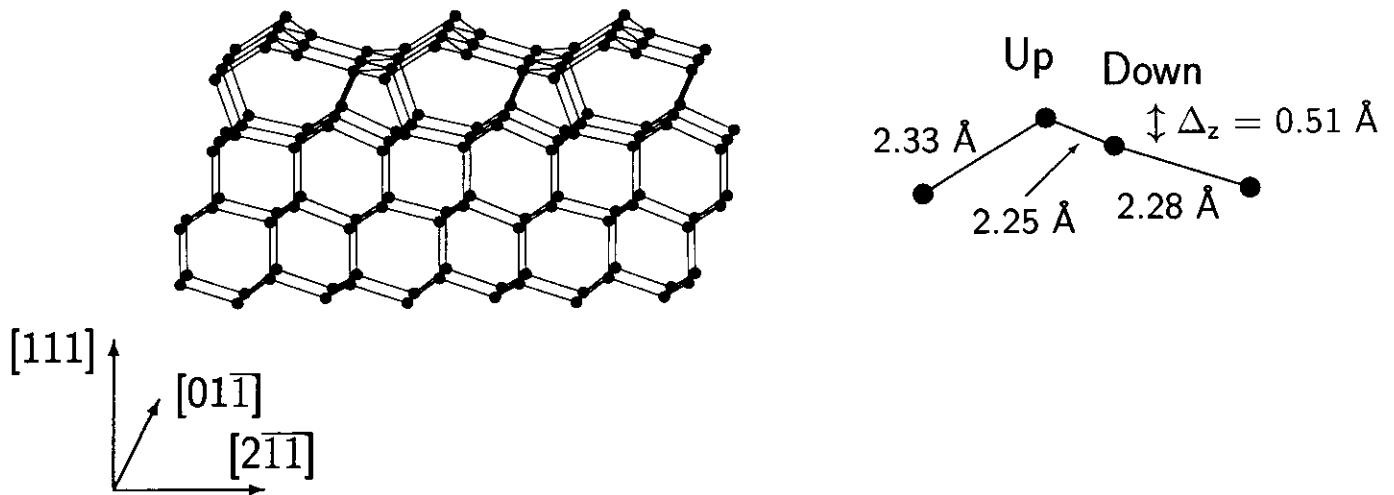
— with E-H Int.      - - - without E-H Int.

● ○ ○ Exp.: D.E. Aspnes and A.A. Sturge, PRB 27, 985 (1983);  
P. Lautenschlager et al., PRB 35, 9174 (1987).

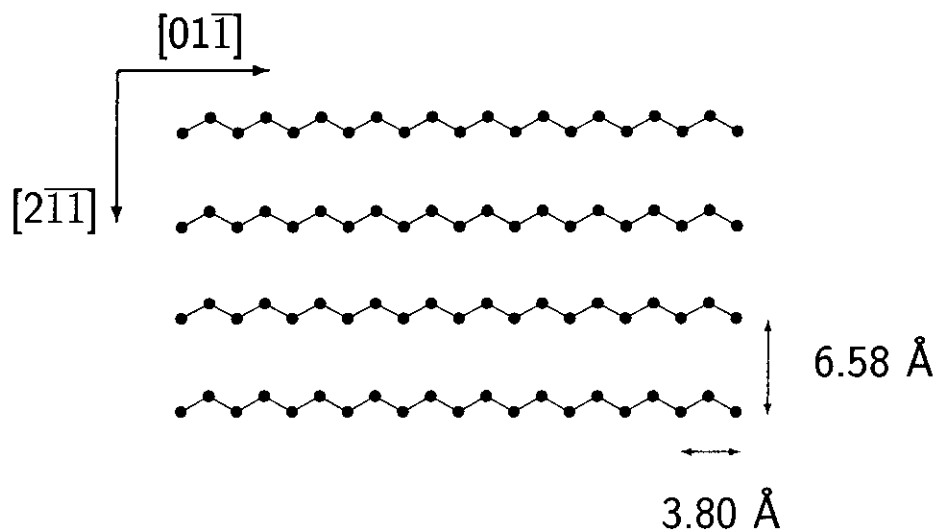
## Structure of the Si(111)-(2×1) Surface:

- $\pi$ -bonded chains ( $\leftrightarrow$  K.C. Pandey, Phys. Rev. Lett. 49, 223 (1982))

Side view:



Top view:

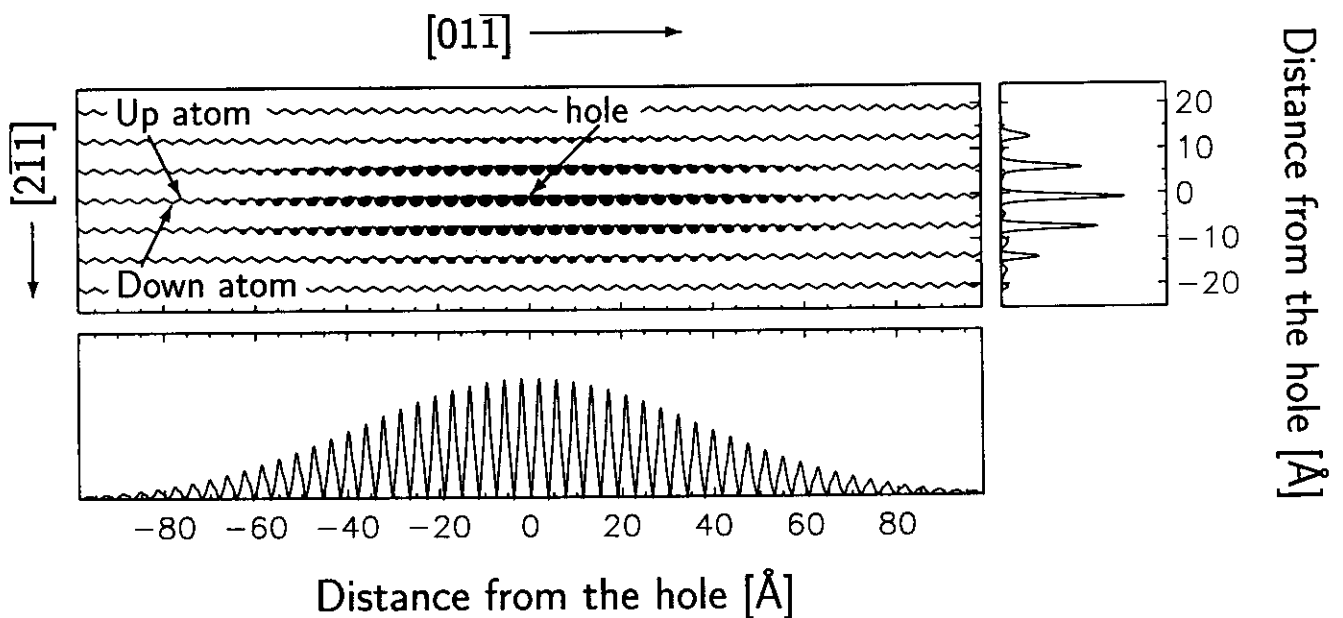


## Wave function of the exciton (0.43 eV) – Top view

$$\chi^S(\mathbf{r}_h, \mathbf{r}_e) = \sum_{v\mathbf{ck}} A_{v\mathbf{ck}}^S \psi_{v\mathbf{k}}^*(\mathbf{r}_h) \psi_{c\mathbf{k}+\mathbf{Q}}(\mathbf{r}_e)$$

$\mathbf{r}_h$  = coordinates of the hole,  $\mathbf{r}_e$  = electron

Distribution of the electron relative to the hole:

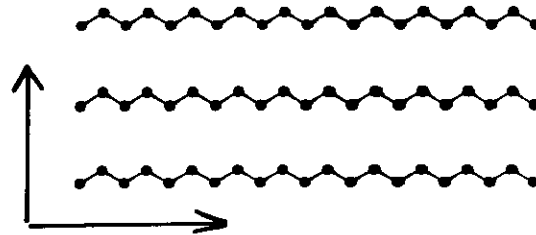


Mean distance Electron—Hole:

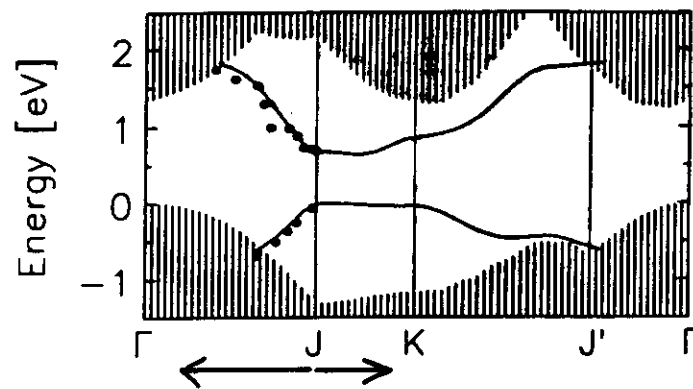
- along the chains: 40 Å (~ Wannier exciton)
- across the chains: 8 Å (~ Frenkel exciton)

# Anisotropy of the Si(111)-(2×1) Surface Exciton

Geometric Structure

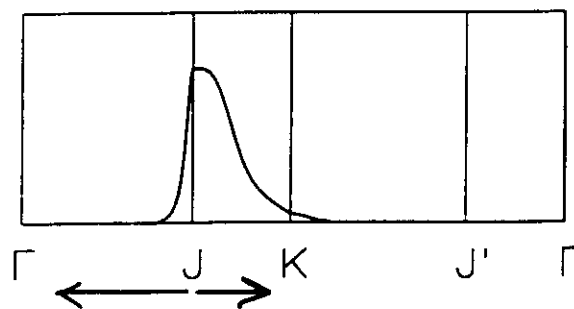


Band Structure

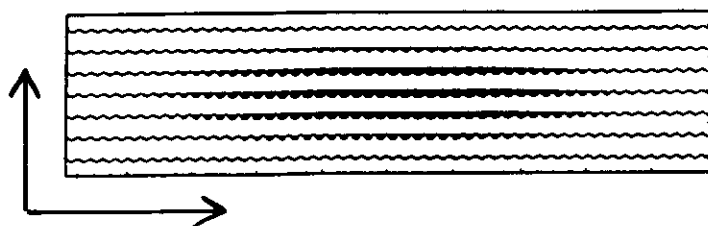


Coupling Coefficients

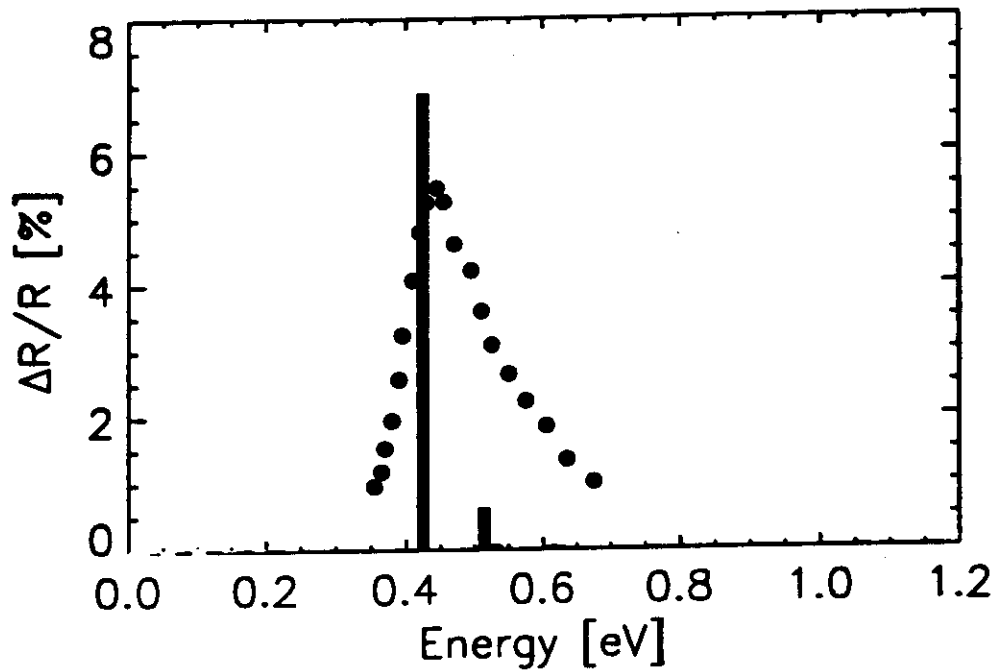
$$A_{\text{up} \rightarrow \text{down}}(\mathbf{k})$$



Exciton Wavefunction



## Si(111)-(2×1): Line Shape of the DRS



Exp.: P. Chiaradia et al., PRL 52, 1145 (1984).

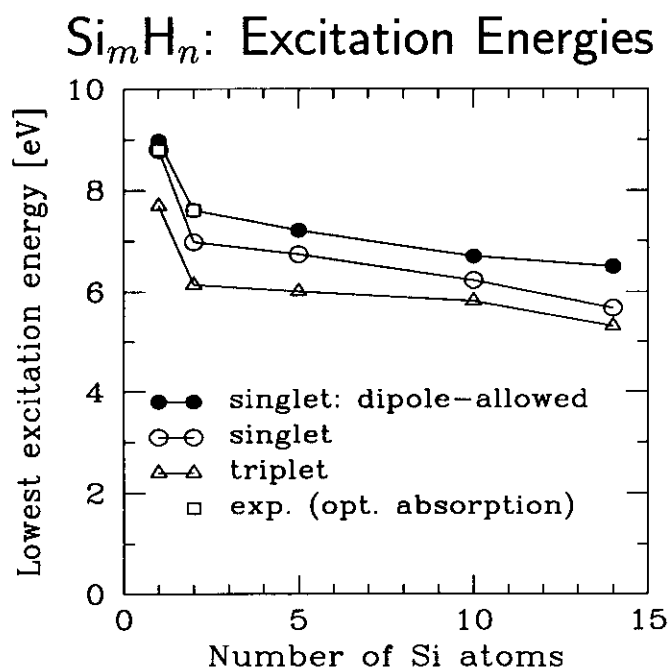
- Theory: Discrete exciton levels.
- Exp.: Broadened spectrum  
(FWHM  $\sim 100$  meV at RT).

$\longleftrightarrow$  Coupling to lattice vibrations.

## Excitation Energies of Atoms and Molecules

|                               | Spin Singlet |                   | Spin Triplet |                   |
|-------------------------------|--------------|-------------------|--------------|-------------------|
| [eV]                          | GW+BSE       | Exp. <sup>a</sup> | GW+BSE       | Exp. <sup>a</sup> |
| He                            | 20.75        | 20.615            | 19.81        | 19.818            |
| Ne                            | 16.95        | 16.848            | 16.71        | 16.668            |
| Ar                            | 11.99        | 11.827            | 11.76        | 11.631            |
| CO                            | 7.9          | 8.03              | 5.5          | 6.01              |
| HCl                           | 7.5          | 8.1               | 6.9          |                   |
| CH <sub>4</sub>               | 8.6          | 8.52              | 8.2          |                   |
| C <sub>2</sub> H <sub>4</sub> | 7.0          | 7.11              | 3.7          | 4.36              |

<sup>a</sup>Exp.: Landolt-Börnstein, Vol. I-1 (Springer, Berlin 1950);  
 G. Herzberg, *Molecular Spectra* (Van Nostrand, N.Y. 1966);  
 L. Serrano-Andres *et al.*, J.Chem.Phys. 98, 3151 (1993).

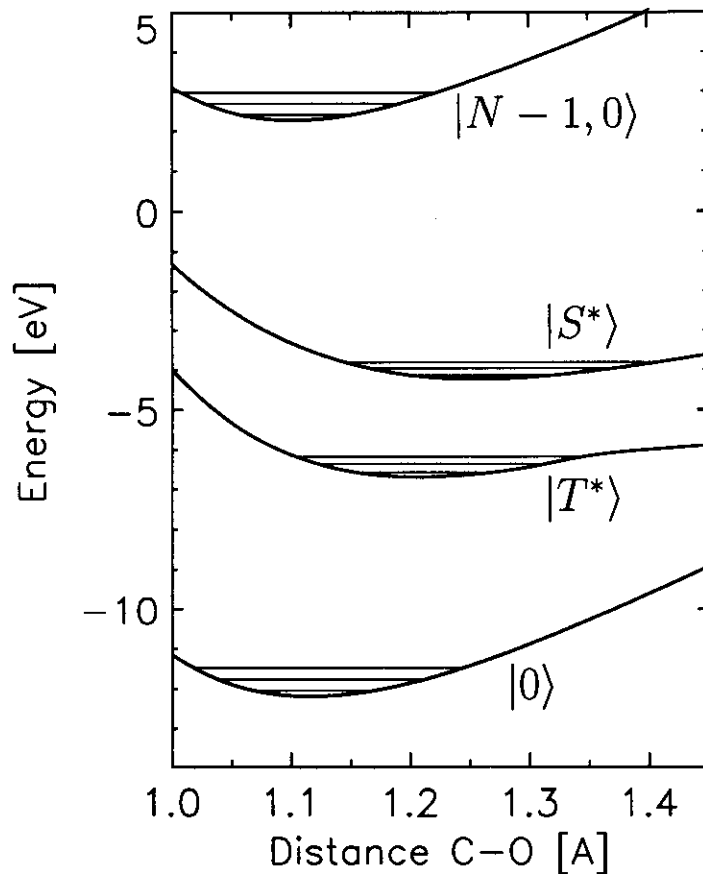


Theor.: M. Rohlfing and S.G. Louie, PRL 80, 3320 (1998).

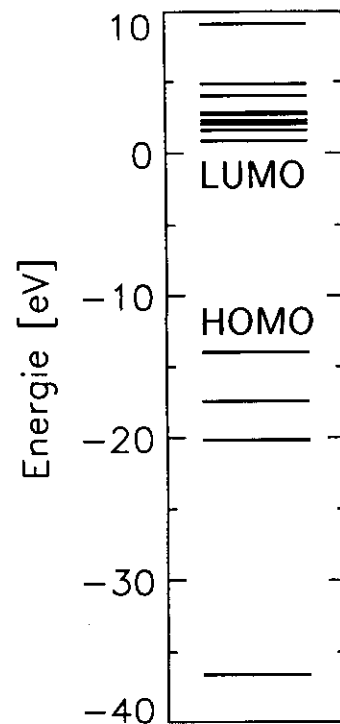
Exp.: U. Itoh *et al.*, J. Chem. Phys. 85, 4867 (1986).

## A Diatomic Molecule: CO

Total Energy:



Single-Particle Spectrum ( $d=1.11\text{\AA}$ ):



| State           | $d_0$ [Å] |       | $\omega$ [cm <sup>-1</sup> ] |      |
|-----------------|-----------|-------|------------------------------|------|
|                 | Theo.     | Exp.  | Theo.                        | Exp. |
| $ 0\rangle$     | 1.11      | 1.128 | 2270                         | 2170 |
| $ T^*\rangle$   | 1.21      | 1.206 | 1750                         | 1743 |
| $ S^*\rangle$   | 1.26      | 1.235 | 1480                         | 1518 |
| $ N-1,0\rangle$ | 1.10      |       | 2240                         |      |

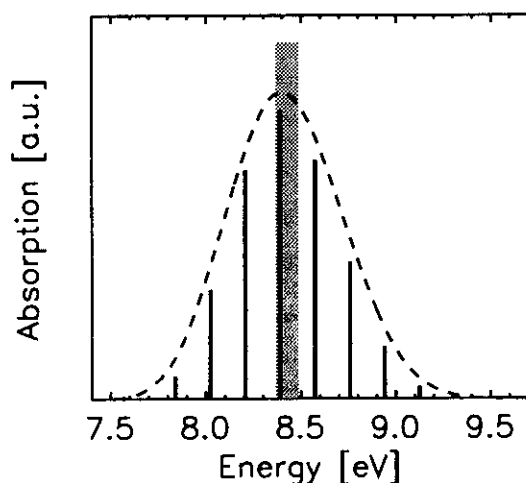
Transitions ( $\nu=0 \rightarrow \nu'=0$ ) [in eV]:

|                                       | Theo. | Exp.  |
|---------------------------------------|-------|-------|
| CO $\rightarrow$ C+O                  | 12.0  | 11.1  |
| $ 0\rangle \rightarrow  T^*\rangle$   | 5.5   | 6.01  |
| $ 0\rangle \rightarrow  S^*\rangle$   | 7.9   | 8.03  |
| $ 0\rangle \rightarrow  N-1,0\rangle$ | 14.3  | 14.01 |
| HOMO-LUMO gap                         | 14.8  |       |

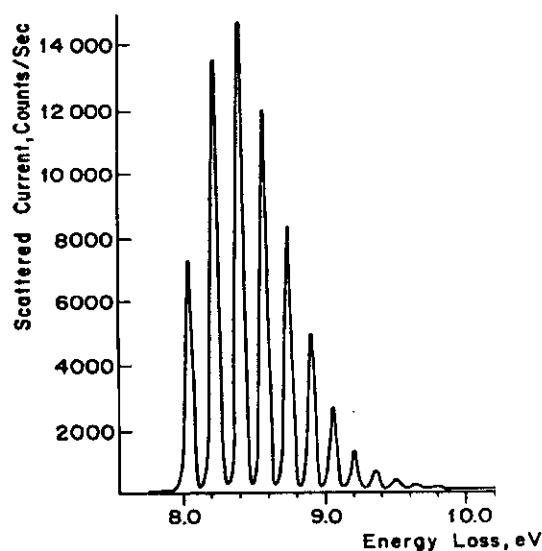
Exp.: K.P. Huber und G. Herzberg, *Molecular Spectra and Molecular Structure*,  
Vol. IV: *Constants of Diatomic Molecules* (Van Nostrand, New York 1979).

## Vibrational structure of the spectrum – CO

Theory:



Exp.:



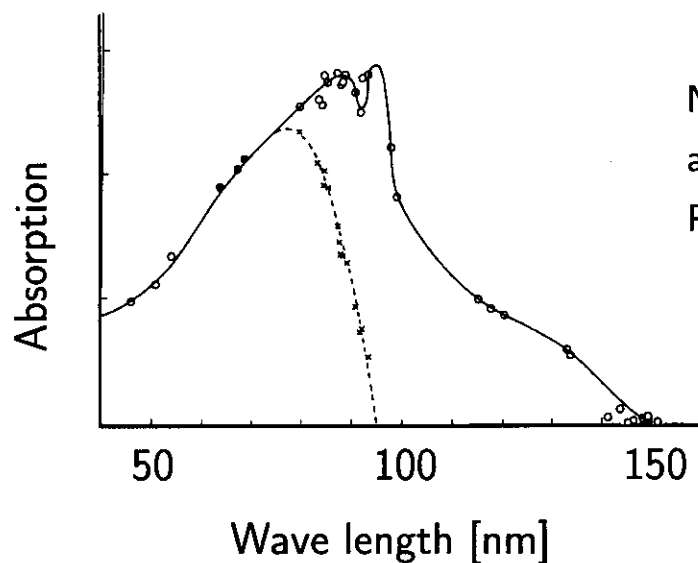
|                        | Theory | Exp. |
|------------------------|--------|------|
| $\Delta E$ [eV]        | 0.18   | 0.17 |
| Width [eV]             | 0.53   | 0.64 |
| Max. Amplitude at [eV] | 8.43   | 8.47 |
| Maximum Peak           | 4      | 3    |

- Peak of maximum amplitude  $\leftrightarrow$  Vertical transition
- Low-energy onset  $\leftrightarrow$  Minimum energy in excited state
- Width  $\leftrightarrow$  Gain in total energy due to relaxation

Exp.: E.N. Lassettre and A. Skerbele, JCP 54, 1597 (1971).

## Absorption spectrum of CH<sub>4</sub> and SiH<sub>4</sub>

CH<sub>4</sub>:

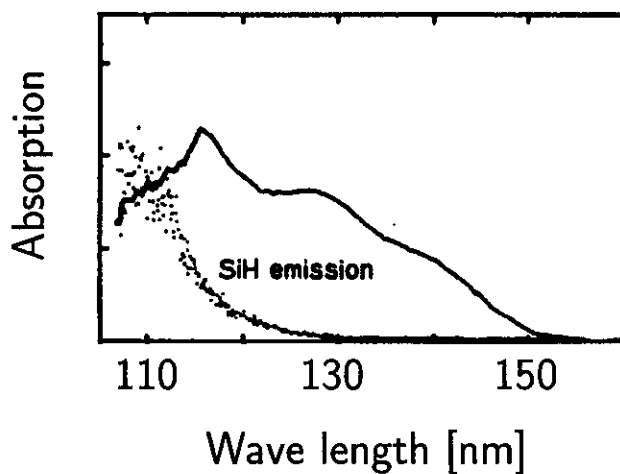


N. Wainfan, W.C. Walker,  
and G.L. Weissler,  
Phys.Rev. 99, 542 (1955).

Onset:  $\lambda=146 \text{ nm} \hat{=} 8.5 \text{ eV}$

First maximum:  $\lambda \sim 125 \text{ nm} \hat{=} 9.9 \text{ eV}$

SiH<sub>4</sub>:



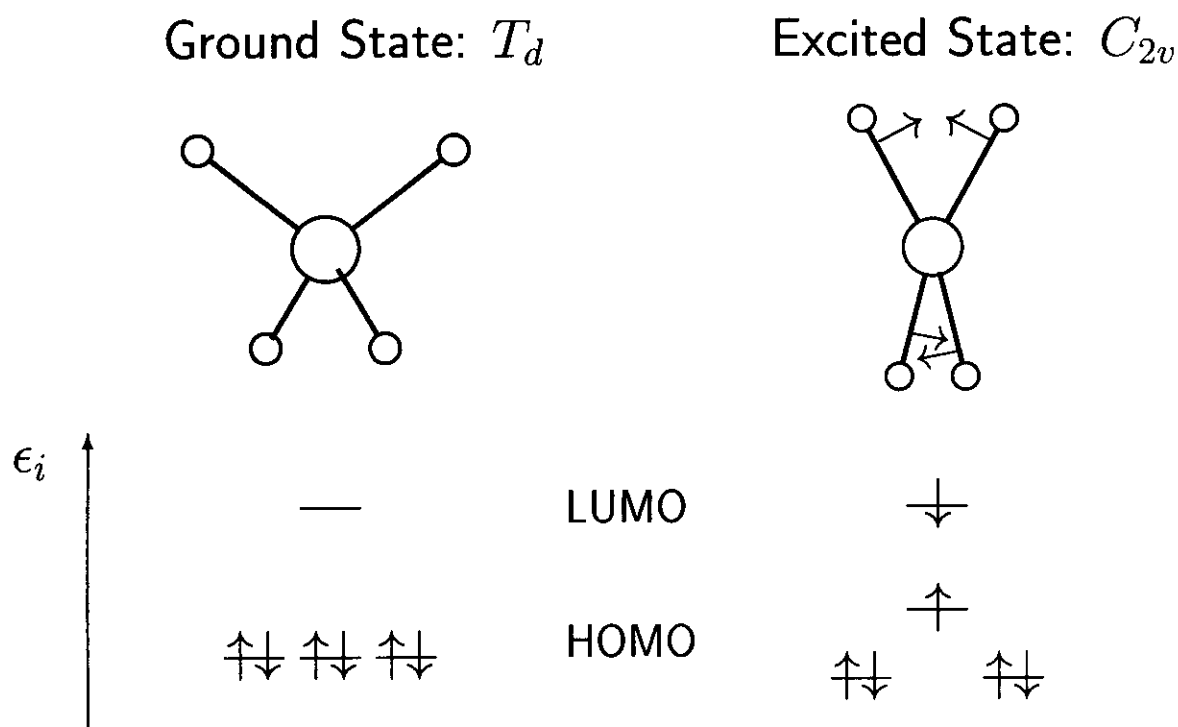
U. Itoh, Y. Toyoshima,  
and H. Onuki,  
JCP 85, 4867 (1986).

Onset:  $\lambda=151 \text{ nm} \hat{=} 8.2 \text{ eV}$

First maximum:  $\lambda=141 \text{ nm} \hat{=} 8.8 \text{ eV}$

# CH<sub>4</sub> and SiH<sub>4</sub>: Geometry of the Excited States

- Increase of the C-H distance
- Distortion / Reduction of the symmetry

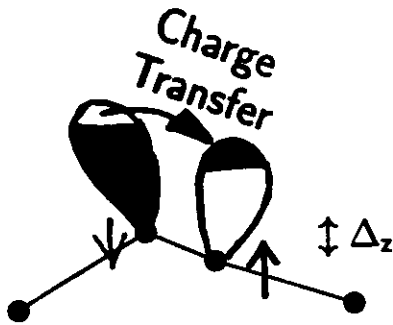


|                    |            | Vert.  | Energy Gain         |                          | Min.   | Exp.  |
|--------------------|------------|--------|---------------------|--------------------------|--------|-------|
|                    |            | Trans. | $d_0 \rightarrow d$ | $T_d \rightarrow C_{2v}$ | Trans. |       |
| CH <sub>4</sub> :  | Ionization | 14.3   | -0.2                | -1.6                     | 12.5   | 12.99 |
|                    | $E_S$      | 10.5   | -0.3                | -1.6                     | 8.6    | 8.52  |
|                    | $E_T$      | 10.1   | -0.4                | -1.5                     | 8.2    |       |
| SiH <sub>4</sub> : | Ionization | 12.6   | -0.1                | -0.6                     | 11.9   |       |
|                    | $E_S$      | 9.2    | -0.2                | -0.7                     | 8.3    | 8.2   |
|                    | $E_T$      | 8.5    | -0.5                | -0.3                     | 7.7    |       |

Exp.: G. Herzberg, *Molecular Spectra and Molecular Structure* (Van Nostrand 1966);  
 U. Itoh *et al.*, J. Chem. Phys. 85, 4867 (1986).

# Si(111)-(2×1) Surface – Interaction with the Structure

Exciton state:

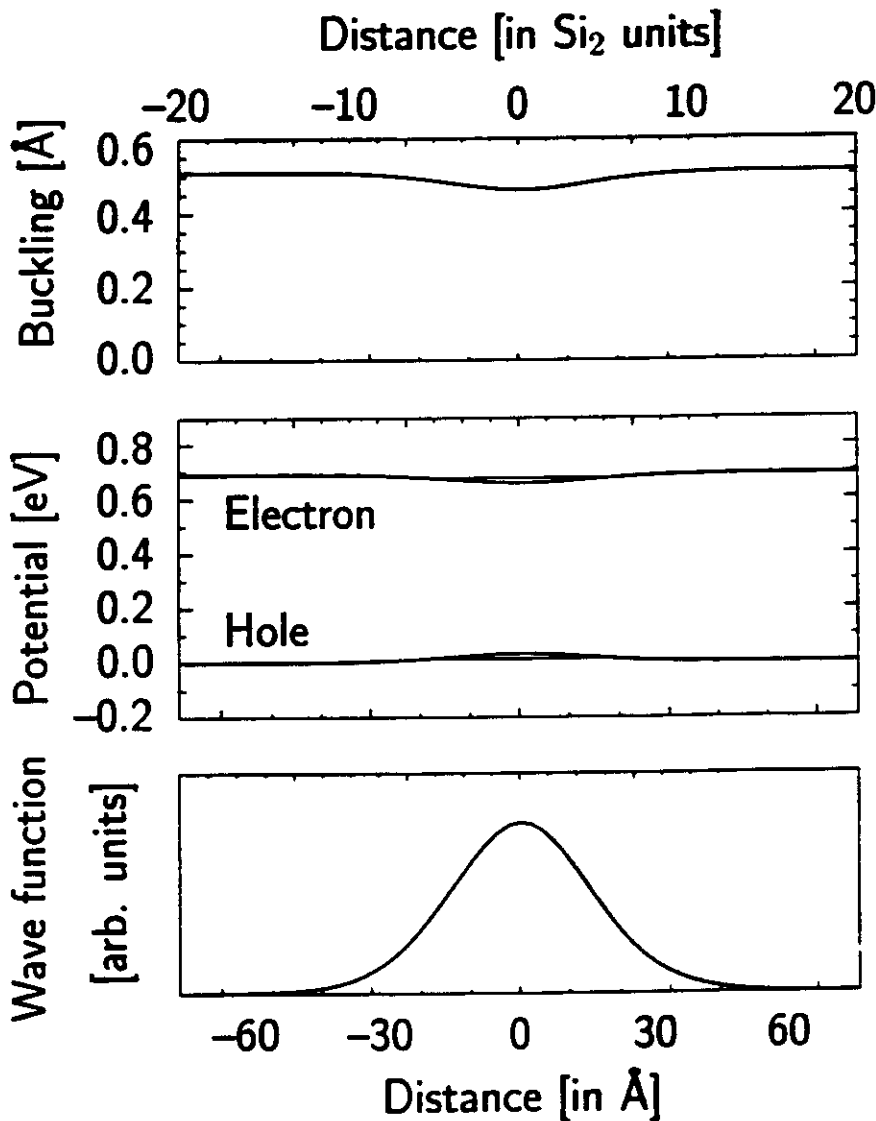


Charge transfer

⇒ Reduction of buckling

⇒ Reduction of  $E_{\text{gap}}$

⇒ Self Trapping



Minimization of  $E_{\text{tot}}^*$ :

$$\Delta z = \underline{-0.05 \text{ \AA}}$$

$$\Delta E_g = \underline{-65 \text{ meV}}$$

New VBM: +13 meV

New CBM:  $E_g - 13 \text{ meV}$

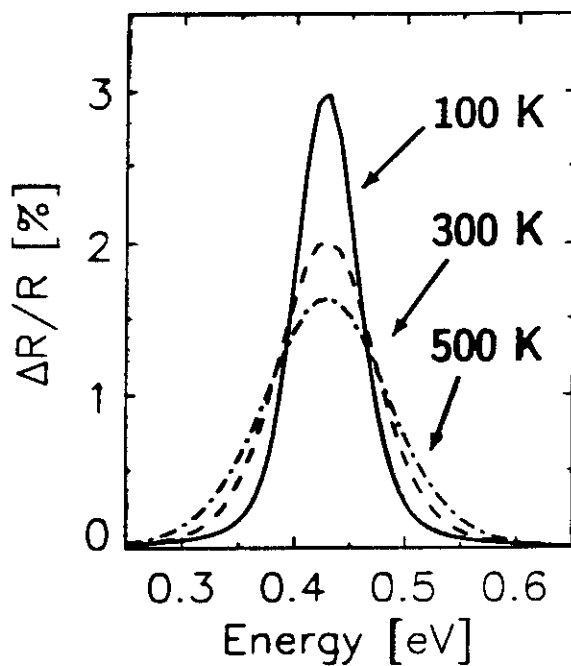
New Exciton:  $E_1 - \underline{41 \text{ meV}}$

Lattice Energy: +17 meV

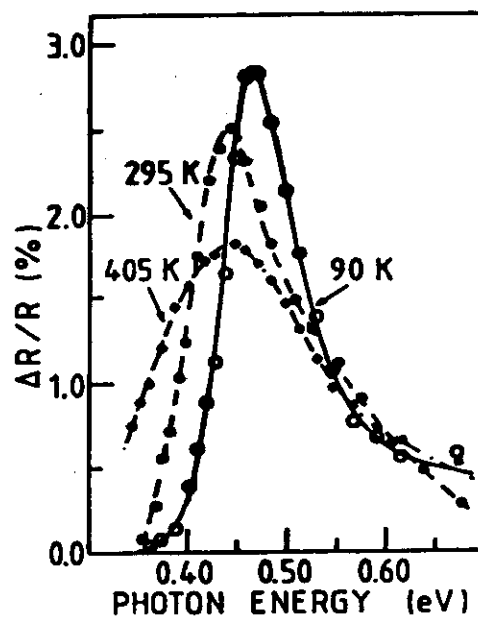
$$\Rightarrow E_{\text{tot}}^* = E_1 - \underline{24 \text{ meV}}$$

# Si(111)-(2×1) Surface Exciton – Spectral Width

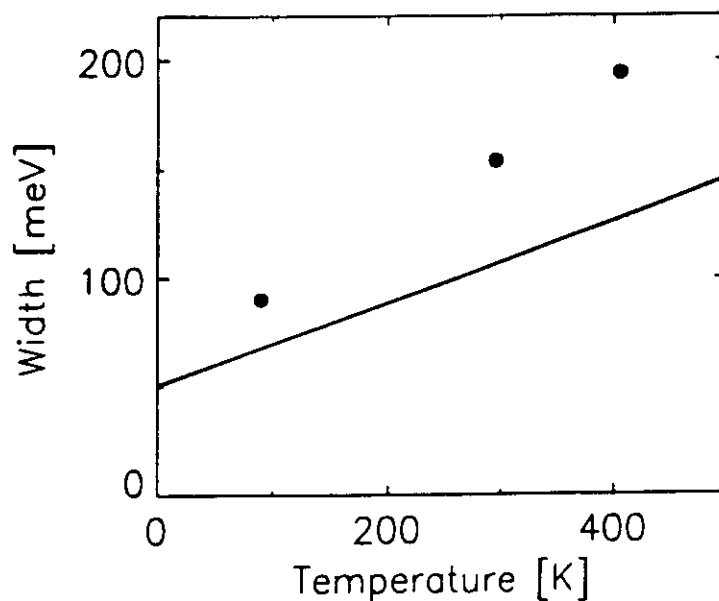
Calculated:



Measured:



Spectral Width (at half maximum):



Exp. (●): F. Ciccacci et al., PRL 56, 2411 (1986).

# Conclusions

- Three-level ab-initio concept:
  - Ground state (LDA)
  - $G_1$  (GWA); QP spectrum
  - Electron-hole interaction +  $G_2$ ;  
Electron-hole Excitations; optical spectrum
- Semiconductor surface Si(111)-(2×1):
  - Bound surface exciton
  - Enhanced electron-hole interaction
- Molecules:
  - Interrelation between excitations and geometry
- Outlook:
  - Excitations of molecules adsorbed at surfaces