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SEISMIC WAVE INVERSION FOR EARTHQUAKE SOURCE AND NUCLEAR EXPLOSIONS STUDY

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I. Formal description of seismic source

The description of seismic source we will consider is based on the formalism developed by Backus and Mulcahy, 1976.

Statement of the problem.

Motion equation

$$\rho \ddot{u}_i = \sigma_{ij,j} + f_i \tag{1.1}$$

Hook's law for isotropic medium

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij} \tag{1.2}$$

Initial conditions

$$\dot{\mathbf{u}} \equiv \mathbf{u} \equiv 0, t < 0 \tag{1.3}$$

Boundary conditions

$$\sigma_{ij} n_j |_{S_0} = 0 \tag{1.4}$$

Here $\mathbf{u}$ – displacement vector; σ_{ij} – elements of symmetric 3x3 stress tensor; i,j=1,2,3 and the summation convention for repeated subscripts is used; $\sigma_{ij,j} = \sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j}$; ε_{ij} – elements of symmetric 3x3 strain tensor and $\varepsilon_{ij} = 0.5(u_{i,j} + u_{j,i})$; ρ - density; f_i – components of external force; n_j – components of the normal to the free surface S_0.

Solution of the problem (1.1)-(1.4) can be given by formula

$$u_i(\mathbf{x},t) = \int_0^T d\tau \int_\Omega G_{ij}(\mathbf{x},\mathbf{y},t-\tau) f_j(\mathbf{y},\tau) dV_y \tag{1.5}$$

or

$$u_i(\mathbf{x},t) = \int_0^T d\tau \int_\Omega H_{ij}(\mathbf{x},\mathbf{y},t-\tau) \dot{f}_j(\mathbf{y},\tau) dV_y \tag{1.6}$$

Here G_{ij} is the Green's function,

$$H_{ij}(\mathbf{x},\mathbf{y},t) = \int_0^t G_{ij}(\mathbf{x},\mathbf{y},\tau) d\tau , \tag{1.7}$$

$\mathbf{x} \in \Omega$ and $0 < t < T$ are the space region and time interval where $\dot{f}$ is not identically zero.

Sources of seismic disturbances

We will consider internal sources only (earthquakes and explosions). In this case any external forces are absent. We must then set $\mathbf{f} \equiv 0$ in equation (1.1), so that the only solution that satisfies the homogeneous initial (1.3) and boundary (1.4) conditions, as well

as Hook's law (1.2) will be $\mathbf{u} \equiv 0$. Non-zero displacements cannot arise in the medium, unless at least one of the above conditions is not true.
Following Backus and Mulcahy, 1976, we assume seismic motion to be caused by a departure from ideal elasticity (from Hook's law) within some volume of the medium Ω at some time interval $0 < t < T$.

Let $\mathbf{u}(\mathbf{x},t)$ be the actual displacements, $\sigma(\mathbf{x},t)$ - correspondent stresses, if Hook's law is valid, $\mathbf{s}(\mathbf{x},t)$ - actual stresses.
Let the difference

$$\Gamma(\mathbf{x},t) = \sigma(\mathbf{x},t) - \mathbf{s}(\mathbf{x},t), \tag{1.8}$$

called the *stress glut tensor* or *moment tensor density*, is not identically zero for $0 < t < T$ and $\mathbf{x} \in \Omega$.
T we define as source duration, and Ω - source region. Within this region and time interval (and only there) the tensor $\dot{\Gamma}(\mathbf{x},t)$ is not identically zero as well.

Replacing $\sigma(\mathbf{x},t)$ by $\mathbf{s}(\mathbf{x},t)$ in equation (1.1), using definition (1.8) and the absence of external forces ($\mathbf{f} \equiv 0$) we can rewrite the motion equation (1.1) in form

$$\rho \ddot{u}_i = s_{ij,j}$$

or

$$\rho \ddot{u}_i = \sigma_{ij,j} + g_i \tag{1.9}$$

where

$$g_i = -\Gamma_{ij,j} \ . \tag{1.10}$$

Equation (1.10) defines the equivalent force $\mathbf{g}$. Using formula (1.6) with f_i replaced by g_i, definition (1.10) and Gauss theorem we have for displacements

$$u_i(\mathbf{x},t) = \int_0^T d\tau \int_\Omega H_{ij,k}(\mathbf{x},\mathbf{y},t-\tau)\dot{\Gamma}_{jk}(\mathbf{y},\tau)dV_y, \tag{1.11}$$

where H_{ij} is differentiated with respect to y_k.
If the inelastic motions are concentrated at a surface Σ, then

$$u_i(\mathbf{x},t) = \int_0^T d\tau \int_\Sigma H_{ij,k}(\mathbf{x},\mathbf{y},t-\tau)\dot{\Gamma}_{jk}(\mathbf{y},\tau)d\Sigma_y. \tag{1.12}$$

Relation of stress glut (moment tensor density) with classic definition of moment tensor M :

$$\mathbf{M} = \int_0^T dt \int_\Omega \dot{\Gamma}(\mathbf{y},t)dV_y \quad . \tag{1.13}$$

Normalizing moment tensor we define seismic moment M_0 :

$\mathbf{M}=M_0\mathbf{m}$, where tensor $\mathbf{m}$ is normalized by condition $\mathrm{tr}(\mathbf{m}^\mathrm{T}\mathbf{m}) = \sum_{i,j=1}^{3} m_{ij}^2 = 2$, $\mathbf{m}^\mathrm{T}$ is transposed tensor $\mathbf{m}$.

Stress glut moment for special types of seismic sources

1. Discontinuity of displacement $\Delta\mathbf{u}$ at a surface Σ in isotropic medium (stress is continuous):

$$\Gamma_{ij}(\mathbf{x},t) = \lambda \Delta u_k(\mathbf{x},t) n_k(\mathbf{x})\delta_{ij} + \mu[n_i(\mathbf{x})\Delta u_j(\mathbf{x},t) + n_j(\mathbf{x})\Delta u_i(\mathbf{x},t)]. \tag{1.14}$$

Here $\mathbf{n}(\mathbf{x})$ is the normal to the surface Σ, and seismic disturbances are given by formula (1.12).

2. In the case of tangential (shear) dislocation we have
$\Delta u_k n_k \equiv 0$ and formula (1.14) takes form

$$\Gamma_{ij}(\mathbf{x},t) = \mu[n_i(\mathbf{x})\Delta u_j(\mathbf{x},t) + n_j(\mathbf{x})\Delta u_i(\mathbf{x},t)]. \quad (1.15)$$

3. Instant point tangential dislocation occurred in the point **x=0** at time t=0:

$$\dot{\Gamma}_{ij}(\mathbf{x},t) = M_0 m_{ij}\delta(t)\delta(\mathbf{x}), \quad (1.16)$$

where $m_{ij} = n_i a_j + n_j a_i$, $\mathbf{a} = \Delta\mathbf{u}/|\Delta\mathbf{u}|$ and $M_0 = \mu\ |\Delta\mathbf{u}|$.

Phenomena of matrix **m**

Tr**m** = 0. The eigenvalues of matrix **m** are: 1, -1 and 0. The eigenvector correspondent to 1 defines the direction of maximum extension, and the eigenvector correspondent to -1 defines the direction of maximum compression. Such a source is called double couple.

As it follows from formula (1.12) an instant point double couple excites a displacement field of the form

$$u_i(\mathbf{x},t) = M_0 H_{ik,l}(\mathbf{x},\mathbf{0},t)m_{kl}. \quad (1.17)$$

We have for Fourier transforms **H**(**x**,**y**,ω) and **G**(**x**,**y**,ω) from equation (1.7):

$$\mathbf{H}(\mathbf{x},\mathbf{y},\omega) = \frac{1}{\mathrm{i}\omega}\mathbf{G}(\mathbf{x},\mathbf{y},\omega), \quad (1.18)$$

where i is the imaginary unit, and ω is angular frequency.
As result the spectrum of displacements is given by formula

$$u_i(\mathbf{x},\omega) = \frac{1}{\mathrm{i}\omega}M_0 m_{kl}G_{ik,l}(\mathbf{x},\mathbf{0},\omega). \quad (1.19)$$

Relation between the displacement field and stress glut moments

We assume that following product can represent the time derivative of stress glut tensor:

$$\dot{\Gamma}(\mathbf{x},t) = f(\mathbf{x},t)\mathbf{m}, \quad (1.20)$$

where $f(\mathbf{x},t)$ is non-negative function and **m** is a uniform normalized moment tensor.
The moment $f^{(l,n)}_{k_1\dots k_l}(\mathbf{q},\tau)$ of spatial degree l and temporal degree n with respect to point **q** and instant of time τ is a tensor of order l and is given by formula

$$f^{(l,n)}_{k_1\dots k_l}(\mathbf{q},\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_{k_1} - q_{k_1})\cdots(x_{k_l} - q_{k_l})(t-\tau)^n dt, \quad (1.21)$$

$k_1,\dots,k_l$=1,2,3.

Replacing in equation (1.11) H_{ij}(**x**,**y**,t-τ) by its Taylor series in powers of **y** and in powers of τ, we get:

$$u_i(\mathbf{x},t) = \sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n}{l!n!}m_{jk}f^{(l,n)}_{k_1\dots k_l}(\mathbf{0},0)\frac{\partial^n}{\partial t^n}\frac{\partial}{\partial y_{k_1}}\cdots\frac{\partial}{\partial y_{k_l}}\frac{\partial}{\partial y_k}H_{ij}(\mathbf{x},\mathbf{y},t)\Big|_{\mathbf{y}=\mathbf{0}}. \quad (1.22)$$

Using formulae (1.18) and (1.22) we have following equation for the spectrum of displacements:

$$u_i(\mathbf{x},\omega) = \sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n}{l!n!}m_{jk}f^{(l,n)}_{k_1\dots k_l}(\mathbf{0},0)(\mathrm{i}\omega)^{n-1}\frac{\partial}{\partial y_{k_1}}\cdots\frac{\partial}{\partial y_{k_l}}\frac{\partial}{\partial y_k}G_{ij}(\mathbf{x},\mathbf{y},\omega)\Big|_{\mathbf{y}=\mathbf{0}}. \quad (1.23)$$

Here we assume that the point **y=0** and the instant t=0 belong to the source region and the time of the source activity respectively.

When the spectra of displacements u_i(**x**,ω) and Green's function G_{ij}(**x**,**y**,ω) have been low pass filtered, the terms in equation (1.23) start to decrease with l and n increasing at least as rapidly as $(\omega T)^{l+n}$ (T is the source duration, and $\omega T<1$), and one might then restrict to considering finite sums only.

In the following sections we will take into account in formula (1.23) only the first terms for $l+n \le 2$.

II. Source inversion in moment tensor approximation

The first term in (1.23) corresponding to l=0, n=0, describes the spectra of displacements $u_i(\mathbf{x},\omega)$ excited by an instant point source (compare with formula (1.19) taking into account that seismic moment is equal to zero moment of function $f(\mathbf{x},t)$: $M_0=f^{(0,0)}$). For a source with nonzero size and duration this term approximates $u_i(\mathbf{x},\omega)$ with high accuracy for periods much longer then source duration. Performing the inversion of long period seismic waves we describe the earthquake by an instant point source. As it was mentioned in previous section, an instant point source can be given by moment tensor - a symmetric 3x3 matrix $\mathbf{M}$. Seismic moment M_0 is defined by equation $M_0 = \sqrt{\frac{1}{2}\mathrm{tr}(\mathbf{M}^T\mathbf{M})}$, where $\mathbf{M}^T$ is transposed moment tensor $\mathbf{M}$, and $\mathrm{tr}(\mathbf{M}^T\mathbf{M}) = \sum_{i,j=1}^{3} M_{ij}^2$. Moment tensor of any event can be presented in the form $\mathbf{M} = M_0\mathbf{m}$, where matrix $\mathbf{m}$ is normalized by condition $\mathrm{tr}(\mathbf{m}^T\mathbf{m}) = 2$.

We'll consider a double couple instant point source (a pure tangential dislocation) at a depth h. Such a source can be given by 5 parameters: double couple depth, its focal mechanism which is characterizing by three angles: strike, dip and slip or by two unit vectors (direction of principal tension **T** and direction of principal compression **P**) and seismic moment M_0. Four of these parameters we determine by a systematic exploration of the four dimensional parametric space, and the 5-th parameter M_0 - solving the problem of minimization of the misfit between observed and calculated surface wave amplitude spectra for every current combination of all other parameters.

Under assumptions mentioned above the relation between the spectrum of displacements $u_i(\mathbf{x},\omega)$ and moment tensor $\mathbf{M}$ can be expressed by formula (1.19) rewritten below in slightly different form:

$$u_i(\mathbf{x},\omega) = \frac{1}{\mathrm{i}\,\omega}[M_{jl}\frac{\partial}{\partial y_l}G_{ij}(\mathbf{x},\mathbf{y},\omega)] \qquad (2.1)$$

i,j = 1,2,3 and the summation convention for repeated subscripts is used. $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ in equation (2.1) is the spectrum of Green function for the chosen model of medium and wave type (see Levshin, 1985; Bukchin, 1990), $\mathbf{y}$ - source location. We will discuss the inversion of surface wave spectra, so $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ is the spectrum of surface wave Green function. We assume that the paths from the earthquake source to seismic stations are relatively simple and are well approximated by weak laterally inhomogeneous model (Woodhouse, 1974; Babich *et al.*, 1976). The surface wave Green function in this approximation is determined by the near source and near receiver velocity structure, by the mean phase velocity of wave, and by geometrical spreading. The amplitude spectrum $|u_i(\mathbf{x},\omega)|$ defined by formula (2.1) does not depend on the average phase velocity of the wave. In such a model the errors in source location do not affect the amplitude spectrum (Bukchin, 1990). The average phase velocities of surface waves are usually not well known. For this reason as a rule we use only amplitude spectra of surface waves for determining source parameters under consideration. We use observed surface wave phase spectra only for very long periods.

Surface wave amplitude spectra inversion

If all characteristics of the medium are known the representation (2.1) gives us a system of equations for parameters defined above. Let us consider now a grid in the space of these 4 parameters. Let the models of the media be given. Using formula (2.1) we can calculate the amplitude spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed amplitude spectra give us a residual $\varepsilon^{(i)}$ for every point of observation, every wave and every frequency ω. Let $u^{(i)}(\mathbf{x},\omega)$ be any observed value of the spectrum, i = 1,...,N; $\varepsilon^{(i)}_{\text{amp}}$ - corresponding residual of $|u^{(i)}(\mathbf{x},\omega)|$. We define the normalized amplitude residual by formula

$$\varepsilon_{\text{amp}}(h,\mathbf{T},\mathbf{P}) = \left[\left(\sum_{i=1}^{N} \varepsilon^{(i)\,2}_{\text{amp}}\right) \Big/ \left(\sum_{i=1}^{N} |\,u^{(i)}(\mathbf{x},\omega\,|^2\right)\right]^{1/2}. \qquad (2.2)$$

The optimal values of the parameters that minimize ε_{amp} we consider as estimates of these parameters. We search them by a systematic exploration of the four-dimensional parameter space. To characterize the degree of resolution of every of these source characteristics we calculate partial residual functions. Fixing the value of one of varying parameters we put in correspondence to it a minimal value of the residual ε_{amp} on the set of all possible values of the other parameters. In this way we define one residual function on scalar argument and two residual functions on vector argument corresponding to the scalar and two vector varying parameters: $\varepsilon_h(h)$, $\varepsilon_{\mathbf{T}}(\mathbf{T})$ and $\varepsilon_{\mathbf{P}}(\mathbf{P})$. The value of the parameter for which the corresponding function of the residual attains its minimum we define as estimate of this parameter. At the same time these functions characterize the degree of resolution of the corresponding parameters. From geometrical point of view these functions describe the lower boundaries of projections of the 4-D surface of functional ε on the coordinate planes. A sketch illustrating the definition of partial residual functions is given in Fig. 1.
Here one of 4 parameters is picked out as 'parameter 1', and one of coordinate axis corresponds to this parameter. Another coordinate axis we consider formally as 3-D space of the rest 3 parameters. Plane Σ is orthogonal to the axis 'parameter 1' and cross it in a point p_0. Curve L is the intersection of the plane Σ and the surface of functional ε. As one can see from the figure the point $\varepsilon_1(p_0)$ belong to the boundary of projection of the surface of functional ε, and at the same time it corresponds to a minimal value of the residual ε on the set of all possible values of the other 3 parameters while 'parameter 1' is equal to the value p_0.
So, as it is accepted in engineering we characterize our surface by its 4 projections on coordinate planes.

It is well known that the focal mechanism cannot be uniquely determined from surface wave amplitude spectra. There are four different focal mechanisms radiating the same surface wave amplitude spectra. These four equivalent solutions represent two pairs of mechanisms symmetric with respect to the vertical axis, and within the pair differ from each other by the opposite direction of slip.

To get a unique solution for the focal mechanism we have to use in the inversion additional observations. For these purpose we use very long period phase spectra of surface waves or polarities of P wave first arrivals.

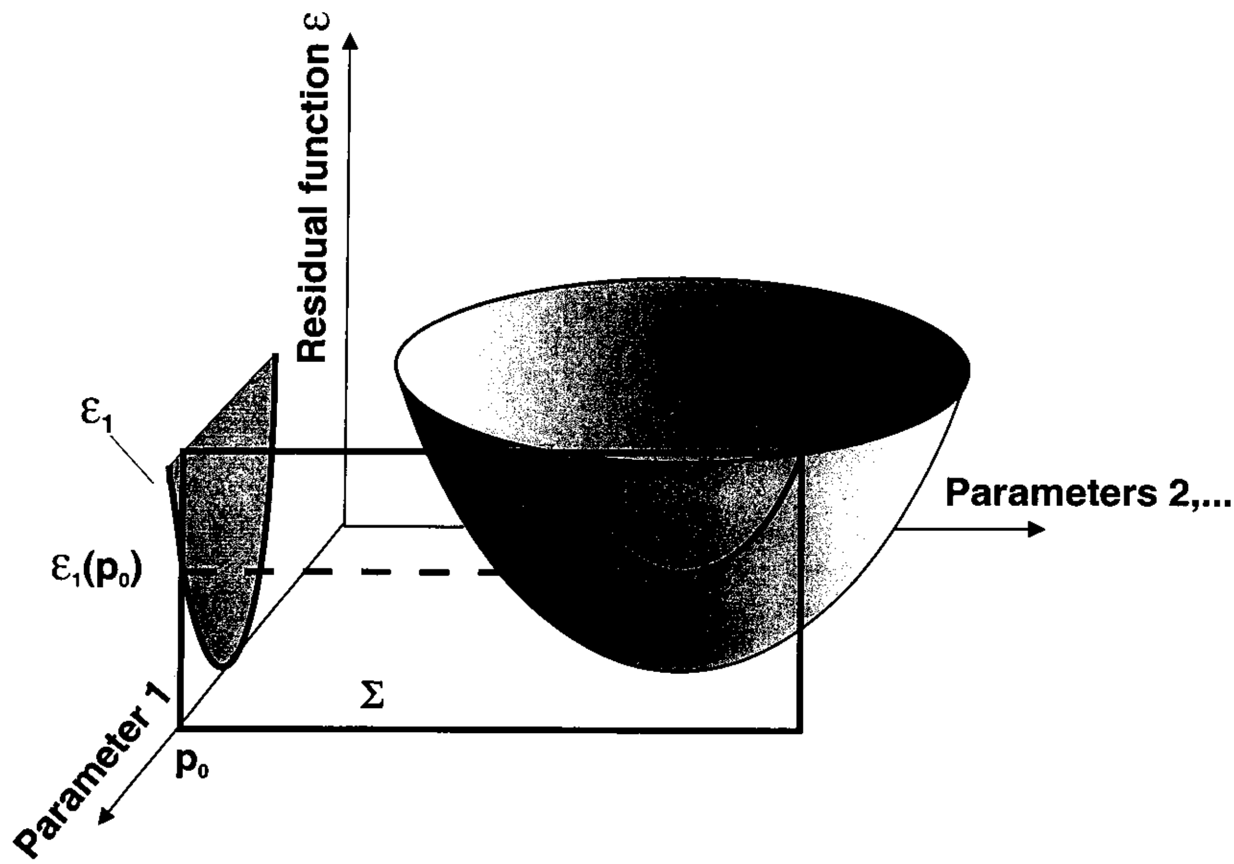


Fig. 1. A sketch illustrating the definition of partial residual functions.

Joint inversion of surface wave amplitude and phase spectra

Using formula (2.1) we can calculate for chosen frequency range the phase spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed phase spectra give us a residual $\varepsilon_{ph}^{(i)}$ for every point of observation, every wave and every frequency ω. We define the normalized phase residual by formula

$$\varepsilon_{ph}(h, \varphi, \mathbf{T}, \mathbf{P}) = \frac{1}{\pi}\left[\left(\sum_{i=1}^{N} \varepsilon_{ph}^{(i)2}\right) / N\right]^{1/2} . \tag{2.3}$$

We determine the joint residual ε by formula

$$\varepsilon = 1 - (1 - \varepsilon_{ph})(1 - \varepsilon_{amp}) . \tag{2.4}$$

To characterize the resolution of source characteristics we calculate partial residual functions in the same way as was described above.

Joint inversion of surface wave amplitude spectra and P wave polarities

Calculating radiation pattern of P waves for every current combination of parameters we compare it with observed polarities. The misfit obtained from this comparison we use to calculate a joint residual of surface wave amplitude spectra and polarities of P wave first arrivals. Let ε_{amp} be the residual of surface wave amplitude spectra, ε_{p} - the residual of P wave first arrival polarities (the number of wrong polarities divided by the full number of observed polarities), then we determine the joint residual ε by formula

$$\varepsilon = 1 - (1 - \varepsilon_{p})(1 - \varepsilon_{amp}) . \tag{2.5}$$

For this type of inversion we calculate partial residual functions to characterize the resolution of parameters under determination in the same way as it was described for two first types.

Before inversion we apply to observed polarities a smoothing procedure, which we will describe here briefly.

Let us consider a group of observed polarities (+1 for compression and -1 for dilatation) radiated in directions deviating from any medium one by a small angle. This group is presented in the inversion procedure by one polarity prescribing to this medium direction. If the number of one of two types of polarities from this group is significantly larger then the number of opposite polarities, then we prescribe this polarity to this medium direction. If no one of two polarity types can be considered as preferable, then all these polarities will not be used in the inversion. To make a decision for any group of n observed polarities we calculate the sum $m = n_+ - n_-$, where n_+ is the number of compressions and $n_- = n - n_+$ is the number of dilatations. We consider one of polarity types as preferable if $|m|$ is larger then its standard deviation in the case when +1 and -1 appear randomly with this same probability 0.5. In this case n_+ is a random value distributed following the binomial low. For its average we have $M(n_+) = 0.5n$, and for dispersion $D(n_+) = 0.25n$. Random value m is a linear function of n_+ such that $m = 2n_+ - n$. So following equations are valid for the average, for the dispersion, and for the standard deviation σ of value m

$M(m) = 2M(n_+) - n = n - n = 0$, $D(m) = 4D(n_+) = n$, and $\sigma(m) = \sqrt{n}$.

As a result, if the inequality $|m| \geq \sqrt{n}$ is valid then we prescribe +1 to the medium direction if $m > 0$, and -1 if $m < 0$.

Example of application

We illustrate the technique by results of its application for a study of Vrancea earthquake, 30.05.90, M_s=6.7. Using frequency-time and polarization analysis programs we analyzed fundamental Love and Rayleigh modes recorded by IRIS and GEOSCOPE networks. We selected for moment tensor and source depth inversion records of 14 stations. We used the signals of a good quality and normal polarization. The distribution of selected stations with respect to the epicenter is given in Fig. 1.

Analyzing the long period part of the spectra (periods from 100 to 170 seconds) we determined the following focal mechanism of the source: strike 225°, dip 60°, and rake 105°. The stereographic projection of nodal planes on the lower hemisphere and seismic moment, obtained by surface wave amplitude spectra inversion and by joint inversion of surface wave amplitude spectra and first arrival polarities are shown in Fig. 3. and Fig.4 respectively. As one can see, the best double couple obtained by joint inversion is the same as one of four equivalent solutions in Fig. 3. The procedure of polarity smoothing is illustrated by Fig. 5 (the value of angle was taken equal to 10 °). The resolution maps for main double couple axes obtained by two different inversions are given in figures 6 and 7. In the first inversion (Fig. 6) the first arrival polarities were used as additional observations to the surface wave amplitude spectra. I the second inversion we used as additional observations surface wave phase spectra for periods from 150 to 170 seconds. The best solutions obtained by both inversions are the same, but the resolution of focal mechanism is significantly higher when the first arrival polarities were used. The source depth was estimated at 105 km. The depth resolution curve by joint inversion of surface wave amplitude spectra and first arrival polarities is shown in Fig.8.

Map, centered at45.87:26.67

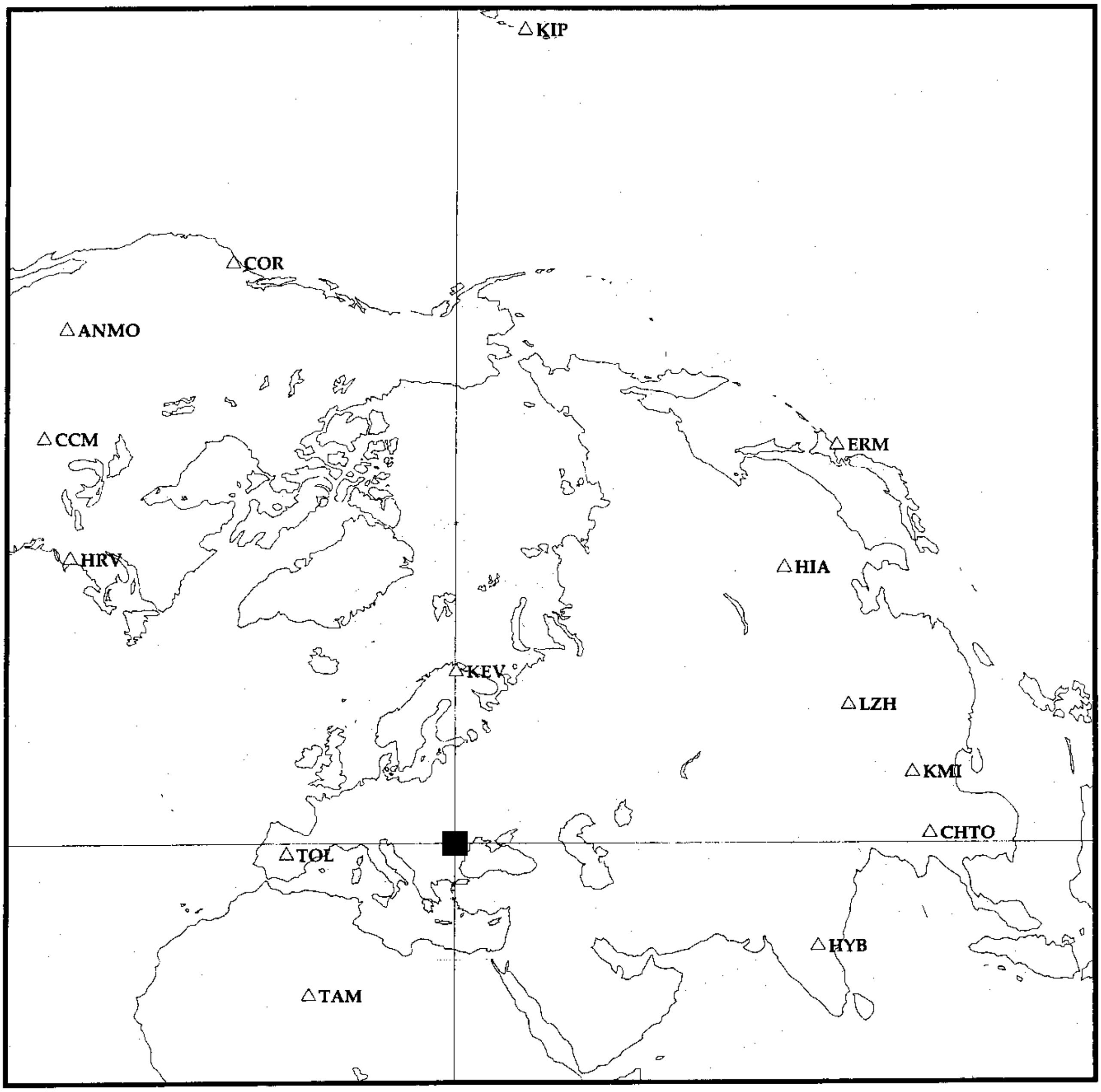


Fig. 2. Vrancea, 90 earthquake. Distribution of stations used for long period surface waves inversion.

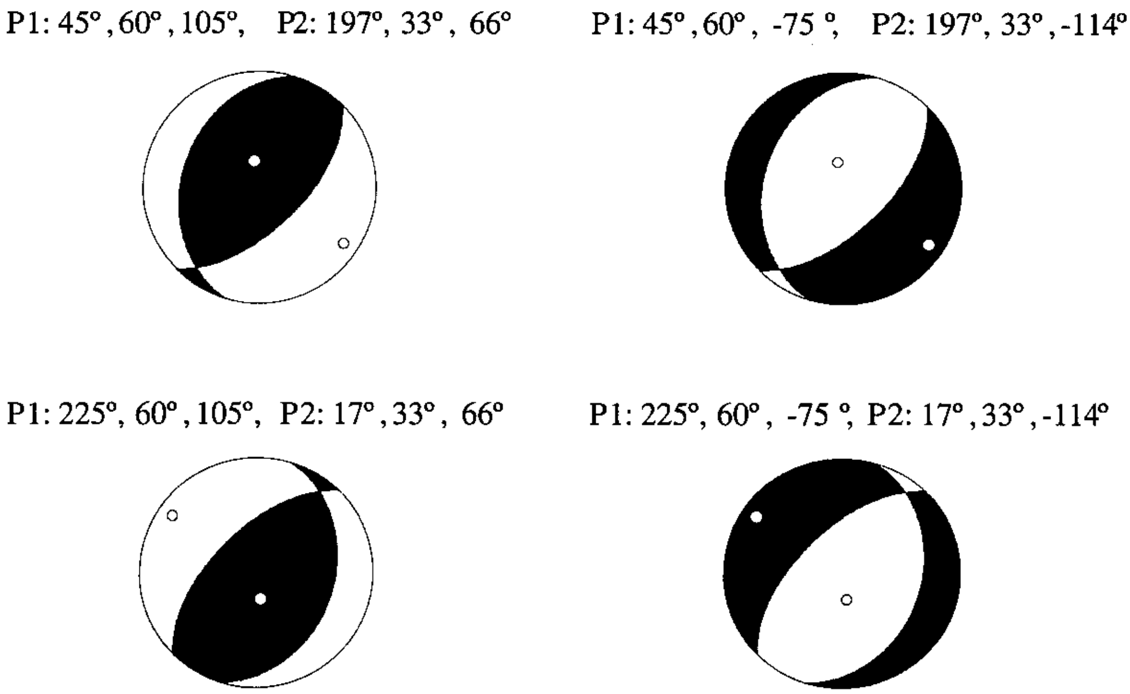


Fig. 3. Four equivalent solutions
from surface wave amplitude spectra inversion
M0=0.26E+20N·m

P1: 225°, 60°, 105°, P2: 17°, 33°, 66°

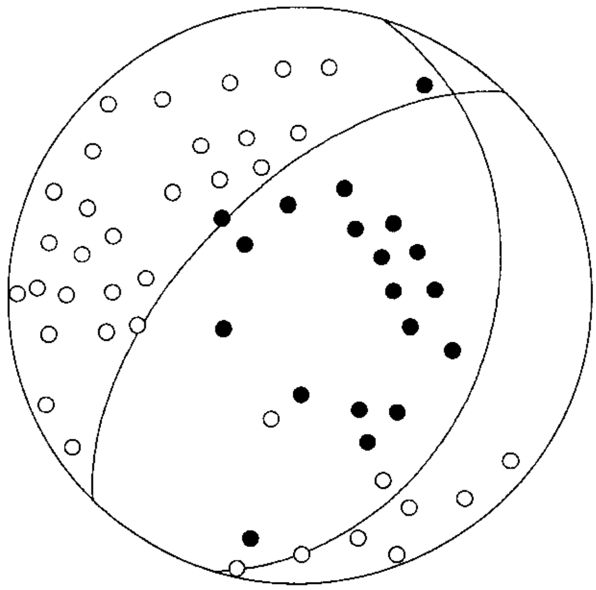
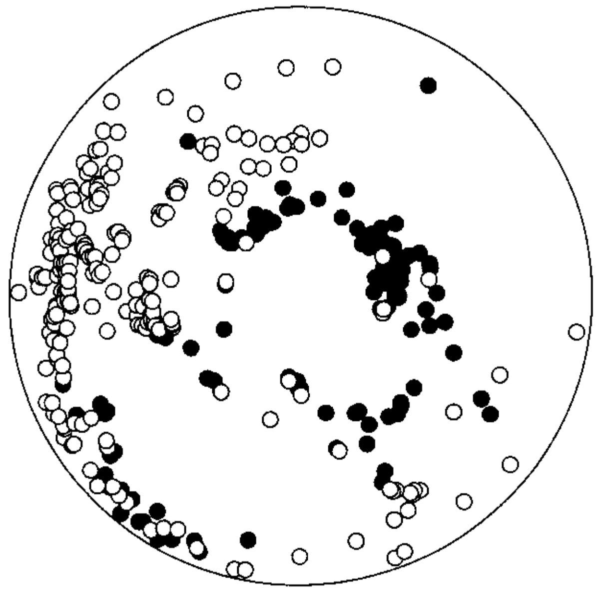


Fig. 4. Best double couple from joint inversion of surface wave amplitude spectra and first arrival polarities M0=0.26E+20N·m

Original first arrival polarities



Selected and rarefied first arrival polarities

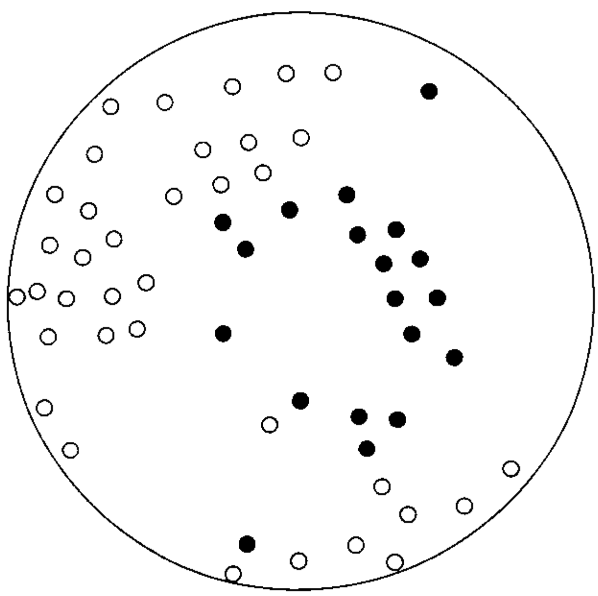


Fig. 5. First arrival polarities before and after smoothing

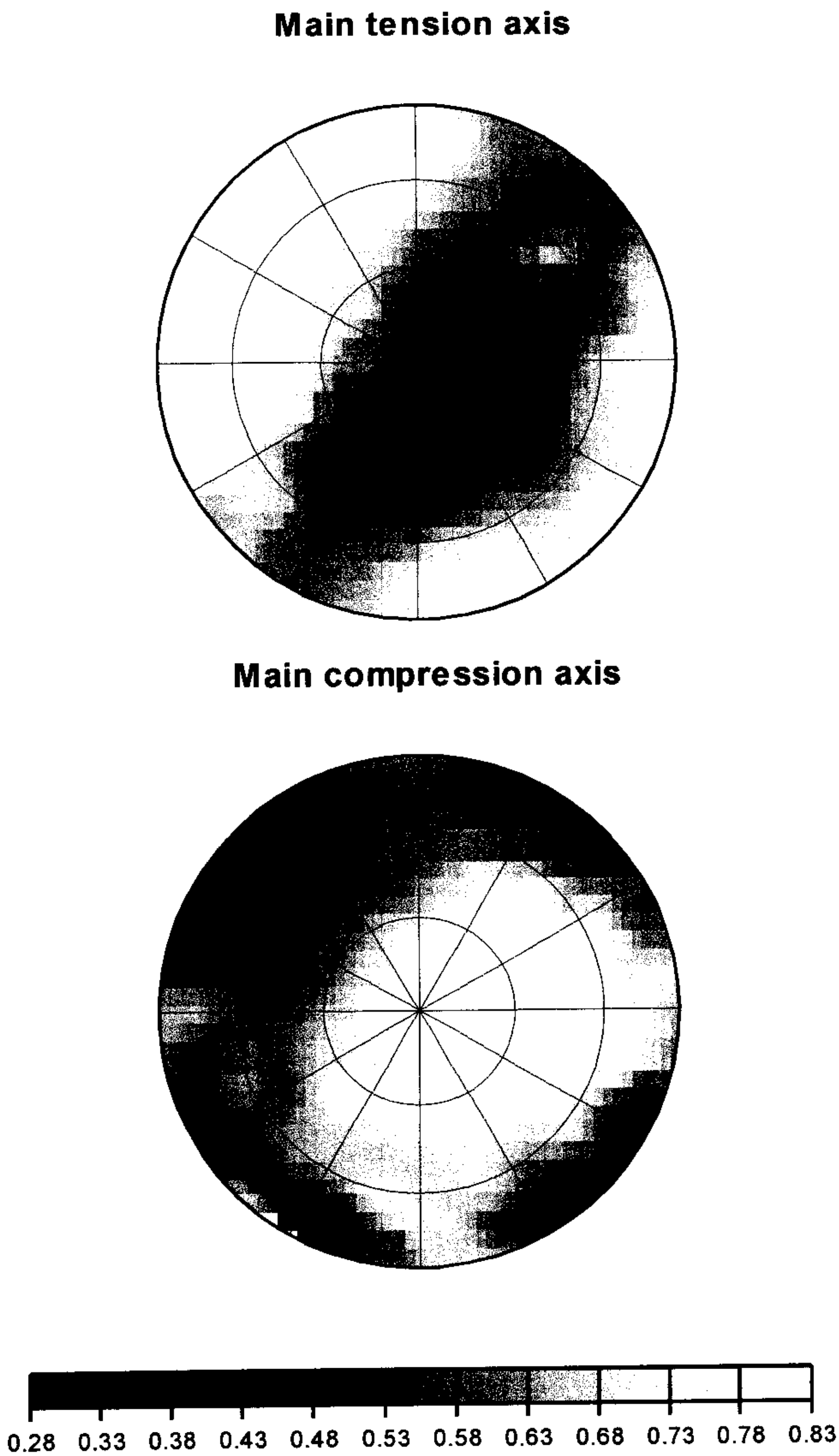


Fig. 6. Resolution of main axes by joint inversion
of surface wave amplitude spectra and first arrival polarities

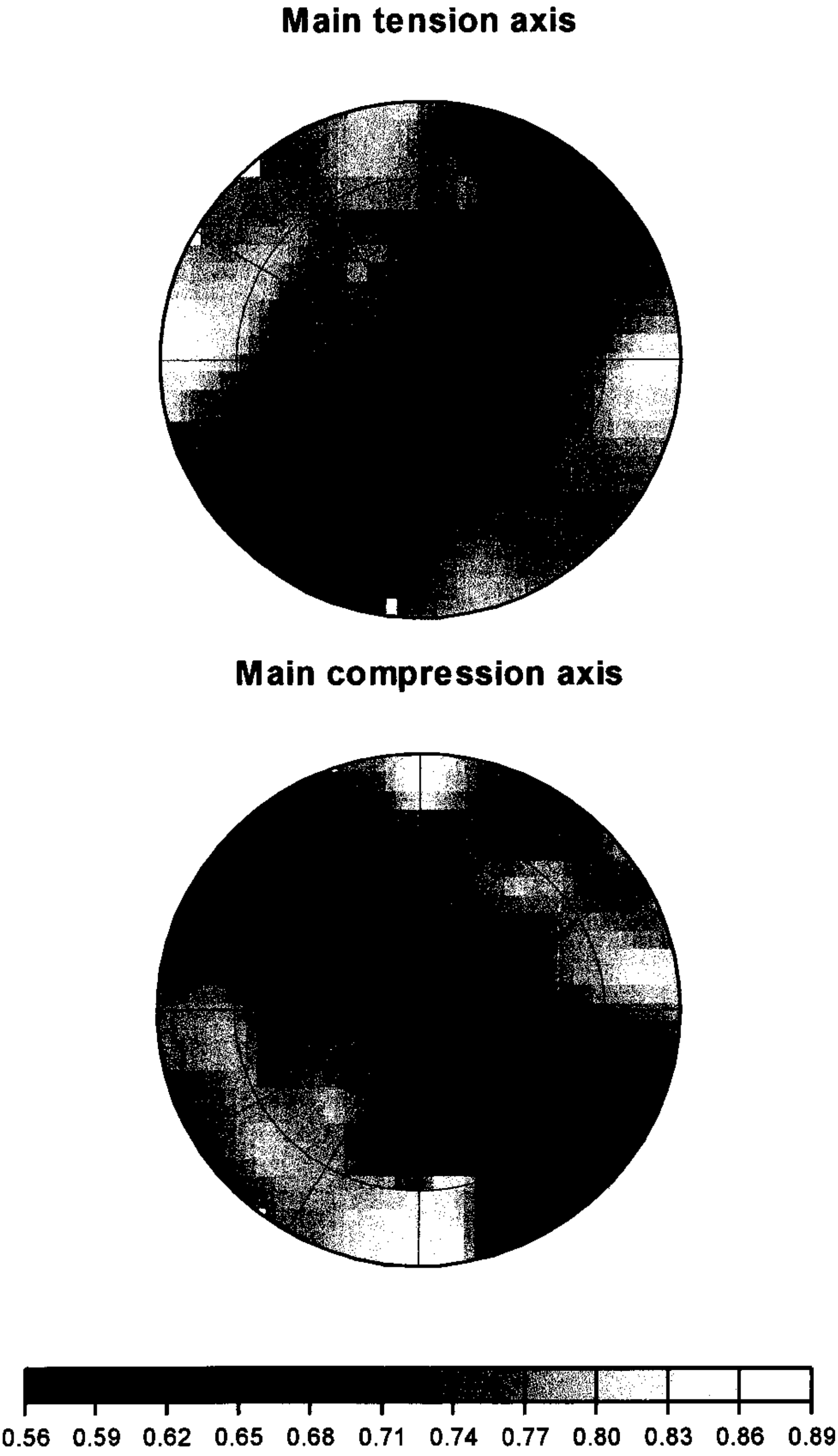


Fig. 7. Resolution of main axes by joint inversion of surface wave amplitude and long period phase spectra

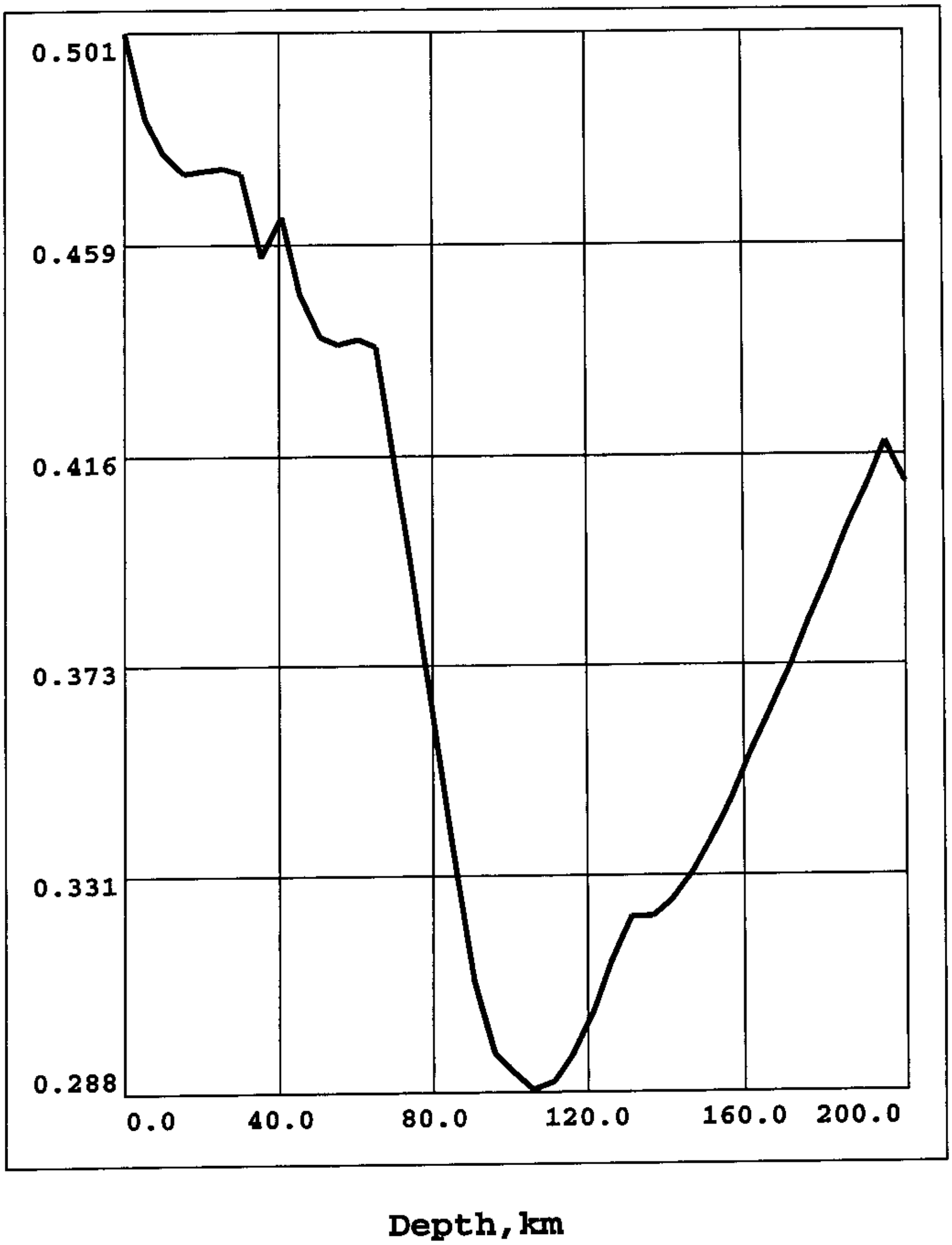


Fig. 8. Resolution of best double couple depth by joint inversion of surface wave amplitude spectra and first arrival polarities

III. Second moments approximation. Characteristics of source shape and evolution in time.

We present here a technique based on the description of seismic source distribution in space and in time by integral moments (see Bukchin *et al.*, 1994; Bukchin, 1995; Gomez, 1997 a, b). We assume that the time derivative of stress glut tensor $\dot{\Gamma}$ can be represented in form (1.20). Following Backus and Mulcahy, 1976 we will define the source region by the condition that function $f(\mathbf{x},t)$ is not identically zero and the source duration is the time during which nonelastic motion occurs at various points within the source region, i.e., $f(\mathbf{x},t)$ is different from zero.

Spatial and temporal integral characteristics of the source can be expressed by corresponding moments of the function $f(\mathbf{x},t)$ (Backus, 1977; Bukchin *et al.*, 1994). These moments can be estimated from the seismic records using the relation between them and the displacements in seismic waves, which we will consider later. In the general case stress glut moments of spatial degree 2 and higher are not uniquely determined by the displacement field. But in the case when equation (1.20) is valid such uniqueness takes place (Bukchin, 1995).

Following equations define the spatio-temporal moments of function $f(\mathbf{x},t)$ of total degree (both in space and time) 0, 1, and 2 with respect to point $\mathbf{q}$ and instant of time τ.

$$f^{(0,0)} = \int_V dV \int_0^\infty f(\mathbf{x},t)dt, \quad f_i^{(1,0)}(\mathbf{q}) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_i - q_i)dt,$$

$$f^{(0,1)}(\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(t-\tau)dt, \quad f^{(0,2)}(\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(t-\tau)^2 dt,$$

$$f_i^{(1,1)}(\mathbf{q},\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_i - q_i)(t-\tau)dt, \tag{3.1}$$

$$f_{ij}^{(2,0)}(\mathbf{q}) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_i - q_i)(x_j - q_j)dt$$

Using these moments we will define integral characteristics of the source. Source location is estimated by the spatial centroid $\mathbf{q}_c$ of the field $f(\mathbf{x},t)$ defined as

$$\mathbf{q}_c = \mathbf{f}^{(1,0)}(\mathbf{0}) / M_0 , \tag{3.2}$$

where $M_0 = f^{(0,0)}$ is the scalar seismic moment.

Similarly, the temporal centroid τ_c is estimated by the formula

$$\tau_c = f^{(0,1)}(0) / M_0 . \tag{3.3}$$

The source duration is Δt estimated by $2\Delta\tau$, where

$$(\Delta\tau)^2 = f^{(0,2)}(\tau_c) / M_0 . \tag{3.4}$$

The spatial extent of the source is described by matrix $\mathbf{W}$,

$$\mathbf{W} = \mathbf{f}^{(2,0)}(\mathbf{q}_c) / M_0 . \tag{3.5}$$

The mean source size in the direction of unit vector $\mathbf{r}$ is estimated by value $2l_r$, defined by formula

$$l_r^2 = \mathbf{r}^T \mathbf{W} \mathbf{r}, \tag{3.6}$$

where $\mathbf{r}^T$ is the transposed vector. From (3.5) and (3.6) we can estimate the principal axes of the source. There directions are given by the eigenvectors of the matrix $\mathbf{W}$, and the lengths are defined by correspondent eigenvalues: the length of the minor semi-axis is equal to the least eigenvalue, and the length of the major semi-axis is equal to the greatest eigenvalue.

In the same way, from the coupled space time moment of order (1,1) the mean velocity $\mathbf{v}$ of the instant spatial centroid (Bukchin, 1989) is estimated as

$$\mathbf{v} = \mathbf{w} / (\Delta\tau)^2 , \qquad (3.7)$$

where $\mathbf{w} = \mathbf{f}^{(1,1)}(\mathbf{q}_c, \tau_c) / M_0$.

Now we will consider the low frequency part of the spectra of the i^{th} component of displacements in Love or Rayleigh wave $u_i(\mathbf{x},\omega)$. It is assumed that the frequency ω is small, so that the duration of the source is small in comparison with the period of the wave, and the source size is small as compared with the wavelength. It is assumed that the origin of coordinate system is located in the point of spatial centroid $\mathbf{q}_c$ (i.e. $\mathbf{q}_c = \mathbf{0}$) and that time is measured from the instant of temporal centroid, so that $\tau_c = 0$. With this choice the first degree moments with respect to the spatial origin **x**=**0** and to the temporal origin t=0 are zero, i.e. $\mathbf{f}^{(1,0)}(\mathbf{0}) = \mathbf{0}$ and $f^{(0,1)}(0) = 0$.

Under this assumptions, taking into account in formula (1.23) only the first terms for $l + n \leq 2$ we can express the relation between the spectrum of displacements $u_i(\mathbf{x},\omega)$ and the spatio-temporal moments of the function $f(\mathbf{x},t)$ by following formula (Bukchin,1995)

$$u_i(\mathbf{x},\omega) = \frac{1}{\mathrm{i}\omega} M_0 M_{jl} \frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega) + \frac{1}{2\mathrm{i}\omega} f_{mn}^{(2,0)}(\mathbf{0}) M_{jl} \frac{\partial}{\partial y_m}\frac{\partial}{\partial y_n}\frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega)$$

$$- f_m^{(1,1)}(\mathbf{0},0) M_{jl} \frac{\partial}{\partial y_m}\frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega) + \frac{\mathrm{i}\omega}{2} f^{(0,2)}(0) M_{jl} \frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega) , \qquad (3.8)$$

i,j,l,m,n = 1,2,3 and the summation convention for repeated subscripts is used. $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ in equation (3.8) is the spectrum of Green function for the chosen model of medium and wave type. We assume that the paths from the earthquake source to seismic stations are well approximated by weak laterally inhomogeneous model. Under this assumption the amplitude spectrum $|u_i(\mathbf{x},\omega)|$ defined by formula (3.8) does not depend on the average phase velocity of the wave, and the errors in source location do not affect the amplitude spectrum (Bukchin, 1990).

If all characteristics of the medium, depth of the best point source and seismic moment tensor are known (determined, for example, using the spectral domain of longer periods) the representation (3.8) gives us a system of linear equations for moments of the function $f(\mathbf{x},t)$ of total degree 2. But as we mentioned considering moment tensor approximation the average phase velocities of surface waves are usually not well known. For this reason, we use only amplitude spectrum of surface waves for determining these moments, in spite of non-linear relation between them.

Let us consider a plane source. All moments of the function $f(\mathbf{x},t)$ of total degree 2 can be expressed in this case by formulas (3.2)-(3.7) in terms of 6 parameters: Δt - estimate of source duration, $l_{\max}$ - estimate of maximal mean size of the source, φ_l - estimate of the angle between the direction of maximal size and strike axis, $l_{\min}$ - estimate of minimal mean size of the source, v - estimate of the absolute value of instant centroid mean velocity **v** and φ_v - the angle between **v** and strike axis.

Using the Bessel inequality for the moments under discussion we can obtain the following constrain for the parameters considered above (Bukchin, 1995):

$$v \Delta t^2 \left(\frac{\cos^2 \varphi}{l_{\max}^2} + \frac{\sin^2 \varphi}{l_{\min}^2} \right) \leq 1 , \qquad (3.9)$$

where φ is the angle between major axis of the source and direction of **v**.

Assuming that the source is a plane fault and representation (1.20) is valid let us consider a rough grid in the space of 6 parameters defined above. These parameters have to follow inequality (3.9). Let models of the media be given and the moment tensor be fixed as well as the depth of the best point source. Let the fault plane (one of two nodal planes) be identified. Using formula (3.8) we can calculate the amplitude spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed amplitude spectra give us a residual $\varepsilon^{(i)}$ for every point of observation, every wave and every frequency ω. Let $u^{(i)}(\mathbf{r},\omega)$ be any observed value of the spectrum, $i = 1,\ldots,N$; $\varepsilon^{(i)}$ - corresponding residual of $|u^{(i)}(\mathbf{r},\omega)|$. We define the normalized amplitude residual by formula

$$\varepsilon(\Delta t, l_{max}, l_{min}, \varphi_l, v, \varphi_v) = \left[\left(\sum_{i=1}^{N} \varepsilon^{(i)2} \right) \Big/ \left(\sum_{i=1}^{N} |u^{(i)}(\mathbf{r},\omega|^2 \right) \right]^{1/2} . \quad (3.10)$$

The optimal values of the parameters that minimize ε we consider as estimates of these parameters. We search them by a systematic exploration of the six dimensional parameter space. To characterize the degree of resolution of every of these source characteristics we calculate partial residual functions in the same way as was described in previous section. We define 6 functions of the residual corresponding to the 6 varying parameters: $\varepsilon_{\Delta t}(\Delta t)$, $\varepsilon_{l_{max}}(l_{max})$, $\varepsilon_{l_{min}}(l_{min})$, $\varepsilon_{\varphi_l}(\varphi_l)$, $\varepsilon_v(v)$ and $\varepsilon_{\varphi_v}(\varphi_v)$. The value of the parameter for which the corresponding function of the residual attains its minimum we define as estimate of this parameter. At the same time these functions characterize the degree of resolution of the corresponding parameters.

Example of application

We illustrate the technique by results of its application for a study of the same Vrancea earthquake, 30.05.90, which we considered above. To estimate duration and geometry of the source we have used amplitude spectra of fundamental modes of Love and Rayleigh waves in the spectral domain from 40 to 60 seconds. We fixed source depth, focal mechanism and seismic moment obtained from analysis of long period surface wave spectra considered above. The plane dipping to the North-East was identified as a fault plane. Results of direct trial of possible values of the unknown parameters are shown in Fig. 10.

Residual function for the integral estimate of duration attains its minimum at 8 seconds. The residual function for the integral estimate of the instant centroid velocity attains its minimum between 3 and 4 km/s.

For the integral estimate of the main axis was obtained the value 40 km. The residual function for the integral estimate of the minimal size of the source attains its minimum between 0 and 20 km.

The residual functions for φ_l and φ_v defining the direction of main axis of the source and the direction of the instant centroid velocity are given by two last curves in Fig.10. These residuals were calculated for all possible values of angles φ_l and φ_v while other parameters were fixed equal to their estimates obtained before. Angles are measured in the footwall of the fault plane clockwise round from the strike axis. φ_l varies from -90° to 90° and $\varepsilon_{\varphi_l}(\varphi_l)$ attains its minimum at -25°; φ_v varies from 0° to 360° and residual functions $\varepsilon_{\varphi_v}(\varphi_v)$ attain its minimum at 155°. Taking into account that two directions of maximum source size differing from each other by 180° are equivalent, one can see that estimated instant centroid velocity is directed along estimated source major axis.

As a result of surface wave spectra analysis we produced a model of the source. A scheme of this model is given in Figure 11. The ellipse in the fault plane represents here the integral estimates of source geometry. Vector **v** is the direction of the instant centroid velocity. Vector **E** is directed to the East, and vector **S** is directed to the South. Stereographic projection of nodal planes on the lower hemisphere defining the focal mechanism is shown at the same figure.

Map, centered at 45.87:26.67

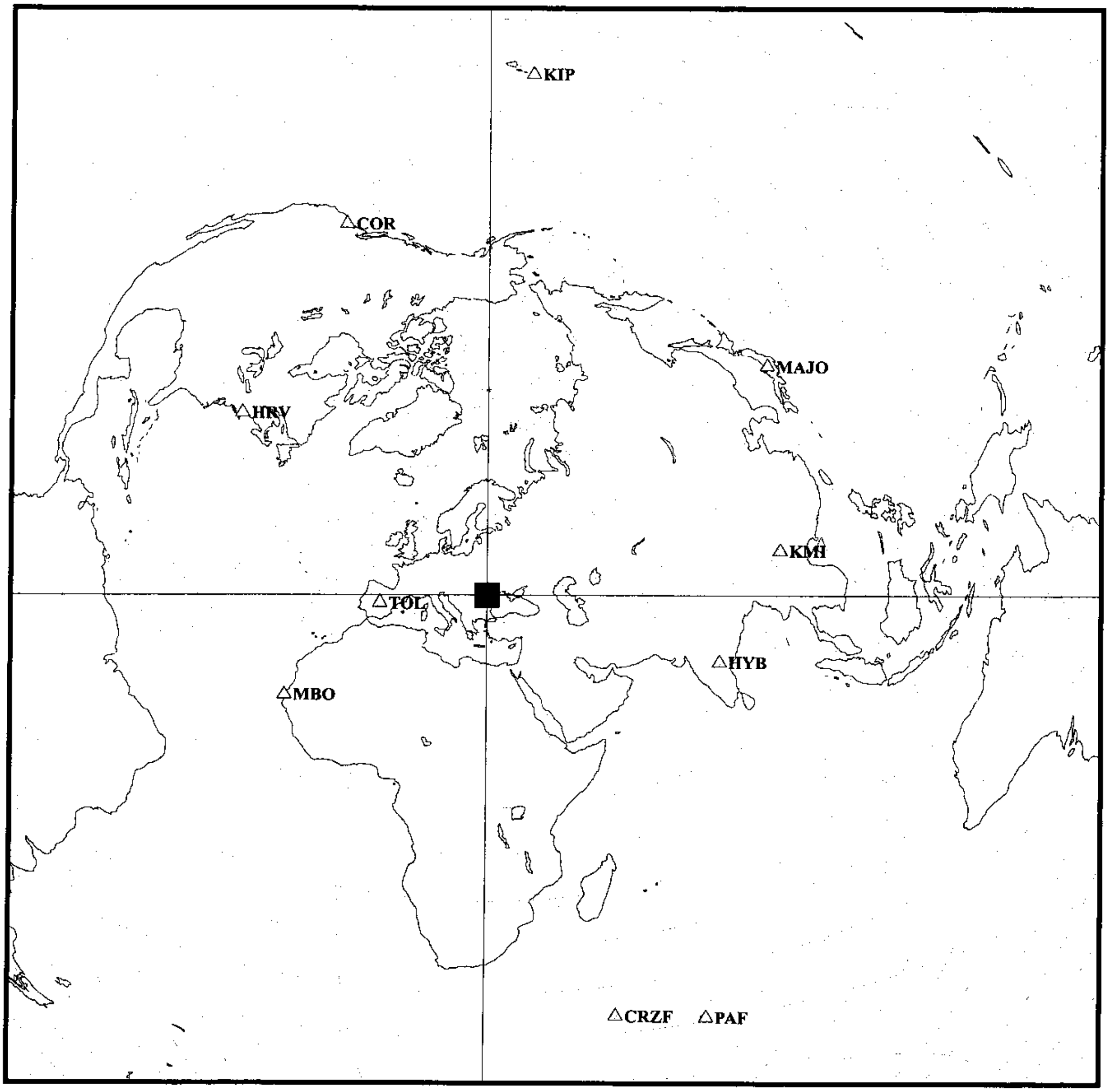


Fig. 9. Vrancea, 90 earthquake. Distribution of stations used for intermediate period surface waves inversion.

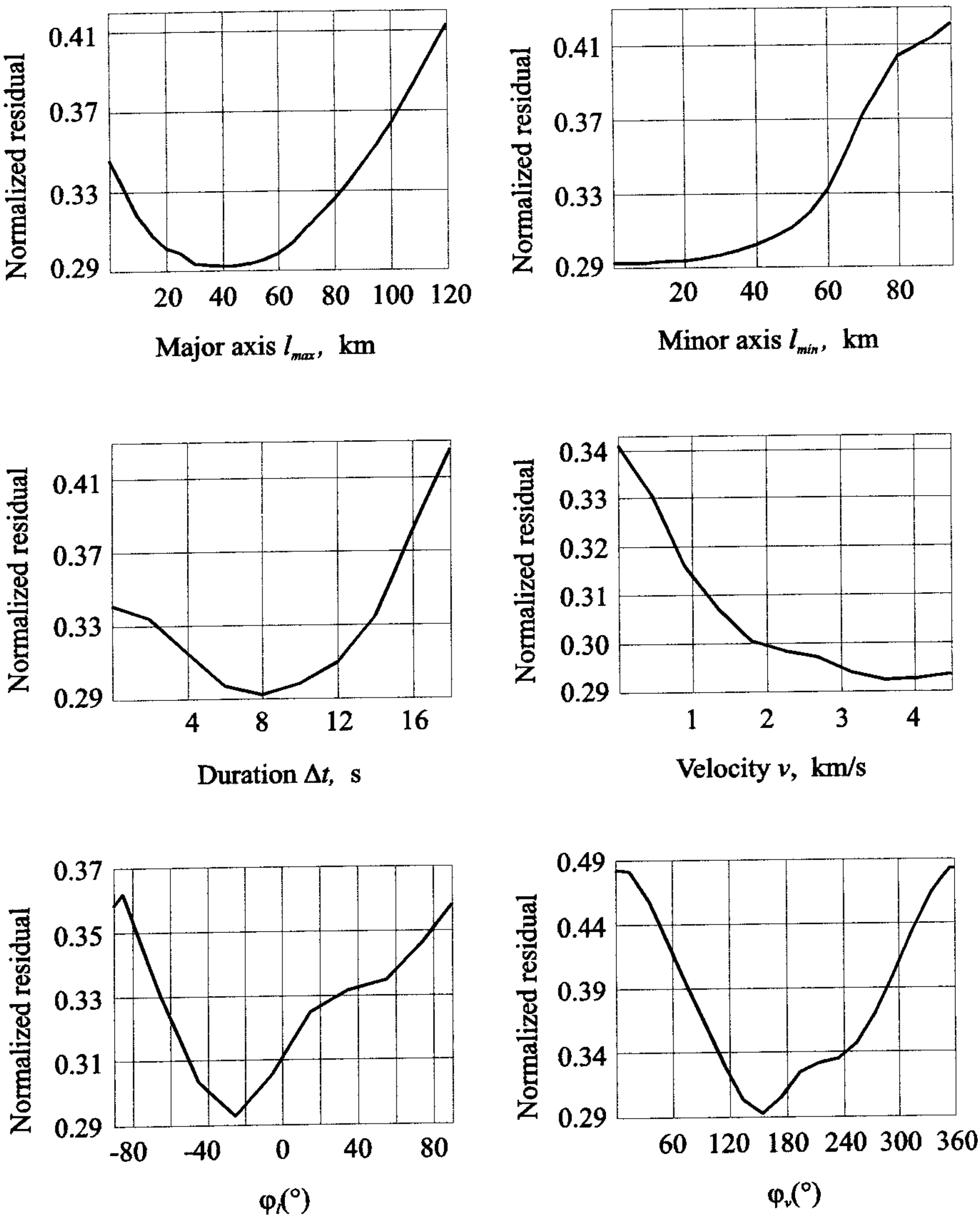


Fig. 10. Partial residual functions of varying parameters

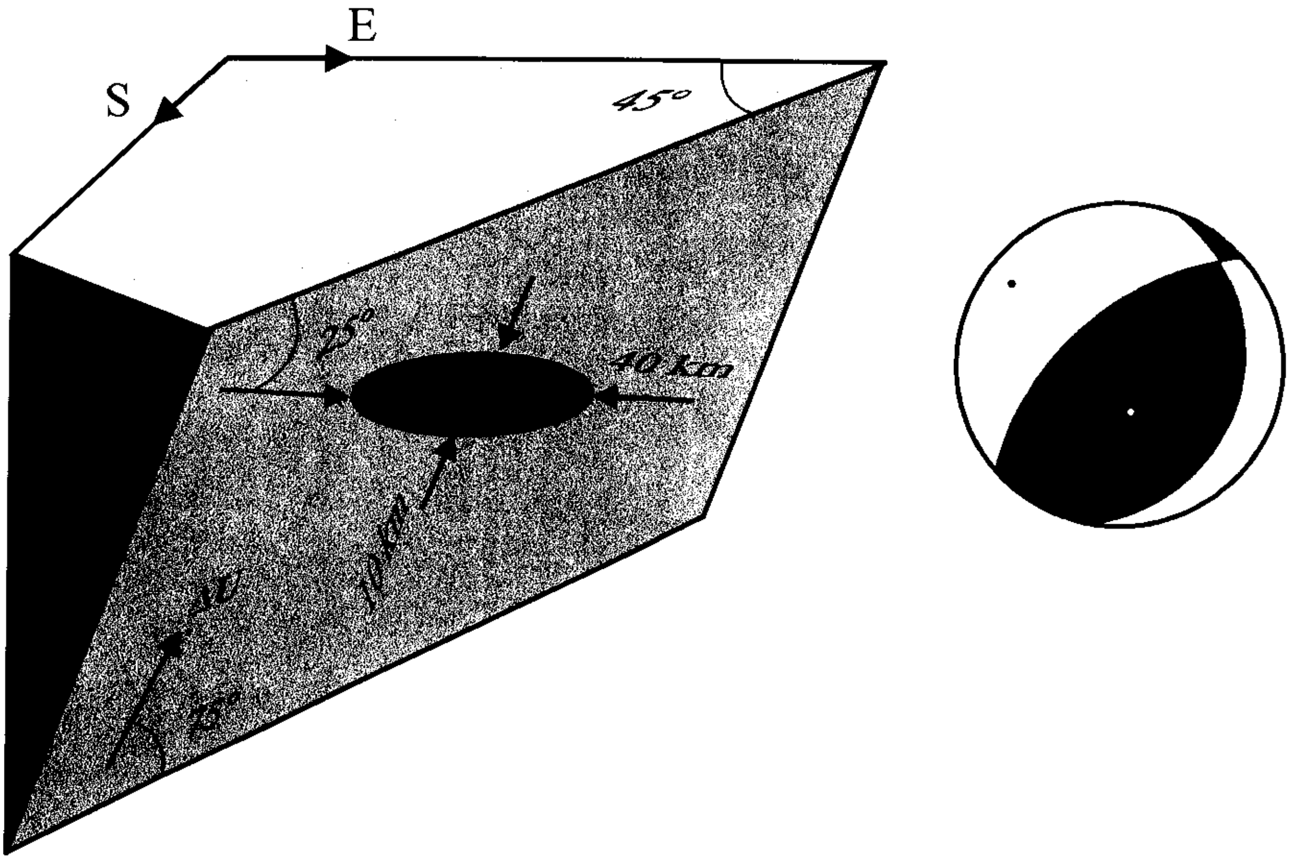


Fig. 11. Scheme of source model and focal mechanism.
ΔU - slip vektor, V - direction of instant centroid velocity.

IV. Study of nuclear explosions by joint inversion of surface wave amplitude spectra and P wave first arrival polarities.

This section is based on the paper by Bukchin *et al.*, 2000 submitted to PAGEOPH. A combination of isotropic tensor, modelling an explosion, and pure double couple, modelling the tectonic moment release, is considered as the moment tensor of seismic source. The explosion is located at free surface, and the depth of double couple is a varying parameter. Such a source can be characterized by six parameters - seismic moments of double couple and explosion, three parameters determining the double couple focal mechanism, and its depth.

We determine these source characteristics by minimizing the misfit between synthetic and observed Love and Rayleigh surface waves spectra and polarities of P-waves first arrivals. The optimal values of the parameters that minimize the misfit we consider as estimates of these parameters. We search them by a systematic exploration of parameter space. To characterize the degree of resolution of every of these source characteristics, we calculate partial residual functions as it was described above.

We applied the method described herein to a small set of data recorded regionally between 1990 and 1996 following events (seven nuclear explosions, three earthquakes, Fig. 12) that occurred on the Lop Nor test site in Western China. The explosions all possess a significant non-zero isotropic component and the estimated depth of the double couple component of the moment tensor, presumably the result of tectonic release, lies between about 0 and 3 km. For the earthquakes studied, the isotropic component is indistinguishable from zero and the depths of the sources are estimated at 3, 17 and 31 km. The data set we have studied, although still very small, suggests that certain source characteristics (namely, double couple depth and the ratio of the isotropic to nonisotropic components of seismic moment) may prove useful in discriminating explosions from shallow earthquakes.

Description of technique

As it was mentioned in section II an instant point source can be described by the moment tensor - a symmetric 3x3 matrix $\mathbf{M}$. Seismic moment M_0 is defined by equation $M_0 = \sqrt{\tfrac{1}{2}\mathrm{tr}(\mathbf{M}^\mathrm{T}\mathbf{M})}$, where $\mathrm{tr}(\mathbf{M}^\mathrm{T}\mathbf{M}) = \sum_{i,j=1}^{3} M_{ij}^{\;2}$. Moment tensor of any event can be presented in the form $\mathbf{M} = M_0\mathbf{m}$, where matrix $\mathbf{m}$ is normalized by condition $\mathrm{tr}(\mathbf{m}^\mathrm{T}\mathbf{m}) = 2$.

We consider the event under study as a sum of earthquake (0-trace moment tensor $\mathbf{M}_{qu}$) and explosion (moment tensor $\mathbf{M}_{ex}$). Moment tensor $\mathbf{M}$ of such an event is given by sum $\mathbf{M} = \mathbf{M}_{qu} + \mathbf{M}_{ex}$.

Let $\mathbf{I}$ be an identity 3x3 matrix. Then $\mathbf{M}_{ex} = \sqrt{\tfrac{2}{3}}M_{0ex}\mathbf{I}$, where M_{0ex} is the seismic moment of the explosion. For the earthquake $\mathbf{M}_{qu} = M_{0qu}\mathbf{m}$, where M_{0qu} is the seismic moment of the earthquake, and $\mathbf{m}$ is a normalized moment tensor, such that $\mathrm{tr}\,\mathbf{m} = 0$ and $\mathrm{tr}(\mathbf{m}^\mathrm{T}\mathbf{m}) = 2$.

Let us consider a 6-D linear Euclidean space of symmetric 3x3 matrixes $\mathbf{M}$, and let the scalar product of two vectors $(\mathbf{M},\mathbf{N})$ is defined by formula

$$(\mathbf{M},\mathbf{N}) = \sum_{i,j} M_{ij}N_{ij} = \mathrm{tr}(\mathbf{M}^\mathrm{T}\mathbf{N})\,. \qquad (4.1)$$

Isotropic tensors $\mathbf{M}_{ex}$ form a 1-D subspace which is orthogonal to 5-D linear subspace of zero trace tensors $\mathbf{M}_{qu}$. This follows from definition (4.1) and from relations:

$$(\mathbf{M}_{qu},\mathbf{M}_{ex}) = \sqrt{2/3}\, M_{0qu} M_{0ex} \mathrm{tr}(\mathbf{m}^{\mathrm{T}}\mathbf{I}) = \sqrt{2/3}\, M_{0qu} M_{0ex} \mathrm{tr}\mathbf{m} = 0. \quad (4.2)$$

Then the scalar moment of combined event can be expressed by formula

$$\begin{aligned} M_0 &= \sqrt{1/2(\mathbf{M},\mathbf{M})} = \sqrt{1/2((\mathbf{M}_{qu}+\mathbf{M}_{ex}),(\mathbf{M}_{qu}+\mathbf{M}_{ex}))} \\ &= \sqrt{1/2(\mathbf{M}_{qu},\mathbf{M}_{qu}) + 1/2(\mathbf{M}_{ex},\mathbf{M}_{ex})} = \sqrt{M_{0qu}{}^2 + M_{0ex}{}^2}. \end{aligned} \quad (4.3)$$

So 6-D vector $\mathbf{M}$ is a sum of two orthogonal vectors $\mathbf{M}_{qu}$ and $\mathbf{M}_{ex}$, and seismic moments of the explosion component M_{0ex} and of the earthquake component M_{0qu} can be expressed by total scalar moment M_0 and the angle φ between 6-D vectors $\mathbf{M}_{qu}$ and $\mathbf{M}$ which determines the ratio of the seismic moments of the isotropic and double couple components:

$$M_{0qu} = M_0 \cos\varphi \quad (4.4)$$

$$M_{0ex} = M_0 \sin\varphi \quad (4.5)$$

$$\tan\varphi = \frac{M_{0ex}}{M_{0qu}}. \quad (4.6)$$

We call φ the isotropic angle, for want of a better term, because $\varphi=0$ corresponds to a pure earthquake, $\varphi=90°$ corresponds to a pure explosion.

Let us consider a seismic source as a combination of isotropic tensor, modelling an explosion located at a zero depth, and pure double couple point source at a depth h, modelling the tectonic moment release. Both explosion and earthquake are considered as an instant sources. Such a source can be given by 6 parameters: described above angle φ, double couple depth, its focal mechanism which is characterizing by three angles: strike, dip and slip or by two unit vectors (direction of principal tension **T** and direction of principal compression **P**) and seismic moment M_0. Five of these parameters we determine by a systematic exploration of the five dimensional parametric space, and the 6-th parameter M_0 - solving the problem of minimization of the misfit between observed and calculated surface wave amplitude spectra for every current combination of all other parameters.

Under assumptions mentioned above the relation between the spectrum of the displacements $u_i(\mathbf{x},\omega)$ in any surface wave and the total moment tensor $\mathbf{M}$ can be expressed by following formula

$$u_i(\mathbf{x},\omega) = \frac{1}{\mathrm{i}\omega}[M_{qu\,jl}\frac{\partial}{\partial y_{qu_l}}G_{ij}(\mathbf{x},\mathbf{y}_{qu},\omega) + M_{ex\,jl}\frac{\partial}{\partial y_{ex_l}}G_{ij}(\mathbf{x},\mathbf{y}_{ex},\omega)] \quad (4.7)$$

i,j = 1,2,3 and the summation convention for repeated subscripts is used. $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ in equation (4.7) is the spectrum of Green function for the chosen model of medium and wave type (see section II), $\mathbf{y}$ - source location (we assume that the explosion and earthquake have this same horizontal coordinates, but different depth: h for earthquake and 0 for explosion). We make the same assumption as in section II: the paths from the earthquake source to seismic stations are relatively simple and are well approximated by weak laterally inhomogeneous model. We remind that the surface wave Green function in this approximation is determined by the near source and near receiver velocity structure, by the mean phase velocity of wave, and by geometrical spreading. The amplitude spectrum

$|u_i(\mathbf{x},\omega)|$ defined by formula (4.7) does not depend on the average phase velocity of the wave. In such a model the errors in source location do not affect the amplitude spectrum. Average phase velocities of surface waves are usually not well known. For this reason, we use only amplitude spectrum of surface waves for determining source parameters under consideration. An example of fit to the surface wave amplitude spectra is shown in Fig. 13. If all characteristics of the medium are known the representation (4.7) gives us a system of equations for parameters defined above. Let us consider now a grid in the space of these 5 parameters. Let the models of the media be given. Using formula (4.7) we can calculate the amplitude spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed amplitude spectra give us a residual $\varepsilon^{(i)}$ for every point of observation, every wave and every frequency ω. Let $u^{(i)}(\mathbf{x},\omega)$ be any observed value of the spectrum, i = 1,...,N; $\varepsilon^{(i)}$ - corresponding residual of $|u^{(i)}(\mathbf{x},\omega)|$. We define the normalized amplitude residual by formula similar to formula (2.2)

$$\varepsilon(h,\varphi,\mathbf{T},\mathbf{P}) = \left[\left(\sum_{i=1}^{N} \varepsilon^{(i)2}\right) \Big/ \left(\sum_{i=1}^{N} |u^{(i)}(\mathbf{x},\omega|^2\right)\right]^{1/2} . \qquad (4.8)$$

The optimal values of the parameters that minimize ε we consider as estimates of these parameters. We search them by a systematic exploration of the five dimensional parameter space. To characterize the degree of resolution of every of these source characteristics we calculate partial residual functions in the same way as in previous sections. We define two residual functions on scalar argument and two residual functions on vector argument corresponding to the two scalar and two vector varying parameters: $\varepsilon_h(h)$, $\varepsilon_\varphi(\varphi)$, $\varepsilon_\mathbf{T}(\mathbf{T})$ and $\varepsilon_\mathbf{P}(\mathbf{P})$. The value of the parameter for which the corresponding function of the residual attains its minimum we define as estimate of this parameter.

To improve the resolution of focal mechanism we use in the inversion polarities of P wave first arrivals as we did it for earthquake study considered above. Before inversion we apply the smoothing procedure described in section II to the observed polarities. In calculating the radiation pattern of P waves for a set of source parameters, we assume that the waves radiated by the isotropic (i.e., explosion) and nonisotropic (i.e., tectonic release or earthquake) source arrive simultaneously. This assumption can be abolished if the observed signs of the P first arrivals would be substituted by polarities measured from long period P wave spectra (Bukchin *et al.*, 1997).

Description of applications

We utilized IRIS and GEOSCOPE broadband digital seismograms and NEIC bulletins for 14 events that occurred at the Lop Nor test site in China from 1990 through 1996. Eight of these events are nuclear explosions, the other six are natural earthquakes. However, only 10 events with high signal-to-noise surface waves recorded at several stations were selected for study. The location of the selected events (7 explosions and 3 earthquakes) are given in Fig. 12.

We estimated the source parameters using the spectra of Love and Rayleigh fundamental waves in the spectral domain for periods ranging from 20 s to 70 s. Love and Rayleigh fundamental modes were extracted by using frequency-time analysis (FTAN) and floating filtering. Only records in which the surface wave polarization pattern did not exhibit significant azimuthal anomalies (< 15°) were used for further analysis. Some examples of normalized amplitude spectra are shown in Fig. 13. Note that the amplitude of the Love waves, which cannot be excited by a pure isotropic source, is comparable with the amplitude of the Rayleigh waves.

The focal mechanisms for the earthquakes and for the double couple components of the explosions are shown in Fig. 12.

The double couple depth is well resolved for all events studied and varies from 0 to 3 km for explosions and for the earthquakes we obtained depths 3 km, 31 km, and 17 km. The partial residual function of double couple depth is presented in Fig. 14 for all events.

Fig. 15 shows an example of the resolution of the double couple focal mechanism for the large explosion that occurred on 05.10.93 with $M_b = 5.9$ and $M_s = 4.7$. This figure displays the partial residual function of principal axes orientation. The principal compression axis is resolved better than the principal tension axis.

The number of surface wave records that we selected for the studied events was small, particularly for the earthquakes on the test site. For some events, only a few polarities of P wave first arrivals were available. As a result, the double couple focal mechanism was very well resolved only for the earthquake on 21.01.90, but for half of the events the principal compression axis, which trends in the SE-NW direction, was resolved well. Considering the focal mechanisms of these events, and the solutions obtained by Gao and Richards (1994), we found that while they vary widely, there is a moment tensor characteristic which is quite stable for all events. This characteristic is the orientation of the principal axes of 2 x 2 minor of the moment tensor corresponding to the horizontal coordinates $\{M_{xx}; M_{xy}; M_{yx}; M_{yy}\}$. This minor describes the horizontal deformations on the horizontal plane. We found that for all events, horizontal compression is dominant, and the principal horizontal compression axis deviates from the average direction (with an azimuth of 60°) by no more than 12°. These observations allow us to improve the estimates of the isotropic angle φ by assuming that the orientation of the horizontal deformations is a stable characteristic for the region under study. The average direction of horizontal compression is deviated from the fault visible in Fig. 12 by about 45°. With this assumption, we applied an a priori constraint on the possible double couple focal mechanisms. This constraint was formulated as a condition that the azimuth of the principal compression axis of the horizontal minor of the moment tensor cannot differ from 60° by more than by ±45°. This constraint is weak, but it reduces the number of possible focal mechanisms in half. Although the constraint is heuristic, we can present a special case when it is exact. This is when the deviatoric tectonic stresses in the region are horizontal and the direction mentioned above is the direction of principal compression. Then the constraint follows from the principle of positive deformation energy: $\sum_{i,j=1}^{3} S_{ij} M_{ij} \geq 0$, where **S** is the regional stress tensor, and **M** is moment tensor of event in the region.

When we applied this constraint, the resolution of the angle φ defining the seismic moments ratio was improved for three of seven studied explosions but the minimums of the curves did not vary appreciably. The curves of partial residual functions of the isotropic angle φ for all explosions are shown in Figure 16. Those for the three earthquakes are displayed in Fig. 17. We applied the same approach for the two earthquakes occurred in Gansu province, China (Tianzhu, 1996, M_s = 4.9, depth 12 km, and Yongden, 1995, M_s = 5.4, depth 6 km) studied by Lasserre *et al.*,2000 under the assumption of pure double couple source. The curves of partial residual functions of the isotropic angle φ are shown in Fig. 17. The explosions, with perhaps one exception (05.10.93), all display substantial non-zero isotropic angles φ ranging from about 10° to 50°, corresponding to M_{0ex}/M_{0qu} ranging from about 20% to 120%. In contrast, the isotropic angles for the earthquakes are all estimated to be approximately 0.

Fig. 18 illustrates the effect of adding of an isotropic component to a double couple source model on fitting of theoretical and observed data in the case of an explosion (on 10.06.94). Theoretical Rayleigh wave amplitude spectra and P wave first arrival polarities are compared here with observations for two different moment tensors. One of them (a,b) was obtained by minimizing joint residual for a fixed zero isotropic angle (double couple source model). Another one (c, d) is a result of a similar minimization, but for varying isotropic angle (30° is the estimated optimal value). As one can see the fitting of P wave first arrival polarities is good for both source models, but the fitting of Rayleigh wave amplitude spectra is much better in the case of non-zero isotropic component.

The results above are consistent with the hypothesis that motivates this study. Namely, that for the events we analyzed on the Lop Nor test site surface wave amplitude spectra combined with polarities of P wave first arrivals can be used to discriminate explosions from earthquakes based on source characteristics alone (a combination of the double couple depth and the ratio of the isotropic to nonisotropic moments). But it must be tested if the method can be applied to events with smaller moments and if the method is transportable to other regions.

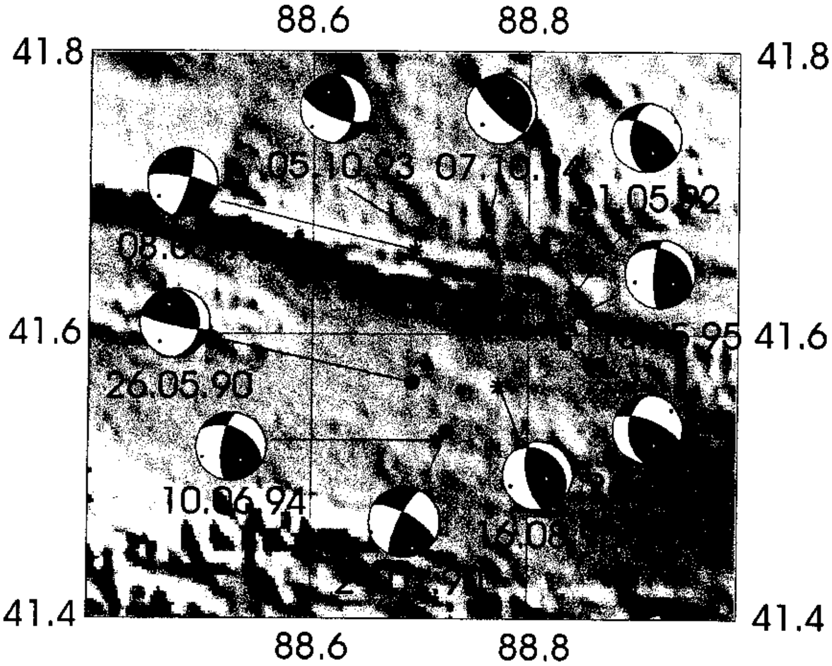


Figure 12. Epicenters and focal mechanisms of earthquakes (circles) and explosions (stars). Focal mechanisms for explosions are given for their tectonic release.

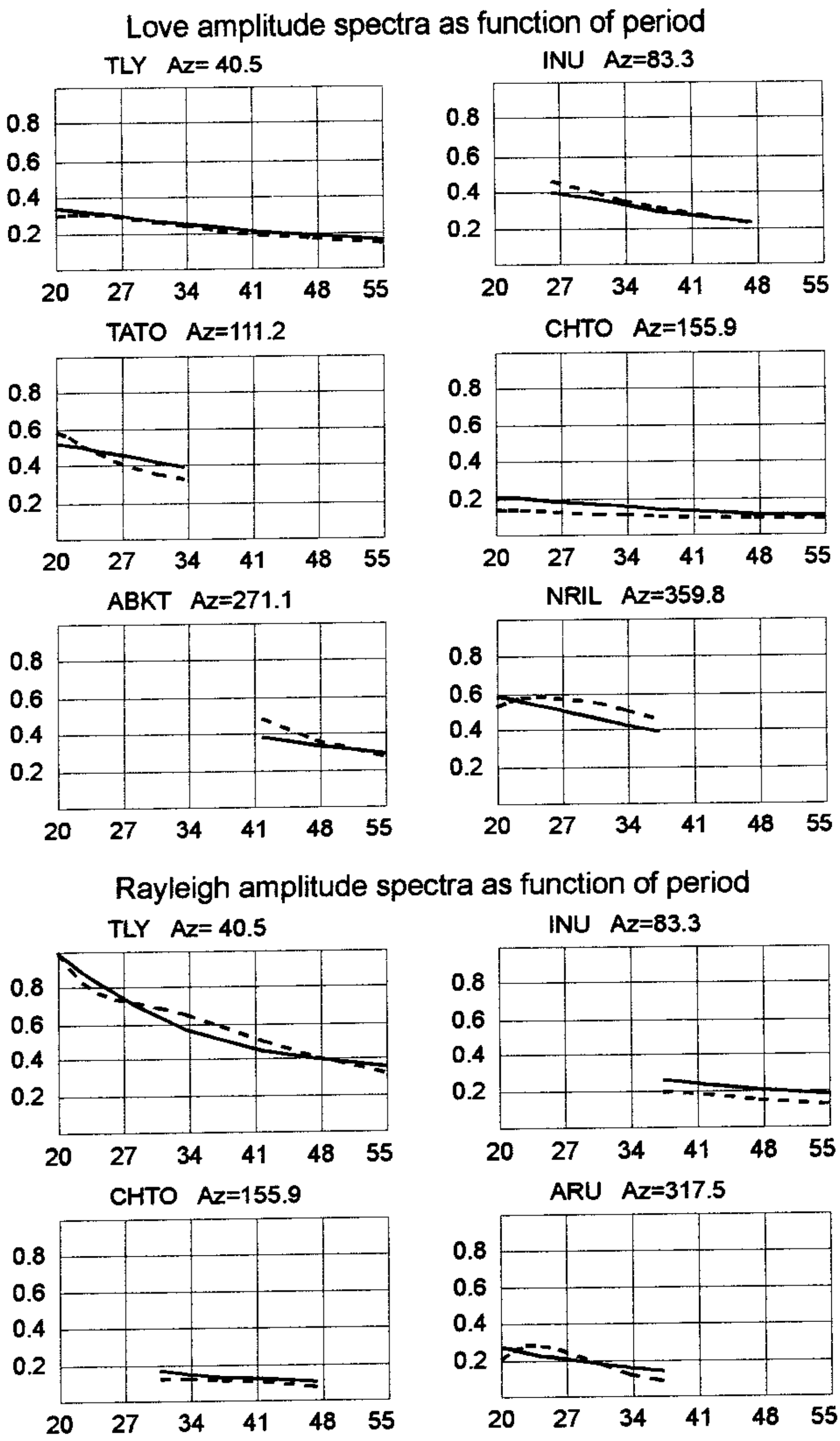


Figure 13. Comparison of observed (dashed) and computed (solid) Rayleigh and Love wave amplitude spectra for the explosion on 5 October, 1993.

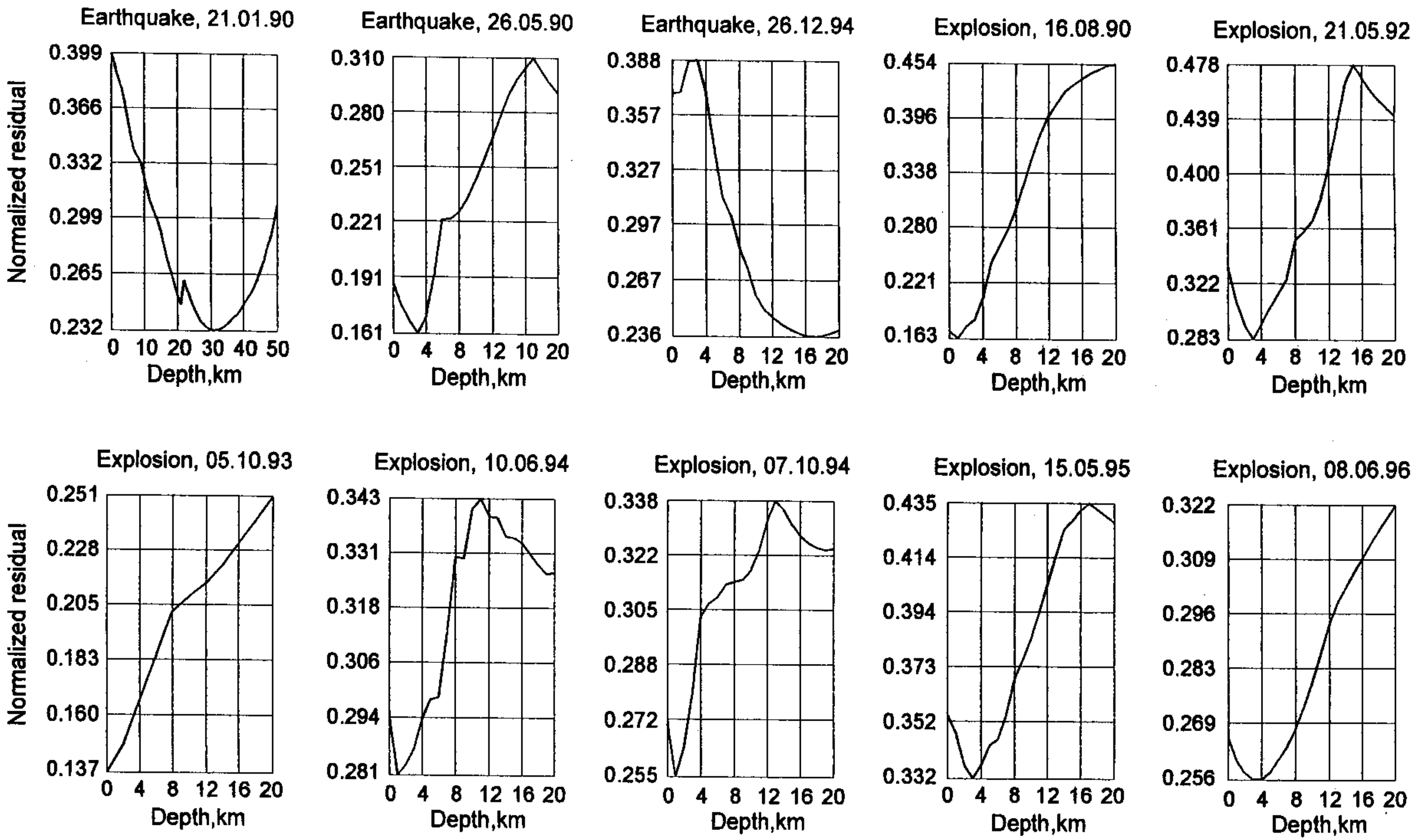


Fig. 14. Normalized residual as function of double couple depth.

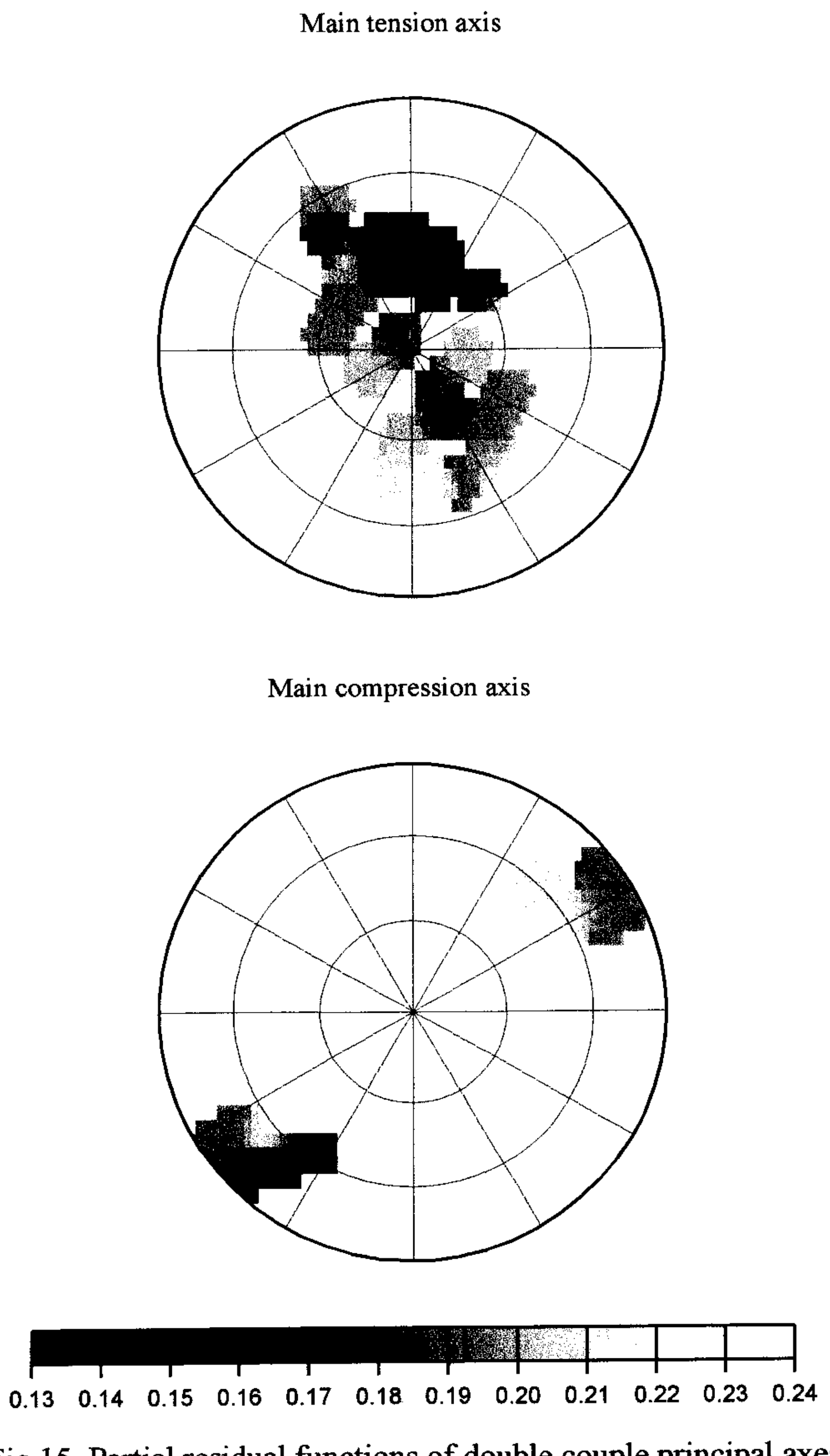


Fig.15. Partial residual functions of double couple principal axes orientation for the explosion on 5 October, 1993.

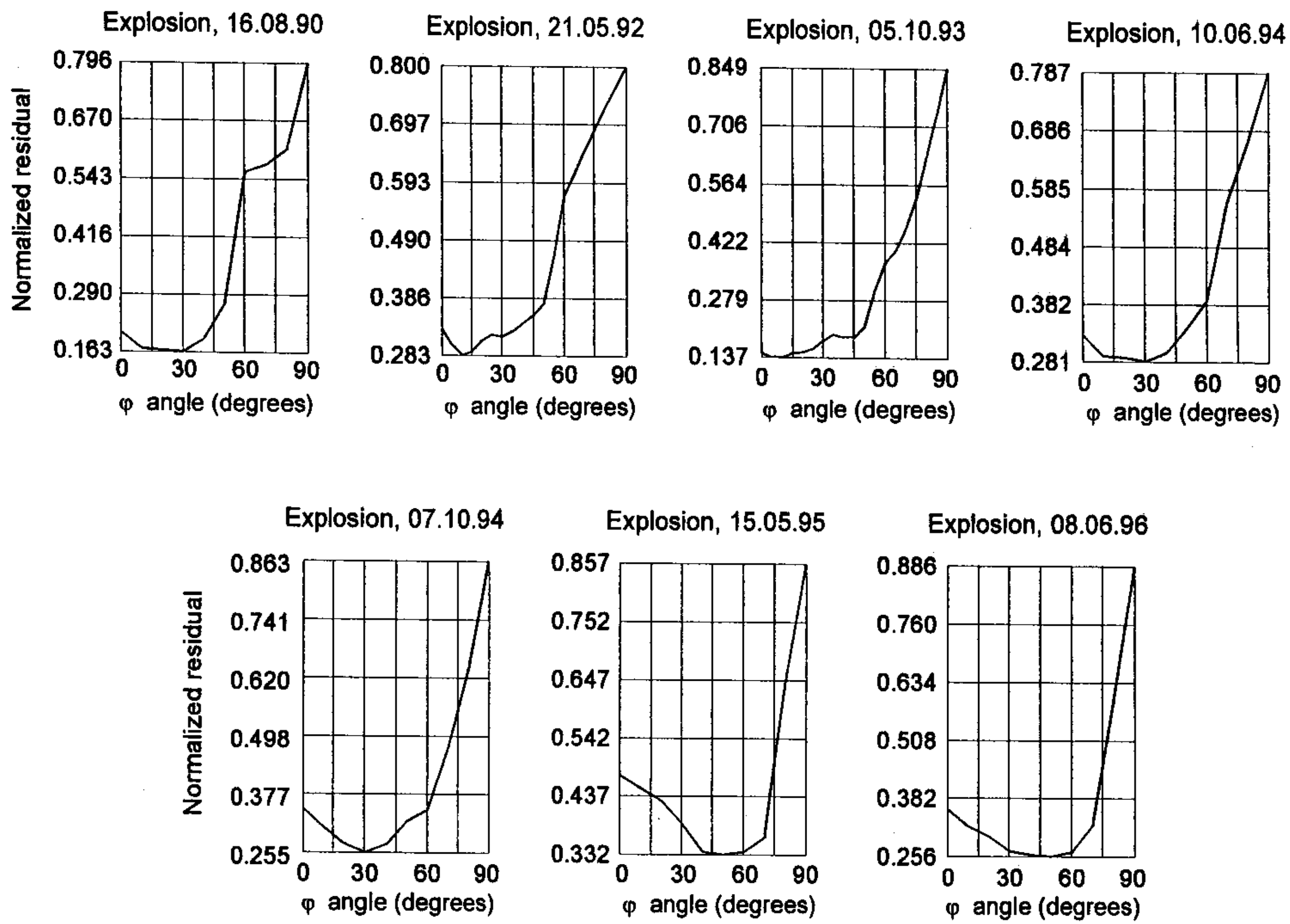


Fig. 16. Normalized residual as function of the φ angle that determines the ratio of seismic moments of isotropic and double couple components .

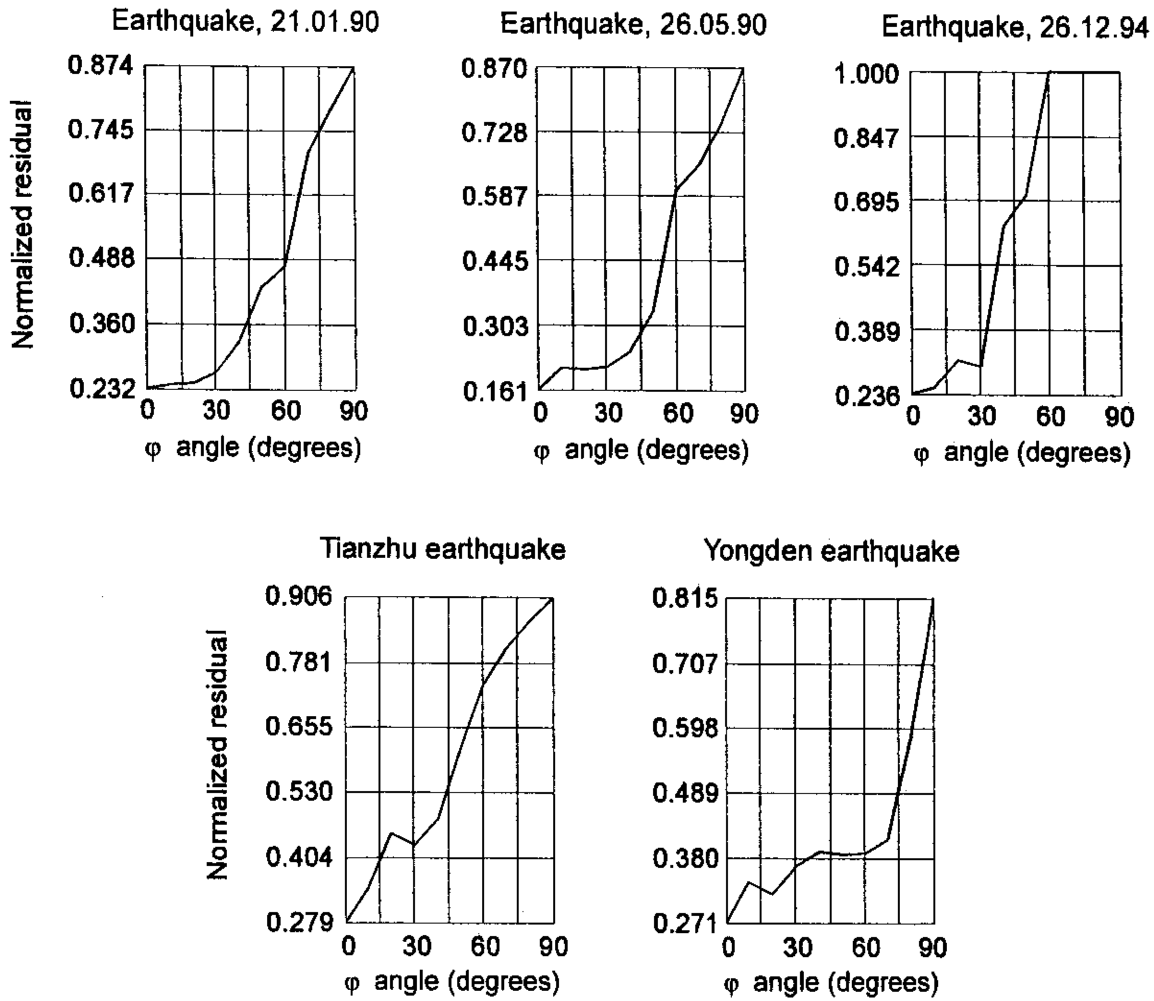


Fig. 17. Normalized residual as function of φ angle for studied earthquakes.

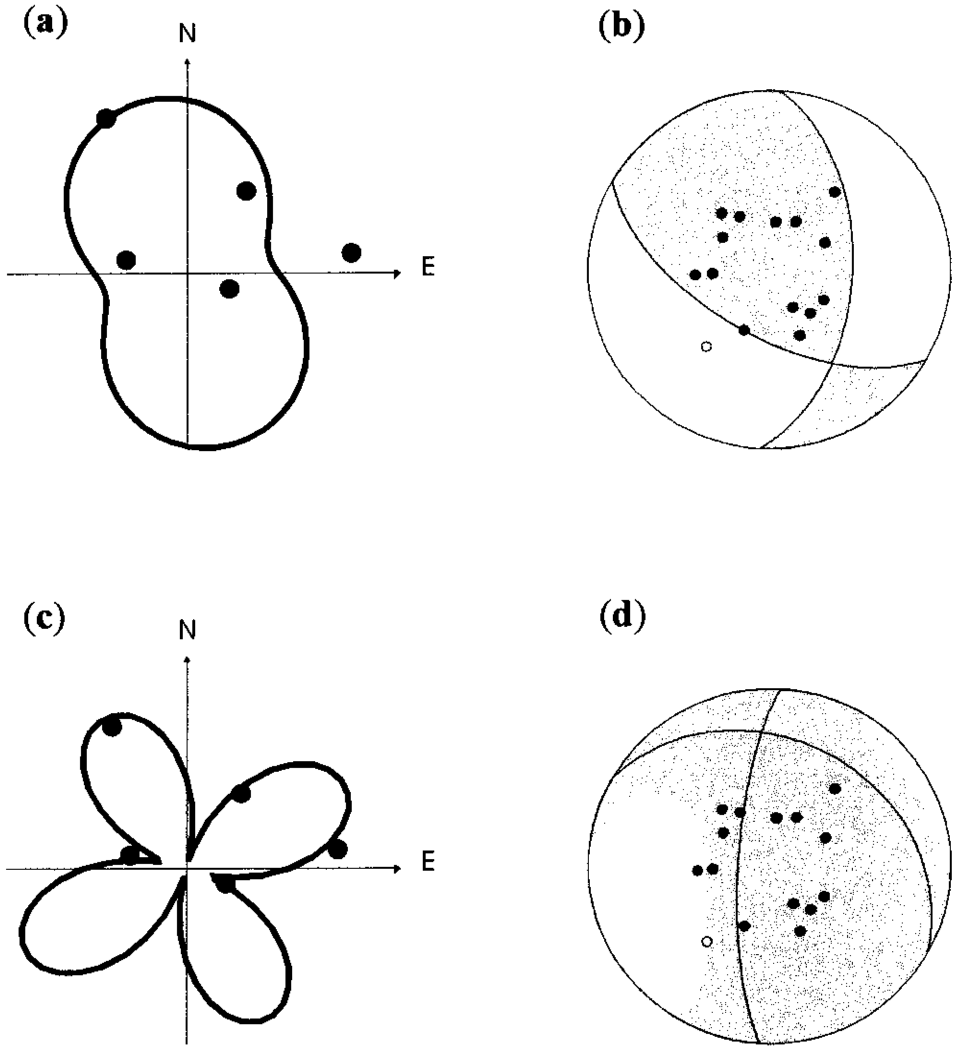


Fig. 18. Comparison of observed (circles) and synthetic (a, c) Rayleigh wave amplitude spectra for period 26 s and (b, d) first arrival polarities for explosion, 10.06.94. Rayleigh amplitude spectra are recalculated for laterally homogeneous media for source-receiver distance 3000 km.
(a, b) - moment tensor is estimated by minimizing of joint residual of surface wave amplitude spectra and first arrival polarities for fixed zero isotropic angle. (c, d) - moment tensor is estimated by the same minimization for varying isotropic angle (optimal value is 30°). Directions of compression waves radiation are shaded. Nodal planes of double couple component are shown by solid lines.

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