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"Generalized theorem for no ground magnetic effect of vertical currents connected with Pedersen currents in the uniform-conductivity ionosphere"

" Ground magnetic effect of field-aligned currents connected with ionospheric currents "

presented by :

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GENERALIZED THEOREM FOR NO GROUND MAGNETIC EFFECT OF VERTICAL CURRENTS CONNECTED WITH PEDERSEN CURRENTS IN THE UNIFORM-CONDUCTIVITY IONOSPHERE

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ABSTRACT

For knowing the ground magnetic effect of a three-dimensional current system in the ionosphere-magnetosphere, we can apply a theorem which states that no ground magnetic effect is produced by the combination of vertical currents into or out of the ionosphere and the associated Pedersen currents in the ionosphere, if the electric conductivity is uniform all over the ionosphere. This theorem is shown to hold for both plane and spherical ionosphere models with any distribution of vertical currents into and out of the ionosphere. The ground magnetic effect of an actual three-dimensional current in the ionosphere-magnetosphere can be known with the aid of this theorem and the corrections for non-vertical incidence of field-aligned currents and non-uniformity of the ionospheric conductivity.

1. Introduction

The ground magnetic effect of a three-dimensional electric current system in the ionosphere and magnetosphere (containing field-aligned currents in the magnetosphere and horizontal ionospheric currents) can sometimes be easily known with the aid of a useful theorem[1], which states that the ground magnetic effect of a vertical current into or out of the ionosphere is exactly cancelled out by that of a uniformly diverging or converging current in the ionosphere connected with the inflowing or outgoing vertical current. If the ionospheric conductivity is uniform, the diverging or converging current in the ionosphere is a pure Pedersen current. Hence, the above theorem can be stated that no ground magnetic field is produced by a combination of vertical current and the associated Pedersen current in the uniform-conductivity ionosphere.

The purpose of this paper is to show that this theorem holds for both plane and spherical ionosphere models with any distribution of vertical currents into or out of the ionosphere. In the evaluation of ground magnetic effect of an actual three-dimensional current system in the ionosphere-magnetosphere, it is necessary to correct for the non-vertical flow of field-aligned currents and for the non-uniformity of the ionospheric conductivity.

2. Plane Ionosphere Model of Uniform Electric Conductivity

Fig. 1 shows a vertical line current and the associated Pedersen current

1) Fukushima, N., Rept. Ionos. Space Res. Japan, 23, 219 (1969); Radio Sci., 6, 269 (1971).

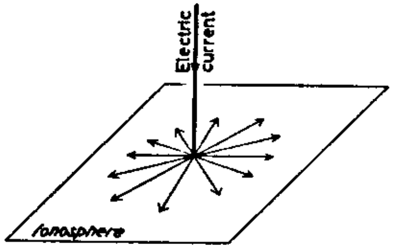

Fig. 1. An electric current is incident on an ionospheric plane of uniform electric conductivity and is converted into uniformly diverging Pedersen currents in the ionosphere.

diverging on a plane ionosphere with uniform conductivity. A cylindrical coordinate (r, ϕ, z) is taken here; the vertical downward current flows along the z-axis at $r=0$, and the diverging electric current flows on the plane $z=0$. The ionospheric current $I(i_r, i_\phi, 0)$ at a point $(r, \phi, 0)$ is given by

$$i_r(r,\phi,0)=I_0/(2\pi r), \qquad i_\phi(r,\phi,0)=0, \qquad i_z(r,\phi,0)=0. \tag{1}$$

Since the electric current system of Fig. 1 is symmetric around the z-axis, the magnetic field produced by this current system must be everywhere in the azimuthal direction. The magnetic field $H(0, H_\phi, 0)$ at a point (r, ϕ, z) is dependent on r and z. According to the Maxwellian theorem,

$$2\pi r H_\phi(r, z)=i(z), \tag{2}$$

where $i(z)$ is the total intensity of electric current passing upward through the circular plane of radius r at a height z, namely $i(z>0)=-I_0$ and $i(z<0)=0$. Therefore,

$$\left.\begin{aligned} &H_\phi(r, z)=-I_0/(2\pi r) \text{ independent of } z \text{ everywhere in the region } z>0,\\ &H_\phi(r, z)=0 \text{ everywhere in the region } z<0, \text{ i.e. below the ionosphere.}\end{aligned}\right\} \tag{3}$$

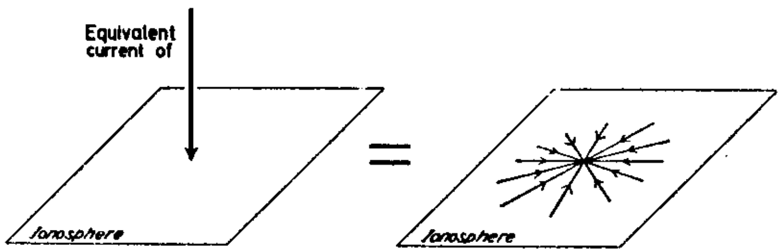

Fig. 2. The equivalent current for a vertical current flowing into the ionosphere is a current uniformly converging towards the point of intersection.

Since the current system of Fig. 1 produces no ground magnetic effect, the equivalent current for a vertical downward current into the ionosphere can be given by a uniformly converging current in the ionosphere, as illustrated by Fig. 2.

3. Spherical Ionosphere Models of Uniform Electric Conductivity

3.1. *Incidence and outflow of vertical currents at the antipodal points*

We first discuss a simple case illustrated by Fig. 3. The spherical coordinates (r, θ, ϕ) are taken with their origin at the earth's center, and a vertical current flows into and out of the ionosphere (of radius r_0) along the $\theta=0$ and $\theta=\pi$ axes, respectively. The θ-component of the ionospheric current is independent of ϕ-value because of the assumption of uniform electric conductivity all over the ionosphere. The magnetic field produced by the current system of Fig. 3 is everywhere in the ϕ-direction, and H_ϕ is given through the Maxwellian theorem as

$$\left.\begin{aligned} &H_\phi(r, \theta)=-I_0/(2\pi r \sin\theta) && \text{for } r>r_0,\\ &H_\phi(r, \theta)=0 && \text{inside the ionosphere, } r<r_0.\end{aligned}\right\} \tag{4}$$

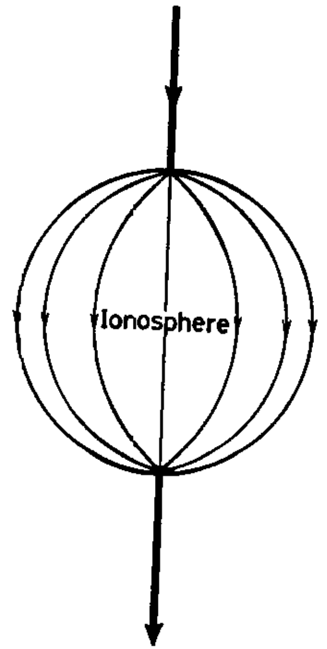

Fig. 3. An electric current flowing vertically in and out of the antipodal points of the spherical ionosphere.

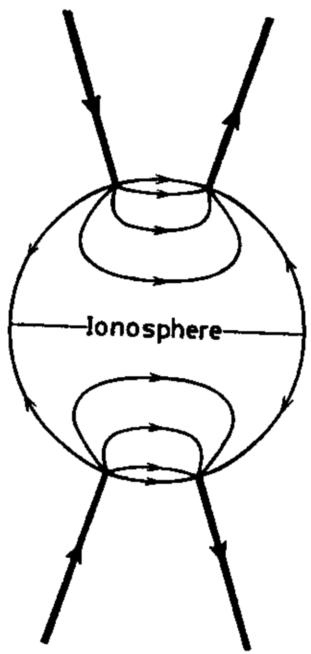

Fig. 4. Pairs of vertical electric currents flowing into and out of the ionosphere in high latitudes.

From this relation, it is evident that a current system of Fig. 4 also produces no magnetic effect below the ionosphere[2)], because this system is a simple superimposition of two systems, each of which is a similar one as given in Fig. 3.

3.2. *Case of axial asymmetry of vertical currents*

For a case in which the inflow and outflow of vertical electric currents are not at the antipodal points of the ionosphere, the following consideration can be made. As seen in Fig. 5, current system (A) is thought to consist of (A1) and (A2); the former pattern being illustrated more in detail by Fig. 6, in which a line current of intensity I_0 flow vertically into the ionosphere of radius r_0, and electric currents flow vertically outward with uniform density $I_0/(4\pi r_0^2)$ from all over the ionospheric surface. Since this current system is axisymmetric around the line connecting the earth's center and the incoming line current, no magnetic field is produced inside the ionosphere. Hence, both (A1) and (A2), and (A) of Fig. 5 produce no ground magnetic effect. This leads to an important general conclusion that the ground magnetic effect never appears for any distribution of vertical currents into and out of the ionosphere with uniform conductivity, insofar as the amounts of incoming and outgoing currents are equal. Current system (A) of Fig. 5 is only a special simple example of the generalized configuration with multiple line currents into and out of the ionosphere with uniform electric conductivity.

The current system of Fig. 6 produces a magnetic field outside the ionosphere, and it can easily be obtained in the following way. We take the spherical coordinates (r, θ, ϕ) with their origin at the earth's center, and the $\theta=0$ axis coincident with the vertical line current flowing into the ionosphere. For each of the three regions, i.e. $r\cos\theta > r_0$, $r_0 > r\cos\theta > -r_0$, and $r\cos\theta < -r_0$, circular planes L1, L2, and L3 are drawn, respectively, whose centers are located on the $\theta=0$ and π axis with radius $r\sin\theta$. Plane L2 intersects the ionosphere at colatitude θ_0, but planes L1 and L3 do not. The amount of electric currents i (consisting of

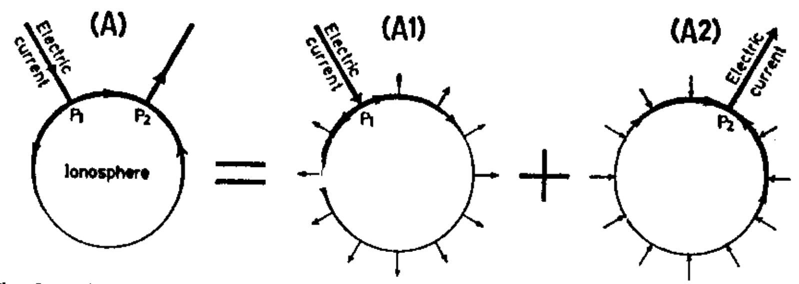

Fig. 5. Principle used to decompose any distribution of incoming and outgoing vertical currents into axisymmetric patterns.

2) The writer is grateful to Dr. V. M. Vasyliunas for suggesting this way of proof in a private discussion during the Upper Atmospheric Current and Electric Fields Symposium, Boulder, Colorado in August, 1970.

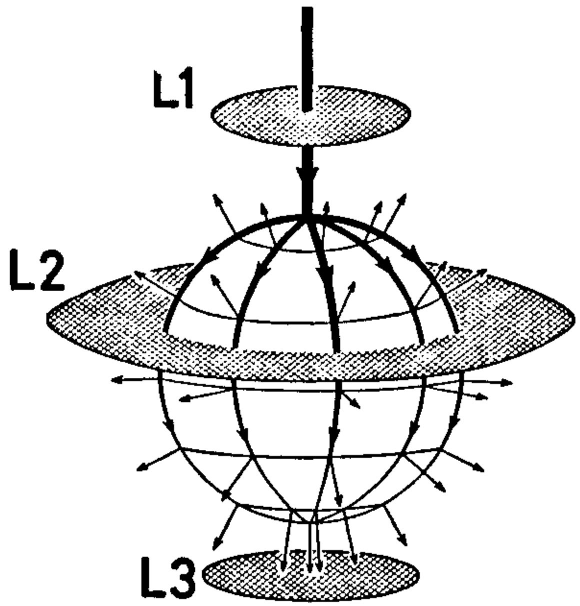

Fig. 6. A vertical line current flowing into the ionosphere with a simultaneous outflow of uniform vertical currents from all over the ionosphere. Planes L1, L2, and L3 are drawn to calculate the magnetic effect above the ionosphere.

the vertical line current into the ionosphere or the associated Pedersen currents in the ionosphere, and the vertical outgoing currents from the ionospheric surface) passing through these planes is

$$\left.\begin{aligned}
&i(\text{through plane L1}) = \frac{I_0}{2}(1-\cos\theta) - I_0 = -\frac{I_0}{2}(1+\cos\theta),\\
&i(\text{through plane L2}) = \frac{I_0}{2}(\cos\theta_0 - \cos\theta) - \frac{I_0}{2}(1+\cos\theta_0) = -\frac{I_0}{2}(1+\cos\theta),\\
&i(\text{through plane L3}) = -\frac{I_0}{2}(1+\cos\theta).
\end{aligned}\right\} \quad (5)$$

The value of $H_\phi(r, \theta)$ is obtained by dividing the above value by $2\pi r\sin\theta$, according to the Maxwellian theorem, i.e.

$$H_\phi(r, \theta) = -\frac{I_0(1+\cos\theta)}{4\pi r\sin\theta} = -\frac{I_0}{4\pi r}\cot\frac{\theta}{2} \quad (6)$$

at all places outside the ionosphere. The magnetic field produced by current system (A) of Fig. 5 is a sum of the contributions from (A1) and (A2), each of which is easily known referring to Fig. 6 and eq. (6).

4. Corrections Required for Non-Vertical Flow of Field-Aligned Currents and for Non-Uniformity of Ionospheric Conductivity

In the actual three-dimensional electric current in the ionosphere and magnetosphere, the field-aligned current from the magnetosphere is not vertical, and this current flows into the ionosphere of non-uniform conductivity. Referring to Fig. 7, we see that current system (B) consists of (B1) and (B2) systems. Hence, the correction for the non-vertical incidence of a field-aligned line current is to add the magnetic field produced by current system (B1). Current system (B2) consists of a vertical line current into the ionosphere and the real current in the ionosphere. Note here that current system (B2) produces no ground magnetic effect, if the ionospheric conductivity is uniform and the Hall current is disregarded.

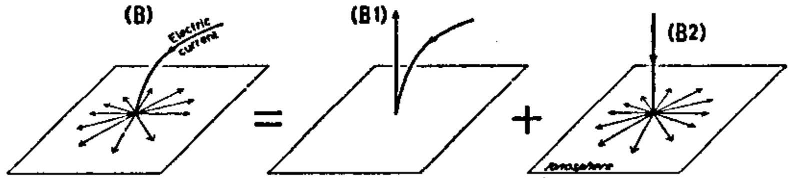

Fig. 7. Correction for the curvature of an electric current flowing into an ionospheric plane. If the ionospheric conductivity is uniform, the ground magnetic effect of (B) is the same as that of (B1), because (B2) produces no magnetic field below the ionosphere.

For a general case when the ionospheric conductivity is non-uniform, the pattern of real current in the ionosphere depends on the actual conductivity distribution as well as the condition for electric charge accumulation or dissipation at the boundary of conductivity discontinuity[3]. The equivalent current system for the ground magnetic effect of such a current system (consisting of a vertical line current and real currents in the ionosphere) has been studied for some special cases of conductivity distribution in the ionosphere[3-6]. The generalized theorem described in this paper is very useful in studying the relation between three-dimensional current in the magnetosphere-ionosphere and its two-dimensional equivalent current for ground magnetic effect.

3) Fukushima, N., Rept. Ionos. Space Res. Japan, **28**, 139, 147, 195, 207 (1974); **29**, 31, 39 (1975).
4) Leont'yev, S. V., W. B. Lyatskiy, and Yu. P. Mal'tsev, Geomagn. i Aeronomiya, **14**, 112 (1974).
5) Lyatskiy, W. B., Yu. P. Mal'tsev, and S. V. Leont'yev, Planet. Space Sci., **21**, 329 (1973).
6) Lyatskiy, W. B. and Yu. P. Mal'tsev, Geomagn. i Aeronomiya, **15**, 118 (1975).

GROUND MAGNETIC EFFECT OF FIELD-ALIGNED CURRENTS CONNECTED WITH IONOSPHERIC CURRENTS

— Fundamental Theorems and Their Applications —

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CONTENTS

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— Fundamental Theorems and Their Applications —

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ABSTRACT

For studying the magnetic effect of electric currents flowing in the ionosphere and magnetosphere, the following fundamental theorem can be applied. It states that the ground magnetic effect of vertical currents into or out of the ionosphere is perfectly cancelled out by that of the Pedersen currents in the ionosphere, if the electric conductivity is uniform throughout the ionosphere. This paper presents a proof of the fact that this theorem holds for both the plane ionosphere model and the spherical ionosphere model for any distribution of vertical currents.

The actual ground magnetic effect of the magnetospheric field-aligned currents and the associated ionospheric currents can then be obtained by estimating the following three effects, i.e.

(1) Curvature of the field-aligned currents in the magnetosphere,
(2) Nonuniformity of the ionospheric conductivity, and
(3) Hall currents in the ionosphere.

Effect (1) can be easily calculated through Biot-Savart's law. For the discussion of effects (2) and (3), it is necessary to know whether the electric currents do or do not tend to flow along the geomagnetic field-lines at the boundaries of conductivity discontinuity in the ionosphere, in order to preserve the electric current continuity. Two extreme cases are dealt with, i.e. when electric charges can escape freely out of the ionosphere (open field-line case), and when field-aligned currents do not flow at all, thus keeping a balance between the charge accumulation due to the primary Pedersen and Hall currents and the charge dissipation through the secondary Pedersen and Hall currents (closed field-line case). Some simple examples of cases of nonuniform ionospheric conductivity are introduced, which have been described in a recent series of works concerning the relationship between three-dimensional electric currents in the ionosphere-magnetosphere and their equivalent two-dimensional currents for ground magnetic variations.

1. Introduction

Ground geomagnetic variations are caused by electric currents flowing in the space in and above the ionosphere, along with currents in the earth's interior; the latter being induced by the magnetic field variation caused by currents flowing horizontally in the ionospheric E-layer and those flowing in the magnetosphere along and across geomagnetic field-lines. The magnetic effect produced by electric currents of any configuration can be calculated in principle by Biot-Savart's law. However, the ground magnetic effect of the combination of field-aligned currents in the magnetosphere and horizontal ionospheric currents can sometimes be easily known with the aid of the theorems described in this paper.

In the study of world geomagnetic variations, the ordinary procedure is to draw an *equivalent overhead current system* from the analysis of observed geomagnetic variations over the world, under the fundamental assumption that the ground geomagnetic effect is produced entirely by electric currents flowing immediately above the earth. Although the actual electric currents responsible for ground magnetic variations must be three-dimensional throughout the ionosphere and magnetosphere, it is mathematically possible to draw a two-dimensional *equivalent current system* on a spherical surface (say, at the level of the ionosphere) covering the earth, which produces the same magnetic effect on the earth's surface as that of actual three-dimensional currents. It is impossible in principle to obtain the actual electric current distribution in the ionosphere-magnetosphere, so long as we deal only with geomagnetic field variations on the earth's surface.

We deal here with a stationary state of electric currents in the ionosphere and magnetosphere. Hence the contribution from the induced currents in the earth's interior need not be considered. The ionospheric currents consist of Pedersen and Hall currents; the latter currents are divergence-free and closed in the ionosphere. The field-aligned currents in the magnetosphere are connected with the Pedersen currents in the ionosphere to preserve the continuity of the electric currents. This paper deals with the magnetic effect of a three-dimensional current system, which consists of *field-aligned currents in the magnetosphere and the Pedersen currents in the ionosphere*. The objective of this paper is to obtain an *equivalent overhead current* for the magnetic effect of such a three-dimensional current system in the ionosphere-magnetosphere.

The electric conductivity of the ionosphere is not at all uniform all over the earth. However, useful theorems are introduced for a simple case of uniform conductivity in this paper, and later it is shown how to take the effect of conductivity nonuniformity into consideration.

2. Plane Ionosphere Model of Uniform Electric Conductivity

2.1. Incidence of a vertical current to a horizontal conducting plane

Fig. 1 shows an electric current system in which a vertical line current is converted into diverging Pedersen currents on a plane ionosphere. We take here a cylindrical coordinate (r, ϕ, z); the vertical downward current flows along the z-axis at $r = 0$, and the diverging electric currents flow on the plane $z = 0$. The ionospheric current density $\boldsymbol{i}(i_r, i_\phi)$ at the point $(r, \phi, 0)$ is given by

$$i_r(r, \phi, 0) = I_0/(2\pi r), \qquad i_\phi(r, \phi, 0) = 0. \tag{1}$$

The magnetic field $H(H_r, H_\phi, H_z)$ produced by the electric current system of Fig. 1 in the region $z > 0$ (i.e. above the ionospheric plane) is given by

$$H_r(z>0) = 0, \quad H_\phi(z>0) = I_0/(2\pi r), \quad H_z(z>0) = 0, \tag{2}$$

whereas the magnetic field in the region $z < 0$ (i.e. below the ionospheric plane) vanishes. Although this result can be obtained from calculations using Biot-Savart's law, the same result can be proved also in the following ways.

(a) The electric current system of Fig. 1 can be thought to be a summation of elementary electric circuits, each of which consists of a vertical downward current and a radial outward current of intensity $I_0 d\phi/(2\pi)$ flowing in a sector of longitude ϕ and $\phi + d\phi$. The magnetic potential at $(r_0, \phi_0, -z_0)$ for the elementary circuit in longitude ϕ_0 or $\phi_0 \pm \pi$ is 0, because the solid angle subtended at the point $(r_0, \phi_0, -z_0)$ by this elementary current circuit is 0. Since the electric current system of Fig. 1 is symmetric around the z-axis, the magnetic potential at point $(r_0, \phi_0, -z_0)$ produced by the elementary circuit in the meridional plane of $\phi = \phi_0 + d\phi$ is always cancelled by that of $\phi = \phi_0 - d\phi$. Hence, from the axial symmetry of the electric current distribution of Fig. 1, the magnetic potential is 0 everywhere below the $z = 0$ plane. Therefore the current system of Fig. 1 produces no magnetic field below the plane $z = 0$.

(b) Another simple explanation for the above result is as follows. Since the electric current of Fig. 1 has no azimuthal component, the associated magnetic field must be in the azimuthal direction, i.e. the magnetic field has only the ϕ-component, at both above and below the plane of $z = 0$. The magnetic field at the point (r_0, ϕ_0, z_0) is denoted here by $H(0, H_\phi, 0)$, where H_ϕ is dependent on r_0 and z_0. Applying the Maxwellian theorem on a circle of $r = r_0$, we see

$$2\pi r_0 H_\phi(r_0, z_0) = \boldsymbol{i}(z_0), \tag{3}$$

where $\boldsymbol{i}(z_0)$ is the total intensity of electric current passing upward through the plane of $z = z_0$, namely $\boldsymbol{i}(z>0) = -I_0$ and $\boldsymbol{i}(z<0) = 0$. Therefore, in the region $z > 0$, $H_\phi = -I_0/(2\pi r_0)$, whereas no magnetic field is produced at $z < 0$.

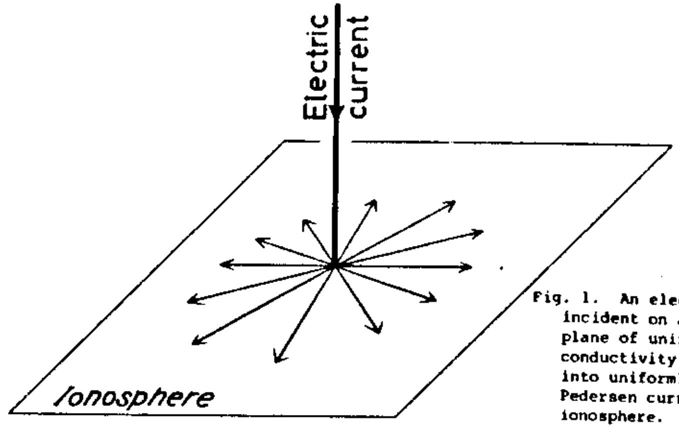


Fig. 1. An electric current is incident on an ionospheric plane of uniform electric conductivity and is converted into uniformly diverging Pedersen currents in the ionosphere.

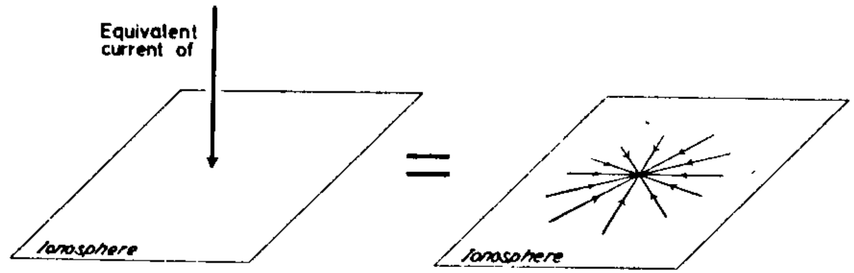


Fig. 2. The equivalent current for a vertical current flowing into the ionosphere is a current uniformly converging towards the point of intersection.

2.2. Equivalent horizontal current for a vertical line current

The above result can be applied to geophysical problems in the following way. If the ionosphere can be approximated by an infinite plane with a uniform electric conductivity, the magnetic effect does not appear below the ionosphere, when a line current flows vertically into the ionosphere, insofar as we are concerned only with the Pedersen current in the ionosphere. This can be restated in such a way that the ground magnetic effect of a vertical line current is completely cancelled out by the uniformly diverging current in the ionosphere from the foot of the vertical incoming current. In other words, *the equivalent current for a vertical downward current flowing into the ionosphere is a uniformly converging current in the ionosphere,* as illustrated by Fig. 2. This is a fundamental theorem in the discussion of the relationship between a three-dimensional current and its equivalent two-dimensional current. An intuitive but non-exact proof for this theorem was earlier shown by the present author (Fukushima, 1969, 1971).

2.3. Equivalence in the ground geomagnetic effect of Chapman-Vestine's and Birkeland-Alfvén's current systems for polar magnetic storms

By means of the theorem shown in Fig. 2, it is possible to give an intuitive explanation for the reason why the same ground magnetic effect is produced by the two different current systems for magnetic storms (Fukushima, 1969, 1971), i.e. the current system in the ionosphere connected with an auroral electrojet (depicted by Chapman, Vestine and others), and the electrojet connected with field-aligned currents from and to the magnetosphere (first proposed by Birkeland and later advocated by Alfvén and his coworkers). The field-aligned currents of the Birkeland-Alfvén current system are approximated for simplicity by vertical currents, and the ionosphere is approximated by a conducting plane. As will be seen in Fig. 3, the Birkeland-Alfvén current system (A) is thought to be a superimposition of currents (B) and (C), where (C) is the Chapman-Vestine current system. Pattern (B) can be further decomposed to (B1) and (B2), both of which produce no magnetic effect below the ionosphere. Thus, both the Chapman-Vestine current system and Birkeland-Alfvén current system produce the same magnetic field on the earth.

Above the ionosphere, however, these two current systems give rise to a different magnetic effect, since current system (B) produces a magnetic field above the ionosphere. It is thus concluded that we cannot judge which one, (A) or (C), is more realistic during polar substorms, insofar as we deal only with the magnetic field data observed on the ground. Observations in and above the ionosphere are required to determine the real current distribution in the

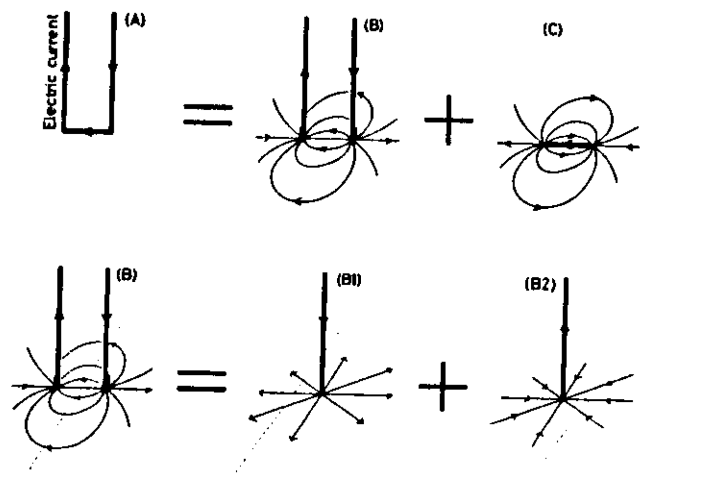


Fig. 3. The Birkeland-Alfvén current system (A), for a polar substorm, is the sum of current systems (B) and (C), where (C) is the Chapman-Vestine current system. Current system (B) = (B1) + (B2) produces no magnetic effect below the ionosphere.

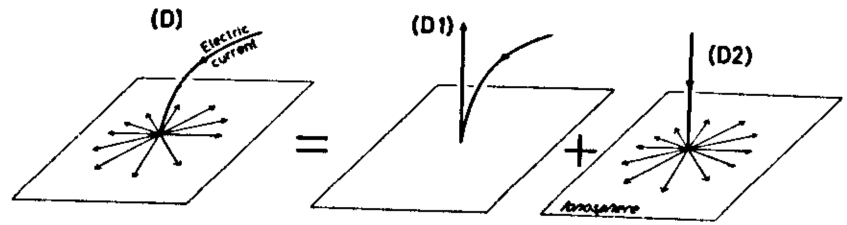


Fig. 4. Correction for the curvature of an electric current flowing into an ionospheric plane. The ground magnetic effect of (D) is the same as that of (D1), because (D2) produces no magnetic field below the ionosphere.

ionosphere and magnetosphere, which results in the ground magnetic variations. Field-aligned currents in the magnetosphere can be detected through the magnetic field observations with three-component magnetometers, such as those on board the TRIAD satellite. However there is a possibility of inferring the field-aligned current distribution from a comparison of low-altitude satellite and ground observations of the geomagnetic field, even if the satellite measures only the scalar magnitude of magnetic field above the ionosphere (Fukushima, 1975c,d; Kawasaki and Fukushima, 1975).

2.4. Correction for non-vertical incidence of field-aligned currents

Although field-aligned currents flow into or out of the ionosphere nearly vertically in high latitudes, it is still necessary to correct for the effect of their oblique incidence. This correction is easily made according to Fig. 4. If the ionospheric conductivity were uniform, (D2) of Fig. 4 would produce no magnetic effect on the ground. Hence, the correction for the non-vertical incidence of field-aligned currents requires the addition of the magnetic effect of current system (D1) of Fig. 4, which can be easily calculated by means of Biot-Savart's law.

If the field-aligned currents flow into the ionosphere in a sheet of infinite extent along the x-axis, the magnetic field of the (D1)-type sheet current does not leak down to the ground, as is readily seen in Fig. 5.

3. Spherical Ionosphere Model of Uniform Electric Conductivity

3.1. Incidence and outflow of vertical currents at the antipodal points on the spherical ionosphere

We discuss first the simple case illustrated by Fig. 6. The spherical coordinates (r, θ, ϕ) are taken with their origin at the earth's center, and a vertical line current flows into and out of the ionosphere (of radius r_0) along the $\theta = 0$ and $\theta = \pi$ axes, respectively. The θ-component of the ionospheric current is independent of ϕ-value, because of the assumption of uniform electric conductivity throughout the ionosphere. By means of the method written in 2.1.(b), the magnetic effect vanishes everywhere inside the ionosphere. Outside the ionosphere, the magnetic field $H(0, 0, H_\phi)$ is given by

$$H_\phi(r, \theta) = -I_0/(2\pi r \sin\theta) \qquad \text{for} \quad r > r_0. \qquad (4)$$

From this conclusion, it can be said that the case of Fig. 7 also produces no magnetic effect inside the ionosphere, because the current system of Fig. 7 is a superimposition of two systems, each of which is a system shown by Fig. 6.

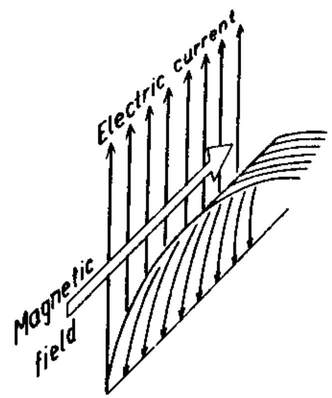


Fig. 5. The magnetic field of a field-aligned sheet current of uniform current density and infinite extent, which does not leak down to the ground.

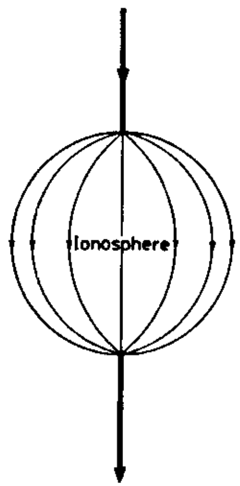


Fig. 6. An electric current flowing vertically in and out of the antipodal points of the spherical ionosphere.

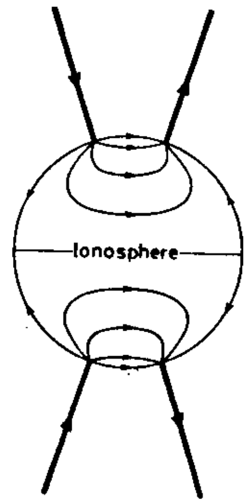


Fig. 7. Pairs of vertical electric currents flowing into and out of the ionosphere in high latitudes.

The current system of Fig. 7 reminds us of the field-aligned currents flowing from the magnetosphere into the dawn regions of high latitudes and out of the ionosphere from the dusk regions. If the incoming and outgoing currents were vertical, and the ionospheric conductivity were uniform over the entire ionosphere, no magnetic field would be produced on the earth. In reality, the ground magnetic effect comes from corrections for the curvature of field-aligned currents and the nonuniformity of the ionospheric conductivity.

3.2. Case of axial asymmetry of vertical currents

For a case in which the inflow and outflow of vertical electric currents are not at the antipodal points on the ionosphere of uniform electric conductivity, the following calculations can be made. As seen in Fig. 8, current system (E) is thought to be the sum of (E1) and (E2). In current system (E1), electric currents flow out vertically with uniform density from all over the ionosphere to compensate for the inflowing vertical current at the point P_1, whereas in current system (E2) vertical currents flow uniformly into the ionosphere and a vertical outward line current is at P_2. The vertical current density on the surface of the ionosphere in systems (E1) and (E2) is $I_0/(4\pi r_0^2)$, where r_0 is the radius of the ionosphere, and I_0 is the intensity of the vertical line current at P_1 and P_2. In current system (E1), if we take the spherical coordinates (r, θ, ϕ) with their origin at the earth's center and $\theta = 0$ axis along the vertical current as shown in Fig. 9, the overall current distribution is axisymmetric around the $\theta = 0$ and π axis. Hence, similar to the calculation mentioned in 3.1, the magnetic field inside the ionosphere associated with current system (E1) vanishes everywhere. The magnetic field outside the ionosphere has only the ϕ-component, and the value $H_\phi(r, \theta)$ is obtained as follows.

We divide the whole space outside the ionosphere into three regions, as shown in Fig. 9; i.e. Region I ($r\cos\theta > r_0$), Region II ($r_0 > r\cos\theta > -r_0$), and Region III ($r\cos\theta < -r_0$). In each region we consider circular planes L1, L2, and L3, whose centers are on the $\theta = 0$ and π axis and whose radius is $r\sin\theta$. Planes L1 and L3 do not intersect the ionosphere, and plane L2 intersects the ionosphere at colatitude θ_0. The amount of electric currents i, which pass through these planes (including the vertical line current and the associated ionospheric Pedersen currents) are

$$\left.\begin{array}{ll} i = \frac{I_0}{2}(1-\cos\theta) - I_0 = -\frac{I_0}{2}(1+\cos\theta) & \text{through the plane L1,} \\ i = \frac{I_0}{2}(\cos\theta_0 - \cos\theta) - \frac{I_0}{2}(1+\cos\theta_0) = -\frac{I_0}{2}(1+\cos\theta) & \text{through the plane L2, and} \\ i = -\frac{I_0}{2}(1+\cos\theta) & \text{through the plane L3.} \end{array}\right\} \quad (5)$$

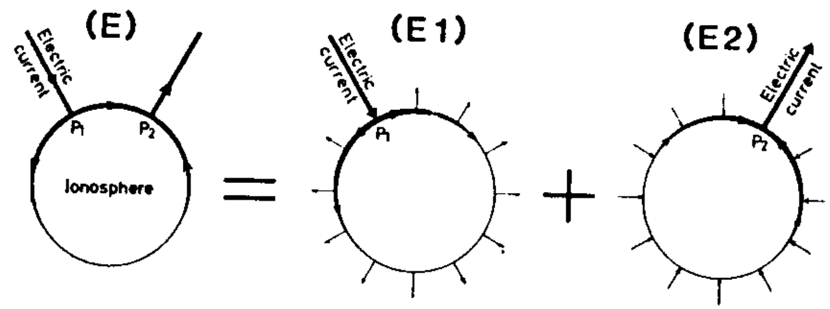


Fig. 8. Principle used to decompose any distribution of incoming and outgoing vertical currents into axisymmetric patterns.

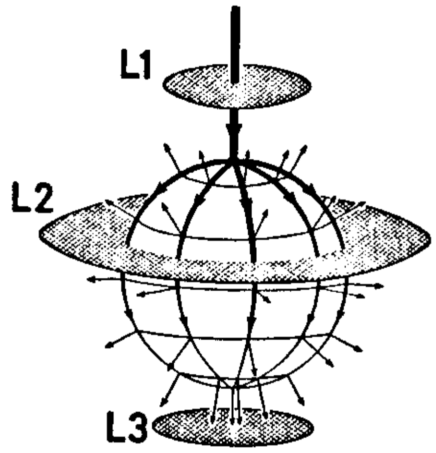


Fig. 9. A vertical line current flowing into the ionosphere with a simultaneous outflow of uniform-density vertical currents from all over the ionosphere. Planes L1, L2 and L3 are drawn for calculating the magnetic effect above the ionosphere.

The value of $H_\phi(r,\theta)$ is obtained by dividing these values by $2\pi r \sin\theta$, according to the Maxwellian theorem, i.e.

$$H_\phi(r,\theta) = -\frac{I_0(1+\cos\theta)}{4\pi r \sin\theta} = -\frac{I_0}{4\pi r}\cot\frac{\theta}{2} \qquad (6)$$

throughout Regions I, II, and III. The magnetic field outside the ionosphere is everywhere westward in the case of Fig. 9.

The above conclusions can be extended also to the cases in which a number of vertical line currents are flowing into and out of the ionosphere, where the total amount of inflowing currents is equal to that of outflowing currents to preserve the divergence-free condition of the ionospheric currents. We obtain the following important general theorem that *the magnetic effect vanishes everywhere below the ionosphere if the ionospheric conductivity is uniform for any distribution of vertical incoming and outgoing currents into and out of the ionosphere*, insofar as we deal with the combination of vertical currents and the associated Pedersen currents in the ionosphere.

3.3. Geophysical applications to field-aligned currents in high latitudes and in middle latitudes

(a) High-latitude field-aligned currents

Recent observations by satellites show the permanent existence of field-aligned currents into and out of the ionosphere in high latitudes. The theorem described in 3.2 shows that the ground magnetic effect would not appear if the ionospheric conductivity were uniform and the currents flowing into or out of the ionosphere were vertical. In order to obtain the actual magnetic effect of the field-aligned currents combined with the ionospheric Pedersen currents, we need the corrections for non-vertical incidence of field-aligned currents and nonuniformity of ionospheric conductivity. The effect of ionospheric conductivity nonuniformity is dealt with later in Section 4, while the oblique incidence effect is discussed below.

Referring to Fig. 10, the effect of non-vertical incidence of field-aligned currents is corrected by adding the magnetic effect of current system (F1). Although the magnetic effect of currents flowing at a great distance may be neglected in practical calculations of ground magnetic effect, it is also necessary for an accurate calculation to know how the field-aligned currents form a closed electric circuit in the distant magnetosphere, i.e. whether it is connected with a partial ring current in the equatorial plane or with some other currents (Kamide and Fukushima, 1972; Fukushima, 1972; Fukushima and Kamide, 1973b). It is worth noting that the principal contribution to the observed

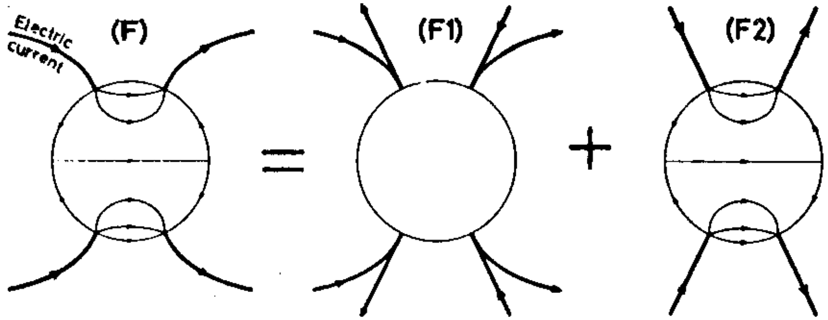


Fig. 10. Correction for the curvature effect of field-aligned currents into and from the high-latitude ionosphere. The ground magnetic effect of (F) is the same as that of (F1), because (F2) produces no magnetic field inside the ionosphere.

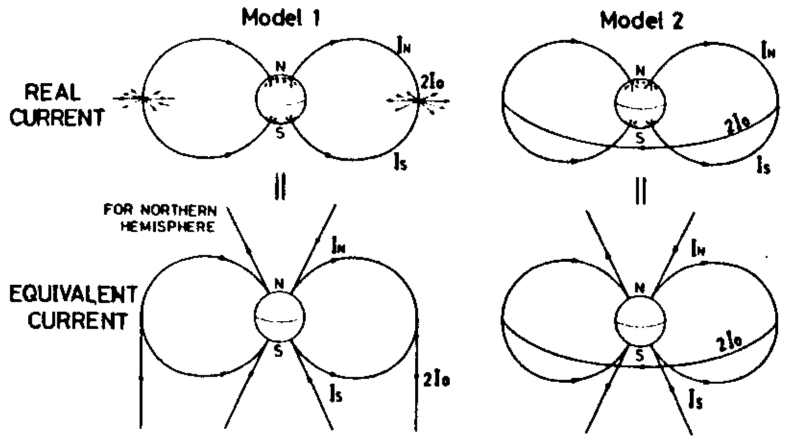


Fig. 11. Two models of field-aligned currents in the magnetosphere, with and without a westward flowing partial ring current in the equatorial plane (after Fukushima and Kamide, 1973b).

asymmetry of the depression of the horizontal component of the geomagnetic field in low latitudes during disturbed periods does not come from the longitudinal inequality of equatorial ring-current intensity but from the curvature of field-aligned currents in the magnetosphere (Fukushima, 1973; Fukushima and Kamide, 1973a,b,c). If the field-aligned currents from or to the distant equatorial plane were connected with a uniformly converging or diverging current on the equatorial plane, such as the current associated with the plasma convection in the magnetosphere, the ground magnetic effect of the converging or diverging currents in the equatorial plane would be given by that of a semi-infinite line current perpendicular to the equatorial plane, as illustrated by Fig. 11. Here the theorem of Fig. 2 is again used to replace the equatorial currents by a line current perpendicular to the equatorial plane.

(b) Middle latitude field-aligned currents

It has been widely advocated by a number of research workers that a field-aligned current must be flowing in the magnetosphere between the northern and southern hemispheres to connect the Sq current vortices in the daytime middle latitudes; the direction of this field-aligned current is from the winter hemisphere to the summer hemisphere, as will be seen in Fig. 12. If the electric conductivity of the ionosphere is assumed to be uniform for simplicity, the ground magnetic effect of current system (G) is equal to that of (G1); the latter is easily calculated by Biot-Savart's law. We see here that current system (G) or (G1) produces a westward magnetic field all over the dayside hemisphere of the earth, the equivalent current of which is a northward current. It is worth noting that the westward/eastward shift of the Sq current vortices in the summer/winter hemisphere might be attributable to the superimposition of the north-south current on the symmetric Sq current vortices of ionospheric origin. In other words, it may be unnecessary to think that the ionospheric Sq current vortices in the northern and southern hemispheres show a seasonal displacement in their locations.

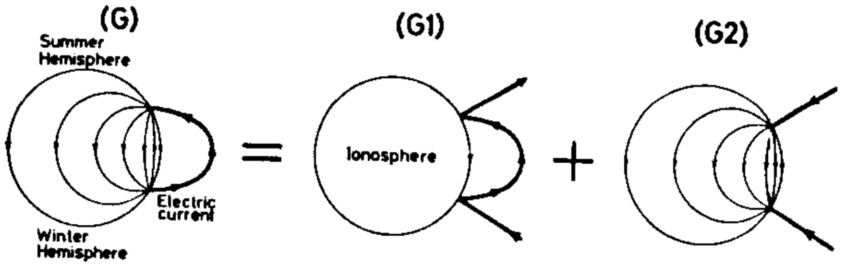


Fig. 12. Middle-latitude field-aligned current in the solstitial seasons. The ground magnetic effect of (G) is the same as that of (G1) if the ionospheric conductivity is uniform.

4. Correction for Nonuniformity of Ionospheric Conductivity

Throughout the preceding discussions, the electric conductivity of the ionosphere was assumed to be uniform all over the earth. In such a case, field-aligned currents in the magnetosphere are connected only with the Pedersen currents in the ionosphere, and the ground magnetic effects of these two currents completely cancel each other out if the field-aligned currents are vertical. The Hall currents in the ionosphere are divergence-free, and they contribute wholly to the magnetic field variations on the earth's surface. Hence, if we may neglect the contributions from electric currents in the distant magnetosphere, the ground magnetic effect is thought to consist of (i) the Hall currents in the ionosphere, (ii) the effect of non-vertical incidence of field-aligned currents, and (iii) the nonuniformity effect of the ionospheric conductivity.

As to the first point, since the distribution of Hall currents in the ionosphere can be easily known if the ionospheric conductivity is uniform, it is possible to estimate the ground magnetic effect due to the Hall currents. As for the second point, the effect of non-vertical incidence of field-aligned currents has already been discussed in 2.4 and 3.3.

In the case of nonuniform conductivity distribution in the ionosphere, the Pedersen and Hall currents in the ionosphere are not easily separable. It is also necessary to consider how electric currents preserve their continuity at the boundaries of conductivity discontinuity in the ionosphere, i.e. whether electric currents do or do not tend to flow out of the ionosphere along geomagnetic field lines. In the former case, there is no excess electric charge at the boundaries of conductivity discontinuity (in other words, the electric charges escape as field-aligned currents, thus not producing any excess charge), whereas in the latter case excess charges are accumulated in order to keep the electric current continuous in the ionosphere across the boundaries of conductivity discontinuity.

4.1. Two extreme cases of geomagnetic field-line conditions, closed and open

Referring to an actual model of the earth's environmental space, if the distributions of the ionospheric conductivity and of the primary field-aligned currents are symmetric with respect to the geomagnetic equator, the secondary field-aligned currents from the conjugate points of the northern and southern hemispheres cancel each other out, along the closed geomagnetic field lines. If the field lines are open to the magnetotail, on the other hand, as those from the polar-cap regions inside the auroral oval, the electric current may flow without being cancelled by the current from the opposite hemisphere. Therefore we will consider the following two extreme cases for the stationary state, i.e.

(i) the *closed* field-line case .... the electric current does not flow at all along geomagnetic field lines. (From the standpoint of an electric circuit, the circuit between the conjugate points is open in the the magnetosphere.)

(ii) the *open* field-line case .... the electric current flows freely along the geomagnetic field lines. (The electric circuit from a point on the ionosphere is connected with an infinite capacitance in the magnetosphere.)

These are the extreme cases, but in an actual flow of field-aligned currents, the situation will be between these two cases, because charged particles as carriers of electric currents drift across the geomagnetic field lines in the magnetosphere. The electric conductivity along the geomagnetic field lines must be studied in greater detail in the future.

By means of the simple assumption that the field-aligned currents are allowed to flow freely (open field-line case) or are completely suppressed (closed field-line case), the equivalent current for the ground magnetic variations has been obtained for some special distributions of ionospheric conductivity when a vertical line current or sheet current is incident on the ionosphere, or when a horizontal uniform external electric field is applied (Fukushima, 1974a,b,c d,e; 1975a,b).

Before introducing some examples, it is worthwhile mentioning first a very special case in which the ionospheric conductivity is independent of longitude but dependent on the distance from the foot of the incident current on the ionosphere, and the field-aligned currents from the conductivity discontinuity boundaries are assumed to be completely suppressed. In these cases, the excess electric charges appear in the ionosphere at the circular boundaries of conductivity discontinuity, thus keeping the radial Pedersen current in the ionosphere continuous across the boundaries of conductivity discontinuity. Since the current distribution is axisymmetric in the ionosphere, the resultant Pedersen current distribution is exactly the same as that in the case of uniform electric conductivity shown in Fig. 1 or Fig. 6. Hence, this special case produces no magnetic effect on the ground.

4.2. Equivalent current pattern when the ionospheric conductivity is different on both sides of the foot of a field-aligned line current

We deal here with a plane ionosphere model. The height-integrated Pedersen conductivity of the ionosphere is assumed to be σ_1 in the region $0<\phi<\pi$, and $k_1\sigma_1$ in the region $\pi<\phi<2\pi$, with $k_1>1$, as shown in current system (H) of Fig. 13.

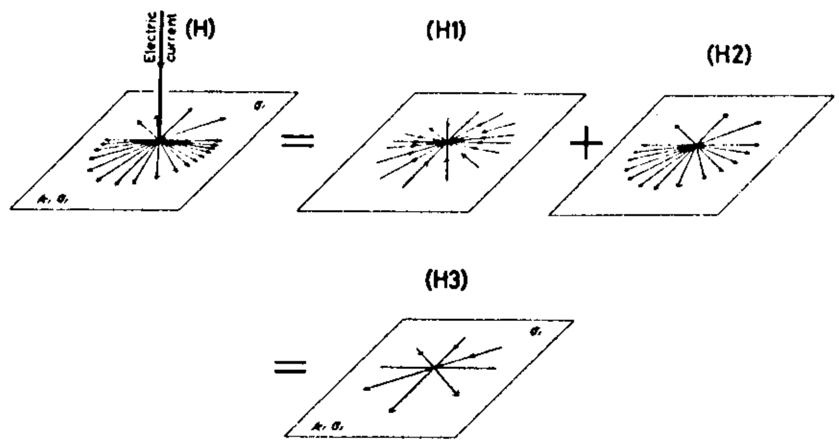


Fig. 13. The current pattern for two regions of ionospheric plane with different electric conductivities. The equivalent current system for (H) is given by (H3).

If we denote the intensity of vertical incident current by I_o, $I_o/(k_1+1)$ is converted to the Pedersen currents in the region $0<\phi<\pi$, and $I_o k_1/(k_1+1)$ in the region $\pi<\phi<2\pi$. The Pedersen currents do not cross the boundary of ionospheric conductivity discontinuity. With the aid of the theorem shown in Fig. 2, the equivalent current of system (H) is a current converging towards the foot of the field-aligned current in the region of lower conductivity, whereas it is a diverging current in the region of greater conductivity, as given by (H3) of Fig. 13. The current intensity per radian in the ionosphere in the (H3) current is $(k_1-1)I_o/\{2\pi(k_1+1)\}$.

When the primary field-aligned current is a sheet current of uniform density and infinite extent along the $\phi=0$ and π lines, the equivalent current of field-aligned sheet current and associated Pedersen currents is a uniform parallel current perpendicular to the conductivity-discontinuity boundary flowing from the region of lower conductivity to that of higher conductivity (Fukushima, 1974b).

If we take the Hall currents also into account, the pattern of equivalent current depends on the condition whether the magnetic field-lines are open or closed at the boundary of conductivity discontinuity. For simplicity, the magnetic field lines are assumed to be vertical. If the magnetic field lines are open, the divergence-free condition of the circular Hall currents requires that the vertical currents flow at the boundary of conductivity discontinuity. These vertical currents can be replaced by horizontal equivalent currents, according to the theorem given by Fig. 2. The total summation of these horizontal equivalent currents is a circular current, whose direction is opposite in the $0<\phi<\pi$ and $\pi<\phi<2\pi$ regions. The sum of the primary Hall currents and these equivalent currents is a divergence-free circular current in the ionosphere. Hence, if the height-integrated Hall conductivities in the $0<\phi<\pi$ and $\pi<\phi<2\pi$ regions are denoted respectively by σ_2 and $k_2\sigma_2$, an *apparent Hall conductivity* throughout the plane ionosphere can be defined, and in this case of open field-lines it is expressed as $\sigma_2(1+k_2)/2$, i.e. the arithmetical average of the Hall conductivity values in the two regions (Fukushima, 1974a).

On the other hand, if the magnetic field lines are closed on the $\phi=0$ and π boundaries of ionospheric conductivity discontinuity, excess electric charges are produced at the boundaries in the stationary state, so as to balance the charge accumulation by the primary Hall currents and the dissipation through the secondary Pedersen and Hall currents in the ionosphere. In this case, it can be shown that the effect of the secondary Pedersen currents is to make the sum of the primary Hall and secondary Pedersen currents a circular current; the apparent Hall conductivity over the entire ionosphere is given by $\sigma_2(k_2+k_1)/(1+k_1)$, and this value is smaller than in the case of open field lines when $k_2>1$. On the other hand, the effect of the secondary Hall currents is to *intensify the equivalent current* for the primary field-aligned current and Pedersen currents by the factor $[1+\{(k_2-1)\sigma_2\}^2/\{(k_1+1)\sigma_1\}^2]$, with some similarity to the apparent enhancement of electric conductivity along the geomagnetic equator (Fukushima, 1974a).

4.3. Equivalent current pattern when a vertical line current is incident at the center of a belt of enhanced ionospheric conductivity

This case refers to the incidence of a field-aligned current into the auroral zone, the conductivity of which is much greater than in the surrounding region. For simplicity, the electric conductivity is assumed to be uniform in each region outside and inside the auroral zone, and the magnetic field is taken to be vertical. We will consider a plane ionosphere model also in this case. The pattern of equivalent currents depends on the condition of magnetic field lines, whether they are open or closed. Figs. 14, 15 and 16 show respectively the examples of equivalent current patterns, for the cases where (1) both the poleward and

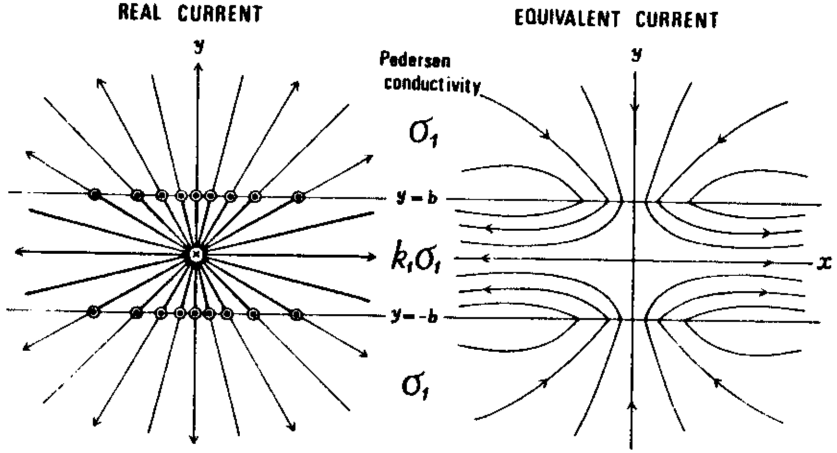


Fig. 14-1. Comparison of the real Pedersen currents (with associated field-aligned currents in the open field-line case) and their equivalent current pattern, for $k_1=2$. The amount of electric current between adjacent stream-lines in the equivalent current pattern is 1/4 of the real Pedersen currents of the region $|y|>b$.

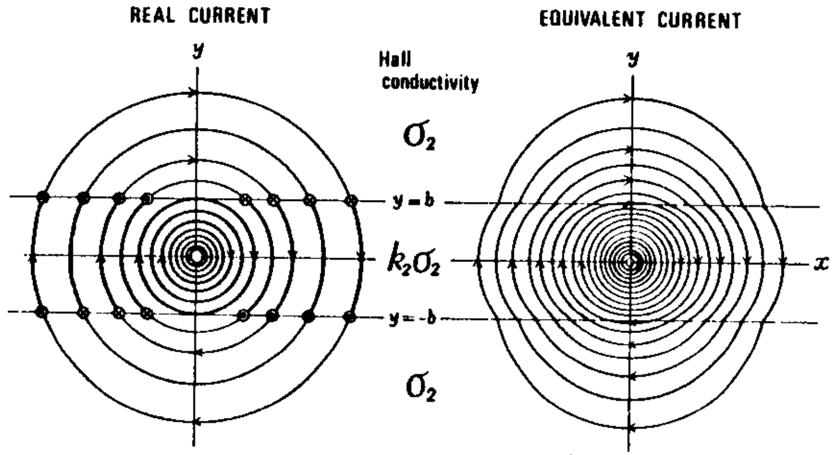


Fig. 14-2. Comparison of the real Hall currents (with associated field-aligned currents in the open field-line case) and their equivalent current pattern, for $k_2=3$. The amount of electric current between adjacent streamlines in the equivalent current pattern is the same as that of the real Hall currents of the region $|y|>b$. The current amount between adjacent streamlines in Fig. 14-2 is σ_2/σ_1 times that of Fig. 14-1 in the region $|y|>b$.

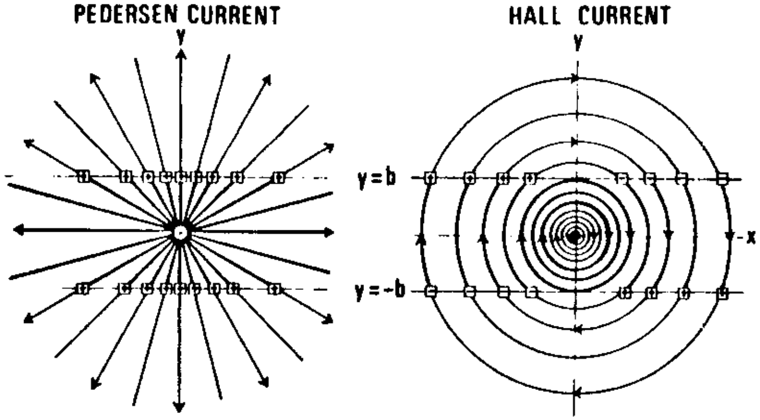


Fig. 15-1. Horizontal Pedersen and Hall currents in the ionosphere and associated electric charge accumulation (shown by squares) on $y = b$ and $y = -b$ boundaries of electric conductivity discontinuity in the case of closed field lines, when a downward line current flows into the ionosphere at (0,0).

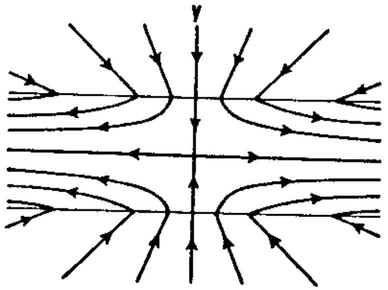


Fig. 15-2. Equivalent current pattern for the vertical line current at (0,0) and associated Pedersen currents, when the Pedersen conductivity is ∞ inside the region $|y| < b$. The amount of electric current between adjacent stream lines is $I_0/16$, where I_0 is the intensity of vertical incoming current at (0,0).

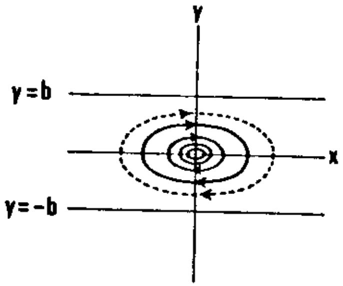


Fig. 15-3. Equivalent current pattern which appears in addition to Fig. 15-2, when the Hall conductivity inside the region $|y| < b$ is ∞. The amount of electric current between adjacent solid streamlines is $(I_0/8\pi)k_2\sigma_2/(k_1\sigma_1)$.

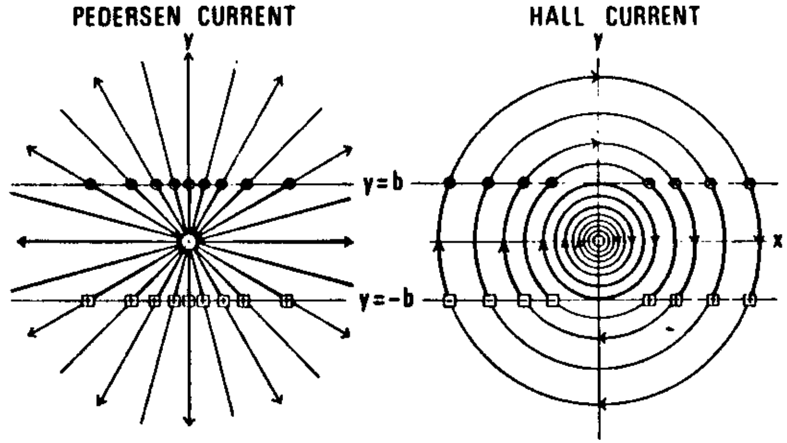


Fig. 16-1. Horizontal Pedersen and Hall currents in the ionosphere and associated field-aligned currents (at $y = b$ boundary) and electric charge accumulation (at $y = -b$ boundary), when a vertical line current flows into the ionosphere at (0,0).

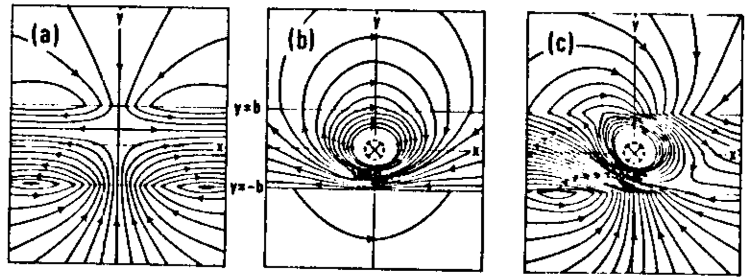


Fig. 16-2

(a) Equivalent current pattern for a line current of intensity I_0 at (0,0) and the associated field-aligned and ionospheric currents, when the Pedersed conductivity is ∞ inside the region $|y| < b$. The amount of electric current between adjacent streamlines is $I_0/16$.

(b) Equivalent current pattern which appears in addition to (a), when the Hall conductivity of the ionosphere inside the region $|y| < b$ is ∞. The amount of electric current between adjacent streamlines is $(I_0/8\pi)k_2\sigma_2/(k_1\sigma_1)$.

(c) Total equivalent overhead current system produced by a vertical incoming line current at (0,0), when both Pedersen and Hall conductivities are infinitely great and their ratio is unity. This current pattern is the superimposition of (a) and (b).

equatorward boundaries are open lines, (ii) both boundaries are closed, and (iii) the poleward boundary is open and the equatorward boundary is closed (Fukushima, 1974e, 1975a, 1975b). In each case, the pattern of electric currents can be shown to consist of two kinds of current patterns; one is a group of currents which converge to and diverge from the foot of the primary field-aligned current and its mirror point(s) with respect to the boundary or boundaries of the conductivity discontinuity, and the other is a group of currents which are concentric around the foot of the primary field-aligned current and its mirror point(s).

5. Concluding Remarks

This paper summarizes the results described in a recent series of papers concerning the relationship between three-dimensional electric currents in the ionosphere-magnetosphere and their equivalent currents; the latter gives the same magnetic effect on the earth as that of the former, but the effect of these two current systems is different above the ionosphere. We cannot infer the three-dimensional current distribution in the ionosphere-magnetosphere unless we compare the observational results from above the ionosphere. The series of works summarized in this paper will contribute to some extent to the work of inferring the three-dimensional current system from ground magnetic observations.

The main conclusions described in this paper may be summarized as follows. If the ionospheric conductivity is uniform all over the ionosphere, the ground magnetic effect of vertical currents flowing into or out of the ionosphere is perfectly cancelled out by that of the associated Pedersen currents in the ionosphere. Therefore, the magnetic effect of field-aligned currents and associated Pedersen currents originates from the effect of non-vertical incidence of field-aligned currents and that of nonuniformity of the ionospheric conductivity. The magnetic effect of Hall currents, however, always appears on the ground.

When there are field-aligned currents into and out of the ionosphere, the ground magnetic variations at a place of observation are not entirely due to the overhead currents flowing in the ionosphere above the station. Recent incoherent radar observations in high latitudes have detected a noticeable difference between the observed horizontal current in the ionosphere and the equivalent overhead current responsible for the simultaneous ground magnetic variations. It is necessary to know such a difference between real and equivalent currents also in middle and low latitudes. A detailed study of the dynamic behavior of ionospheric changes in middle and low latitudes accompanying geomagnetic bays will also contribute to finding the difference of real and equivalent currents, because the height-distribution of ion and electron densities in the ionosphere must be changed by the real electric current flowing in the ionosphere.

NOTE The present paper includes the summary of the results described in the following papers.

[A] Preliminary proof for the fundamental theorem stating the cancellation of ground magnetic effects by a verical line current and horizontal Pedersen currents is shown in

Fukushima, N., Equivalence in ground geomagnetic effect of Chapman-Vestine's and Birkeland-Alfvén's electric current systems for polar magnetic storms, Rept. Ionos. Space Res. Japan, 23, 219-227 (1969).

Fukushima, N., Electric current systems for polar substorms and their magnetic effect below and above the ionosphere, Radio Science, 6, 269-275 (1971).

Final proof for the fundamental theorem is written in

Fukushima, N., Generalized theorem for no ground magnetic effect of vertical currents connected with Pedersen currents in the uniform-conductivity ionosphere, Rept. Ionos. Space Res. Japan, 30, 35-40 (1976).

[B] The low-latitude DS(H) is discussed in connection with field-aligned currents in the following papers.

Fukushima, N., Polar magnetic substorms, Planet. Space Sci., 20, 1443-1454 (1972).

Fukushima, N., Apparent increase in low-latitude DS(H) unrelated to high-latitude geomagnetic activity, Rept. Ionos. Space Res. Japan, 27, 53-56 (1973).

Fukushima, N., and Y. Kamide, Contribution to low-latitude geomagnetic DS(H) from field-aligned currents in the magnetosphere, Rept. Ionos. Space Res. Japan, 27, 57-58 (1973a).

Fukushima, N., and Y. Kamide, Partial ring current models for worldwide geomagnetic disturbances, Rev. Geophys. Space Phys., 11, 795-853 (1973b).

Fukushima, N., and Y. Kamide, Contribution of magnetospheric field-aligned current to geomagnetic bays and Sq fields: A comment on partial ring-current models, Radio Science, 8, 1013-1017 (1973c).

Kamide, Y. and N. Fukushima, Positive geomagnetic bays in evening high-latitudes and their possible connection with partial ring current, Rept. Ionos. Space Res. Japan, 26, 79-101 (1972).

[C] Examples of the relationship between the vertical field-aligned currents with associated ionospheric currents and their equivalent current systems for the case of nonuniform electric conductivity of the ionosphere are shown in

Fukushima, N., Equivalent current pattern for ground geomagnetic effect when the ionospheric conductivity is discontinuous at the foot of a field-aligned current, Rept. Ionos. Space Res. Japan, 28, 139-146 (1974a).

Fukushima, N., Equivalent current pattern for ground geomagnetic effect when the ionospheric conductivity is discontinuous at the foot of field-aligned current sheet, Rept. Ionos. Space Res. Japan, 28, 147-151 (1974b).

Fukushima, N., Ground magnetic effect of two-dimensional vertical field-aligned current sheet pair, Rept. Ionos. Space Res. Japan, 28, 195-200 (1974c).

Fukushima, N., Deflection of the equivalent current across the ionospheric conductivity discontinuity under uniform electric field, Rept. Ionos. Space Res. Japan, 28, 201-206 (1974d).

Fukushima, N., Equivalent current pattern for a field-aligned current into the auroral belt of enhanced electric conductivity, Part I. Case of no charge accumulation at the auroral-zone boundaries, Rept. Ionos. Space Res. Japan, 28, 207-213 (1974e).

Fukushima, N., Equivalent current pattern for a field-aligned current into the auroral belt of enhanced electric conductivity, Part II. Case of charge accumulation at the auroral-zone boundaries, Rept. Ionos. Space Res. Japan, 29, 31-38 (1975a).

Fukushima, N., Equivalent current pattern for a field-aligned current into the auroral belt of enhanced electric conductivity, Part III. Case of open field lines on the poleward side and closed field lines on the equatorward side of the auroral belt, Rept. Ionos. Space Res. Japan, 29, 39-47 (1975b).

Fukushima, N., and K. Kawasaki, A simplified model of field-aligned current sheet pairs along the auroral oval in connection with geomagnetic S_q^p-field, Rept. Ionos. Space Res. Japan, 28, 83-88 (1974).

Kawasaki, K., and N. Fukushima, Equivalent current pattern of ionospheric and field-aligned currents generated by geomagnetic S_q^p-field with closed equatorward auroral boundary, Rept. Ionos. Space Res. Japan, 28, 187-194 (1974).

[D] Discussion on ΔB above the ionosphere is given in

Fukushima, N., Field-aligned current as a possible source for ΔB observed by low-altitude satellites, Rept. Ionos. Space Res. Japan, 29, 51-57 (1975d).

Kawasaki, K. and N. Fukushima, Derivation of field-aligned current distribution from scalar B observed by low-altitude satellites, Rept. Ionos. Space Res. Japan, 29, 58-64 (1975).

Remark on Geomagnetic Bays in Middle and Low Latitudes

This paper deals with the non-uniqueness of electric current distribution in the ionosphere-magnetosphere when we infer it from the geomagnetic variation observed on the ground alone. Fig. 17 shows three possible models of electric current flow in and above the ionosphere responsible for geomagnetic bays. In middle and low latitudes, at the time of positive geomagnetic bays ($\Delta H > 0$), the electric current in the ionosphere can be either eastward or westward or almost absent; the current direction may change even during the course of a bay. There would be two kinds of study to determine the ionospheric current in middle and low latitudes, i.e., (1) the comparison of magnetic field variations below and above the ionosphere (with the aid of rockets and satellites), and (2) the observation of the time-variation in the ionospheric N(h)-profile (through repeated sweep-frequency ionosonde measurement of h'-f traces) or charged-particle drift in the ionospheric region (by radar technique).

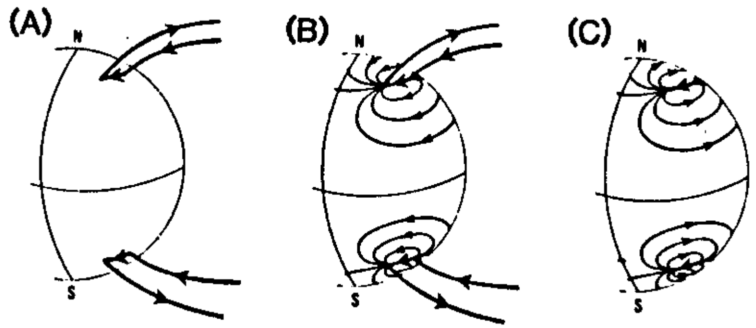


Fig. 17. Schematic illustration of three different kinds of electric current-systems for geomagnetic bay; (A) the Birkeland-Alfvén system, (B) the addition of the Pedersen current in the ionosphere to the Birkeland-Alfvén system, (C) the two-dimensional Chapman-Vestine current-system.

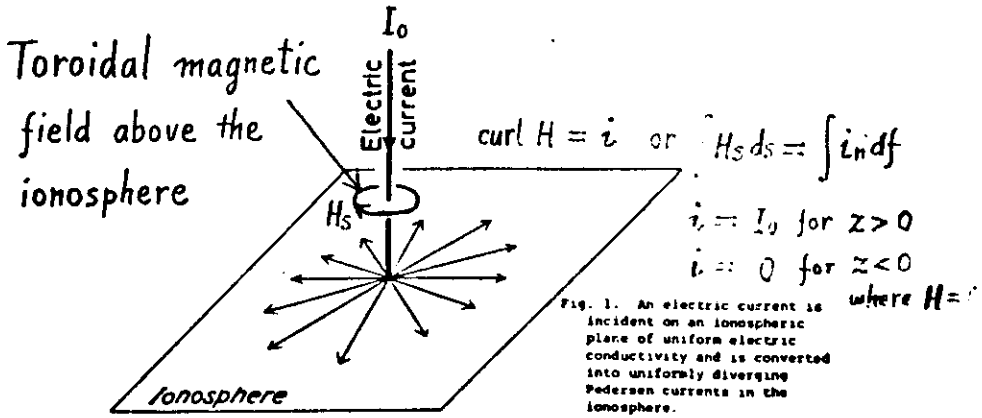


Fig. 1. An electric current is incident on an ionospheric plane of uniform electric conductivity and is converted into uniformly diverging Pedersen currents in the ionosphere.

2.1. Incidence of a vertical current to a horizontal conducting plane

Fig. 1 shows an electric current system in which a vertical line current is converted into diverging Pedersen currents on a plane ionosphere. We take here a cylindrical coordinate (r, ϕ, z); the vertical downward current flows along the z-axis at r=0, and the diverging electric currents flow on the plane z=0. The ionospheric current density $i(i_r, i_\phi)$ at the point $(r, \phi, 0)$ is given by

$$i_r(r, \phi, 0) = I_0/(2\pi r), \quad i_\phi(r, \phi, 0) = 0. \qquad (1)$$

The magnetic field $H(H_r, H_\phi, H_z)$ produced by the electric current system of Fig. 1 in the region z>0 (i.e. above the ionospheric plane) is given by

$$H_r(z > 0) = 0, \quad H_\phi(z > 0) = I_0/(2\pi r), \quad H_z(z > 0) = 0, \qquad (2)$$

whereas the magnetic field in the region z<0 (i.e. below the ionospheric plane) vanishes.

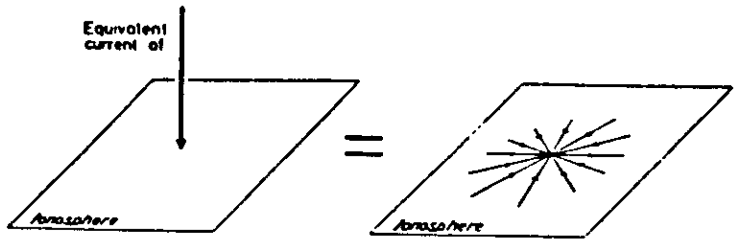


Fig. 2. The equivalent current for a vertical current flowing into the ionosphere is a current uniformly converging towards the point of intersection.

THE BIRKELAND SYMPOSIUM ON AURORA AND MAGNETIC STORMS
18-22 September 1967, Sandefjord, Norway

Sydney CHAPMAN (1918, 1927, 1935, 1961)

Hannes ALFVÉN (1939, 1940)

Average Disturbance $= \frac{1}{n}\sum_n$ (Polar Elementary Storm)

$= \frac{1}{n}\sum_n$ (Polar Substorm)

BRITISH SCHOOL

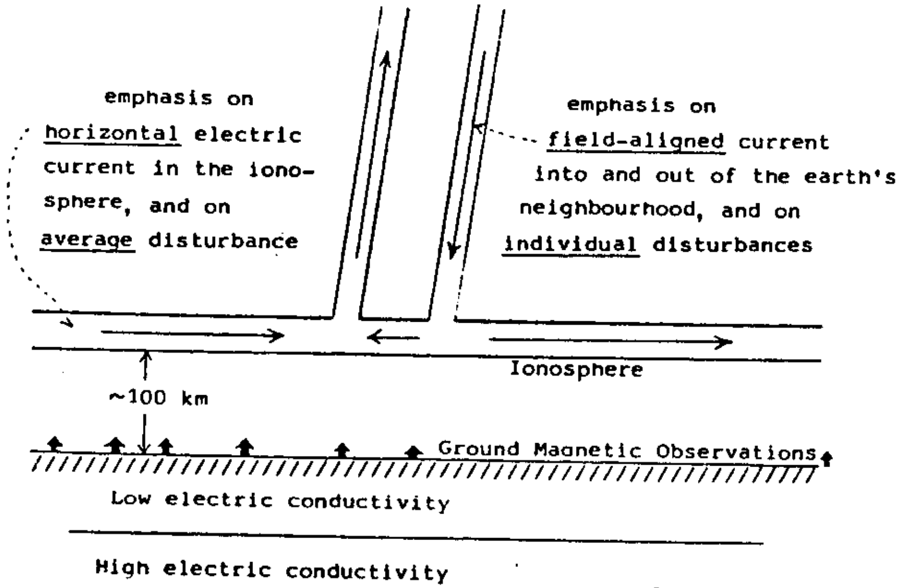
Experimental detection of the ionosphere: Appleton (1925)

Prediction of the existence of the current-flowing layer: Stewart (1882)

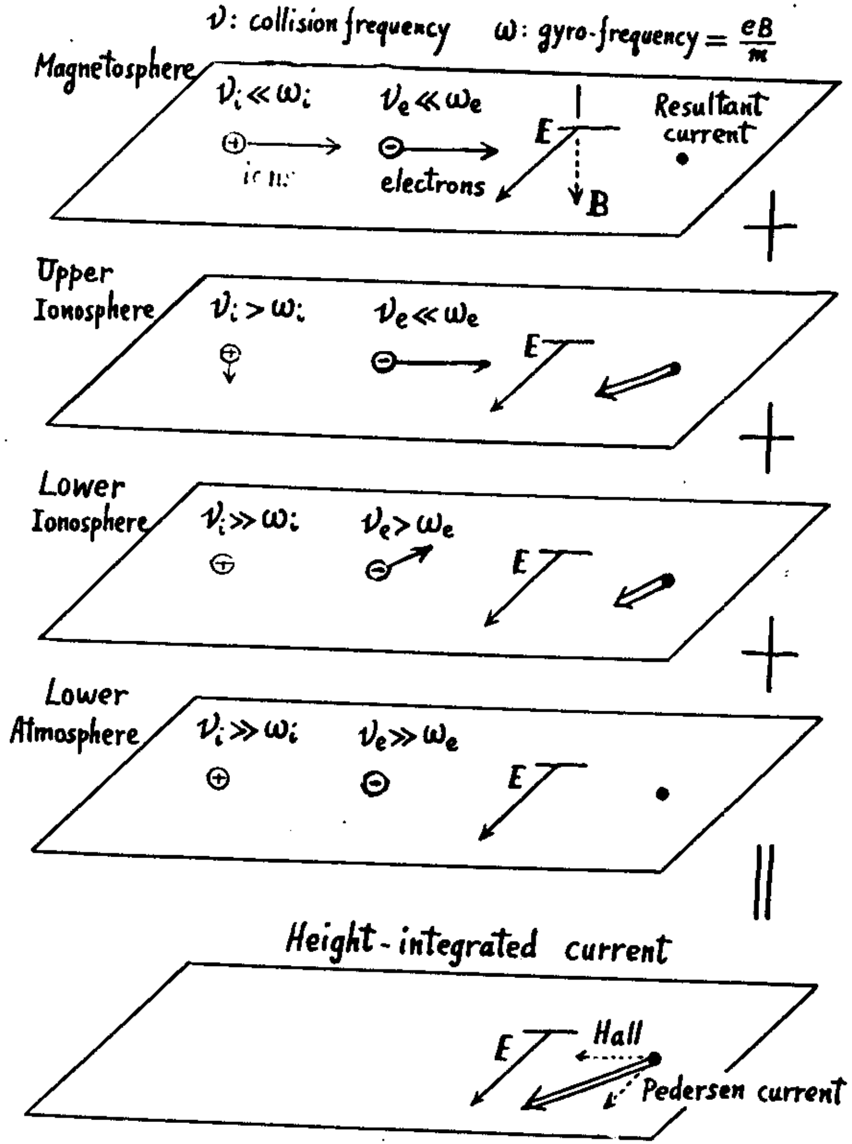
SCANDINAVIAN SCHOOL

Kr. Birkeland's Norwegian Aurora Polaris Expedition

(1901, 1908, 1913 publications)

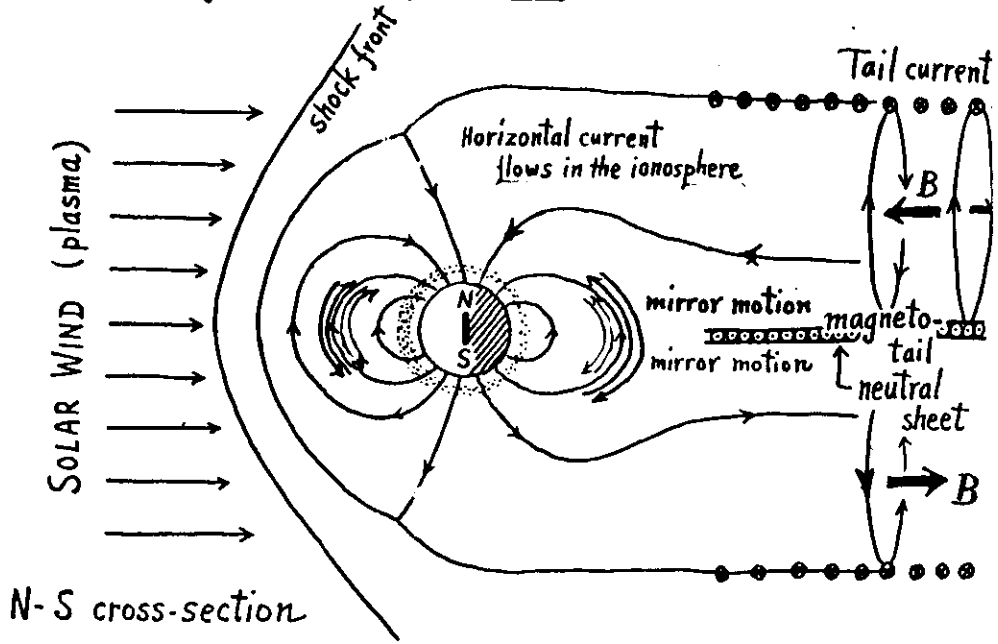


Relationship between Electric Field and Current in the Earth's Atmosphere



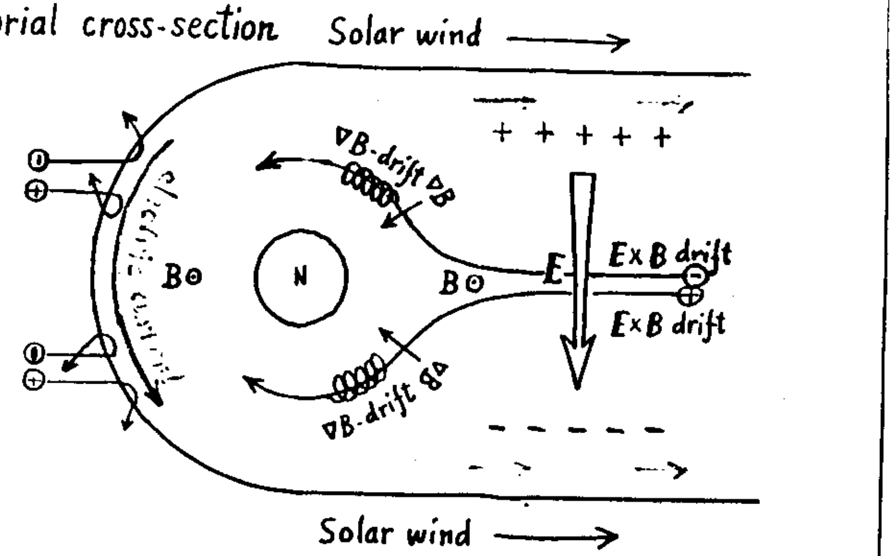
Currents Responsible for Geomagnetic Variations

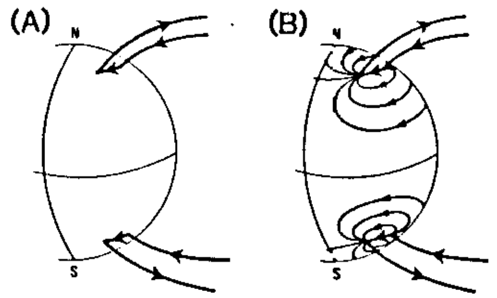
Role of Geomagnetic Field in Space Science



N-S cross-section

Equatorial cross-section





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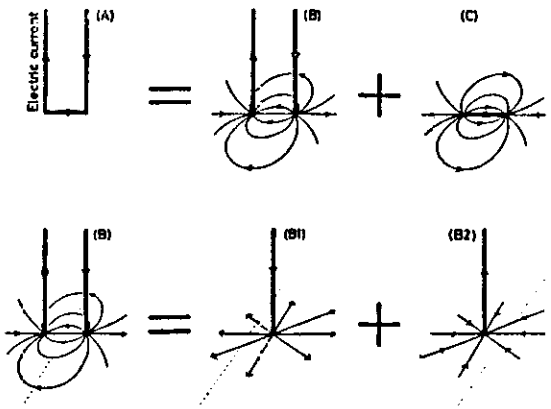

Fig. 3. The Birkeland-Alfvén current system (A), for a polar substorm, is the sum of current systems (B) and (C), where (C) is the Chapman-Vestine current system. Current system (B) = (B1) + (B2) produces no magnetic effect below the ionosphere.