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STUDY OF THERMAL CONDUCTIVITY AND MOISTURE CONTENT OF LOOSE MATERIALS (SOIL)

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Study of Thermal Conductivity And Moisture Content Of Loose Materials (Soil)

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ABSTRACT

Assuming a regular geometry of finite-spaced spherical particles (λ_2) a theory for the effective thermal conductivity (λ_e) of all kind of two phase materials is developed. The flux modification is carried out by considering the effective neighbouring interactions in the solution of Poisson's equation. The theory is extended to estimate the λ_e of loose and granular materials assuming an effective continuous media approximation. The theory is applied to moist and dry (loose) two phase systems considering dispersion of solid phase in effective continuous media formed by moist air and solid phase. A comparison of calculated values of λ_e with the reported experimental results shows a good agreement.

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1. INTRODUCTION

The effective properties of heterogeneous multi-phase media has been estimated in the literature [1-10] using three different methods i.e. method of generalized function [1-4], the method of multipole expansion [5-8] and volume averaging approach [9,10]. The generalized function technique uses perturbation expansion of locally varying functions, the multipole expansion method considers the sources distributed periodically in a continuous media and the volume averaging method applies the property of statistical homogeneity and local heterogeneity of multiphase media. However, these approaches are applicable mostly to composites and material mixtures [10-17] assuming either a random or a periodic geometry. Further the lattice-type periodic model formed by finite-spaced, point-spherical and interacting solid particles [18] has been explored to estimate the λ_e of all kind of two phase materials. However, the common loose materials like beads, sand, soil etc., have a variable porosity ranging from .3 to .7. The λ_e of such loose materials has been widely measured but the theoretical investigations [9,14,15,18] are disappointing. The continuous medium at such large porosity is contributed by both the phases, solid phase and air phase. We, therefore, in the present endeavour consider an effective continuous media [19] composed by equal volume fraction of solid and gas phases. By allowing small dispersions of air, solid or vapour phase in the effective continuous media we produce the porosity of dry or moist soil.

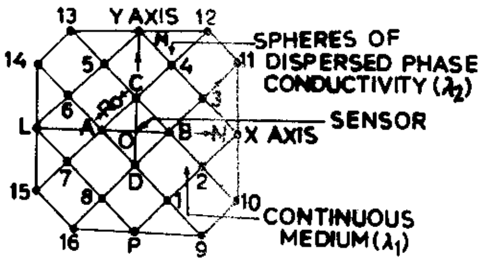

13
Y AXIS
12
14
SPHERES OF
DISPERSED PHASE
CONDUCTIVITY (λ_2)
5
C
4
11
6
3
SENSOR
L
A
O
B
N
X AXIS
7
D
2
15
8
1
10
CONTINUOUS
MEDIUM (λ_1)
16
P
9

Fig.1

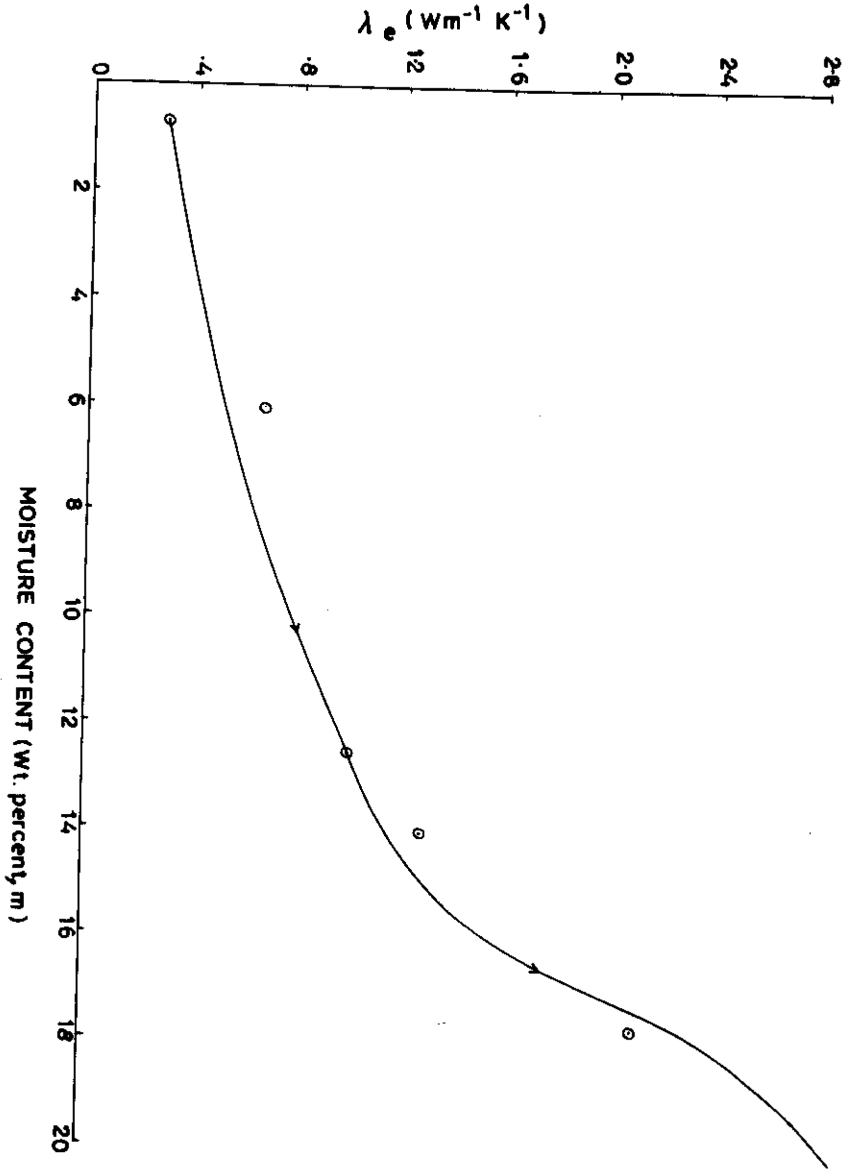
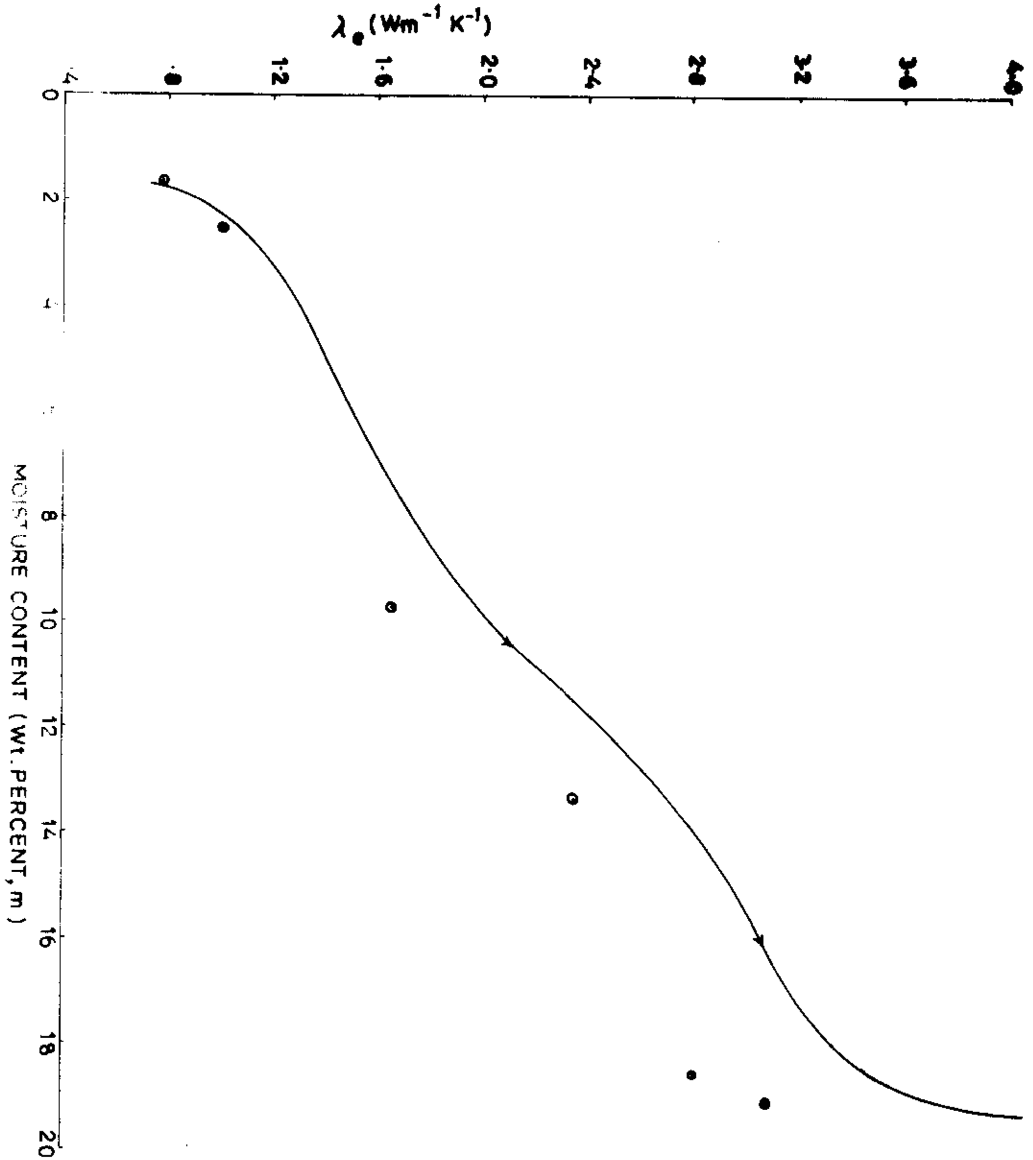

λ_e (Wm^{-1} K^{-1})
0
·4
·8
1·2
1·6
2·0
2·4
2·8
2
4
6
8
10
12
14
16
18
20
MOISTURE CONTENT (Wt. percent, m)

Fig.3



λe (Wm-1 K-1)
1·2
1·6
2·0
2·4
3·2
0
2
8
10
12
14
16
18
20
MOISTURE CONTENT (Wt. PERCENT, m)

2. THEORY

2.1. Effective Thermal Conductivity Of Two Phase Medium.

We consider a steady line source in a cylindrical container filled with a two phase medium and assume that there is a homogeneous cubic dispersion of finite spaced, point spherical and interacting soil particles of phase λ_2 in the continuous phase (air) λ_1, within the two phase sample. We consider a position dependent conductivity function $\lambda(\vec{R})$ where

$$\lambda(\vec{R}) = \lambda_1 + \delta(\vec{R}, \vec{r}_i)(\lambda_2 - \lambda_1)$$

such that

$$\delta(\vec{R},\vec{r}_i) = 1 \text{ for } |\vec{R}-\vec{r}_i| < a_i \qquad (1)$$
$$= 0 \text{ for } |\vec{R}-\vec{r}_i| > a_i$$

Here a_i is the radius of point spheres. The flux $\vec{J}$ and potential Φ at point $\vec{R}$ satisfy

$$\vec{J}(\vec{R}) = -\lambda(\vec{R}) \vec{\nabla} \Phi(\vec{R}) \qquad (2)$$

In steady state div $\vec{J} \to 0$. Using eq. (1) for $\lambda(\vec{R})$ in eq. (2) and applying steady state condition the div $\vec{J} = 0$ transforms into a Poisson equation [18]. Using Green function method the solution of Poisson equation is given by

$$\Phi(\vec{R}) = \Phi_0(\vec{R}') + \frac{1}{4\pi\lambda_1} \int_V \frac{1}{|\vec{R}'-\vec{R}|} \vec{\nabla} \cdot \left\{\delta(\vec{R}',\vec{r}_i)(\lambda_2-\lambda_1) \vec{\nabla}\Phi(\vec{R}')\right\} dV \qquad (3)$$

Considering only nonvanishing part of the solution (3) evaluated over the considered cubic geometry round the line source (fig. 1) yields a field distribution at the source [18] given as

$$\vec{E}(0) = \vec{E}_0(0) + \sum_i \frac{\lambda_2-\lambda_1}{4\pi\lambda_1} \int_s \vec{\nabla}\left\{\frac{1}{\vec{r}_i} [\vec{\nabla}\Phi(\vec{r}_i).\vec{ds}]\right\} \qquad (4)$$

Considering the relevant interactions the field modification $\langle d\vec{E} \rangle = \langle \vec{E} - \vec{E}_0 \rangle$ is estimated directly from the point spheres of phase λ_2 situated in X-Y plane surrounding the sensor. Using volume averaging approach the effective thermal conductivity λ_e is given by the eq. (2) as

$$\lambda_e = \frac{\langle \vec{J} \rangle}{\langle \vec{E}_0 \rangle \mp \langle d\vec{E} \rangle} \qquad (5)$$

where $\langle d\vec{E} \rangle$ is change in field because of dispersion (as given by eq. (4)). However, the initial field at the source before any dispersion is made is $\vec{E}_0$. Hence

$$\langle \vec{J} \rangle = \lambda_1 \vec{E}_0 \qquad (6)$$

Using eqs. (5) and (6) one finds

$$\lambda_e = \frac{\lambda_1}{\left[1 \mp \frac{\langle d\vec{E} \rangle}{\langle \vec{E}_0 \rangle}\right]} \qquad \begin{array}{l} -\text{ ve for } \lambda_2 > \lambda_1 \\ +\text{ ve for } \lambda_2 < \lambda_1 \end{array}$$

As $\frac{\lambda_{air}}{\lambda_{solid}}$ in soil is almost zero, the field modification

$\langle d\vec{E}\rangle$ is limited to the overlapping contribution from few nearest neighbours. Summing the eq. (4) upto fourth nearest neighbours [18] we find

$$\lambda_e = \lambda_1 \left\{ 1 \mp 3.844 \left(\frac{\lambda_2 - \lambda_1}{\lambda_2 + 2\lambda_1}\right) \psi^{2/3} \right\}^{-1} \qquad (7)$$

when $\frac{\langle dE\rangle}{\langle E_o\rangle}$ is less than unity eq. (7) may be written as

$$\lambda_e = \lambda_1 \left\{ 1 \pm 3.844 \left(\frac{\lambda_2 - \lambda_1}{\lambda_2 + 2\lambda_1}\right) \psi^{2/3} \right\} \qquad (8)$$

+ for $\lambda_2 > \lambda_1$

− for $\lambda_2 < \lambda_1$

Expression (8) is applicable at small dispersions of phase λ_2. At large dispersions eq. (7) should be used. When ψ is small, λ_2 is solid and λ_1 is air eq. (8) is written as [18]

$$\lambda_e = \lambda_a \left\{ 1 + 3.844\, \psi_s^{2/3} \right\} \qquad (9) \quad (.5 > \psi_s > 0)$$

However, when λ_2 is air and λ_1 is solid eq. (8) transforms into

$$\lambda_e = \lambda_s \left\{ 1 - 1.545\, \psi_a^{2/3} \right\} \qquad (10) \quad (.5 > \psi_a > 0)$$

where a and s stand for air and solid phases.

2.2. Thermal Conductivity of Effective Continuous Media.

As eqs. (9) and (10) are valid at small dispersions, we consider small n successive dispersions each of value $\delta\psi_a$ of air phase in continuous solid phase (λ_2). Under the limit $n\delta\psi_a \to 0.5$ the resulting media would be effective continuous media [19]. The thermal conductivity of this medium following eq. (10) is

$$\lambda_{ec} = \lambda_s \left(1 - 1.545\, \delta\psi_a^{2/3}\right)^n \quad (\text{for } n\delta\psi_a = 0.5) \qquad (11)$$

λ_{ec} can be also estimated by making n successive dispersions each of value $\delta\psi_s$ in the continuous air phase. Under the limit $n\delta\psi_s \to 0.5$ eq. (9) leads to

$$\lambda_{ec} = \lambda_a \left(1 + 3.844\, \delta\psi_s^{2/3}\right)^n \quad \text{for } (n\delta\psi_s = 0.5) \qquad (12)$$

The self consistency of eqs. (11) and (12) for $\delta\psi_s = \delta\psi_a = \delta\psi$ leads to

$$\lambda_{ec}^2 = \lambda_a\lambda_s \left\{ 1 + 2.299\, \delta\psi^{2/3} - 5.939\, (\delta\psi^{2/3})^2 \right\}^n \qquad (13)$$

For the rapid convergence of series expansion of eq. (13) we need that n should lie in the vicinity of unity [19]. Thus eq. (13) for n > 1 yields

$$\lambda_{ee}^2 \geq \lambda_a\lambda_s \left\{ 1 + 2.99\, (n\delta\psi^{2/3}) - 5.939\, \frac{(n\delta\psi^{2/3})^2}{n} \right\} \qquad (14)$$

Averaging the function $n\delta\psi^{2/3}$ for n > 1 in the region n = 1 to n = 1.5 we find

$$\langle n\delta\psi^{2/3}\rangle = .676763 \qquad (\text{for } n\delta\psi \to 0.5)$$

This on substitution in eq. (14) yields,

$$\lambda_{ec} > 0.6132\, (\lambda_a\lambda_s)^{1/2} \qquad (15)$$

2.3. Application To Loose Materials.

The loose two phase system is formed by allowing small dispersions of solid or gas phase (ξ_s or ξ_a) in the effective continuous media to produce a porosity range of .3 to .7. The λ_e of loose two phase system [19] for such small dispersions is given through eqs. (9) and (10) as

$$\lambda_e = \lambda_{ec}\,(1 + 3.844\,\xi_s^{2/3}) \qquad (\xi_s = \psi_s - 0.5) \qquad (16a)$$

and

$$\lambda_e = \lambda_{ec}\,(1 - 1.545\,\xi_a^{2/3}) \qquad (\xi_a = \psi_a - 0.5) \qquad (16b)$$

2.4. Application To Moist Loose Materials.

The addition of moisture in a loose material replaces the air in void space of the sample. The thermal conductivity of void when the sample is completely dry is that of dry air. A gradual addition of moisture content in the sample increase the thermal conductivity of void air. The process is similar to the dispersion of water vapour in a continuous air media. The thermal conductivity of the moist air (λ_{ma}) in void space is given through eqs. (7) and (9) as

$$\lambda_{ma} = \lambda_a \left\{1 - 3.844\left(\frac{\lambda_w - \lambda_a}{\lambda_w + 2\lambda_a}\right)\psi_{ma}^{2/3}\right\}^{-1} \qquad (18a)$$

$$(\text{for } \psi_{ma} \ll 0.5)$$

At saturation the air in void space is completely replaced by water and thus λ_{ma} tends to λ_w. Here ψ_{ma} is the volume

fraction of moisture with respect to void volume (air). If ψ_m be the volume fraction of moisture and ψ_a that of air (in air saturated sample) the ψ_{ma} is given as

$$\psi_{ma} = [\psi_m / \psi_a]$$

where $\psi_m = [m/M]\ \psi_a$

Therefore $\psi_{ma} = (m/M)$.

When $(m/M) \to 1$, $\lambda_{ma} = \lambda_w \left\{1 + 3.844\left(\frac{\lambda_a - \lambda_w}{\lambda_a + 2\lambda_w}\right)(1 - \psi_{ma})^{2/3}\right\}^{-1}$ (18b)

m and M represent the varying moisture content and the moisture content at saturation (by weight percent) respectively.

Thus the thermal conductivity of effective continuous media changes with a change in the thermal conductivity of void air. Following eqs. (15) and (19) we find

$$\lambda_{ec} \geq .6132\,(\lambda_{ma}\lambda_s)^{1/2} \qquad (19)$$

The volume fraction of solid phase remains constant even upto saturation. When $\psi_s > 0.5$, we assume a small dispersion of solid phase equivalent to $\xi_s = \psi_s - 0.5$ in the effective continuous media formed by the moist air and solid phase. Following eq. (10) the effective thermal conductivity of the sample is given by

$$\lambda_e = \lambda_{ec}\left\{1 - 3.844\left(\frac{\lambda_s - \lambda_{ec}}{\lambda_s + 2\lambda_{ec}}\right)\xi_s^{2/3}\right\}^{-1} \qquad (20)$$

3. COMPARISON WITH EXPERIMENTAL RESULTS AND DISCUSSIONS

The effective thermal conductivity of loose two phase materials is estimated using relevant expressions in Table 1. We find that the estimated λ_e values using eqs.(16a) and (16b) are better adopted to experimental results as compared to the estimations carried using any other expression. Here in most of the cases the estimated and reported values of λ_e differ (systems 2-7, 9-11) with in 0-5 percent. The average deviation is about $\pm$ 4.5 percent.

Estimations for λ_e of moist samples are performed using two set of datas for moist sand. The estimation of λ_e for moist samples requires the calculation of λ_{ma} and λ_{ee} prior to the calculation of λ_e. We have calculated the values of λ_e using eqs. (18) to (20), for moist Indian desert sand[20] and moist Italian sand[21]. The variation in λ_e with moisture content is plotted in figures 2 and 3. In case of Indian desert sand the theoretical curve represents nearly a mean graph of experimental results. While in case of Italian sand the estimated values are little larger as compared to the reported experimental results. The deviation in case of Italian sand is about 16 per cent.

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Figure Captions

FIG 1. A lattice plane of dispersed spheres around the sensor in X-Y plane.

FIG 2. Variation in the effective thermal conductivity of Italian sand[21] with moisture content. Theoretical λ_e curve →— Experimental points ⊙ .

FIG 3. Variation in the effective thermal conductivity of Rajasthan Desert sand with moisture content [20] . Theoretical λ_e curve →— Experimental points ⊙ .

Table 1. Comparison of calculated and experimental values of λ_e ($Wm^{-1}K^{-1}$) for various kind of loose materials.

System	Loose material system	λ_s	λ_g	ψ_g	λ_e using Maxwell formula	Lichtnecker formula	Kumar et al's formula	Present expressions (16a) & (16b).	λ_e (Experimental)
1	Zirconia powder/Air[23]	1.998	0.021	0.53	0.073	0.178	0.134	0.115	0.119
2	Zirconia powder/Air[24]	1.998	0.0297	0.42	0.141	0.341	0.258	0.254	0.229
3	Glass beads/Air[25]	1.200	0.0275	0.35	0.174	0.321	0.247	0.231	0.220
4	Miami silt loam/Air[26]	2.93	0.0234	0.448	0.072	0.746	0.254	0.222	0.221
5	Dune sand/Air[27]	3.344	0.026	0.325	1.74	0.515	0.403	0.346	0.387
				0.3876	1.73	0.509	0.391	0.342	0.375
				0.4052	1.67	0.467	0.357	0.336	0.336
				0.4297	1.59	0.415	0.316	0.299	0.312
				0.4394	1.56	0.396	0.301	0.288	0.302
				0.4502	1.52	0.376	0.285	0.275	0.289
				0.460	1.50	0.358	0.279	0.262	0.274
6	Dune sand/Air[28]	3.344	0.026	0.480	1.43	0.325	0.244	0.232	0.239
				0.485	1.41	0.317	0.238	0.222	0.220
7	Mud powder[28]	5.70	0.026	0.496	2.32	0.393	0.295	0.259	0.256
8	Brick sand/Air[28]	2.95	0.026	0.450	0.659	0.295	0.214	0.196	[illegible]
9	Concrete (dune)/air[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	Cement (loose)/Air[29]	2.60	0.026	0.56	0.087	0.197	0.148	0.121	[illegible]
11	Lime (loose)/Air[28]	0.75	0.026	0.73	0.051	0.064	0.051	0.068	[illegible]