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(SUMMARIES AND CONTRIBUTIONS)

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SUM RULES AND TRANSITION DENSITIES

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A survey of total and partial nuclear cross section sum rules for electron scattering is presented. Partial sum rule relations are applied to connect transition densities as well as static multipole moments, transition probabilities and radii of low-lying states of anharmonic vibrator nuclei.

By summing the electroexcitation cross section $d\sigma$ over all final states it is possible to use closure to eliminate the final states and to relate observable quantities to ground state expectation values[1,2]. In particular

$$(1) \qquad \int_0^\infty d\omega \, \frac{d\sigma(q,\omega,\theta)}{d\sigma_{\text{no structure}}} = A(q) + B(q) \tan^2 \frac{\theta}{2} ,$$

where

$$(2) \qquad A(q) = \sum_n \left\{ \frac{q_\mu^4}{q^4} |\rho_{no}(q)|^2 - \frac{q_\mu^2}{q^2} |\underset{\sim}{j}^{tr}_{no}(q)|^2 \right\}; \; B(q) = \sum_n |\underset{\sim}{j}^{tr}_{no}(q)|^2 .$$

The sum rule quantities A and B may be measured separately as a function of momentum transfer q and are related to photonuclear sum rules at the photon point ($q \to \omega$).

More general sum rules are obtained by weighing the cross section with additional powers of the excitation energy, ω^k. For physical reasons it is useful to obtain separate sum rules for Coulomb and transverse current contributions.

There is some experimental information[3] on the k = 0 sum rule, recent theoretical estimates[4] find that statistical and dynamical correlations tend to cancel. The effects of exchange currents on sum rules have been studied in a simple Fermi gas model[5], finding modifications of the q-dependence in addition to the standard enhancement of the photonuclear Thomas-Reiche-Kuhn sum rule.

More detailed information is obtained by a decomposition of the sum rule relation into partial waves and isoscalar and isovector components. Assuming that the isoscalar density, which is little affected by exchange currents, commutes with the nucleon-nucleon potential, the following relation may be derived[6] (note k = 0 in the following)

$$2\sum_n \omega_{no} \,\mathrm{Re}\left\{ \varrho^*_{no}(q')\,\varrho_{no}(q) \right\} = \frac{\hbar^2}{m}\, q\cdot q'\, \varrho_{oo}(q'-q)\,. \tag{3}$$

From this "progenitor" sum rule one can obtain all previous static multipole sum rules as well as the more recent dynamical relations of Fallieros et al.[7,8]. While previous sum rule relations connected experimental quantities to ground state expectation values, we have recently generalized the sum rule technique and applied it to transition densities of the low-lying collective states of anharmonic vibrator nuclei[9]. We obtain, between general states J_i and J_f, the following potential wave decomposition corresponding to Eq.(3)

$$\langle J_f \| [\hat{F}^{l_1}(q'), [\hat{H}, \hat{F}^{l_2}(q)]]^L \| J_i \rangle = \sqrt{2L+1}\sum_n \left(\omega_{ni} \begin{Bmatrix} J_f & l_1 & J_n \\ l_2 & J_i & L \end{Bmatrix} F^{l_1}_{fn}(q')\, F^{l_2}_{ni}(q) + i \leftrightarrow f \right) \tag{4}$$

where $F^{l}_{nm}(q)$ are the form factors of multipolarity l for a transition between states m and n (Note: all operators are coupled to good angular momentum). Specializing the relation to quadrupole transitions ($l_1 = l_2 = \ell = 2$) and after a Fourier transformation, we derive a relation between transition and static densities

$$\sum_n \sqrt{2L+1}\,(\omega_{ni}+\omega_{nf}) \begin{Bmatrix} J_f & \ell & J_n \\ \ell & J_i & L \end{Bmatrix} M^{\ell}_{fn}\, \varrho^{\ell}_{ni}(r) = (-)^{J_i+J_f+1}\, \frac{e\hbar^2(2\ell+1)}{2m\sqrt{4\pi}} \begin{pmatrix} \ell & \ell & L \\ 0 & 0 & 0 \end{pmatrix} r^{\ell-2}\left(L(L+1)+2\ell r\frac{\partial}{\partial r}\right)\varrho^{L}_{fi}(r) \tag{5}$$

where M^{ℓ}_{nm} are the multipole moments of order ℓ between states n and m.

Applying the exhaustion hypothesis of the sum rule within the collective band, it is possible to solve a system of equations between those states and to relate transition strengths, radii and densities to static values.

While recent collective interpretations of electron scattering on anharmonic vibrator nuclei have met with considerable success[10,11], this contribution shows that such results do not depend on the details of the (collective) model, but follow from relatively general sum rule relations.

The predictions obtained have been compared to the results of recent experiments of Neuhausen[12] for various Zn isotopes. In view of the weakness of the exhaustion hypothesis the overall agreement is surprisingly good. Fig.1. shows the static density ϱ_{oo} as well as the transition densities ϱ_{10} and ϱ_{20} to the first ("one boson") and second ("two boson") quadrupole states, compared to the experimental data for ^{64}Zn. The additional fluctuations at small radii are of less practical importance than indicated in the figure, because physical quantities weigh the densities with higher powers of the radius, e.g., r^4 for transition probabilities.

These deviations are, however, typical of some remaining shell structure, particularly in the interior of the nucleus, as has been demonstrated in RPA calculations[13]. (Note that the relatively good agreement between sum rule prediction and transition density of the surface vibration mode indicates that the transition density of the giant quadrupole mode should have a similar behaviour, because the sum of both should fulfil the sum rule!)

Fig.2. demonstrates, that the present experimental data show a phase transition in Zn isotopes, favouring clearly the prolate shape in ^{64}Zn and an oblate shape in ^{68}Zn. In particular, the ratio of transition radii of two- to one-boson states, $(R_{20}/R_{10})^2$, increases from the sum rule prediction 0.76 for ^{64}Zn to 1.19 for ^{68}Zn, in precise agreement with the data.

The results show that electroexcitation data in connection with sum rule assumptions make it possible to get information on nuclear shapes and sizes in excited states, in addition to ground state expectation and transition quantities.

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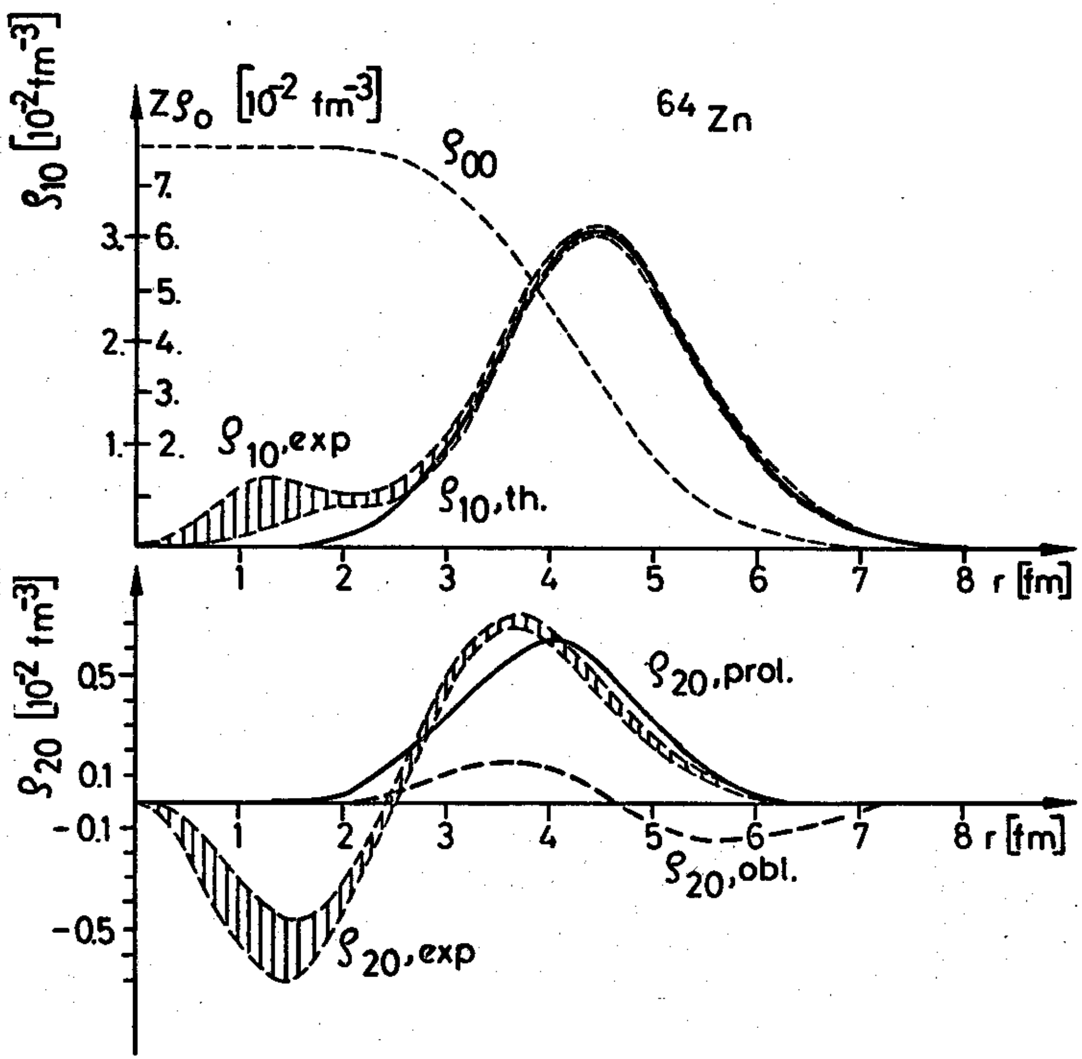

Fig. 1

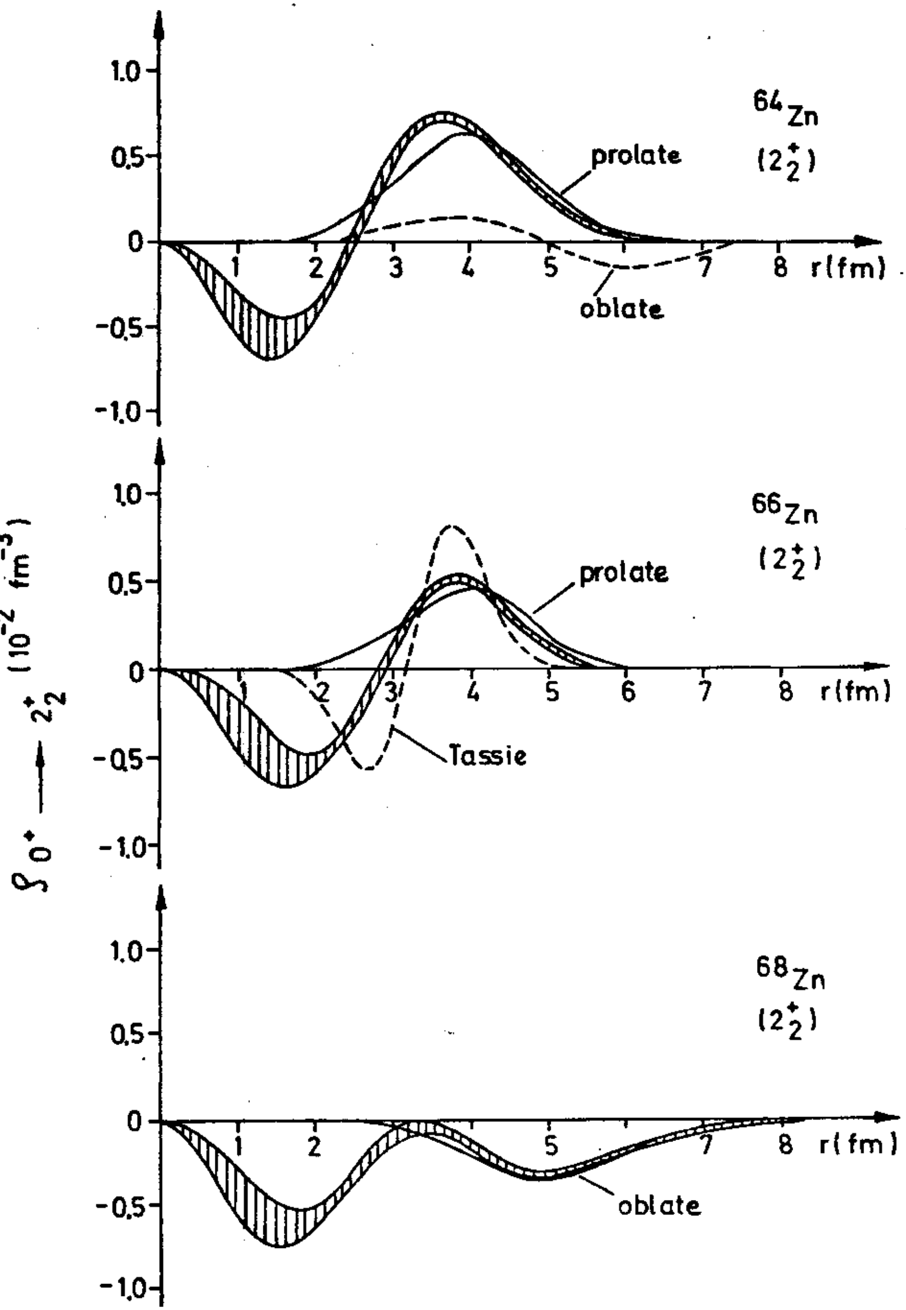
1.0
0.5
0
-0.5
-1.0
1 2 3 4 5 6 7 8 r(fm)
^{64}Zn
(2_2^+)
prolate
oblate
^{66}Zn
(2_2^+)
prolate
Tassie
^{68}Zn
(2_2^+)
oblate
$\rho_{0^+ \longrightarrow 2_2^+}$ $(10^{-2}\ fm^{-3})$

Fig. 2