

INTERNATIONAL ATOMIC ENERGY AGENCY  
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION



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SMR/147-45

COLLEGE ON SOIL PHYSICS

15 April - 3 May 1985

COLLOQUIUM ON ENERGY FLUX AT THE SOIL ATMOSPHERE INTERFACE

6 - 10 May 1985

HEAT TRANSPORT IN SOIL

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## 7 Heat transport in soil

### 7.1 INTRODUCTION

An important aspect of the behaviour and use of soils is their thermal regime. This regime is characterized to a large extent by the temperature. Many processes occurring in soils are strongly influenced by temperature. This is true especially for biological processes, such as germination of seeds, plant growth, root development and activity, microbial activity, etc. Physical and chemical processes influenced by temperature are frost heaving, weathering, decomposition of organic matter, etc.

Soil temperatures are determined by the transport processes of heat within the soil and by exchange of heat between the soil and the atmosphere. There are three basically different processes whereby heat can be transported: conduction, convection and radiation. Conduction of heat occurs by transmission of thermal energy of motion from one microscopic particle to another. Transport of heat by a fluid in motion is called convection. Heat convection with accompanying phase changes can increase heat transfer tremendously. This is especially true for water, which has very high values of latent heat of condensation/evaporation and freezing/melting. Radiation is transfer of thermal energy from a body to its surroundings by electromagnetic waves. Thus, in contrast to conduction and convection, radiation can occur through a vacuum.

Transport of heat within soils can occur by conduction and by convection, with or without latent heat transport. Heat conduction is governed by the thermal soil properties, volumic heat capacity and heat conductivity. The thermal exchange processes at the soil surface are dominated by the meteorological conditions and occur by radiation, conduction and convection, with or without phase changes. The thermal soil properties are strongly dependent on water content. A complete description of the thermal regime of soils is very complex and falls outside the scope of this text. Only the main aspects will be presented.

### 7.2 THERMAL SOIL PROPERTIES

#### 7.2.1 Volumic heat capacity

The change of heat content of soil divided by volume and by change of temperature is called volumic heat capacity,  $C_h$ . It is expressed in  $\text{J m}^{-3} \text{K}^{-1}$  (Section 4.3). The volumic heat capacity of a soil can be obtained by summing the contributions of the different soil components:

$$C_h = \sum_i C_{h,i} \phi_i = \sum_i \rho_i c_i \phi_i \quad (7.1)$$

where  $C_{h,i}$  and  $c_i$  are the volumic and specific heat capacities, and  $\phi_i$  and  $\rho_i$  are the volume fraction and density of component  $i$ . The volumic heat capacities of various soil components are presented in Table 7.1.

Table 7.1 Thermal properties of soil components at 10 °C

	$c$ $\text{kJ kg}^{-1} \text{K}^{-1}$	$C_h$ $\text{MJ m}^{-3} \text{K}^{-1}$	$\lambda$ $\text{J m}^{-1} \text{s}^{-1} \text{K}^{-1}$ ( $\text{W m}^{-1} \text{K}^{-1}$ )
quartz	0.76	2.0	8.8
clay minerals	0.73	2.0	2.9
organic matter	1.8	2.5	0.25
water	4.2	4.2	0.57
ice (0 °C)	2.1	1.9	2.18
air (saturated with water vapour)	1.0	0.0013	0.025

#### 7.2.2 Heat conductivity

The heat conductivity,  $\lambda$ , of a soil is defined as the heat flux density by conduction through the soil divided by the temperature gradient. It is expressed in  $\text{J m}^{-1} \text{s}^{-1} \text{K}^{-1}$  (Section 4.3). The heat conductivity has no simple relationship with the heat conductivities of the individual soil components, because the conduction of heat takes place through all kinds of sequences of the conducting materials, in series and in parallel. The value of  $\lambda$  depends highly on the way in which the best conducting mineral particles are interconnected by the less conducting water phase and are separated by the poorly conducting gas phase.

Figure 7.1 shows the heat conductivity of a soil as a function of volume fraction of liquid. At very low water contents,  $\lambda$  is generally smaller than  $0.5 \text{ J m}^{-1} \text{s}^{-1} \text{K}^{-1}$ . The heat transport then takes place mainly through the narrow points of contact between the soil particles. The contribution of the soil air is very small due to the very low heat conductivity of air. A small increase in water content of a dry soil causes only a modest increase of  $\lambda$ , because this water forms thin films around the soil particles. Further increases in water content cause a sharp increase of the heat conductivity, because water has a much higher heat conductivity than air and this water collects around the contact points between the soil particles. Water around contact points forms very effective 'bridges' for conduction of heat. Upon further increase of the water content, the value of  $\lambda$  increases ever more gradually, because the conducting cross-sectional area of the water 'bridges' increases ever more slowly. The maximum value of  $\lambda$  is reached at water saturation. For mineral soils this value is generally between 1.5 and  $2.0 \text{ J m}^{-1} \text{s}^{-1} \text{K}^{-1}$ .

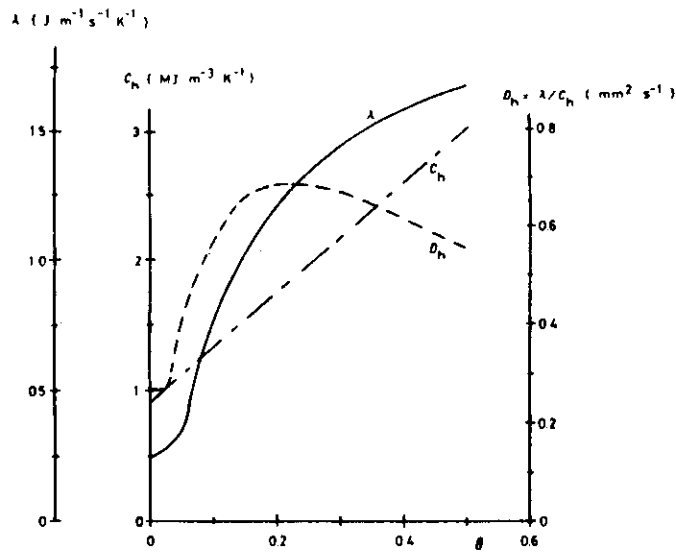


Figure 7.1 Thermal soil properties as a function of volume fraction of liquid.

### 7.3 HEAT CONDUCTION IN DRY SOIL

In Section 4.3, one-dimensional conduction of heat was described with the equation

$$\frac{\partial T}{\partial t} = \frac{\lambda}{C_h} \frac{\partial^2 T}{\partial s^2} \quad (4.31)$$

This equation is valid if  $\lambda$  is independent of position, which is usually not true. The following sections contain solutions of this equation for three simple situations, all involving dry soil for which Equation 4.31 is valid.

#### 7.3.1 Steady heat conduction

With steady heat conduction the temperature, per definition, does not change in time, i.e.  $\partial T / \partial t = 0$ . Since  $\lambda$  and  $C_h$  are constant and non-zero for dry soil, Equation 4.31 reduces to:

$$\frac{d}{ds} \left( \frac{dT}{ds} \right) = 0 \quad (7.2)$$

Integrating twice yields:

$$T = C_1 s + C_2 \quad (7.3)$$

The integration constants,  $C_1$  and  $C_2$ , can be determined if the temperature is known for two values of  $s$ .

#### 7.3.2 Cyclic variation of surface temperature

In nature, soil temperatures fluctuate due to more or less cyclic variations of the surface temperature,  $T_{0,t}$ . These variations are the diurnal (day-night) and the annual (summer-winter) cycles. Both can be approximated by a sinus function (Figure 7.2). The mathematical derivation of the solution of Equation 4.31 for these cyclic boundary conditions will not be given here. The solution for a soil with a constant 'average' temperature,  $T_{av}$ , is:

$$T_{s,t} = T_{av} + A_0 \exp(-s/d) \sin(\omega t - s/d) \quad (7.4)$$

where  $T_{s,t}$  is the temperature at depth  $s$  and time  $t$ ,  $A_0$  is the amplitude of the sinusoidal temperature variation at the soil surface,  $\omega$  is the angular frequency of the temperature variation (wave),  $t$  is time and  $d$  is the damping depth. The angular frequency is

$$\omega = \frac{2\pi}{t_c} \quad (7.5)$$

where  $t_c$  is the time needed to complete one cycle of the wave.

The sinusoidal temperature fluctuation at the surface penetrates into the soil by heat conduction. However, due to the finite heat diffusivity of the soil, the amplitude of the heat wave decreases with soil depth (Figure 7.2). In Equation 7.4, this phenomenon is accounted for by the introduction of the so-called damping depth,  $d$ , which is the depth where the amplitude of the temperature fluctuation has decreased to  $A_0/e \approx 0.37 \times A_0$ .

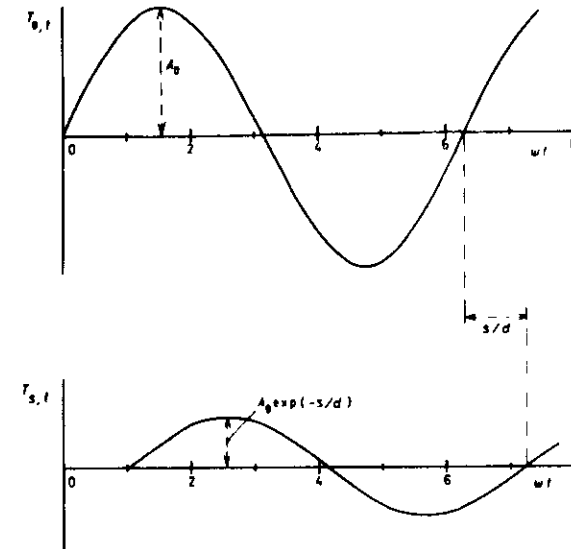


Figure 7.2 Cyclic variation of temperature at the soil surface and at depth  $s$ .

The damping depth depends on the thermal properties of the soil and on the angular frequency of the temperature variation according to

$$d = \sqrt{\frac{2\lambda}{\omega C_h}} = \sqrt{\frac{2D_h}{\omega}} \quad (7.6)$$

The finite heat diffusivity of the soil also causes a phase shift between the heat wave at the soil surface and at a certain depth  $s$  (Figure 7.2). This is represented in Equation 7.4 by the variable  $-s/d$  in the sinus function ( $-s/d$  is the phase angle).

### 7.3.3 Step-increase of surface temperature

If a soil profile has a uniform initial temperature ( $T = T_i$  for  $s \geq 0$  and  $t = 0$ ) and the surface temperature is suddenly increased and then maintained at that higher temperature ( $T = T_0$ , for  $s = 0$  and  $t > 0$ ), heat will penetrate into the soil from the surface. This so-called step-increase hardly occurs in nature; it is artificial. However, with the boundary conditions just stated, Equation 4.31 can be solved analytically. The solution is similar to that for infiltration of water in soil (Section 5.3.3.3 and Appendix 2):

$$\frac{T - T_i}{T_0 - T_i} = \operatorname{erfc}\left(\frac{s}{2\sqrt{(\lambda/C_h)t}}\right) \quad (7.7)$$

where  $\frac{T - T_i}{T_0 - T_i}$  is the temperature change ( $T - T_i$ ) at time  $t$  and distance  $s$  expressed as a fraction of the temperature change at the surface ( $T_0 - T_i$ ). Since  $T$  can vary only between  $T_i$  and  $T_0$ , it can easily be checked that this fraction can only vary between 0 and 1. It is, therefore, often called dimensionless temperature. The value of  $\operatorname{erfc}(u)$  for any positive value of  $u$  can be found from Figure 5.16. Consistent with the above conclusion,  $\operatorname{erfc}(u)$  varies between 0 and 1.

## 7.4 THE HEAT BALANCE OF THE SOIL SURFACE

As mentioned in Section 7.3, the temperature at the soil surface varies periodically. During daylight, most of the incoming direct and indirect short-wave radiation is absorbed by the soil surface, the remainder is reflected back into the atmosphere. During darkness as well as daylight, the soil surface loses energy by emitting long-wave heat radiation. The net radiation flux density,  $f_n$  ( $\text{J m}^{-2} \text{s}^{-1} = \text{W m}^{-2}$ ), is the difference between the unreflected incoming short-wave radiation and the outgoing long-wave heat radiation. Normally, during the day  $f_n > 0$ , and at night  $f_n < 0$ . The value of  $f_n$  can be measured directly or estimated from meteorological data.

The net radiation flux density is used for:

- evaporation of water at the soil surface ( $f_e$ )
- transpiration of water by the vegetative cover ( $f_t$ )
- heating of the soil ( $f_s$ )
- heating of the atmospheric air by conduction at the soil surface ( $f_a$ ).

The heat balance of the soil can thus be described by:

$$f_n = f_e + f_t + f_s + f_a \quad (7.8)$$

The distribution of  $f_n$  over  $f_e$ ,  $f_t$ ,  $f_s$  and  $f_a$  is important, because  $f_e$  and  $f_t$  determine the amount of water that can be evaporated, whereas  $f_s$  determines the soil temperature and, thus, the growth of crops.

Generally, it is not easy to calculate the distribution of  $f_n$  over the four terms in Equation 7.8, because the four processes are strongly interrelated. An estimate of this distribution is more easily made for the following two special situations:

- A rather dry, bare topsoil. Here, transpiration is absent and evaporation from the soil surface is negligible. Thus Equation 7.8 can be simplified to  $f_n = f_s + f_a$ . The distribution of  $f_n$  over  $f_s$  and  $f_a$  can then be estimated on the basis of calculations of heat conduction in dry soil and turbulent heat transport in the atmosphere. Such calculations yield, on average, a ratio of 1:1 between  $f_s$  and  $f_a$ . The exact value of this ratio depends on the thermal properties of the soil and the wind velocity.
- A rather wet soil. Here, evaporation of water consumes a significant part of the net radiation flux density. The soil acts mainly as a heat reservoir, absorbing heat during the day and discharging it again at night. As a result, the soil temperature changes little over a complete diurnal cycle, making  $f_s$  negligible over that interval. The net radiation flux density during one daily cycle thus will be distributed over  $f_e$ ,  $f_t$  and  $f_a$ . The relative magnitudes of  $f_e$ ,  $f_t$  and  $f_a$  can be estimated theoretically, but this will not be discussed here further.

## 7.5 THERMAL REGIME OF UPPER SOIL LAYERS

The temperature fluctuations in the soil profile resulting from the cyclic temperature variations at the soil surface were discussed in Section 7.3.2. The influence of  $\lambda$  and  $C_h$  on these fluctuations was also described. This insight, together with the heat balance concept, can be used to characterize the thermal regime of the upper soil layers.

In dry soil, evaporation consumes very little energy from the net radiation, leaving nearly all energy available for heating the soil. At the same time, the heat penetrates into the soil only slowly, due to the low values of the heat diffusivity and damping depth (Figure 7.1). This combination of circumstances causes a large accumulation of heat in the surface layer with a relatively low heat capacity, resulting in a rapid

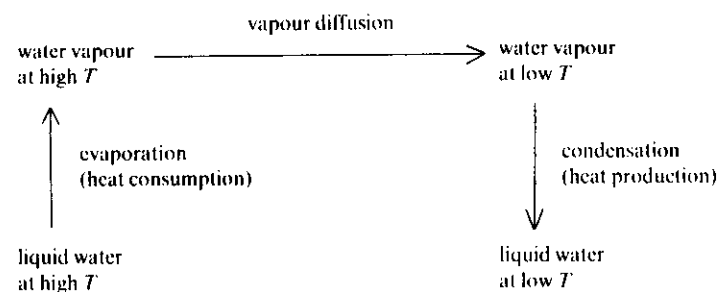
rise of the soil surface temperature during daytime. At night, the surface of a dry soil cools down more rapidly than a wet soil surface. Thus, a dry soil is subject to much stronger temperature variations in the upper layer than a wet soil. Due to the heat exchange between the soil and the atmosphere, the air layers above dry soil also will have large temperature fluctuations. This explains the large differences between day- and night-temperatures in deserts.

In temperate climates, dry soils will be warmer than wet soils during day-time. In early spring, this offers more favourable conditions for the germination of seeds and early development of crops. On the other hand, dry surfaces will cool down more strongly during the night and have a higher incidence of night frost. Thus, for a given climate, differences in thermal regime of the surface layers are determined to a large extent by differences in water content.

The surface layer of a wet soil requires much incoming radiation to heat up, because

- much of the incoming radiation is used for evaporation
- the remaining heat is distributed over a large depth, due to the large damping depth
- the heat remaining in the topsoil causes a relatively small increase of temperature, due to the high value of the heat capacity.

The latent heat of evaporation of water is high. Therefore, much heat is consumed where water vapour is formed and produced where it condenses. The net effect of this is an often large heat flux density in the direction of the vapour flux, as follows:



One might say that the effective heat conductivity has increased. This heat transport associated with the water vapour diffusion will eventually wipe out the temperature gradient, unless it is maintained by outside influences.

From: B.A. Kimball and R.D. Jackson. Soil heat flux. In: Modification of the aerial environment of plants. ASAE Monograph No. 2. USA. 1979.  
and W.R. van Wijk. Physics of plant environment. North-Holland Publishing Co. Amsterdam, 1963.

If the heat diffusivity  $D_h$  is relatively constant with depth and time, it can be computed from the ratio of the measured amplitudes  $A_1$  and  $A_2$  of a sinusoidal temperature wave at two depths  $z_1$  and  $z_2$  according to

$$D_h = (\omega/2) [(z_2 - z_1) / \ln(A_1/A_2)]^2$$

The velocity of penetration of the maximum (or minimum) of a sinusoidal temperature wave into a soil is

$$v_T = \sqrt{2\omega D_h}$$

Figure 3 and 4 show the diurnal and annual temperature wave for different depths.

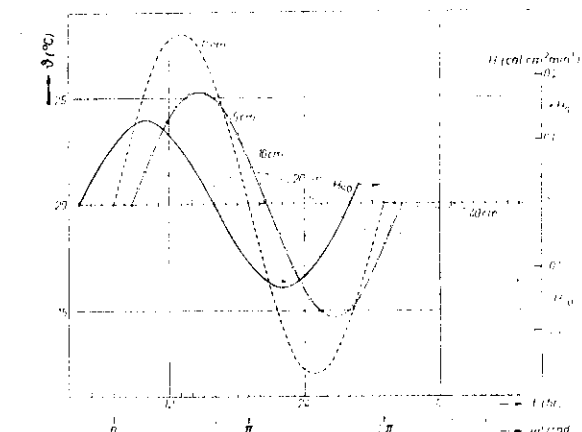


Fig. 3 Variation of temperature with depth for a sand soil. The full curve is the sensible heat flux density into the soil  $H_{s0}$  at the soil surface (right hand vertical scale). The dotted curves represent the temperatures at various depths (left hand vertical scale). A value of  $\lambda = 0.0042 \text{ cal cm}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{C}^{-1}$  and  $C = 0.5 \text{ cal cm}^{-3} \text{ } ^\circ\text{C}^{-1}$  has been assumed resulting in a damping depth  $D = 15.2 \text{ cm}$ .

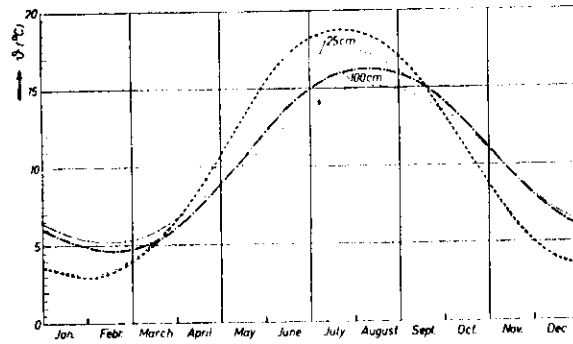


Fig. 4. Average annual soil temperature variations at two different depths for two stations, Wageningen and de Bilt, in the Netherlands.

----- } Wageningen  
----- } de Bilt

The heat flux density at the soil surface for a sinusoidal temperature fluctuation is:

$$f_h(o,t) = -\lambda(dT/dz)_{z=0} \\ = A_0 \sqrt{\omega \lambda C_h} \sin(\omega t + \pi/4)$$

Thus, the phase-angle of the heat flux density wave is  $\pi/4$  "ahead" of the temperature wave, as indicated by the solid curve in Figure 3. For a given heat flux density the amplitude of the temperature is inversely proportional to the value of  $\sqrt{\omega \lambda C_h}$ , the so-called thermal inertia,  $I_h$ , of the soil:

$$\frac{A_1}{A_2} = \frac{\sqrt{\omega \lambda_2 C_{h2}}}{\sqrt{\omega \lambda_1 C_{h1}}} = \frac{I_{h2}}{I_{h1}}$$

For the same heat flux density (meteorological conditions) at the soil surface the ratio between the amplitude of the surface temperature fluctuations of a homogeneous soil with and without a surface layer with different thermal properties is:

$$\frac{A_1}{A} = Y = \frac{I_h}{I_r}$$

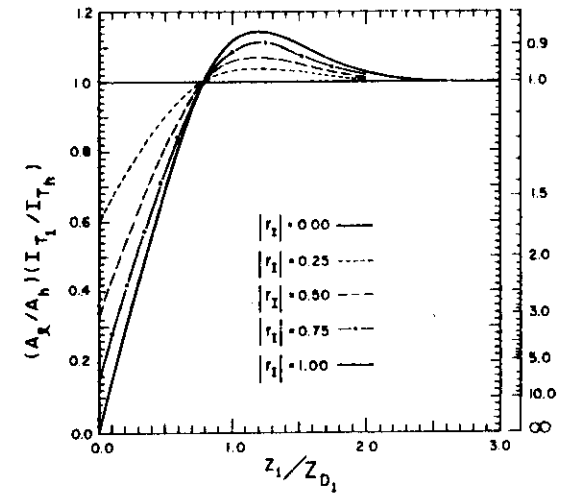


FIG. 5. Bracketed term from equation (3.4-30) for determining the ratio of surface temperature amplitude between layered and homogeneous soil. The axis is the ratio between the thickness of the upper layer,  $z$ , and its damping depth,  $Z_D$ .  $r_T$  is the ratio between the difference and the sum of the thermal inertias of the two layers,  $(I_T - I_{T1})/(I_T + I_{T1})$ . The left-hand ordinate scale is for negative  $r_T$  while the right-hand scale is for positive  $r_T$  (redrawn from van Wijk and Dijkshoof, 1963).

The parameter  $Y$  is plotted in Figure 5. This information can be used to analyse the influence of a dry or tilled surface layer, mulch, etc. on the thermal behavior of an otherwise homogeneous soil. For most field situations, the thermal inertia of the upper layer will be smaller than the inertia of the underlying soil, so  $r_T$  will usually be negative and then left ordinate will apply. For further details, see caption of Figure 5 and original publication in Van Wijk.

Below are tabulated thermal properties of a dry clay and a wet sand for the diurnal and annual cycle with  $\omega = 7.27 \times 10^{-5} \text{ s}^{-1}$  and  $\omega = 1.99 \times 10^{-7} \text{ s}^{-1}$ , respectively.

	Dry clay		Wet sand	
$C_h$ [MJm <sup>-3</sup> K <sup>-1</sup> ]	1.3		2.9	
$\lambda$ [Wm <sup>-1</sup> K <sup>-1</sup> ]	0.25		2.2	
$D_h$ [mm <sup>2</sup> s <sup>-1</sup> ]	0.19		0.76	
	diurnal	annual	diurnal	annual
$d$ [cm]	7.2	138	14.5	276
$v_T$ [cm h <sup>-1</sup> ]	1.9	0.094	3.8	0.20
$I_h$ [Wm <sup>-2</sup> K <sup>-1</sup> ]	4.9	0.25	21.5	1.1