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SECOND WORKSHOP ON MATHEMATICS IN INDUSTRY

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COMPENSATION

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3. Compensation

Design objective: Not instantaneous, but merely asymptotic decoupling of the output from the disturbance

$$\lim_{t\to\infty} y(t) = 0 \quad \text{if} \quad x(0) = 0$$

A-priori-information about the disturbance: Given a dynamic model for the disturbance: $\dot{\tilde{x}} = \tilde{A}\tilde{x}$ plus a mapping of the solution space into the v-space. Combined mathematical model for plant and disturbance

$$\dot{x}_1 = A_{11}x_1 + \underbrace{A_{12}x_2}_{= Dv} + Bu \qquad y = C_1 x_1 \qquad (1)$$

$$\dot{x}_2 = A_{22}x_2$$

x_1 = state of the plant, x_2 = "state" of the disturbance

v is a deterministic time signal depending upon finitely many "hidden" parameters (example: v = const, v periodic with known period). Problem: Supplement (1) by a feedback/observer structure (2)

$$u = -Ly - F\hat{x}, \quad \dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u + \hat{K}y$$

such that the closed loop system defined by (1) plus (2) has these properties:

(i) $y(t) := C_1 x_1(t) \to 0$ for $t \to \infty$, irrespective of the initial state of $x = (x_1, x_2)$ and $\hat{x}$,

(ii) $(x_1, \hat{x}) \to 0$ for $t \to \infty$ if $x_2(t) \equiv 0$, irrespective of the initial state of $x_1, \hat{x}$.

Application of the separation principle is based on the following

<u>Lemma 1</u>. Assume that the undisturbed system $\dot{x}_1 = A_{11} x_1 + Bu$, $y = C_1 x_1$ is observable. Then one can replace the disturbance model by an equivalent one such that the augmented system

$\dot{x}_1 = A_{11} x_1 + A_{12} x_2 + Bu$, $y = C_1 x_1$, $\dot{x}_2 = A_{22} x_2$

is observable.

Reduced problem: Construct state feedback law

$$u = -Fx = -F_1 x_1 - F_2 x_2$$

such that properties (i) + (ii) hold.

<u>Theorem 1</u>. Assume that $\dot{x}_1 = A_{11} x_1 + Bu$, $y = C_1 x_1$ is stabilizable and that none of its transmission zeros coincides with an eigenvalue of A_{22}. Then compensation of the disturbance is possible iff the linear matrix equations

$$A_{11} X - X A_{22} + A_{12} - BY = 0, \quad C_1 X = 0 \qquad (3)$$

(unknowns X: type $n_1 \times n_2$, Y: type $m \times n_2$, $n_1 = \dim x_1$, $n_2 = \dim x_2$, $m = \dim u$)

admit a solution.

<u>Proof</u> (of the if-part). Choose F_1 such that $\sigma(A_{11} - BF_1) \subset \mathbb{C}^-$. Put $F_2 := Y - F_1 X$. Rewrite (3)

$(A_{11} - BF_1)X - XA_{22} + A_{12} - BF_2 = 0$, $C_1 X = 0$

feedback law $u = -F_1 x_1 - F_2 x_2$ generates the closed loop

$\dot{x}_1 = (A_{11} - BF_1) x_1 + (A_{12} - BF_2) x_2$, $\dot{x}_2 = A_{22} x_2$

Introduce new coordinates: $x_1 =: x_1' + X x_2$. The equation for the closed loop can be written as

$\dot{x}_1' = (A_{11} - BF_1) x_1$, $\dot{x}_2 = A_{22} x_2$, $y = C_1 x_1'$