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SECOND WORKSHOP ON MATHEMATICS IN INDUSTRY

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NONINTERACTING CONTROL

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4. Noninteracting control

Given an affine control system

$$\dot{x} = a(x) + B(x)u = a(x) + \sum_{i=1}^{m} b_i(x)u_i =: f(x,u)$$

plus scalar outputs $\gamma_j(x), j = 1, \dots, k$.

Special cases:

i) Linear systems: $a(x) = Ax$, $B(x) = B = \text{const}$.

$\gamma_j(x) =$ j-th component of Cx, hence

$\gamma_j(x) = \tilde{c}_j^T x$, $\tilde{c}_j =$ j-th row of C

ii) Bilinear system: $a(x) = Ax$, $B(x)$ linear in x

Lie-derivative of a scalar function $\alpha(x)$ along a vector-field $f(x) = \begin{pmatrix} f_1(x) \\ \vdots \\ f_n(x) \end{pmatrix}$

$$L_f \alpha := \sum_{i=1}^{n} \alpha_{x_i} f_i = \text{grad}\,\alpha \cdot f$$

Iterates: $L_f^0 \alpha := \alpha$, $L_f^{\rho+1} \alpha = L_f(L_f^\rho \alpha)$

Special case $\alpha(x) = a^T x$, $f(x) = Ax \Rightarrow L_f^\rho \alpha = a^T A^\rho x$. !

Interpretation of $L_f \alpha$: Derivative of α "along" the vector field f, i.e.: $x(t)$ solution of $\dot{x} = f(x) \Rightarrow$

$$\frac{d}{dt} \alpha(x(t)) = (L_f \alpha)(x(t)).$$

From now on: $f = f(x,u)$. If α is a function of x alone then $L_f \alpha$ is well defined. The iterate $L_f^\rho \alpha$ is well defined if $L_f^{\rho-1} \alpha$ does not depend upon u.

Definition 1: The characteristic number ρ_i of the i-th output $\gamma_i(x)$ is the largest non negative number such that $L_f^\rho \gamma_i$ is independent from u for $\rho \le \rho_i$.

$f = a + \sum_i b_i u_i \Rightarrow L_f^\rho \gamma_i = L_a^\rho \gamma_i$ for $\rho \le \rho_i$ and

$$L_f(L_f^{\rho_i} \gamma_i) = L_f(L_a^{\rho_i} \gamma_i) = L_a^{\rho_i+1} \gamma_i + \sum_i u_i L_{b_i}(L_a^{\rho_i} \gamma_i)$$

von Definition 1: $c_i := (L_{b_1}(L_a^{\rho_i} \gamma_i), \dots, L_{b_m}(L_a^{\rho_i} \gamma_i)) \ne 0$.

Hypothesis: The k vectors $c_1, \dots, c_k$ are linearly independent (for all x) ($\Rightarrow m \ge k$).

<u>Proposition 1.</u> For a linear system $\dot{x} = Ax + Bu$, $y = Cx$:
All outputs have characteristic number 0 and the general hypothesis holds iff CB has linearly independent rows.

<u>Proof</u>: Let $\tilde{c}_i^T$ be the rows of C, then $g_i(x) = \tilde{c}_i^T x$

g_i has characteristic number 1 if

$(L_{b_1} g_i, \dots, L_{b_m} g_i) = (\tilde{c}_i^T b_1, \dots, \tilde{c}_i^T b_m) = c_i^T B \neq 0$

General hypothesis: The $c_i^T B$ are linearly independent.

Define F, G such that

$$c_i F = L_a^{\rho_i+1} g_i, \qquad c_i G = e_i \quad (= (0, \dots, 0, \overset{\overset{i}{\downarrow}}{1}, 0, \dots 0))$$

F, G are smooth functions of x.

Introduce a new control variable u' via feedback transformation

$$u = -F(x) + G(x)u'.$$

Then

<u>Proposition 2.</u> $\tilde{f}(x; u') = f(x; -F(x) + G(x)u')$

$L_{\tilde{f}}^{\rho} g_i$ independent from u' if $\rho \leq \rho_i$

$$(L_{\tilde{f}}^{\rho_i+1} g_i)(x, u') = u_i', \quad i = 1, \dots, m$$

The input / output behaviour of the feedback-transformed system

$$\dot{x} = f(x; -F(x) + G(x)u'), \quad y = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} = \begin{pmatrix} g_1(x) \\ \vdots \\ g_m(x) \end{pmatrix}$$

Let $u'(t)$ be an arbitrary control function, $x(t)$ a solution of the differential equation and $y_i(t) = g_i(x(t))$. Then

$$\frac{d^{\rho_i+1}}{dt^{\rho_i+1}} y_i(t) = u_i'(t).$$