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DRAINAGE OF AGRICULTURAL LANDS

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Because we are interested in the drainage of agricultural land and hence we consider drainage as the removal of excess subsurface water by means of conduits or other conveying devices; it follows that we are concerned with water tables, movement of water through soil, and the relationship that exist between water tables and crops.

In many areas both surface and subsurface drainage may be required.

Surface drainage is accomplished by open ditches and lateral drains.

Subsurface drainage is accomplished by a system of open ditches and buried tube drains into which water seeps by gravity. Water collected in drains is coveyed to a suitable outlet.

Drainage problems differ widely because of the varied nature of physical conditions and crops to be grown. Besides the crops the following factors have to be taken into account : soils, precipitation, topography.

Excessive soil water reduces the exchange of air between soil and atmosphere. Therefore, wet soil conditions are generally accompanied by O_2 deficiency. A considerable amount of O_2 is required in the soil for mineralization of nutrient elements from organic matter by microbiological activity. Deficient aeration reduces this microbiological activity, decreasing the rate at which NH_3^+ and NO_3^- are supplied. Consequently, a tendency towards N deficiency exists in waterlogged soils.

All biological processes are strongly influenced by temperature. Wet soils have a large heat capacity and considerable amounts of heat are required to raise their temperature. Therefore, wet soils are cold and crop growth starts later and is slower than in dryer soils.

The direct aim of drainage systems in humid regions is to lower the moisture content of the upper layers so air can penetrate more easily to the roots, and transport of CO_2 produced by roots, micro-organisms, and chemical reactions is facilitated. Lowering soil moisture content also results in a change in heat budget and higher soil temperatures. This change can be expected to occur in well-drained soils, especially in the spring.

Although the depth of the ground-water table has no direct influence on crop growth, it indirectly determines the prevailing moisture conditions and therefore has an influence on water supply, aeration conditions, and heat properties in soils.

Numerous laboratory and field experiments on the effect of water-table depth on crop yields have been conducted at various locations. The main reason for this possibly is that the water-table depth is easily determined compared to determining other soil properties such as aeration or thermal conductivity.

8.1 INTRODUCTION

Until recently, all over the world, the only common practice of controlling the water table was by a system of open ditches. In modern agriculture many of these systems have been, or are now being, replaced by pipe drains

In any system of drains one may distinguish between (Fig.1):
- field drains or field laterals, usually parallel drains whose function is to control the groundwater depth;
- collector drains, whose function is to collect water from the field drains and to transport it to the main drains;
- main drains, whose function is to transport the water out of the area.

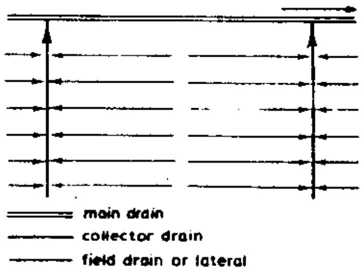

Fig.1. Drain functions.

There is not always a sharp distinction between the functions of the drains. For instance all field and collector drains also have a transport function, and all the collector and main drains also control the groundwater depth to some extent.

The discussion in this chapter will be restricted to parallel field drains. Figure 2 shows a cross-section of the laterals in Fig.1. The water table is usually curved, its elevation being highest midway between the drains. The factors which influence the height of the water table are:
- precipitation and other sources of recharge
- evaporation and other sources of discharge
- soil properties
- depth and spacing of the drains
- cross-sectional area of the drains
- water level in the drains

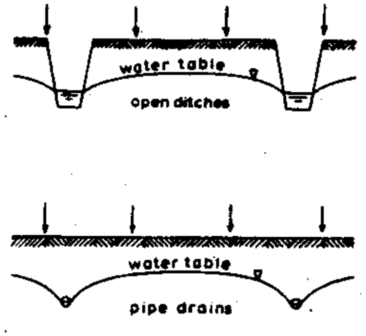

Fig.2. Cross-section of laterals showing a curved water table under influence of rainfall.

In this chapter the above factors are interrelated by drainage equations, based on two assumptions, viz.:
- two-dimensional flow, i.e. the flow is identical in any cross-section perpendicular to the drains;
- a uniform distribution of the recharge, steady or non-steady, over the area between the drains.

Most of the equations discussed in this chapter are moreover based on the Dupuit-Forchheimer assumptions . Consequently they have to be considered as approximate solutions only. Such approximate solutions, however, are generally accepted as having such a high degree of accuracy that their application in practice is completely justified.

A distinction is made between steady state and non-steady state drainage formulas. The steady state formulas (Sect.8.2) are derived under the assumption that the recharge intensity equals the drain discharge rate and consequently that the water table remains in position. The non-steady state drainage equations (Sect. 8.3) consider the fluctuations of the water table with time under influence of a non-steady recharge.

8.2 STEADY STATE DRAINAGE EQUATIONS

8.2.1 HORIZONTAL FLOW TO DITCHES REACHING AN IMPERVIOUS FLOOR

It is recalled from Chap.6, Vol.I that under the assumptions of one-dimensional horizontal flow, implying parallel and horizontal streamlines, the flow to vertically walled ditches reaching an impervious floor (Fig.3a) can be described by the so-called Donnan equation (DONNAN, 1946)

$$R = q = \frac{4K(H^2 - D^2)}{L^2} \qquad (1)$$

where

R = recharge rate per unit surface area (m/day)

q = drain discharge rate per unit surface area (m/day)

K = hydraulic conductivity of the soil (m/day)

H = height above the impervious floor of the groundwater table midway between two drains (m)

D = height above the impervious floor of the water level in the drains = thickness of aquifer below drain level (m)

L = drain spacing (m)

which has also been derived by HOOGHOUDT (1936).

Equation 1 may be rewritten as

$$q = \frac{4K(H+D)(H-D)}{L^2} \qquad (2)$$

Setting (Fig.3a) $h = H-D$ and $H+D = 2D+h$, where h is the watertable height above drain level at midpoint, i.e. the hydraulic head for subsurface flow into drains (m), Eq. 2 then changes into

$$q = \frac{8K(D+\frac{1}{2}h)h}{L^2} \qquad (3)$$

The factor $D+\frac{1}{2}h$ in Eq.3 can be considered to represent the average thickness of the soil layer through which the flow takes place (aquifer), symbolised by $\overline{D}$. Introducing $\overline{D}$ into Eq.3 yields

$$q = \frac{8K\overline{D}h}{L^2} \qquad (4)$$

where $K\overline{D}$ = transmissivity of the aquifer (m^2/day).

Equation 3 can be written as follows

$$q = \frac{8KDh + 4Kh^2}{L^2} \qquad (5)$$

Setting D = 0 gives

$$q = \frac{4Kh^2}{L^2} \qquad (6)$$

Equation 6 apparently represents the horizontal flow above drain level. This equation is known as the Rothe equation. It seems to have been derived as early as 1879 by Colding in Denmark.

If D is large compared with h, the second term in the numerator of the right hand side of Eq.5 can be neglected against the first term, giving

$$q = \frac{8KDh}{L^2} \qquad (7)$$

Equation 7 and the first term of Eq.5 apparently represent the horizontal flow below drain level.

The above considerations permit the conception of a two-layered soil with interface at drain level. Accordingly Eq.5 may be rewritten as

$$q = \frac{8K_bDh + 4K_ah^2}{L^2} \qquad (8)$$

where

K_a = hydraulic conductivity of the layer above drain level (m/day)

K_b = hydraulic conductivity of the layer below drain level (m/day)

8.2.2 PRINCIPLES OF THE HOOGHOUDT EQUATION

If the ditches do not reach the impervious floor, the flow lines will not be parallel and horizontal but will converge towards the drain (radial flow). In this region the flow system cannot be simplified to a flow field with parallel and horizontal streamlines without introducing large errors.

The radial flow causes a lengthening of the flow lines. This lengthening causes a more than proportional loss of hydraulic head since the flow velocity in the vicinity of the drains is larger than elsewhere in the flow region. Consequently, the elevation of the water table will be higher when the vertically walled ditches are replaced by pipe drains, the drain level remaining the same.

HOOGHOUDT (1940) derived a flow equation for the flow as presented in Fig.3b, in which the flow region is divided into a part with horizontal flow and a part with radial flow.

If the horizontal flow above drain level is neglected, the flow equation for a uniform soil reads

$$h = \frac{qL}{K} F_H \tag{9}$$

and

$$F_H = \frac{(L-D\sqrt{2})^2}{8DL} + \frac{1}{\pi} \ln \frac{D}{r_o\sqrt{2}} + f(D,L) \tag{10}$$

where

r_o = radius of the drains

$f(D,L)$ = a function of D and L, generally small compared with the other terms in Eq.10; it can therefore usually be ignored (LABYE, 1960).

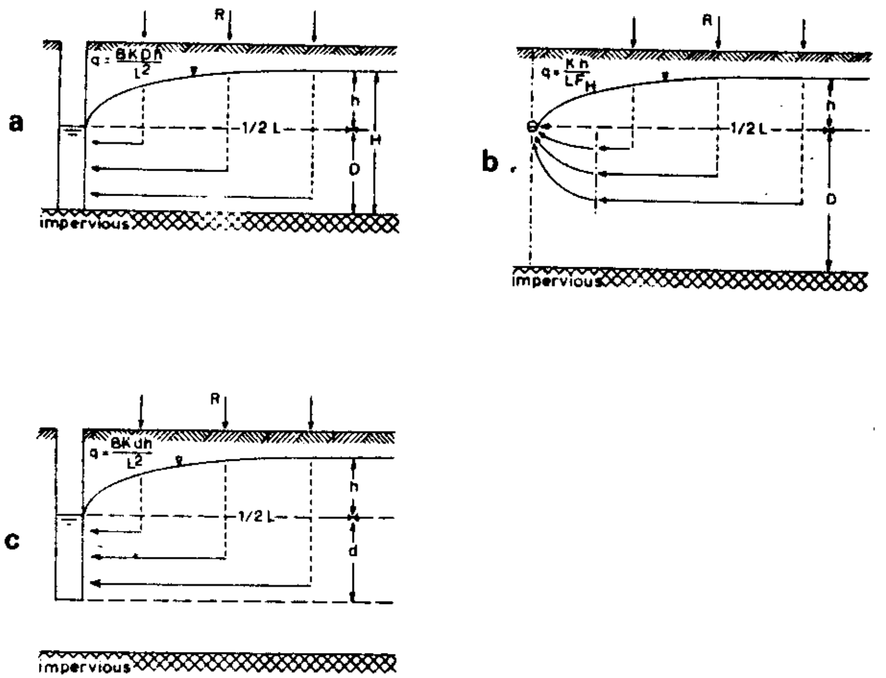

Fig.3. The concept of the equivalent depth to transform a combination of horizontal and radial flow into an equivalent horizontal flow.

The first term of the right hand member of Eq.10 pertains to horizontal flow, the second and the third term to radial flow.

Instead of working with Eqs.9 and 10, HOOGHOUDT considered it more practical to have a formula similar to the equations given in the previous section. To account for the extra resistance caused by the radial flow, he introduced a reduction of the depth D to a smaller equivalent depth d. By so doing, the flow pattern is replaced by a model with horizontal flow only (Fig.3c). If we consider only the flow below drain level, Eq.7 is reduced to

$$q = \frac{8Kdh}{L^2} \tag{11}$$

where $d < D$. This equation must be made equivalent to Eq.9. Solving the latter equation for q and equating the result with Eq.11 results in the equation for the equivalent depth

$$d = \frac{L}{8F_H} \tag{12}$$

The factor d is like F_H a function of L, D and r_o, as may be seen from Eqs.10 and 12. Values of d for r_o = 0.1 m and various values of L and D are presented in Table 1. For other drain diameters Fig.14 can be used, which will be explained in Sect.8.2.9.

In order to take radial flow into account the d-value can be introduced into all equations of Sect.8.2.1. When introduced in Eq.8 it yields

$$q = \frac{8K_b dh + 4K_a h^2}{L^2} \tag{13}$$

Equation 13 is called the Hooghoudt equation.

Discussion

In Eq.10 the first term in the right hand member pertains to the horizontal flow region. Comparison with Eq.7 proves that the horizontal flow is taken over a distance $L-D\sqrt{2}$ instead of L, and that the radial flow consequently is taken over a distance of $\frac{1}{2}D\sqrt{2}$ to both sides of the drains.

If we neglect f(D,L) in Eq.10 and set

$$F_h = \frac{(L-D\sqrt{2})^2}{8DL} \qquad (14)$$

and

$$F_r = \frac{1}{\pi} \ln \frac{D}{r_o\sqrt{2}} \qquad (15)$$

Eq.10 may be written as

$$F_H = F_h + F_r$$

Consequently Eq.9 changes into

$$h = \frac{qL}{K} F_h + \frac{qL}{K} F_r = h_h + h_r \qquad (16)$$

Thus the total hydraulic head is the sum of the hydraulic heads h_h and h_r required for horizontal and radial flow respectively.

Table 1. Values for the equivalent depth d of Hooghoudt (r_o = 0.1 m, D and L in m)

L→	5 m	7.5	10	15	20	25	30	35	40	45	50
D											
0.5 m	0.47	0.48	0.49	0.49	0.49	0.50	0.50				
0.75	0.60	0.65	0.69	0.71	0.73	0.74	0.75	0.75	0.75	0.76	0.76
1.00	0.67	0.75	0.80	0.86	0.89	0.91	0.93	0.94	0.96	0.96	0.96
1.25	0.70	0.82	0.89	1.00	1.05	1.09	1.12	1.13	1.14	1.14	1.15
1.50		0.88	0.97	1.11	1.19	[illegible]	1.28	1.31	1.34	1.35	1.36
1.75		0.91	1.02	1.20	1.30	1.39	1.45	1.49	1.52	1.55	1.57
2.00			1.08	[illegible]	[illegible]	1.5	1.57	1.62	1.66	1.70	1.72
2.25			1.13	1.34	1.50	1.69	1.69	1.76	1.81	1.84	1.86
2.50				1.38	1.57	1.69	1.79	1.87	1.94	1.99	2.02
2.75				1.42	1.63	1.76	1.88	1.98	2.05	2.12	2.18
3.00				1.45	1.67	1.83	1.97	2.08	2.16	2.23	2.29
3.25				1.48	1.71	1.88	2.04	2.16	2.26	2.35	2.42
3.50				1.50	1.75	1.93	2.11	2.24	2.35	2.45	2.54
3.75				1.52	1.78	1.97	2.17	2.31	2.44	2.54	[illegible]
4.00					1.81	2.02	2.22	2.37	2.51	2.62	2.71
4.50					1.85	2.08	2.31	2.50	2.63	2.76	2.87
5.00					1.88	2.15	2.38	2.58	2.75	2.89	3.02
5.50						2.20	2.43	2.65	2.84	3.00	3.15
6.00							2.48	2.70	2.92	3.09	3.26
7.00							2.54	2.81	3.03	3.24	3.43
8.00							2.57	2.85	3.13	3.35	3.56
9.00								2.89	3.18	3.43	3.66
10.00									3.23	3.48	3.74
∞	0.71	0.93	1.14	1.53	1.89	2.24	2.58	2.91	3.24	3.56	3.88

Table 1. (cont.)

L ⟶	50	75	80	85	90	100	150	200	250
D									
0.5	0.50								
1	0.96	0.97	0.97	0.97	0.98	0.98	0.99	0.99	0.99
2	1.72	1.80	1.82	1.82	1.83	1.85	1.00	1.92	1.94
3	2.29	2.49	2.52	2.54	2.56	2.60	2.72	2.70	2.83
4	2.71	3.04	3.08	3.12	3.16	3.24	3.46	3.58	3.66
5	3.02	3.49	3.55	3.61	3.67	3.78	4.12	4.31	4.43
6	3.23	3.85	3.93	4.00	4.08	4.23	4.70	4.97	5.15
7	3.43	4.14	4.23	4.33	4.42	4.62	5.22	5.57	5.81
8	3.56	4.38	4.49	4.61	4.72	4.95	5.68	6.13	6.43
9	3.66	4.57	4.70	4.82	4.95	5.23	6.09	6.63	7.00
10	3.74	4.74	4.89	5.04	5.18	5.47	6.45	7.09	7.53
12.5		5.02	5.20	5.38	5.56	5.92	7.20	8.06	8.68
15		5.20	5.40	5.60	5.80	6.25	7.77	8.84	9.64
17.5		5.30	5.53	5.76	5.99	6.44	8.20	9.47	10.4
20			5.62	5.87	6.12	6.60	8.54	9.97	11.1
25			5.74	5.96	6.20	6.79	8.99	10.7	12.1
30							9.27	11.3	12.9
35							9.44	11.6	13.4
40								11.8	13.8
45								12.0	13.8
50								12.1	14.3
60									14.6
∞	3.88	5.38	5.76	6.00	6.26	6.82	9.55	12.2	14.7

8.2.4 PRINCIPLES OF THE KIRKHAM EQUATION

KIRKHAM (1958) gives an analytical solution for a problem similar to Hooghoudt's, viz. two-dimensional flow, a regularly distributed rainfall over the area, and drains not reaching an impervious floor. If the flow above the drains is ignored, Kirkham's solution can be written in a form similar to Eq.9

$$h = \frac{qL}{K} F_K \qquad (17)$$

and

$$F_K = \frac{1}{\pi}\left[\ln \frac{L}{\pi r_o} + \sum_{n=1}^{\infty} \frac{1}{n} \left(\cos \frac{2n\pi r_o}{L} - \cos n\pi\right)\left(\coth \frac{2n\pi D}{L} - 1\right)\right] \qquad (18)$$

Values of F_K are given in Table 2. It is found that the F_K values of Kirkham are very close to the F_H values of Hooghoudt, so that both the Hooghoudt and the Kirkham equations give almost identical results (WESSELING, 1964).

Table 2. Values of F_K according to Toksöz and Kirkham.

L/D → $D/2r_o$	100	50	25	12.5	6.25	3.125	1.5625	0.78125
8192	-	-	-	-	-	-	-	2.654
4096	-	-	-	-	-	-	2.65	2.43
2048	-	-	-	-	-	2.66	2.43	2.21
1024	-	-	-	-	2.84	2.45	2.21	1.99
512	-	-	-	3.40	2.63	2.23	1.99	1.76
256	-	-	4.76	3.19	2.40	2.01	1.76	1.54
128	-	7.64	4.53	2.96	2.19	1.78	1.54	1.32
64	13.67	7.43	4.31	2.74	1.96	1.57	1.32	1.10
32	13.47	7.21	4.09	2.52	1.74	1.35	1.10	0.88
16	13.27	6.99	3.86	2.30	1.52	1.13	0.88	0.66
8	13.02	6.76	3.64	2.08	1.30	0.90	0.66	0.44
4	12.79	6.54	3.42	1.86	1.08	0.68	0.44	-
2	12.57	6.32	3.20	1.63	0.85	0.46	-	-
1	12.33	6.08	2.95	1.40	0.62	-	-	-
0.5	12.03	5.77	2.66	1.11	-	-	-	-
0.25	11.25	5.29	2.20	-	-	-	-	-

In the solution represented by Eq.17 the flow in the upper region has been neglected (Fig.8). In a later paper KIRKHAM (1960) reported that, if vertical flow is assumed in this region, the hydraulic head should be multiplied by $(1-q/K)^{-1}$. Since this term relates to the flow in the layer above drain level, the general equation for a two-layer problem is (WESSELING, 1964)

$$h = \frac{qL}{K_b} \frac{1}{1-q/K_a} F_K \qquad (19)$$

where K_a is the hydraulic conductivity above drain level and K_b below that drain level. The boundary between the two layers must, as in the Hooghoudt solution, coincide with the drain level (Fig.8).

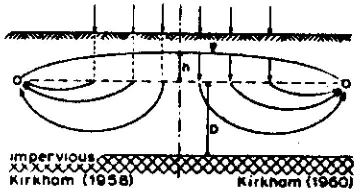

Fig.8.
Two-dimensional flow pattern according to the analytical solutions of KIRKHAM (1958, 1960).

8.2.6 PRINCIPLES AND APPLICATION OF THE DAGAN EQUATION

Analogous to the method of Hooghoudt, DAGAN (1964) thought the flow to be composed of a radial flow in the area between the drain and a distance $\frac{1}{4}D\sqrt{2}$ away from the drain, and an intermediate, though mainly horizontal, flow in the area between the $\frac{1}{4}D\sqrt{2}$ plane and the midplane between the drains.
The Dagan equation, in a form similar to the Hooghoudt and Kirkham equations, reads

$$h = \frac{qL}{K} F_D \qquad (20)$$

The expression for F_D is

$$F_D = \frac{1}{4}\left(\frac{L}{2D} - \beta\right) \qquad (21)$$

where $\beta = \frac{2}{\pi} \ln(2\cosh \frac{\pi r_o}{D} - 2)$ (22)

In Fig.10 the term β has been presented as a function of $\frac{\pi r_o}{D}$. Note that β-values are negative. With the aid of this figure the application of Dagan's equation is easy.

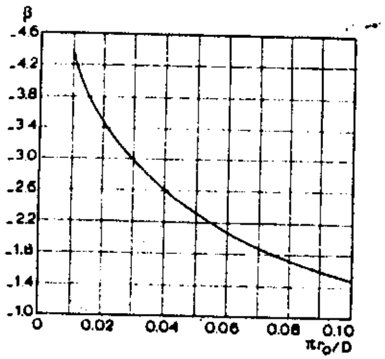

Fig.10.
Nomograph for the determination of β in the Dagan equation (DAGAN, 1964).

8.3 NON-STEADY STATE DRAINAGE EQUATIONS

8.3.1 INTRODUCTION

In areas with periodic irrigations or high intensity rainfall, the assumption of a steady recharge is no longer justified. Under these conditions non-steady state solutions of the flow problem must be applied. Non-steady state solutions are indispensable when actual, non-steady water table elevations and drain discharges, as obtained from field data, must be evaluated (Chap.26, Vol. III).

It is recalled from Chap.6 (Vol.I) that the differential equation for non-steady state flow, as derived on the basis of the Dupuit-Forchheimer assumption, can be written as

$$KD \frac{\partial^2 h}{\partial x^2} = \mu \frac{\partial h}{\partial t} - R \qquad (32a)$$

or, when the recharge rate R equals zero

$$KD \frac{\partial^2 h}{\partial x^2} = \mu \frac{\partial h}{\partial t} \qquad (32b)$$

where

KD = transmissivity of the aquifer (m^2/day)
R = recharge rate per unit surface area (m/day)
h = hydraulic head as a function of x and t (m)
x = horizontal distance from a reference point, e.g. ditch (m)
t = time (days)
μ = drainable pore space (dimensionless, m/m)

8.3.2 PRINCIPLES OF THE GLOVER-DUMM EQUATION

DUMM (1954) used a solution for Eq.32b found by Glover who assumed an initial horizontal groundwater table at a certain height above the drain level. The solution describes the lowering of the groundwater table - which does not remain horizontal - as a function of time, place, drain spacing and soil properties. The initial horizontal water table is thought to have been the result of an instantaneous rise caused by rainfall or irrigation, which instantaneously recharged the groundwater. Later DUMM (1960) assumed that the initial water table is not completely flat but has the shape of a fourth degree parabola, which resulted in a slightly different formula.

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The main part of the text is reproduced from the lecture notes of the International Course on Land Drainage, Wageningen, The Netherlands.

Figure 16 depicts the condition before and just after an instantaneous rise of a horizontal groundwater table. The initial and boundary conditions for which Eq. 32b must be solved are:

$t = 0,\qquad h = R_i/\mu = h_o,\qquad 0 < x < L$ (initial horizontal groundwater table)

$t > 0,\qquad h = 0,\qquad x = 0,\ x = L$ (water in drains remains at zero level = drain level)

where

R_i = instantaneous recharge per unit surface area (m)

h_o = height above drain level of the initial horizontal water table.

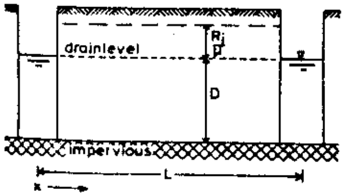

Fig.16.
Boundary conditions for the Glover-Dumm equation with initial horizontal water table.

The solution of Eq.32b for these conditions may be found in CARSLAW and JAEGER (1959)

$$h(x,t) = \frac{4h_o}{\pi} \sum_{n=1,-3,5,}^{\infty} \frac{1}{n} e^{-n^2\alpha t} \sin \frac{n\pi x}{L} \qquad (33)$$

where

$$\alpha = \frac{\pi^2 KD}{\mu L^2} \text{(reaction factor, day}^{-1}\text{)} \qquad (34)$$

For the height of the water table midway between the drains at any time t, $h_t = h(\frac{1}{2}L,t)$, one may substitute $x = \frac{1}{2}L$ into Eq.33 yielding

1) $$h_t = \frac{4}{\pi} h_o \sum_{n=1,-3,5,}^{\infty} \frac{1}{n} e^{-n^2\alpha t} \qquad (35)$$

Apparently the value of each term of Eq.35 decreases with increasing n. If $\alpha t > 0.2$ the second and next term will be comparatively small and may be neglected. Equation 35 then reduces to

$$h_t = \frac{4}{\pi} h_o e^{-\alpha t} = 1.27\ h_o e^{-\alpha t} \qquad (36)$$

2) Under the assumption of an initial water table having the shape of a <u>fourth degree parabola</u>, Eq.36 changes into (DUMM, 1960)

$$h_t = 1.16\ h_o e^{-\alpha t} \qquad (37)$$

The only difference between Eq.36 and Eq.37 is a change of the shape factor $\frac{4}{\pi} = 1.27$ in 1.16.

Substituting Eq.34 into Eq.37 and solving for L yields

$$L = \pi \left[\frac{KDt}{\mu}\right]^{\frac{1}{2}} \left[\ln 1.16 \frac{h_o}{h_t}\right]^{-\frac{1}{2}} \qquad (38)$$

which is called the Glover-Dumm equation.

As the Glover-Dumm equation does not take into account a radial resistance of flow towards drains not reaching an impermeable layer, the thickness of the aquifer D is often replaced by the d-value of Hooghoudt to account for the convergency of the flow in the vicinity of the drains. This substitution is justified since the flow paths for steady and non-steady flow may be considered at least similar, although not exactly identical.

Thus Eq.34 becomes

$$\alpha = \frac{\pi^2 Kd}{\mu L^2} \text{ (day}^{-1}\text{)} \qquad (39)$$

and Eq.38 changes into

$$L = \pi \left[\frac{Kdt}{\mu}\right]^{\frac{1}{2}} \left[\ln 1.16 \frac{h_o}{h_t}\right]^{-\frac{1}{2}} \qquad (40)$$

This may be called the modified Glover-Dumm equation.