IC/74/76
INTERNAL REPORT
(Limited distribution)

International Atomic Energy Agency

and

United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

T O P I C A L   M E E T I N G

ON THE PHYSICS OF COLLIDING BEAMS

20 - 22 June 1974

(SUMMARIES AND CONTRIBUTIONS)

MIRAMARE - TRIESTE

July 1974

THE BREAKDOWN OF SCALING IN ASYMPTOTICALLY FREE THEORIES

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I. THE ROAD TO ASYMPTOTIC FREEDOM.

- In QUANTUM Field Theory B.J. Scaling requires

  $\Rightarrow$ Asymptotic Freedom $\approx$ Strong Interactions "turn off" at short distances. (Callan & Gross)

- Asymptotic Freedom $\Rightarrow$ Non-Abelian Gauge Theories of Strong Int. (Coleman Gross)

- Gauge Theories are Asym. Free (Gross w/ CB; Politzer; 't Hooft)

  $\Rightarrow$ BJ Scaling with $\ln Q^2$ corrections + Parton Model at short distances.

II. Structure of Asym. Free Gauge Theories

- Choice of Gauge Group, Color
- Low Energy Crisis. Containment.

III. Deep Inelastic Scattering.

- Where is Asymptopia?
- Predictions for the Moments of the Structure Function
- Sum Rules. + Corrections.
- Behavior of $F(x, Q^2)$, Size of Scaling Deviations. Threshold Behavior.
- Extrapolation to $\rightarrow$ threshold $\rightarrow$ Hadronic Form Factor

  $\searrow$ Deep Limit

. The Road To Asymptotic Freedom.

Renormalizable Quantum Field Theory

Other approach have problems with unitarity or locality.

characterized by $\begin{cases} m \sim \text{masses} \\ g \sim \text{dimensionless coupling constants.} \end{cases}$

At short distances (large momenta) masses can be neglecte

But: dont get scaling (i.e. dimensional analysis) since
a) Hidden Scale: Inherent in defining g. (μ).
b) Scale or dimension of Operators depends on the interaction.

$$J_\mu(x)\, J_\nu(0) \underset{x\to 0}{\sim} \sum_N C_N(x^2, g)\, x^{\mu_1}\cdots x^{\mu_N}\, \mathcal{O}_{\mu\nu\mu_1\cdots\mu_N}(0)$$

$$\int_0^1 dx\, x^N F(x, q^2) \underset{q^2\to\infty}{=} \tilde{C}_N(q^2, g)\, \langle H | \mathcal{O}^N | H \rangle$$

$x = \frac{q^2}{2\nu}$ $\longrightarrow \left(\frac{1}{q^2}\right)^{\frac{1}{2}\gamma^N}$ γ^N = ANAMALOUS DIMENSION OF $\mathcal{O}_N$.

B.-J. Scaling: $\gamma^N = 0$

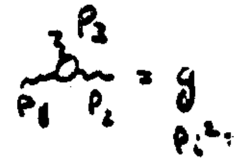
Parton Model: $\tilde{C}_N$ same structure as Free Field The of Quarks.

Renormalization Group.

Only scale present at large momenta (neglecting ma is renormalization scale parameter μ used to:

define: couplings: $g = g(\mu)$

scale of operators: $Z_\psi, Z_{O_N}, \cdots$



Change of μ (equivalent to a change of scale of all mom is equivalent to a change of g:

to a change of scale of all operators.

$$\boxed{\tilde{C}_N\left(\frac{q^2}{\mu^2}, g\right) = \tilde{C}_N\left(1, \bar{g}\left(\frac{q^2}{\mu^2}\right)\right) \exp \int_0^{\frac{1}{2}\ln\frac{q^2}{\mu^2}} \gamma_N(\bar{g}(t))\,dt.}$$

$\mu^2 \to q^2\mu^2.$

$g = g(\mu) \to \bar{g}(q^2)$

Dynamics at momenta $\approx q^2$ governed by $\bar{g}\left(\frac{q^2}{\mu^2}\right)$.

Change of $\bar{g}(q^2)$ dictated by renormalization gro equation:

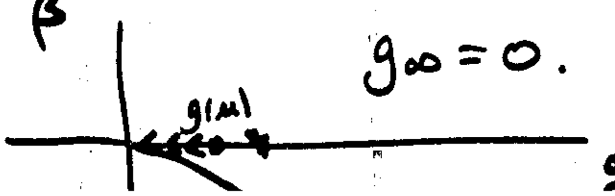
$t = \frac{1}{2}\ln\frac{q^2}{\mu^2}$ $\quad \boxed{\frac{d\bar{g}}{dt} = \beta(\bar{g}) = \frac{1}{2}b_0\bar{g}^3 + b_1\bar{g}^5 + \cdots}$

$$\bar{g}(t) \xrightarrow[t\to\infty]{} g_\infty \qquad \beta(g_\infty) = 0 \qquad \beta'(g_\infty) < 0$$

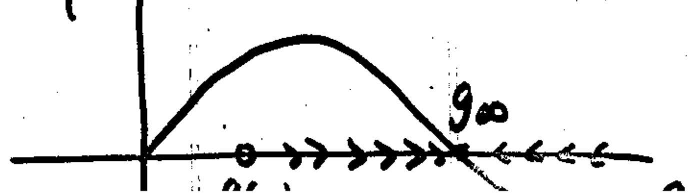
$q^2 \to \infty$

ASYMPTOTICALLY FREE

$g_\infty = 0.$



INFRA RED STABL



o: $\tilde{C}_N\left(\frac{q^2}{\mu^2}, g\right) \longrightarrow \tilde{C}_N(1, g_\infty)\left(\frac{\mu}{q}\right)^{\gamma_N(g_\infty)} e^{\int_0^{\ln q/\mu}[\gamma_N(\bar{g}(t)) - \gamma_N(g_\infty)]}$

n Asymptotically free $g_\infty \neq 0$ ≈ 1 : ~~Naive~~ Large Deviations From Parton Model. } Power deviations From B.J. Scaling. $\approx\left(\frac{\mu^2}{q^2}\right)^{1}$

less $g_\infty \approx$ very small $\frac{1}{10}$?)

symptotically Free $\longrightarrow \tilde{C}_N(1,0)\ \exp\int_0^{\ln q/\mu} \gamma_N(\bar{g}(t))dt$

$=0$

$(0)=0$

Free Field Theory Structure

Only Logarithmic Deviations From B.J. Scaling.

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jorken Scaling IMPLIES Asymptotic Freedom (Callan Gross)

i.e. $\gamma_N(g_\infty) = 0 \implies g_\infty = 0$. B.J Scaling + Parton model

: 1

Asymptotic Freedom $\implies$ Non-Abelian Gauge Theories of the Strong Int. (Coleman-Gross)

-Abelian Gauge Theories (Politzer Gross-Wilczek)

e. Yukawa, Scalar, Abelian Gauge theories $\neq$ Asym. Free

Charge is screened by vacuum polarization.

ED: At short distances there is less screening, so

MODELS OF THE STRONG INTERACTIONS

At short distances we want to see "free" quarks

$\Longrightarrow$ Gauge mesons must be neutral with respect to $SU(3) \times$

$\Longrightarrow$ MUST HAVE COLOR GAUGE GROUP

$$\Psi = \underbrace{\begin{pmatrix} p_1 & n_1 & \lambda_1 \\ p_2 & n_2 & \lambda_2 \\ & \vdots & \\ p_m & n_m & \lambda_m \end{pmatrix}}_{SU(3) \times SU(3)} \Big\} \text{ Strong Gauge Group } G$$

$\mathcal{L}$ invariant under $SU(3) \times SU(3) \times G$

E.g. $G = SU(3)'$: Red-white-Blue quarks.

$$\alpha = \frac{\bar{g}^2}{4\pi^2} \sim \frac{4}{9} \frac{1}{\ln Q^2/\mu^2}$$

+ Leading short distance Behavior of $SU(3) \times SU(3)$ Current

$\equiv$ FREE QUARK MODEL.

$$\frac{4}{9} = \frac{6}{11 C_2(G) - 4T(R)} \text{ - is crucial.}$$

Unless (F... α ... until Q^2 ...

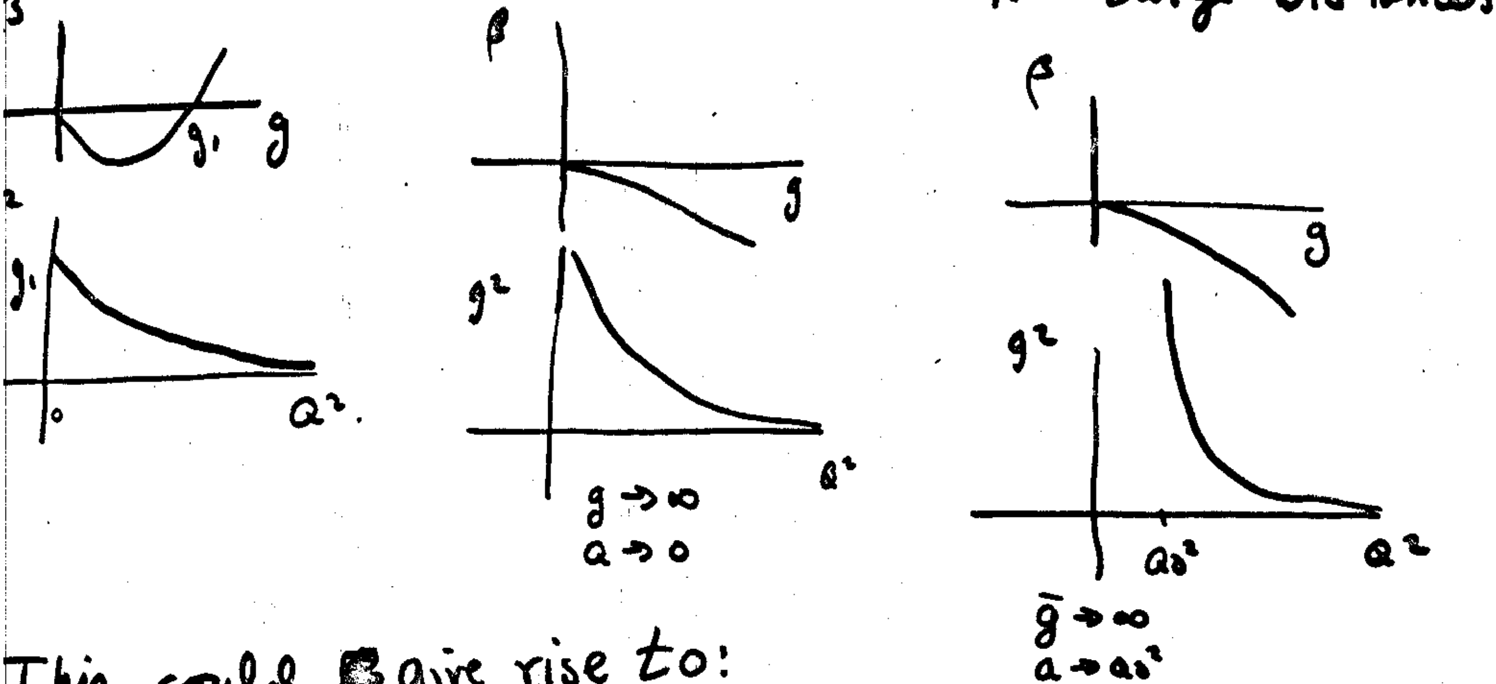
nce $\boxed{\Rightarrow \alpha(Q^2 = 10) = .2}$

IF INSTEAD of $4/9$ we had $9/4$ then:

$\alpha(Q^2 = 1000) = .3$ $\qquad$ $\alpha(Q^2 = 10) = 1$

Low Energy Behavior. $\left\{ \begin{array}{l} \text{Bound States} \\ \text{Confinement} \end{array} \right\}$.

As $Q \to 0$ $\bar{g}^2(Q)$ increases: Strong + Infrared sing
At Large Distances



This could give rise to:

A. Dynamical Spontaneous Symmetry Breaking
B. Containment of the Quarks - Gluons.
only color Singlets = Asymptotic States.

Realized in an Asym. Free $(\bar{\psi}\psi)^2$
2 Dim. Model. (Gross + Neveu).

Deep Inelastic Scattering in AFGT.

Since: $\gamma_N(g^2) = \gamma_N g^2 + \cdots$ $\quad + \quad \bar{g}^2(q^2) \to \frac{\text{const.}}{\ln q^2}$

$$\Rightarrow C_i^{(N)}\left(\frac{q^2}{\mu^2}, g\right) \xrightarrow[q^2\to\infty]{} C_i^{(N)}(1,0)(\ln q^2)^{-A_N}.$$

STRUCTURE OF THE FREE QUARK MODEL - UP TO CALCULABLE $\bar{g}^2$ corrections.

CALCULABLE LOGARITHMIC DEVIATIO FROM SCALING.

$$_N = \int_0^1 dx\, X^N F(x,q^2) = \sum_i C_i^{(N)}(1, \bar{g}^2)(\ln q^2)^{-A_N}$$

$A_0 = 0 \quad A_N \approx G(4\ln N - .69) \sim .6\ln N - .1 \quad + O\left(\frac{M^2}{q^2}\right).$

A. Where Is Asymptopia?

Require : $\quad Q^2 \gg M^2 \quad$ masses ≈ 0

$$\bar{\alpha} = \frac{g^2}{4\pi^2} \to \frac{4}{9\ln\left(\frac{Q^2}{\mu^2}\right)} \ll 1$$

$\mu = 1$ RWB model $\quad \bar{\alpha}(5\ \text{Bev}^2) \simeq \frac{1}{4} \qquad \frac{M^2}{Q^2} \approx .2$

$\bar{\alpha}(50\ \text{Bev}^2) \approx .1 \qquad \approx .0$

Above theorems are asymptotic expansions in $\bar{\alpha}$.

Sum Rules

ll parton ~~m~~-or light cone model sum rules are
symptotic theorems for the moments of
i.

rrections of order $\alpha(Q^2)$. These will ~~have~~ be
calculable, and have a characteristic ω dependen

$$\frac{\int_0^1 F_L(\cancel{x},q^2)\,x^N dx}{\int_0^1 F_T(x,q^2)\,x^N dx} \longrightarrow \alpha C_N\left(1+O(\alpha)\right)$$

Calculable (at least for nonsinglet F's

$$\Rightarrow F_L^{NS}(\omega,Q^2) = \alpha(Q^2)\,C_2(R)\,\frac{1}{\omega^2}\int_1^{\omega} d\omega'\,\omega'\,F_T^{NS}(\omega',Q^2) + O(\alpha^2)$$

$$\int_0^1 dx\left[F_3^{\nu p}+F_3^{\nu n}\right] = -6\left[1+\frac{5}{8}C_2(R)\,\alpha(Q^2)\right] + O(\alpha^2)$$

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lew Sum Rule. FOR $O_{\mu\nu}$!

$$\int_0^1 dx\left[F_2^{ep}+F_2^{en}\right] = 2\langle Q^2\rangle \times \frac{T(R)}{2C_2(R)+T(R)} + O\left[\alpha^{.4}+\alpha^{.6}+\alpha\right]$$

(.16 in RWB)

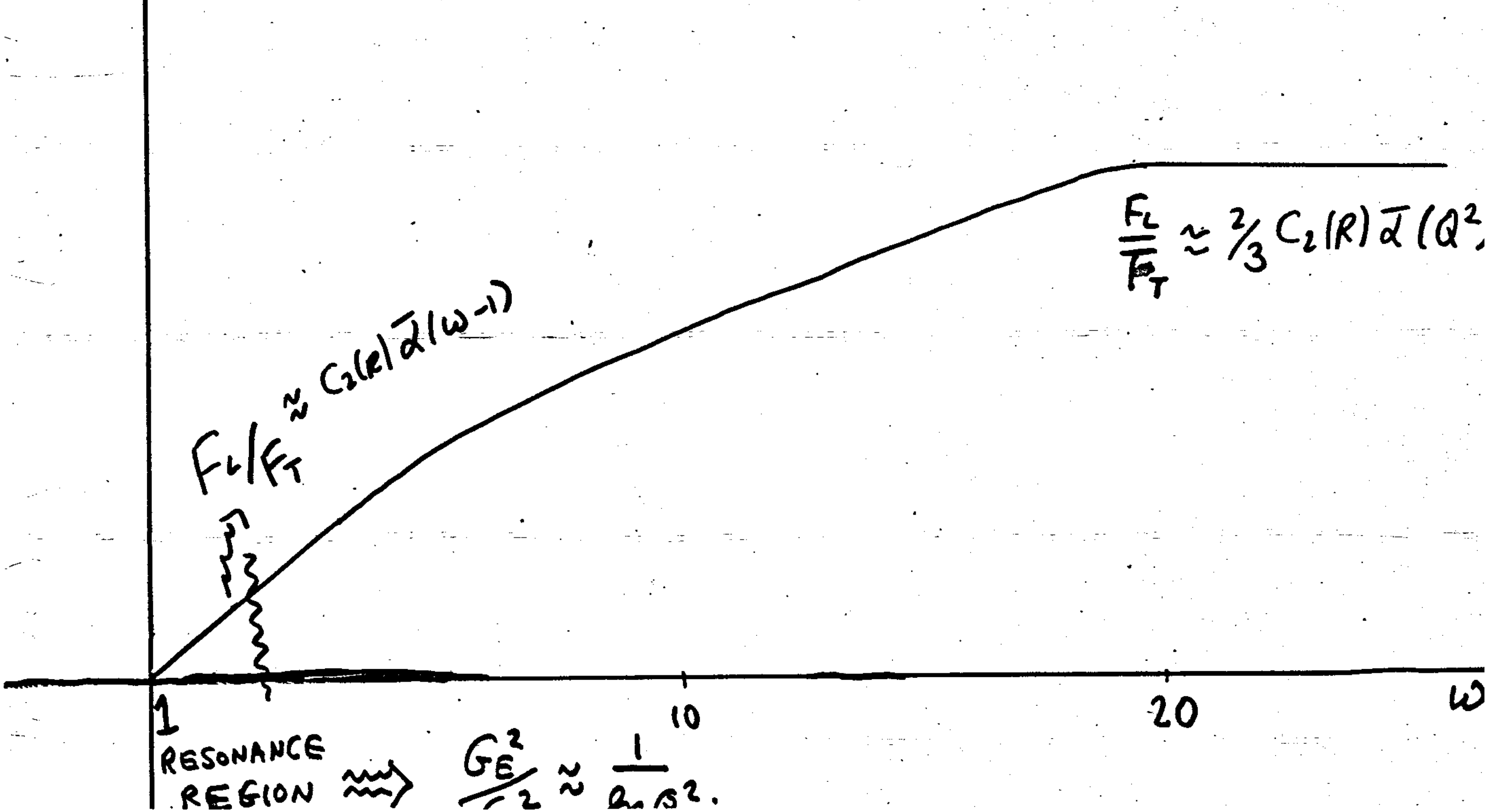
FRACTION OF P IN

$$R = \frac{F_L(\omega, Q^2)}{F_T(\omega, Q^2)}$$

NON SINGLET

$$\bar{\alpha}(Q^2) \approx \frac{1}{9 \ln Q^2} \qquad C_2(R) = \frac{1}{3}.$$

$$F_L(\omega, Q^2) = C_2(R)\, \bar{\alpha}(Q^2) \frac{1}{\omega^2} \int_1^{\omega'} d\omega' \omega' F_T(\omega', Q^2)$$



caling Deviations

$$\int_0^1 dx\, x^N F(x,Q^2) \underset{Q^2\to\infty}{\sim} (\ln Q^2)^{-A_N} C_N\left(1+O_N(\alpha)\right)$$

$$A_N^{N.S.} = G\left[1-\frac{2}{(N+2)(N+3)}+4\sum_{k=0}^{N}\frac{1}{k+2}\right] \underset{N\to\infty}{\sim} 4G\ln N - .69$$

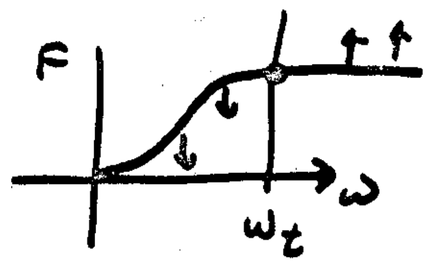
$$A_N^{SING.} = 0 \quad N=0 \; ; \quad \underset{N\to\infty}{\to} A_N^{NS} + O\left(\frac{1}{N^2 \ln N}\right); \quad \text{Pole at } N=-1.$$

oments are difficult to use as test of AFGT:

A_N, for small N, are small.

Require $F(\omega, Q^2)$ for large ω.

At least $\omega \gtrsim \omega_{transition}$.



But $\omega_t \cong \text{const}\left(\frac{1}{\alpha(Q^2)}\right)^{1.5}$; $\omega_t(50) \approx 20$

$\omega_{EXP}^{(Q^2)} \leq \frac{2\nu_{MAX}}{Q^2}$ $\quad \omega_{MAX}(50) \leq \frac{\nu_{MAX}}{25}$ | 6 at NAL.

ASYMPTOTIC EXTRAPOLATION FORMULA.

e $\underset{\alpha(Q'^2)\ll 1}{F(\omega, Q'^2)}$ to determine C_N.

$$F(\omega,Q^2) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} ds\, \omega^{s}\, e^{-A(s)\ln\frac{\ln Q^2}{\ln Q'^2}} \int_1^{\omega} d\omega'\, F\left(\frac{\omega}{\omega'}, Q'^2\right)$$

$$\boxed{F(\omega,Q^2) = \int_1^{\omega'} \frac{d\omega'}{\omega'} F_2\left(\frac{\omega}{\omega'}, Q'^2\right) T\left(\frac{\ln Q^2}{\ln Q'^2}, \omega'\right)}$$

RMITY: Corrections: $\ln^3 J \cdot \alpha(Q^2)$ | $\frac{\alpha(Q^2)}{J^3}$

$(\ln\ln\omega)^3 \ll \ln Q^2$

Utility

1. Need only know $F(w', t')$ for $1 \leq w' \leq w$ to determine $F(\omega, t)$. Good – since exp. limit for given q^2 is: $1 \leq w \leq \frac{2 \nu_{MAX}}{q^2}$.

2. For small $\underline{w}$ – need only know A_N for large

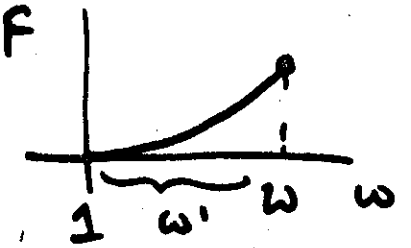
$\Rightarrow$ a. Common A_N, Common threshold behavior

$\frac{\ln \frac{t}{x}}{\ln w}$ independent of quantum numbers of F!

b. $\boxed{\Gamma \text{ is simple } \simeq \left(\frac{t}{t'}\right)^{.696} \frac{(\ln w)^{p-1}}{w\,\Gamma(p)} \left[1 + \sum C_n (\ln\right.}$

$$p = 4G \ln \frac{t}{t'} = 4G \ln \left\{ \frac{\ln \frac{q^2}{\mu^2}}{\ln \frac{q'^2}{\mu^2}} \right\}.$$

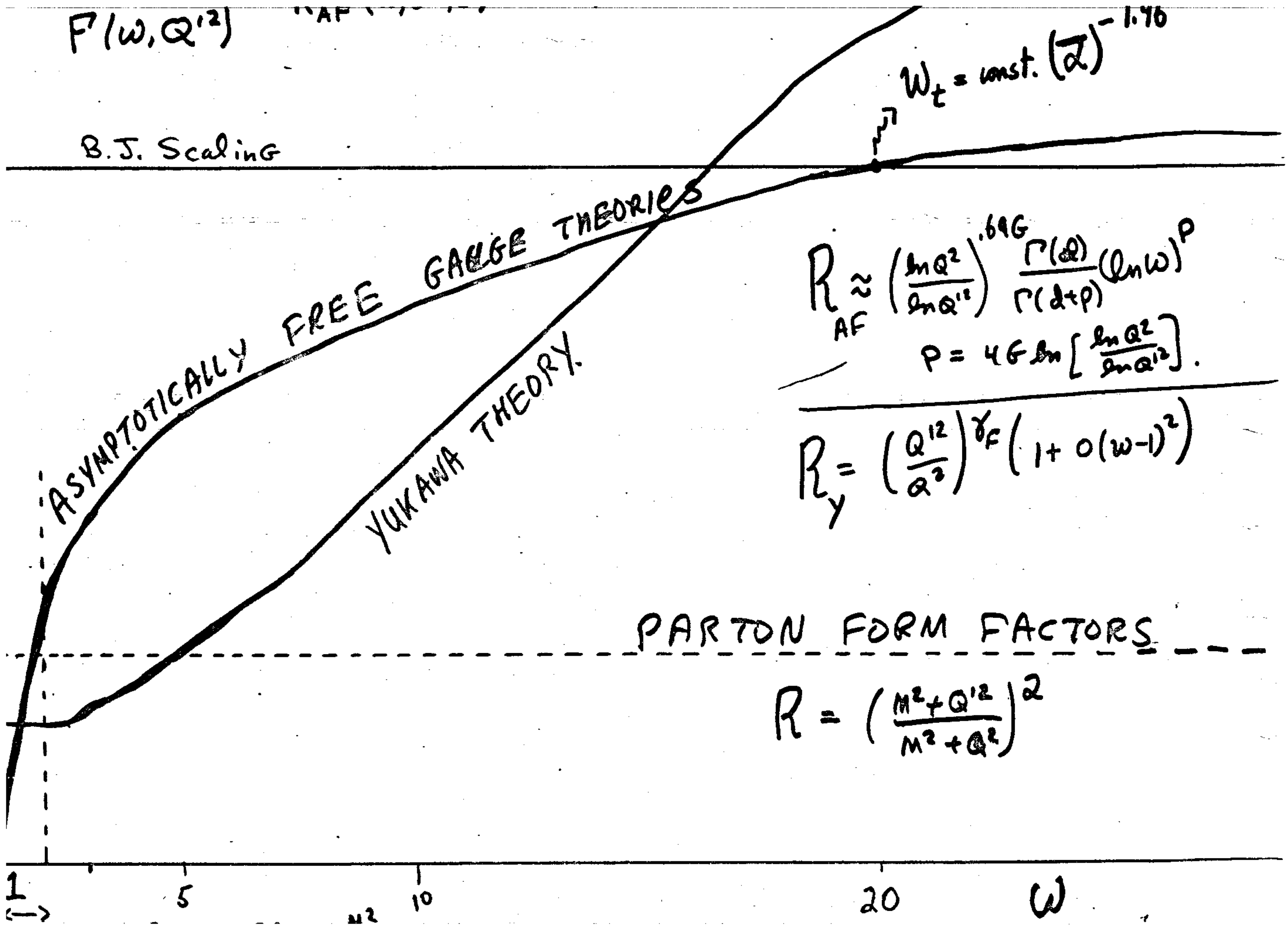
c. Enhanced Scaling violations $w \approx 1$, since integrate over region where $F \to 0$ $w \to 1$.



THRESHOLD FORMULA

If $F(w, t') \approx c(w-1)^{d}(1 + O(w-1))$ then:

$$\boxed{R(w; q^2, q'^2) = \frac{F(w\,t)}{F(w, t')} = \left(\frac{t}{t'}\right)^{.696} \frac{\Gamma(1+d)}{\Gamma(1+d+p)} (\ln w)^{p} \left[w \text{ old}\right.}$$



F(ω, Q'²)
$W_t = \text{const.}\,(\bar{\alpha})^{-1.46}$
B.J. ScalinG
ASYMPTOTICALLY FREE GAUGE THEORIES
YUKAWA THEORY.
$R_{AF} \approx \left(\frac{\ln Q^2}{\ln Q'^2}\right)^{.69G} \frac{\Gamma(\alpha)}{\Gamma(\alpha+p)} (\ln \omega)^p$
$p = 4G \ln\left[\frac{\ln Q^2}{\ln Q'^2}\right].$
$R_Y = \left(\frac{Q'^2}{Q^2}\right)^{\gamma_F}\left(1 + O(\omega-1)^2\right)$
PARTON FORM FACTORS
$R = \left(\frac{M^2+Q'^2}{M^2+Q^2}\right)^2$
1
5
10
20
ω

HADRONIC FORM FACTORS

Near threshold, $w=1$, ~~the~~ F receives contributions from nucleon resonances.

Here we use $w = \frac{2\nu + m^2}{q^2}$.

Consider contribution to $\bar{F}$ - from resonance mass m_R:

$$F_2^R \equiv \nu G^2(q^2)\,\delta(\nu - \nu_R) \qquad \nu_R = \tfrac{1}{2}(m_R^2 - m^2 + q^2).$$

IF the res = nucleon ($m_r = m$); then:

$$G^2 = \frac{G_E^2 + \frac{q^2}{4M^2} G_M^2}{1 + \frac{q^2}{4M^2}} \xrightarrow[q^2\to\infty]{} G_M^2(q^2)$$

IF: $\frac{G_E}{G_M} \sim \text{const}$ $q^2 \to \infty$.

Positivity:

$$\int_1^{1+\frac{M_0^2}{q^2}} dw\, F_2(q^2, w) > G^2(q^2).$$

$q^2 \to \infty$, integral restricted to $w \approx 1 \Rightarrow$ Can use threshold formula; so:

$$\boxed{G^2(q^2) < \text{const.} \left(\frac{t}{t'}\right)^{.696} \frac{\Gamma(d+2)}{\Gamma(d+2+p)} \left(\frac{S_0}{q^2}\right)^{d+1+p}}$$

Duality: (a la Bloom + Gilman).

$$\frac{\int d\omega\, F_R}{\int d\omega\, F} \approx \text{const. ind. of } q^2, \text{ even when } q^2 \to \infty.$$

Appears to work in SLAC region.
Find $S_0 = (1.23)^2\ \mathrm{GeV}^2$ best choice.

Then!

$$\boxed{\frac{G_M^2(q^2)}{G_M^2(q'^2)} \approx \left(\frac{\ln \frac{q^2}{\mu^2}}{\ln \frac{q'^2}{\mu^2}}\right)^{.696} \frac{\Gamma(d+2)}{\Gamma(d+2+p)} \left(\frac{S_0}{q^2}\right)^p \left(\frac{q'^2}{q^2}\right)^{1+} \qquad q^2, q'^2 \gg \mu^2}$$

Note:

1. Since all operators, $\underset{\sim}{8} + \underset{\sim}{1}$, have same γ_N when $N \to$
   $\Rightarrow$ the above holds for nucleon iso-vector & axial vector scalar form factors

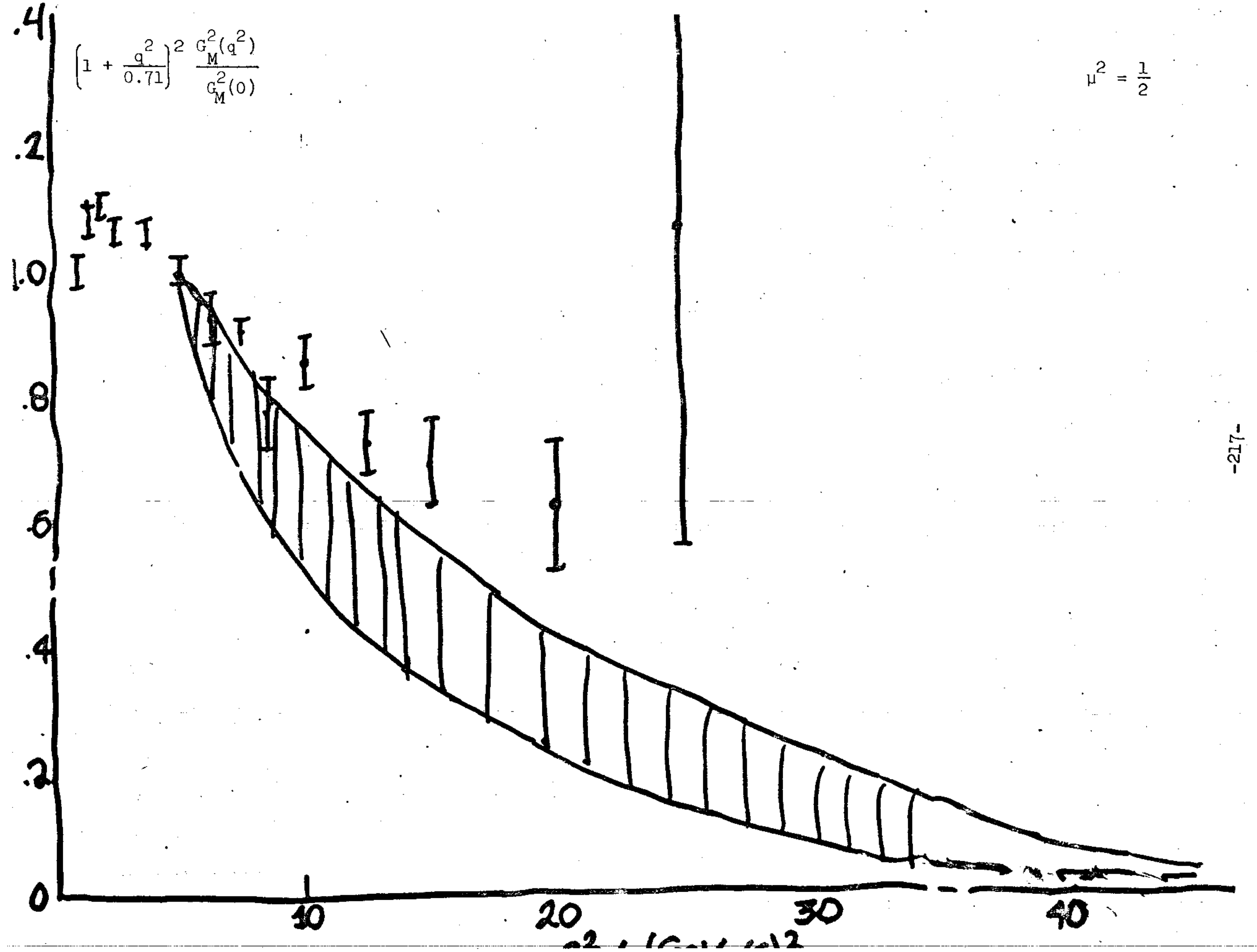
If do similar anaylsis for $F_L \sim G_E^2$; & use fact that: $\frac{F_L}{F_T} \underset{\omega \to 1}{\sim} (1-\omega) \frac{1}{\ln q^2} \Rightarrow \frac{G_E^2}{G_M^2} \simeq \frac{1}{\ln q^2}$.

I.E. <u>FORM FACTOR SCALING (UP TO LOGS) IS A CONSEQUENCE OF ASYMPTOTIC FREEDOM</u>.

<u>Dipole</u> - put in (thru d). Presumably

$G \sim \frac{1}{(q^2)^2} \cdot \Sigma \ln q^2 \longrightarrow$ corrections in AN AFGT.

$\longrightarrow N = 3\,9'\mathrm{s}$



$\left(1 + \frac{q^2}{0.71}\right)^2 \frac{G_M^2(q^2)}{G_M^2(0)}$
$\mu^2 = \frac{1}{2}$
.4
.2
1.0
.8
.6
.4
.2
0
10
20
30
40

Summary

Theoretical:

1. BJ Scaling $\Rightarrow$ Asym. Free Gauge Theories $\Rightarrow$ $\left\{ \begin{array}{l} \text{BJ Scaling to within } \ln Q^2. \\ \text{Parton model at short distances} \end{array} \right.$

2. By looking at short distances one can "see" $\left\{ \begin{array}{l} \text{the Constituents of the Hadrons} \\ + \text{ Nature of the Strong Interactions.} \end{array} \right\}$

3. Asym. Freedom may provide mechanism for 1) Dyn. Sym. Breaking or 2) Containment of Quarks + Gluons.

4. Other Applications: 1. Use Weak Int. to probe short Distance Behavior of Strong Int
$\rightsquigarrow \ln\left(\frac{M_W^2}{M_H^2}\right) \approx 10$. $\Delta I = \frac{1}{2}$ Rule
(B. Lee + M. Gaillard / Altarelli + Maiani).
2. Form Factors
Large Angle Elastic
Large $P_\perp$ Inclusive.
⋮

Exp. Tests

1. Expects substantial deviations from scaling, Q^2: 10–50 for small $\omega = \frac{2\nu}{Q^2}$.

2. Requires: $\left\{ \begin{array}{l} \text{High Precision } 1\sim5\% \\ \text{Low } \omega \end{array} \right\}$

Ability to determine the ω-dependence of $\frac{F_L}{F_T}$, scaling violations, etc..