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" SPRING COLLEGE ON GEOMAGNETISM AND AERONOMY "

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" Morphology of the geomagnetic field "
" Origin of the Earth's magnetic field "
" Energy sources of the Earth's magnetic field "
" Reversals of the Earth's magnetic field "
" Secular variation & changes in the length of the day "

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These are preliminary lecture notes, intended for distribution to participants only.

Lecture 1

Morphology of the geomagnetic field

1.1 Introduction

The total intensity

F , the declination D and the inclination I completely define the magnetic field at any point, although other components are often used. The horizontal and vertical components of F are denoted by H and Z . H may be further resolved into two components X and Y , X being the component along the geographical meridian and Y the orthogonal component. Figure 5.1 illustrates these different

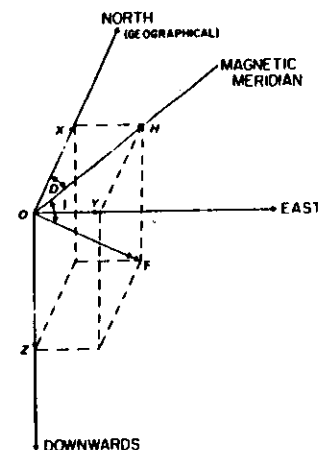


Fig. 5.1

magnetic elements. They are simply related to one another by the following equations,

$$H = F \cos I, \quad Z = F \sin I, \quad \tan I = Z/H \quad (5.1)$$

$$X = H \cos D, \quad Y = H \sin D, \quad \tan D = Y/X \quad (5.2)$$

$$F^2 = H^2 + Z^2 = X^2 + Y^2 + Z^2 \quad (5.3)$$

The variation of the magnetic field over the earth's surface is best illustrated by isomagnetic charts, i.e. maps on which lines are drawn through points at which a given magnetic element has the same value. Contours of equal intensity in any of the elements X , Y , Z , H or F are called isodynamics. ~~Figure 5.2 illustrates these different~~

1.2 The field of a uniformly magnetized sphere

Consider therefore the field of a uniformly magnetized sphere whose magnetic axis runs north-south, and let P be any external point distant r from the centre O and θ the angle NOP , i.e. θ is the magnetic co-latitude (see figure 1.7). If m is the magnetic moment of a geocentric dipole directed along the axis, the potential at P is

$$V = \frac{m \cos \theta}{4\pi r^2} \quad (1.4)$$

The inward radial component of force corresponding to the magnetic

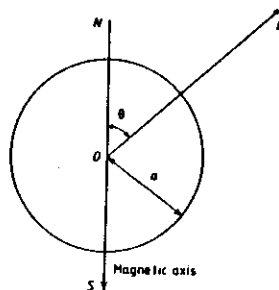


Figure 1.7

component Z is given by

$$Z = -\mu_0 \frac{\partial V}{\partial r} = \frac{\mu_0 m \cos \theta}{2\pi r^3} \quad (1.5)$$

and the component at right angles to OP in the direction of decreasing θ ,

corresponding to the magnetic component H , by

$$H = -\mu_0 \frac{1}{r} \frac{\partial V}{\partial \theta} = \frac{\mu_0 m \sin \theta}{4\pi r^3} \quad (1.6)$$

where μ_0 is the permeability of free space.

The inclination I is then given by

$$\tan I = Z/H = 2 \cot \theta \quad (1.7)$$

and the magnitude of the total force F by

$$F = (H^2 + Z^2)^{1/2} = \frac{\mu_0 m}{4\pi r^3} (1 + 3 \cos^2 \theta)^{1/2} \quad (1.8)$$

Thus intensity measurements are a function of latitude.

1.3 Spherical harmonic analysis of the Earth's magnetic field

Assuming that there is no magnetic material near the ground, the Earth's magnetic field can be derived from a potential function V which satisfies Laplace's equation and can thus be represented as a series of spherical harmonics

$$V = \frac{\mu_0}{4\pi} \sum_{n=1}^{\infty} \sum_{m=0}^n P_n^m(\cos \theta) \left\{ \left[c_n^m \left(\frac{r}{a} \right)^n + (1 - c_n^m) \left(\frac{a}{r} \right)^{n+1} \right] A_n^m \cos m\phi \right. \\ \left. + \left[s_n^m \left(\frac{r}{a} \right)^n + (1 - s_n^m) \left(\frac{a}{r} \right)^{n+1} \right] B_n^m \sin m\phi \right\} \quad (1.9)$$

Written in this form the coefficients A_n^m and B_n^m have the dimensions of magnetic field, c_n^m and s_n^m are numbers lying between 0 and 1, and represent the fractions of the harmonic terms $P_n^m(\cos \theta) \cos m\phi$ and $P_n^m(\cos \theta) \sin m\phi$ in the expansion of V which, on the surface of the sphere ($r = a$), are due to matter outside the sphere. There is no term with $n=0$, which would correspond to a magnetic monopole within the Earth. It is also assumed that there are no electric currents flowing across the surface of the Earth: if there were they would set up a non-potential field and thus contribute a part of the Earth's magnetic field which could not be represented by equation (1.9).

The potential V cannot be measured directly; what can be determined are the three components of force $X = (\mu_0/r)(\partial V/\partial \theta)$ (horizontal, northward), $Y = (-\mu_0/r \sin \theta)(\partial V/\partial \phi)$ (horizontal, eastward) and $Z = \mu_0(\partial V/\partial r)$ (vertical, downward) at the Earth's surface, $r = a$. Z (at $r = a$) may be expanded as a series of spherical harmonics

$$Z = \mu_0 \frac{\partial V}{\partial r} = \sum_{n=1}^{\infty} \sum_{m=0}^n P_n^m(\cos \theta) [\alpha_n^m \cos m\phi + \beta_n^m \sin m\phi] \quad (1.10)$$

and the coefficients α_n^m , β_n^m determined from the observed values of Z .

By differentiating equation (1.9) with respect to r and then writing $r = a$, we have

$$Z = \mu_0 \frac{\partial V}{\partial r} = \sum_{n=1}^{\infty} \sum_{m=0}^n P_n^m(\cos \theta) \{ [nc_n^m - (n+1)(1-c_n^m)] A_n^m \cos m\phi \\ + [ns_n^m - (n+1)(1-s_n^m)] B_n^m \sin m\phi \} \quad (1.11)$$

The coefficients of each separate harmonic term for each n and m must be equal in the two expansions of Z given by equations (1.10) and (1.11). Hence

$$\alpha_n^m = [nc_n^m - (n+1)(1-c_n^m)] A_n^m \\ \beta_n^m = [ns_n^m - (n+1)(1-s_n^m)] B_n^m \quad (1.12)$$

Again from an analysis of the observed values of X and Y , the coefficients in the following two expansions derived from equation (1.9) may be obtained

$$Y_{r=a} = \left(-\frac{\mu_0}{r} \frac{\partial V}{\partial \phi} \right)_{r=a} \\ = \frac{1}{\sin \theta} \sum_{n=1}^{\infty} \sum_{m=0}^n P_n^m(\cos \theta) (mA_n^m \sin m\phi - mB_n^m \cos m\phi) \quad (1.13)$$

$$X_{r=a} = \left(\frac{\mu_0}{r} \frac{\partial V}{\partial \theta} \right)_{r=a} \\ = \sum_{n=1}^{\infty} \sum_{m=0}^n \frac{d}{d\theta} P_n^m(\cos \theta) (A_n^m \cos m\phi + B_n^m \sin m\phi) \quad (1.14)$$

From a knowledge of the coefficients A_n^m , B_n^m , α_n^m and β_n^m , equations (1.12) determined c_n^m and s_n^m . Gauss found from data available at that time that $c_n^m = s_n^m = 0$, i.e. the source of the Earth's magnetic field is entirely internal. The coefficients of the field of internal origin are

$$g_n^m = (1 - c_n^m) A_n^m \quad h_n^m = (1 - s_n^m) B_n^m \quad (1.15)$$

and are known as Gauss coefficients. If the external field is negligible, equations (1.15) reduce to $g_n^m = A_n^m$, and $h_n^m = B_n^m$. Values of these coefficients are

Lecture 2

Origin of the Earth's magnetic field

A review of theories that have been shown to fail has been given by
D.J. Stevenson, Icarus 22, 403, 1974.

References for the geodynamo

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2.1 The homogeneous dynamo equations

The dynamo problem involves the solution of a highly complicated system of coupled partial differential equations - electrodynamic, hydrodynamic and thermodynamic. Elsasser (1954) showed by a dimensional analysis that in geophysical and astrophysical problems the displacement current and all purely electrostatic effects are negligible, as are all relativistic effects of order higher than U/c where U is the fluid velocity. Thus the electromagnetic field equations are the usual Maxwell equations

$$\nabla \times E = -\partial B / \partial t \quad (1.19)$$

$$\nabla \times B = \mu_0 j \quad (1.20)$$

$$\nabla \cdot B = 0 \quad (1.21)$$

where B and E are the magnetic and electric fields respectively and j the electric current density. The electromotive forces which give rise to j are due both to electric charges and to motional induction so that the total current j is given by

$$j = \sigma(E + U \times B). \quad (1.22)$$

Assuming the electrical conductivity σ to be constant, taking the curl of equation (1.20), and using equations (1.22) and (1.19), E can be eliminated, leading to the equation

$$\nabla \times (\nabla \times B) = \mu_0 \sigma \left(-\frac{\partial B}{\partial t} + \nabla \times (U \times B) \right) \quad (1.23)$$

Since $\nabla \times (\nabla \times B) = \nabla(\nabla \cdot B) - \nabla^2 B = -\nabla^2 B$, on using equation (1.21), we finally obtain

$$\partial B / \partial t = \nabla \times (U \times B) + v_m \nabla^2 B \quad (1.24)$$

where

$$v_m = 1 / \mu_0 \sigma \quad (1.25)$$

is the magnetic diffusivity. Equations (1.21) and (1.24) give the relationships between B and U which have to be satisfied from electromagnetic considerations. The term $\nabla \times (U \times B)$ in equation (1.24) is the source term, and represents the physical process by which magnetic induction is 'created' through the flow of fluid across lines of force. The term $v_m \nabla^2 B$ represents the tendency for the field to decay through ohmic dissipation by the electric currents supporting the field. ~~The balance between these two terms, none~~

Consider special cases of equation (1.24)

Magnetized at rest ($U=0$)

Magnetized has negligible electrical resistance ($v_m=0$)

Frozen fields. Magnetic Reynolds number R_m .

To the electromagnetic equations must be added the hydrodynamical equation of fluid motion in the Earth's core (the Navier-Stokes equation) together with the equation of continuity, which, for an incompressible fluid (the speed of flow is much less than the speed of sound in the Earth's core) reduces to

$$\nabla \cdot U = 0. \quad (1.29)$$

The Navier-Stokes equation is

$$\rho \left(\frac{\partial U}{\partial t} + (U \cdot \nabla)U + 2\Omega \times U - v \nabla^2 U \right) - \frac{1}{\mu_0} (\nabla \times B) \times B = -\nabla p + \rho \nabla W \quad (1.30)$$

where U is the velocity relative to a system rotating with angular velocity Ω , p the pressure, W the gravitational potential (in which is absorbed the centrifugal force) and ρ and v the density and kinematic viscosity, respectively. Equations (1.24) and (1.30) contain only the vectors U and B and are the basic equations of field motion.

Most dynamo models use large-scale, highly ordered fluid motions, i.e. motions in which the characteristic length of the velocity field is not much less than the radius of the Earth. In the early 1950s several attempts were made to produce models in which turbulent (i.e. random and small-scale) velocities might act as dynamos. The modern theory, which has been called mean field electrodynamics, has been developed independently by Moffatt (1970) in Britain and by Krause, Rädler and Steenbeck in Germany. An account of the German work has been given in a recent book by Krause and Rädler (1980).

In mean field dynamo models the velocity U and magnetic field B are each represented as the sum of a statistical average and a fluctuating part. We thus write

$$U = U_0 + u \quad \langle u \rangle = 0 \quad (1.31)$$

$$B = B_0 + b \quad \langle b \rangle = 0. \quad (1.32)$$

The average fields U_0 and B_0 are assumed to vary on a length scale L , while the fluctuating fields u and b (with zero statistical average) are assumed to vary on a length scale l ($l \ll L$). This separates the velocity and magnetic fields into mean, slowly varying and fluctuating parts. The induction equation may then be divided into its mean and fluctuating parts

$$\partial B_0 / \partial t = \nabla \times (U_0 \times B_0) + \nabla \times \varepsilon + v_m \nabla^2 B_0 \quad (1.33)$$

$$\partial b / \partial t = \nabla \times (U_0 \times b) + \nabla \times (u \times B_0) + \nabla \times G + v_m \nabla^2 b \quad (1.34)$$

where

$$\varepsilon = \langle u \times b \rangle \quad \text{and} \quad G = u \times b - \langle u \times b \rangle. \quad (1.35)$$

ε can be regarded as an extra mean electric force arising from the interaction of the turbulent motion and magnetic field. If the velocity field is isotropic, it can be shown that

$$\varepsilon = \alpha B_0 - \beta \nabla \times B_0 \quad (1.36)$$

where α and β depend on the local structure of the velocity field.

The induction equation (1.33) satisfied by the mean field is

$$\partial B_0 / \partial t = \nabla \times (\alpha B_0 + U_0 \times B_0) + (v_m + \beta) \nabla^2 B_0. \quad (1.37)$$

The term αB_0 represents an electric field parallel to B_0 . The quantity β is often replaced by a total diffusivity $v_m + \beta$. Parker (1955) first drew attention to the possibility that $\varepsilon = \alpha B_0$, and Steenbeck and Krause (1966) christened this the α effect. A key concept in the theory is the helicity defined as $u \cdot (\nabla \times u)$. Parker (1955, 1970) showed that convective fluid motions having non-zero helicity could distort lines of magnetic force in such a way as to produce a regeneration mechanism. ~~Another vanishing helicity indicates that the velocity~~

There are two possible types of dynamo using the α effect - α^2 dynamos and $\alpha\omega$ dynamos. In an α^2 dynamo the α effect generates poloidal field from toroidal field and generates toroidal field from poloidal field. The toroidal field has lines of force that lie on spherical surfaces and has no component external to the core. The poloidal field has a radial component in general, and joins continuously with the external, observed field. The α effect can also be used in conjunction with a large-scale shear flow (the ω effect) to produce an $\alpha\omega$ dynamo. Parker (1955, 1971, 1979) is primarily responsible for the development of dynamo models of this type, in which an α effect from cyclonic turbulence generates poloidal field from toroidal field, and differential rotation creates toroidal field from poloidal field thereby completing the cycle.

A differential rotation by itself cannot produce a poloidal field from a toroidal field, only a toroidal field from a poloidal one. However, if there is an α effect as well as a strong differential rotation, the latter may dominate in producing a toroidal field from a poloidal field. For a dynamo of the α^2 type, the poloidal and toroidal fields are of the same order of magnitude, whereas in the case of an $\alpha\omega$ -type dynamo, the toroidal field is much stronger than the poloidal one.

Lecture 3

Energy sources of the Earth's magnetic field

1. Structure of the Earth

Information from seismology. Velocity-depth curves and free oscillation data \rightarrow density, pressure, g & elastic parameters.

$$\text{Velocity of P waves } v_p = \sqrt{\frac{\kappa_s + \frac{4}{3}\mu}{\rho}}$$

$$\text{Velocity of S waves } v_s = \sqrt{\frac{\mu}{\rho}}$$

κ_s is the bulk modulus (or adiabatic incompressibility)

defined as

$$\frac{1}{\kappa_s} = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial p} \right)_s$$

μ is the modulus of rigidity, ρ density and p pressure.

N.B.
$$\phi = v_p^2 - \frac{4}{3} v_s^2 = \frac{\kappa_s}{\rho} = \left(\frac{\partial p}{\partial \rho} \right)_s$$

2. Thermal evolution of the Earth

Heat sources of embryo Earth

Need to know the adiabatic and liquidus gradients in the fluid outer core.

3. Constitution of the core

Shock wave data.

Outer core, ρ & $\kappa_s <$ those of Fe under equivalent conditions (by $\sim 10\%$)

Inner core, ρ & $\kappa_s \simeq$ those of Fe

Light element in outer core - Si, S, O?

Possibility of K (& hence ^{40}K) in outer core.

4. Energetics of the Earth's Core

Possible driving force for the geodynamo

- (i) Precession
- (ii) Thermal convection
- (iii) Core formation
- (iv) Seismic activity

Under (ii) consider latent heat released by formation of solid inner core

Under (iii) consider "compositionally-driven" convection - the gravitational energy released by the separation of matter in the core to form the solid inner core.

5. References

J.A. Jacobs, in Physics of the Earth's Interior, 1980 LXXVIII Corso Soc. Italiana di Fisica, Bologna, Italy.

J.A. Jacobs, The Earth's Core, Acad. Press, 1975
(2nd edition in press).

Lecture 4. Reversals of the Earth's Magnetic Field

Topics covered will include

1. Field reversals or self-reversal
2. The morphology of geomagnetic reversals
Field intensity and direction during a polarity transition
Mean frequency of reversals
3. Geomagnetic excursions (aborted reversals?)
The Laschamp, Lake Mungo & Mono Lake excursions
The Gothenburg "flip"
4. Models for reversals
The disc dynamo
Models of Cox, Parker, Hoffman, Olson
Statistical Analyses
Secular variation, reversals and polarity bias
5. Magnetostratigraphy
The polarity time scale
6. Reversals and other geophysical phenomena
Magnetic reversals and changes in climate
Magnetic reversals and ice ages
Magnetic reversals and ^{faunal}~~faunal~~ extinctions

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Lecture 5. Secular variation & changes in the length of the day

5.1 Core motions

The induction equation is

$$\dot{\mathbf{B}} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \nu_m \nabla^2 \mathbf{B} \quad (5.1)$$

In the frozen flux approximation (i.e. treating the core as a perfect conductor), we ignore diffusion & equation (5.1) becomes

$$\begin{aligned} \dot{\mathbf{B}} &= \nabla \times (\mathbf{u} \times \mathbf{B}) \\ &= (\mathbf{B} \cdot \nabla) \mathbf{u} - \mathbf{B} (\nabla \cdot \mathbf{u}) - (\mathbf{u} \cdot \nabla) \mathbf{B} + \mathbf{u} (\nabla \cdot \mathbf{B}) \end{aligned}$$

Take the radial component at the MCB where $\mathbf{u}_r = 0$

Then since $\nabla \cdot \mathbf{B} = 0$, we have

$$\dot{B}_r = -B_r (\nabla_H \cdot \mathbf{u}) - (\mathbf{u} \cdot \nabla_H) B_r \quad (5.2)$$

where

$$\nabla_H = \nabla - \hat{r} (\hat{r} \cdot \nabla)$$

From a knowledge of surface magnetic data, find B_r & \dot{B}_r at the MCB and hence solve equation (5.2) for \mathbf{u} .

There have been several attempts to infer quantitative estimates of core motions from geomagnetic data. There are two main problems. First, extrapolation of the geomagnetic field from the Earth's surface to the MCB is an unstable process. Second, even if the magnetic field and its secular change at the MCB were perfectly resolved, the velocity field could not be deduced uniquely.

Backus showed that a necessary condition for the frozen flux assumption to hold is that

$$\int_{S_i} \dot{B}_r ds = 0 \quad (5.3)$$

where S_i is a patch of the MCB bounded by a null flux curve (contour on which $B_r = 0$). There appear to be about five null flux curves on the MCB at present.

From equation (5.2), where $\nabla_H B_r = 0$ (i.e. at local maxima, minima and saddle points of the radial field),

$$\nabla_H \cdot u = - \frac{\dot{B}_r}{B_r} \quad (5.4)$$

$\nabla_H \cdot u$ measures convergence or divergence of flow at a point, and, for an incompressible flow, this indicates upwelling or downwelling of fluid. This is important for the thermodynamics of the core - no upwelling indicating that the core is stably stratified near the MCB.

Bloxham & Gubbins (Geophys. J. Roy. Astr. Soc. 84, 139, 1986) tested the frozen flux hypothesis and found evidence for flux diffusion.

Bloxham & Gubbins (Nature 317, 777, 1985) obtained models of the magnetic field at the MCB at selected epochs from 1715 to 1980. Westward drift occurs only in certain well defined regions of the core - in the northern Pacific Ocean there is a slow eastward drift.

5.2 Variations in the length of the day (l.o.d.)

Earth's angular velocity Ω not constant

Changes in rate of spin $1/\Omega$ i.e. changes in l.o.d. and also changes in the direction of Ω i.e. the Earth wobbles.

Number of peaks in the frequency spectrum of the Earth's rate of spin.

Three distinct components have been reorganized.

Steady increase in the l.o.d. ($\sim 2 \times 10^{-3}$ s / century) - tidal friction

Seasonal fluctuations ($\sim 10^{-3}$ s) - oceanic currents and atmospheric winds.

Less regular variations ($\sim 5 \times 10^{-3}$ s) having time ^{scales} ~~periods~~ of the order of years (The "decade" fluctuations).

It has been suggested that these decade fluctuations are caused by the transfer of angular momentum between the solid mantle and liquid core.

Need some form of core-mantle coupling - possible mechanisms are inertial, electromagnetic, and topographic. Maximum torque required to act across the MCB to ^{account} ~~account~~ for the decade fluctuation in the l.o.d. is $\sim 10^{18}$ Nm, corresponding to an average tangential stress of 2×10^{-3} Nm⁻² on the MCB. The various possible mechanisms for core-mantle coupling have been reviewed by Rochester (Phil. Trans. Roy. Soc. London A 313, 95, 1984).

Le Mouél et al. (Nature 290, 763, 1981) found that l.o.d.

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variations lagged geomagnetic variations by 10 - 15 years. Such a correlation is opposite to that proposed by other workers (eg. Runcorn, Phil. Trans. Roy. Soc. London A 306, 261, 1982 and Backus, Geophys. J. Roy. Astr. Soc. 74, 713, 1983 where changes in the westward drift follow L.O.D. changes, Courtillot et al. (Nature 297, 386, 1982) suggested that decade fluctuations in climate and L.O.D. are both correlated with changes in the Earth's magnetic field.

