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INTRODUCTION TO EARTHQUAKE SOURCE MECHANISM. KINEMATICS

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16. SOURCE MECHANISM

16.1 REPRESENTATION OF THE SOURCE. KINEMATIC AND DYNAMIC MODELS

We have seen in chapter 15, that earthquakes are produced by fractures in the Earth crust. In Reid's model of elastic rebound, faulting is caused by the sudden release of accumulated elastic strain when strength of the material is overcome. In seismology the problem of source mechanism consists in relating observed seismic waves with the parameters that describe the source. In the direct problem, theoretical seismic wave displacements are determined from source models and in the inverse problem parameters of source models are derived from observed wave displacements. The first step in both problems is to define the seismic source by a mechanical model that represents the physical fracture. These models or representations of the source are defined by parameters whose number depends on its complexity. Simple models are defined by a few parameters while more complex ones require a larger number (Madariaga, 1983; Udías, 1991; Koyama, 1997).

Fracture process can be approached in two different ways, kinematic and dynamic. Kinematic models of the source consider the slip of the fault without relating it to the stresses that cause it. Fracture process is described purely by the slip vector as a function of coordinates on the fault plane and time. From this type of models, it is a relatively simple problem to determine the corresponding elastic displacement field. The second approach considers the complete fracture process relating fault slip to acting stress on the focal region. A complete dynamic description must be able to describe fracture from material properties of the focal region and stress conditions. Dynamic models present greater difficulties and their solutions, in many cases, can only be found by numerical methods.

16.2. EQUIVALENT FORCES. POINT SOURCE

The first mathematical formulation of the mechanism of earthquakes was presented by Nakano (1923) using the ideas already developed by Lamb (1904) and Love (1945). Nakano used the point source approximation, valid if observation points are at a sufficiently large distance compared with source dimensions and wave lengths also large. Thus he represented the source by a system of body forces acting at a point. Since these forces must represent the fracture phenomenon they are called equivalent forces.

The problem may be stated as follows. Let us consider an elastic medium of volume V surrounded by a surface S. In its interior there is a small region of volume V_0, surrounded by a surface Σ, that we will called the focal region, where fracture takes place (Fig. 16.1). This process can be represented by a distribution of body forces $\mathbf{F}(\xi_i,t)$ acting per unit volume inside V_0. If it is assumed that no other body forces are present (gravity, etc.), the equation of motion (2.54) can be written as,

$$\int_{V-V_0} [\rho (x_i,t) - \tau_{ij,j}(x_i,t)]dV = \int_{V_0} F(\xi_i,t)\, dV \qquad (16.1)$$

Where ξ_i are the coordinates inside the focal region and x_i those outside. Elastic displacements and stresses are only considered outside the focal region. From the static case (2.57), body forces F_i are formally related to stresses inside V_0 by,

$$F_i = - \tau_{ij,j} \qquad (16.2)$$

In the case of a point source, if volume V is an infinite medium, equation (16.1), according to (2.57), is given by,

$$\rho_i - \tau_{ij,j} = F_i \qquad (16.3)$$

Where F_i are forces at a point that is selected as the origin of x_i coordinates where elastic displacements $\mathbf{u}(\mathbf{x},t)$ are evaluated. These forces are the limit of the forces acting on V_0 as it tends to zero,

$$F_i(t) = \lim_{V_0 \Rightarrow 0} \int_{V_0} F_i(\xi_k,t)\, dV \qquad (16.4)$$

For a homogeneous medium, equation (16.3) can be expressed in terms of displacements as (2.59),

$$\rho_{i} - C_{ijkl} u_{k,lj} = F_i \tag{16.5}$$

This equation allows the determination of elastic displacements $\mathbf{u}(\mathbf{x},t)$ produced by a force or system of forces $\mathbf{F}$ acting at the origin of coordinates. In the inverse problem, from observed elastic displacements we can obtain certain characteristics of these forces.

Formulation using Green's function

A more convenient formulation of the problem can be obtained using the representation theorem in terms of Green's function (section 2.8). According to (2.88), if body forces are limited to the focal region V_0 (Fig. 16.2) and on its surface Σ stresses and displacements are null, we obtain for a volume V surrounded by a surface S,

$$u_i = \int_{-\infty}^{\infty} d\tau \int_{V_0} F_k G_{ki} dV + \int_{-\infty}^{\infty} d\tau \int_S [G_{ji} T_j - u_j C_{jkln} G_{li,n} \nu_k] \, dS \tag{16.6}$$

Where $T_i = \tau_{ij} \nu_j$ is the stress vector, ν_i the normal to surface element dS and G_{ki} Green's function of the medium, defined by equation (2.76) in section 2.7. Green's function, a tensor, is continuous through the whole volume V and represents the propagating effect in the medium. As we saw in section 2.8, Green function is the solution of the equation of motion for an impulsive force and depends on the characeristics (C_{ijkl} and ρ) of the medium. If the medium is infinite, conditions on surface S are homogeneous (stress and displacement are null) and equation (16.6) becomes, (Fig. 16.3),

$$u_i(x_s,t) = \int_{-\infty}^{\infty} d\tau \int_{V_0} F_k(\xi_s,\tau) G_{ki}(x_s,t;\xi_s,\tau) \, dV \tag{16.7}$$

Function G_{ki} acts as a "propagator" of the effects of forces F_k, from the points where they are acting (ξ_s inside V_0) to points x_s outside V_0 where elastic displacement u_i are produced. For a point focus in the origin of coordinates we have,

$$u_i(x_s,t) = \int_{-\infty}^{\infty} F_k(\tau)G_{ki}(x_s,t-\tau)\, d\tau \tag{16.8}$$

Elastic displacements are given now by the time convolution of the forces acting at the focus with Green's function of the medium.

From the point of view of the representation of the seismic source by equivalent forces, there are two ways to find elastic displacements. The first consists in solving directly the equation of motion (16.1). This implies solving a second order inhomogeneous differential equation for displacements or solving a homogeneous equation and introducing the forces as boundary conditions. In both cases, the problem is not easy. The second consists in using equations (16.6), (16.7) or (16.8). In this case we have to determine previously Green's function. Since Green's function is the solution of the equation of motion, this equation must be solved anyway. However, the advantage of the second approach is that for a given medium the equation of motion must be solved only once to find Green's function, while in the first it must be solved for each system of forces. For example, in the point source problem, the first approach requires for each system of forces the solution of equation (16.5) in the same medium. In the second approach, equation (2.77) is solved only once to find Green's function. Then for each system of forces, we apply equation (16.8), that is a convolution of each system of forces with Green's function.

Single and double couple

Several systems of forces have ben proposed to represent [illegible] source of an earthquake. For point sources, the most common [illegible] those of a couple of forces (SC, single couple) and two [illegible] perpendicular to each other without resulting moment (DC, [illegible] couple) (Fig. 16.4 and 16.5). The second system is also equi[illegible] to two linear dipoles of forces (arm in the same direction [illegible] forces) corresponding to pression and tension acting at 45° [illegible] couples. Both models were thought to represent a shear fr[illegible] but, as we will see, this is only true for the secon[illegible] extended sources, models used distributions of single or double couples on a plane surface.

For a point source, elastic displacements due to a couple of forces can be derived from those from a single force. If u_i^1 are the displacements due to a force acting at the origin in x_1 direction, those of a couple of forces on the plane (x_1, x_2) with forces along x_1 axis and arm in x_2 direction, (Fig. 16.4) are derived taking a Taylor expansion for each force of the couple, displaced s/2 from the origin along the x_2 axis. For the force in the positive direction of x_1 and shifted s/2 from the origin in the positive direction of x_2, elastic displacement is,

$$u_i^+ = u_i^1 + \frac{s}{2} u_{i,2}^1 \qquad (16.9)$$

For the force in the negative direction of x_1 and shifted s/2 in the negative direction of x_2, displacement is,

$$u_i^- = -u_i^1 + \frac{s}{2} u_{i,2}^1 \qquad (16.10)$$

Displacement due to a single couple is the sum of both,

$$u_i^{SC} = s\, u_{i,2}^1 \qquad (16.11)$$

For a double couple in the (x_1, x_2) plane, with forces in the direction of x_1 and x_2 (Fig.16.5), using (16.11), elastic displacement is given by,

$$u_i^{DC} = s\,(u_{i,2}^1 + u_{i,1}^2) \qquad (16.12)$$

If we substitute (16.11) in (16.8), we obtain the displacement due to a SC, as defined above, in terms of Green's function,

$$u_i^{SC} = \int_{-\infty}^{\infty} M(\tau)\ G_{i1,2}(t-\tau)\ d\tau \qquad (16.13)$$

Where $M(t) = F(t)s$ is the moment of the couple. For a DC in the x_1 and x_2 directions, using (16,12), we obtain,

$$u_i^{DC} = \int_{-\infty}^{\infty} M(\tau)\ [G_{i1,2}(t-\tau) + (G_{i2,1}(t-\tau)]\ d\tau \qquad (16.14)$$

For a SC in an arbitrary orientation, with forces in the direction of unit vector **l** and arm in that of **n**, where **n.l** = 0, and a DC with the second couple with forces in the direction of **n** and arm in that of **l**, general expressions are,

$$u_i^{SC} = \int_{-\infty}^{\infty} M\ l_k n_l\ G_{ik,l}\ d\tau \qquad (16.15)$$

$$u_i^{DC} = \int_{-\infty}^{\infty} M\ (l_k n_l + n_k l_l)\ G_{ik,l}\ d\tau \qquad (16.16)$$

Let us consider now two perpendicular linear dipoles with opposite sign. The linear dipole with forces in positive direction corresponds to tension and in negative direction to pressure. If the forces are in the direction of x_1' and x_2', in a similar form as in (16.12), elastic displacement are,

$$u_i'^{TP} = s\ (u'^{1}_{i,1} - u'^{2}_{i,2}) \qquad (16.17)$$

If the system of coordinates (x_1', x_2') in (16.17) is rotated 45° with respect to (x_1, x_2) of equation (16.12), both expressions can be shown to be equivalent. If tension and pressure forces are defined by scalar moment M and unit vectors **T** and **P**, in a similar form as in (16.16), elastic displacement in terms of Green's function is,

$$u_i^{TP} = \int_{-\infty}^{\infty} M\,(T_k T_l - P_k P_l)\, G_{ik,l}\, d\tau \qquad (16.18)$$

For the two equivalent systems, relations between the unit vectors **P**, **T** and **n**, **l** are,

$$\mathbf{P} = \frac{1}{\sqrt{2}}\,(\mathbf{n} - \mathbf{l}) \qquad (16.19)$$

$$\mathbf{T} = \frac{1}{\sqrt{2}}\,(\mathbf{n} + \mathbf{l}) \qquad (16.20)$$

$$\mathbf{B} = \mathbf{n} \times \mathbf{l} = \mathbf{P} \times \mathbf{T} \qquad (16.21)$$

Where **B** is the unit vector normal to the plane of the forces. This vector is known as the null axis, since there is no component of forces in its direction. In this way, for a DC point source, we can define two orthogonal systems of axes in the direction of unit vectors (**n**, **l**, **B**) and (**P**, **T**, **B**), to specify the orientation of the source. We will see that the second system corresponds to the principle axes of stress.

16.3 FRACTURES AND DISLOCATIONS

If an earthquake is produced by fracture of the Earth crust, a mechanical representation of its source can be done in terms of fractures or dislocations in an elastic medium. The theory of elastic dislocations was developed by Volterra in 1907 and is discussed in Love (1945). Its first applications to the problem of the seismic source are by Vvedenskaya (1956), Keylis-Boros (1956), Steketee (1958), Knopoff and Gilbert (1960) and Burridge and Knopoff (1964).

A dislocation consists in an internal surface inside an elastic medium across which there exists a discontinuity of displacement or strain. Here we will consider only displacement dislocations, that is, those where there is discontinuity of displacement but stress is continuous. The problem will be formulated using the representation theorem in terms of Green's function (2.88). The focal region consists in an internal surface

Σ with two sides (positive and negative). This surface can be considered as derived from the focal volume V_0 that is flattened to form a surface with both sides together without any volume. Coordinates on this surface are ξ_i and the normal at each point $n_i(\xi_k)$. From one side to the other of this surface there is a discontinuity in displacement or slip, so that (Fig.16.6),

$$u_i^+(\xi_k,t) - u_i^-(\xi_k,t) = \Delta u_i(\xi_k,t) \tag{16.22}$$

Where the plus and minus signs refer to the displacement at each side of the surface Σ. If there are no body forces (**F** = 0), stresses are continuous through Σ (their integral is null) and the conditions on the external surface S are homogeneous (all integrals on S are null), then equation (2.88) results in,

$$u_n(x_s,t) = \int_{-\infty}^{\infty} d\tau \int_{\Sigma} \Delta u_i(\xi_s,\tau) C_{ijkl} n_j(\xi_s) G_{nk,l}(\xi_s,\tau;\ x_s,t)\, dS \tag{16.23}$$

In consequence, the seismic source is represented by a dislocation or discontinuity in displacement given by the slip vector $\Delta\mathbf{u}$ on the surface Σ, which corresponds to the relative displacement between the two sides of a fault. This is, then, a non-elastic displacement that once produced doesn't go back to the initial position. In the most general case, $\Delta\mathbf{u}(\xi_i,\tau)$ can have a different direction for each point ξ_i of the surface Σ and in each of these points varies with time, starting with a zero value at t = 0, to a maximum value at a certain time. The normal to the surface Σ, given by unit vector $\mathbf{n}(\xi_i)$, can have different direction at points of the surface, but usually is considered to be constant, that is, Σ is a plane. Green's function **G** includes the propagation effects of the medium from points (ξ_i) of surface Σ to points (x_i) where elastic displacements (u_i) are evaluated. To solve the problem, according to equation (16.23) we must know first the derivatives of Green's function for the medium, which are also known as excitation functions.

Equation (16.23) corresponds to a kinematic model of the source, that is, a model in which elastic displacements **u** are derived from slip vector $\Delta\mathbf{u}$, which represents non-elastic displacement of the two sides of a fault of surface Σ. Slip is assumed to be known and is not derived from stress conditions in the focal region as it is done in dynamic models. In equation (16.23) derivatives of Green function includes derivatives of delta function. If we change the order of integration, integral on time of the product of **Δu** with the derivatives of the delta function results in time derivatives $\Delta\dot{u}$, that is, slip velocity. Thus, elastic displacements depend not on slip but on slip velocity. This means that the source radiates elastic energy only while it is moving. When motion at the source stops it ceases to radiate energy.

As a particular case, let us consider an isotropic medium of coefficients λ and μ, a plane surface Σ (**n** constant) and constant slip **Δu** with the same direction defined by unit vector **l**. The integrand of (16.23) becomes,

$$\Delta u(t)\ [\lambda\ l_k n_k \delta_{ij} + \mu(l_i n_j + l_j n_i)]\ G_{ni,j} \qquad (16.24)$$

Geometry of the source is now defined by the orientation of the two unit vectors **n** and **l**. These two vectors, referred to the geographic system of axes (North, East, Nadir), define the orientation of the source, namely, **n** orientation of the fault plane and **l** that of slip (Fig. 16.7). Since they are unit vectors, each has only two independent components. For $i = j$, expression (16.24) gives component of displacement normal to the fault plane which implies changes in volume. If **l** and **n** are perpendicular, slip is along the fault plane, there are no changes in volume, and it represents a shear fracture. In this case (**n**.**l** = 0), there are only three independent components of **n** and **l**.

In the kinematic model of a dislocation on a plane surface with constant slip, the parameters of the source are the following: λ and μ elastic coefficients of the focal region, four independent components of **n** and **l** defining the orientation of the fault plane and slip, the magnitude of the slip Δu and the area S of the fault. There are eight parameters, that added to the four of the hypocenter (ϕ_0, λ_0, h, t_0) sum up to twelve. If the source is a shear fracture the number of parameters is only ten.

16.4 GREEN FUNCTION FOR AN INFINITE MEDIUM

The problem of determining Green's function is not an easy one and depends on the characteristics of each medium. As we saw in section 2.7, Green's function is the solution of the equation of direction cosines, we obtain,

$$G_{i1} = \frac{1}{4\pi\rho} \left[\frac{1}{r^3}(3\gamma_1\gamma_i - \delta_{1i}) \int_{r/\alpha}^{r/\beta} \tau\ \delta(t-\tau)\ d\tau + \right.$$

$$\left. + \frac{1}{r\alpha^2}\gamma_1\gamma_i\ \delta(t-\frac{r}{\alpha}) - \frac{1}{r\beta^2}(\gamma_1\gamma_i - \delta_{1i})\ \delta(t-\frac{r}{\beta}) \right] \tag{16.56}$$

This equation correspond to a force in the x_1 direction. We can generalize this result for Green's function corresponding to a force in arbitrary direction given by the vector ν_j,

$$G_{ij} = \frac{1}{4\pi\rho} \left[\frac{1}{r^3}(3\gamma_i\gamma_j - \delta_{ij}) \int_{r/\alpha}^{r/\beta} \tau\ \delta(t-\tau)\ d\tau + \right.$$

$$\left. + \frac{1}{r\alpha^2}\gamma_i\gamma_j\ \delta(t-\frac{r}{\alpha}) - \frac{1}{r\beta^2}(\gamma_i\gamma_j - \delta_{ij})\ \delta(t-\frac{r}{\beta}) \right] \tag{16.57}$$

This is the expression for Green's function for an infinite, homogeneous, isotropic elastic medium with velocities α and β. This is a very important result in elastodynamics, which gives the elastic displacement field for the most fundamental type of source. It constitutes the basic building block in seismic source studies.

A similar fundamental problem is the static solution for a constant force acting at a point in the direction of unit vector ν_j. For an infinite homogeneous isotropic elastic medium, static displacements are solution of equation,

$$\alpha^2\ \nabla(\nabla.\mathbf{u}) - \beta^2\ \nabla\times(\nabla\times\mathbf{u}) = \frac{\mathbf{F}}{\rho}$$

The solution can be found in a similar way as in the previous problem expressing displacement and force in terms of potentials that leads to equations of the form of Poisson's equation (Lay and Wallace, 1995). The result for the displacement, in terms of direction cosines γ_i, is,

$$u_{ij} = \frac{F}{8\pi\rho r}\left[\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)\gamma_i\gamma_j + \left(\frac{1}{\beta^2} + \frac{1}{\alpha^2}\right)\delta_{ij}\right] \qquad (16.58)$$

Subindex j indicates the direction of the force and as in (16.57), displacements are given by a tensor. This expression, known as Sommigliana's tensor, is the fundamental equation in elastostatics.

16.5. SEPARATION OF NEAR AND FAR FIELDS

The first term of Green's function in equation (16.57) depends with distance as r^{-3} and the other two as r^{-1}. Thus, displacement represented by the first term attenuates with distance more rapidly and for this reason is called the near field. This term depends on both α and β and is a displacement mixed of P and S motion. The second and third terms constitute the far field where P and S waves are separated. In both cases, near and far fields, displacements have two parts, one, called radiation pattern, depends on the direction cosines and express the spatial distribution of amplitudes, and other which depends on time or wave form.

Near field

The time dependence of the near field (16.57) can be rewritten as (Knopoff, 1967),

$$\int_{r/\alpha}^{r/\beta} \tau\,\delta(t-\tau)d\tau = \int_{-\infty}^{\infty} \tau\,\delta(t-\tau)[H(\tau - \frac{r}{\alpha}) - H(\tau - \frac{r}{\beta})]d\tau \quad (16.59)$$

Where H(t) is the step or Heaviside's function. According to the properties of step and delta functions, the integral in (16.59) results,

$$\int_{-\infty}^{\infty} [\quad] \, d\tau = t\,H(t - \frac{r}{\alpha}) - t\,H(t - \frac{r}{\beta}) \qquad (16.60)$$

Each of these two terms can be written as,

$$t\,H(t - \frac{r}{\alpha}) = (t - \frac{r}{\alpha})\,H(t - \frac{r}{\alpha}) + \frac{r}{\alpha}\,H(t - \frac{r}{\alpha}) \qquad (16.61)$$

Term $(t - r/\alpha)\,H(t - r/\alpha)$ is a ramp function of slope unity. The complete expression for the near field can be written as,

$$G_{ij}^{NF} = \frac{1}{4\pi\rho}(3\gamma_i\gamma_j - \delta_{ij})\left\{ \frac{1}{r^3}\left[(t - \frac{r}{\alpha})H(t - \frac{r}{\alpha}) - (t - \frac{r}{\beta})H(t - \frac{r}{\beta})\right] + \frac{1}{r^2}\left[\frac{1}{\alpha}H(t - \frac{r}{\alpha}) - \frac{1}{\beta}H(t - \frac{r}{\beta})\right]\right\} \qquad (16.62)$$

Time dependence of the near field has now two parts that depend on distance as r^{-3} and r^{-2}, both depending on velocities of P and S waves. The first part is the difference between two ramp functions and the second the difference between two step functions of different amplitude. The result is shown in figure 16.12. The part that depends on r^{-3} is formed by a ramp of unit slope starting at $t = r/\alpha$ until $t = r/\beta$. From this time on the displacement has a constant amplitude of $1/\beta - 1/\alpha$. The part that depends on r^{-2} is a step function starting at $t = r/\alpha$ and amplitude $1/\alpha$ followed at $t = r/\beta$ by one of amplitude $1/\alpha - 1/\beta$. Displacement in the near field has a part that remains constant with time. The radiation pattern is common to the complete near field displacement.

Far field

The far field (part that depends on $1/r$) of Green's function is formed by separate P and S waves,

$$G^{P}_{ij} = \frac{1}{4\pi\rho\alpha^2 r} \gamma_i \gamma_j \; \delta(t - \frac{r}{\alpha}) \qquad (16.63)$$

$$G^{S}_{ij} = \frac{-1}{4\pi\rho\beta^2 r} (\gamma_i \gamma_j - \delta_{ij}) \; \delta(t - \frac{r}{\beta}) \qquad (16.64)$$

Time dependence is in both cases a delta function. The far field is, then, formed by two impulses that propagate with velocities α and β, that is, P and S waves of impulsive form (Fig. 16.13).

The radiation patterns for P and S waves are different (16.63 and 16.64). To represent radiation patterns we take polar coordinates (r,θ) with center at the focus and considered the distribution of normalized amplitudes. If the force is in x_1 direction, normalized components of displacement of P waves in (x_1,x_3) plane are given by (Fig. 16.14),

$$G^{P}_{11} = \gamma_1 \gamma_1 = \cos\theta \cos\theta \qquad (16.65)$$

$$G^{P}_{31} = \gamma_3 \gamma_1 = \cos\theta \,\mathrm{sen}\,\theta \qquad (16.66)$$

It can be easily seen that displacement is in radial direction as expected for P waves and its modulus,

$$|G^{P}_{i1}| = \cos\theta \qquad (16.67)$$

For S waves, normalized displacement components are,

$$G^{S}_{11} = -(\gamma_1 \gamma_1 - 1) = \mathrm{sen}\,\theta \,\mathrm{sen}\,\theta \qquad (16.68)$$

$$G^{S}_{31} = -\gamma_1 \gamma_3 = -\,\mathrm{sen}\,\theta \cos\theta \qquad (16.69)$$

Resulting displacement is in transversal direction and its modulus is,

$$|G^S_{i1}| = \sin\theta \qquad (16.70)$$

Displacements of P and S waves are in radial and transversal direction as corresponding for each type of waves. Giving values to θ from 0 to 360 degrees, we obtain radiation patterns. In both cases the pattern has two lobes (Fig. 16.15). For P waves, displacements are radial, in the right lobe outwards, that is, compressions and in the left inward or dilatations, with maxima at $\theta = 0$ and π (Fig. 16.15a). For S waves, displacements are transversal that converge toward the direction of the force in both lobes with maxima at $\theta = \pi/2$ and $3\pi/2$ (Fig. 16.15b). P waves have a nodal plane (x_2,x_3), normal to the force and S waves the plane (x_1,x_2) that contains the force.

16.6 SHEAR DISLOCATION OR FRACTURE. POINT SOURCE

Let us consider the seismic source represented by a shear dislocation fracture, with fault plane Σ of area S and normal **n**, slip $\Delta u(\xi,\tau)$ in the direction of unit vector **l**, contained on the plane so that **l** and **n** are perpendicular (**n** . **l** = 0). For an infinite, homogeneous isotropic medium, displacement according to (16.23) and (16.24) is,

$$u_k = \int_{-\infty}^{\infty} d\tau \int_{\Sigma} \Delta u\, \mu\, (l_i n_j + l_j n_i)\, G_{ki,j}\, dS \qquad (16.71)$$

If distance from observation point to the source is large in comparison with the source dimensions $(r \gg \Sigma)$ and the wave lengths are also large, the problem can be approximated by a point source and equation (16.71) takes the form,

$$u_k(t) = \mu S\, (l_i n_j + l_j n_i) \int_{-\infty}^{\infty} \Delta u(\tau) G_{ki,j}(t-\tau)\, d\tau \qquad (16.72)$$

Displacements are given by time convolution of slip with the derivatives of Green's function.

For the far field, Green's functions for P and S waves are given by (16.63) and (16.64). Derivatives for P waves are,

$$G^P_{ki,j} = \frac{1}{4\pi\rho\alpha^2} \frac{\partial}{\partial\xi_j}\left[\frac{1}{r} \gamma_i\gamma_k \delta(t - \frac{r}{\alpha}) \right] \qquad (15.73)$$

If in the derivatives we keep only the terms that depend on the least negative power of r (1/r),

$$\frac{1}{r} \gamma_i\gamma_k \frac{\partial}{\partial\xi_j} \delta(t - \frac{r}{\alpha}) = - \frac{1}{r\alpha} \gamma_i\gamma_k \dot{\delta}(t - \frac{r}{\alpha}) \frac{\partial r}{\partial\xi_j}$$

Substituting the direction cosine we obtain for P waves

$$G^P_{ki,j} = \frac{1}{4\pi\rho\alpha^3 r} \gamma_i\gamma_k\gamma_j \dot{\delta}(t - \frac{r}{\alpha}) \qquad (16.74)$$

In a similar form for S waves,

$$G^S_{ki,j} = \frac{-1}{4\pi\rho\beta^3 r} (\gamma_i\gamma_k - \delta_{ik}) \gamma_j \dot{\delta}(t - \frac{r}{\beta}) \qquad (16.75)$$

This approximation is consistent with the far field. Now, we substitute (16.74) and (16.75) in (16.72) and take into account the property of the derivative of the delta function,

$$\int_{-\infty}^{\infty} \Delta u(\tau) \dot{\delta}(t - \frac{r}{\alpha} - \tau)\, d\tau = \Delta \dot{u}(t - \frac{r}{\alpha}) \qquad (16.76)$$

The final result, after this substitution, gives for displacements of the P and S waves in the far field,

$$u_j^P = \frac{\mu S}{4\pi\rho\alpha^3 r} (n_k l_i + n_i l_k) \gamma_i \gamma_k \gamma_j \Delta\dot{u}(t - \frac{r}{\alpha}) \quad (16.77)$$

$$u_j^S = \frac{\mu S}{4\pi\rho\beta^3 r} (n_k l_i + n_i l_k)(\delta_{ij} - \gamma_i \gamma_j)\gamma_k \Delta\dot{u}(t - \frac{r}{\beta}) \quad (16.78)$$

It is important to notice, as we have mentioned in section 16.3, that elastic displacements depend on slip velocity or slip rate. The source radiates elastic energy only while it is moving and ceases when it stops. If the source time function is a step function $\Delta u(t) = \Delta u \; H(t)$, its derivative is the delta function, and we obtain from (16.77) and (16.78),

$$u_j^P = \frac{M_0}{4\pi\rho\alpha^3 r} (n_k l_i + n_i l_k) \gamma_i \gamma_k \gamma_j \delta(t - \frac{r}{\alpha}) \quad (16.79)$$

$$u_j^S = \frac{M_0}{4\pi\rho\beta^3 r} (n_k l_i + n_i l_k)(\delta_{ij} - \gamma_i \gamma_j)\gamma_k \delta(t - \frac{r}{\beta}) \quad (16.80)$$

Where we have substituted seismic moment $M_0 = \mu \Delta u S$ (15.23). Therefore, for a step source time function, elastic displacements in the far field for P and S waves are impulses that arrive at a distance r at times, $t = r/\alpha$ and $t = r/\beta$.

Radiation pattern

The radiation pattern consists in the spatial distribution of amplitudes around the source. Let us consider a shear fracture on plane (x_1,x_2), with slip in x_1 direction, that is, $\mathbf{n} = (0,0,1)$ and $\mathbf{l} = (1,0,0)$ (Fig. 16.16). In a similar form as we did for Green's function, normalized displacements in (x_1,x_3) plane in polar coordinates (r,θ) for P waves are,

$$u_1^P = 2\,\gamma_1^2\,\gamma_3 = \text{sen}\;2\theta\;\cos\theta \qquad (16.81)$$

$$u_3^P = 2\,\gamma_3^2\,\gamma_1 = \text{sen}\;2\theta\;\text{sen}\;\theta \qquad (16.82)$$

And for S waves,

$$u_1^S = (1 - 2\gamma_1^2)\,\gamma_3 = -\cos 2\theta\;\text{sen}\;\theta \qquad (16.83)$$

$$u_3^S = (1 - 2\gamma_3^2)\,\gamma_1 = \cos 2\theta\;\cos\theta \qquad (16.84)$$

We can see that displacement of P waves is in radial direction and that of S waves in transversal. If we define components u_r and u_θ in these two directions, we obtain,

$$u_r^P = \text{sen}\;2\theta \qquad (16.85)$$

$$u_\theta^S = \cos 2\theta \qquad (16.86)$$

In both cases, the radiation pattern has four lobes or quadrants. For P waves lobes have alternating direction of motion, outward or positive (compression) and inward or negative (dilatation). There are two nodal planes, (x_1,x_2) and (x_3,x_2), the first corresponds to the fault plane and the second, normal to this and to the direction of $\Delta\mathbf{u}$, is called the auxiliary plane. Maxima of displacement are at 45 degrees of directions of $\mathbf{l}$ and $\mathbf{n}$ (Fig. 16.17a). In the four lobes of the radiation pattern of S waves, motion change direction. Maxima coincide with directions of $\mathbf{l}$ and $\mathbf{n}$ and nodal planes are at 45 degrees from them (Fig. 16.17b). In both cases, radiation pattern is symmetrical and we can interchange $\mathbf{n}$ and $\mathbf{l}$ with the same result. This is consequence of the symmetrical form respect to $\mathbf{n}$ and $\mathbf{l}$ of expression (16.71). For this reason, radiation patterns of P and S waves do not identified the fault plane from the auxiliary plane. This ambiguity is present in the methods to determine the orientation of the fault plane from far field displacements of P and S waves.

To study the radiation pattern in three dimensions we use spherical coordinates (r,θ,ϕ). The focus is located at the center of a sphere of unit radius (focal sphere) and displacements are evaluated for points of its surface. A system of Cartesian coordinates (x_1, x_2, x_3) with origin at the center of the sphere, called the source system, is defined so that (x_1, x_2) is the fault plane, **n** is in x_3 direction and **l** in that of x_1. For a point on the surface of the sphere, direction cosines of r with respect to the three axes are,

$$\gamma_1 = \text{sen}\ \theta \cos \phi \tag{16.87}$$

$$\gamma_2 = \text{sen}\ \theta\ \text{sen}\ \phi \tag{16.88}$$

$$\gamma_3 = \cos \theta \tag{16.89}$$

Where θ is measured from x_3 and ϕ from x_1. At each point of the spherical surface, we define a system of Cartesian coordinates with unit vectors (e_r, e_θ, e_ϕ) in the direction of the increments of r, θ, and ϕ. Components of displacements in these directions correspond to P, and two components of S waves respectively (Fig. 16.18). Normalized amplitudes are given by,

$$P : u_r = \text{sen}\ 2\theta \cos \phi \tag{16.90}$$

$$S1 : u_\theta = \cos 2\theta \cos \phi \tag{16.91}$$

$$S2 : u_\phi = \cos \theta\ \ \text{sen}\ \phi \tag{16.92}$$

Displacement of P waves have two nodal planes, $\theta = \pi/2$, (x_1, x_2) and $\phi = \pi/2$, (x_2, x_3). Displacement of S1 component have a nodal plane for $\phi = \pi/2$, (x_2, x_3) and is null also for points of intersection of the surface of the focal sphere and the [illegible] angle $\theta = \pi/4$ and $3\pi/4$. Displacement of S2 have two nodal planes for $\theta = \pi/2$, (x_1, x_2) and $\phi = 0$, (x_1, x_3).

Geometry of a shear fracture

As we have seen, orientation of a shear fracture is [illegible] two orthogonal unit vectors **n** and **l**. These two vectors must be expressed in relation to the geographical reference system defined by the axis (x_1, x_2, x_3), positive in North, East and Nadir directions. With respect to these axes we define now the spherical coordinates θ, measured from x_3 and ϕ from x_1 on the (x_1, x_2) (horizontal) plane. In reference to this system the vector **n** is given by,

$$n_1 = \text{sen}\,\theta_n \cos\phi_n \tag{16.93}$$

$$n_2 = \text{sen}\,\theta_n\,\text{sen}\,\phi_n \tag{16.94}$$

$$n_3 = \cos\theta_n \tag{16.95}$$

And in a similar way for vector **l** (l_1, l_2, l_3). Since **l** and **n** are orthogonal unit vectors only three components are independent. We saw in section 15.1 that a fault or shear fracture can be also defined by angles ϕ, δ, λ (Fig. 15.1). These angles can be expressed in terms of those defining the geographical orientation of vectors **l** and **n** (Fig. 16.19),

$$\phi = \phi_n + \frac{\pi}{2} \tag{16.96}$$

$$\delta = \theta_n \tag{16.97}$$

$$\lambda = \text{sen}^{-1}\left(\frac{\cos\theta_l}{\text{sen}\,\theta_n}\right) \tag{16.98}$$

Components of **n** and **l** referred to geographical axes (x_1, x_2, x_3) can be written in terms of ϕ, δ, λ, in the form,

$$n_1 = -\ \text{sen}\delta\ \text{sen}\phi \qquad (16.99)$$

$$n_2 = \text{sen}\delta\ \cos\phi \qquad (16.100)$$

$$n_3 = -\cos\delta \qquad (16.101)$$

$$l_1 = \cos\lambda\ \cos\phi + \cos\delta\ \text{sen}\lambda\ \text{sen}\phi \qquad (16.102)$$

$$l_2 = \cos\lambda\ \text{sen}\phi - \cos\delta\ \text{sen}\lambda\ \cos\phi \qquad (16.103)$$

$$l_3 = -\ \text{sen}\lambda\ \text{sen}\delta \qquad (16.104)$$

In every case, orientation of the source is given uniquely by three parameters, namely, ϕ, δ, λ, or θ_n, ϕ_n, θ_l. Since, as we mentioned already, there is always an ambiguity respect to vectors **n** and **l**, source orientation given by (θ_n, ϕ_n), and (θ_l, ϕ_l), doesn't assume a distinction between fault and auxiliary planes. Given in terms of ϕ, δ, λ, we assumed that this is the orientation of the fault plane. If we don't know which one is the fault plane, we must give values of ϕ, δ, λ for both nodal planes.

Displacement of P waves and SV and SH components of S waves can be referred to the geographical coordinate axes through the direction of the seismic ray. In the focal sphere, this is a straight line from the center to the surface. If ϕ is the azimuth of the ray measured from North and i the take-off angle of the ray measured from the downward vertical, direction cosines of the ray respect to the geographical axes (North, East, Nadir) are,

$$\gamma_1 = \text{sen}\,i\ \cos\phi \qquad (16.105)$$

$$\gamma_2 = \text{sen}\,i\ \text{sen}\phi \qquad (16.106)$$

$$\gamma_3 = \cos i \qquad (16.107)$$

Components of P, SV, and SH displacements along the geographical axes are (Fig. 16.20),

$$P_1 = P \text{ seni } \cos\phi \qquad SV_1 = SV \cos i \cos\phi \qquad SH_1 = -SH \text{ sen}\phi$$

$$P_2 = P \text{ seni sen}\phi \qquad SV_2 = SV \cos i \text{ sen}\phi \qquad SH_2 = SH \cos\phi$$

$$P_3 = P \cos i \qquad SV_3 = -SV \text{ seni} \qquad SH_3 = 0$$

Where P, SV and SH amplitudes depend on the location of the observation point with respect to the source orientation. For points on the focal sphere, observation point is given by (ϕ, i) and orientation of the source by (ϕ, δ, λ) or $(\theta_n, \phi_n, \theta_1)$.

16.7 SOURCE TIME FUNCTION

The source time function (STF) $\Delta u(t)$ represents the slip dependence on time and is an important characteristic of focal mechanism. We have already considered the most simple STF, namely, the step function. There are other functions, some commonly used, including the step function, are, (Fig. 16.21),

$$\Delta u(t) = \Delta u \, H(t) \tag{16.108}$$

$$\Delta u(t) = \begin{cases} \Delta u \dfrac{t}{\tau}, & 0 \le t \le \tau \\ \Delta u, & t > \tau \end{cases} \tag{16.109}$$

$$\Delta u(t) = \Delta u \, H(t)(1 - e^{-t/\tau}), \tag{16.110}$$

In all cases, the slip of the fault starts at $t = 0$, and once it reaches its maximum value Δu, it stays constant. The fault doesn't return to its initial state. In the first (16.108), $\Delta u(t)$ has the form of a step or Heaviside function where the slip reaches its maximum value instantaneously at time, $t = 0$. In the second (16.109), $\Delta u(t)$ increases linearly from $t = 0$ to $t = \tau$, and at that time reaches its maximum value. This STF introduces a new parameter of the source, namely, τ time it takes slip to reach its maximum value, or rise time. In the third (16.110), $\Delta u(t)$ is a continuous function for $t > 0$. Slip reaches asymptotically with time its maximum value. For the rise time, $\Delta u(\tau) = 0.63\Delta u$.

We have seen in equations (16.77) and (16.78) that elastic

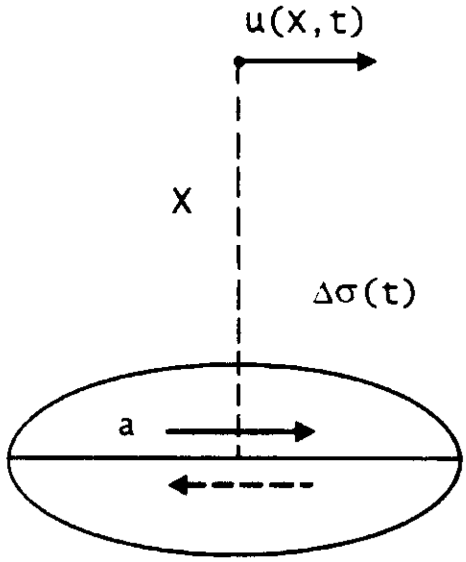
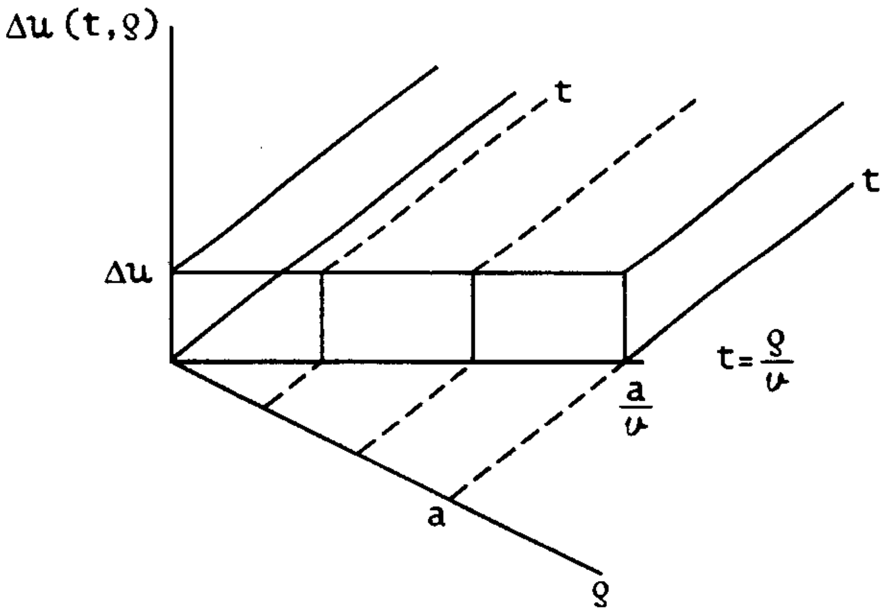


fig - 18.7



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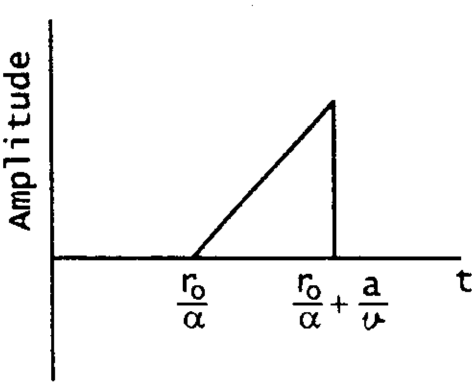

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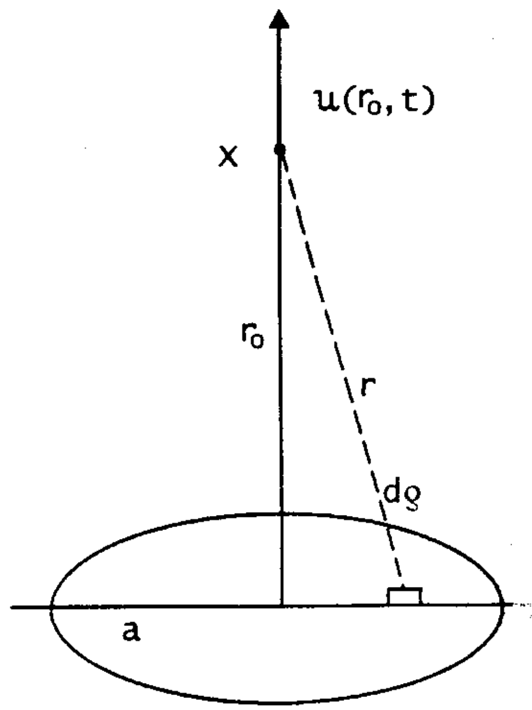


(a)

fig - 18.9

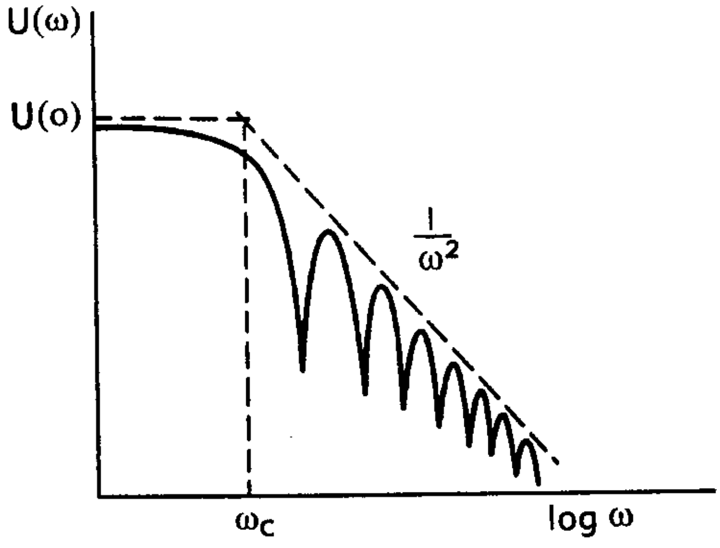


fig - 18.4

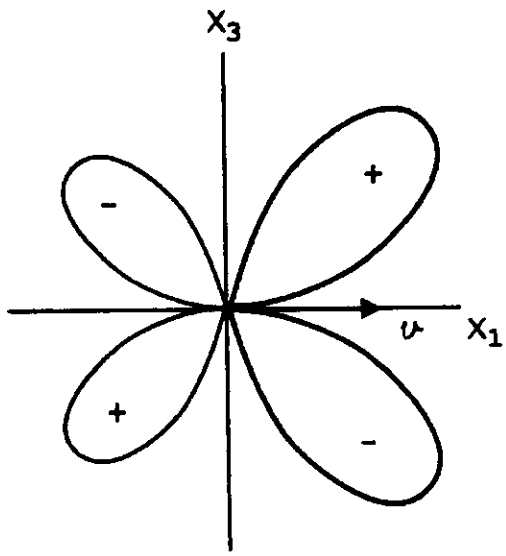


fig - 18.5

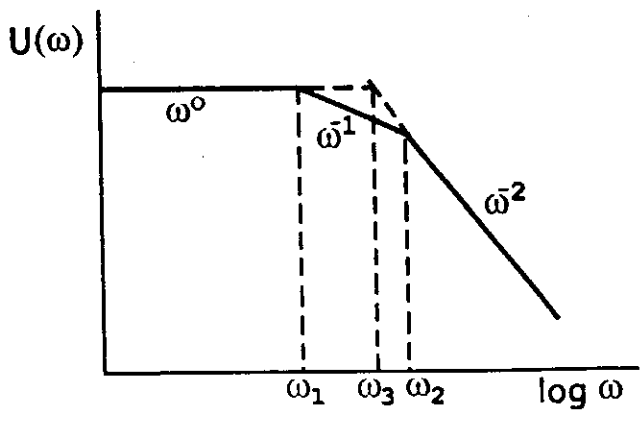


fig - 18.6

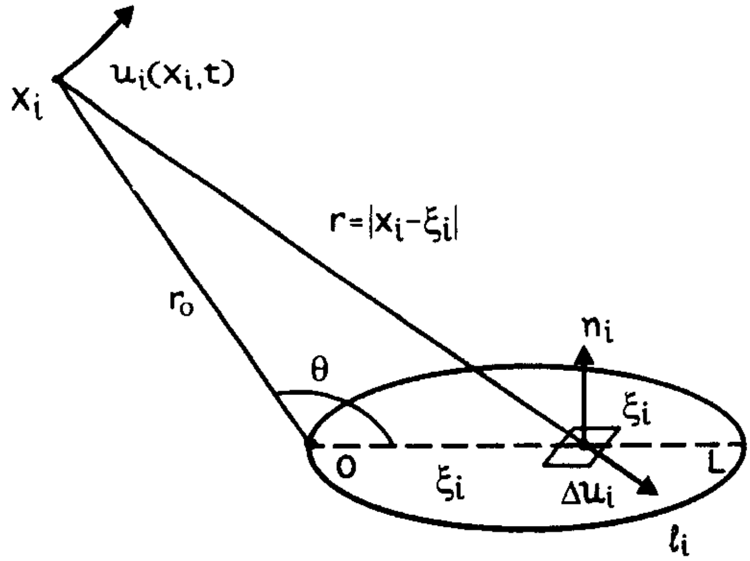
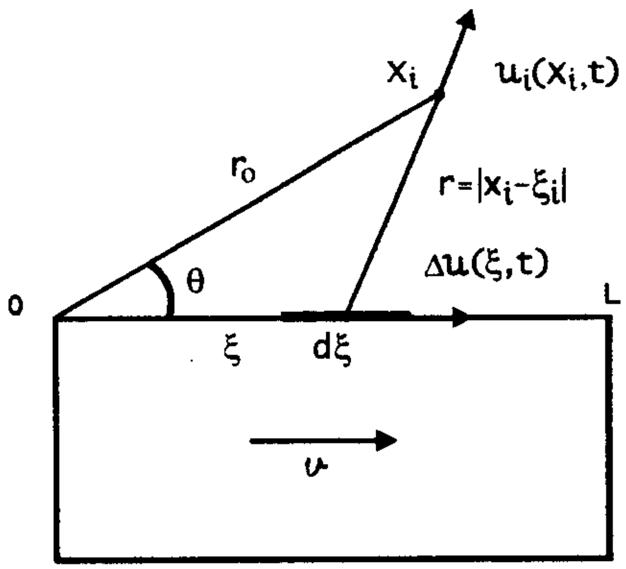


fig - 18.1



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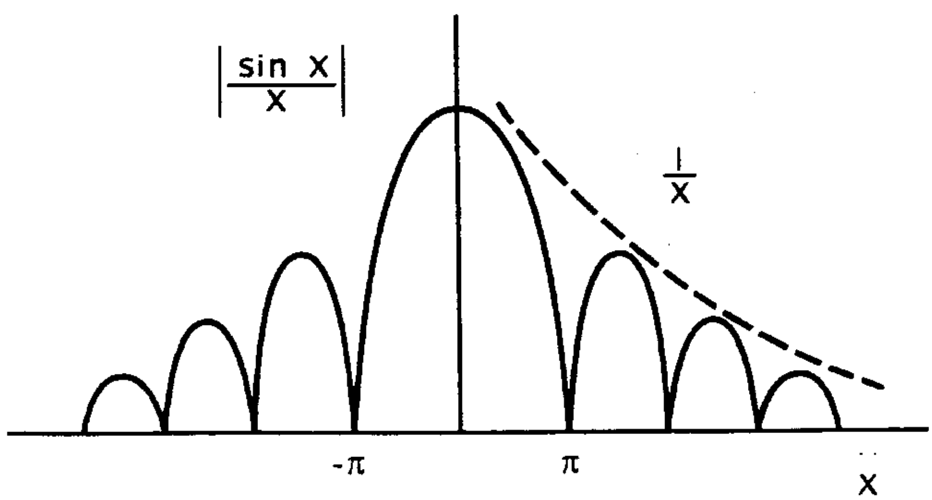

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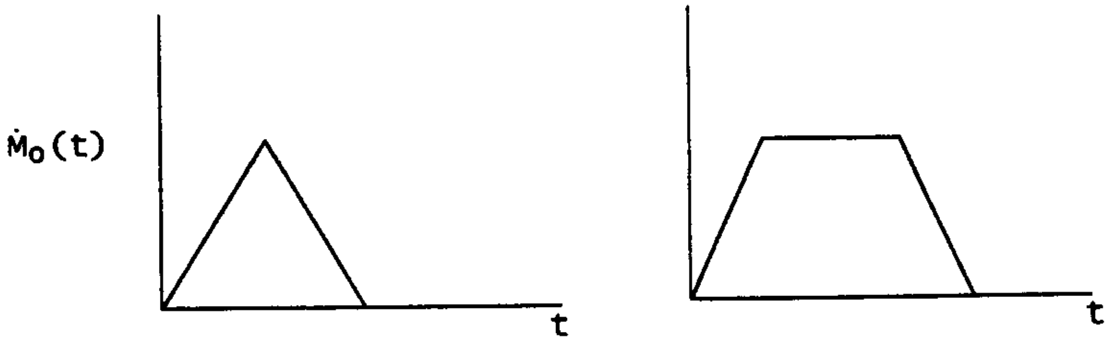

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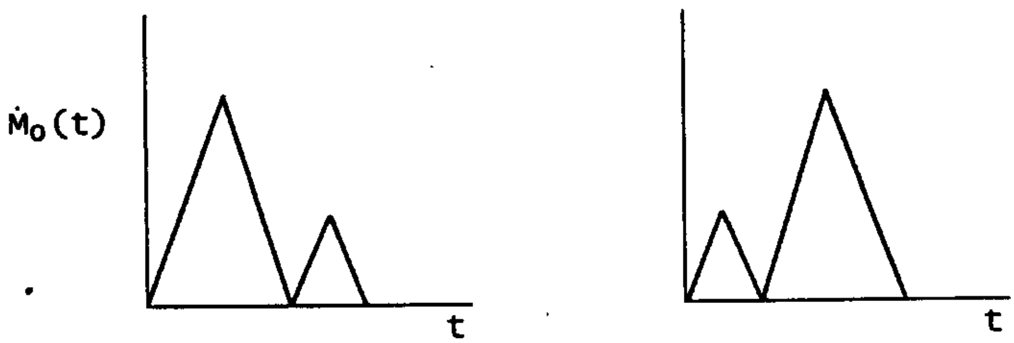

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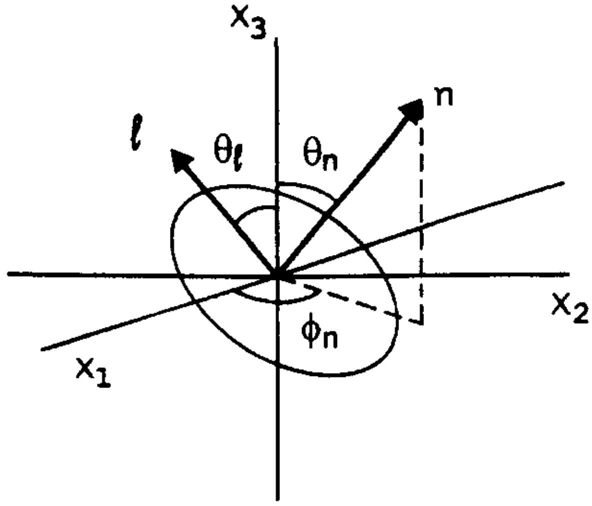

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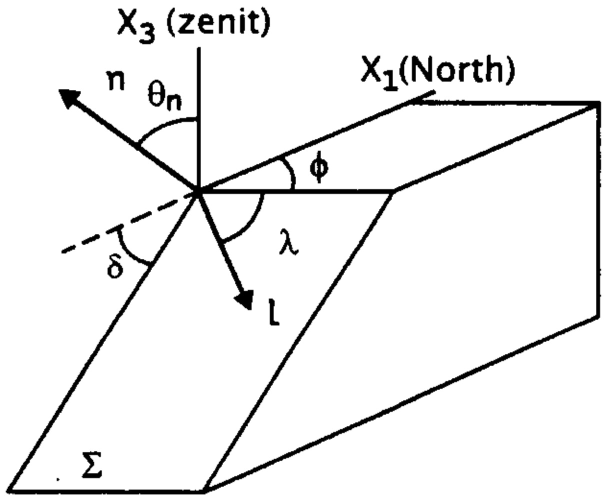

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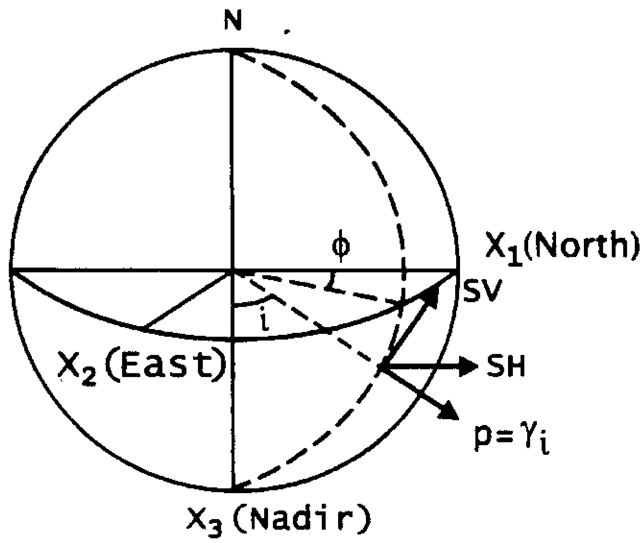

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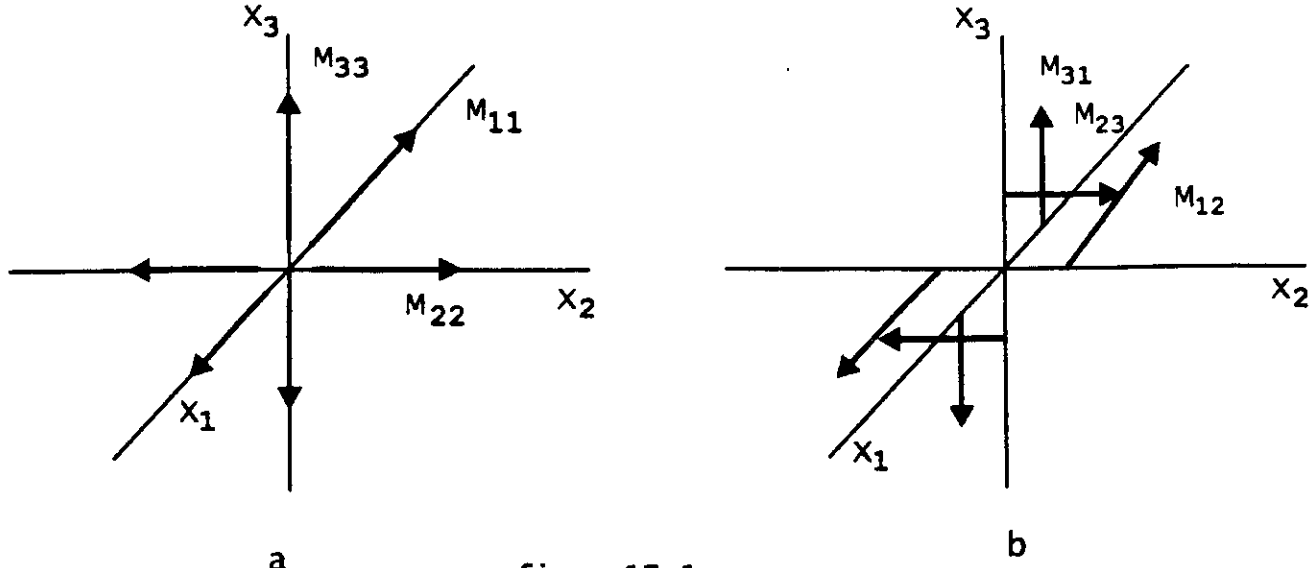

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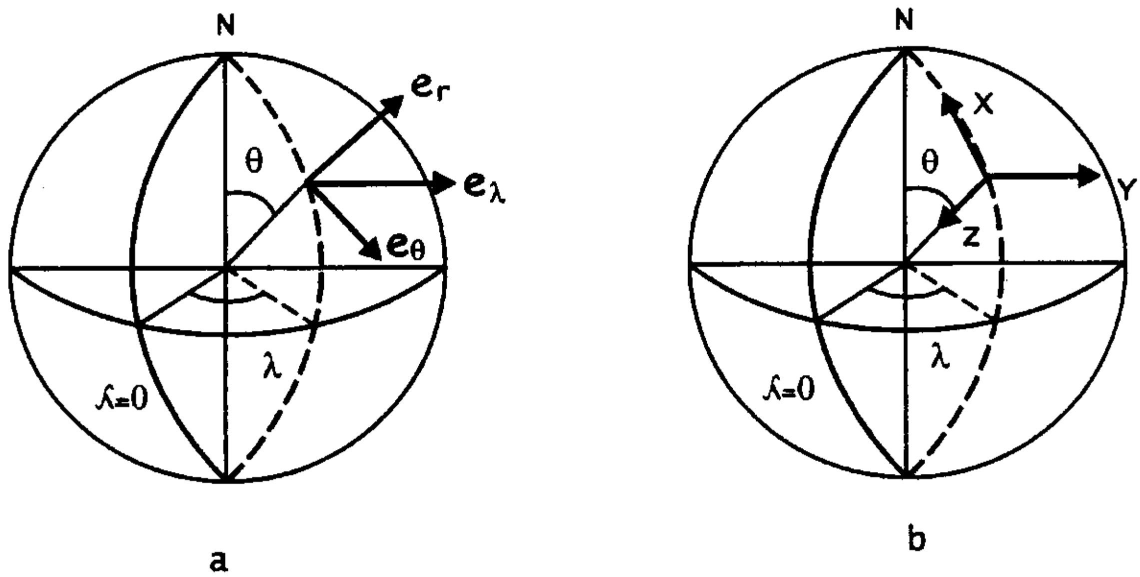

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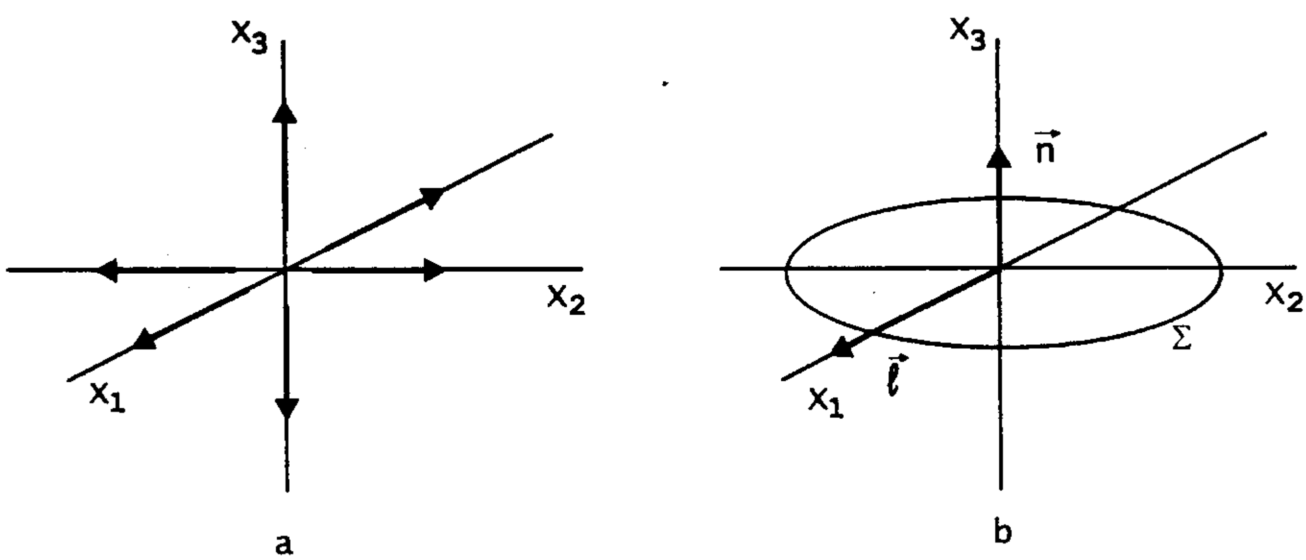
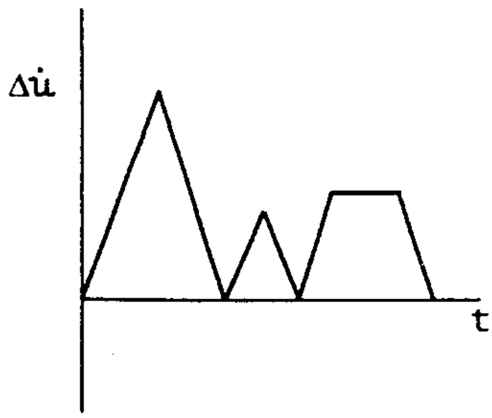


fig - 17.3



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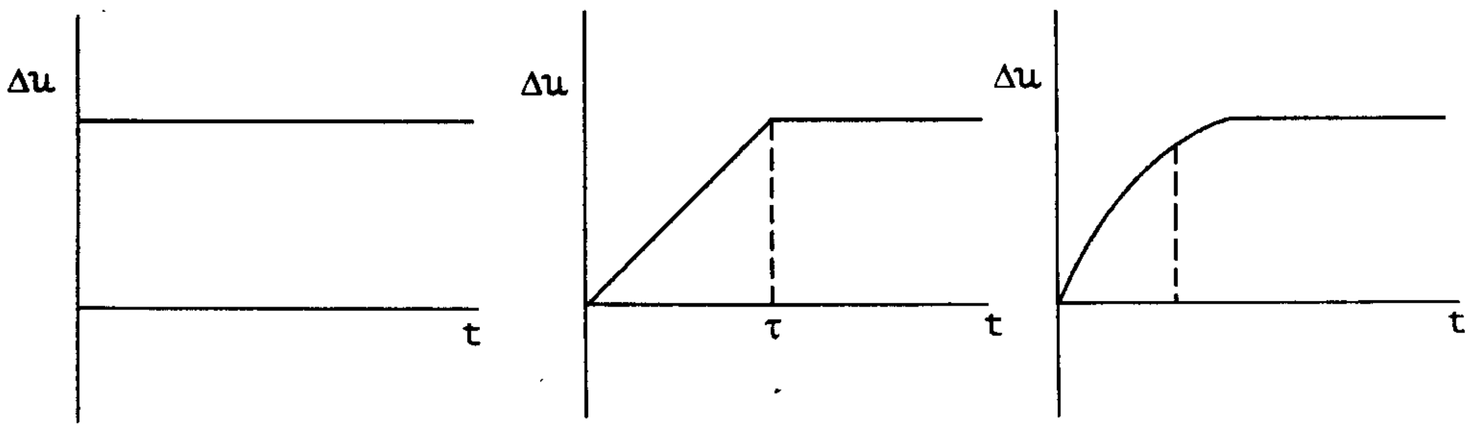

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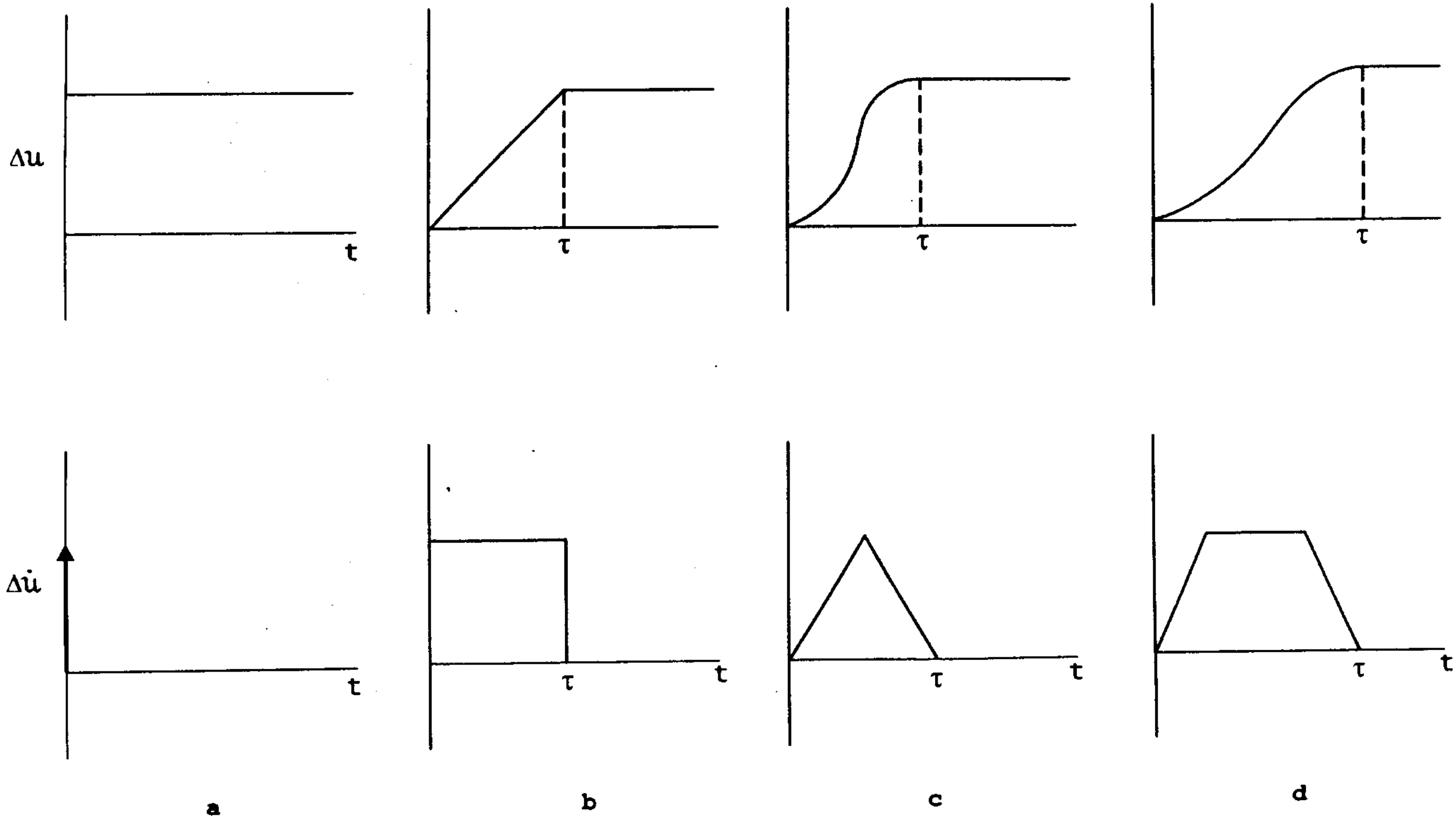

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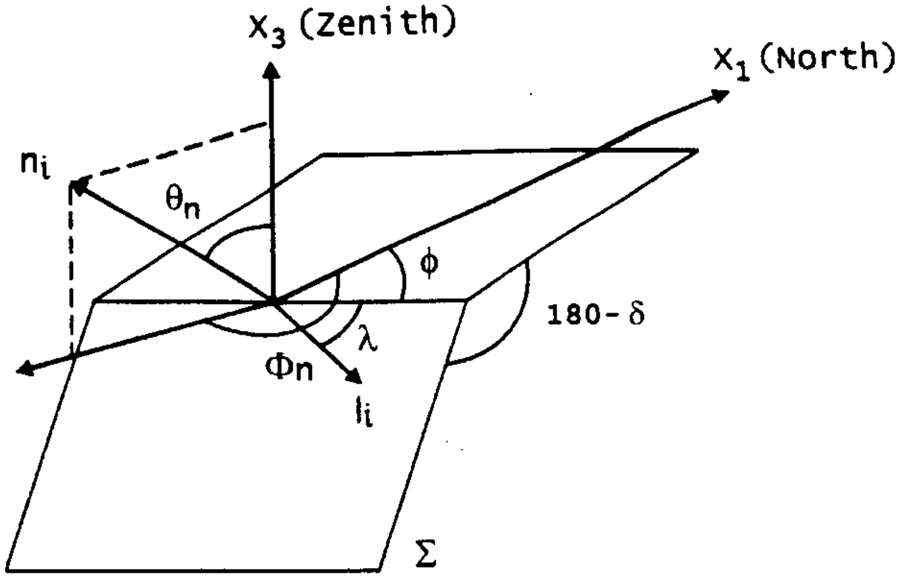

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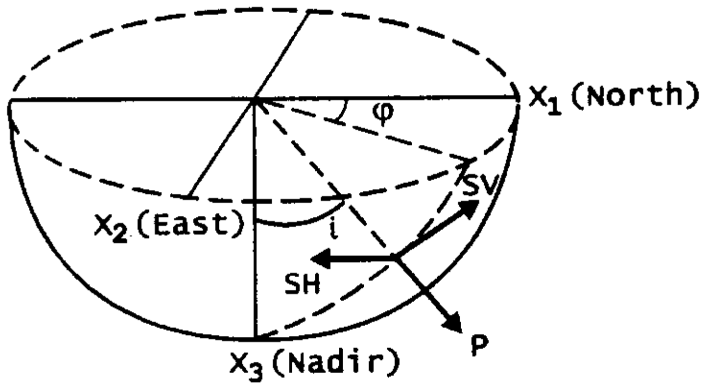

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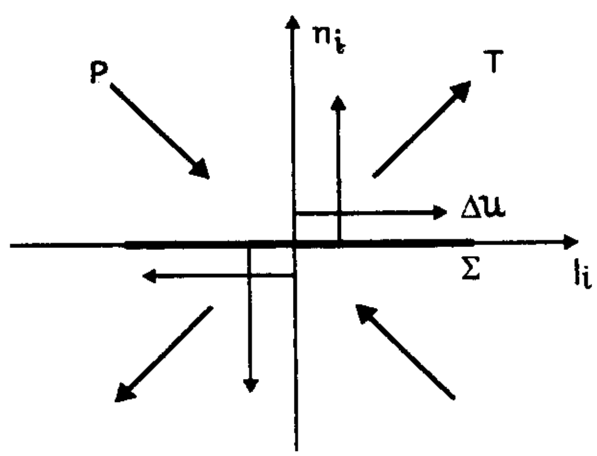

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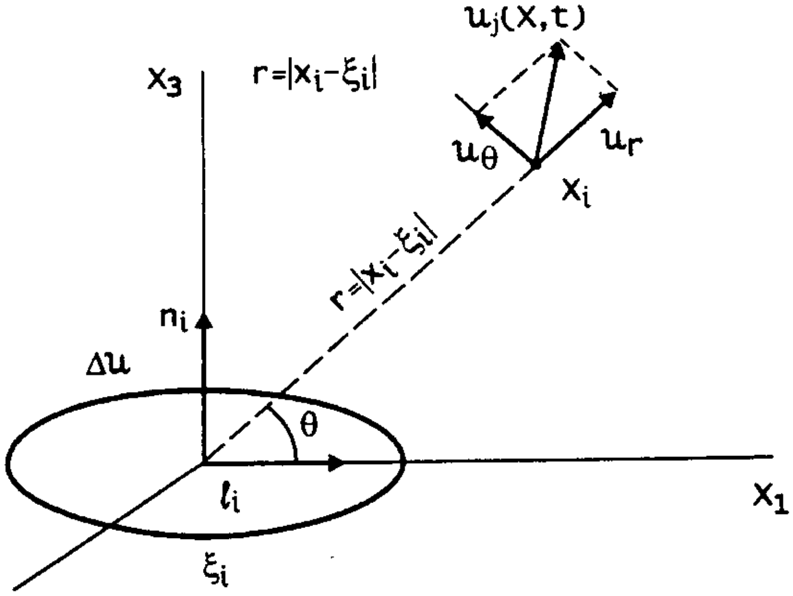

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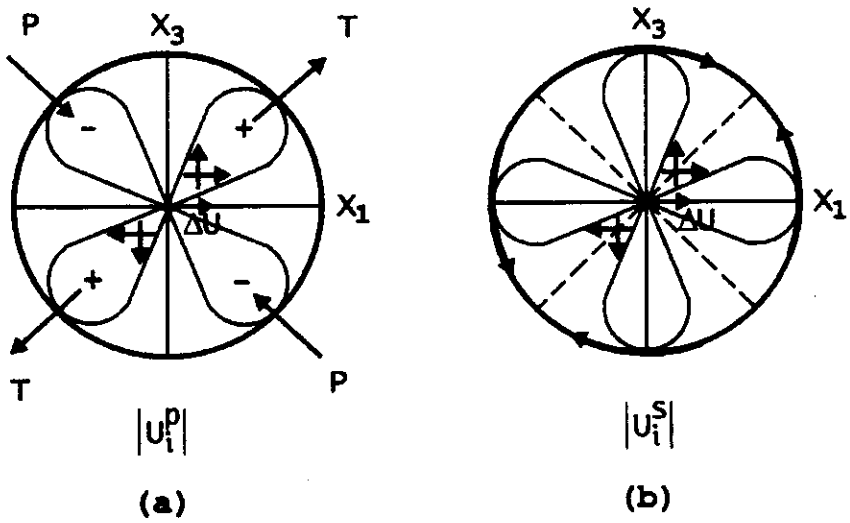

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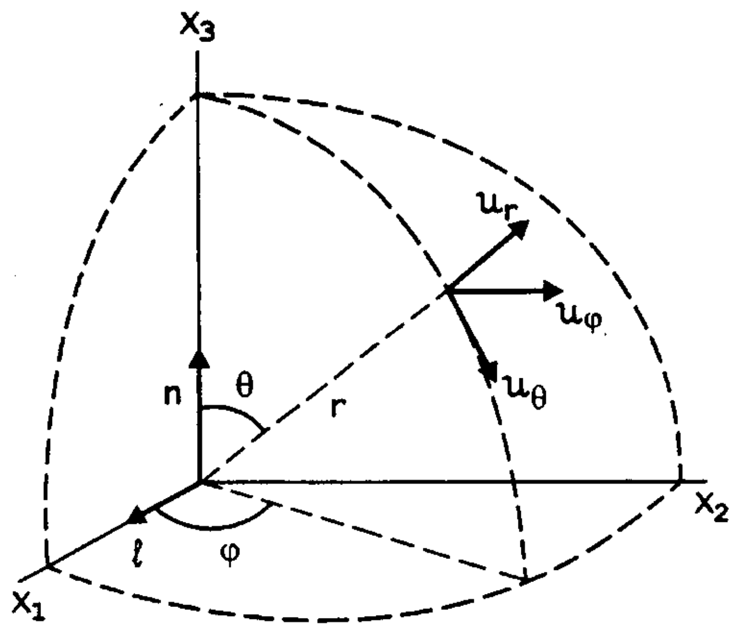

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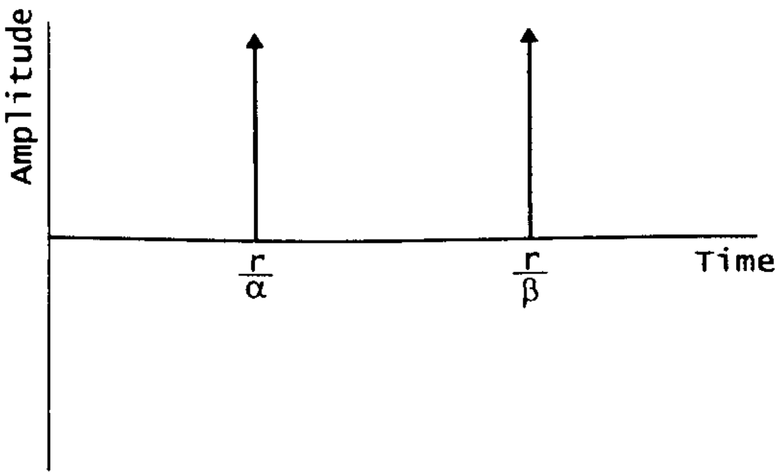

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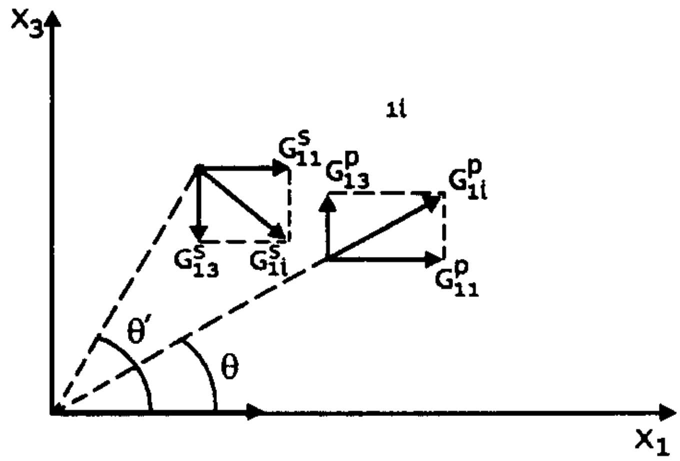

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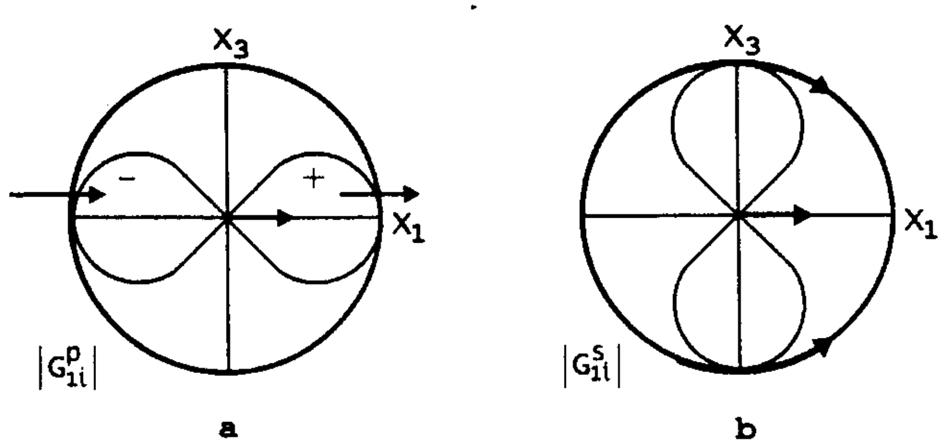

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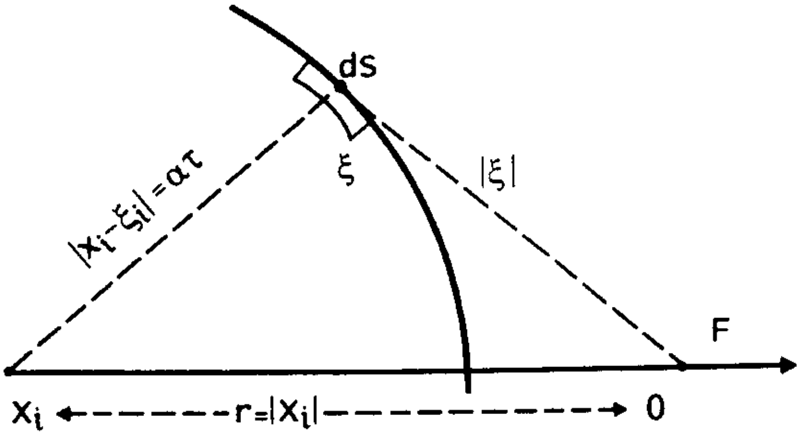

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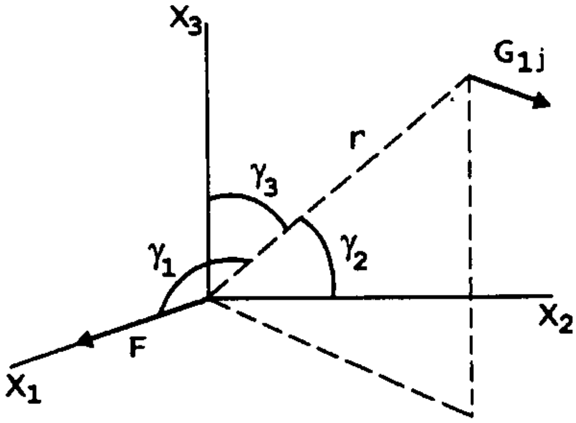

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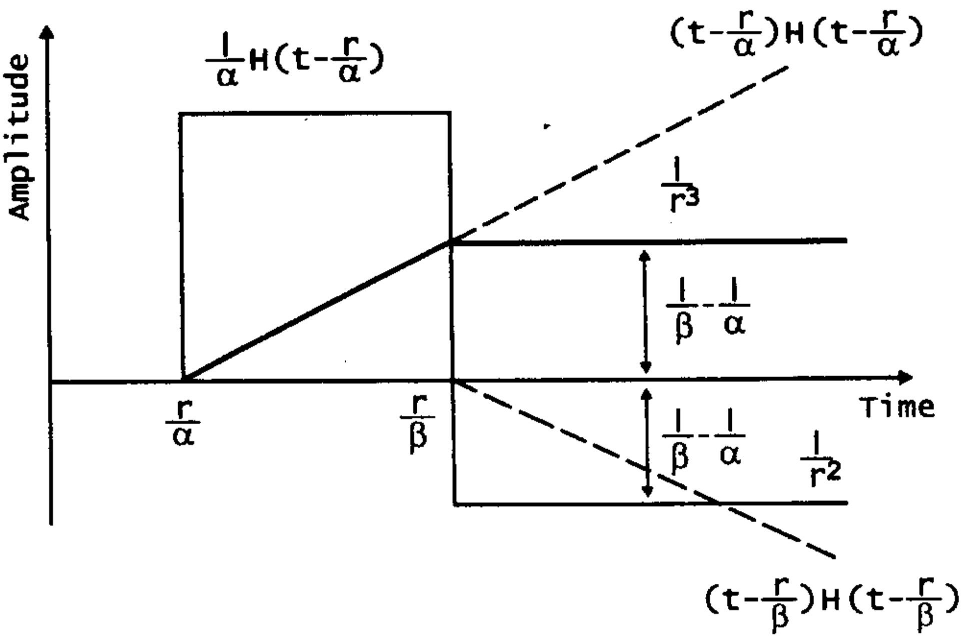

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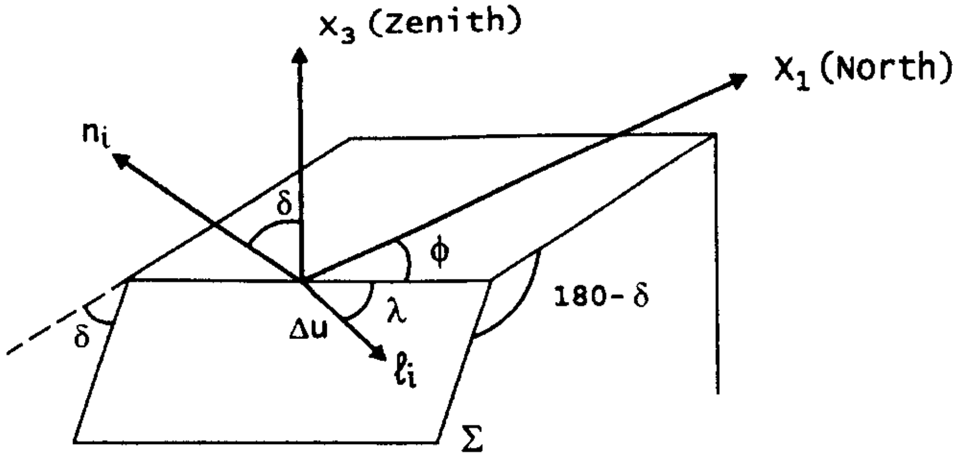

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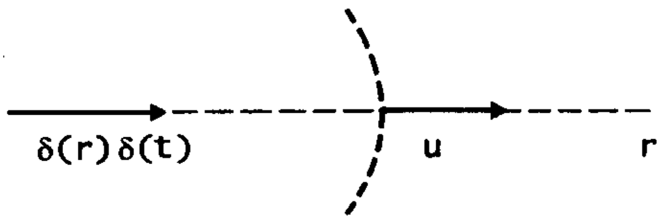

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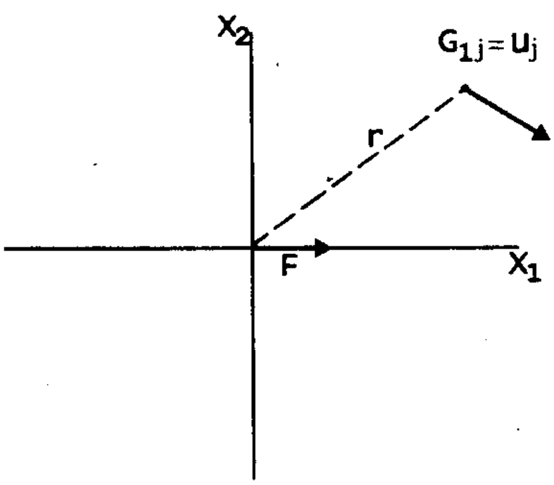

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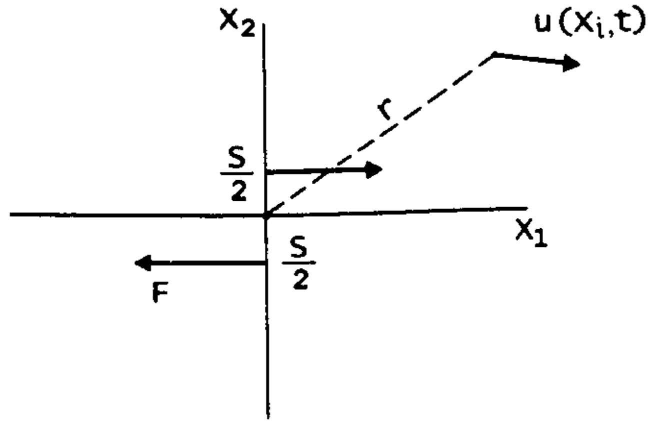

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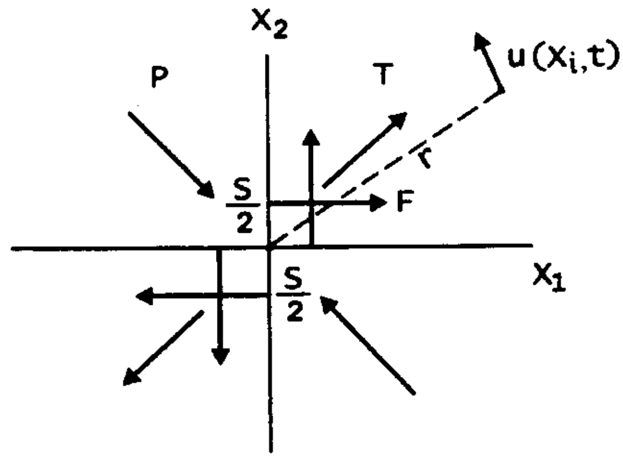

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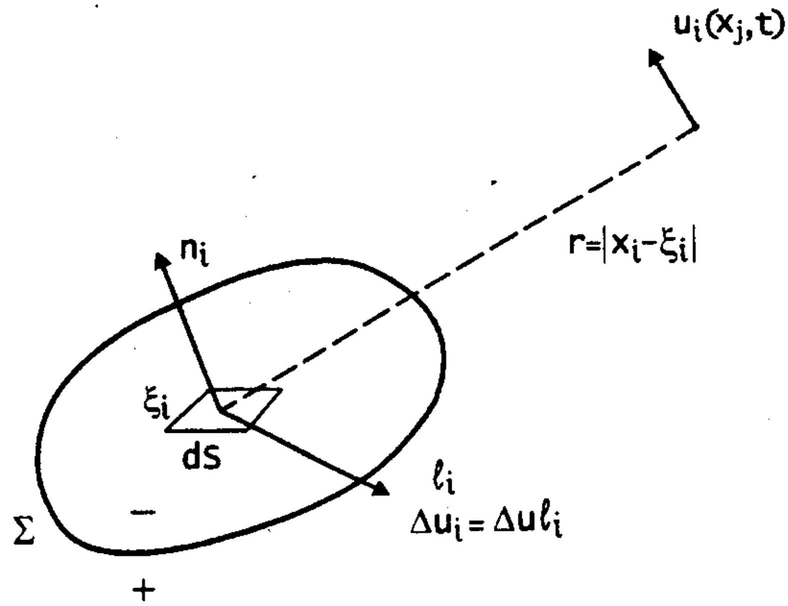

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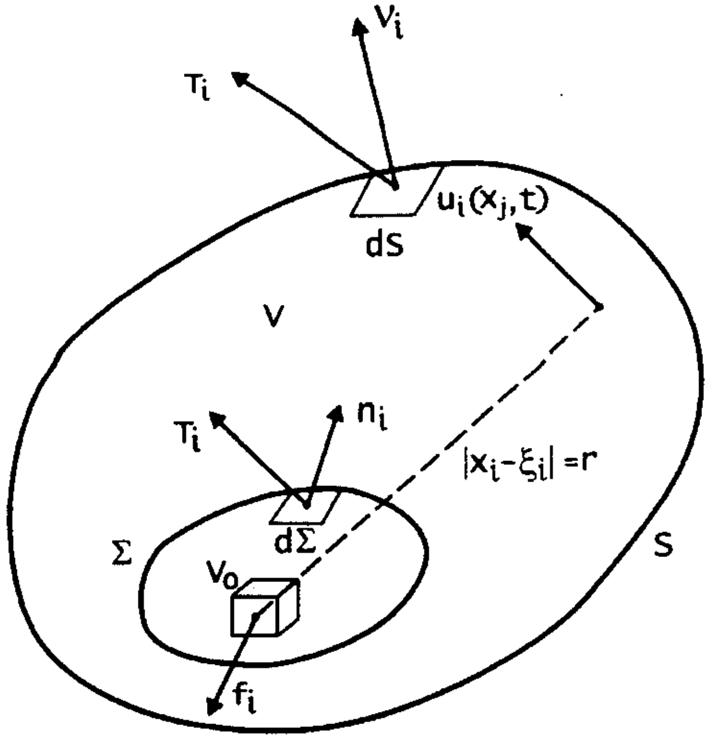

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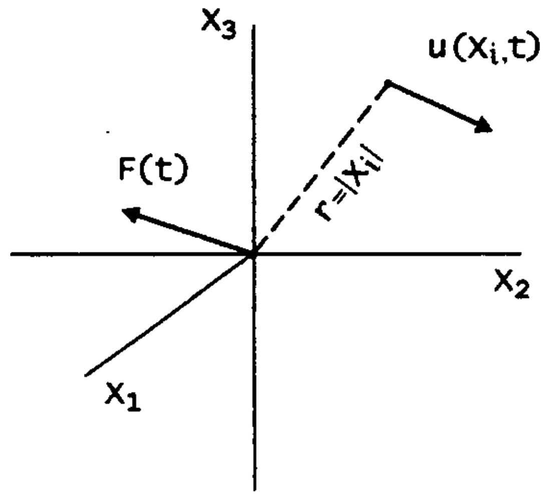
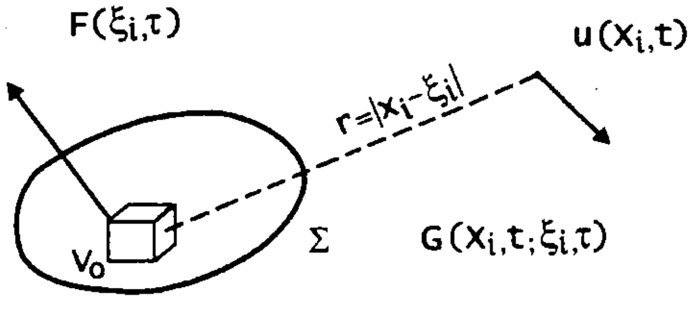


fig - 16.2



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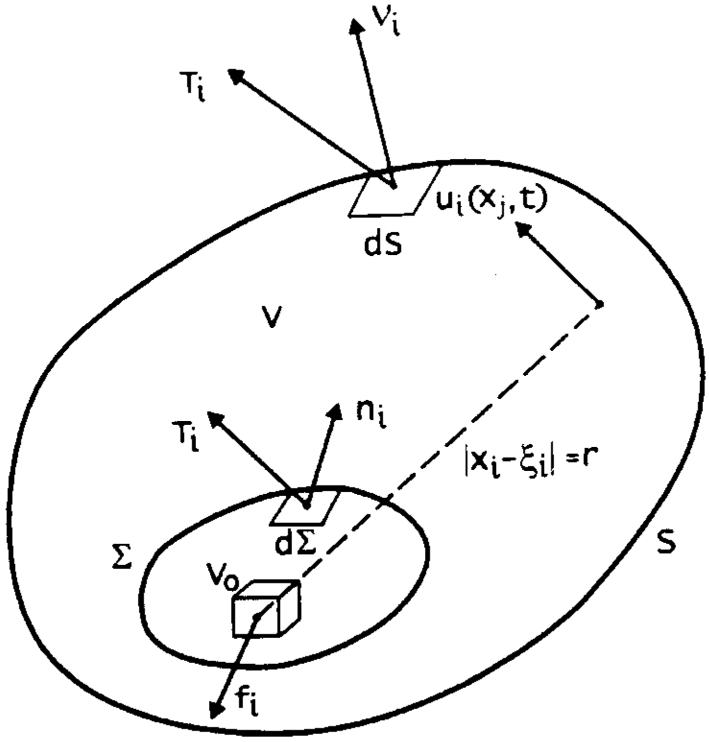

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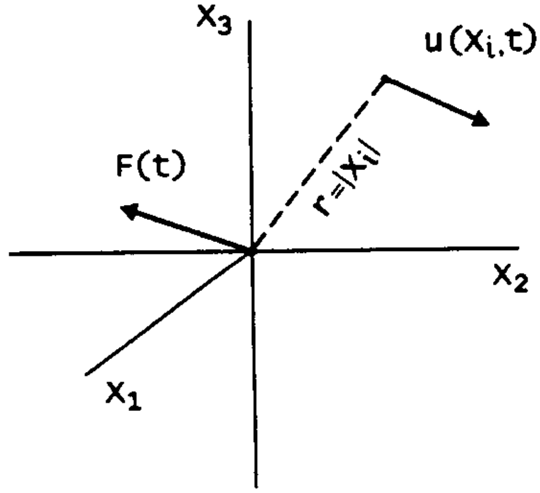

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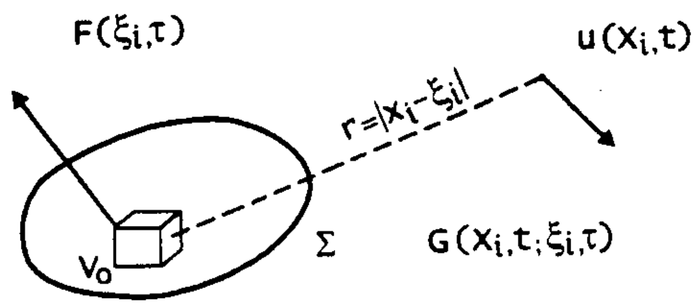

fig - 16.3

$$\Delta\dot{u}(\rho,t) = \Delta V \left[H\left(t - \frac{\rho}{v}\right) - H\left(t + \frac{\rho}{v} - \frac{a}{v}\right)\right] H(a - \rho) \qquad (18.40)$$

In this model, slip velocity takes a constant value $\Delta\dot{u} = \Delta V$ in each point of the fault as rupture front arrives at $t = \rho/v$ that remains constant as long as $t < (a - \rho)/v$ and ceases for $t \geq (a - \rho)/v$. This means that slip velocity lasts in each point of the fault until fracture stops at the border $\rho = a$ (Fig. 18.10). The way to stop motion in the fault is not physically possible, since it implies that information of the stop of the fault at the border reaches instantaneously (with infinite velocity) all points of the fault. This can be solved by introducing a finite velocity, for example, velocity of P waves to bring the information of the stop of rupture at the border to all points of the interior of the fault. This can be done by substituting the second term inside the square brackets of (18.40) by $H[t - a/v - (a - \rho)/\alpha]$. Now slip velocity becomes zero at each point of the fault as a P wave arrives from the fault border, once motion has stop. This wave is called the healing front, since it heals the fault stopping its motion. In kinematic models healing of rupture inside the fault must be introduced in some way.

Models represented by (18.32) and (18.40) have slip and slip velocity time dependence of a step function. As in point sources, we can introduce a rise time in these models. As rupture front reaches each point of the interior of the fault, slip velocity starts to increase from zero to a maximum value during a time τ. Slip velocity is brought down to zero in each point of the fault as a healing front arrives from the border of the fault where it has stopped. More realistic kinematic models can be established with different shapes, generally rectangular, where slip and slip velocities decrease gradually as rupture near to the border of the fault and fracture velocity, maximum slip and slip velocity and rise time vary along the fault plane (Archuleta, 1984; Mendoza and Hartzell, 1989). In general, kinematic models of faulting can be made to correspond quite realistically to conditions on a fault but they are not completely exempt of certain arbitrariness. Some conditions on the faulting process must be imposed a priori, such as velocity of rupture propagation, stopping at the border and healing process.

$$\int_{-\infty}^{\infty} f(x)\, \delta(ax - b)\, dx = \frac{1}{a} f(\frac{b}{a}) \qquad (18.36)$$

The integral of (18.35) can be extended to the interval $(-\infty, \infty)$ since it is zero outside the interval (0, a), and thus applying (18.36), we obtain,

$$u(r_0,t) = 2\pi\Delta u\, v^2\, H(t - \frac{r_0}{\alpha})\, (t - \frac{r_0}{\alpha})\{1 - H[v(t - \frac{r_0}{\alpha}) - a]\} \qquad (18.37)$$

According to this expression,

$$t \le \frac{r_0}{\alpha} \text{ and } t \ge \frac{r_0}{\alpha} + \frac{a}{v} \; : \; u(r_0, t) = 0 \qquad (18.38)$$

$$\frac{r_0}{\alpha} < t < \frac{r_0}{\alpha} + \frac{a}{v} \; : \; u(r_0,t) = 2\pi\Delta u v^2 (t - \frac{r_0}{\alpha}) \qquad (18.39)$$

Displacement starts at $t = r_0/\alpha$ and increases linearly with time until $t = a/v + r_0/\alpha$ when it drops to zero (Fig. 18.9b). According to the approximation used ($r = r_0$), time of the discontinuity corresponds to the arrival of the signal from the stop of the fracture at the border ($\rho = a$), which is called the stopping phase. Displacement drops discontinuously to zero and velocity and acceleration becomes infinite.

In Savage's model slip passes instantaneously from zero to its maximum value in each point of the fault as rupture propagates from the center outwards. Slip velocity is a pulse that propagates in the same way until it reaches the border of the fault. Since elastic displacements depend on slip velocity, other models specify directly this value (as was done for STF in Section 16.6) (Molnar et al., 1973). For a circular fault the slip velocity can be expressed as, In the center of the fault ($\rho = 0$), slip at $t = 0$, passes instantaneously from zero to Δu. For a value at distance ρ ($0 < \rho < a$) from the the center, slip is 0 until $t = \rho/v$ when it becomes Δu. For $\rho = a$, rupture stops and for $\rho \geq a$, slip is zero for all values of t (Fig. 18.8).

For a point of observation on x_3 axis, that is, over the center of the fault at distance r_0 (Fig. 18.9a), the form of P waves according to (18.2) is given by,

$$u(r_0,t) = \int_0^{2\pi}\int_0^a \Delta\dot{u}\ (\rho,\ t - \frac{r}{\alpha})\ \rho\ d\rho d\phi \tag{18.33}$$

Since rupture propagates in ρ direction with velocity v, as in (18.13), slip can be written as $\Delta u(t - r/\alpha - \rho/v)$. Substituting in (18.33) and in an approximation for $r_0 \gg a$, $r = r_0$, after integration over ϕ, we have,

$$u(r_0,t) = 2\pi \int_0^a \Delta\dot{u}\ (t - \frac{r_0}{\alpha} - \frac{\rho}{v})\rho\ d\rho \tag{18.34}$$

Taking the time derivative in (18.32) and substituting in (18.34), displacement is given by,

$$u(r_0,t) = 2\pi\Delta u\ H(t - \frac{r_0}{\alpha})\int_0^a \delta(t - \frac{\rho}{v} - \frac{r_0}{\alpha})[1 - H(\rho - a)\rho\ d\rho \tag{18.35}$$

To evaluate this integral we use the relation, Where "a" is the radius of the fault. The spectrum (18.29) has a flat part at low frequencies as they tend to zero and decreases as ω^{-2} for high frequencies, starting at the corner frequency ω_c = b. If stress drop is not total, for ε small ($\varepsilon \simeq 0.01$), spectrum decreases as ω^{-1}. Fault radius can be deduced from the corner frequency of S waves,

$$a = \frac{2.33\beta}{\omega_c} \tag{18.31}$$

Brune's model is commonly used to obtain fault dimensions from spectra of S waves for small to moderate size earthquakes ($M < 6$) for which the circular fault is a good approximation. We have mentioned (section 15.1) that earthquakes take place in the brittle part of the crust (about 20 km thickness) or seismogenic layer. For dimensions less than 20 km ($M < 6$), fault planes are contained inside the seismogenic layer. Fractures start at a point and grow unhindered in all directions with near circular form ($L \simeq W$) and can be approximated by Brune's model. Larger earthquakes have larger dimensions, since its width is limited to about 20 km, its length must be larger than its width ($L > W$), In these cases, Haskell's rectangular model is a better approximation.

17.4 NUCLEATION, PROPAGATION AND STOP OF RUPTURE

Haskell's model doesn't include the effect of either the beginning or nucleation of rupture or its stop or arrest. The first kinematic model that include both effects was proposed by Savage (1966). Savage's model consists in an elliptical fault in which slip begins in one of the foci and stop when it reaches the border of the ellipse. The model can be simplified for a circular fault of radius a, where slip Δu (constant for all points) begins at the center, propagates radially with constant rupture velocity v and circular rupture fronts and stops at the circular border. We use polar coordinates (ρ, ϕ) on the fault plane, ρ with origin at the center of the fault and ϕ measured from x_1. Then, time dependence of slip is a step function and slip is only a function of ρ given in the form,

$$\Delta u(\rho,t) = \Delta u \; H\left(t - \frac{\rho}{v}\right)[1 - H(\rho - a)] \tag{18.32}$$

$$\Delta\sigma(x,t) = \Delta\sigma \; H(t - \frac{x}{\beta}) \qquad (18.25)$$

Shear displacement Δu, for $x = 0$, is obtained by integration of (18.25), since in this case, $\sigma = \mu \partial u / \partial x$,

$$\Delta u(t) = H(t) \frac{\Delta\sigma}{\mu} \beta t \qquad (18.26)$$

Its Fourier transform is,

$$\Delta U(\omega) = - \frac{\Delta\sigma\beta}{\mu\omega^2} \qquad (18.27)$$

The effective stress is the difference between tectonic stress σ_0 acting on the fault plane and friction stress σ_f, ($\Delta\sigma = \sigma_0 - \sigma_f = \varepsilon\sigma_0$). This coincides with the stress drop defined in (15.25) with $\sigma_1 = \sigma_f$. For total stress drop ($\Delta\sigma = \sigma_0$ and $\varepsilon = 1$) displacement of S waves in the far field at distance r, not including radiation pattern and dependence with distance, is,

$$u(t) = \frac{\Delta\sigma\beta}{\mu} (t - \frac{r}{\beta}) \, e^{-b(t - \frac{r}{\beta})} \qquad (18.28)$$

Its spectrum is,

$$U(\omega) = \frac{\Delta\sigma\beta}{\mu} \frac{1}{\omega^2 + b^2} \qquad (18.29)$$

$$b = \frac{2.33\beta}{a} \qquad (18.30)$$

$$\text{P:}\quad \omega_1 = \frac{\alpha}{2L} \qquad\qquad \text{S:}\quad \omega_1 = \frac{3.6\beta}{L}$$

$$\omega_2 = \frac{2.4\alpha}{W} \qquad\qquad \omega_2 = \frac{4.1\beta}{W}$$

$$\omega_3^2 = \frac{2.9\alpha^2}{LW} \qquad\qquad \omega_3^2 = \frac{14.8\beta^2}{LW}$$

Corner frequencies of P waves are always lower than those of S. Usually, observed corner frequencies ω_c correspond to ω_3, and from this value we can obtain source dimensions,

$$\sqrt{LW} = \frac{1.7\alpha}{\omega_c^P} = \frac{3.8\beta}{\omega_c^S} \tag{18.24}$$

Difference between ω_1 and ω_2 depends on the relation between L and W. If $W \ll L$, that is, fault is long and narrow, the difference is large and if $L \approx W$, the three frequencies practically coincide.

18.3 CIRCULAR FAULT. BRUNE'S MODEL

Another fundamental model of an extended seismic source is that of a circular fault known as Brune's model (Brune, 1972). This model consists in a circular fault plane with finite radius on which a shear stress pulse is applied instantaneously (Fig 18.7). Since this model specifies stress on the fault, this is not exactly a kinematic model. Because the stress pulse is applied instantaneously on the whole fault area, there is no fracture propagation. The shear pulse generates a shear wave that propagates perpendicularly to the fault plane. Adapting Brune's notation to the one we have used, we call $\Delta\sigma$ to Brune's effective shear stress and Δu to the displacement on the fault plane (this is, for $x = 0$, where x is the distance normal to the fault plane). The stress pulse has a time dependence given by a step function and for a distance x is,

$$D(\omega) = \frac{\sin[\omega L/2c\ (c/v - \cos\theta)]\ (c/v + \cos\theta)}{\sin[\omega L/2c\ (c/v + \cos\theta)]\ (c/v - \cos\theta)} \qquad (18.23)$$

Where c is the wave velocity. This function has a series of maxima and minima for frequencies that depend on L and v and can be used to determine source dimensions and velocity of fracture propagation. This is easier for surface waves, since θ represents the azimuth at the focus with respect to the trace of the fault.

Another effect of equation (18.20) is on the form of the radiation pattern (section 16.5, figure 16.17). If wave length is much larger than source dimensions ($\lambda \gg L$), X tends to zero and senX/X is unity for all values of θ. Amplitudes are not affected and radiation pattern corresponds to that of a point source. If wave length is of the same order as the dimensions ($\lambda \approx L$), amplitudes are affected by the factor senX/X that depends on θ and the radiation pattern is modified. According to (18.19), this factor is maximum for $\theta = 0$ and minimum for $\theta = \pi$, that is, amplitudes are larger in the same direction as fracture propagation ($\theta = 0$) and smaller in the opposite direction ($\theta = \pi$) (Fig. 18.5). This effect is called focusing of energy in the direction of fracture propagation and is a phenomenon present in all propagating sources.

The kinematic model of a rectangular unilateral fracture with constant rupture velocity has shown us the effects of source dimensions in the radiated displacement field. Amplitude spectra of displacements have constant value proportional to seismic moment in low frequencies and decrease with frequency in high frequencies starting at the corner frequency. If the STF includes a rise time, this decrease corresponds to $1/\omega^2$. Radiation pattern is also affected by dimensions, with greater energy radiated in azimuth corresponding to the direction of rupture propagation.

Haskell's model with bilateral fracture, rupture velocity $v = 0.9\beta$ and STF given by (18.22) has two corner frequencies ω_1 and ω_2 instead of one (Savage, 1972). For frequencies between zero and ω_1, spectrum is flat, between ω_1 and ω_2 decreases as ω^{-1} and for frequencies higher than ω_2 decreases as ω^{-2} (Fig. 18.6). A third corner frequency ω_3 is defined by the intersection of the flat part and the decay as ω^{-2}. For P and S waves ω_1, ω_2 and ω_3 are given by, Where we have replaced $i = e^{i\pi/2}$. The form of the amplitude spectrum depends on the factor senX/X. We have discussed the form of this function in Section 12.2. It has value unity for $X = 0$, roots for X equal to integer multiples of π, and its envelope decreases as $1/X$ (Fig. 18.3). Since for fixed values of θ and L, X depends on ω, in the limit when ω tends to zero (low frequencies), the factor equals unity and for high frequencies its enveloped decreases with $1/\omega$.

The form of the amplitude spectrum depends also on the form of $\Delta U(\omega)$, the transform of the source time function (STF) (18.20). If $\Delta u(t) = \Delta u H(t)$ its transform is $\Delta U(\omega) = \Delta u/i\omega$. From (18.20) we obtain that $U(\omega)$ is proportional to the seismic moment ($M_0 = \mu LW\Delta u$) for the limit of low frequencies and decreases as $1/\omega$ for high frequencies. If the STF has a rise time τ that Δu takes in getting its maximum value at each point of the fault plane (Section 16.7), the spectrum depends on the transform of the STF. For example, transforms of STF given by (16.109) and (16.110), are,

$$\text{(a)}\quad \Delta u(t) = \begin{cases} \Delta u\, t/\tau_0 : & 0<t<\tau \\ \Delta u : & t\geq t \end{cases};\quad \Delta U(\omega) = \frac{\Delta u(1 - e^{-i\omega\tau})}{\omega^2\tau} \qquad (18.21)$$

$$\text{(b)}\quad \Delta u(t) = \Delta u\, H(t)(1-e^{-t/\tau});\quad \Delta U(\omega) = \frac{\Delta u}{(1 + i\omega\tau)i\omega} \qquad (18.22)$$

In both cases, transforms depend on $1/\omega^2$. If we substitute these values of $\Delta U(\omega)$ in (18.20), envelope of $U(\omega)$ decreases with frequency as $1/\omega^2$. If we represent the spectrum respect to logarithm of frequency, its form is a flat part in low frequencies and from a certain frequency ω_c, called the corner frequency, its envelope is a straight line of slope -2 (Fig. 18.4). This form of the spectrum is due to the combined effect of source dimensions and rise time. If we consider a particular case for $\theta = \pi/2$, and that ω_c corresponds to $X = \pi/2$, we obtain $\omega_c = 2v/L$, that is, corner frequency is proportional to the inverse of source length. Observed spectra of seismic waves show these characteristics indicating the finite dimensions of the source and presence of rise time (Aki, 1967).

Influence of source dimensions can be isolated by means of the directivity function $D(\omega)$, defined by Ben-Menahem (1961), as the quotient of spectral amplitudes of waves that leave the source in opposite directions, that is, with angles θ and $\theta + \pi$. According to (18.19) and (18.20), this quotient is,

$$\int_{-\infty}^{\infty} \Delta\dot{u}(t - d)\, e^{-i\omega(t-d)}\, dt = i\omega\, \Delta U(\omega)\, e^{-i\omega d} \qquad (18.16)$$

Where $\Delta U(\omega)$ is the transform of $\Delta u(t)$, the transform of $\Delta\dot{u}(t)$ is $i\omega\ \Delta U(\omega)$ and that of $\Delta u(t - d) = \Delta U(\omega)\exp(-i\omega d)$. Therefore, equation (18.15) becomes,

$$U(x_i,\omega) = W\ i\omega\ \Delta U(\omega)\ e^{-i\omega r_0/\alpha} \int_0^L e^{-i\frac{\xi\omega}{\alpha}\left(\frac{\alpha}{v} - \cos\theta\right)} d\xi \qquad (18.17)$$

To evaluate the integral in (18.17), we substitute $b = -\omega/\alpha(\alpha/v - \cos\theta)$, and obtain,

$$\int_0^L e^{ib\xi}\, d\xi = \frac{2}{b}\ \text{sen}\left(\frac{bL}{2}\right) e^{i\frac{bL}{2}} = L\ \frac{\text{sen}X}{X}\ e^{iX} \qquad (18.18)$$

Where,

$$X = \frac{bL}{2} = -\frac{\omega L}{2\alpha}\left(\frac{\alpha}{v} - \cos\theta\right) \qquad (18.19)$$

Final form for the transform of elastic displacements of P waves $U(x_i,\omega)$, according to (18.17), is,

$$U(x_i,\omega) = WL\omega\ \Delta U(\omega)\ \frac{\text{sen}X}{X}\ e^{-i\left(\frac{\omega r_0}{\alpha} - X - \frac{\pi}{2}\right)} \qquad (18.20$$

18.2 RECTANGULAR FAULT. HASKELL'S MODEL

A simple kinematic model of finite dimensions, known as Haskell's model, is a rectangular fault of length L and width W where slip Δu propagates only along L direction with constant velocity v (slip moves instantaneously along W) (Haskell, 1964). Coordinate along L is ξ, with origin in one end of the fault and Δu has only one component (Fig. 18.2). Fractures that propagate only in one sense (from 0 to L) are called unilateral fracture and in both senses (from 0 to L/2 and from 0 to -L/2) bilateral. For unilateral fracture, according to (18.6), prescinding from radiation pattern, dependence on distance and the other factors of (18.1), the form of P waves in the far field, is given by,

$$u(x_i,t) = W \int_0^L \Delta\dot{u}\left(\xi,\ t - \frac{r_0 - \xi\cos\theta}{\alpha}\right) d\xi \qquad (18.12)$$

If slip moves in the positive direction of ξ with constant fracture velocity v, then, $\Delta u(\xi,t) = \Delta u(t - \xi/v)$ and we obtain,

$$u(x_i,t) = W \int_0^L \Delta\dot{u}\left(t - \frac{r_0}{\alpha} - \frac{\xi}{\alpha}\left(\frac{\alpha}{v} - \cos\theta\right)\right) d\xi \qquad (18.13)$$

If we substitute

$$d = \frac{r_0}{\alpha} + \frac{\xi}{\alpha}\left(\frac{\alpha}{v} - \cos\theta\right) \qquad (18.14)$$

The Fourier transform of $u(x_i,t)$ is,

$$U(x_i,\omega) = W \int_0^L d\xi \int_{-\infty}^{\infty} \Delta\dot{u}(t-d)\, e^{-i\omega(t-d)}\, dt \qquad (18.15)$$

But we have that,

18. MODELS OF FRACTURE

18.1 SOURCE DIMENSIONS. KINEMATIC MODELS

In chapters 16 and 17 we have considered in some detail the characteristics and displacement field corresponding to point sources. A more complete representation of the seismic source must include its dimensions and consider its effects on wave radiation. First considerations of dimensions of the seismic focus proposed models consisting in spherical cavities of finite radius with uniform distribution of stresses on its surface (Jeffreys, 1931; Nishimura, 1937; Scholte, 1962).

The first models for extended sources of shear fracture were kinematic models consisting in slip that propagates with constant velocity over a surface with finite area. Ben Menahem (1961, 1962) described extended sources by distributions of single and double couples propagating with a certain velocity over a rectangular surface, and determined the corresponding displacements of body and surface waves. Berckhemer (1962) studied the effect of circular fractures of finite radius that propagates from its center on the width of time pulses. Burridge and Knopoff (1964) treated shear dislocations that propagate on a certain area and showed their equivalence with propagating double couples. Haskell (1964, 1966) proposed a rectangular model of fracture and Savage (1966) an elliptical fault and studied the effects on spectra of body and surface waves. Brune (1970) presented a model with shear stresses suddenly applied on a circular fault and studied elastic displacements in near and far fields. More recent kinematic models include propagating shear fractures on finite faults with variable slip, rupture velocity and rise times (Hartzell, 1989).

Let us consider first some general characteristics of kinematic models of extended sources represented by a surface Σ over which a shear dislocation $\Delta\mathbf{u}(\xi_i)$ propagates with constant velocity v in one direction, from the origin ($\xi_i = 0$) to a final point over a distance L (Fig. 18.1) (Aki and Richards, 1980). Velocity of fracture propagation is assumed to be constant and less than velocity of wave propagation ($v < \beta < \alpha$), that is, we treat subsonic fractures (a common value is $v = 0.7\beta$). From equations (16.15) and (16.69), displacements of P waves in the far field for an infinite, homogeneous, isotropic medium can be written as,

$$u_P = A\,[\text{sen}^2 i\,(\cos^2\phi\; m_{11} + \text{sen}^2\phi\; m_{22} + \text{sen}2\phi\; m_{12}) + \cos^2 i\; m_{33} + \text{sen}2i\,(\cos\phi\; m_{13} + \text{sen}\phi\; m_{23})] \qquad (17.47)$$

$$u_{SV} = B\,[\tfrac{1}{2}\,\text{sen}2i\,(\cos^2\phi\; m_{11} + \text{sen}^2\phi\; m_{22} - m_{33} + \text{sen}2\phi\; m_{12}) + \cos 2i\,(\cos\phi\; m_{13} + \text{sen}\phi\; m_{23})] \qquad (17.48)$$

$$u_{SH} = B\,[\text{sen}\,i\,(\tfrac{1}{2}\,\text{sen}2\phi\; m_{22} - \tfrac{1}{2}\,\text{sen}2\phi\; m_{11} + \cos 2\phi\; m_{12}) + \cos i\,(\cos\phi\; m_{23} - \text{sen}\phi\; m_{13})] \qquad (17.49)$$

These three equations are for a general form of moment tensor without assuming any particular condition.

17.6. TEMPORAL DEPENDENCE

For a point source, moment tensor, if all components have the same time dependence, the source time function is given by $M_0(t)$. As we saw in (17.39) and (17.40), displacements depend on moment rate $\dot{M}(t)$ and its time dependence is also called the STF. This function represents the form moment rate changes with time and its integral or area under its curve is the scalar seismic moment M_0.

As for time dependence of slip rate, for a simple source a commonly used function for $\dot{M}_0(t)$ is a triangle or a trapezoid (Fig. 17.7). We have discussed the properties of this STF in section 16.6 (Fig. 16.22). Since the size of an earthquake is given by $\dot{M}_0$, the same size will result with two different functions of moment rate one with greater modulus and shorter duration and another with smaller modulus and longer duration. We must remember that this is a point source representation, duration of the moment rate function is related in an extended source with time duration of the fracture process and thus with its dimensions. For a complex source, moment rate may be represented by several triangles of different size (Fig. 17.8). Relative size of these sources show how the moment radiation with time or moment release takes place. In some cases, the greatest part of moment release occurs in the first part of the shock and a minor part follows later. (Fig. 17.8a). Other possibility is that there is

$$u_P = A\,\gamma_i \gamma_j\, m_{ij} \qquad (17.41)$$

$$u_{SV} = -B\,SV_i \gamma_j\, m_{ij} \qquad (17.42)$$

$$u_{SH} = B\,SH_i \gamma_j\, m_{ij} \qquad (17.43)$$

Where SV_i and SH_i are unit vectors in SV and SH directions and we have taken into account that $SV_i\gamma_i = 0$ and $SH_i\gamma_i = 0$. Since γ_i is a unit vector in the ray direction, P displacements are in its same direction and those of S are perpendicular (Fig.16.20). If the problem is referred to geographical axes (North, East, Nadir), for an homogeneous medium, γ_i, SV_i and SH_i can be given in terms of azimuth ϕ and take-off angle i of the ray,

$$\gamma_1 = \text{sen } i \cos\phi$$
$$\gamma_2 = \text{sen } i \text{ sen }\phi \qquad (17.44)$$
$$\gamma_3 = \cos i$$

$$SV_1 = \cos i \cos\phi$$
$$SV_2 = \cos i \text{ sen }\phi \qquad (17.45)$$
$$SV_3 = -\text{sen } i$$

$$SH_1 = -\text{sen }\phi$$
$$SH_2 = \cos\phi \qquad (17.46)$$
$$SH_3 = 0$$

Displacements u_P, u_{SV} and u_{SH} can finally be given in terms of i and ϕ (Fig. 17.6) and components of the moment tensor, substituting equations (17.44) to (17.46) in equations (17.41) to (17.43),

17.5. DISPLACEMENTS DUE TO A POINT SOURCE

According to (17.10), displacements due to a point source can be expressed by a time convolution of the moment tensor with derivatives of Green function,

$$u_i(x_n,t) = \int_{-\infty}^{\infty} M_{kj}(\tau)\, G_{ik,j}(t - \tau)\, d\tau \qquad (17.37)$$

Derivatives of Green function for P and S waves in the far field for an infinite, homogeneous isotropic medium are given by (16.74) and (16.75). For P waves, time convolution according to (16.76) is given by,

$$\int_{-\infty}^{\infty} M_{ij}(\tau)\, \dot{\delta}(t - \frac{r}{\alpha} - \tau)\, d\tau = \dot{M}_{ij}(t - \frac{r}{\alpha}) \qquad (17.38)$$

If we separate the modulus and time dependence from orientation in the form $M_{ij}(t) = M_0(t)\, m_{ij}$, according to (16.77) and (16.78), we obtain elastic displacements of P and S waves in the far field,

$$u_k^P = \frac{\dot{M}_0(t - \frac{r}{\alpha})}{4\pi\rho\alpha^3 r}\, \gamma_i \gamma_j \gamma_k\, m_{ij} \qquad (17.39)$$

$$u_k^S = \frac{\dot{M}_0(t - \frac{r}{\beta})}{4\pi\rho\beta^3 r}\, (\delta_{ik} - \gamma_i \gamma_k)\gamma_j\, m_{ij} \qquad (17.40)$$

Elastic displacements depend on time derivative of moment or moment rate. Displacement for P waves and for SV and SH components of S waves are, (calling A and B the factors in (17.39) and (17.40)), The source represented by DC + CLVD corresponds to a shear fracture in which, during the rupture process, the shear modulus in the focal region changes suddenly. This separation represents the best solution that maximize the DC part of the source.

In conclusion, a seismic point source of general type can be represented by the moment tensor. This source may involve changes in volume, shear fracture and sudden changes in rigidity at the source, and thus be separated in the form,

$$\mathbf{M} = \mathbf{M}^{0} + \mathbf{M}^{DC} + \mathbf{M}^{CLVD} \qquad (17.36)$$

This partition separates shear fracture, considered as the standard model for the source of earthquakes, from other effects that may be also present. The isotropic part is in many problems assumed to be zero as a previous condition. Deviatoric source is and formed by the sum DC + CLVD. Deviation from a pure DC is sometimes represented by $\delta = |\sigma_3/\sigma_1|$, greatest and least eigenvalues. For pure DC, $\delta = 1$.

When the moment tensor is obtained from observations, presence of non-DC components may be due to errors in observations or in propagation effects that have not been taken into account, rather than from the source itself. There is always a certain amount of ambiguity between effects that are due to the source and to propagation. Perfect separation of these two effects is not always possible.

don't impose any condition and result for some earthquake sourcesin the presence of certain amounts of non isotropic and non double-couple components. For this reason, is convenient to separate the moment tensor in three parts, one isotropic, corresponding to changes in volume, one of pure shear fracture or double couple (DC) and a third that may be of different kinds (Strelitz, 1989). This analysis is called partition or separation of the moment tensor and can be expressed by,

$$\mathbf{M} = \mathbf{M}^{0} + \mathbf{M}^{DC} + \mathbf{M}^{R} \qquad (17.33)$$

The isotropic part (17.28) has been already defined. Partition of the deviatoric part $(\mathbf{M}^{DC} + \mathbf{M}^{R})$ can be made in several ways. The simplest is to separate this part into two DC, major and minor. To do this we take into account that for a deviatoric tensor $\sigma_2 = -\sigma_1 - \sigma_3$, and obtain,

$$\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & -\sigma_1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\sigma_3 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} \qquad (17.34)$$

The two DC have different orientation, the major DC with moment $M_0 = \sigma_1$ and the minor with $M_0 = \sigma_3$.

A more efficient separation is that proposed by Randall and Knopoff (1970),

$$\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(\sigma_1-\sigma_3) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2}(\sigma_1-\sigma_3) \end{pmatrix} + \begin{pmatrix} -\sigma_2/2 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & -\sigma_2/2 \end{pmatrix} \qquad (17.35)$$

As before $\sigma_2 = -\sigma_1 - \sigma_3$. The first term is a DC source. The second is called a compensated linear vector dipole (CLVD). ITs physical meaning is a sudden change in the shear modulus in a direction normal to the fault plane, without changes in volume.

pressure and tension axes, **P** and **T**. The third axis corresponding to zero eigenvalue is the null axis **B**. In terms of unit vectors **P** and **T**, the moment tensor is given by,

$$M_{ij} = M_0 \,(T_i T_j - P_i P_j) \qquad (17.32)$$

This result is analogous to that found in section 16.1, regarding the equivalence of a double couple with pressure and tensions forces at 45 degrees from the couples.

17.4 TYPES OF SOURCES AND SEPARATION OF THE MOMENT TENSOR

We have already said that the moment tensor represents a very general type of source. The analysis of its eigenvalues indicates, in each case, the type of source. The most general case corresponds to three different eigenvalues $\sigma_1 \neq \sigma_2 \neq \sigma_3$, whose sum is not zero, $\sigma_1 + \sigma_2 + \sigma_3 \neq 0$. Then, the source has changes in volume and after separation of the isotropic part (17.29), the deviatoric part is of a general type and no necessarily a shear fracture or double couple.

If $\sigma_1 = \sigma_2 = \sigma_3$, as we have seen, the source is an isotropic expansion or contraction depending on the sign. In each case $\sigma_1 + \sigma_2 + \sigma_3$ represents the increase or decrease in volume. For positive sign the source represents an explosion.

For sources without net volume changes, $\sigma_1 + \sigma_2 + \sigma_3 = 0$, moment tensor is purely deviatoric. This conditions is often impose in earthquake sources. In this case, only two of the eigenvalues are independent, since $\sigma_2 = - \sigma_1 - \sigma_3$.

For a shear fracture or double couple source, the moment tensor is deviatoric and must satisfy the condition $\sigma_3 = - \sigma_1$ and $\sigma_2 = 0$.

Earthquake sources are supposed to be shear fractures or nearly so. However, this may not be always the case and presence of changes in volume cannot be completely ruled out. Methods of inversion of moment tensor from observations

$$\sigma_0 = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) \qquad (17.28)$$

If we subtract this from tensor M_{ij}, we obtain the deviatoric tensor M'_{ij} whose sum of the diagonal elements is always zero and doesn't include changes in volume,

$$M'_{ij} = M_{ij} - \delta_{ij}\, \sigma_0 \, . \qquad (17.29)$$

According to (17.29), the moment tensor can be separated into two tensors, one isotropic, $M^0_{ij} = \delta_{ij}\, \sigma_0$ and the other deviatoric M'_{ij},

$$M_{ij} = M^0_{ij} + M'_{ij} \qquad (17.30)$$

Changes in volume are, thus separated, from other parts of the moment tensor. Moment tensor that represent an explosive source (17.19) is purely isotropic and that for a shear fracture (17.22) purely deviatoric.

If we represent the moment tensor for an explosive source referred to its principal axis we obtain the same result as in (17.19). An explosive source is purely isotropic and any reference system is equivalent to the principal axes. For the shear fracture of (17.22) (fault plane normal to x_3 and slip in x_1 direction), the eigenvalues of matrix (17.22) are 1, -1, 0. The eigenvectors are found by substitution in (17.25) resulting for $\sigma = 1$, $(1/\sqrt{2}, 0, 1/\sqrt{2})$ and for $\sigma = -1$, $(1/\sqrt{2}, 0, -1/\sqrt{2})$.The tensor referred to its principal axes is,

$$M_{ij} = M_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad (17.31)$$

This tensor represent two linear dipoles of positive and negative forces or tension and pressure forces along the principal axes, that is, in (x_1, x_3) plane at 45 degrees from the direction of slip. Thus, for a shear fracture, eigenvectors corresponding to eigenvalues σ_1 and σ_3 define the principal axes of stress or

17.3. EIGENVALUES AND EIGENVECTORS

As we saw in the discussion of stress and strain tensors (Section 2.1), we can also make an eigenvalue and eigenvector analysis of the moment tensor. Since this tensor is symmetric its eigenvalues are real and its eigenvectors mutually orthogonal. They satisfy the equation,

$$(M_{ij} - \delta_{ij}\, \sigma)\, \nu_k = 0 \qquad (17.25)$$

Where the three eigenvalues σ_1, σ_2, σ_3 are the roots of the cubic equation resulting from putting equal to zero the determinant of (17.25). Substituting each eigenvalue in (17.25) we obtain the three eigenvectors ν_k^1, ν_k^2, ν_k^3. which form the principal axes. In reference to these axes, the moment tensor have the form,

$$M_{ij} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}. \qquad (17.26)$$

In this system, moment tensor is formed by three linear dipoles in the direction of the principal axes and thus represent the principal stresses. If we order the eigenvalues $\sigma_1 > \sigma_2 > \sigma_3$, then, σ_1 corresponds to the greatest stress, σ_3 to the least stress, and σ_2 to the intermediate stress. The sum of the elements of the principal diagonal is the first invariant of the tensor and has the same value for any reference system,

$$M_{11} + M_{22} + M_{33} = \sigma_1 + \sigma_2 + \sigma_3 \qquad (17.27)$$

This sum represents the change in volume, as we saw for the explosive source. Thus we can define the isotropic part of the moment tensor as, The sum of the principal diagonal is null, indicating that there is no change in volume.

Unit vectors **n** and **l** referred to geographical reference system can be written in terms of angles θ_n, ϕ_n and θ_l, ϕ_l, respectively (Fig. 17.4). By substitution in (17.21), expressions for the components of **n**, according to (16.85), (16.86) and (16.87) and similarly for **l**, we obtain for the six components of the normalize moment tensor,

$$
\begin{aligned}
m_{11} &= 2\,\text{sen}\theta_n \cos\phi_n \,\text{sen}\theta_l \cos\phi_l \\
m_{22} &= 2\,\text{sen}\theta_n \,\text{sen}\phi_n \,\text{sen}\theta_l \,\text{sen}\phi_l \\
m_{33} &= 2\cos\theta_n \cos\theta_l \\
m_{12} &= \text{sen}\theta_l \cos\phi_l \,\text{sen}\theta_n \,\text{sen}\phi_n + \text{sen}\theta_l \,\text{sen}\phi_l \,\text{sen}\theta_n \cos\phi_n \\
m_{13} &= \text{sen}\theta_l \cos\phi_l \cos\theta_n + \cos\theta_l \,\text{sen}\theta_n \cos\phi_n \\
m_{23} &= \text{sen}\theta_l \,\text{sen}\phi_l \cos\theta_n + \text{sen}\phi_n \,\text{sen}\theta_n \cos\theta_l
\end{aligned}
\qquad (17.23)
$$

As we have seen, a shear fracture can be also specified by angles ϕ, δ and λ (Section 16.6). Using the relation between ϕ, δ, λ and n_i and l_i equations (16.99) to (16.104), from (17.21), the components of the moment tensor are (Fig. 17.5),

$$
\begin{aligned}
m_{11} &= -\,\text{sen}\delta \cos\lambda \,\text{sen}2\phi - \text{sen}2\delta \,\text{sen}^2\phi \,\text{sen}\lambda \\
m_{22} &= \text{sen}\delta \cos\lambda \,\text{sen}2\phi - \text{sen}2\delta \cos^2\phi \,\text{sen}\lambda \\
m_{33} &= \text{sen}2\delta \,\text{sen}\lambda \\
m_{12} &= \text{sen}\delta \cos\lambda \cos2\phi + 1/2\,\text{sen}2\delta \,\text{sen}2\phi \,\text{sen}\lambda \\
m_{13} &= -\,\text{sen}\lambda \,\text{sen}\phi \cos2\delta - \cos\delta \cos\lambda \cos\phi \\
m_{23} &= \cos\phi \,\text{sen}\lambda \cos2\delta - \cos\delta \cos\lambda \,\text{sen}\phi
\end{aligned}
\qquad (17.24)
$$

In (17.23), expressions are symmetric with respect to **n** and **l** and don't imply a selection of the fault plane from the two possible planes. In (17.24), equations are related to the orientation of motion on the selected fault plane (Fig. 17.5). Naturally, the result is the same for values of ϕ, δ, λ, of the second plane.

An explosive source may be considered as an expansion along the three coordinates axes. This situation is represented by linear dipoles (**l** and **n** in the same direction) along each axis, that is, (1,0,0), (0,1,0) and (0,0,1). The moment tensor is the sum of the three and using (17.18) we obtain (Fig. 17.3a),

$$m_{ij} = K\,\Delta u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (17.19)$$

Where $K = \lambda + 2/3\ \mu$ is the bulk modulus (2.22). The sum of elements of the principal diagonal gives the volume increase per unit volume.

$$m_{11} + m_{22} + m_{33} = 3\,K\,\Delta u \qquad (17.20)$$

Shear fracture

In a shear fracture slip, Δ**u** is along the fault plane, that is, **n** and **l** are perpendicular. Using equation (17.18) and the definition of M_0 after integration over the source surface of area S, the moment tensor for a point source is,

$$M_{ij} = M_0(l_i n_j + l_j n_i) \qquad (17.21)$$

For a particular case when the fault plane is plane (x_1, x_2), that is, **n** = (0,0,1) and the slip in the x_1 direction, **l** = (1,0,0), (Fig. 17.3b), we obtain,

$$M_{ij} = M_0 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad (17.22)$$

$$M_{11} = M_{xx} = M_{\theta\theta}$$

$$M_{22} = M_{yy} = M_{\phi\phi}$$

$$M_{33} = M_{zz} = M_{rr}$$

$$M_{12} = M_{xy} = - M_{\theta\phi}$$

$$M_{13} = M_{xz} = M_{r\theta}$$

$$M_{23} = M_{yz} = - M_{r\phi}$$

17.2. MOMENT TENSOR AND ELASTIC DISLOCATIONS

If we compare equations (17.9) and (16.15), we can define the moment tensor density corresponding to a dislocation with slip $\Delta \mathbf{u}$ on a surface Σ of normal $\mathbf{n}$,

$$m_{ij} = C_{ijkl} \, \Delta u_k \, n_l \tag{17.16}$$

and for an isotropic medium,

$$m_{ij} = \lambda \, n_k \Delta u_k \, \delta_{ij} + \mu(\Delta u_i n_j + \Delta u_j n_i) \tag{17.17}$$

If the slip direction is given by unit vector $\mathbf{l}$ equation [illegible] becomes,

$$m_{ij} = \Delta u \, [\lambda \, l_k n_k \, \delta_{ij} + \mu(l_i n_j + l_j n_i)] \tag{17.18}$$

From this expression we can find the moment tensor for [illegible] types of sources, specifying orientations of n_i and l_i.

Explosive source

Moment tensor density m_{ij} corresponds to the first non zero term in Taylor's expansion of (17.12), and thus is called the first order moment tensor. This term is associated to the first derivatives of Green's functions. We can also derive moment tensors of higher order that are associated to higher derivatives of Green functions. For example, the second order moment tensor, third term in (17.12), which represents its variations with space.

According to (17.15), components of m_{ij} correspond to force couples or dipoles. Components m_{11}, m_{22} and m_{33} are linear dipoles without moment, that is, the arm is in the same direction as the forces. The other components have the arm perpendicular to the forces and are couples with moment (Fig. 17.1). Condition of zero net moment implies that the tensor is symmetric $m_{ij} = m_{ji}$, couples with opposite moment must be equal. We have seen that Green's function represents displacements due to impulsive forces, their derivatives, represent displacements due to couples or dipoles of impulsive forces. In consequence, according to equations (17.8) and (17.9), elastic displacements are given by convolution of distributions of dipoles or couples of forces representing the source (moment tensor) with displacements due to couples of impulsive forces (derivatives of Green's function).

Components of moment tensor are expressed with relation to a coordinate system of reference, usually, the geographic system, with origin at the focus of the earthquake. For example, Cartesian coordinate system (x_1, x_2, x_3) or (x, y, z), positive in direction North, East, Nadir (Fig. 17.2b) or also North, West, Zenith. Other system also used is referred to geocentric spherical coordinates of the focus (r, θ, ϕ) where r, is in the radial direction, θ geocentric colatitude and ϕ geocentric longitude. In the focus a Cartesian coordinate system is formed with unit vectors $\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_\phi$, (in direction of positive increments of r, θ, ϕ). This system have positive axes in direction Zenith, South, East (Fig. 17.2a). Correspondence of the six components of the moment tensor in the three systems is, The physical meaning of the moment tensor can be understood in relation to the equivalent body forces. According to (16.7), elastic displacements are given by,

$$u_i(x_s,t) = \int_{-\infty}^{\infty} d\tau \int_{V_0} F_k(\xi_s,\tau) G_{ik}(x_s,t;\xi_s,\tau) dV \qquad (17.11)$$

If we make a Taylor's expansion of G_{ik} around the origin, $\xi_k = 0$, the first three terms are,

$$G_{ik}(\xi_s) = G_{ik}(0) + \xi_s \frac{\partial G_{ik}}{\partial \xi_s} + \frac{1}{2} \xi_n \xi_s \frac{\partial^2 G_{ik}}{\partial \xi_n \partial \xi_s} + \ldots \qquad (17.12)$$

Taking only the first two terms and substituting in (17.11), the first term is zero by the condition that the sum of internal forces must be zero,

$$\int_{V_0} F_k(0)\ G_{ik}(0,\mathbf{x})\ dV = 0 \qquad (17.13)$$

Therefore, we obtain,

$$u_i = \int_{-\infty}^{\infty} d\tau \int_{V_0} F_k \xi_j\ G_{ik,j}\ dV \qquad (17.14)$$

By comparison of (17.14) with (17.8), we find,

$$m_{jk} = \xi_j F_k \qquad (17.15)$$

$$u_i = \int_{-\infty}^{\infty} m_{kj} \, G_{kj} \, d\tau + \int_{-\infty}^{\infty} d\tau \int_{V_0} m_{kj} \, G_{ik,j} \, dV$$

In the absence of external forces and torques, the sum of all internal forces and moments are null, then by an appropriate chose of the origin of coordinates, $m_{kj} G_{kj} = 0$, and we obtain,

$$u_i = \int_{-\infty}^{\infty} d\tau \int_{V_0} m_{kj} \, G_{ik,j} \, dV \tag{17.8}$$

If the moment tensor is defined only on a surface Σ, we use m_{ij}, the moment tensor density per unit surface,and we write (17.8) as a surface integral,

$$u_i = \int_{-\infty}^{\infty} d\tau \int_{\Sigma} m_{kj} \, G_{ik,j} \, dS \tag{17.9}$$

Equations (17.8) and (17.9) show that elastic displacements outside the focal region can be derived from the seismic moment tensor and the derivatives of Green's function integrated over the focal region (V_0 or Σ). Since we have not specified its form, m_{ij} can represent a very general type of source. It corresponds to any system of internal body forces according to (17.6), provided the net effect of their sum and the sum of their moments are null. The moment tensor is, thus, a very convenient form to represent the source of an earthquake in a general way.

For a point source, equations (17.8) and (17.9) can be written in a compact form using an asterisk to express time convolution,

$$u_i = M_{kj} * G_{ik,j} \tag{17.10}$$

displacements depend on slip velocity $\Delta\dot{u}$. For this reason, time dependence of slip velocity is often also called STF. For the first two models (16.108 and (16.109) we obtain,

$$\Delta\dot{u}(t) = \Delta V\ \delta(t) \tag{16.111}$$

$$\Delta\dot{u}(t) = \Delta V\ [H(t) - H(t - \tau)] \tag{16.112}$$

In these two models for $t = 0$, slip velocity jumps instantaneously from 0 to its maximum value ΔV (Fig. 16.22a, b). In the first, slip velocity is an impulse and in the second it has a duration τ with constant value. More realistic is to define a STF with slip velocity that increases from zero to its maximum value and then decreases to zero after a time τ. A model that satisfies these conditions is a triangular function (Fig. 16.22c),

$$\Delta\dot{u} = \begin{cases} 0 & t < 0 \\ \Delta V\,\dfrac{2t}{\tau}\,; & 0 \le t \le \dfrac{\tau}{2} \\ \Delta V\,\dfrac{2(\tau - t)}{\tau}\,; & \dfrac{\tau}{2} < t \le \tau \\ 0 & t > \tau \end{cases} \tag{16.113}$$

Slip velocity increases linearly from zero at $t = 0$, to reach its maximum value (ΔV) at $t = \tau/2$ and then decreases to zero for $t = \tau$. In the first part of the process, slip acceleration (Δ) is positive and in the second negative. If we want to increases duration of the source process we can use a STF of trapezoidal form (Fig. 16.22d). In this case, slip velocity maintains its maximum value during a certain time before decreasing to zero value at $t = \tau$.

The models of STF we have mentioned represent simple sources consisting of a single event. A complex source can be represented by a STF consisting of several triangles or trapezoids of different height. In this way we represent with a point source a mechanism that has several accelerations ($\Delta > 0$), decelerations ($\Delta < 0$) and stops ($\Delta u^{\cdot} = 0$), during the total process of fracture (Fig. 16.23).

16.7 EQUIVALENCE BETWEEN FORCES AND DISLOCATIONS

We have seen that the source of earthquakes can be represented by systems of forces (16.7) or by displacement dislocations