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SECOND WORKSHOP ON MATHEMATICS IN INDUSTRY

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CONSTRUCTION OF OUTPUT REGULATORS

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2. Construction of output regulators

Given a system $\dot{x} = Ax + Bu + Dv$, $y = Cx$

Total input: (u, d)

u = control, d = (external) disturbance

Output control (output regulation) + strategy for specia- living u to a control action $u(t)$ in each moment t depending on past observations $y(s)$, $s \leq t$, only. Objective: To reduce the influence of , v on z.

Typical example of an admissible strategy: A feedback/observer structure. The control action is generated by a linear system where the output of the given system serves as input:

$$u = -L - F\hat{x}, \quad \dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u + \hat{K}y.$$

1. Stabilisation of a system by means of output feedbacks.

Theorem 1. If the matrices F, K are chosen such

that $\sigma(A-BF) \subset \mathbb{C}^-$, $\sigma(A-KC) \subset \mathbb{C}^-$

then the closed loop defined in terms of the relations

$$\dot{x} = Ax + Bu, \quad \dot{\hat{x}} = A\hat{x} + Bu + K(y - C\hat{x}), \; u = -F\hat{x}, y = Cx$$

is asymptotically stable

Proof Put $e := x - \hat{x}$, regard (x, e) as state variable of the closed loop. (x, e) satisfies the d.e.

$$\dot{x} = (A - BF)x + BFe, \quad \dot{e} = (A - KC)e.$$

2. Problem: Stabilization plus disturbance decoupling

Given $\dot{x} = Ax + Dv$, $y = Cx$. The out-

Definition 1. put y is decoupled from the input v if the following holds. The solution of $\dot{x} = Ax + Dv(t)$, $x(0) = 0$, for any $v(\tau)$, satisfies $y(t) := Cx(t) = 0$ for all t

Remind on some results and definitions of Sec. 1:

$\mathcal{V}^-$ largest subspace $\mathcal{V}$ of $\ker C$ with this property

$(A - BF)\mathcal{V} \subseteq \mathcal{V}$, $\sigma(A - BF) \subset \mathbb{C}^-$ for some F.

Later: $\mathcal{S}^-$ smallest space $\supseteq \operatorname{Im}(D)$: $(A - KC)\mathcal{S} \subseteq \mathcal{S}$, $\sigma(A - KC) \subset \mathbb{C}^-$.

(Th)eorem 2. Assume that the system is stabilizable and that $\mathcal{D} := \mathrm{Im}(D) \subseteq \mathcal{V}^- (\subseteq \ker C)$. Then one can stabilize the system and decouple simultaneously y from v by state feedback

Proof. Use the input/output normal form with $\mathcal{V}^-$ instead of $\mathcal{V}^*$, i.e. write the system in the form

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13}\\ 0 & A_{22} & A_{23}\\ 0 & 0 & A_{33}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} + \begin{pmatrix}B_1\\ 0\\ B_3\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix} + \begin{pmatrix}*\\ *\\ 0\end{pmatrix} v, \quad y = C_3 x_3$$

N.B.: The disturbance range is contained in $\mathcal{V}^- = \{(x_1, x_2, x_3) : x_3 = 0\}$

$$(A-BF)\mathcal{V}^- \subseteq \mathcal{V}^- \iff A - BF = \begin{pmatrix} \tilde{A}_{11} & \multicolumn{2}{c}{*} \\ 0 & \tilde{A}_{22} & \\ 0 & 0 & \tilde{A}_{33}\end{pmatrix}$$

Input/output behavior of the equivalent system:

$$\dot{x}_3 = \tilde{A}_{33} x_3, \quad y = C_3 x_3.$$

Theorem 2. *Hypotheses.* (i) System is stabilizable and detectable, (ii)

$$\mathrm{Im}(D) \subseteq \mathcal{S}^- \subseteq \mathcal{V}^- \subseteq \ker(C)$$

Then one can stabilize and decouple simultaneously by means of output feedback.

under the additional hypothesis $\mathcal{S}^- = \mathcal{V}^-$ and with the help of

Lemma 1 $(A-BF)\mathcal{V} \subseteq \mathcal{V}$ and $(A-KC)\mathcal{V} \subseteq \mathcal{V} \Rightarrow \exists L$ such that $(A-BLC)\mathcal{V} \subseteq \mathcal{V}$.

Proof of Theorem 2 Feedback/observer model for

the controller:

$$\left.\begin{aligned} u &= -Ly - (F-LC)\hat{x} \\ \dot{\hat{x}} &= A\hat{x} + Bu + K(y - C\hat{x}) \\ \dot{x} &= Ax + Bu + Dv, \quad y = Cx \end{aligned}\right\} (*)$$

plant:

Consider $(*)$ as a system with input v, output $y = Cx$

State x, $e := \hat{x} - x$ • Dynamics

$$\dot{x} = Ax - BF\hat{x} + BLCe + Dv = (A-BF)x + B(LC-F)e + Dv \quad (1)$$

$$\dot{e} = (A-KC)e - Dv \quad (2)$$

To be shown: $x(0) = 0, e(0) = 0 \Rightarrow y(t) = Cx(t) = 0$ irrespective of $v(t)$.

Follows from $e(t) \in \mathcal{V}^-$, for all t, Lemma 1, Sec. 1, applied to (2)

$B(LC-F)e(t) \subseteq (A-BLC)\mathcal{V}^- + (A-BF)\mathcal{V}^- \subseteq \mathcal{V}^-$ (hypotheses)

$x(t) \in \mathcal{V}^-$: Lemma 1, Sec. 1, applied to (1).