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OPTIMIZATION OF HYDRO ENERGY STORAGE PLANT SYSTEMS

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OPTIMIZATION OF HYDRO ENERGY STORAGE PLANT SYSTEMS

(Case Studies: Power Plants Gosau - Gosauschmied - Steeg
Power Plant Partenstein)

W.Bauer, H.Gfrerer, E.Lindner, Hj.Wacker

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1. Hydro Energy Production

Production of electric energy by hydro energy plants is of some importance in Austria, cf. figure 1.

year	hydro energy [GWh]	thermal energy [GWh]	imports [GWh]	total [GWh]
196o	1o.345	2.593	851	13.789
197o	19.296	6.22o	1.6o5	27.121
198o	27.o15	9.342	3.492	39.849
1982	28.63o	8.926	3.4o8	4o.964
1983	28.295	8.9o4	4.681	41.88o

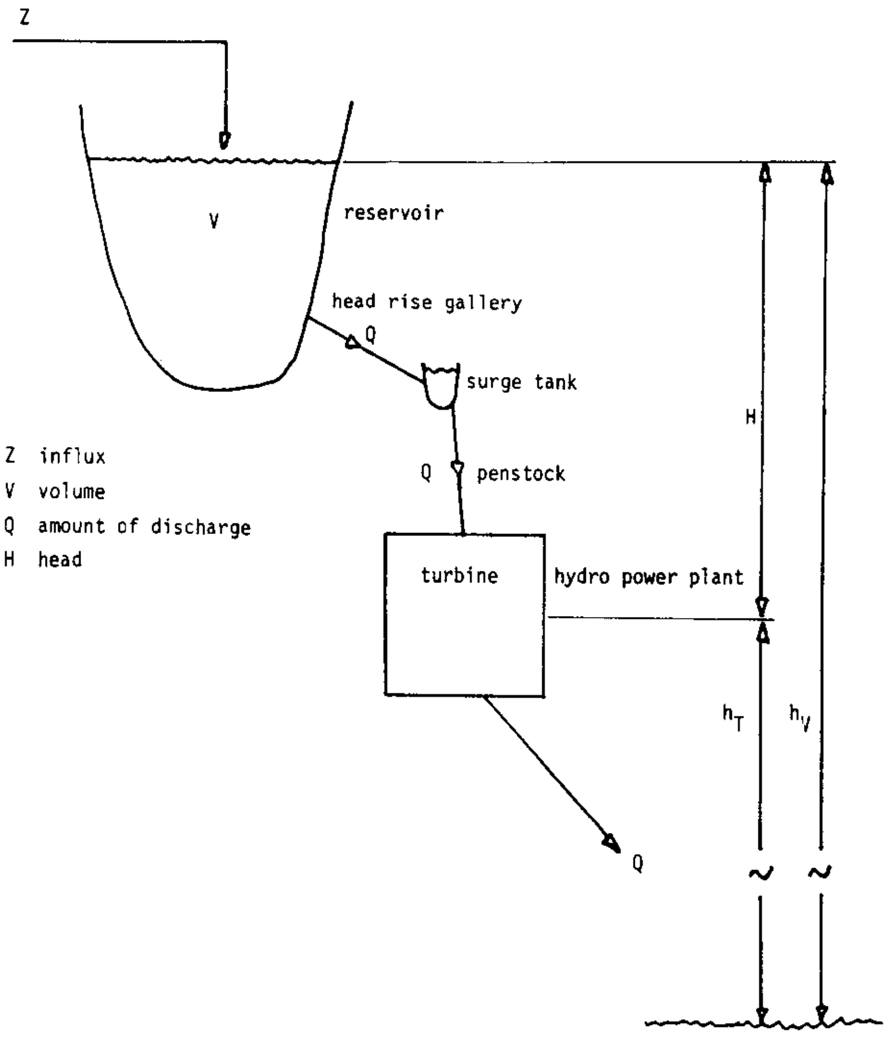
Figure 1: electric energy production/consumption in Austria

Although the ratio hydro energy production versus total energy production decreased from 75 % (196o) to 68 % (1983), river power plants and storage power plants are the main suppliers of electric energy in Austria, cf. [2c]. In 1983 river power plants produced 19.432 GWh whereas storage power plants produced 8.863 GWh. Although the ratio is more than two to one, storage power

plants are more important. Both river and storage power plants have a reservoir, but only the reservoir of a storage power plant can be used for transferring energy production with respect to time. Austrian river power plants are mainly situated at big rivers, e.g. river Danube. Their operation mode is almost completely predetermined by the influx and restrictions due to flood protection, shipping etc.
In this paper our central interest is the optimal control of storage power plants. The objective of our control problem is always the energy produced within a given interval of time, weighted by a nonconstant tariff function. This means, that we want to maximize the monetary value of the energy produced, resp. the income of the power plant company.

2 An Optimization Model for Energy Production by a Single Storage Power Plant (Rated Energy Production)

We will give a description of a single storage power plant.



Z influx
V volume
Q amount of discharge
H head

Figure 2

Figure 2 describes the essentials of a storage power plant system. The inflow Z runs into the reservoir, whose content is V. Water is taken down through a conducting system (head rise gallery, penstock) to the power plant itself. There, the amount of discharge Q leaves the power station after the turbines, where the potential energy is changed into cinetic energy (water → turbine) and afterwards into electric energy (turbine → generator).
For a power plant with a single turbine this can be formalized as follows:

$P_{Th} = g.\rho.H.Q$	theoretic power	[W]
$P \; = P_{Th}.\eta(H,Q)$	actual power	[W]

i.e., the theoretic power is reduced by the efficiency η ($\in \;]0,1[$) due to losses during transmuting the potential into electric energy. g, and ρ, see below.
Our aim is to optimize this system, i.e., we want to maximize the total rated energy production $E(T_B,T_E)$ within the time interval $[T_B,T_E]$. This results in the following model (M1):

Model (M1):

Maximize

(M1.1) $$E(T_B,T_E) = \int_{T_B}^{T_E} \frac{a(t)}{3\,600\,000} .P(t)dt$$

where

(M1.2) $$P(t) = P_{Th}(t).\eta(H(t),Q(t))$$

(M1.3) $$P_{Th}(t) = g.\rho.H(t).Q(t)$$

under the restrictions

(M1.4) $$V(t) = V(T_B) + \int_{T_B}^{t} (Z(\tau)-Q(\tau)-S(\tau))d\tau$$

(reservoir continuity equation)

(M1.5) $$V(T_B) = V_B$$

(M1.6) $V(T_E) = V_E$

(M1.7) $V_{min} \le V(t) \le V_{max}$

(M1.8) $Q_{min} \le Q(t) \le Q_{max}$

(M1.9) $0 \le S(t) \le S_{max}$

(M1.1o) $V(t) = C(h_V(t))$

(M1.11) $H(t) = h_V(t)-h_T$

for $t \in [T_B,T_E]$.

We use the following notations ($t \in [T_B,T_E]$):

$E(T_B,T_E)$	total rated energy production in the time interval $[T_B,T_E]$, ATS ... Austrian Shilling	[ATS]
T_B,T_E	beginning, and ending, point of the time interval under consideration, resp.	[s]
a(t)	tariff at time t, varying on day time, day of the week and season, cf. remark 1. a)	[ATS/kWh]
P(t)	actual power at time t	[Ws]
$P_{Th}(t)$	theoretical power at time t	[Ws]
$\eta(H,Q)$	total efficiency of the power plant cf. section 3.1	[-]
g	acceleration due to gravity, 9,81 m/s^2	
ρ	density of water, 1ooo kg/m^3	
H(t)	head at time t	[m]
Q(t)	amount of discharge through the turbine	[m^3/s]
V(t)	volume of water in the reservoir at time t	[m^3]

Z(t)	influx into the reservoir at time t	[m^3/s]
S(t)	spillage water at time t	[m^3/s]
V_B,V_E	volume of water in the reservoir at T_B, and T_E, resp.	[m^3]
V_{min},V_{max}	minimum, and maximum, volume of the reservoir, resp.	[m^3]
Q_{min},Q_{max}	minimum, and maximum, amount of discharge through the turbine, resp.	[m^3/s]
S_{max}	maximum spillage water	[m^3/s]
$h_V(t)$	absolute height of the surface of the reservoir at time t	[m]
h_T	absolute height of the level, where the water leaves the power plant again	[m]
C	reservoir capacity function cf. remark 1. b	[m^3]

Remark 1:

a) The tariff a(.) is a step function, attaining just five different values a year. Figure 3 gives the tariff in ATS/kWh valid since April 1st, 1985.

	Summer	transition period	winter
high	.433	.546	.571
low	.382	.489	.489

Figure 3: tariff valid since April 1st, 1985 in ATS/kWh

When "high", resp. "low", tariff is valid, is presented in figure 4.

	Summer and transition period	winter
Mo-Fr 6am-1opm	high	high
Sa 6am-1pm	high	high
Sa 1pm-1opm	low	high
So 6am-1opm	low	high
daily 1opm-6am	low	low

Figure 4: periods of "high",and "low", tariff, resp.

The actual value of a(.) gives the price at which an Austrian electric energy supplying company can buy or sell electric energy from or to the Austrian Central Electricity Board. Note also, that most of our results presented in the following parts are substantially depending on this varying tariff.

Figure 5 gives a sketch of the principial behaviour of the tariff function during one week of summer and transition period.

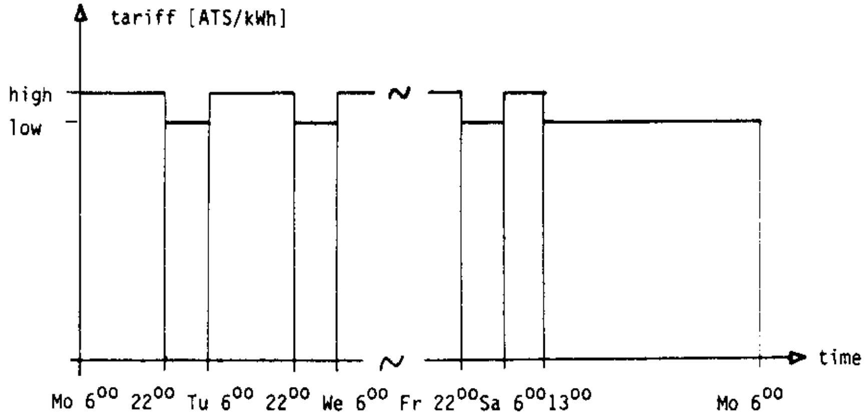


Figure 5

principal behaviour of the tariff function during summer (May - August) and transition period (April, September)

b) The reservoir capacity function C: $[h_{V,min}, h_{V,max}] \to [V_{min}, V_{max}]$ is a positive, strictly increasing smooth function. $h_{V,min}$, and $h_{V,max}$, are the minimum, and maximum, height of the reservoir, with $V_{min} = C(h_{V,min})$, and $V_{max} = C(h_{V,max})$, resp.

As C is bijective, (M1.1o) can be written as

(M1.1o') $h_V(t) = C^{-1}(V(t))$

and - more important - (M1.11) can be written as

(M1.11') $H(t) = C^{-1}(V(t)) - h_T =: f(V(t))$

Therefore, H depends continuously differentiable on V and vice versa, f is a positive, continuously differentiable strictly increasing function in V. Both C and f are nonlinear.

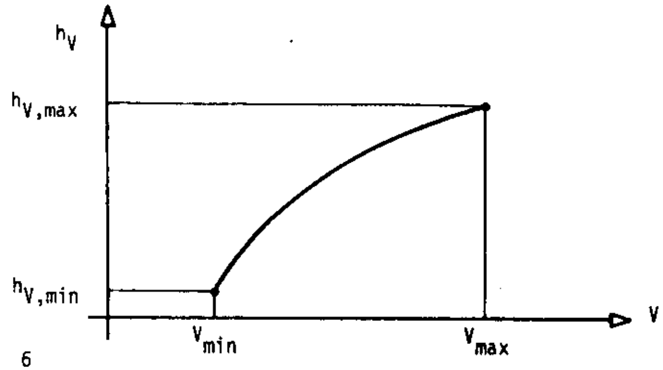


Figure 6

reservoir capacity function C (resp. C^{-1})

Note, that the above figure 6 is interpreted as C^{-1} by a mathematician. An engineer always represents both C and C^{-1} as sketched above. As "height" is a vertical direction it has to be represented by the ordinate ignoring the actual functional formulation.

c) Obviously, in model (M1) there are some equations just written dowr for clarifying the physical background. If we include (M1.2), (M1.3), (M1.11') in (M1.1) we result in

Model (M2):

Maximize

$$(M2.1)\qquad E(T_B,T_E) = \int_{T_B}^{T_E} \frac{a(t)}{3\,600\,000}\cdot g\cdot\rho\cdot f(V(t)).Q(t).\eta(f(V(t)),Q(t))dt$$

under the restrictions

$$(M2.2)\qquad V(t) = V(T_B) + \int_{T_B}^{t} (Z(\tau)-Q(\tau)-S(\tau))\,d\tau$$

$$(M2.3)\qquad V(T_B) = V_B$$

$$(M2.4)\qquad V(T_E) = V_E$$

$$(M2.5)\qquad V_{min} \le V(t) \le V_{max}$$

$$(M2.6)\qquad Q_{min} \le Q(t) \le Q_{min}$$

$$(M2.7)\qquad 0 \le S(t) \le S_{max}$$

for $t \in [T_B,T_E]$

For notations see model (M1) and remark 1. This is a problem of nonlinear optimal control with Q and S as control variables and V as state variable. Because of the tariff function a(.) the objective is nonconvex in any case.

Remark 2:

Both for further analytic and numeric considerations eliminating H in (M1) - as done in (M2) - is to prefer to eliminating V in (M1). This second elimination is also based on (M1.1o), (M1.11),

leading to

$$V(t) = C(H(t)-h_T)$$

Its disadvantage is, that (M1.4) becomes nonlinear: (M1.4) leads to

$$C(H(t)-h_T) = V_B + \int_{T_B}^{t} (Z(\tau)-Q(\tau)-S(\tau))d\tau$$

resp.

$$H(t) = h_T + C^{-1}(V_B + \int_{T_B}^{t} (Z(\tau)-Q(\tau)-S(\tau))d\tau),$$

and thereby increases the complexity of our model.

(M1), and (M2) are models for the simplest case of a storage power plant. In general, a power plant does not have a single turbine, but two or more. Also a chain of hydro energy power plants will be of interest. However, model (M1), and (M2), can easily be generalized.

Remark 3:

Other authors use objectives, which are not based on the price of energy. Bauer et al. [7], [8] consider an objective similar to (M1.1) but drop the tariff, leading to

$$E(T_B,T_E) = \int_{T_B}^{T_E} P(t)dt$$

In Bauer et al. [2] models using objectives of the type

$$Q(t) = \sum_{j=1}^{n} Q_j(t) \to \min,$$

and

$$P(t) = \sum_{j=1}^{n} P_j(t) \to \max,$$

under suitable restrictions on $P_j(t)$, and $Q_j(t)$, resp., are set up.

Concentrating our interest on models of the type (M1), (M2) is guided by applications. The power plants Gosau - Gosauschmied - Steeg and Partenstein are owned by the Upper Austrian Power Plants Company (OKA), whose shares are completely held by the federal government of Upper Austria.
The OKA intends the following production strategy:

production by OKA itself 1/3
joint production with other (Austrian) power companies 1/3
purchase of energy 1/3,

i.e., a ratio of one to one to one, whereas the actual ratio is 13 to 12 to 21. Therefore, it is of interest for the OKA to reduce purchase by producing more energy itself (resp. to reduce the amount of money for purchase). The objective used in (M1), (M2) also is the accounting mode of the Central Electricity Board, the exchange mode between different companies and the internal accounting mode of power plant companies in Austria.

3. The Storage Power Plant Partenstein

The storage power plant Partenstein and its storage lake Langhalsen are situated north of river Danube in Upper Mühlviertel, Upper Austria (cf. figure 7). It was built between 1919 and 1924 and opened on October 3oth,1924 as the first hydroelectric superpower station (at that time). It was designed for supplying Vienna with electric energy by the first 11o kV high-voltage power line in Austria. Because of harnessing river Danube by river power plants and thereby rising the level of the river, the power plant Partenstein had to be reconstructed from 1962 till 1964. Since that time Partenstein has had the following technical equipment:

main part:

2 Francis turbines with vertical shaft and 16,2 MW power each
maximum consumption 13 m^3/s each
2 three-phase synchronous alternators with 21,5 MVA, 55oo V, 5o Hz each

ancillary part:

1 Kaplan turbine with an inclination of 18^o with 2192 kW power
maximum consumption 26 m^3/s
1 three-phase asynchronous alternator with 2,4 MW, 55oo V, 5o Hz

The reservoir Langhalsen is due to a gravity retaining wall at river Große Mühl, some 1o km before it flows into river Danube. Its maximum content is 736 ooo m^3, its level is varying from 451,oo m till 456,2o m.
Water can be drained off by a weir plant at a maximum rate of 4oo m^3/s (two hook-shaped brackets, each 8,o m broad and 9,2 m high). River Große Mühl is the only influx to the reservoir. The water runs at first 5 572 m through a head rise gallery into the surge tank (capacity 1 619 m^3) and then through a penstock 371 m into the power station Partenstein. After the two Francis turbines the water can run either directly into the river Große Mühl (level 291 m) or through the Kaplan turbine into the river Danube (mean level 28o,5 m). The watershed is 5o6 km^2 with a mean annual water freight of 29o mill.m^3.
The power plant Partenstein produces 97 GWh annually on an average. For the data cf. [19]. The power plant Partenstein is part of the group of hydro power plants Upper Mühlviertel, consisting of Partenstein and Ranna. Both are owned by the Upper Austrian Power Plants Company (OKA). Partenstein and Ranna are only joint by a 11o kV power line.

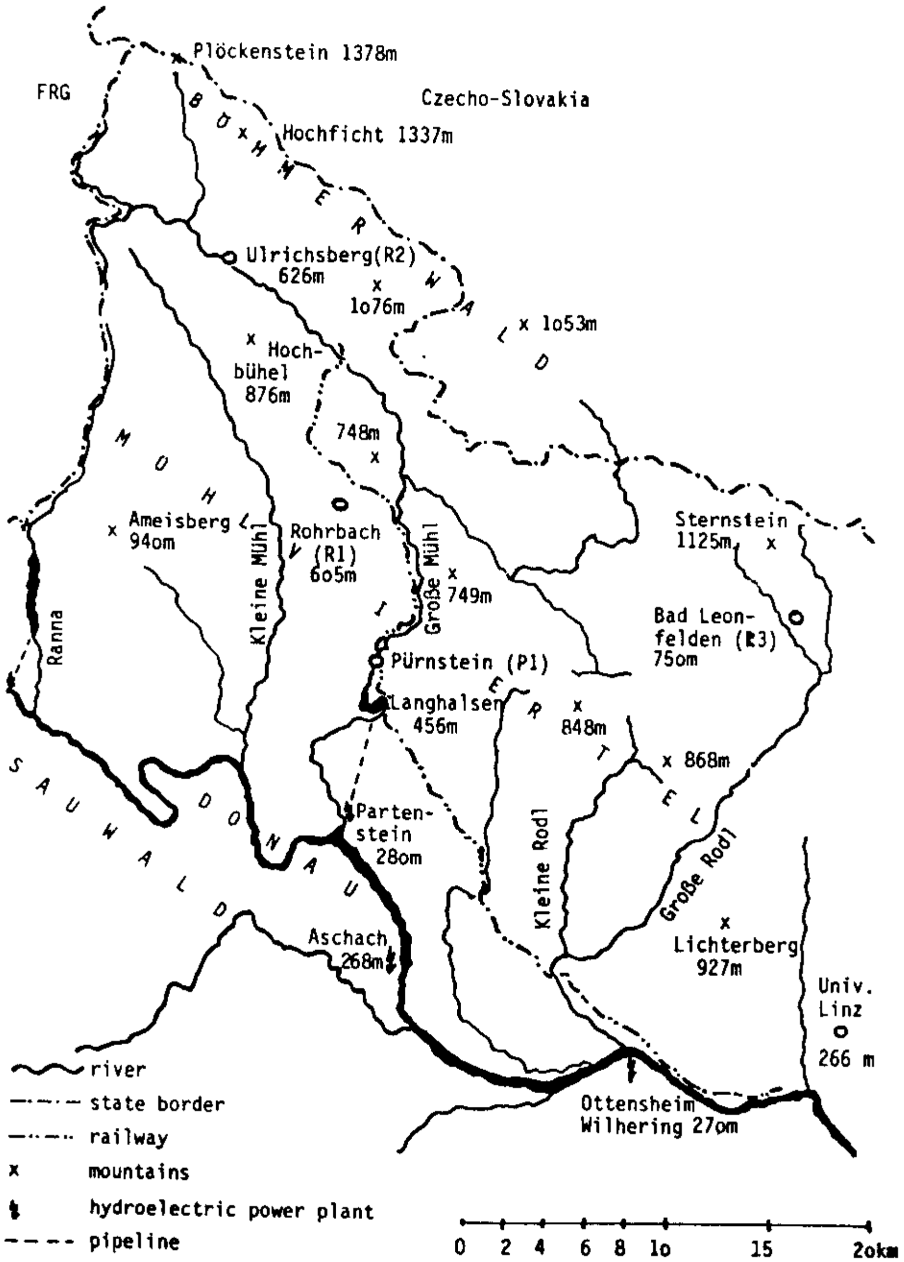


Figure 7

Without modelling the ancillary part we get a model similar to (M2) for Partenstein, but for two turbines.

Model (P1):

Maximize

(P1.1) $$E(T_B,T_E) = \int_{T_B}^{T_E} \frac{a(t)}{3\ 600\ 000} (P_1(t)+P_2(t))dt$$

where

(P1.2) $$P_i(t) = g.\rho.f(V(t)).Q_j(t).\eta_j(f(V(t)),\ Q_1(t),\ Q_2(t)) \qquad i= 1,2$$

subject to

(P1.3) $$V(t) = V(T_B) + \int_{T_B}^{t} (Z(\tau)-Q_1(\tau)-Q_2(\tau)-S(\tau))d\tau$$

(P1.4) $$V(T_B) = V_B$$

(P1.5) $$V(T_E) = V_E$$

(P1.6) $$V_{min} \le V(t) \le V_{max}$$

(P1.7) $$Q_{min,j} \le Q_j(t) \le Q_{max,j}$$

(P1.8) $$0 \le S(t) \le S_{max}$$

for $j = 1,2,\ t \in [T_B,T_E]$

We use the same notations as in (M2), and add subscript j, for denoting turbine j, j = 1,2.

Most of the input parameters in our model are easy to get. We confine our interest to the determination of C resp. f, η_j, and Z.

i) Determining the reservoir capacity function is an easy problem. We used a polynomial approximation, an acceptable result was obtained by classical linear least squares after some initial troubles on the data set. For details see [15, 16].

ii) For determining the efficiency function η_j see 3.1.

iii) Estimating the influx Z to the storage lake Langhalsen was done by two methods. For a model based on Multivariate Normal Distribution and conditional probability see [17]. For a transfer function model see [16], or Oberaigner [21].

3.1 Determination of the Efficiency

The total efficiency η of the power plant is defined as the ratio of produced power and theoretical power. The theoretical power is reduced by losses

i) in the tunnel system conducting the water from the reservoir to the turbines

ii) in the turbines themselves and

iii) in the generators and transformers.

Afterwards the produced electric energy is no more under the responsibility of the power plant. Therefore, we do not consider further losses in the network etc. Only for iii) acceptable information was available from the producers. Hence we had to develop a model for the efficiency including the losses i) and ii) only.

At the beginning of our work no data were available on the amounts of discharge Q_1, and Q_2. Hence, we tried the following ansatz.

From (P1.3) we obtain

$$V'(t) = Z(t)-Q_1(t)-Q_2(t)-S(t) = (C(h_V(t))' =$$

$$= C'(h_V(t)).h_V'(t)$$

If we have two measurements of h_V, and Z, at time t_o and $t_o+\Delta t$ and if there is no spillage water S, we can approximate the total efficiency

$$\eta_{total} = \frac{E_1+E_2}{g\rho H_{av}Q_{av}\Delta t}$$

where

$$H_{av} = \frac{1}{2}\,(h_V(t_o) + h_V(t_o+\Delta t))-h_T$$

$$Q_{av} = \frac{1}{2}\,(Z(t_o) + Z(t_o+\Delta t) - \frac{1}{\Delta t}\,(C(h_V(t_o+\Delta t))-C(h_V(t_o)))$$

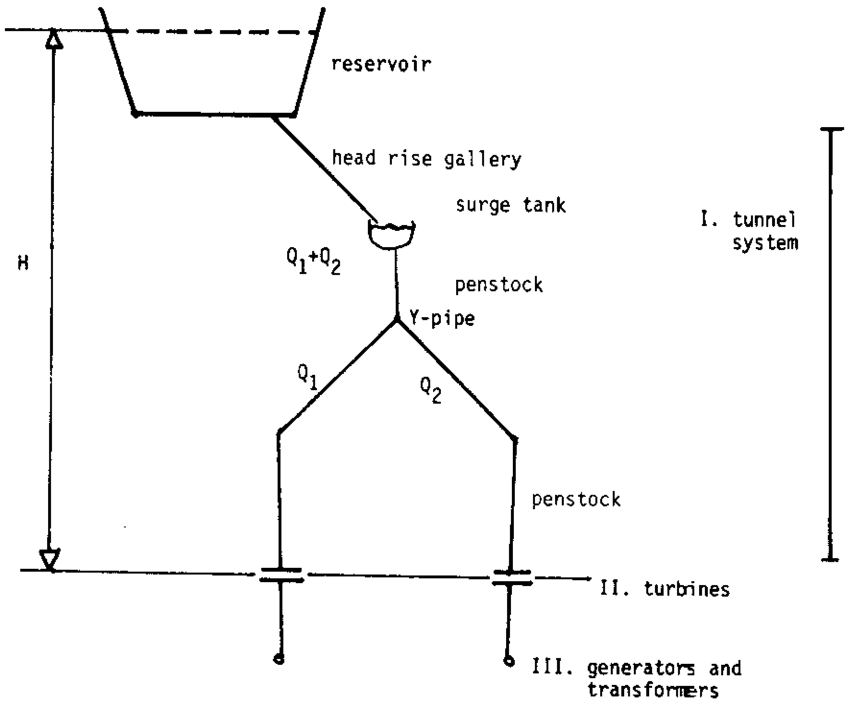
E_j is the energy produced in the interval $[t_o,t_o+\Delta t]$ for $j = 1,2$

This ansatz was not successful as

- it does not contain the partitioning of the total discharge on the two turbines
- the numerical results are not acceptable. Small errors in the measurements of h_V result in enormous errors in Q_{av}, as $C'(h_V) \sim 1,4\ .\ 1o^5$.

Hence, we tried another ansatz.

Figure 8: basic hydrological structure of the power plant Pa-tenstein



We took a model motivated by technical considerations.

i) The water is brought down to the power plant by a single tunnel which is split up into two just before the turbines. By engineering assumption the losses of pressure in the tunnel may be expressed by a fictitious reduction of the head H which is proportional to the square of the discharges in each separate section of the tunnel system. So we get

$H^{(2)} := H - c(Q_1+Q_2)^2$ in the upper tunnel,

$H_i^{(3)} := H^{(2)} - c_i^{(1)} \left[\left(\frac{Q_1+Q_2}{F_o} \right)^2 - \left(\frac{Q_i}{F_i} \right)^2 \right]$ in the Y-pipe of the tunnel and

$H_i^{(4)} := H_i^{(3)} - c_i^{(2)} Q_i^2 =: H_i$ in the part just before turbine i, $i = 1,2$

where F_o, F_1, F_2 denote the circular cross-section of the upper tunnel, the tunnel to the first and to the second turbine, resp. Because of similar construction we assume $c_1^{(1)} = c_2^{(2)} = c^{(1)}$. Summing up all losses we get the following expression for the manometric head H_i before turbine i

$$H_i = H - c_o(Q_1+Q_2)^2 - c_i Q_i^2 \qquad i = 1,2$$

where $c_o = c + c^{(1)}/F_o^2$ and $c_i = c_i^{(2)} - c_i^{(1)}/F_i^2$

Therefore, the power before the turbine i is

$$P_{bT,i} = g \cdot \rho \cdot H_i \cdot Q_i = g \cdot \rho H Q_i \eta_S(H,Q_1,Q_2)$$

where $\eta_S(H,Q_1,Q_2) = 1 - c_o \frac{(Q_1+Q_2)^2}{H} - c_i\, Q_i^2/H$

ii) As in our case there are two identical turbines we assume that the efficiency functions $\eta_{T,1}$ and $\eta_{T,2}$ of the turbines are identical. Significantly the contours of the efficiency of a single turbine are shell like.

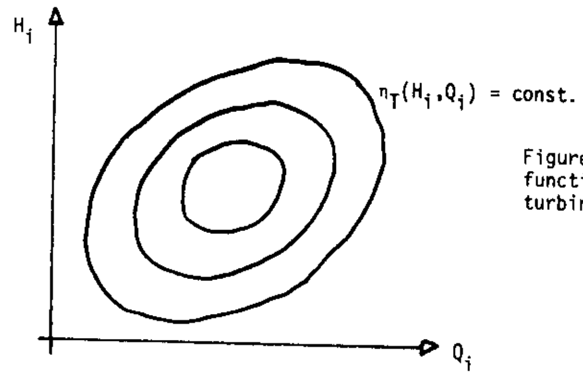


Figure 9: efficiency function of a single turbine

Therefore, we use a bi-quadratic approximation, i.e.

$$\eta_T(H_i,Q_i) = a_o + a_1 Q_i + a_2 H_i + a_3 H_i Q_i + a_4 Q_i^2 + a_5 H_i^2.$$

For the output-power of the turbine we get

$$P_{T,i} = P_{bT,i} \cdot \eta_T(H_i,Q_i) = g \cdot \rho\, H Q_i \eta_S(H,Q_1,Q_2) \cdot \eta_T(H_i,Q_i)$$

iii) Again both generators and transformers are identical. The producers presented a function $\tilde{\eta}_G = \tilde{\eta}_G(P_i)$. As the argument is the output value of the power plant, we computed its inverse $\eta_G = \eta_G(P_{T,i})$.

Putting i.), ii.) and iii.) together we obtain

$P_i = P_{T,i} \cdot \eta_G(P_{T,i})$ with $P_{T,i} = g \cdot \rho \; HQ_i \eta_S(H,Q_1,Q_2) \cdot \eta_T(H_i,Q_i)$

resp.

$$\eta_{Total} = \frac{P_1+P_2}{P_{Th,1}+P_{Th,2}} = \sum_{i=1}^{2} \frac{Q_i \eta_S(H,Q_1,Q_2)\eta_T(H_i,Q_i)\eta_G(P_{T,i})}{Q_1+Q_2}$$

$$= \eta_{Total}(H,Q_1,Q_2)$$

and

$$P_{Total} = g \cdot \rho \cdot H(Q_1+Q_2)\eta_{Total}(H,Q_1,Q_2)$$

To determine η_{Total} means to determine $(a,c) := (a_o,\dots,a_5;c_o,c_1,c_2)$.
In principle this can be done by nonlinear least squares techniques based on measurements for P_{1j}, P_{2j} at the points (H_j, Q_{1j}, Q_{2j}).
In our case there arises the additional difficulty that the points where the measurements (H_j,Q_{1j},Q_{2j}) are taken can be determined only with uncertainty. A first attempt to use standard nonlinear least squares was on obvious failure. Nevertheless, this problem can be handled satisfactorily by the following technique proposed by Schwetlick/Tiller [23].
We consider the unconstrained optimization problem

$$\frac{1}{2}\sum_{j=1}^{l} \begin{pmatrix} r(x^j,\theta)-Y^j \\ x^j-X^j \end{pmatrix}^T W_j \begin{pmatrix} r(x^j,\theta)-Y^j \\ x^j-X^j \end{pmatrix} \to \min$$

where minimization is done with respect to $x^1,\dots,x^l$ and θ. The matrices W_j are appropriately chosen weights, which are symmetric and positive definite. The main problem is the high dimension caused by including the errors of the input data (x^j-X^j) into the standard model. Schwetlick/Tiller overcome this difficulty by reformulating the above optimization problem as a nonlinear least squares problem which is solved in two steps: at first linearization by the Gauß-Newton method and secondly solving the resulting linear minimum norm problem profiting by the special structure of the resulting objective. For details see [23]. We additionally used a damped Gauß-Newton method and regularization for the linear solver.
For Partenstein we chose W_j to be diagonal matrices with the reziprocals of the variance as diagonal elements. Based on 26 measurements we got

$c_o = 0.0106$	$a_o = 99.070$	$a_3 = -0.020$
$c_1 = -0.1283$	$a_1 = 3.098$	$a_4 = 0.172$
$c_2 = -0.1249$	$a_2 = -1.329$	$a_5 = 0.004$

The same method has also successfully been applied to the power plant Gosau. For details see [17].

3.2 Optimization for Partenstein

Up to now we used three different optimization techniques. At first we present two models based on a constant efficiency.

3.2.1 A Decomposition/Convexification Technique

In [10] Gfrerer describes a globally convergent decomposition method for nonconvex optimization problems. He considers separable problems of the following form

$$\begin{aligned} &\text{minimize } f(x) \\ &\text{subject to } Ax = b \\ &\qquad g(x) \le 0 \\ &\qquad x \in K \end{aligned} \qquad (*)$$

where $f \in C^2(\mathbb{R}^n)$, $A \in \mathbb{R}^m_n$ with full rank $m \le n$, $K \subseteq \mathbb{R}^n$ is a convex, compact set with int $K \neq \emptyset$, $g_j \in C^2(\mathbb{R}^n)$ convex on K for $j = 1,\dots,p$, and there exists $x_o \in$ int K so that $Ax_o = b$ and $g(x_o) < 0$.

For fixed $y \in \mathbb{R}^n$ he considers the problem

$$\begin{aligned} &\text{minimize } f(x) + 1/2\,(x-y)^T C(x-y) \\ &\text{subject to } Ax = b \\ &\qquad g(x) \le b \\ &\qquad x \in K \end{aligned} \qquad (**)$$

where C is a positive semidefinite matrix preserving the separable structure of the original problem (*) so that the objective in (**) is strictly convex on K with respect to x. Then problem (**) has an unique solution x(y) and the sequence $(y_k)_{k \in \mathbb{N}}$ defined by $y_{k+1} := x(y_k)$ for $k \in \mathbb{N}$ and $y_1 \in \mathbb{R}^n$ arbitrary has the property that each clusterpoint satisfies the necessary conditions for a local minimizer.

This time the problem of maximizing the rated production of energy was discretized with respect to time, i.e.,

$$[T_B,T_E] = \bigcup_{i=0}^{N-1} [t_{i-1},t_i[\;\cup\; [t_{N-1},t_N]$$

where $t_{i-1} < t_i$ for $i = 1,\dots,N$, according to the tariff function a(.). Assuming the influx and the discharges to be constant on $[t_{i-1},t_i[$, the influx to be known a-priori and $S \equiv 0$ this leads to the following optimization model with (N-1) variables $V_i := V(t_i)$, $i = 1,\dots,N-1$ and (4N-2) inequality constraints.

maximize

$$\frac{\rho \cdot g}{3{,}6.10^6} \sum_{i=1}^{N} \int_{t_{i-1}}^{t_i} a(t_{i-1})\left(\frac{V_{i-1}-V_i}{t_i-t_{i-1}} + Z(t_{i-1})\right).f\left(V_{i-1} + \frac{(V_i-V_{i-1})(t-t_{i-1})}{t_i-t_{i-1}}\right)dt$$

subject to

$$Q_{min} \le \frac{V_{i-1}-V_i}{t_i-t_{i-1}} + Z(t_{i-1}) \le Q_{max} \qquad i = 1,\dots,N$$

$$V_{min} \le V_i \le V_{max} \qquad i = 1,\dots,N-1$$

$$V_o = V_N = V_{max}$$

A separable structure is obtained by introducing new variables V_i^+, and V_i^-. Finally he results in the problem

maximize

$$\frac{\rho\, g}{3{,}6.10^6} \sum_{i=1}^{N} \int_{t_{i-1}}^{t_i} a(t_{i-1})\left(\frac{V_{i-1}^+-V_i^-}{t_i-t_{i-1}} + Z(t_{i-1})\right).$$

$$.f\left(V_{i-1}^+ + (V_i^- - V_{i-1}^+)\,\frac{t-t_{i-1}}{t_i-t_{i-1}}\right) dt$$

subject to

$$Q_{min} \le \frac{V_{i-1}^+-V_i^-}{t_i-t_{i-1}} + Z(t_{i-1}) \le Q_{max} \qquad i = 1,\dots,N$$

$$V_{min} \le V_{i-1}^+ \le V_{max} \qquad i = 2,\dots,N$$

$$\tilde{V}_{min} \le V_i^- \le \tilde{V}_{max} \qquad i = 1,\dots,N-1$$

$$V_i^- = V_i^+ \qquad i = 1,\dots,N-1$$

$$V_o^+ = V_N^- = V_{max}$$

The lower, and upper, bounds $\tilde{V}_{min} < V_{min}$, and $\tilde{V}_{max} > V_{max}$, resp., were introduced to avoid difficulties concerning linear dependence of the gradients of the active constraints at the solution.

For Partenstein he obtained the following results for an optimization period of one week.

Number of time intervals	CPU-time on an IBM 3o31
42	14"
84	28"
168	1'27"
5o4	9'3o"
1oo8	35'11"

The optimal solution looked like as follows

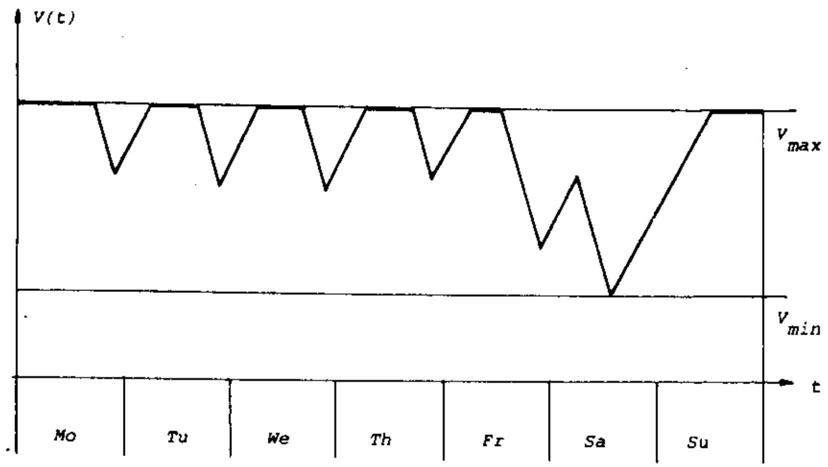


Figure 1o

3.2.2 A Variational Method

It is quite outstanding, that the optimal solution in 3.2.1 seems not to depend on the discretization parameter N. In 3.2.1 model (P1) is mainly seen as a nonlinear nonconvex optimization problem.
This time it will be considered as a control problem. Assuming a constant efficiency and $0 \leq Q_{min} < Z(t) < Q_{max}$ for $t \in [T_B,T_E]$ this results in the control problem (M)

$$\text{maximize} \int_{T_B}^{T_E} a(t)f(V(t))Q(t)dt$$

$$\text{subject to } V'(t) = Z(t)-Q(t)$$

$$V(T_B) = V_B,\ V(T_E) = V_E$$

$$V_{min} \leq V(T) \leq V_{max}$$

$$Q_{min} \leq Q(t) \leq Q_{max}$$

$$\text{for } t \in [T_B,T_E]$$

with $V \in C[T_B,T_E]$ as state variable and $Q \in L_\infty(T_B,T_E)$ as control variable.

Applying the maximum principle for the calculus of variations we get the following necessary conditions that $(V_*(.),\ Q_*(.))$ is a solution of (M): (see Girsanov [14]).

There exist real numbers 1, $\lambda_0 \geq 0$, a function ψ and nonnegative measures μ_1 resp. μ_2 with support $\bar{V}: = \{t \in [T_B,T_E] | V_*(t) = V_{max}\}$ resp. $\underline{V}: = \{t \in [T_B,T_E] | V_*(t) = V_{min}\}$, not all zero, so that

$$-\psi(t) = -1-\lambda_0 \int_t^{T_E} a(\tau)f'(V_*(t))Q_*(\tau)d\tau + \int_t^{T_E} 1d\mu_1(\tau) - \int_t^{T_E} 1d\mu_2(\tau)$$

and for all $Q(.)$ satisfying $Q_{min} \leq Q(t) \leq Q_{max}$ and almost all $T_B \leq t \leq T_E$

$$(\psi(t)-\lambda_0\, a(t)f(V_*(t))).(Q(t)-Q_*(t)) \geq 0.$$

If further

$$V_{min} < V_B < V_{max}, \quad V_{min} < V_E < V_{max}$$

is satisfied, then ψ and λ_0 are not both zero.

Analysing these necessary conditions (see Gfrerer [11]) we obtain, that the optimal solution (V_*,Q_*) is characterised by two time points for each time interval, where the tariff is constant. Hence, the control problem is equivalent to a finite dimensional nonlinear nonconvex optimization problem, which was solved using an algorithm based on imbedding (see Gfrerer et al. [13], CPU-time about 6 min.).

In a further step the assumption $Z(t) < Q_{max}$ for $t \in [T_B,T_E]$ was dropped and spillage water S introduced as an additional control variable. The analogous analysing was done in [15], and finally also resulted in a finite dimensional optimization problem.

3.2.3 A Generalized Gradient Procedure

Assuming $Q_{min} < Z(t) < Q_{max}$ we obtain $S \equiv 0$. Then model (P1) is a special problem of the following general type:

Model (A) $S(x,u) \rightarrow \max!$

$T(x,u) = 0$

$z: = (x,u) \in D_z = D_x \times D_u$

$S: X \times U \rightarrow \mathbb{R}, \quad T: X \times U \rightarrow X$

We assume that

- X Banach space, U reflexive Banach space
- $\bigwedge_{u \in U} \bigvee^{1}_{x \in X} \; x = F(u), \; T(F(u),u) \equiv 0$
- T,S twice Frechet differentiable
- $T'_x(x,u)$ regular

The basic idea is that the procedure yields $z: = \bar{z} + \Delta z$ for a starting point $\bar{z} = (\bar{x},\bar{u})$ with $T(\bar{x},\bar{u}) = 0$ so that $S(z) < S(\bar{z})$, $T(z) = 0$ and $z \in D_z$.

For details see Engl/Wacker/Zarzer [9]. This method was used by Rathmair [22] for Partenstein, resulting only in a slightly different optimal solution than in 3.2.1, see figure 11. Cf. also Wacker [24].

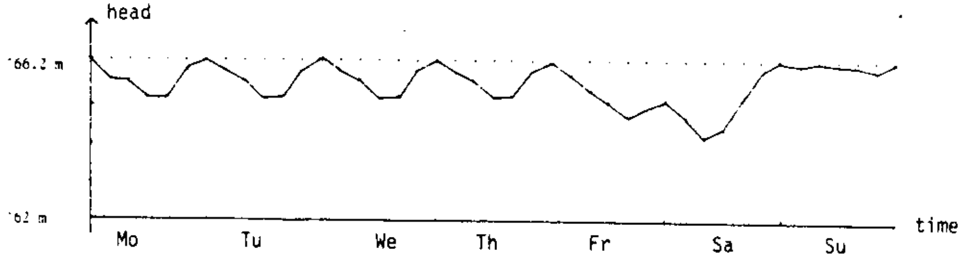


Figure 11: sketch of optimal head for influx lo m^3/s

This indicates that the model in 3.2.2 yields at least the principle behaviour of the optimal solution but needs much less computational effort.

Though the project is not finished we already got some information on the improvements due to our optimization. As the managing engineer told us the mean amount of discharge needed for producing 1 kWh was reduced by about lo percent, i.e., the production was increased by about lo percent.

4 The Power Plant System Gosau

The system is a chain of three storage power plants, situated in southern Upper Austria, namely the Salzkammergut in the calcite ranges of the Austrian Alpes. The following figure 12 gives a sketch of the hydrological system. We again use the notation of (M2) in section 2, Q_L will be explained below. We add subscript 1,2, and 3 for denoting power plant Gosau, Gosauschmied, and Steeg, resp.

The highest point of the system is Lake Gosau, a natural lake whose capacity for storing water was rised by building a dam. The maximum, and the minimum, level of the reservoir are 923,25m, and 875,oo m, resp. Only the upper part of this reservoir, namely till 9o2,5o m, can directly be harnessed while the remaining water can only be used by means of a shaft pump. The differences with respect to the heights in figure 12 are due to technical restrictions, i.e. to ensure sufficient pressure of water in the head rise gallery and in the shaft pump.
Having passed the head rise gallery - and if necessary the shaft pump before - the water runs via the surge tank and the penstock into the pumping turbine of the power plant Gosau. The outlet is directly into the reservoir Gosauschmied. Alternatively this pumping turbine can restore water from the reservoir Gosauschmied back into Lake Gosau.
The special problem of this chain of power plants is partly inherent in the seepage losses Q_L of Lake Gosau. The surrounding area is mainly limestone, that is why there are considerable seepage losses through the bottom and the side walls of the reservoir. It is known that these losses depend on the actual height of the level of Lake Gosau and are part of the influx to the reservoir Gosauschmied (see [4]).
We are not allowed to model spillage water S_1 for Lake Gosau, flood protection purposes have to be covered by a suitable long term drainage through the turbines. Practically this is also done by reducing $h_{V,max,1}$ by 1 m.

After the reservoir Gosauschmied the water runs down the Gosau valley as a natural brook. At the end of this plain valley there is the reservoir Klaushof. Finally the water runs through the head rise gallery, the surge tank and the penstock into the turbines of the power plant Steeg and afterwards into Lake Hallstatt.

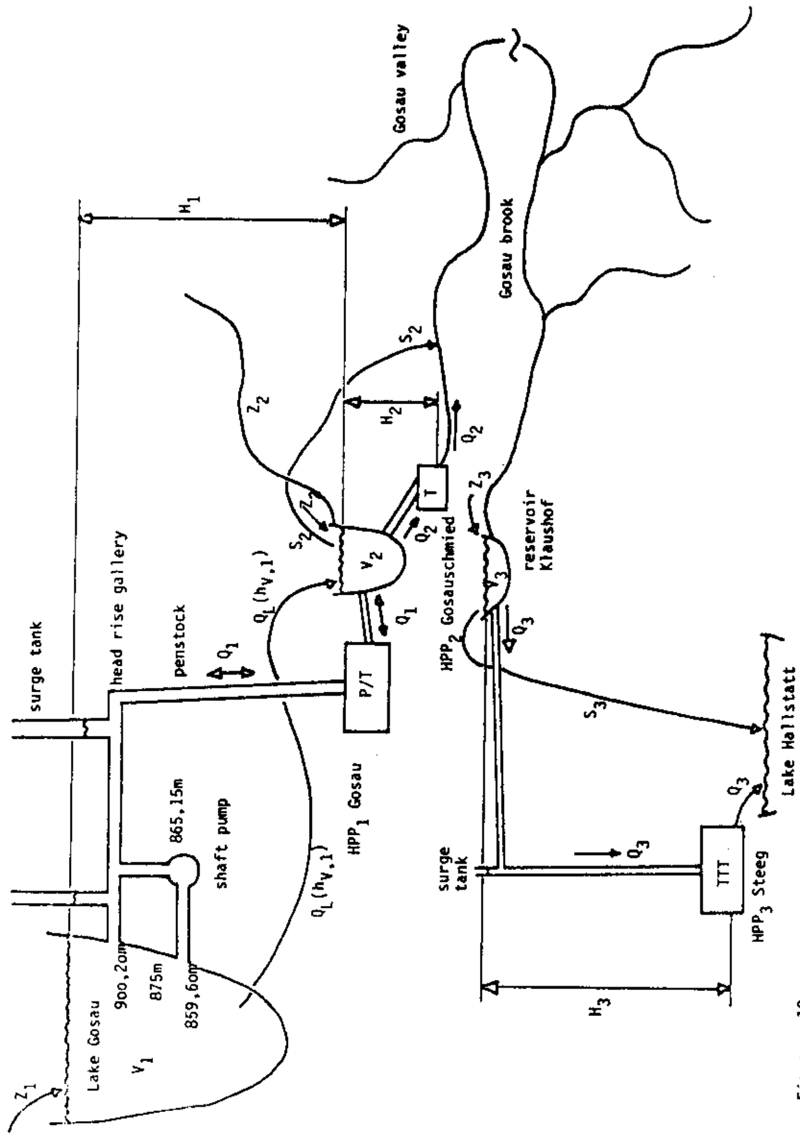


Figure 12

Additionally to the physical constraints we have to consider some other restrictions:

i) There is a supply contract with the ÖBB (Austrian Federal Railway Company), saying that Steeg has to produce 1,5 MW from 6 a.m. till 1o p.m. daily.

ii) Lake Gosau is situated in a touristic area of Austria, therefore, one has to keep in mind the landscape. This results in the restriction, that the level of Lake Gosau must not fall below 915 m from June 15th till October 15th. There also is an agreement with the local tourist office to keep the level of Lake Gosau at least at about 92o/921 m during July and August.

For the system Gosau there are the following data:

power plant Gosau:

one Francis pumping turbine with a maximum power of 5,5 MW
one three-phase synchronous alternator, with 7 MVA

power plant Gosauschmied:

one Kaplan turbine with a maximum power of o,8 MW
one three-phase asynchronous alternator with 1 MVA

power plant Steeg:

one twin Francis turbine with a maximum power of 15 MW
one three-phase synchronous alternator with 15,5 MVA
two Pelton turbines with a maximum power of 2,4 MW each
two one-phase synchronous alternators with 1,5 MVA each

For a more detailed description and more technical details we refer to [1, 4, 18]. In Steeg the supply contract with the ÖBB can only be fulfilled by means of the Pelton turbines. This is their only purpose. Note, that spillage water in Gosauschmied, and Klaushof, is overflowing water.

For the system Gosau we use a model derived from (M2).

Model (SG):

Maximize

(SG.1) $$E(T_B,T_E) = \int_{T_B}^{T_E} \frac{a(t)}{3\,600\,000} (P_1(t)+P_2(t)+P_3(t))dt$$

where

(SG.2) $$P_1(t) = g.\rho.f_1(V_1(t),V_2(t)).Q_1(t).\eta_1(f_1(V_1(t),V_2(t)),Q_1(t))$$

(SG.3) $$P_2(t) = g.\rho.f_2(V_2(t))Q_2(t).\eta_2(f_2(V_2(t)),Q_2(t))$$

(SG.4) $$P_3(t) = g.\rho.f_3(V_3(t))Q_3(t).\eta_3(f_3(V_3(t)),Q_3(t),t)$$

subject to

(SG.5) $$V_1(t) = V_1(T_B) + \int_{T_B}^{t} [Z_1(\tau)-Q_1(\tau)-Q_L(C_1^{-1}(V_1(\tau)))]\, d\tau$$

(SG.6) $$V_2(t) = V_2(T_B) + \int_{T_B}^{t} [Z_2(\tau)+Q_1(\tau)+Q_L(C_1^{-1}(V_1(\tau)))-Q_2(\tau)-S_2(\tau)]d\tau$$

(SG.7) $$V_3(t) = V_3(T_B) + \int_{T_B}^{t} [Z_3(\tau)+Q_2(\tau-\tau_2)+S_2(\tau-\tau_2)-Q_3(\tau)-S_3(\tau)]d\tau$$

(SG.8) $V_i(T_B) = V_{B,i}$

(SG.9) $V_i(T_E) = V_{E,i}$ $\quad i = 1,2,3$

(SG.1o) $V_{min,i} \le V_i(t) \le V_{max,i}$

(SG.11) $Q_{min,i} \le Q_i(t) \le Q_{max,i}$ $\quad i = 1,2$

(SG.12) $Q_{min,3D} \leq Q_3(t) \leq Q_{max,3}$ daily from 6am till 1opm

(SG.13) $Q_{min,3N} \leq Q_3(t) \leq Q_{max,3}$ daily from 1opm till 6am

(SG.14) $0 \leq S_i(t)$ $i = 2,3$

(SG.15) $15.1o^6 \leq V_1(t)$ from June 15th till October 15th

(SG.16) $18.1o^6 \leq V_1(t)$ during July and August

(SG.17) $(V_{max,i} - V_i(t)).S_i(t) = 0$ $i = 2,3$

(SG.17) describes, that spillage water is overflowing water, i.e. water running over the dam. The supply contract with the ÖBB is hidden in (SG.4), (SG.12), and (SG.13). Originally, we had

$Q_3(t) = Q_{31}(t) + Q_{32}(t) + Q_{33}(t)$, with separate upper and lower bounds on $Q_{3j}(t)$, $j = 1,2,3$

$P_3(t) = P_{31}(t) + P_{32}(t) + P_{33}(t)$,

$P_{32}(t) + P_{33}(t) = 1,5.1o^6$ daily from 6am till 1opm (*),

$P_{32}(t) + P_{33}(t) = 0$ daily from 1opm till 6am (**),

and an upper bound on the capacity of the penstock, i.e.

$Q_{31}(t) + Q_{32}(t) + Q_{33}(t) \leq PS_{max}$, where Q_{31}, and Q_{32}, Q_{33}, represent(s) the Francis, and the Pelton, turbine(s), resp. in Steeg. Because of the special situation in Steeg we can calculate $Q^*_{32}(t) = Q^*_{33}(t) = q\ (Q_{31}(t),t)$ a-priori. This leads to (SG) finally by the monotonic behaviour of the friction losses for Steeg. (*), and (**) are still reflected in the split lower bound on $Q_3(t)$ and in the explicit parameter t in η_3.

τ_2 is the mean running time of water from Gosauschmied to Klaushof.

As optimization period $[T_B,T_E]$ we take the hydrological year for Lake Gosau, i.e. September till August. Even a time discretization using just two time points a day (i.e. when the tariff changes) results in a nonlinear optimization

problem with 219o variables and 876o constraints. Hence, for the annual optimization some simplifications had to be met.

For the running time τ_2 is about one hour it was dropped for the long term models. Further, the reservoir Klaushof was neglected because of its extremely small capacity (4ooo m^3). Therefore, $Q_3(t)$ is calculated in dependence on $Q_1(t)$ and $Q_2(t)$. Assuming a constant efficiency at Gosauschmied we determined by the same methods as in 3.2.2 the optimal solution $Q_2^*(t)$. However, even 73o variables are too many.

Hence, for the annual optimization we took advantage of the hydrological rhythm of Lake Gosau. Moreover, the power plant company well-comed the following splitting into three periods:

Phase I:

The earliest starting for sinking the level of Lake Gosau in autumn is fixed at September 1st because of tourism. The sinking process ends at December 31st at the level of 9o2,5 m.

Phase II:

At first the level of 9o2,5 m is kept fixed because of purposes of reserve, only the natural influx is used for production. Then the sinking process is continued using the shaft pump system where both the starting point and the lowest level are determined by the optimization. Phase II ends at June 15th when a level of 915 m must be reached both by the natural influx due to melted snow and by water pumped back from Gosauschmied into Lake Gosau.

Phase III:

Storing water is going on up to a level of Lake Gosau of 92o m. Lake Gosau is kept at this level during summertime. There is no optimization to be done.

However, before starting any optimization code we had to spend considerable time for determining the seepage losses Q_L, the influx, and the efficiencies; for details see Bauer et al. [4], [5], Wacker et al. [25].

Optimization of Phase I:

At first the optimal draining strategy for Lake Gosau alone was obtained by dynamic programming (DP). Just considering the first power plant was motivated by getting an insight in the principle behaviour of the system. Afterwards the optimization problem for all three power plants was considered.

Additionally two refined models were used. Using the decomposition/convexification method, cf. 3.2.1, a weekly optimization with an hourly time grid was done, see Gfrerer [12]. A special nonlinear optimization model for one day was set up which included peak power demands and where no simplifications were met as stated above, see [15]. This was done for justifying our simplifications and for getting detailed information on short term optimization.

Optimization of Phase II:

It is based on two models using DP. The first one is done by time discretization as above, DP was combined with lower- and upper bound techniques and with a special splitting technique, see Bauer et al. [6].
In the second model the variables are certain operation modes. There are ten different ones, where each of them represents a certain mode fixed for one week. We result in a nonlinear nonconvexe discrete optimization problem with 24 variables and 95 restrictions, for details see Bauer [3].

Presentation of Optimization Results:

The managing staff of the power plant system Gosau did not get columns of numbers etc. but a verbal formulation how to obtain the optimal value resp. the optimal control for the power plants.

For the daily optimization we presented two main guidelines:

- Avoid spillage water, at least at Klaushof
- If possible produce energy at the higher tariff

and four additional guiding rules:

- The efficiency functions are not so important, nevertheless the turbines are to be operated near their optimal efficiency value ($\pm$ 1 m^3/s).
- The seepage losses of Lake Gosau are of little interest
- The reservoir Gosauschmied is used as an auxiliary reservoir for Steeg
- Due to the running time τ_2 of one hour Gosauschmied starts producing energy already at 5 am, but stops already at 9pm in contradiction to the guidelines due to the tariff.

Hence, the acceptance of the results was rised as sufficiently many decisions were left to the personal staff of the power plants.

Practical Results:

The results of our optimization models have been put into practise since 1982. We confine ourselves to present some data of the last years made available by the OKA. Production gives the energy produced/used, and pc is the production coefficient of the river power plant Gmunden, indicating the amount of precipitation. During the last four years the results improved significantly, especially taking pc into account.

year	production [GWh]	pc	$\frac{\text{production}}{\text{pc}}$
1968	1,75	-	-
1969	1,55	o,39	3,97
1971	o,65	o,56	1,16
1974	1,28	1,58	o,81
1979	2,o9	1,11	1,88
1982	1,93	o,6o	3,22
1983	2,65	o,78	3,4o
1984	2,55	o,6o	4,25
1985	2,26	o,46	4,91

Figure 13: Results of Phase I (Gosau)

year	production [GWh]	pc	production / pc
1969	-o,2	o,78	-,26
1976	-o,7	o,79	-,89
1983	o,4	1,o1	o,4o
1984	o,7	o,86	o,81
1985	o,6	o,9o	o,67
1986	1,2	1,o8	1,11

Figure 14: Results of Phase II (Gosau)

Acknowledgement:

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