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AUTUMN WORKSHOP

IN

CLOUD PHYSICS AND CLIMATE

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I. Introduction

A cloud model is a set of equations which determine the microphysical, thermodynamic and dynamic parameters within a cloud as functions of space and time. Models are used in two ways; the first is to test, by means of comparisons between model predictions and observations, our descriptions of the cloud processes, and thus our understanding of the underlying physics. The second is to simulate proposed experiments or atmospheric events, and thus to predict outcomes of certain situations which we cannot easily set up in the real world.

No single model can provide a realistic description of all the cloud parameters as they vary over space and time. Models are always approximations, and the proposed use for a model determines the appropriate approximations to be made in it. The set of equations to be used in a particular application is often truncated, with little or no attempt made to realistically simulate processes which are not of direct interest for that application. In these lectures we will identify the various components and governing equations for several models designed for different purposes. A list of references is given at the end in which the interested reader can find detailed discussions of the topics touched upon here.

II. Cloud Models

Table I shows the variables to be determined by a cloud model.

Table I

	Variable	Symbol	Dimensions
Thermodynamic Variables	pressure of cloudy air	p	N/m^2
	density of cloudy air	ρ	kg/m^3
	temperature of cloudy air	T	K or °C
Microphysical Variables	concentration of drops of mass between m_d and $m_d + dm_d$	$n_d(m_d)dm_d$	number/m^3 cloudy air
	concentration of ice particles of mass between m_i and $m_i + dm_i$	$n_i(m_i)dm_i$	number/m^3 cloudy air
Water category variables	specific total water content	Q	kg H_2O/kg cloudy air
	specific humidity	q_v	kg vapor/kg cloudy air
	specific liquid water content	q_l	kg liquid/kg cloudy air
	specific ice content	q_i	kg ice/kg cloudy air
	air velocity components	u,v,w	m/s

Let us begin with an examination of the equations for the thermodynamic variables.

A. Thermodynamics

We assume that both dry air (density ρ_d, pressure p_d) and vapor (density ρ_v, pressure e) behave like ideal gases, so that $q_v = \rho_v/\rho$, etc.

and

$$p = p_d + e \quad : \quad p_d = \rho_d R_d T, \quad e = \rho_v R_v T \qquad (1)\ a)$$

or

$$p \cong \rho_d R_d T_v \qquad (1)\ b)$$

where

$$T_v \equiv T\left[1 + q_v\left(\frac{R_v}{R_d} - 1\right) - q_i - q_l\right] \qquad (2)$$

where R_v and R_d are the gas constants for vapor and dry air. For future reference we also define the saturation vapor pressures e_{si} and e_{sl}. These are the vapor pressures which characterize air in equilibrium over a flat surface of ice or of water. They are given by the Clausius-Clapeyron equation,

$$\frac{de_{sl}}{dT} = \frac{L_v e_{sl}}{R_v T^2} \qquad (3)\ a)$$

$$\frac{de_{si}}{dT} = \frac{L_s e_{si}}{R_v T^2} \qquad (3)\ b)$$

where L_s and L_v(J/kg) are the latent heats of sublimation and of evaporation, respectively. (Numerical values for the temperature dependent latent heats, saturation vapor pressures and other thermodynamic properties of cloudy air may be readily computed from formulae given in Bolton, 1980.)

Equations (1) and (2) are algebraic equations linking values of several thermodynamic parameters at a given point in space and time. The other variables in Table I are determined by differential equations which yield their values at one point to those at other points. The fundamental form for these equations is the following: letting ξ represent a variable in Table I:

$$\frac{D\xi}{Dt} \equiv \frac{\partial\xi}{\partial t} + \underline{V}\cdot\nabla\xi = \text{Source}_{(\xi)} - \text{Sink}_{(\xi)} \qquad (4)$$

where vectors are indicated by underlines; $\underline{\nabla} \equiv (\partial/\partial x, \partial/\partial y, \partial/\partial z)$ and $\underline{V} \equiv (u,v,w)$. The task of a cloud modeller is to represent the physical processes denoted "sources and sinks" in these conservation equations by realistic, yet manageable mathematical functions of the selected variables.

The equation for ρ is derived by writing Equation (4) in the case ξ = mass. Let M be the mass of an air parcel of volume $\forall$. Then $M = \rho\forall$ and

$$\frac{DM}{Dt} = 0 = \forall\frac{D\rho}{Dt} + \rho\frac{D\forall}{Dt} = \forall\left(\frac{\partial\rho}{\partial t} + \underline{V}\cdot\nabla\rho\right) + \rho\forall\underline{\nabla}\cdot\underline{V}$$

or

$$\frac{\partial\rho}{\partial t} + \underline{\nabla}\cdot(\rho\underline{V}) = 0 \qquad (5)$$

To derive an equation for the temperature, we first write Equation (4) in the form of a conservation equation for the moist static energy h, defined here as

$$h \equiv L_v q_v - L_f q_i + gz + c_p T$$

L_f(J/kg) is the latent heat of freezing and C_p(J/kg -°C) is the specific heat of cloudy air. h is approximately conceived in adiabatic processes if $V^2/2 \ll h$, as we shall assume here. We have

$$\frac{Dh}{Dt} = \text{Source}_{(h)} - \text{Sink}_{(h)} \equiv \dot{H} \qquad (6a)$$

where $\dot{H}$(W/kg) represents possible external energy sources and sinks such as the radiative flux divergence or precipitation fallout.

The equation for T is found from the definition of h and Equation (6a): we have

$$\frac{DT}{Dt} = -\frac{gw}{c_p} + \frac{L_f}{c_p}\frac{Dq_i}{Dt} - \frac{L_v}{c_p}\frac{Dq_v}{Dt} + \frac{\dot{H}}{c_p} \qquad (6b)$$

B. Microphysics

In a model to be used for examination of size distributions (called 'spectra') of cloud or precipitation particles we need equations for the explicit microphysical variables $n_d(m_d)$ and $n_i(m_i)$ in Table I. If, on the other hand, the spectra are of less importance than the dynamics or the gross partitioning of water into its three forms, we use the water category variables instead. In this section we discuss the appropriate equations in both cases.

1. Explicit Microphysics

a) Warm Clouds

Putting $\xi = n_d(m_d)$ in Equation (4), we can write

$$\frac{Dn_d}{Dt} = \dot{n}_d\Big)_{\text{activation}} + \dot{n}_d\Big)_{\text{vapor diffusion}} + \dot{n}_d\Big)_{\text{coalescence}} + \dot{n}_d\Big)_{\text{breakup}} + \dot{n}_d\Big)_{\text{sedimentation}} + \dot{n}_d\Big)_{\text{entrainment}} \qquad (7)$$

Let us consider each term on the RHS of this equation.

(1) Activation

Let $N_{CCN}(s_\ell)$ be the concentration of CCN activated in steady state at supersaturation s_ℓ, where

$$s_\ell \equiv e/e_{s\ell} - 1 \qquad (8)$$

There is experimental evidence (Twomey and Wojciechowski, 1969) that

$$N_{CCN}(s_\ell) \cong Cs_\ell^k \qquad (9)$$

where C and k are constants; k~0.4-0.7 and C depends on time and location. Activation is usually assumed to occur instantaneously as the supersaturation changes, and to produce particles of a minimum mass, which we can denote m_1. Similarly, if the saturation is decreasing, droplets with the minimum mass m_1 are assumed to evaporate instantaneously. In other words, in numerical models of activation we usually put

$$\begin{aligned} \dot{n}_d(m_d)\Big)_{activation} &= 0, \quad m_d \neq m_1 \\ \dot{n}_d(m_1)\Big)_{activation} &= \frac{Ds_l}{Dt}\frac{\partial N_{CCN}}{\partial s_l}, \quad s_l \text{ increasing} \\ &= -\frac{1}{\Delta t} n_d(m_1), \quad s_l \text{ decreasing} \end{aligned} \qquad (10)$$

where Δt is the timestep of the computation. See, for example, Clark (1973) or Takahashi (1978) for further details.

(2) Vapor Diffusion

After activation diffusion of water vapor to and from the surface of a droplet causes its radius $r_d(m_d)$ to change. For $r_d \gtrsim 1\mu m$, the rate of change of the droplet radius is approximately

$$\frac{\partial r_d}{\partial t} \cong \frac{D \rho_{v,s,l}\, s_l}{\rho_l r_d} \qquad (11)$$

where ρ_l is the density of liquid water, $D(m^2/s)$ is the diffusion coefficient of vapor in air, and $\rho_{s,l}$ the density of saturated vapor at the environmental temperature T. See, for example, Pruppacher and Klett (1978) for details of the derivation of Eq. (11).

The concentration of droplets of mass in the interval m_d, $m_d + dm_d$ changes with time due to vapor diffusion as follows:

$$\dot{n}_d(m_d)\Big)_{\substack{vapor\\diffusion}} = -\frac{\partial}{\partial m_d}\left(\frac{dm_d}{dt} n_d(m_d)\right) \qquad (12)$$

where $\frac{\partial m_d}{\partial t} = \rho_l 4\pi r_d^2 \frac{\partial r_d}{\partial t}$, $\frac{\partial r_d}{\partial t}$ given by eq. (11).

To compute the RHS of Equation (12) in a numerical model, all droplets are classified into discrete size classes, and then the concentration in each size class is evaluated at each time step by considering how many droplets grew into or out of that class.

For nonprecipitating clouds, when droplet radii remain smaller than ~ 20μm, the rest of the terms in Equation (7) can be ignored. If, however, it is necessary to consider larger drop sizes, these terms become important.

(3) Coalescence

As droplets grow by condensation (Eq. (11)) to sizes at which they begin to fall relative to the air, there are collisions between droplets moving relative to one another. Some of these collisions result in coalescence, and the formation of large drops called precipitation, or raindrops. If $A(m_d)$ is the cross-sectional area and $V_t(m_d)$ the terminal fall velocity of a drop of mass m_d, then in time dt a drop of mass m_d encounters and coalesces with all drops of mass m_d' within a volume we define as

$$K(m_d, m_d')dt \equiv (A(m_d)+A(m_d'))|V_t(m_d) - V_t(m_d')|E(m_d, m_d')dt \quad (13)$$

where the factor $E(m_d, m_d')$ takes into account the deformation of streamlines around the particles and the fact that not all collisions result in coalescence. The rate at which a given particle of mass m_d coalesces with any particle of mass m_d' is thus $n_d(m_d')K(m_d,m_d')dm_d'$.

Then, the mean rate of disappearance via coalescence of drops of mass m_d in the interval $(m_d, m_d + dm_d)$ is $D(m_d) = n_d(m_d)\int^{\infty} K(m_d,m_d')n_d(m_d')dm_d'$. At the same time, new drops are constantly being created in this mass interval by collisions among smaller drops. The mean rate of production by these collisions of droplets in the size range of interest is

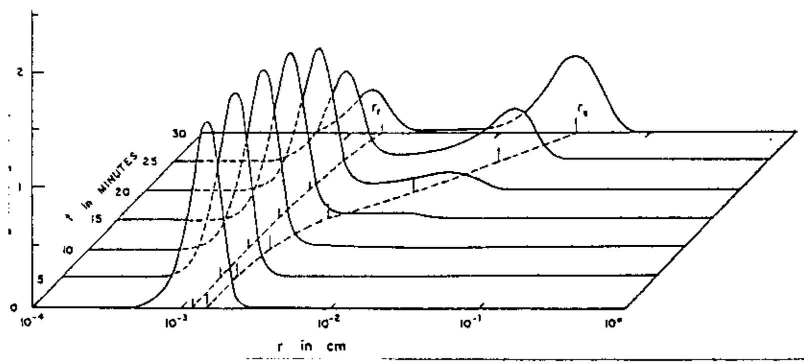
$$P(m_d) \equiv \frac{1}{2}\int_0^{m_d} n_d(m_d')n_d(m_d-m_d')K(m_d', m_d-m_d')dm_d'$$

Thus the third term on the RHS of Equation (7) is

$$\dot{n}_d(m_d)\Big)_{coalescence} = -D(m_d) + P(m_d) = -n_d(m_d)\int_0^{\infty} n_d(m_d')K(m_d, m_d')dm_d' + \frac{1}{2}\int_0^{m_d} n_d(m_d')n_d(m_d-m_d')K(m_d', m_d-m_d')dm_d' \quad (14)$$

Equation (14) is the so-called stochastic collection equation (SCE). See Berry and Rhinehart, 1974; Soong and Ogura, 1973; and Pruppacher and Klett, 1978, for detailed discussions of the terms in Equations (13) and (14). Figure 1 shows an example of the results of calculations of $n_d(m_d,t)$ using the SCE.

Figure 1 Time development of the drop spectrum computed from the SCE, Equation (14). The ordinate is related to $n_d(m_d)$ and the abscissa is r_d. From Berry and Rhinehardt, 1974. Note the depletion of small droplets as coalescence continues.



Because of the complexity of the SCE, a simpler approach to the study of coalescence is often adopted when the goal is to determine (roughly) the rate of change of size of only the precipitation particles, rather than to develop a time-dependent expression for the entire drop population. This second approach, called the 'continuous collection method', begins from Equation (13). In the approximation that a few large collector drops of mass m_d fall through a population of much smaller cloud droplets, the instantaneous rate of growth of the large drops is

$$\frac{\partial m_d}{\partial t} \cong K(m_d, \overline{m_d}') \, q_l \qquad (15)$$

where q_l is the liquid water content in the small droplets and $\overline{m_d}'$ is a mean droplet mass in the cloud.

(4) Breakup

The rate of growth of large droplets is limited by three factors; the rate of supply of liquid water, the break-up of pairs of drops which have recently collided and the spontaneous break-up of individual drops as they oscillate and deform during fall. The second two processes may be represented by the expression

$$\dot{n}_d(m_d)\Big|_{breakup} = -n_d(m_d)\Pi(m_d) + \int_{m_d}^{\infty} n_d(m_d')\, Q_B(m_d', m_d)\, \Pi(m_d')\, dm_d' \qquad (16)$$

where $\pi(m_d)$ is the rate of break-up of a drop of mass m_d and $Q_B(m_d', m_d)$ is the fraction of break-up events of drops of mass m_d' which result in the formation of a daughter

drop of mass m_d. The factors Q_B and π are usually taken from a combination of theory and experiment (Srivastava, 1978; Young, 1975).

Note that coalescence and break-up (Eqs. 14-16) result in the transformation of a large number of cloud droplets into a relatively tiny number of raindrops. To accurately follow this transformation numerically requires great computational precision. Any numerical spreading in the drop distribution will result in apparent production of raindrops in a model of drop spectral evolution.

(5) Sedimentation

The rate of change of drop number due to the sedimentation of precipitation sized particles ($r_d \geq 50$ μm) is given by the expression

$$\dot{n}_d(m_d)\Big|_{sedimentation} = -\frac{\partial}{\partial z}\left(V_t(m_d)\, n_d(m_d)\right) \qquad (17)$$

where the terminal velocities are taken from experimental values (see Pruppacher and Klett for a review of these data.)

(6) Entrainment

Turbulent motions bring about the mixing of environmental air into clouds, or entrainment. Entrainment dilutes the cloud hydrometer populations and can deform the particle spectra via evaporation. In a real cloud, there is no artificial boundary separating the cloud from the environment. In general, however, cloud models which include microphysical detail do not include much dynamic realism, and mixing is usually highly parameterized. One approach to convective cloud modeling is to solve the model equations inside a set volume, called "the cloud", and to parameterize the fluxes (entrainment) over the bounding surface of this volume. For example, it is often assumed that convective clouds behave like self similar plumes or thermals (Morton et al., 1956; Turner, 1962). In this kind of flow the rate of entrainment of environmental fluid into the rising cloud is determined by the assumption that the horizontal velocity at the interface is proportional to the local vertical velocity $w(z)$; i.e.,

$$u_r(z) = \alpha w(z) \tag{18}$$

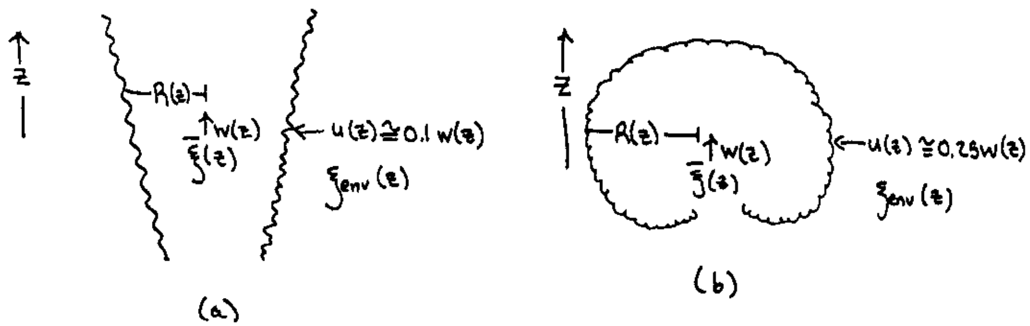
where u_r is the radial velocity, $\alpha \cong 0.1$ for a plume and $\alpha = .25$ for a thermal (see Figure 2). This leads to the expressions

$$\dot{\xi}\Big)_{\substack{\text{entrainment}\\ \text{(plume)}}} = \frac{-2\alpha w}{R}\left(\bar{\xi} - \xi_{\text{environment}}\right); \quad \dot{\xi}\Big)_{\substack{\text{entrainment}\\ \text{(thermal)}}} = -\frac{3\alpha w}{R}\left(\bar{\xi} - \xi_{env}\right) \tag{19}$$

for the plume and thermal, respectively, where $\bar{\xi}$ is the average in-cloud value of any variable obeying a conservation equation (Eq. 4), and the radius of the cloud, R, grows linearly with height z. (See the above references for derivations of Eq. 19.)

Warner (1970, 1973), Lee and Pruppacher (1977) and many others have used these assumptions. Other authors (Telford, 1966; and Lopez, 1973) have assumed the lateral entrainment velocity is a mean turbulent velocity which

Figure 2 Schematic drawings showing the boundaries of (a) a self-similar plume and (b) a self-similar thermal, both at large distances in a neutral environment from the (point) source. The source is (a) constant in time for the plume; (b) a single pulse for the thermal. There is no generation of buoyancy (as in phase change) and the inflow velocity is proportional to the propagation velocity. With these assumptions the horizontal dimension R grows linearly with z in both cases.



can be computed from a conservation equation for turbulent kinetic energy. It is to be noted that difficulties with model predictions (to be discussed below) and recent observational studies of entrainment throw some doubt on the extent to which these model flows are useful analogies. See Simpson (1983) for a review of the successes and failures of plume models in reproducing observed cumulus cloud behavior.

We have briefly discussed each term in Equation (7), which completely describes the microphysics for clouds which do not rise above the 0°C isotherm. We turn now to an even briefer description of the analogous equation for $n_i(m_i)$.

b) Cold Clouds

The conservation equation for $n_i(m_i)$ which is analogous to Equation (7) for $n_d(m_d)$ is

$$\frac{Dn_i}{Dt} = \dot{n}_i\Big)_{nucleation} + \dot{n}_i\Big)_{vapor\ diffusion} + \dot{n}_i\Big)_{riming} + \dot{n}_i\Big)_{ice-ice\ collisions} + \dot{n}_i\Big)_{multiplication} + \dot{n}_i\Big)_{sedimentation} + \dot{n}_i\Big)_{entrainment} \quad (7')$$

The terms in Equation (7') are in general more complicated than their counterparts in Equation (7) because of the complex shapes and density distributions of the solid particles. Moreover, we have fewer observations of glaciation processes than of the corresponding warm cloud processes. Therefore there have been relatively few

attempts to use explicit microphysical models in cold clouds. We shall just touch upon the main features of each term in Equation (7').

(1) Nucleation

It has been customary to assume that the first stage in glaciation is heterogeneous nucleation on ice nuclei (IN) whose concentrations N_{IN} increase exponentially with cooling; (Fletcher, 1962)

$$N_{IN} = A\exp[-\beta T] \quad (20)$$

where T is in °C.

Typical values of the constants are $A \sim 10^{-5}$/liter, $\beta \sim 0.6/°C$. Nucleation is assumed to occur instantaneously as the temperature in a cloudy parcel changes; i.e., in analogy to Equation (10),

$$\dot{n}_i(m_i)\Big)_{nucleation} = 0, \quad m_i \neq m_1$$

$$\dot{n}_i(m_1)\Big)_{nucleation} = \frac{\partial N_{IN}}{\partial T}\frac{DT}{Dt}, \quad T\ \text{decreasing} \quad (10')$$

$$= -\frac{n_i(m_1)}{\Delta t}, \quad T\ \text{increasing}$$

where m_1 is the arbitrarily set minimum ice crystal mass and Δt the time step of the model. We note, however, that frequently there are far more ice particles in clouds than can be explained as descendants of the IN whose concentrations are given by Equation (20). The cause of this discrepancy is not yet known, but the use of Equation (20) to predict the concentration of small ice particles appears questionable until more is understood about glaciation; it may be preferable to use empirically determined concentrations of the small crystals. The uncertainties in our present understanding of glaciation limit the confidence to be placed in numerical models of both naturally and induced precipitation development in cold clouds.

(2) Vapor Deposition

For small ice crystals, the most important process of growth or decay is vapor diffusion to and from the particle surface. Because the surface is not spherical, the vapor flux at the surface is not uniform and the growth equation is somewhat more complicated than Equation (11) for spherical droplets. We have

$$\frac{\partial m_i}{\partial t} \simeq \frac{C\, D\, \rho_{v_{s,i}}\, s_i\, f_v}{\rho_i} \qquad (21)$$

where $s_i \equiv e/e_{si} - 1$, and C is a shape factor. The shape (growth habit) of small ice crystals depends on temperature and vapor pressure; some shapes and the corresponding expressions for C are given in Pruppacher and Klett.

The 'ventilation factor', f_v, is a semiempirical factor representing the effects of particle motion on vapor diffusion to and from its surface. Ventilation

is not important (i.e., $f_v \approx 1$) at the small drop sizes ($r_d \lesssim 50$ µm) for which vapor diffusion is the dominant droplet growth mechanism, and thus f_v was put equal to unity in Equation (11). However, ice crystals grow via vapor diffusion to larger sizes, where ventilation is important. f_v may be expressed as a function of particle shape and mass and the viscosity and vapor diffusion coefficients in the air. See Pruppacher and Klett (1978), for a discussion of the relevant experimental and numerical studies of ventilation. The vapor diffusion term in Equation (7') is

$$\dot{n}_i(m_i)\Big|_{\substack{\text{vapor}\\ \text{diffusion}}} = -\frac{\partial}{\partial m_i}\left(\frac{\partial m_i}{\partial t}\, n_i(m_i)\right) \qquad (12')$$

Vapor deposition is the most important crystal growth mechanism until crystals are around 150-700 microns in lateral dimensions (the threshold depending on habit), when the large crystals begin to fall relative to the smaller ones and particle-particle collisions ensue.

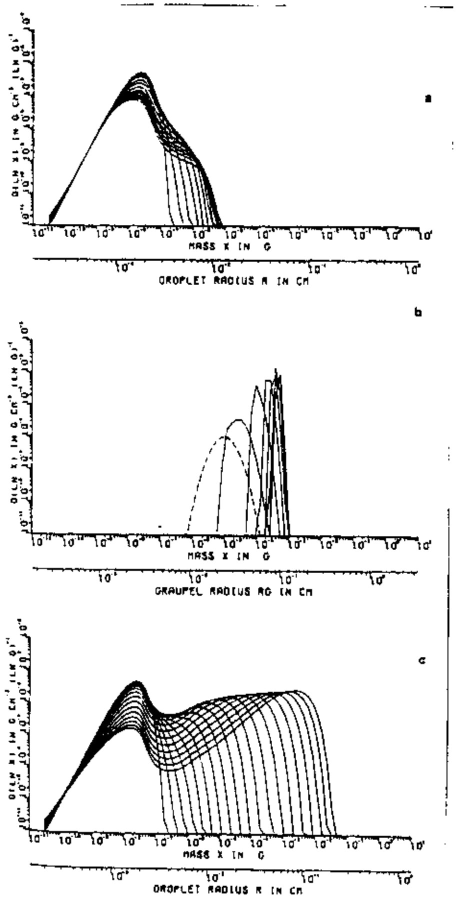
(3) Collection

Ice particles can collide with other ice crystals, to form the low density material called snow, and/or they can collide with supercooled drops to form rimed crystals and eventually graupel and hail. Snow formation is usually represented by a continuous collection model analogous to Equations (13) and (15); the relevant fall velocities and collection efficiencies are determined from experiment (see Pruppacher and Klett, 1978). The rate of growth by riming depends on the rates of shedding and of freezing of any excess water; that is, the efficiency $E(m_i, m_i')$ in the analogue of Equation (13) for ice depends on the liquid water content, the temperatures of the graupel particle and of the environment, the updraft velocity, and the ice concentrations. In general the continuous collection growth model, i.e., Equation (15), is also used to describe the rate of growth of ice particles by riming, although modified SCE's of the type of Equation (14) have been used (Beheng, 1978). (See Figure 3.)

(4) Break-up/multiplication

The discrepancy between observed numbers of ice particles and observed concentrations of ice nuclei has led to several hypotheses for secondary ice multiplication mechanisms. Among these is the Hallett-Mossop process (Hallett and Mossop, 1974), in which riming particles can give rise to small ice splinters if the riming occurs within a narrow temperature interval (roughly -3°C to -8°C) and if the drop spectrum is sufficiently wide. It has been shown (Chisnell and Latham, 1976) that this process gives

Figure 3 Time-dependent evolution of the liquid drop and graupel particle size spectra computed via the SCE. (a) and (b): coalescence with riming, in 200 s. intervals up to 1200 s. The dashed line in (b) is the initial ice particle spectrum. (c) Coalescence only in 100 s. intervals up to 1800 s. (compare with Fig. 1). From Beheng, (1978).



rise to a roughly exponential rate of growth of ice crystal concentrations; the rate of production of small crystals is

$$\dot{n}_i(m_i)\Big)_{\substack{\text{multiplication}\\ \text{(Hallett-Mossop)}}} = \beta\, n_i(m_i) \qquad (23)$$

where the lifetime $1/p_0$ is a complicated function of cloud properties. Ice multiplication is still not sufficiently well understood to allow its detailed description within the framework of a cloud model.

(5) Sedimentation

The convergence of precipitation of ice particles is treated exactly as for liquid precipitation particles (Eq. 17). The terminal velocities are taken from experiment (Again, see Pruppacher and Klett for a review of the experimental literature.)

(6) Entrainment

The comments above on entrainment of liquid particles are relevant also for the ice particles. It has also been recently suggested (Hobbs and Rangno, 1985) that entrainment may play a fundamental role in initiating glaciation by promoting contact nucleation suggesting that the term $\dot{n}_i(m_i)\Big)_{\text{entrainment}}$ may be an important one at small sizes.

It is perhaps clear by now that attempts at including all the important microphysical processes explicitly within a cloud model are hampered by the large uncertainties in many of the numerical parameters, by the complexity of ice phase processes, and by the

wide variety in sizes and types of particles which must be considered and hence the enormous computing time requirements for a complete model. Therefore explicit microphysics is usually included only in models which deal with a limited range of temperatures and water categories (initial condensation stage calculations, for example), and usually these have very simple dynamic equations.

If the details of the microphysics are of less interest than the distribution of water quantities and the larger scale features of cloud evolution, then the processes we have discussed here are represented by simplified terms in parameterized versions of the conservation equations, which we now discuss.

B. Parameterized Microphysics (Water Continuity) Models

The aim of parameterized, or water continuity models, is to represent the integrated size-dependent microphysical processes in such a way as to preserve their major features but to replace the variables $n_d(m_d)$ and $n_i(m_i)$ by the (far simpler) water category variables listed in Table I. (Note that there are other ways to approximate the size dependent microphysical processes: see, for example, Yau and Austin, 1977 and Lopez, 1973.) There are several parameterization schemes (i.e., sets of equations for these variables) often referred to in the literature. In general, these are based on Kessler's original scheme (Kessler, 1969) and differ mainly in their representation of the ice phase processes. The reader is referred to Orville and Kopp (1977), Stephens (1979) and

Lin et al. (1983), for details. As illustration of these models, we briefly describe that of Stephens (1979), as adapted by Taylor and Baker (1985). In this model

$q_\ell = q_c + q_p$	:	q_c is the specific liquid water content in cloud droplets (those which move with the air) and q_p is the specific liquid water content in precipitation drops. These are usually assumed to be exponentially distributed in radius; i.e., $n_d(r_d) = n_o \exp(-\lambda r_d)$.
$q_i = q_s + q_g$	:	q_s is the specific ice content in snow (low density ice) q_g is the specific ice content in graupel (high density ice)

Each water content variable obeys an equation of type (4), in which the sources and sinks are due to (A) microphysical transformations (B) fallout of precipitation and (C) entrainment. These equations, in a one-dimensional entraining plume model, have the following forms:

$$\frac{Dq_v}{Dt} = -G_c + E_c + E_p - S_{init} - V_{sdep} - V_{gdep} - G_s - \frac{2\alpha}{R}(q_v - q_{v_{env}}) \qquad (24)$$

$$\frac{Dq_c}{Dt} = G_c - E_c - Auto - Coll - S_{melt} - G_{rme} - S_{rme} - C_{sfz} - \frac{2\alpha q_c}{R} \qquad (25)$$

$$\frac{Dq_p}{Dt} = -E_c + Auto + Coll + G_{melt} - P_{gcol} - P_{gfz} + P_{flux} - \frac{2\alpha q_p}{R} \qquad (26)$$

$$\frac{Dq_s}{Dt} = G_s + S_{init} + V_{sdep} + S_{rme} - S_{gcon} - S_{melt} - C_{sfz} + S_{flux} - \frac{2\alpha q_s}{R} \qquad (27)$$

$$\frac{Dq_g}{Dt} = G_{rme} + S_{gcon} - V_{gdep} - G_{melt} + P_{gcol} + P_{gfz} + G_{flux} - \frac{2\alpha q_g}{R} \qquad (28)$$

G_c and G_s are condensation onto nonprecipitating liquid and ice, and E_c and E_p are evaporation of cloud and precipitation liquid. Auto is the rate at which cloud liquid is transformed to precipitation

liquid by condensation; Coll is the rate of increase of precipitation liquid via collection of cloud water. Smelt and Gmelt are rates of melting of snow and graupel and Srme, Grme the rates of riming of snow and graupel. Pgcol is the rate of collection of precipitation by graupel. Sinit is the rate of nucleation of snow crystals and Vsdep the rate of vapor deposition on ice crystals (assumed to be hexagonal plates). Csfz is the rate of homogeneous freezing of cloud droplets to form snow and Pgfz the rate of freezing of supercooled precipitation via collision with ice crystals to form graupel. Sgcon is the rate of conversion of snow to graupel. The terms P_{flux}, S_{flux} and G_{flux} are sedimentation of precipitating particles and the terms in α are entrainment terms. Each of these processes is represented as a semiempirical function of the q's , the temperature T, pressure p, and the air velocity $\underline{V}$. Each water quantity (except q_v) represents an integral over many particles of different sizes, and as such the sources and sinks in the conservation equation represent integrals of the size dependent processes over the range of sizes of the particles.

To give the flavor of the kinds of approximations involved, let us consider the first few terms. The net rate of condensation on cloud drops in an explicit microphysical model would be given by

$$G_c - E_c = \frac{4\pi}{\rho} \rho_\ell \int_{\text{cloud droplets}} n_d(m_d) \left(\frac{\partial m_d}{\partial t}\right) dm_d \qquad (29)$$

with $\frac{\partial m_d}{\partial t}$ given by Equation (11), and the supersaturation s_ℓ (defined in Eq. 8) computed as a function of temperature and pressure (thence, temperature and height). In the Taylor and Baker (1985) parameterization, the net condensation rate is computed as a function of $\Delta q_v(z) \equiv q_v(z+\Delta z) - q_v(z)$ where Δz is the minimum resolvable vertical distance. We have:

$$C_c(z) \equiv \frac{\Delta q_v}{\Delta t}, \quad E_c(z) = 0 \quad \text{if} \quad \Delta q_v(z) > 0;$$

$$C_c(z) = 0, \quad E_c(z) = \frac{\Delta q_v}{\Delta t} \quad \text{if} \quad \Delta q_v(z) < 0.$$

where Δt is the timestep for the computation.

Similar simple approximations are applied for the calculation of each of the other parameterized terms in Eqs. (24)-(28). See also Lord et al. (1984) and Yau (1977) for other representations of these terms.

We note that microphysical processes are important in determining vertical accelerations of cloudy parcels, both because of the drag due to hydrometers and because of the latent heat release during phase change. Moreover, since phase change itself depends on the vertical velocity and acceleration, the dynamics modify the microphysics. Therefore the use of different microphysical parameterization schemes in numerical models can produce very different microphysical and dynamical results.

After this very brief look at the thermodynamic and microphysical framework for cloud models, we now turn our attention to the dynamic equations.

C. Cloud Dynamics

There are two kinds of models for air motions in atmospheric studies; kinematic models, in which the air motions (and often the temperature) are prescribed as functions of space and time, and dynamic models, in which these are computed. Kinematic models are useful when the dynamics are well understood and simple (as in a steady state orographic, or wave cloud) or in which the links between dynamics and other cloud properties are not of first importance. Kessler's (1969) original parameterized cloud model was of this type as were those of Scott and Hobbs (1977) and Rutledge and Hobbs (1983). Kinematic models appear particularly useful in cloud chemistry studies, where the focus is on chemical interactions and the air motions are assumed unaffected by the chemistry.

In dynamical models the air velocity components are computed from Equation (4), putting $\xi = \underline{V}$, so that Equation (4) becomes Newton's Second Law. Ignoring molecular effects, we have

$$\frac{D\underline{V}}{Dt} = -\frac{1}{\rho}\underline{\nabla}p + g\hat{\underline{k}} - 2\underline{\Omega} \times \underline{V} \qquad (30)$$

$\Omega(s^{-1})$ is the angular velocity of the earth; $2\underline{\Omega} \times \underline{V}$ is the 'Coriolis' term, which is important only when the time scales of the phenomena of interest are comparable with or longer than the Coriolis time scale (~ 10^4 s in midlatitudes).

We have now written equations for each of the variables in Table I, and therefore in principle we have a system of equations, which can be solved to give all the cloud variables as functions of space and time. However, because turbulent air motions on all spatial scales contribute to $\underline{V}$, a numerical model with finite resolution cannot completely reproduce the turbulent fluctuations in the velocity field or the other variables.

There are several ways to approximate turbulent fluctuations for the purposes of numerical modeling. One convenient method is to use "volume averaging." A grid size (Δx, Δy, Δz) is chosen for the computation and each dependent variable is written as a sum;

$$\xi(x,y,z,t) = \xi_B(x,y,z,t) + \xi_R(x,y,z,t) + \xi'_{sg}(x,y,z,t) \qquad (31)$$

where ξ_B represents the value of the quantity in a "base" (usually cloud-free) state; ξ_R represents that part of the perturbation in ξ characterized by spatial variations on scales larger than (Δx, Δy, Δz), and ξ'_{sg} is the subgridscale, or non resolvable perturbation.

We expand Equation (1) about the base state to find

$$\frac{(T-T_B)}{T_B} = -\frac{(\rho-\rho_B)}{\rho_B} + \frac{(p-p_B)}{p_B} \qquad (1')$$

For shallow convection the last term is much smaller than the others and the equation of state is usually approximated by the first two terms alone.

We substitute Equation (31) in Equations (30) and (5) and average over a grid volume, assuming for convenience that $\bar{\xi}'_{sg} = \underline{V}_B = 0$, (where the overbar indicates the average). We also usually assume the hydrostatic equation holds in the base state; i.e.,

$$\frac{1}{\rho_B}\nabla p_B = -g\hat{k} \qquad (32)$$

Then we have

$$\frac{D\underline{V}_R}{Dt} \equiv \frac{\partial \underline{V}_R}{\partial t} + \underline{V}_R\cdot\nabla\underline{V}_R = -\overline{\underline{V}'_{sg}\cdot\nabla\underline{V}'_{sg}} - 2\underline{\Omega}\times\underline{V}_R - \frac{1}{\rho_B}\nabla p_R - \frac{\rho_R g\hat{k}}{\rho_B} + \frac{\rho_R\nabla p_B}{\rho_B^2} + \frac{\overline{\rho'_{sg}\nabla p'_{sg}}}{\rho_B^2} \qquad (33)$$

which is usually simplified by assuming the last two terms are negligible. Applying the averaging procedure to Equation (5) yields

$$\frac{\partial \rho_R}{\partial t} + \nabla\cdot\left(\rho_B\underline{V}_R + \rho_R\underline{V}_R + \overline{\rho'_{sg}\underline{V}'_{sg}}\right) = 0 \qquad (5')$$

It can be shown (Dutton and Fichtl, 1969) that $\nabla\cdot\underline{V}'_{sg} = 0$. For motions on time scales long compared with the period of Brunt-Vaisala oscillations in the air Equation (5') can be approximated in general by

$$\nabla\cdot(\rho_B\underline{V}_R) = 0 \quad \text{(the "anelastic" approximation)} \qquad (5'')$$

and for the special case of shallow convection by

$$\nabla\cdot\underline{V}_R = 0 \quad \text{(the "incompressible" approximation)} \qquad (5''')$$

Equation (1') (without the pressure term) plus Equation (5''') constitute the "Boussinesq" approximation, in which the fluid (air) is treated as if it were incompressible except that the density is temperature dependent. The fluctuations in density are thus significant only when multiplied by g, the acceleration due to gravity. In this approximation, we can write Equation (4) for a fluctating cloud variable ξ as follows:

$$\frac{\partial \xi_R}{\partial t} + \underline{V}_R\cdot\nabla\xi_R + \nabla\cdot\overline{\underline{V}'_{sg}\xi'_{sg}} = \overline{\mathrm{Source}_{(\xi)} - \mathrm{Sink}_{(\xi)}} \qquad (4')$$

A great deal of attention has been devoted to parameterization of the subgridscale fluxes in recent years; see, for example, Moeng and Arakawa (1980) and Cotton (1975a). A time-honored approximation, used more for its convenience than its realistic representation of the turbulent motions, is that the effects of small scale turbulence on large scale properties are similar to the effects of molecular diffusion, and therefore that

$$\overline{w'_{sg}\,\xi'_{sg}} = -K_\xi \frac{\partial \xi_R}{\partial z}\,, \text{etc} \qquad (34)$$

K_ξ is called an "eddy diffusivity", and is either prescribed or parameterized in terms of local properties of the flow.

The particular form in which the decomposed equations are written depends on the application. We now discuss models of stratiform and then of convective clouds.

D. Stratiform Clouds

In layer clouds the horizontal spatial scale is much larger than the vertical scale, given by the layer depth. Therefore the variation of base state cloud properties in the horizontal is usually ignored completely and the only turbulent fluxes considered are those in the vertical. Moreover, the conditions hold for the Boussinesq approximation, Equation (5'''). Layer cloud (or fog) models are written in one, two or three dimensions. The higher dimensional models rarely include explicit microphysics, and often

are written as models of the cloudtopped boundary layer (including the subcloud region). The equations for a warm cloud take on the form (dropping the subscripts $_R$ and $_{sg}$)

$$\frac{\partial \underline{V}}{\partial t} + \underline{V}\cdot\nabla\underline{V} + \frac{\partial}{\partial z}(\overline{\underline{V}'w'}) = -\frac{1}{\rho_3}\nabla p - \frac{\rho g \hat{k}}{\rho_3} - 2\underline{\Omega}\times\underline{V} \qquad (35)$$

$$\frac{\partial q_v}{\partial t} + \underline{V}\cdot\nabla q_v + \frac{\partial}{\partial z}(\overline{q_v' w'}) = \dot{q}_* \qquad (36)$$

$$\frac{\partial q_\ell}{\partial t} + \underline{V}\cdot\nabla q_\ell + \frac{\partial}{\partial z}(\overline{q_\ell' w'}) = -\dot{q}_* \qquad (37)$$

$$\frac{\partial T}{\partial t} + \underline{V}\cdot\nabla T + \frac{\partial}{\partial z}(\overline{T' w'}) = \frac{\dot{H}}{c_p\left[1 + \frac{L_v}{c_p}\frac{\partial q_*}{\partial T}\right]} \qquad (38)$$

where $q_*(T,p) \equiv \frac{R_d}{R_v}\frac{e_{s\ell}}{p_e}$ and we have assumed the layer is always saturated (see Eq. 31). $\dot{q}_*$ is computed from Equations (3), (32), and (37). The radiative term $\dot{H}$ in Equation (38) is either prescribed or calculated from the temperature and water category distributions. (See Herman and Goody, 1976; Paltridge and Platt, 1976, for example.) In layer clouds entrainment occurs along the upper surface and is sometimes included as a boundary condition. (Self-consistent modeling of entrainment constitutes one of the most active areas of layer cloud studies today.)

E. Convective Clouds

In convective clouds the driving force is the generation of density fluctuations which create large up- and down drafts. These can no longer be treated as small fluctuations on a quiescent state. Moreover, the assumption of infinite horizontally homogeneous layers is of course no longer valid. Thus dynamic modeling of convective clouds requires different approximations than that of stratiform clouds.

Whereas numerical layer cloud models are usually written in an Eulerian (fixed) coordinate system, which allows the properties of the entire cloud to be calculated at each time step, it is often desirable in convective cloud models to use (Lagrangian) coordinates fixed in a moving air parcel, and to follow the evolution of properties inside that parcel while neglecting the rest of the cloud. The Lagrangian approach is particularly convenient if we can assume (a) the cloud properties at a fixed location are independent of time, or equivalently, (b) that the properties inside the moving parcel change with time only because the parcel location changes. Then $D\xi/Dt = \underline{V}\cdot\underline{\nabla}\xi$ on the LHS of equation (4). If the parcel path is along the z-axis, then $\frac{D\xi}{Dt} = w\frac{\partial\xi}{\partial z}$.

Let us consider this simple, often used approximation, applied to a shallow, warm, nonprecipitating convective cloud with parameterized microphysics.

Because we are writing the equations for properties inside a finite parcel, we must consider flow across the boundary of the parcel; i.e., entrainment. Using the self-similar plume approximation (Eq. 19) for the entrainment, the cloud model equations using Stephens' notation (see Eqs. 24-28) become

$$w\frac{\partial w}{\partial z} = \frac{g(\rho_{env}-\rho)}{\rho_{env}} - \frac{2\alpha w^2}{R} = g\frac{(T_{v_{env}}-T_v)}{T_{v_{env}}} - \frac{2\alpha w^2}{R} \quad (39)$$

$$w\frac{\partial q_v}{\partial z} = -G_c + E_c - \frac{2\alpha w}{R}(q_v - q_{v_{env}}) \quad (40)$$

$$w\frac{\partial q_c}{\partial z} = G_c - E_c - \frac{2\alpha w}{R} q_\ell \quad (41)$$

$$w\frac{\partial T}{\partial z} = -\frac{gw}{c_p} - \frac{L_v}{c_p} w\frac{\partial q_v}{\partial z} - \frac{2\alpha w}{R}\left[T - T_{env} + \frac{L}{c_p}(q_v - q_{v_{env}})\right] \quad (42)$$

where $R = 6/5\,\alpha z$, and $\xi_e = \xi_{environment}$.

See Figure 4 for results computed from this model.

To include explicit microphysics we would replace the terms $G_c - E_c$ by an integral as in Equation (29). See Warner (1973), and Lee and Pruppacher (1977), as examples of the use of this model.

If it is necessary to describe phenomena occurring simultaneously in different parts of a cloud, if the time development at fixed locations is important, and/or if the fallout of precipitation through a cloud is of interest, then a Lagrangian model, which follows only one air 'parcel', is inadequate. In this case an Eulerian 1-D, 2-D or 3-D model must be used. See the reviews by Simpson (1976) and Cotton (1975b) for comparisons of various cumulus models.

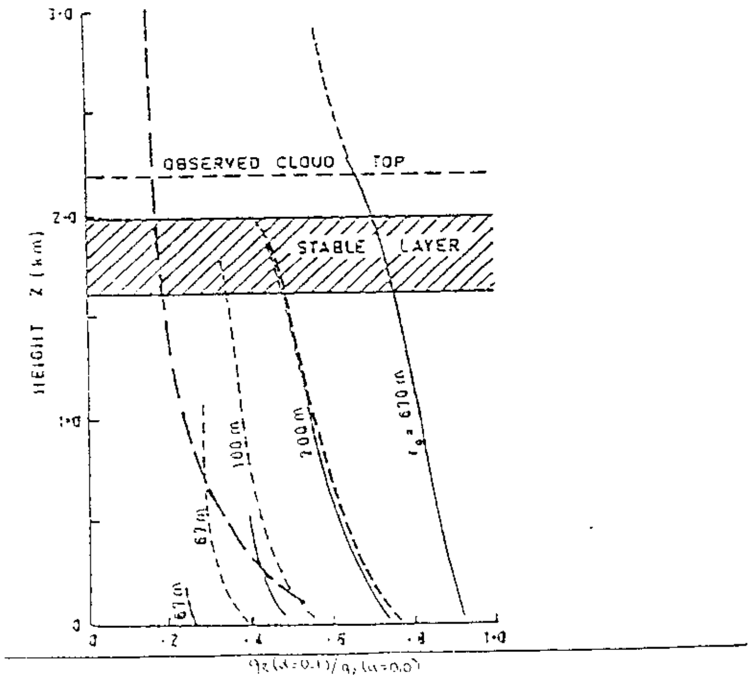
In one-dimensional Eulerian models (Ogura and Takahashi, 1971) the cumulus cloud is usually assumed to grow as a right circular cylinder of radius R. All the dependent variables (ξ = T,u,v,w,q_v,q_i, etc.) inside the cloud are written

$$\xi(z) = \overline{\xi(z)} + \xi'(z)$$

where $\bar{\xi}(z)$ is the horizontal average defined by the equation

$$\bar{\xi}(z) \equiv \frac{1}{\pi R^2}\int_0^{2\pi} d\theta \int_0^R r\,\xi(r,\theta)\,dr \quad (43)$$

Figure 4 Computed cloud properties obtained from Equations (39)-(42) with α=0.1 assuming the cloudy air remains exactly saturated throughout. The abscissa is the ratio of liquid water content, q_ℓ, to the value obtained if there is no entrainment (i.e., α = 0.0). The solid and light dashed curves differ in assumed initial updraft velocity and temperatures and are marked with the value of the initial radius. The lines terminate at the computed cloudtop. The heavy dashed line shows the observed values. From Warner (1970). Note that none of the computed curves comes close to the observations, thus casting doubt on the entrainment hypothesis, Equation (18).



We also define a perimeter average;

$$\tilde{\xi}_R \equiv \frac{1}{2\pi}\int_0^{2\pi} \xi(R,\theta)\,d\theta \qquad (44)$$

and we can define the deviation ξ'' by the equation

$$\xi(r=R) = \tilde{\xi}_R + \xi''.$$

Assuming the anelastic approximation (Eq. 5"), multiplying Equation (4) by ρ_{env} and integrating over the cylinder yields the following conservation equation for $\bar{\xi}$:

$$\frac{\partial\bar{\xi}}{\partial t} + \frac{1}{\rho_{env}}\frac{\partial}{\partial z}\left(\rho_{env}\bar{w}\bar{\xi}\right) + \overbrace{\frac{2}{R}\tilde{u}_R\tilde{\xi}_R}^{A} + \overbrace{\frac{2}{R}\widetilde{u_r''\xi''}}^{B} + \overbrace{\frac{1}{\rho_{env}}\frac{\partial}{\partial z}\left(\rho_{env}\overline{w'\xi'}\right)}^{C} = \text{Source}_{(\xi)} - \text{Sink}_{(\xi)} \qquad (45)$$

where $\tilde{\xi}_R = \xi_{env}$ if $u_r(R) < 0$: $\tilde{\xi}_R = \bar{\xi}$ if $u_r(R) \geq 0$. The terms (A, B, and C) represent entrainment and mixing. A is 'dynamic' entrainment'; inflow due to acceleration in the vertical; term B is lateral mixing and C is vertical mixing. B and C must be further parameterized to be able to solve these equations. Because convective clouds are in general highly turbulent, these terms can be quite large, and their realistic representation in numerical models is therefore important: unfortunately, it is also very difficult.

The approximations involved in (i) averaging over the horizontal cloud area (Eq. 43), which eliminates small scale variability, and (ii) parameterization of the terms B and C, constitute major weaknesses in one-dimensional cloud models. Moreover, the "dynamic entrainment," term A, is due to the constraint that R = constant and is therefore fairly nonphysical. However, despite these shortcomings, the relative ease in constructing and using 1-D models makes them quite attractive for many purposes. Often C is put equal to zero and B is written

$$\frac{2\widetilde{u_r''\xi''}}{R} = -\frac{2\beta}{R}|\bar{w}|(\xi_{env} - \bar{\xi})$$

where β is a constant.

(See Ogura and Takahashi, 1971; Cotton, 1975b.) Then for warm clouds the cloud model becomes

$$\frac{\partial \bar{w}}{\partial t} + \bar{w}\frac{\partial \bar{w}}{\partial z} = -\frac{1}{\rho_{env}}\frac{\partial \bar{p}^*}{\partial z} + g\frac{(\rho_{env} - \bar{\rho})}{\rho_{env}} - \frac{2\beta|\bar{w}|\bar{w}}{R} + \frac{2\tilde{u}_r(\bar{w} - \hat{w}_R)}{R} \quad (47)$$

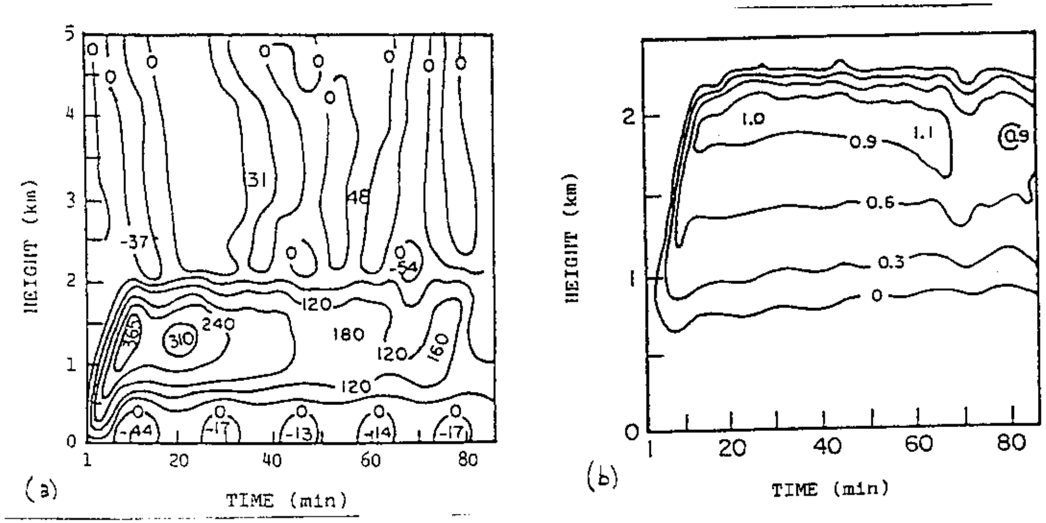
$$\frac{\partial \bar{q}_v}{\partial t} + \bar{w}\frac{\partial \bar{q}_v}{\partial z} = E_c - G_c - \frac{2\beta|\bar{w}|}{R}(\bar{q}_v - q_{v_{env}}) + \frac{2\tilde{u}_r}{R}(\bar{q}_v - \hat{q}_{v_R}) \quad (48)$$

$$\frac{\partial \bar{h}}{\partial t} + \bar{w}\frac{\partial \bar{h}}{\partial z} = -\frac{2\beta|\bar{w}|}{R}(\bar{h} - h_{env}) + \frac{2\tilde{u}_r}{R}(\bar{h} - \hat{h}_R) \quad (49)$$

where $\bar{p}^*$ is the deviation of the average pressure from the hydrostatic value obtained from Equation (32). (See Fig. 5 for some results obtained from this model.)

In higher dimensional models we solve the equations for the variables in Table I without the artifical constraint that the cloud occupy a cylinder or thermal or plume. The Boussinesq approximation is used for shallow cumulus but for deep convection the anelastic approximation Equation (5") is necessary. In general, because of the complexity of the dynamics in higher dimensional models, the microphysics is highly parameterized. (See, for example, Wilhelmson, 1974; Wilhelmson & Gagne, 1982; Soong and Ogura, 1973; Cotton, 1972; Yau, 1980.) Takahashi, (1981) and Hall, (1980) have developed explicit microphysical models of warm precipitating cumulus which are exceptions.

Figure 5 a) Time-dependent vertical velocity (cm/s) computed for a marine cloud from a model based on Equations (47)-(49) with explicit cold cloud microphysics, and the turbulent fluxes parameterized by expressions similar to Equation (34). b) As in a), for liquid water density, ρq_l (g/m^3). From Scott and Hobbs (1977). Note the time scales and the abrupt fall off of vertical velocity near cloudtop, both consequences of the fixed radius assumption.



III. Access to Numerical Models

This concludes our very brief overview of the components of cloud models. We have not dealt at all with many important topics such as modeling radiation, cloud chemistry, or cloud systems, and in the limited space available we have been able only to give very superficial descriptions of the more standard cloud model components. The reader interested in using a numerical cloud model should be aware that many of the models discussed here and in the references are coded and stored on magnetic tape at various European and American universities and atmospheric research centers such as NCAR (USA) and the British Meteorological Office (UK). Scientists at these institutions may be contacted for advice on remote access to their computers and programs, as well as for requests for collaborative projects using these facilities.

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