INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

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WORKSHOP ON OPTICAL FIBER COMMUNICATION
(24 February - 21 March 1986)

PROPAGATION THEORY IN OPTICAL FIBERS

presented by

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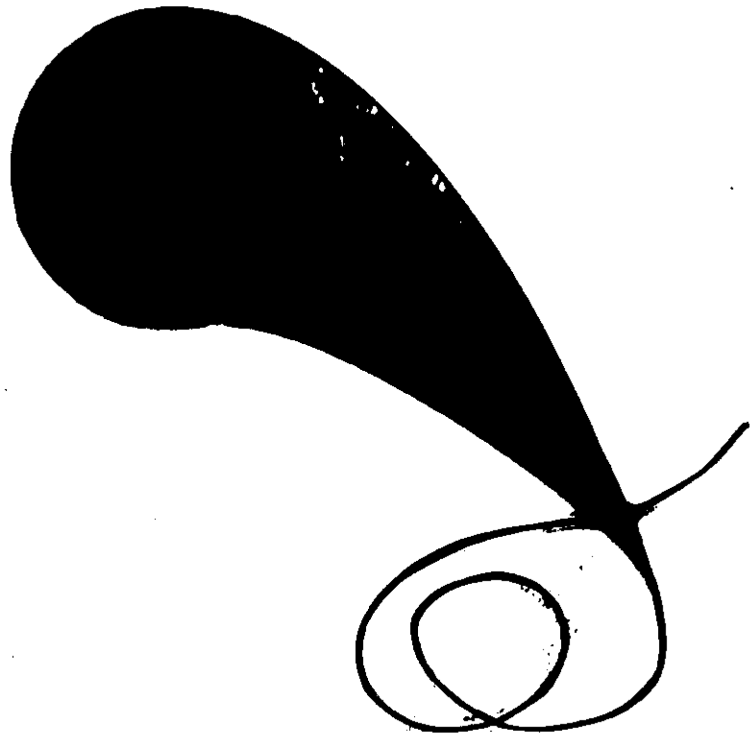
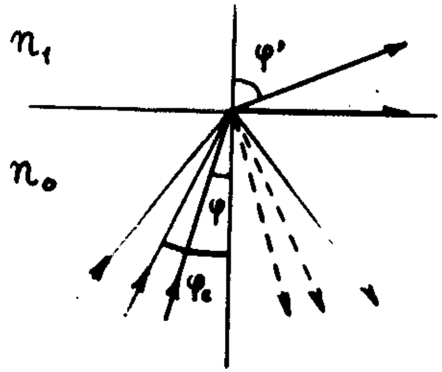
THE OPTICAL FIBRE



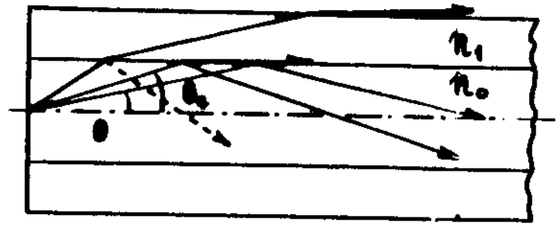
	REFRACTIVE INDEX	DIAMETER
CLADDING	$n_1 \simeq 1.46$	$\sim 125\ \mu m$
CORE	$n_1 + 5\text{-}10‰$	$\sim 5\text{-}10;\ 50\ \mu m$

TOTAL INTERNAL REFLECTION



$n_0 > n_1$

* SNELL'S LAW : $n_0 \sin\varphi = n_1 \sin\varphi'$
* LIMITING CONDITION: $\varphi' = 90° \Rightarrow n_0 \sin\varphi_c = n_1$
* TOTAL REFLECTION FOR $\varphi > \varphi_c$



$\theta = 90° - \varphi$

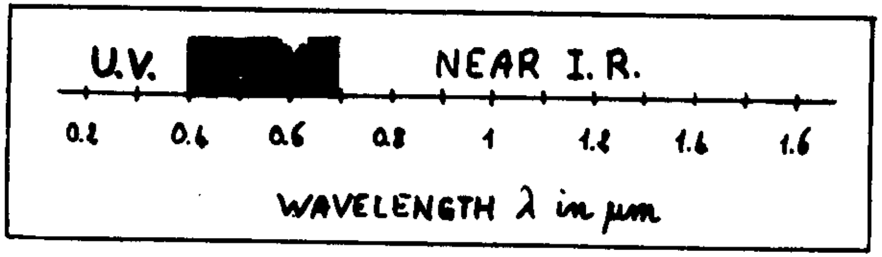
LIMITING ANGLE FOR GUIDED RAYS: $\theta_c = 90° - \varphi_c \Rightarrow \cos\theta_c = \frac{n_1}{n_0}$

GUIDED RAY (TOT. INT. REFLECTION) FOR $\theta < \theta_c$

$$\boxed{\text{NUMERICAL APERTURE: } A = n_0 \sin\theta_c \qquad A = \sqrt{n_0^2 - n_1^2} \simeq \sqrt{2n_1(n_0 - n_1)}}$$

($A \sim 0.1\text{-}0.2$ typic.)

LIGHT: AN ELECTROMAGNETIC WAVE WITH $\nu \sim 10^{14}$ Hz



⇒ Interference And Diffraction Phenomena ⇒

The E.M. Energy Propagates Trough The Fibre According To Well Defined Ways:

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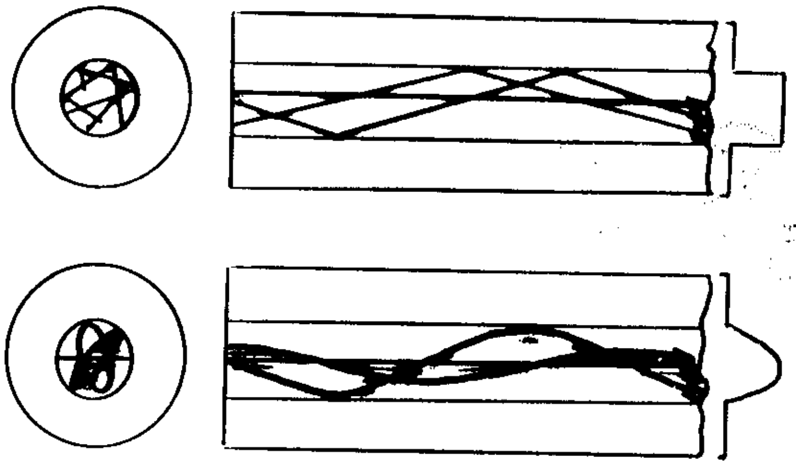
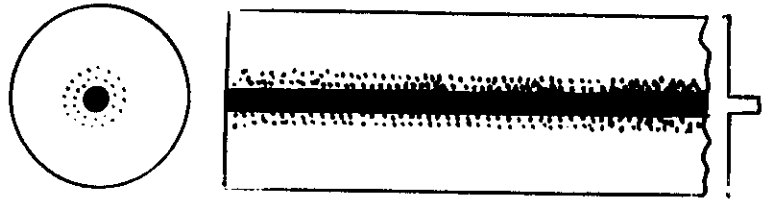
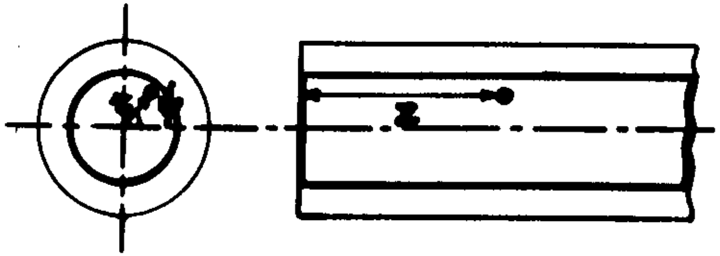
FIBRE TYPES

Multimode Fibre



Single-Mode Fibre





[redacted]

[redacted]

[redacted]

$$\begin{cases} \vec{E}(r,\psi,z;t) = \vec{E}(r)\ \exp[-i(\nu\psi+\beta z-\omega t)] \\ \vec{H}(r,\psi,z;t) = \vec{H}(r)\ \exp[-i(\nu\psi+\beta z-\omega t)] \end{cases}$$

- PROGRESSIVE WAVE : $\exp[-i(\quad \beta z \quad)]$ $(\beta>0)$
- AZIMUTHAL PERIODICITY: $\exp[-i(\nu\psi \quad)]$

$(\nu = 0, 1, 2, \dots)$

RADIAL PART of the FIELD:

$$\boxed{\begin{cases} \vec{E}(r) = \vec{E}_t(r) + \hat{z}\,E_z \\ \vec{H}(r) = \vec{H}_t(r) + \hat{z}\,H_z \end{cases}}$$

SCALAR WAVE EQUATION: $\vec{E}_t \to \phi(r)$

$$\boxed{\phi''(r) + \frac{\phi'(r)}{r} + \left[\left(\frac{2\pi}{\lambda}\right)^2 n^2(r) - \beta^2 - \left(\frac{\nu\pm 1}{r}\right)^2\right]\phi(r) = 0}$$

$$\begin{cases} E_z = -\frac{i}{k}\left(E_r' \pm \frac{\nu\pm 1}{r}\,E_r\right) \\ H_z = -\frac{i}{k}\left(H_r' \pm \frac{\nu\pm 1}{r}\,H_r\right) \end{cases} \qquad \left[k(r) = \frac{2\pi}{\lambda} \dots\right.$$

$$E_z(r) = \begin{cases} A_0\,\phi_0(r) & ,\ r<a \\ A_1\,\phi_1(r) & ,\ r>a \end{cases}$$

CONTINUITY CONDITIONS at $r=a$:

$$\frac{\phi_0'(a)}{\phi_0(a)} = \frac{\phi_1'(a)}{\phi_1(a)}$$

CHARACTERISTIC EQUATION

FOR EACH ν-VALUE → TWO FAMILIES OF SOLUTIONS:

$[\nu \pm 1]$ [redacted] (UPPER SIGN)

[redacted] (LOWER SIGN)

• $\nu = 0$ $\begin{cases} EH_{0\delta} \equiv \underline{TE_{0\delta}} \; : \; E_z = E_r = H_\psi = 0 \\ HE_{0\delta} \equiv \underline{TM_{0\delta}} \; : \; H_z = H_r = E_\psi = 0 \end{cases}$

• $\nu \neq 0$ $\begin{cases} EH_{\nu\delta} \; : \; H_z = -i\sqrt{\frac{\varepsilon}{\mu}}\, E_z \, ; \; E_r = -iE_\psi \, ; \; H_r = -iH_\psi \\ HE_{\nu\delta} \; : \; H_z = i\sqrt{\frac{\varepsilon}{\mu}}\, E_z \, ; \; E_r = iE_\psi \, ; \; H_r = iH_\psi \end{cases}$

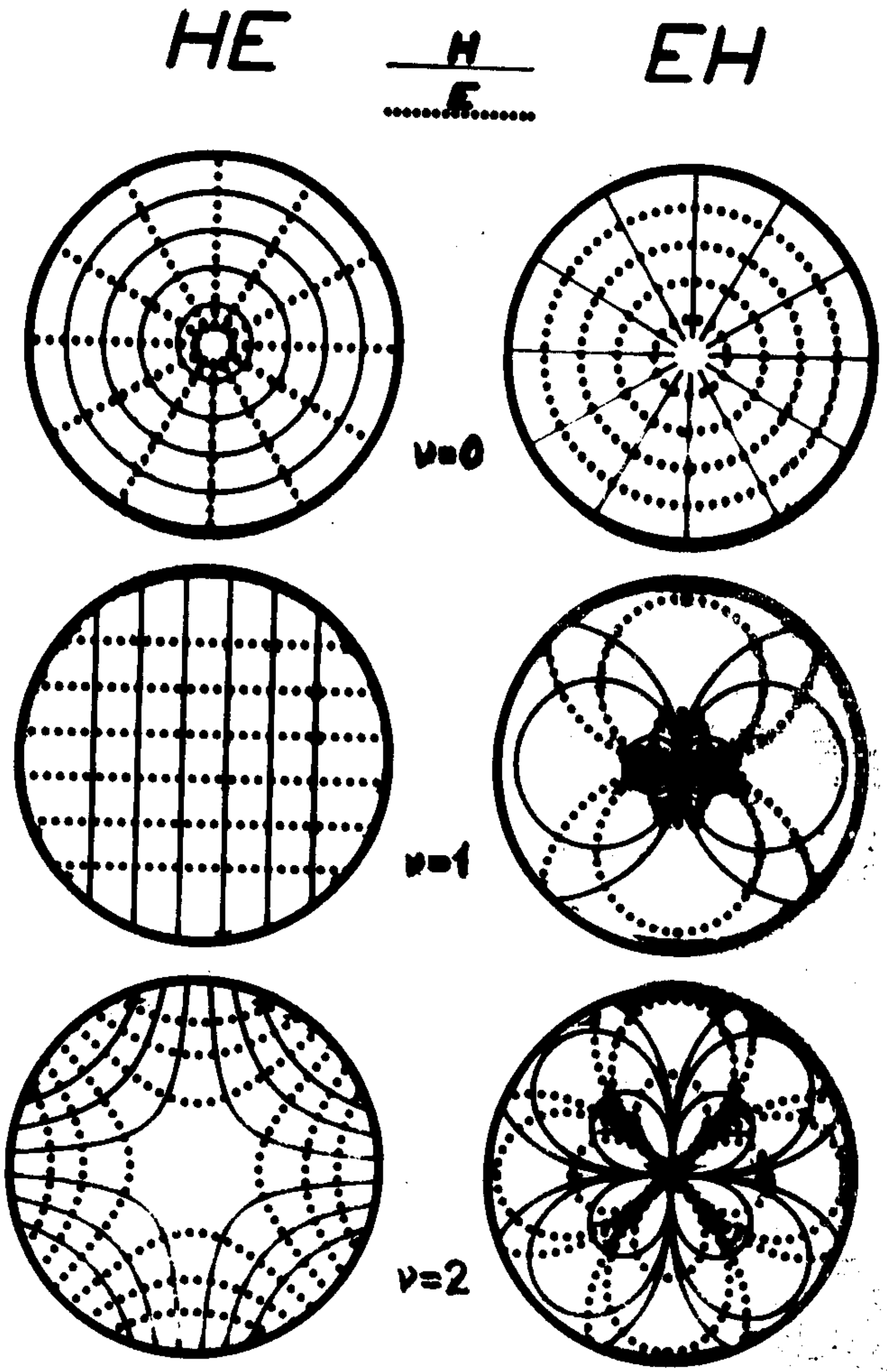
$$H_r = -\sqrt{\frac{\varepsilon}{\mu}}\, E_\psi \quad ; \quad H_\psi = \sqrt{\frac{\varepsilon}{\mu}}\, E_r$$

$$|H_z| \ll |H_r|, |H_\psi| \quad ; \quad |E_z| \ll |E_r|, |E_\psi|$$

$$\vec{H} \simeq \sqrt{\frac{\varepsilon}{\mu}}\, \hat{z} \times \vec{E}$$

LINES OF FORCE OF $\vec{E}$: $r(\psi)$

(Real Parts) $\dfrac{E_r}{E_\psi} = \dfrac{r'}{r} = \begin{cases} -\tan \nu\psi \rightarrow EH_{\nu\delta} \\ +\tan \nu\psi \rightarrow HE_{\nu\delta} \end{cases}$



MODE DEGENERACY: LP MODES

- $HE_{\nu+1\,\delta}$ and $EH_{\nu-1\,\delta}$ FOLLOW THE SAME EQUATION

→ THEY HAVE THE SAME β and PROPERTIES

COMPOSED MODE:

[redacted]

LINEARLY POLARIZED

- $\nu=0$ ONLY ONE MODE $LP_{0\delta} \equiv HE_{1\delta}$

- $\nu=1$ THREE MODES COMBINED:

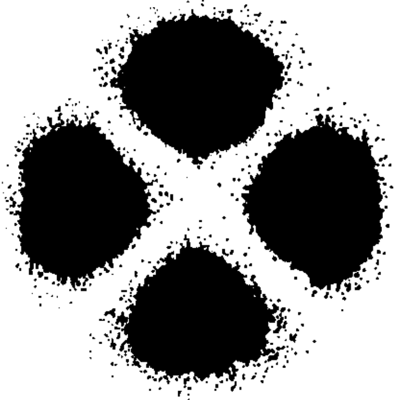
$$LP_{1\delta} \equiv \begin{cases} HE_{2\delta} + TE_{0\delta} \\ HE_{2\delta} + TM_{0\delta} \end{cases}$$

PHYSICAL MEANING OF ν and δ

- 2ν NUMBER OF MAXIMA OF THE FIELD INTENSITY IN A TURN AROUND THE FIBRE AXIS
- δ NUMBER OF MAXIMA OF THE FIELD INTENSITY IN RADIAL DIRECTION

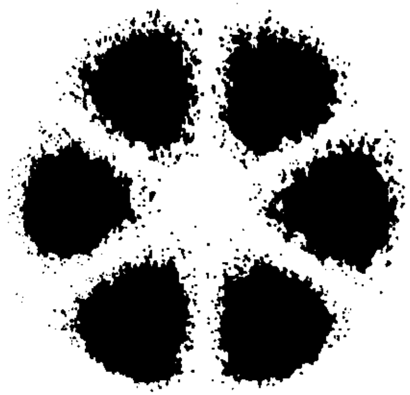
LP_{01}
(HE_{11})

LP_{11}
$(TE_{01}+TM_{01}+HE_{21})$

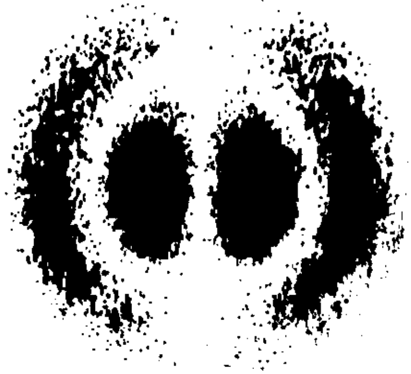


LP_{21}
$(EH_{11}+HE_{31})$

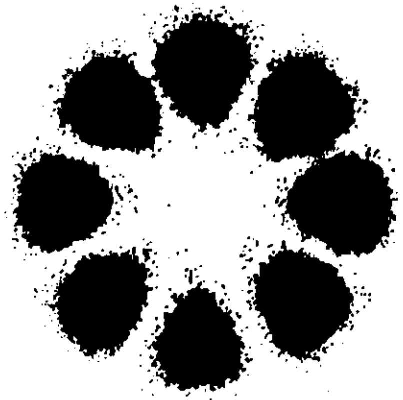
LP_{02}
(HE_{12})



LP_{31}
$(EH_{21}+HE_{41})$



LP_{12}
$(TE_{02}+TM_{02}+HE_{22})$

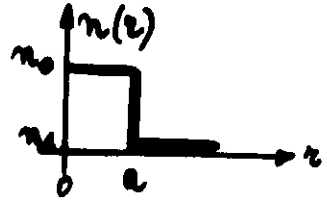


LP_{41}
$(EH_{31}+HE_{51})$

LP_{22}
$(EH_{12}+HE_{32})$

STEP INDEX FIBRES

- $k^2(r)=\left(\frac{2\pi}{\lambda}\right)^2 n^2(r) \rightarrow n^2(r)=\begin{cases} n_0^2 & , r<a \\ n_1^2 & , r>a \end{cases}$



- $LP_{\nu\delta}$:

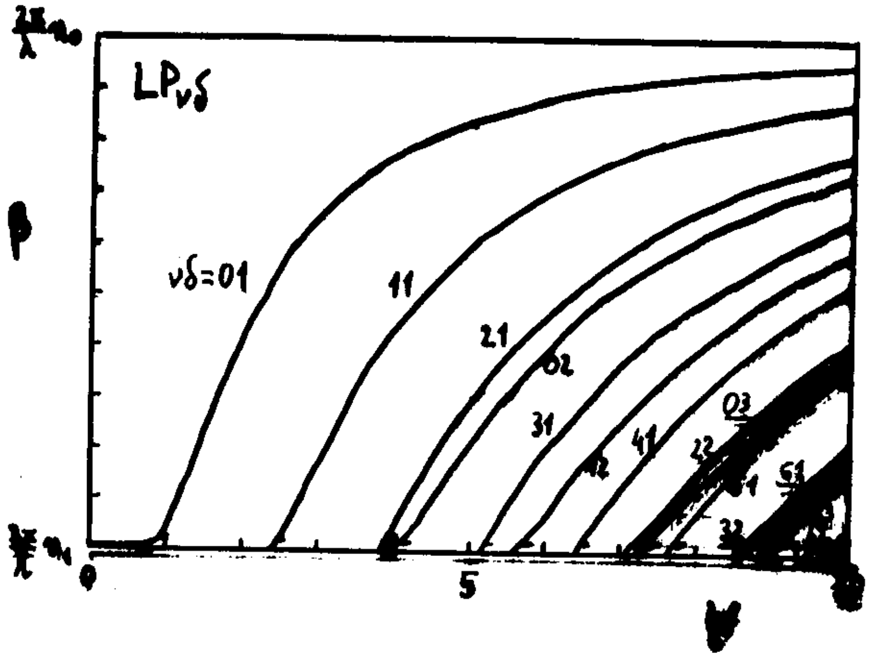
$$E_t \propto \begin{cases} \phi_0(r)=J_\nu(\kappa r), r<a & - \left[\kappa=\sqrt{\left(\frac{2\pi}{\lambda}n_0\right)^2-\beta^2}\right] \\ \phi_1(r)=K_\nu(\gamma r), r>a & - \left[\gamma=\sqrt{\beta^2-\left(\frac{2\pi}{\lambda}n_1\right)^2}\right] \end{cases}$$

- $a^2(\kappa^2+\gamma^2)=V^2$

$$\boxed{V=\underbrace{\frac{2\pi}{\lambda}a\sqrt{n_0^2-n_1^2}=\frac{2\pi}{\lambda}a\,NA}_{\text{NORMALIZED FREQUENCY}}}$$

CHARACTERISTIC EQUATION

$$\frac{\phi_0'(a)}{\phi_0(a)}=\frac{\phi_1'(a)}{\phi_1(a)} \Rightarrow \beta(V) \text{ for each } LP_{\nu\delta} \text{ mode}$$



CUT-OFF

$LP_{\nu\delta}$ MODE IS GUIDED ONLY IF:

$$V > V_{\nu\delta}^{co} \quad \text{or} \quad \lambda < \lambda_{\nu\delta}^{co} = \frac{2\pi a A}{V_{\nu\delta}^T}$$

[redacted] CUT-OFF WAVELENGTH OF $LP_{\nu\delta}$ MODE

IF $V < V_{11}^{co}$ ($\lambda > \lambda_{11}^{co}$) FIBRE GUIDES ONLY [redacted] : FUNDAMENTAL MODE

FIBRE IS **SINGLE-MODE**
(OTHERWISE IT IS MULTIMODE)

SINGLE-MODE FIBRES

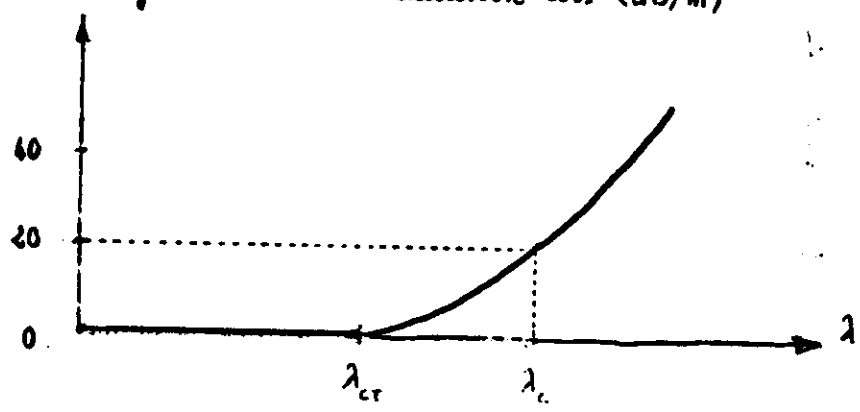
CUT-OFF WAVELENGTH

SINGLE-MODE FIBRE if $V < V_{11}^T$

$V = \frac{2\pi a}{\lambda} A$ (A- numerical apeture $\sqrt{n_0^2 - n_1^2} \simeq 2n_1\Delta n$)

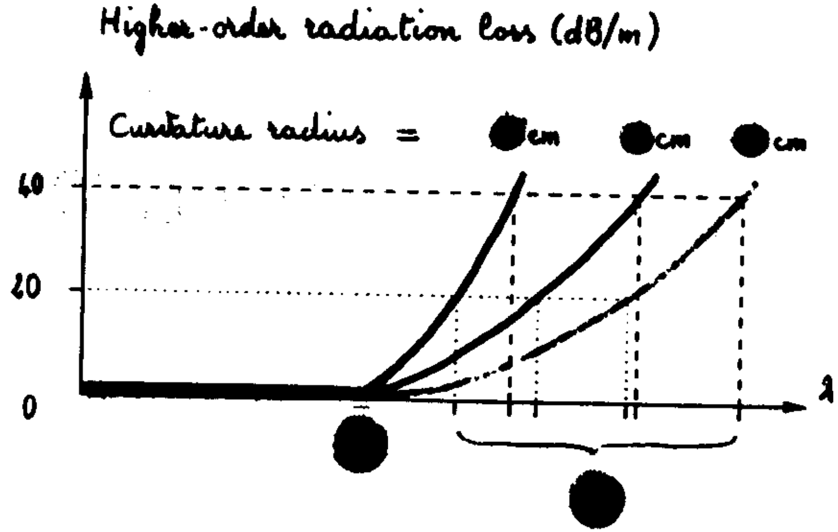
$V_{11}^T = \frac{2\pi a}{\lambda_{CT}} A \Rightarrow \lambda > \lambda_{CT}$

- higher-order mode theoretical cut-off wavelength



MULTIMODE | ? | SINGLE-MODE

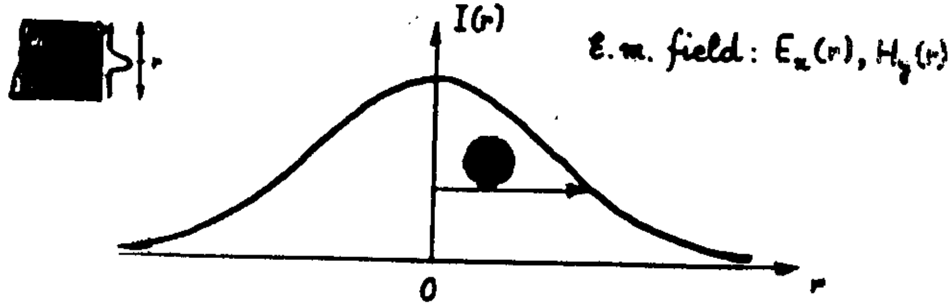
- higher-order mode effective cut-off wavelength



CCITT Normalization : [redacted]

Curvature radius	=	[redacted]	cm
Sample length	=	[redacted]	m

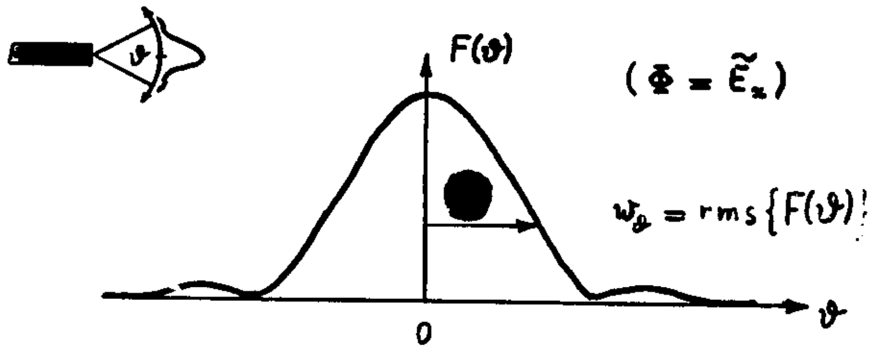
NEAR-FIELD INTENSITY: $I(r) \propto E_x^2(r), H_y^2(r)$



* Mode (field) radius (spot-size) 1

[redacted]

FAR-FIELD INTENSITY: $F(\vartheta) \propto |\Phi(\vartheta)|^2$



* Mode radius (spot-size) 2

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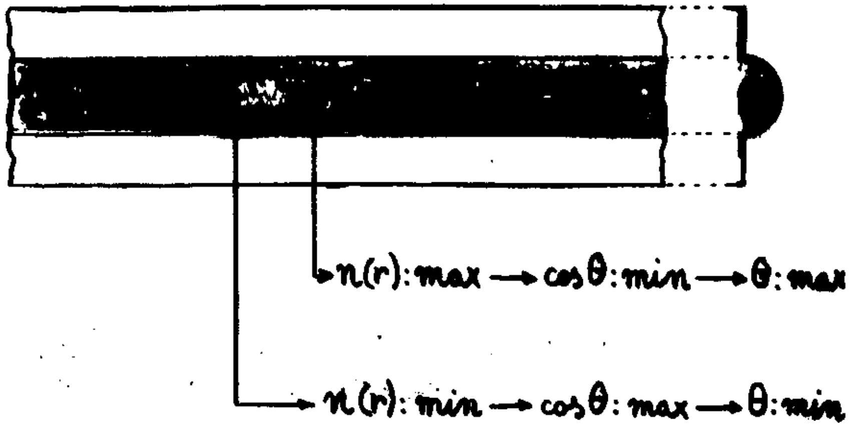
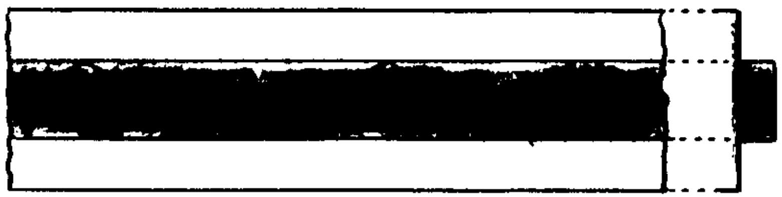
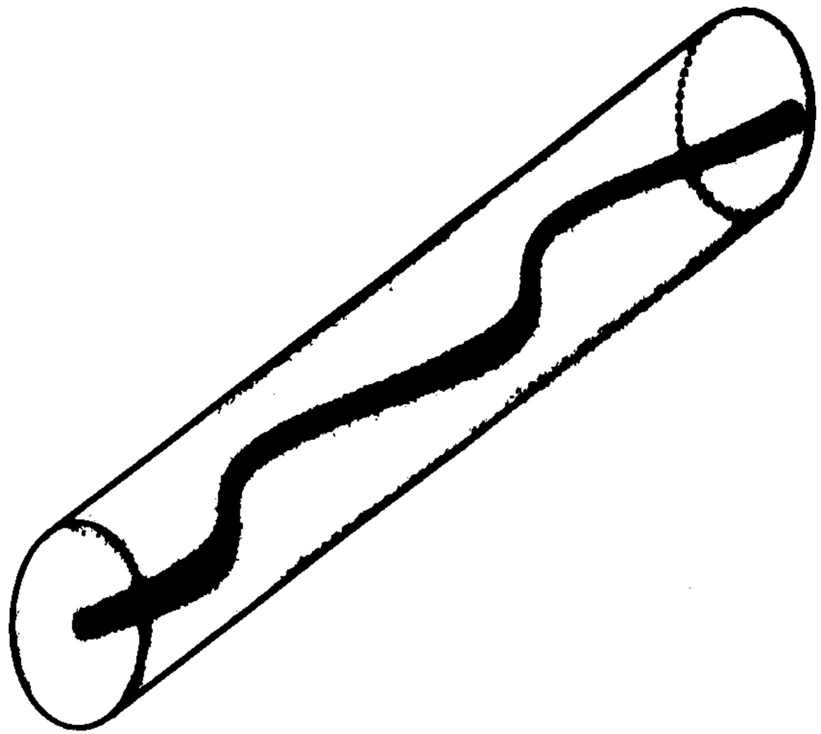
MULTIMODE FIBRES

$$\beta = \frac{2\pi}{\lambda} n(r) \cos\theta = \text{constant}$$

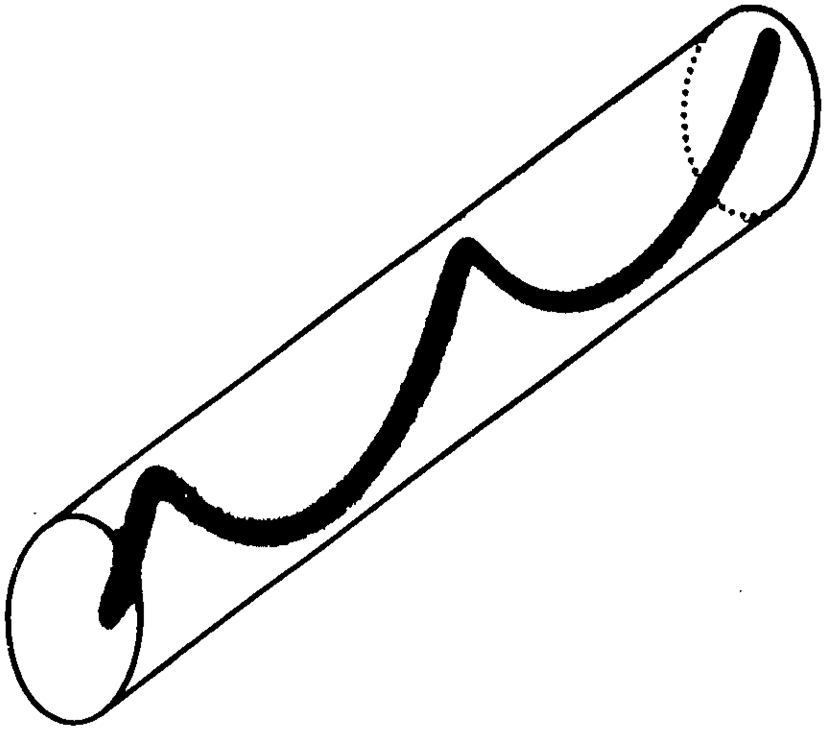




$n(r) = \text{const.} \rightarrow \theta = \text{const.}$



RAGGIO MERIDIANO IN FIBRA
A PROFILO PARABOLICO



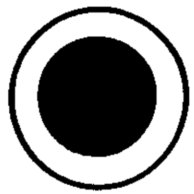
RAGGIO SGHEMBO IN FIBRA
A PROFILO PARABOLICO

CONGRUENZA DI RAGGI

(INSIEME DEI RAGGI CON STESSI ν e β)

- OGNI RAGGIO OTTENUTO DA UN ALTRO MEDIANTE:
  - TRASLAZIONE LUNGO L'ASSE DELLA FIBRA

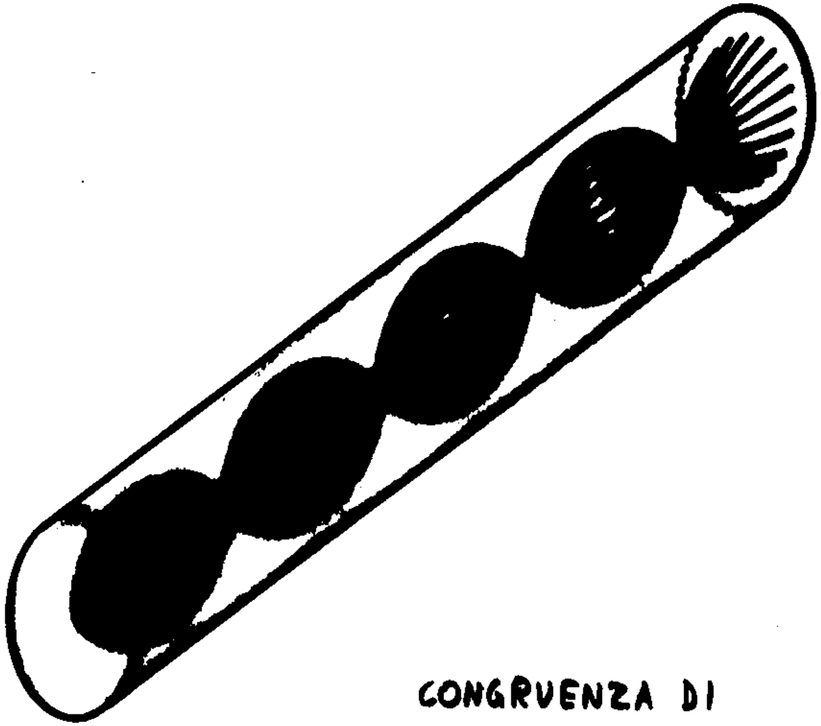
  - ROTAZIONE ATTORNO ALL'ASSE DELLA FIBRA



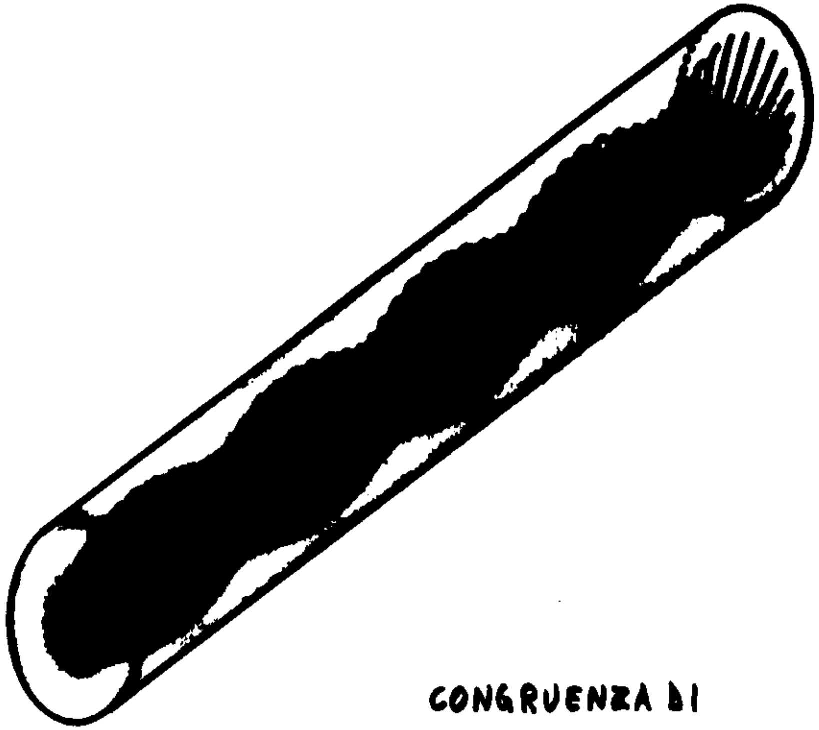
- TALI RAGGI HANNO TUTTI
  LE STESSE PROPRIETA':

PER $\lambda \ll a$ $(V \gg 1)$ CORRISPONDE:

A <u>UN MODO $\leftrightarrow$ UNA CONGRUENZA DI RAGGI</u>



CONGRUENZA DI
RAGGI MERIDIANI IN FIBRA
A PROFILO PARABOLICO



CONGRUENZA DI
RAGGI SGHEMBI IN FIBRA
A PROFILO PARABOLICO

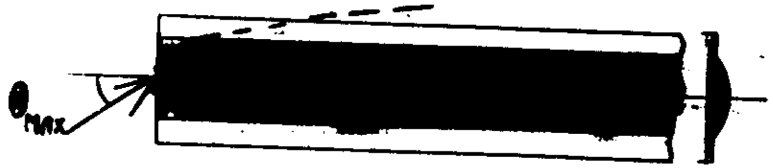
FIBRE NUMERICAL APERTURE

$$A = n \cdot \sin \theta_{MAX}$$

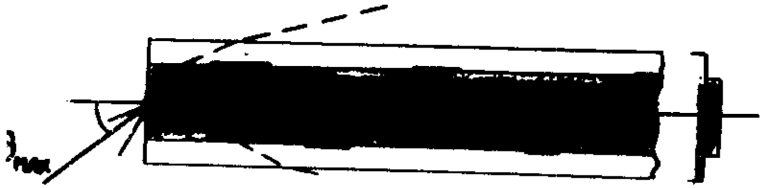
(CUTOFF COND.: $\beta = \frac{2\pi}{\lambda} n(r) \cos\theta_{MAX} = \frac{2\pi}{\lambda} n_{MIN}$)

$$\boxed{A(r) = \sqrt{n^2(r) - n^2_{MIN}}}$$

$\left(A_M = \sqrt{n^2_{MAX} - n^2_{MIN}} = \begin{cases} \sim 0.1 & \text{SINGLE-MODE} \\ \sim 0.2 & \text{MULTIMODE} \end{cases}\right)$



GRADED-INDEX PROF. FIBRES: CONTIN. INTERN. REFRACTION



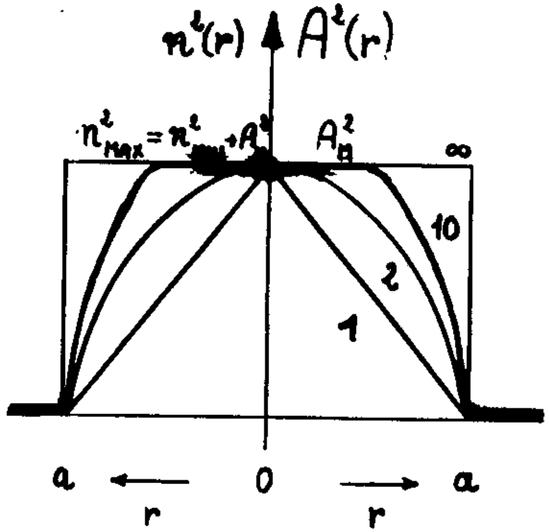
STEP-INDEX PROF. FIBRES: TOTAL INTERN. REFLECTION

$$V = \frac{2\pi}{\lambda} a A_M$$

• "α"- PROFILES:

$$n^2(r) = n^2_{MIN} + A^2_M\left[1 - \left(\frac{r}{a}\right)^\alpha\right] \quad r \leq a$$

- $\alpha = 2$: PARABOLIC PROFILE
- $\alpha \to \infty$: STEP PROFILE



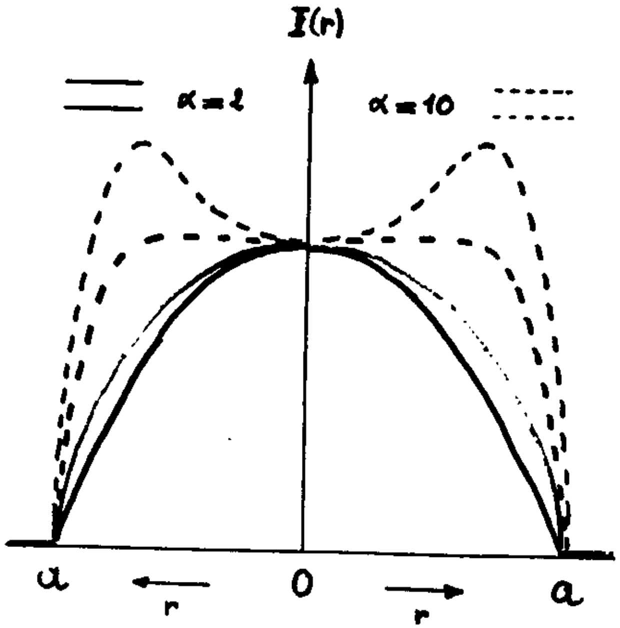
NEAR-FIELD INTENSITY

(MULTIMODE FIBRES WITH $R(x) = R_0$)

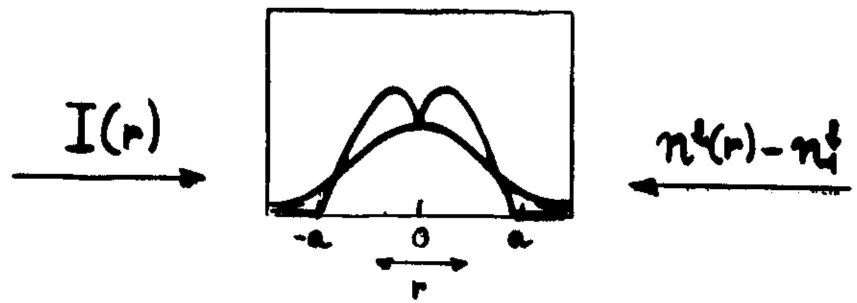
- $I(r) \propto [n^2(r) - n^2_{MIN}]$: Guided Rays Only
- $I(r) \propto \left(\dfrac{n^2(r) - n^2_{MIN}}{\sqrt{1 - r^2/a^2}} \right)$: Leaky Rays Too

➡ Measurement Of $n^2(r)$

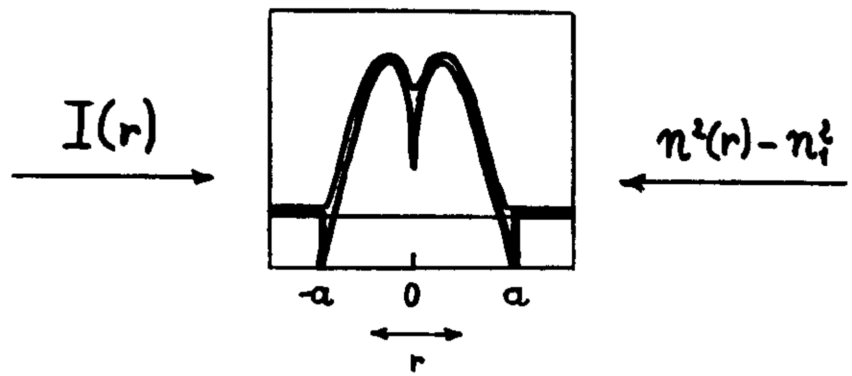
▼



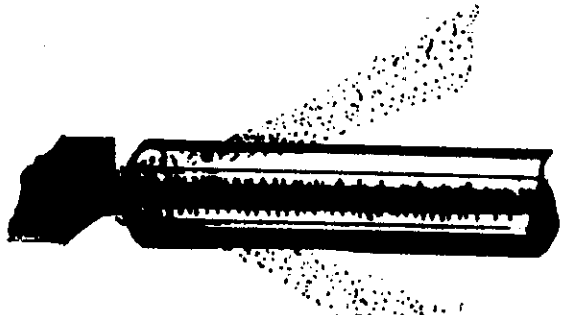
• FEW-MODE FIBRES



• FIBRES OF PARTICULAR PROFILES



LAUNCHING AND COUPLING PROBLEMS



$$W_e = W_o + W_i$$

EMITTED POWER ➡ GUIDED POWER ✚ RADIATED (LOST) POWER

* LAUNCHING EFFICIENCY: $\Lambda = W_o/W_e$ (% typ.)
* COUPLING LOSS: $L = W_i/W_e$ (dB typ.)

SINGLE-MODE FIBRES:

$I_e(r)$ – Emitted power radial distribution

$\boxed{\Lambda \text{ Depends On The “Likeness” Of } I_e(r) \text{ And } I(r)}$

$$\Lambda = [\text{Overlap integral between } I_e \text{ and } I]^2$$

* Ty[illegible]

SOURCE-FIBRE COUPLING

* **LAUNCHING IN MULTIMODE FIBRES**

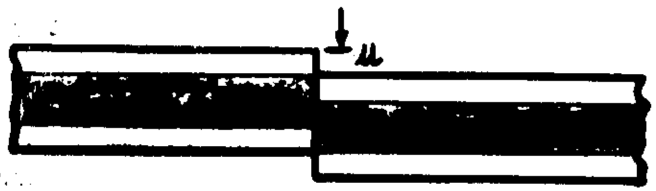
$$\Lambda \propto \frac{A_M^2}{A_{sorg}^2} \cdot R_e \cdot f(\text{PROF.}) \cdot a^2,$$

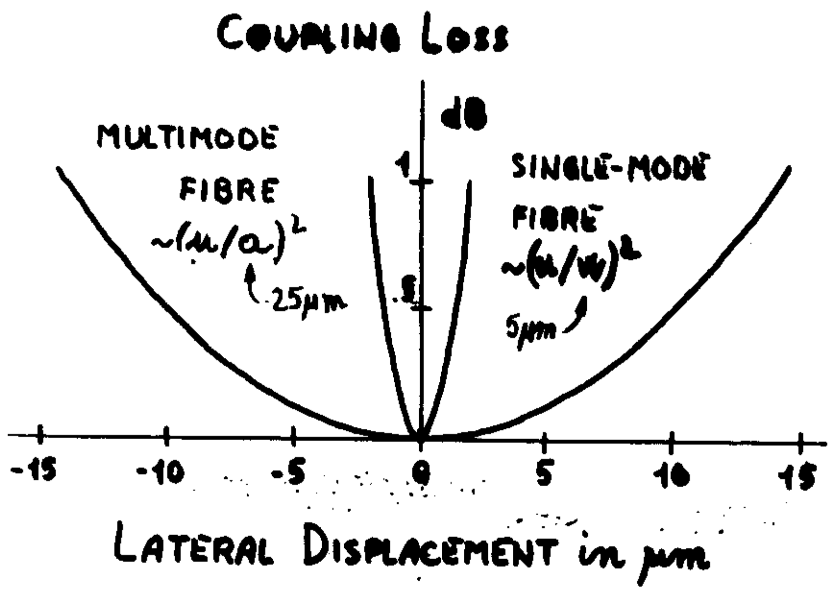
$$f(\text{PROF.}) = \begin{cases} 1/2 & \text{PARABOLIC PROFILE} \\ 1 & \text{STEP PROFILE} \end{cases}$$

* **TYPICAL VALUES:**

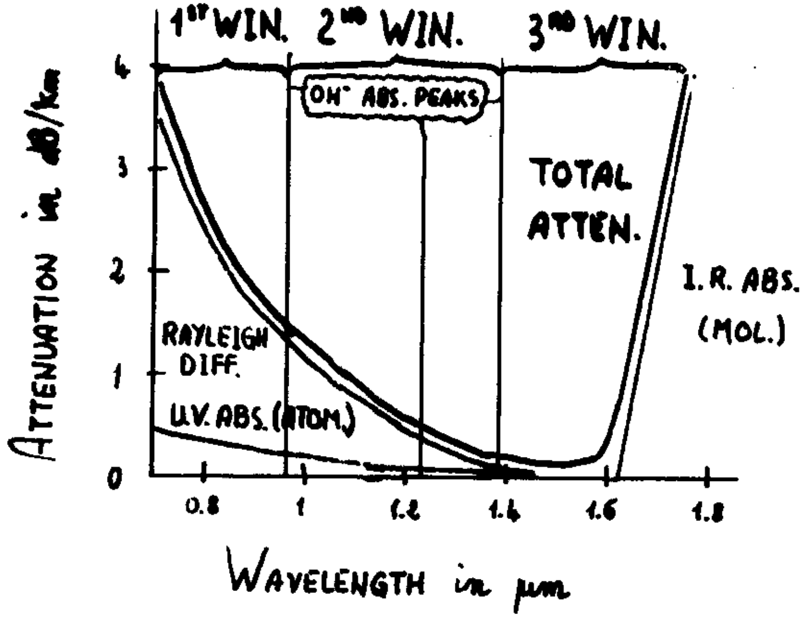
$$\Lambda = \begin{cases} \sim 2\% - \text{LED Sources} \\ \sim 50\% - \text{Semiconductor Laser Sources} \end{cases}$$

COUPLING PROBLEMS





ATTENUATION INTRINSIC FACTORS



ATTENUATION EXTRINSIC FACTORS

* Ion Absorption:

  OH^-, Fe^{++}, Ni^{+++}, Cu^{++}, Cr^{+++}, Mn^{+++}, ...

  (Loss: 1-5 dB/km per part per billion)

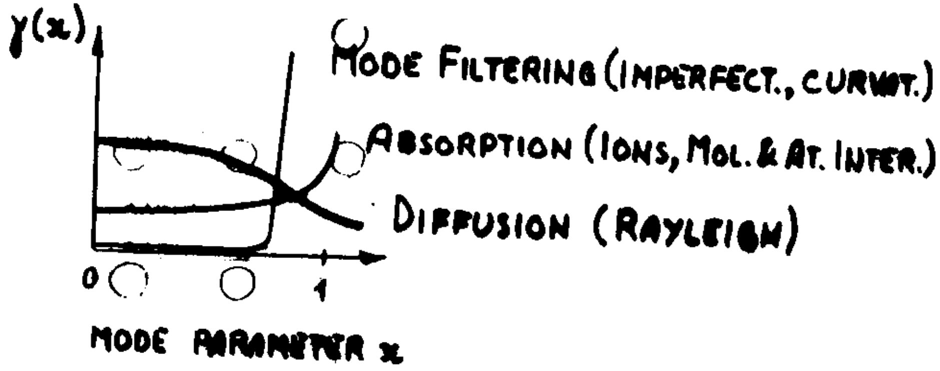
* Diffusion By Optical Inhomogeneities

* Mode-Filtering By Physical Imperfections

* Mode-Conversion By Microbending

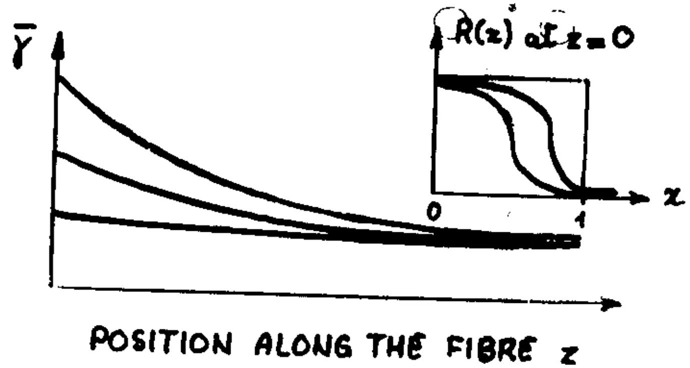
POWER ATTENUATION

* Each Mode Is Attenuated In A Different Way: $\gamma(x)$ - x-Mode Attenuation Constant



* Average Attenuation: $\bar{\gamma} \equiv \langle \gamma(x) \rangle$ In $R(x)$
  - $\bar{\gamma}$ Depends On Launching Conditions: $R(x)$ At $z=0$
  - $\bar{\gamma}$ Depends On z-Coordinate Too

    for large z: $\bar{\gamma} \rightarrow \gamma_{MIN}(x)$



MODE CONVERSION

- CAUSED BY MICROBENDING,
  SMALL IMPERFECTIONS,
  - - - - - .
- PRODUCES: OPTICAL LOSSES,
  (CONVERS. TO RADIAT. MODES)
  MODIFICATION ON DISTRIB.
  (CONVERS. VS. GUIDED MODES)

- SINGLE-MODE FIBRES

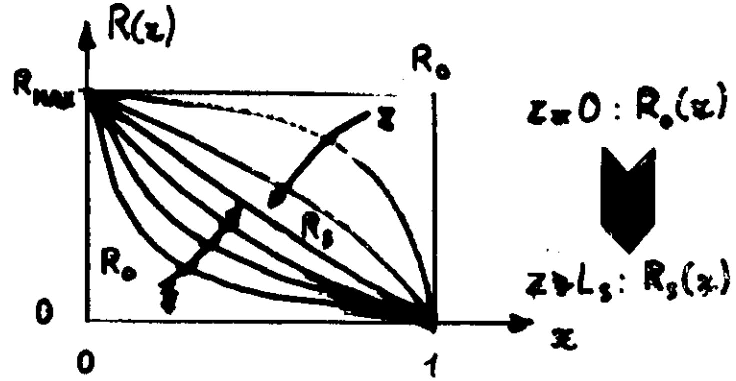
– MODE CONVERSION PRODUCES LOSSES ONLY

– LOSS COEFFICIENT $\propto w^p$ ($p \sim 2-4$)

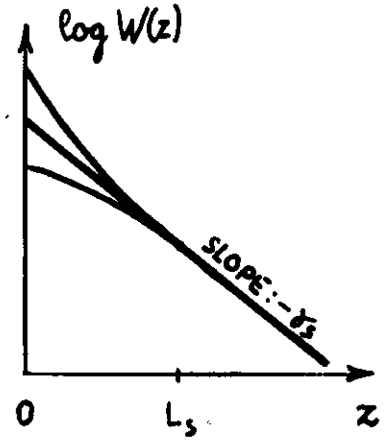
(INCREASING FUNCTION OF MODE RADIUS)

MULTIMODE FIBRES

* for every launched $R(x)$, a steady state is reached after a length L_s: $R_s(x)$, where each mode has the same attenuation: γ_s



* $R_s(x)$: steady state mode distribution
* L_s : steady state length
* γ_s : steady state attenuation coefficient



for $z \gg L_s$:

$R(x) \sim R_s(x) \cdot e^{-\gamma_s z}$

$W(z) \sim e^{-\gamma_s z}$

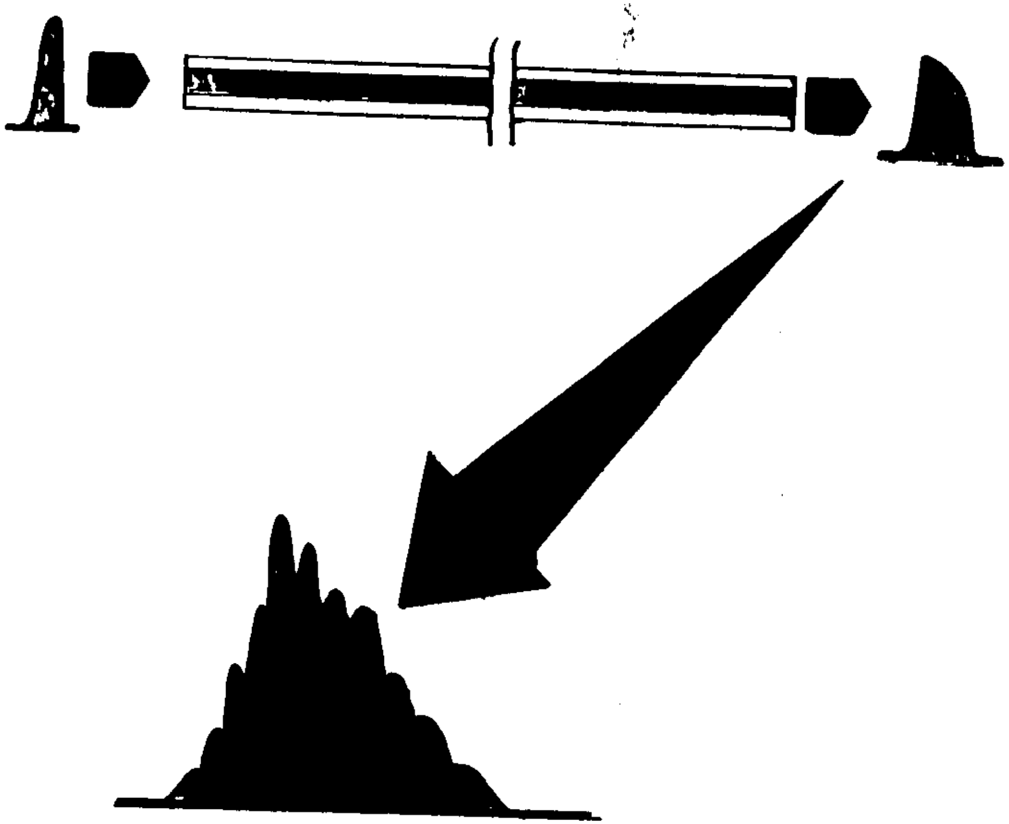
INFORMATION TRANSMISSION CAPACITY

PULSE DISPERSION

* MODAL DISPERSION
  - MODE COUPLING
  - MODE FILTERING
  - OPTIMIZED PROFILE

* CHROMATIC DISPERSION
  - WAVEGUIDE DISPERSION
  - MATERIAL DISPERSION
  - ZERO-DISPERSION

PULSE DISTORTION



* travelling time depends on mode:
  ▶ INTERMODAL DISPERSION
* travelling time depends on λ:
  ▶ INTRAMODAL DISPERSION

CHROMATIC DISPERSION

MODE TRAVELLING TIME DEPENDS on λ

τ : TRAVELLING TIME IN 1km OF FIBRE

$D = d\tau/d\lambda$: **CHROMATIC DISPERSION**

PULSE BROADENING:

$$\Delta t = L \cdot \Delta\lambda \cdot D$$

L : FIBRE LENGTH
$\Delta\lambda$: SPECTRA WIDTH OF THE OPT. SOURCE

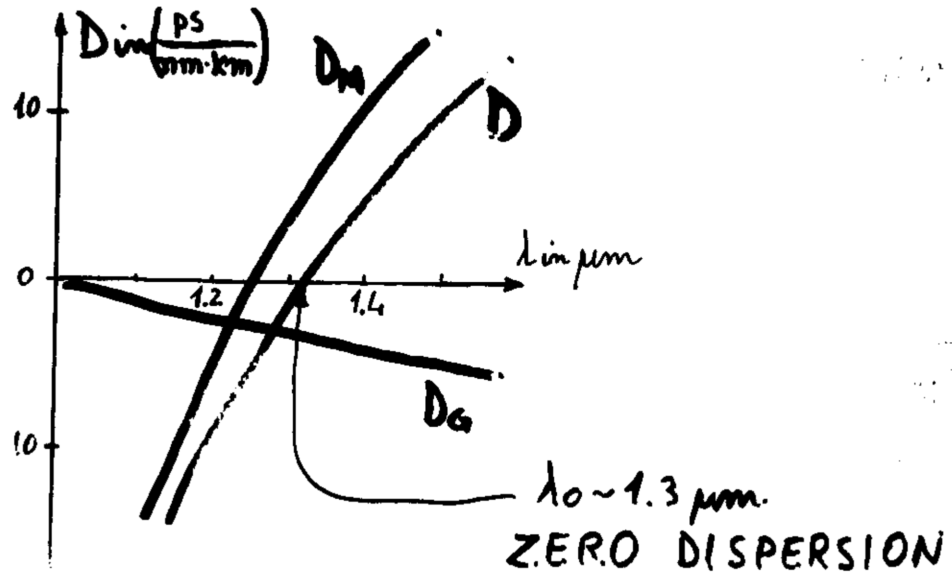
BANDWIDTH OF SINGLE-MODE FIBRES

	λ in μm	D in $\frac{ps}{nm \cdot km}$	BW in GHz·km
LTIMODE SER: $\Delta\lambda \sim 2nm$	1.3	~1	~100
	1.5	~10	~10
[illegible] SER: [illegible]	1.[illegible]	~1	~2000
	[illegible]	~1[illegible]	~2[illegible]

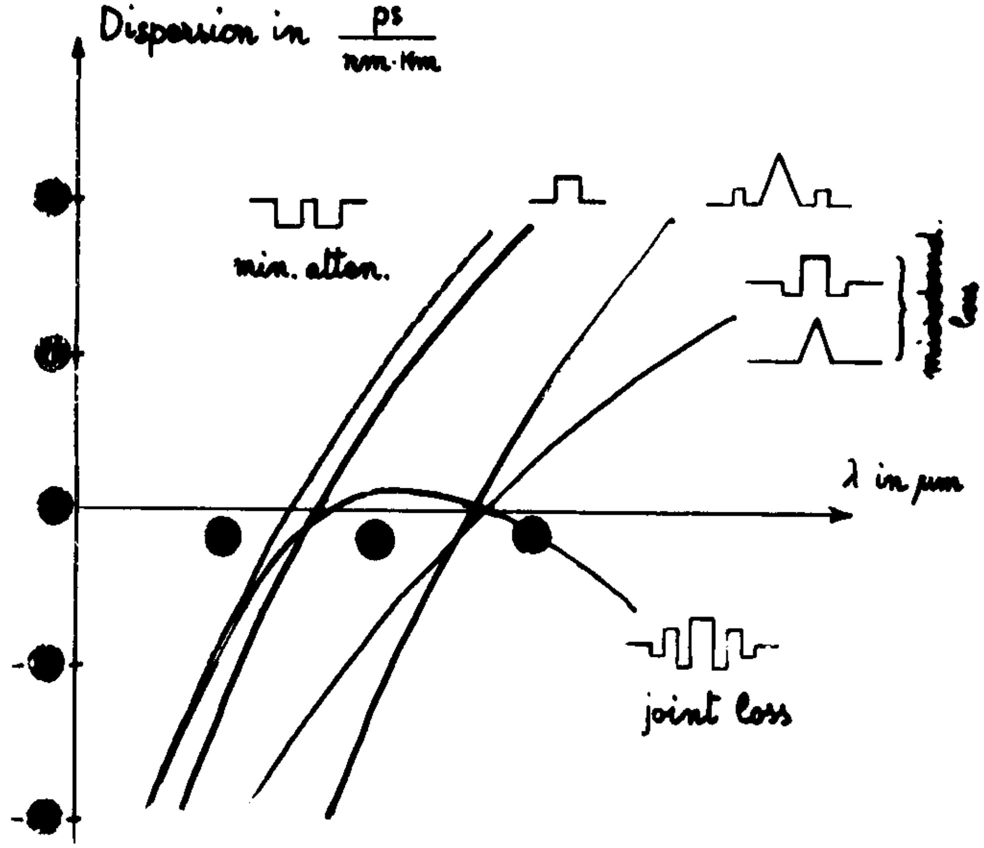
- τ DEPENDS on λ :
  1. THROUGH THE GUIDE EFFECT
  2. BECAUSE THE MATERIAL IS DISPERSIVE $(n = n(\lambda))$

➡ D HAS TWO CONTRIBUTIONS:

1. GUIDE DISPERSION : D_G
2. MATERIAL DISPERSION: D_M



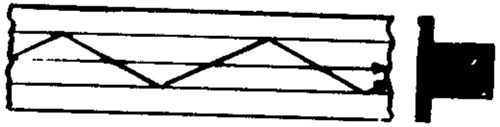
DISPERSION OPTIMIZATION



INTERMODAL DISPERSION

(Travelling Time Depends On Modes)

* Step Profile

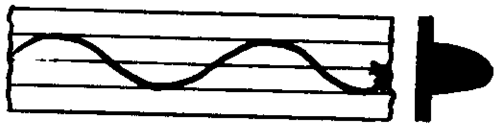


$\Delta t \propto (\text{Num. Apert.})^2$

~ 40 ns/Km

BW $\sim$ 5 MHz·Km

* Parabolic Profile

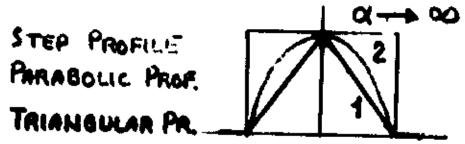


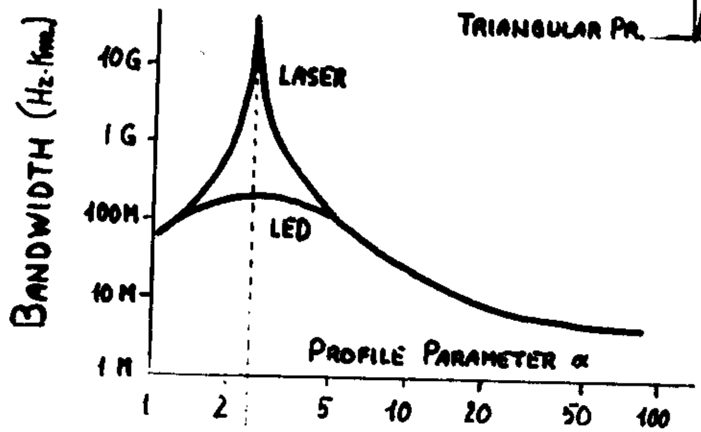
$\Delta t \propto (\text{Num. Apert.})^4$

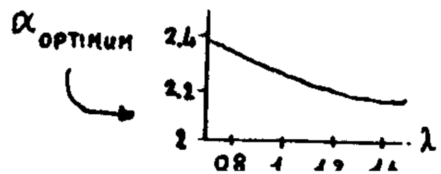
$\sim .4$ ns/Km

BW $\sim$ 500 MHz·Km

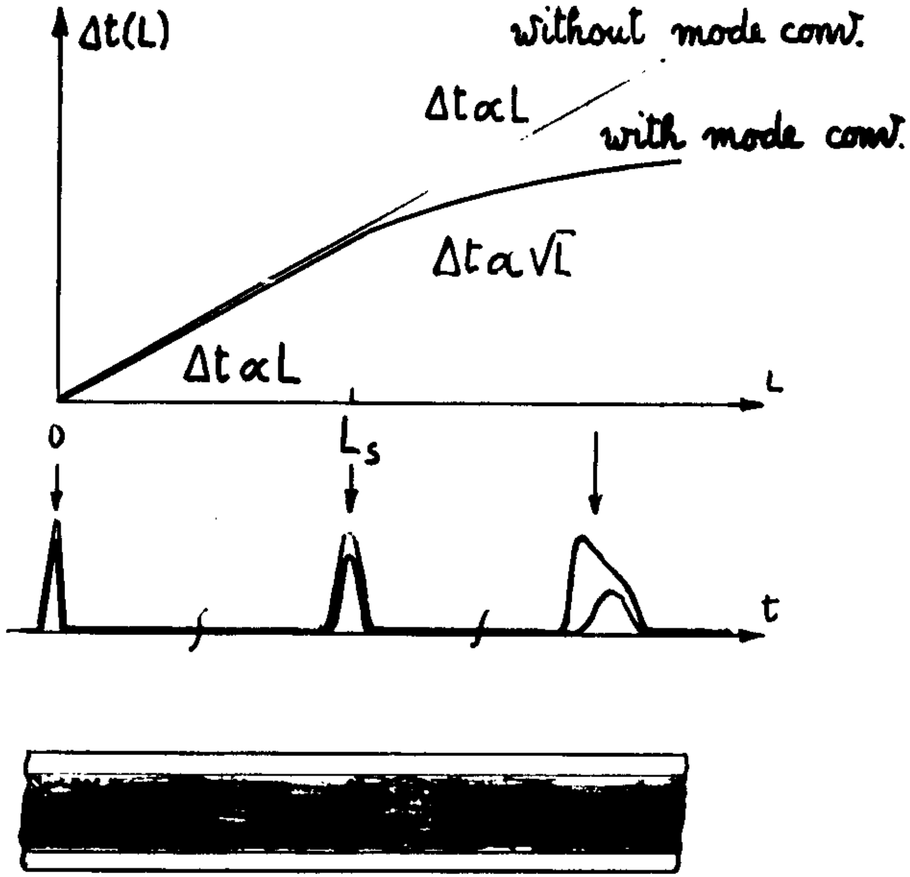
* "α"- Profiles







MODE CONVERSION EFFECTS ON PULSE DISTORTION



* Statistical equalization of optical paths