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" SPRING COLLEGE ON GEOMAGNETISM AND AERONOMY "

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" U.S. Standard Atmosphere "

presented by :

J.A. KLOBUCHAR
Ionospheric Physics Division
Air Force Geophysics Laboratory
Air Force Base
Hanscom, MA 01731
U.S.A.

U.S. STANDARD ATMOSPHERE, 1976

Abstract

The *U.S. Standard Atmosphere, 1976*, which is a revision of the *U.S. Standard Atmosphere, 1962*, was generated under the impetus of increased knowledge of the upper atmosphere obtained over the past solar cycle. Above 50 km, this Standard is based on extensive new rocket data and theory for the mesosphere and lower thermosphere, and on the vast resources of satellite data for the thermosphere acquired over more than one complete solar cycle. This Standard is identical with the ICAO Standard (1964) up to 32 km and the ISO Standard (1973) to 50 km. Part 1 gives the basis for computation of the main tables of atmospheric properties, including values of physical constants, conversion factors, and definitions of derived properties. Part 2 describes the model and data used up to 85 km, in the first section; and the model and data used above 85 km, in the second section. The theoretical basis of the high-altitude model is given in an appendix. Part 3 contains information on minor constituents in the troposphere, stratosphere, and mesosphere. The main tables of atmospheric properties to 1000 km are given in Part 4. The international system of metric units is used.

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PART 1

Defining Constants and Equations

1.0 INTRODUCTION

The U.S. Standard Atmosphere, 1976 is an idealized, steady-state representation of the earth's atmosphere from the surface to 1000 km, as it is assumed to exist in a period of moderate solar activity. For heights from the surface to 51 geopotential kilometers (km′), the tables of this standard are identical with those of the U.S. Standard Atmosphere, 1962 (COESA 1962) and are based on traditional definitions. These definitions, especially for heights below 20 km′, do not necessarily represent an average of the vast amount of atmospheric data available today from observations within that height region. For heights from 51 km′ to 84.852 km′ (i.e., 51.413 to 86 geometric kilometers), the tables are based upon the averages of present-day atmospheric data as represented by the traditional type of defining parameters. These include the linearly segmented temperature-height profile, and the assumption of hydrostatic equilibrium, in which the air is treated as a homogeneous mixture of the several constituent gases.

At greater heights, however, where dissociation and diffusion processes produce significant departures from homogeneity, the definitions governing the Standard are more sophisticated than those used at lower altitudes. In this high-altitude regime, the hydrostatic equation, as applied to a mixed atmosphere, gives way to the more general equation for the vertical component of the flux for individual gas species (Colegrove et al. 1965; Keneshea and Zimmerman 1970), which accounts for the relative change of composition with height. This flux equation simplifies to the hydrostatic equation for the special case when the atmospheric gases remain well mixed, as is the situation below 86 km.

The temperature-height profile between 86 and 1000 km is not expressed as a series of linear functions, as at lower altitudes. Rather, it is defined in terms of four successive functions chosen not only to provide a reasonable approximation to observations, but also to yield a continuous first derivative with respect to height over the entire height regime.

Observational data of various kinds provide the basis for independently determining various segments of this temperature-height profile. The observed temperatures at heights between 110 and 120 km were particularly important in imposing limits on the selection of the temperature-height function for that region, while the observed densities at 150 km and above strongly influenced the selection of both the temperature and the extent of the low-temperature isothermal layer immediately above 86 km.

In spite of the various independent data sets upon which the several temperature-height segments are based, it is desirable, for purposes of mathematical reproducibility of the tables of this Standard, to express the temperature in a series of consecutive height functions from the surface to 1000 km, with the expression for each successive function depending upon the end-point value of the preceding function, as well as upon certain terms and coefficients peculiar to the related height interval. This total temperature-height profile applied to the fundamental continuity models (i.e., the hydrostatic equation and the equation of motion), along with all the ancillary required constants, coefficients, and functions, defines the U.S. Standard Atmosphere, 1976. The specification of this definition without any justification in terms of observed data is the purpose of Section 1.

1.1 INTERNATIONAL SYSTEM OF UNITS

The 1976 U. S. Standard Atmosphere is defined in terms of the International System (SI) of Units (Mechtley 1973). A list of the symbols, names, and the related quantities of the applicable basic and derived SI units, as well as of the non-standard metric units and the English unit employed in this Standard is presented in table 1.

1.2 BASIC ASSUMPTIONS AND FORMULAS

1.2.1 ADOPTED CONSTANTS.—For purposes of computation it is necessary to establish numerical values for various constants appropriate to the earth's atmosphere. The adopted constants are grouped into three categories. Category I includes those constants which are common to many branches of the physical and chemical sciences, and are here considered to be fundamental constants. Some of these may be multi-valued as in the case of M_i representing the molecular weight of the ith gas species. Category I includes three single-valued and one multi-valued constant. Category II includes those constants which, in addition to the

TABLE 1.—Units Applicable to the U.S. Standard Atmosphere 1976

	Symbol	*Name*	*Quantity*
Basic SI			
	m	meter	length
	kg	kilogram	mass
	s	second	time
	K	kelvin	thermodynamic temperature
	mol	mole	the amount of a substance
Derived SI			
	N	newton	force ($kg{\cdot}m/s^2$)
	Pa	pascal	pressure (N/m^2)
	J	joule	work, energy or quantity of heat ($N{\cdot}m$)
	W	watt	rate of energy (or heat) transfer (J/s)
Non-Standard			
	mb	millibar	pressure 100 (N/m^2)
	torr at 0°C	torr	pressure 133.322 (N/m^2)
	°C	Celsius degree	temperature kelvin minus 273.15
English			
	ft	foot	length 0.3048 m*

* exact definition

category I constants and a suitable set of equations, are sufficient to define that portion of the 1976 Standard Atmosphere below 86 km. This category includes nine single-valued and three multi-valued constants. Category III includes all the remaining constants which, along with category-I and category-II constants and the related equations plus an expansion of that set are necessary to define that portion of the 1976 Standard Atmosphere above 86 km. This category includes 7 single-valued and 11 multi-valued constants.

The constants, with appropriate dimensions and symbols, are listed according to categories in three successive sections of table 2.

The definition as well as the authority for the value of each constant is discussed separately from the tabular listing. The multi-valued constants, with one exception, have only their general symbol and dimensions listed in table 2, while the multiple values of these constants, i.e., one value for each of several gas species, or one value for each of several height levels, are listed in tables 4 through 7.

Discussion of the Adopted Values of the Primary Constants:

Category I Constants

k The Boltzmann constant, $k = 1.380622 \times 10^{-23}$ N·m/K, is theoretically equal to the ratio R^*/N_A, and has a value, consistent with the carbon-12 scale as cited by Mechtly (1973).

TABLE 2.—Adopted constants

A. Category I Constants

Symbol	*Value*
k	1.380622×10^{-23} N·m/K
M_i	the set of the first 10 values (kg/kmol) listed in table 3
N_A	6.022169×10^{26} kmol^{-1}
R^*	8.31432×10^{3} N·m/(kmol·K)

B. Category II Constants

Symbol	*Value*
F_i	the set of the 10 values (dimensionless) listed in table 3
g_0	9.80665 m/s^2
g_0'	9.80665 m^2/(s^2·m′)
H_b	the set of eight values (km′) listed in table 4
$L_{M,b}$	the set of seven values (K/km′) listed in table 4
P_0	1.013250×10^5 N/m^2 (or Pa)
r_0	6.356766×10^3 km
T_0	288.15 K
S	110 K
β	1.458×10^{-6} kg/(s·m·K$^{1/2}$)
γ	1.40 (dimensionless)
σ	3.65×10^{-10} m

C. Category III Constants

Symbol	*Value*
a_i	the set of 5 values (m^{-1}·s^{-1}) listed in table 6
b_i	the set of 5 values (dimensionless) listed in table 6
K_7	1.2×10^2 m^2/s
K_{10}	0.0 m^2/s
$L_{K,b}$	the set of 2 values (K/km) listed in table 5
$n(O)_7$	8.6×10^{16} m^{-3}
$n(H)_{11}$	8.0×10^{10} m^{-3}
q_i	the set of 4 values (km^{-3}) listed in table 7
Q_i	the set of 4 values (km^{-3}) listed in table 7
T_9	240.0 K
T_∞	1000.0 K
u_i	the set of 4 values (km) listed in table 7
U_i	the set of 4 values (km) listed in table 7
w_i	the set of 4 values (km^{-3}) listed in table 7
W_i	the set of 4 values (km^{-3}) listed in table 7
Z_b	the set of 6 values (km) listed in table 5
α_i	the set of 6 values (dimensionless) listed in table 6
ϕ	7.2×10^{11} m^{-2}·s^{-1}

M_i The set of values of molecular weights M_i listed in table 3 is based upon the carbon-12 isotope scale for which $C^{12} = 12$. This scale was adopted in 1961 at the Montreal meeting of the International Union of Pure and Applied Chemistry.

N_A The Avogadro constant, $N_A = 6.022169 \times 10^{26}$ kmol^{-1}, is consistent with the carbon-12 scale and is the value cited by Mechtly (1973).

R^* The gas constant, $R^* = 8.31432 \times 10^3$ N·m (kmol·K), is consistent with the carbon-12 scale, and is the value used in the 1962 Standard. This value is not exactly consistent with the cited values of k and N_A.

Category II Constants

F_i The set of values of fractional-volume concentrations F_i listed in table 3 is assumed to represent the relative concentrations of the several gas species comprising dry air at sea level. These values are identical to those given in the 1962 Standard (COESA 1962), and except for minor modifications which are based upon CO_2 measurements by Keeling (1960), these values are the same as those given by Glueckauf (1951), and are based upon the earlier work of Paneth (1939).

g_0 The quantity g_0 (= 9.80665 m/s^2) represents the sea-level value of the acceleration of gravity adopted for this Standard. This value is the one originally adopted by the International Committee on Weights and Measures in 1901 for 45° latitude, and even though it has since been shown to be too high by about five parts in ten thousand (List 1968), this value has persisted in meteorology and in some standard atmospheres as the value associated with 45° latitude, even though it applies more precisely to a latitude of 45° 32′ 33″.

g_0' The dimensional constant g_0' selected to relate the standard geopotential meter to geometric height is numerically equal to g_0, but with appropriately different dimensions. This constant implicitly defines one standard geopotential meter as the vertical increment through which one must lift one kilogram to increase its potential energy by 9.80665 joules. The geometric length of this vertical increment varies inversely with the height-dependent value of g.

H_b Each of the members of the set of geopotential-height values H_b listed in table 4 represents the base of one of eight successive atmospheric layers. The pairs of values of H_b and $L_{M,b}$ are based partly on tradition and partly on present-day observations. The first five of these pairs are identical to those of the first five layers of the 1962 Standard, while the remaining two values of both H_b and $L_{M,b}$ have been newly selected to provide a reasonable fit to the presently available atmospheric data. The first two values of the related sets have their origin in one of the earliest aeronautical standard atmospheres (Toussaint 1919), and were approximated in the first U.S. Standard Atmosphere (Diehl 1925).

TABLE 3.—Molecular weights and assumed fractional-volume composition of sea-level dry air

Gas species	Molecular weight M_i (kg/kmol)	Fractional volume F_i (dimensionless)
N_2	28.0134	0.78084
O_2	31.9988	.209476
Ar	39.948	.00934
CO_2	44.00995	.000314
Ne	20.183	.00001818
He	4.0026	.00000524
Kr	83.80	.00000114
Xe	131.30	.000000087
CH_4	16.04303	.000002
H_2	2.01594	.0000005

TABLE 4.—The defined reference levels and gradients of the linearly segmented temperature-height profile from the surface to 86 geometric kilometers

Subscript b	Geopotential height H_b (km′)	Molecular-scale temperature gradient $L_{M,b}$ (K/km′)	Form of function relating T to H
0	0	−6.5	Linear
1	11	0.0	Linear
2	20	+1.0	Linear
3	32	+2.8	Linear
4	47	0.0	Linear
5	51	−2.8	Linear
6	71	−2.0	Linear
7	84.8520		

Note: These values plus T_0, the defined sea-level value of T, equal to $T_{M,0}$ completely specify the geopotential-height profile of T_M from the surface to 86 geometric kilometers.

$L_{M,b}$ Each member of the set of seven gradients $L_{M,b} = dT_M/dH$ [i.e., of molecular scale temperature T_M (Minzner and Ripley 1956) with respect to geopotential H] listed in table 4 represents the fixed value appropriate throughout its related layer, H_b to H_{b+1}.

P_0 The standard sea-level atmospheric pressure P_0 equal to 1.013250×10^5 Pa (or N/m^2) was adopted in 1947 in

Resolution 164 of the International Meteorological Organization, and corresponds to the pressure exerted by a column of mercury 0.760 m high, having a density of 1.35951×10^4 kg/m^3 and subject to an acceleration due to gravity of 9.80665 m/s^2. This equivalency definition was adopted by the International Commission on Weights and Measures in 1948.

r_0 The effective earth's radius for purposes of calculating geopotential at any latitude is readily obtained from equations given by Harrison (1968). The value of r_0 (= 6856.766 km) used in this Standard corresponds to the latitude for which $g = 9.80665$ m/s^2.

T_0 The standard sea-level temperature T_0 is 288.15 K. This value is based upon two international agreements. The first of these is Resolution 192 of the International Commission for Air Navigation which in 1924 adopted 15 C as the sea-level temperature of The International Standard Atmosphere. This value has been retained unchanged in all known standard atmospheres since that date. The second agreement is that of the 1954 Tenth General Conference on Weights and Measures which set the fixed point of the Kelvin temperature scale at the triple-point temperature 273.16 K, which is 0.01 K above the ice-point temperature at standard sea-level pressure.

S The Sutherland constant, S = 110 K, (Hilsenrath et al. 1955) is a constant in the empirical expression for dynamic viscosity.

β The quantity, $\beta = 1.458 \times 10^{-6}$ kg/(s · m · K$^{1/2}$), (Hilsenrath et al. 1955) is a constant in the expression for dynamic viscosity.

The ratio of specific heat of air at constant pressure to the specific heat of air at constant volume is a dimensionless quantity with an adopted value $\gamma = 1.400$. This is the value adopted by the Aerological Commission of the International Meteorological Organization, in Toronto in 1948.

The mean effective collision diameter σ ($= 3.65 \times 10^{-10}$ m) of molecules is a quantity which varies with gas species and temperature. The adopted value is assumed to apply in a dry, sea-level atmosphere. Above 85 km the validity of the adopted value decreases with increasing altitude (Hirschfelder et al. 1965; Chapman and Cowling 1960) due to the change in atmospheric composition. For this reason the number of significant figures in tabulations of quantities involving σ is reduced from that used for other tabulated quantities at heights above 86 km.

Category III Constants

a_i The quantity a_i represents a set of five values of species-dependent coefficients listed in table 6. Each of these values is used in a particular function for designating the height-dependent, molecular-diffusion coefficient D_i *for* the related gas species. (See b_i.)

TABLE 5.—The reference levels and function designations for each of the four segments of the temperature-height profile between 86 and 1000 km, with gradients specified for two linear segments, and with an intermediate reference height for the adopted atomic-hydrogen number-density value

Subscript b	Geometric height Z_b (km)	Kinetic-temperature gradient $L_{K,b}$ (K/km)	Form of function relating T to Z
7	86	0.0	linear
8	91		elliptical
9	110	12.0	linear
10	120		exponential
11	500		
12	1000		

Note: These specifications, along with a defined value of temperature at 110 km, and the temperature at 86 km (84.8520 km) given in table 4, plus the requirement of a continuous first derivative, dT/dZ, above 86 km, define the temperature-height profile between 86 and 120 km. The definitive form of the exponential function eq (31) is required to complete the specification of the temperature-height profile from 120 to 1000 km. (See Appendix B for the derivation of the elliptical segment given by eq (27)).

b_i The quantity b_i represents a set of five values of species-dependent exponents listed in table 6. Each of these values is used, along with the corresponding value of a_i, in eq (8) for designating the height-dependent, molecular-diffusion coefficient for the related gas species. The particular values of a_i and b_i adopted for this Standard have been selected to yield a height variation of D_i assumed to be realistic.

K_7 The quantity $K_7 = 1.2 \times 10^2$ m^2/s is the adopted value of the eddy-diffusion coefficient K, at $Z_7 = 86$ km and in the height interval from 86 up to 91 km. Beginning at 91 km and extending up to 115 km, the value of K is defined by eq (7b). At 115 km the value of K equals K_{10}.

K_{10} The quantity $K_{10} = 0.0$ m^2/s is the adopted value of the eddy-diffusion coefficient K at $Z_{10} = 120$ and throughout the height interval from 115 km to 1000 km.

$L_{K,b}$ The two-valued set of gradients $L_{K,b} = dT/dZ$ listed in table 5 was specifically selected for this Standard to represent available observations. Each of these two values of $L_{K,b}$ is associated with the entire extent of a corresponding layer whose base is Z_b and whose top is Z_{b+1}.

$n(O)_7$ The quantity, $n(O)_7$ ($= 8.6 \times 10^{16}$ m^{-3}), is the number density of atomic oxygen assumed for this Standard to exist at $Z_7 = 86$ km. This value of atomic oxygen number density, along with other defined constants, leads to number densities of N_2, O_2, Ar, and He at 86 km. (See Appendix A.)

$n(H)_{11}$ The quantity, $n(H)_{11}$ ($= 8.0 \times 10^{10}$ m^{-3}), is the assumed number density of atomic hydrogen at height $Z_{11} = 500$ km, and is used as the reference value in computing the height profile of atomic hydrogen between 150 and 1000 km.

q_i The quantity q_i represents the first set of six species-dependent sets of coefficients or terms (i.e., sets of q_i, Q_i, u_i, U_i, w_i, and W_i), the corresponding members of all six of which are simultaneously used in an empirical expression [eq (37)] for the vertical transport term $v_i/(D_i + K)$ in the vertical flux equation for the particular gas species. The species-dependent values of all six sets have been selected for this Standard to adjust number-density profiles of the related gas species to particular boundary conditions at 150 and 450 km, as well as at 97 km in the case of atomic oxygen. These boundary conditions all represent observed or assumed average conditions. These six sets of values are listed in table 7.

Q_i The quantity Q_i represents the second set of the six sets of constants described along with q_i above.

T_9 The quantity T_9 (= 240.0 K) represents the kinetic temperature at $Z_9 = 110$ km. This temperature has been adopted along with the gradient $L_{K,9}$ (= 12 K/km) to generate a linear segment of $T(Z)$ for this Standard between 110 and 120 km. This segment of $T(Z)$ represents a mean of observed temperature-height data for the corresponding height region.

T_∞ The quantity, T_∞ (= 1000 K) represents the exospheric temperature, i.e., the asymptote which the exponential function representing $T(Z)$ above 120 km closely approaches at heights above about 500 km, where the mean free path exceeds the scale height. The value of T_∞ adopted for this Standard is assumed to represent mean solar conditions

u_i The quantity u_i represents the third set of the six sets of constants described along with q_i above.

U_i The quantity U_i represents the fourth set of the six sets of constants described along with q_i above.

TABLE 6.—A set of species-dependent, thermal-diffusion coefficients and two other sets of species-dependent constants required in specifying the height-dependent function of the molecular-diffusion coefficient for the several species listed

Gas	α_i (dimensionless)	a_i (m^{-1} · s^{-1})	b_i (dimensionless)
N_2	0.00	......	......
O	0.00	6.986×10^{20}	0.750
O_2	0.00	4.863×10^{20}	0.750
Ar	0.00	4.487×10^{20}	0.870
He	−0.40	1.700×10^{21}	0.691
H	−0.25	3.305×10^{21}	0.500

TABLE 7.—Values of six sets of species-dependent coefficients applicable to the empirical expression representing the flux term $v_i/(D_i + K)$ in the equation for number density of the four species listed

Gas	Q_i (km^{-3})	q_i (km^{-3})	U_i (km)	u_i (km)	W_i (km^{-3})	w_i (km^{-3})
O	-5.809644×10^{-4}	-3.416248×10^{-3}*	56.90311	97.0	2.706240×10^{-5}	5.008765×10^{-4}
O_2	1.366212×10^{-4}	0	86.000	......	8.333333×10^{-5}	......
Ar	9.434079×10^{-5}	0	86.000	......	8.333333×10^{-5}	......
He	-2.457369×10^{-4}	0	86.000	......	6.666667×10^{-4}	......

* This value of q_i applies only for $86 \leq Z \leq 97$ km. For $Z > 97$ km, $q_i = 0.0$ km^{-3}.

w_i The quantity w_i represents the fifth set of the six sets of constants described along with q_i above.

W_i The quantity W_i represents the sixth set of the six sets of constants described along with q_i above.

Z_b The quantity Z_b represents a set of six values of Z for b equal to 7 through 12. The values Z_7, Z_8, Z_9, and Z_{10} correspond successively to the base of successive layers characterized by successive segments of the adopted temperature-height function for this Standard. The fifth value, Z_{11}, is the reference height for the atomic hydrogen calculation, while the sixth value, Z_{12}, represents the top of the region for which the tabular values of the Standard are given. These six values of Z_b, along with the designation of the type of temperature-height function associated with the first four of these values, plus the related value of $L_{K,b}$, for the two segments having a linear temperature-height function, are listed in table 5.

α_i The quantity α_i represents a set of six adopted species-dependent, thermal-diffusion coefficients listed in Table 6.

ϕ The quantity ϕ ($= 7.2 \times 10^{11}\ \mathrm{m^{-2} \cdot s^{-1}}$) for the vertical flux is chosen as a compromise between the classical Jeans escape flux for $T_\infty = 1000$ K, with corrections to take into account deviations from a Maxwellian velocity distribution at the critical level (Brinkman 1971), and the effects of charge exchange with H^+ and O^+ in the plasmasphere (Tinsley 1973).

1.2.2 EQUILIBRIUM ASSUMPTIONS.—The air is assumed to be dry; and at heights sufficiently below 86 km, the atmosphere is assumed to be homogeneously mixed with a relative-volume composition leading to a constant mean molecular weight M. The air is treated as if it were a perfect gas, and the total pressure P, temperature T, and total density ρ at any point in the atmosphere are related by the equation of state, i.e., the perfect gas law, one form of which is

$$P = \frac{\rho \cdot R^* \cdot T}{M} \tag{1}$$

where R* is the universal gas constant. An alternate form of the equation of state, in terms of the total number density N and the Avogadro constant N_A is

$$P = \frac{N \cdot R^* \cdot T}{N_A}. \tag{2}$$

This form represents the summation of P_i the partial pressures of the individual gas species, where P_i is related to n_i the number density of the ith gas species in the following expression:

$$P_i = n_i \cdot k \cdot T \tag{3}$$

where k is the Boltzmann constant.

Within the height region of complete mixing, the atmosphere is assumed to be in hydrostatic equilibrium, and to be horizontally stratified so that dP, the differential of pressure, is related to dZ, the differential of geometric height, by the relationship

$$dP = -g \cdot \rho \cdot dZ \tag{4}$$

where g is the height-dependent acceleration of gravity. The elimination of ρ between eq (1) and (4) yields another form of the hydrostatic equation, which serves as the basis for the low-altitude pressure calculation:

$$d\ln P = \frac{dP}{P} = \frac{-g \cdot M}{R^* \cdot T} \cdot dZ. \tag{5}$$

Above 86 km the hydrostatic equilibrium of the atmosphere gradually breaks down as diffusion and vertical transport of individual gas species lead to the need for a dynamically oriented model including diffusive separation. Under these conditions it is convenient to express the height variations in the atmospheric number density in terms of the vertical component of the flux of the molecules of individual gas species (Colgrove et al. 1965). In terms of the ith gas species, this expression is

$$n_i \cdot v_i + D_i \cdot \left(\frac{dn_i}{dZ} + \frac{n_i \cdot (1 + \alpha_i)}{T} \cdot \frac{dT}{dZ} + \frac{g \cdot n_i \cdot M_i}{R^* \cdot T}\right) + K \cdot \left(\frac{dn_i}{dZ} + \frac{n_i}{T} \cdot \frac{dT}{dZ} + \frac{g \cdot n_i \cdot M}{R^* \cdot T}\right) = \phi \tag{6}$$

where

v_i = the vertical transport velocity of the ith species,

D_i = the height-dependent, molecular-diffusion coefficient of the ith species diffusing through N_2,

α_i = the thermal-diffusion coefficient of the ith species.

M_i = the molecular weight of the ith species,

M = the molecular weight of the gas through which the ith species is diffusing, and

K = the height-dependent, eddy-diffusion coefficient.

The function K is defined differently in each of three height regions:

1. For $86 \leq Z < 95$ km,

$$K = K_7 = 1.2 \times 10^2\ \mathrm{m^2/s} \tag{7a}$$

2. For $95 \leq Z < 115$ km

$$K = K_7 \cdot \exp\left[1 - \frac{400}{400 - (Z - 95)^2}\right] \tag{7b}$$

3. For $115 \leq Z < 1000$

$$K = K_{10} = 0.0\mathrm{m^2/s}. \tag{7c}$$

The function D_i is defined by

$$D_i = \frac{a_i}{\Sigma n_i} \cdot \left(\frac{T}{273.15}\right)^{b_i} \tag{8}$$

where a_i and b_i are the species-dependent constants defined in table 6, while T and Σn_i are both altitude-dependent quantities which are specified in detail below. The values of D_i, determined from these altitude-dependent quantities and the defined constants a_i and b_i, are plotted in figure 1 as a function of altitude, for each of four species, O, O_2, Ar, and He. The value of D_i for atomic hydrogen, for heights just below 150 km, is also shown in figure 1. This same figure contains a graph of K as a function of altitude. It is apparent that, for heights sufficiently below 90 km, values of D_i are negligible compared with K, while above 115 km, the reverse is true. In addition, it is known that the flux velocity v_i for the various species becomes negligibly small at altitudes sufficiently below 90 km.

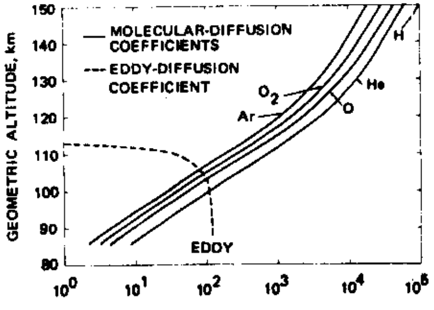

Figure 1. Molecular-diffusion and eddy-diffusion coefficients as a function of geometric altitude.

The information regarding the relative magnitudes of v_i, D_i, and K permits us to consider the application of eq (6) in each of several regimes. One of these regimes is for heights sufficiently below 90 km, such that v_i and D_i are both extremely small compared with K. Under these conditions, eq (6) reduces to the following form of the hydrostatic equation:

$$\frac{dn_i}{n_i} + \frac{dT}{T} = \frac{-g \cdot M}{R^* \cdot T} \cdot dZ. \tag{9}$$

Since the left-hand side of this equation is seen through eq (3) to be equal to dP_i/P_i, eq (9) is seen to be the single-gas equivalent to eq (5). Consequently, while eq (6) was designed to describe the assumed equilibrium conditions of individual gases above 86 km, it is apparent that eq (6) also describes such conditions below that altitude, where the partial pressure of each gas comprising the total pressure varies in accordance with the mean molecular weight of the mixture, as well as in accordance with the temperature and the acceleration of gravity. Nevertheless, eq (5), expressing total pressure, represents a convenient step in the development of equations for computing total pressure versus geometric height, when suitable functions are introduced to account for the altitude variation in T, M, and g.

It has been customary in standard-atmosphere calculations, to effectively eliminate the variable portion of the acceleration of gravity from eq (5) by the transformation of the independent variable Z to geopotential altitude H, thereby simplifying both the integration of eq (5) and the resulting expression for computing pressure. The relationship between geometric and geopotential altitude depends upon the concept of gravity.

1.2.3 GRAVITY AND GEOPOTENTIAL ALTITUDE—Viewed in the ordinary manner, from a frame of reference fixed in the earth, the atmosphere is subject to the force of gravity. The force of gravity is the resultant (vector sum) of two forces: (a) the gravitational attraction in accordance with Newton's universal law of gravitation, and (b) the centrifugal force, which results from the choice of an earthbound, rotating frame of reference.

The gravity field, being a conservative field, can be derived conveniently from the gravity potential energy per unit mass, that is, from the geopotential Φ. This is given by

$$\Phi = \Phi_G + \Phi_C \tag{10}$$

where Φ_G is the potential energy, per unit mass, of gravitational attraction, and Φ_C is the potential energy, per unit mass, associated with the centrifugal force. The gravity, per unit mass, is

$$\mathbf{g} = \nabla\Phi \tag{11}$$

where $\nabla\Phi$ is the gradient (ascendant) of the geopotential. The acceleration due to gravity is denoted by g and is defined as the magnitude of $\mathbf{g}$; that is,

$$g = |\mathbf{g}| = |\nabla\Phi|. \tag{12}$$

When moving along an external normal from any point on the surface Φ_1 to a point on the surface Φ_2 infinitely close to the first surface, so that $\Phi_2 = \Phi_1 + d\Phi$, the incremental work performed by shifting a unit mass from the first surface to the second will be

$$d\Phi = g\cdot dZ. \tag{13}$$

Hence,

$$\Phi = \int_0^Z g\cdot dZ. \tag{14}$$

The unit of measurement of geopotential is the standard geopotential meter (m′) which represents the work done by lifting a unit mass 1 geometric meter, through a region in which the acceleration of gravity is uniformly 9.80665 m/s².

The geopotential of any point with respect to mean sea level (assumed zero potential), expressed in geopotential meters, is called geopotential altitude. Therefore, geopotential altitude H is given by

$$H = \frac{\Phi}{g_0'} = \frac{1}{g_0'}\cdot\int_0^Z g\cdot dZ \tag{15}$$

and is expressed in geopotential meters (m′) when the unit geopotential g_0' is set equal to 9.80665 m²/(s²·m′).

With geopotential altitude defined as in eq (15), the differential of eq (15) may be expressed as

$$g_0'\cdot dH = g\cdot dZ. \tag{16}$$

This expression is used in eq (5) to reduce the number of variables prior to its integration, thereby leading to an expression for computing pressure as a function of geopotential height.

The inverse-square law of gravitation provides an expression for g as a function of altitude with sufficient accuracy for most model-atmosphere computations:

$$g = g_0\cdot\left(\frac{r_0}{r_0+Z}\right)^2 \tag{17}$$

where r_0 is the effective radius of the earth at a specific latitude as given by Lambert's equations (List 1968). Such a value of r_0 takes into account the centrifugal acceleration at the particular latitude. For this Standard, the value of r_0 is taken as 6,356,766 m, and is consistent with the adopted value of g_0 = 9.80665 m/s² for the sea-level value of the acceleration of gravity. The variation of g as a function of geometric altitude is depicted in figure 2.

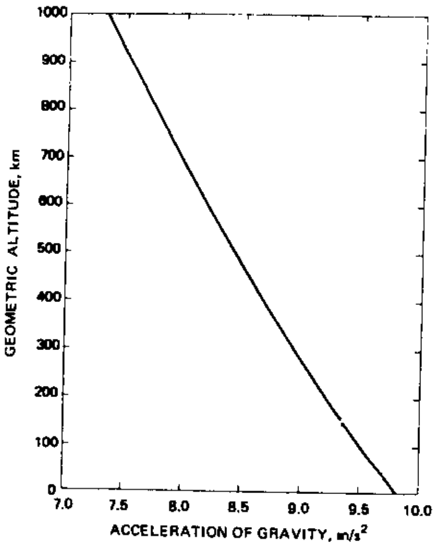

Figure 2. Acceleration of gravity as a function of geometric altitude

Integration of eq (15), after substitution of eq (17) for g, yields

$$H = \frac{g_0}{g_0'}\cdot\left(\frac{r_0\cdot Z}{r_0+Z}\right) = \Gamma\cdot\left(\frac{r_0\cdot Z}{r_0+Z}\right) \tag{18}$$

or

$$Z = \frac{r_0\cdot H}{\Gamma\cdot r_0 - H} \tag{19}$$

where $\Gamma = g_0/g_0' = 1$ m′/m.

Differences between geopotential altitudes obtained from eq (18) for various values of Z, and those computed from the more complex relationship used in developing the U.S. Standard Atmosphere, 1962, are small. For example, values of H computed from eq (18) are approximately 0.2, 0.4, and 33.3 m greater at 90, 120, and 700 km, respectively, than those obtained from the relationship used in the 1962 Standard.

The transformation from Z to H in eq (5) makes it necessary for the altitude variation of T as well as any variation in M between the surface and 86 km also to be defined in terms of H. It is convenient therefore to determine the sea-level value of M as well as the extent of any height dependence of this quantity between the surface and 86 km. Then, for this low-altitude regime, the two variables T and M are combined with the constant M_0 into a single variable T_M, which is then defined as a function of H.

1.2.4 Mean Molecular Weight.—The mean molecular weight M of a mixture of gases is by definition

$$M = \frac{\Sigma(n_i\cdot M_i)}{\Sigma n_i} \tag{20}$$

where n_i and M_i are the number density and defined molecular weight, respectively, of the ith gas species. In that part of the atmosphere between the surface and about 80 km altitude, mixing is dominant, and the effect of diffusion and photochemical processes upon M is negligible. In this region the fractional composition of each species is assumed to remain constant at the defined value F_i, and M remains constant at its sea-level value M_0. For these conditions n_i is equal to the product of F_i times the total number density N, so that eq (20) may be rewritten as

$$M = M_0 = \frac{\Sigma[F_i\cdot N(Z)\cdot M_i]}{\Sigma[F_i\cdot N(Z)]} = \frac{\Sigma(F_i\cdot M_i)}{\Sigma F_i}. \tag{21}$$

The right-hand element of this equation results from the process of factoring $N(Z)$ out of each term of both the numerator and the denominator of the preceding fraction, so that, in spite of the altitude dependence of N, M is seen analytically to equal M_0 over the entire altitude region of complete mixing.

When the defined values of F_i and M_i (from table 3) are introduced into eq (21), M_0 is found to be 28.9644 kg/kmol. At 86 km (84.852 km′), however, the defined value of atomic-oxygen number density (8.6×10^{16} m^{-3}) is seen, in Appendix A, to lead to a value of M = 28.9522 kg/kmol, about 0.04 percent less than M_0. To produce a smooth transition from this value of M to M_0, the altitude profile of M has been arbitrarily defined at intervals of 0.5 km′ for altitudes between 79.006 and 84.852 km′, in terms of the ratio M/M_0 as given in table 8. These ratio values have been interpolated from those initially selected for intervals of 0.5 geometric kilometers between 80 and 86 km to satisfy the boundary conditions of $M = M_0$ = 28.9644 at 80 km, and M = 28.9522 at 86 km, and to satisfy a condition of smoothly decreasing first differences in M within the height interval 80 to 86 km.

These arbitrarily assigned values of M/M_0 may be used for correcting a number of parameters of this Standard if the tabulations are to correctly fit the model in the fifth and perhaps in the fourth significant figures within this height region. This after-the-fact correction is required because these values of M/M_0 were not included in the program used for computing the tables of this Standard below 86 km, and hence, the tabulations of some of the properties may show a discontinuity of up to 0.04 percent between 85.5 and 86 km. This situation exists particularly for four properties in addition to molecular weight, i.e., kinetic temperature, total number density, mean free path, and collision frequency. For these five parameters the discrepancy in the tables between 80 and 86 km can be readily remedied by a simple multiplication or division: tabulated values of M, T, and L must be multiplied by the corresponding values of M/M_0 from table 8; tabulated values of N and ν must be divided by the corresponding values of M/M_0.

Three other properties, dynamic viscosity, kinematic viscosity, and thermal conductivity, which are tabulated only for heights below 86 km, have similar discrepancies for heights immediately below 86 km. These values are not so simply corrected, however, because of the empirical nature of their respective defining functions. Rather, these quantities must be recalculated in terms of a suitably corrected set of values of T, if the precisely correct values are desired for geometric altitudes between 80 and 86 km.

Table 8.—Molecular-weight ratio geopotential and geometric altitudes in meters

H	Z	M/M_0	Z	H	M/M_0
79000	79994.1	1.000000	80000	79005.7	1.000000
79500	80506.9	0.999996	80500	79493.3	0.999996
80000	81019.6	0.999988	81000	79980.8	0.999989
80500	81532.5	0.999969	81500	80468.2	0.999971
81000	82045.4	0.999938	82000	80955.7	0.999941
81500	82558.6	0.999904	82500	81443.0	0.999909
82000	83071.5	0.999864	83000	81930.2	0.999870
82500	83584.8	0.999822	83500	82417.8	0.999829
83000	84098.0	0.999778	84000	82904.4	0.999786
83500	84611.4	0.999731	84500	83391.4	0.999741
84000	85124.8	0.999681	85000	83878.4	0.999694
84500	85638.4	0.999679	85500	84365.2	0.999641
			86000	84852.0	0.999579

1.2.5 Molecular-Scale Temperature vs. Geopotential Altitude 0.0 to 84.8520 Km—The molecular-scale temperature T_M (Minzner et al. 1958) at a point is defined as the product of the kinetic temperature T times the ratio M_0/M, where M is the mean molecular weight of air at that point, and M_0 (=28.9644 kg/kmol) is the sea-level value of M discussed above. Analytically,

$$T_M = T\cdot\frac{M_0}{M}. \tag{22}$$

When T is expressed in the Kelvin scale, T_M is also expressed in the Kelvin scale.

The principle virtue of the parameter T_M is that it combines the variable portion of M with the variable T into a single new variable, in a manner somewhat similar to the combining of the variable portion of g with Z to form the new variable H. When both of these transformations are introduced into (5), and when T_M is expressed as a linear function of H, the resulting differential equation has an exact integral. Under these conditions, the computation of P versus H becomes a simple process not requiring numerical integration. Traditionally, standard atmospheres have defined temperature as a linear function of height to eliminate the need for numerical integration in the computation of pressure versus height. This Standard follows the tradition to heights up to 86 km, and the function T_M versus H is expressed as a series of seven successive linear equations. The general form of these linear equations is

$$T_M = T_{M,b} + L_{M,b} \cdot (H - H_b) \qquad (23)$$

with the value of subscript b ranging from 0 to 6 in accordance with each of seven successive layers. The value of $T_{M,b}$ for the first layer ($b = 0$) is 288.15 K, identical to T_0, the sea-level value of T, since at this level $M = M_0$. With this value of $T_{M,b}$ defined, and the set of six values of H_b and the six corresponding values of $L_{M,b}$ defined in table 4, the function T_M of H is completely defined from the surface to 84.8520 km′ (86 km). A graph of this function is compared with the similar function of the 1962 Standard in figure 3. From the surface to the 51-km′ altitude, this profile is identical to that of the 1962 Standard. The profile from 51 to 84.8520 km′ was selected by Task Group I, and abbreviated tables of thermodynamic properties of the atmosphere based upon this profile were published by Kantor and Cole (1973).

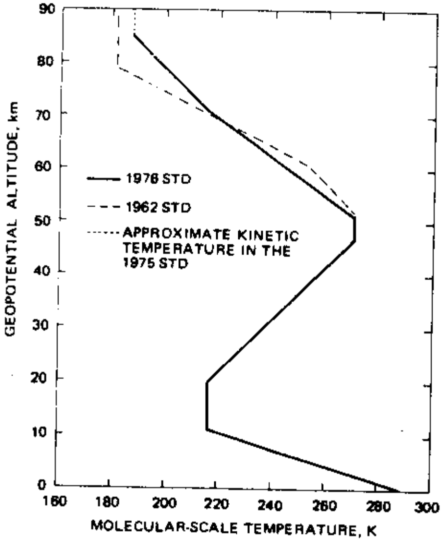

Figure 3. Molecular-scale temperature as a function of geopotential altitude

1.2.6 KINETIC TEMPERATURE VERSUS GEOMETRIC ALTITUDE, 0.0 TO 1000 km—Between the surface and 86-km altitude, kinetic temperature is based upon the defined values of T_M. In the lowest 80 kilometers of this region, where M is constant at M_0, T is equal to T_M in accordance with (22). Between 80 and 86 km, however, the ratio M/M_0 is assumed to decrease from 1.000000 to 0.9995788, as indicated in table 8, such that the values of T correspondingly decrease from those of T_M. Thus, at $Z_7 = 86$ km, a form of eq (22) shows that T_7 has a value 186.8673 K, i.e., 0.0787 K smaller than that of T_M at that height.

At heights above 86 km, values of T_M are no longer defined, and geopotential is no longer the primary argument. Instead, the temperature-altitude profile is defined in terms of four successive functions, each of which is specified in such a way that the first derivative of T with respect to Z is continuous over the entire altitude region, 86 to 1000 km. These four functions begin successively at the first four base heights, Z_b listed in table 5, and are designed to represent the following conditions:

A. An isothermal layer from 86 to 91 km
B. A layer in which $T(Z)$ has the form of an ellipse from 91 to 110 km;
C. A constant, positive-gradient layer from 110 to 120 km; and
D. A layer in which T increases exponentially toward an asymptote, as Z increases from 120 to 1000 km.

86 to 91 km

For the layer from $Z_7 = 86$ km to $Z_8 = 91$ km, the temperature-altitude function is defined to be isothermally linear with respect to geometric altitude, so that the gradient of T with respect to Z is zero (see table 5). Thus, the standard form of the linear function, which is

$$T = T_b + L_{K,b} \cdot (Z - Z_b) \qquad (24)$$

degenerates to

$$T = T_7 = 186.8673 \text{ K} \qquad (25)$$

and by definition

$$\frac{dT}{dZ} = 0.0 \text{ K/km}. \qquad (26)$$

The value of T_7 is derived from one version of eq (22) in which T_M is replaced by $T_{M,7}$, a value determined in 1.2.5 above, and in which M/M_0 is replaced by M_7/M_0 with a value of 0.9995788 in accordance with values of M_0 and M_7 discussed in 1.3.3 below. Since T is defined to be constant for the entire layer, Z_7 to Z_8, the temperature at Z_8 is $T_8 = T_7 = 186.8673$ K, and the gradient dT/dZ at Z_8 is $L_{K,8} = 0.0$ K/km, the same as for $L_{K,7}$.

91 to 110 km

For the layer $Z_8 = 91$ km to $Z_9 = 110$ km, the temperature-altitude function is defined to be a segment of an ellipse expressed by

$$T = T_c + A \cdot \left[1 - \left(\frac{Z - Z_8}{a} \right)^2 \right]^{1/2} \qquad (27)$$

where
$T_c = 263.1905$ K, $A = -76.3232$ K, $a = -19.9429$ km, and Z is limited to values from 91 to 110 km.

Eq (27) is derived in Appendix B from the basic equation for an ellipse, to meet the values of T_8 and $L_{K,8}$ derived above, as well as the defined values $T_9 = 240.0$ K, and $L_{K,9} = 12.0$ K/km, for $Z_9 = 110$ km.

The expression for dT/dZ related to eq (27) is

$$\frac{dT}{dZ} = \frac{-A}{a} \cdot \left(\frac{Z - Z_8}{a} \right) \cdot \left[1 - \left(\frac{Z - Z_8}{a} \right)^2 \right]^{-1/2}. \qquad (28)$$

110 to 120 km

For the layer $Z_9 = 110$ km to $Z_{10} = 120$ km, $T(Z)$ has the form of (24), where subscript b is 9, such that T_b *and* $L_{K,b}$ are, respectively, the defined quantities T_9 and $L_{K,9}$, while Z is limited to the range 110 to 120 km. Thus,

$$T = T_9 + L_{K,9} \, (Z - Z_9) \qquad (29)$$

and

$$\frac{dT}{dZ} = L_{K,9} = 12.0 \text{ K/km}. \qquad (30)$$

Since dT/dZ is constant over the entire layer, $L_{K,10}$, the value of dT/dZ at Z_{10}, is identical to $L_{K,9}$, i.e., 12.0 K/km, while the value of T_{10} at Z_{10} is found from eq (29) to be 360.0 K.

120 to 1000 km

For the layer $Z_{10} = 120$ to $Z_{12} = 1000$ km, $T(Z)$ is defined to have the exponential form (Walker 1965)

$$T = T_\infty - (T_\infty - T_{10}) \cdot \exp\,(-\lambda\xi) \qquad (31)$$

such that

$$\frac{dT}{dZ} = \lambda \cdot (T_\infty - T_{10}) \cdot \left(\frac{r_0 + Z_{10}}{r_0 + Z} \right)^2 \cdot \exp\,(-\lambda \cdot \xi) \qquad (32)$$

where

$\lambda = L_{K,9}/(T_\infty - T_{10}) = 0.01875$, and
$\xi = \xi\,(Z) = (Z - Z_{10}) \cdot (r_0 + Z_{10})/(r_0 + Z)$.

In the above expressions, T_∞ equals the defined value 1000 K. A graph of T versus Z from 0.0 to 1000 km altitude is given in figure 4. The upper portion of this profile was selected by Task Group III to be consistent with satellite drag data (Jacchia 1971), while the mid-portion, particularly between 86 and 200 km and the overlap to 450 km was selected by Task Group II (Minzner et al. 1974) to be consistent with observed temperature and satellite observations of composition data (Hedin et al. 1972).

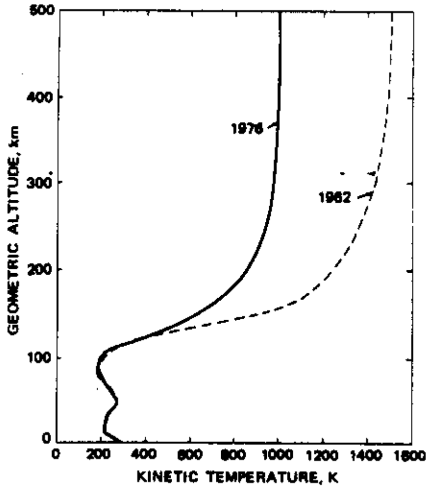

Figure 4. Kinetic temperature as a function of geometric altitude

1.3 COMPUTATIONAL EQUATIONS

The tables of this Standard have been computed in two height regions, 0 to 84.852 km′ (86 km), and 86 to 1000 km, because the computations for each region are based on compatible but different sets of initial conditions. These two different sets of initial conditions lead to two different computational procedures. Consequently, the following discussion of computational equations, which is presented according to a series of atmospheric parameters, does not necessarily follow in the order in which the calculation is actually performed for each altitude region. The equations used for computing the various properties of the atmosphere for altitudes below 86 km are, with certain noted exceptions, equivalent to those used in the 1962 standard, and the various equations involving T_M came from expressions used in the ARDC Model Atmosphere, 1956 (Minzner and Ripley 1956).

1.3.1 PRESSURE.—Three different equations are used for computing pressure P in various height regimes of this Standard. One of these equations applies to heights above 86 km, while the other two apply to the height regime from the surface up to 86 km, within which the argument of the computation is geopotential. Consequently, expressions for computing pressure as a function of geopotential altitude stem from the integration of eq (5) after replacing $g \cdot dZ$ by it equivalent $g_0' \, dH$ from eq (16), and after replacing the ratio M/T by its equivalent, M_0/T_M in accordance with eq (22). Two forms result from this integration, one for the case when $L_{M,b}$ for a particular layer is not equal to zero, and the other when the value $L_{M,b}$ is zero. The first of these two expressions is

$$P = P_b \cdot \left[\frac{T_{M,b}}{T_{M,b} + L_{M,b} \cdot (H - H_b)} \right]^{\left[\frac{g_0' \cdot M_0}{R^* \cdot L_{M,b}} \right]} \tag{33a}$$

and the latter is

$$P = P_b \cdot \exp \left[\frac{- g_0' \cdot M_0 \, (H - H_b)}{R^* \cdot T_{M,b}} \right]. \tag{33b}$$

In these equations g_0', M_0, and R^* are each defined single-valued constants, while $L_{M,b}$ and H_b are each defined multi-valued constants in accordance with the value of b as indicated in table 4. The quantity $T_{M,b}$ is a multi-valued constant dependent on $L_{M,b}$ and H_b. The reference-level value for P_b for $b = 0$ is the defined sea-level value, $P_0 = 101325.0$ N/m² (1013.250 mb). Values of P_b for $b = 1$ through $b = 6$ are obtained from the application of the appropriate member of the pair eq (33a) and (33b) for the case when $H = H_{b+1}$.

These two equations yield the pressure for any desired geopotential altitude from sea level to H_7, where H_7 is the geopotential altitude corresponding to the geometric altitude $Z_7 = 86$ km. Pressures for H from 0 to -5 km′ may also be computed from eq (33a) when subscript b is zero.

For Z equal to 86 km and above, the value of pressure is computed as a function of geometric altitude Z, and involves the altitude profile of kinetic temperature T rather than that of T_M, in an expression in which the total pressure P is equal to the sum of the partial pressures for the individual species as expressed by eq (3). Thus, for $Z = 86$ to 1000 km,

$$P = \Sigma P_i = \Sigma n_i \cdot k \cdot T = \frac{\Sigma n_i \cdot R^* \cdot T}{N_A}. \tag{33c}$$

In this expression

k = the Boltzmann constant, defined in table 2a,

T = $T(Z)$ defined in eq (25), (27), (29), and (31) for successive layers, and

Σn_i = the sum of the number densities of the individual gas species comprising the atmosphere at altitude Z above 86 km, as described below.

Neither n_i, the number densities of individual species, nor Σn_i, the sum of the individual number densities, is known directly. Consequently, pressures above 86 km cannot be computed without first determining n_i for each of the significant gas species.

1.3.2 NUMBER DENSITY OF INDIVIDUAL SPECIES.—The values of n_i, the number densities of individual species, have not been presented in the detailed tables of this Standard for low altitudes where it is assumed that complete mixing keeps F_i, the fractional concentrations of the individual species, at the sea-level value. For altitudes below 80 km, the altitude profile of number density for any particular major species i is equal to F_i times the altitude profile of the total number density N, a quantity which is tabulated in this Standard, in accordance with eq (41) below. Thus, for $Z < 80$ km,

$$n_i = F_i \cdot N \tag{34}$$

where values of F_i for the various species are defined in table 3. For altitudes between 80 and 86 km, the value of n_i determined by eq (34) and the tabulated values of N will need to be increased by the factor M_0/M to be rigorously correct in accordance with the discussion in 1.2.4. At altitudes above 86 km, however, the model assumes the existence of various processes which lead to particular differing height variations in the number-density values of several individual species, N_2, O, O_2, Ar, He, and H, each governed by eq (6). Ideally, the set of equations eq (6), each member of which is associated with a particular species, should be solved simultaneously, since the number densities of all the species are coupled through the expressions for molecular diffusion which are included in eq (6). Such a solution would require an inordinate amount of computation, however, and a simpler approach was desired. This was achieved with negligible loss of validity by some simplifying approximations, and by calculating the number densities of individual species one at a time in the order $n(N_2)$, n (O), $n(O_2)$, n(Ar), n (He), and n(H). For all species except hydrogen (which is discussed in the section on atomic hydrogen) we divide eq (6) by n_i, and integrate directly to obtain the following set of simultaneous equations, one for each gas species:

$$n_i = n_{i,7} \cdot \frac{T_7}{T} \cdot \exp \left\{ - \int_{Z_7}^{Z} \left[f(Z) + \left(\frac{v_i}{D_i + K} \right) \right] dZ \right\}. \tag{35}$$

In this set of equations

$n_{i,7}$ = the set of species-dependent, number-density values for $Z = Z_7 = 86$ km, one member for each of the five designated species, as derived in Appendix A and listed in table 9,

T_7 = 186.8673 K, the value of T at Z_7, as specified in eq (25),

T = $T(Z)$ defined in eq (25), (27), (29), and (31) for the appropriate altitude regions.

$f(Z)$ = the function written as eq (36) below, and

$\dfrac{v_i}{D_i + K}$ = the set of empirical functions written as eq (37) below:

For $f(Z)$ we have

$$f(Z) = \frac{g}{R^* \cdot T} \cdot \left(\frac{D_i}{D_i + K} \right) \cdot \left[M_i + \frac{M \cdot K}{D_i} + \frac{\alpha_i \cdot R^*}{g} \cdot \frac{dT}{dZ} \right] \tag{36}$$

TABLE 9.—Number densities of various species at 86-km altitude

Species	Number density (m^{-3})
N_2	1.129794×10^{20}
O	8.6×10^{16}
O_2	3.030898×10^{19}
Ar	1.351400×10^{18}
He	7.5817×10^{14}

where

D_i = $D_i(Z)$ as defined by eq (8) for the ith species,

K = $K(Z)$ as defined by eq (7a), (7b), and (7c),

M_i = the molecular weight of the ith species as defined in table 3,

α_i = the thermal diffusion coefficient for the ith species as defined in table 4,

dT/dZ = one of eq (26), (28), (30), or (32), as appropriate to the altitude region, and

M = $M(Z)$ with special considerations mentioned below.

For $[v_i/(D_i + K)]$ we have the following set of empirical expressions.

$$\frac{v_i}{D_i + K} = Q_i \cdot (Z - U_i)^2 \cdot \exp [- W_i \cdot (Z - U_i)^3] + q_i \cdot (u_i - Z)^2 \cdot \exp [- w_i \cdot (u_i - Z)^3]. \tag{37}$$

This set of equations, while representing a function of both D_i and K, involves a series of six other coefficients which, for each of four species, have been empirically selected to adjust the number-density profile of the related species to particular values in agreement with observations. The defined values of the six sets of species-dependent coefficients, Q_i, q_i, U_i, u_i, W_i, and w_i used in eq (37) are listed in table 7. The values of q_i and U_i were selected so that for O_2, Ar, and He, the quantity $v_i/(D_i + K)$ becomes zero at exactly 86 km. For atomic oxygen, however, all six of these coefficients contribute to maximizing this quantity for $Z = 86$ km.

Molecular Nitrogen.—Molecular nitrogen (N_2) is the first species for which n is calculated since, on the average, the distribution of N_2 is close to that for static equilibrium, and hence, for this species, we may neglect the transport velocity, thereby eliminating the term $[v_i/(D_i + K)]$ from that version of eq (35) applying to N_2. This species is dominant up to and above the turbopause, and its molecular weight is close to the mean molecular weight in the lower thermosphere, where mixing still dominates the distribution process. We approximate the effect of mixing up to 100 km by two additional adjustments to eq (35), both adjustments implicit in $f(Z)$; i.e., neglecting K and replacing M_i by the mean molecular weight M which, for the altitude region, 86 to 100 km, is approximated by M_0. With these three adjustments, that version of eq (35) applying to N_2 reduces to

$$n(N_2) = n(N_2)_7 \cdot \frac{T_7}{T} \cdot \exp \left\{ - \int_{Z_7}^{Z} \frac{M \cdot g}{R^* \cdot T} \cdot dZ \right\} \tag{38}$$

where

$$M = M_0 \text{ for } Z \leq 100 \text{ km, and}$$
$$M = M(N_2) \text{ for } Z > 100 \text{ km.}$$

Figure 5 shows a graph of $n(N_2)$ versus Z.

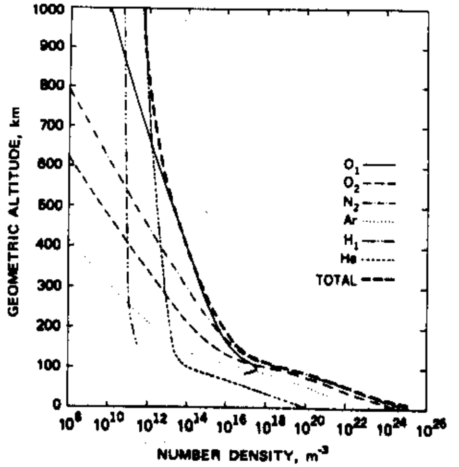

Figure 5. Number density of individual species and total number density as a function of geometric altitude

The species O, O_2, Ar, *and* He.—As noted above, after the calculation of $n(N_2)$ has been performed, the values of n_i for the next four species are calculated from eq (35) in the order O, O_2, Ar, and He.

In the case of O and O_2, the problem of mutual diffusion is simplified by considering N_2 as the stationary background gas (as described in the previous section). For Ar and He, which are minor constituents in the lower thermosphere, it is more realistic to use the sum of the number densities of N_2, O, and O_2 as the background gas in evaluating the molecular-diffusion coefficient D_i, and the mean-molecular weight M, except below 100 km, where M is taken to be the sea-level value M_0. This latter choice is to maintain consistency with the method for calculating $n(N_2)$.

In eq (37), defining $[v_i \ (D_i + K)]$, the coefficients Q_i, q_i, U_i, u_i, W_i, and w_i, which, except for q_i are constant for a particular species, are each adjusted such that appropriate densities are obtained at 450 km for O and He, and at 150 km for O, O_2, He, and Ar. The constant q_i, and hence the second term of eq (37) is zero for all species except atomic oxygen, and is also zero for atomic oxygen above 97 km; the extra term for atomic oxygen is needed below 97 km to generate a maximum in the density-height profile at the selected height of 97 km. This maximum results from the increased loss of atomic oxygen by recombination at lower altitudes. The flux terms for O and O_2 are based on, and lead (qualitatively) to the same results as those derived from the much more detailed calculations by Colegrove et al. (1965), and Keneshea and Zimmerman (1970).

A further computational simplification is realized above 115 km where the eddy diffusion coefficient becomes zero. For these altitudes, the set of equations represented by eq (35) becomes uncoupled, and each member reduces to the sum of the barometric equation for the particular species plus the thermal-diffusion term and the velocity term. In the case of O, O_2, and Ar, the thermal diffusion term is zero. Also, as will be seen in Part 2, the velocity term $[v_i \ (D_i + K)]$, becomes small above 120 km and, with the exception of atomic hydrogen, each species considered is nearly in diffusive equilibrium at these heights. For the present model, however, this situation becomes exactly true only at altitudes above 150 km.

The altitude profile of number density for each of the species O, O_2, Ar, and He is given in figure 5, along with that for N_2.

Atomic Hydrogen.—For various reasons, the height distribution of the number density of atomic hydrogen $n(H)$ is defined only for heights from 150 to 1000 km. Below 150 km, the concentration of H is negligible compared with the concentrations of O, O_2, Ar, and He. The defining expression for $n(H)$, like the expression for $n(N_2)$, $n(O)$, etc., is derived from eq (6). The solution for $n(H)$, however, is expressed in terms of the vertical flux $n(H) \cdot v(H)$, represented by ϕ, rather than in terms of $v(H)$, because it is the flux which is considered known for H. In this model only that contribution to ϕ due to planetary escape from the exosphere is considered.

Since K is zero, for the altitude region of interest, the particular version of eq (6) applied to H is correspondingly simplified, and one possible solution to the resulting expression is

$$n(H) = \left[n(H)_{11} - \int_{Z_{11}}^{Z} \frac{\phi}{D(H)} \cdot \left(\frac{T}{T_{11}}\right)^{1+\alpha(H)} \cdot (\exp \tau) \cdot dZ \right] \cdot \left(\frac{T_{11}}{T}\right)^{1+\alpha(H)} \cdot (\exp - \tau) \quad (39)$$

where

$n(H)_{11}$ = $8.0 \times 10^{10} \ m^{-3}$, the number density of H at $Z_{11} = 500$ km, as defined in table 2,

$D(H)$ = The molecular diffusion coefficient for hydrogen given by eq (8) in which the values of a_i and b_i for hydrogen are as defined in table 6,

ϕ = $7.2 \times 10^{11} \ m^{-2} \cdot s^{-1}$, the vertical flux of H, as defined in table 2,

T = $T(Z)$ as specified by eq (31),

T_{11} = 999.2356 K, the temperature derived from eq (31) for $z = Z_{11}$,

$\alpha(H)$ = The thermal diffusion coefficient for H, −0.25 (dimensionless), as defined in table 6, and

τ = $\tau(Z)$ defined in eq (40) below.

$$\tau = \int_{Z_{11}}^{Z} \frac{g \cdot M(H)}{R^* \cdot T} \cdot dZ. \quad (40)$$

Because $D(H)$ becomes very large compared with ϕ for heights above 500 km, the value of the integral term in eq (39) can be neglected at these heights, and atomic hydrogen is then essentially in diffusive equilibrium. Figure 5 depicts the graph of $n(H)$ as a function of Z.

Eq (35) through (39) permit the calculation of the number densities of the species N_2, O, O_2, Ar, He, and H, for heights above 150 km, and of the first five of these species for heights between 86 and 150 km, where $n(H)$ is insignificant compared with $n(N_2)$. These number densities permit the calculation of several atmospheric parameters in the height region 86 to 1000 km. The first is mean molecular weight using eq (20). These values of M, along with those implicit in table 8, for Z from 80 to 86 km, plus the invariant value, M_0, for heights from 0 to 80 km, are shown in figure 6.

The number densities of the several species also permit us now to compute total pressure for heights

from 86 to 1000 km, using eq (33c). Figure 7 depicts these values as well as those for heights below 86 km computed from eq (33a) and (33b). Finally, these individual number densities permit the calculation of total number density, $N = \Sigma n_i$, at least at heights of 86 to 1000 km.

1.3.3. TOTAL NUMBER DENSITY.—From eq (2), (22), and (33c) it is apparent that total number density N, the number of neutral atmospheric gas particles per unit volume of the atmosphere may be expressed in any one of the three equivalent forms following:

$$N = \frac{M_0 \cdot N_A \cdot P}{T_M \cdot R^* \cdot M} = \frac{N_A \cdot P}{R^* \cdot T} = \Sigma n_i. \quad (41)$$

The three forms are selected to satisfy three types of calculations: (*a*) Those depending upon values of T_M, (*b*) Those depending upon values of T, and (*c*) Those depending upon values of Σn_i. This format will be followed in specifying the computational equations, insofar as possible, for the balance of the quantities discussed in this section. A graph of the altitude variation of N is presented in figure 5, along with the number densities of individual species. That portion of (41) involving T_M is of particular interest in calculating N for heights from 0 to 86 km.

1.3.4 MASS DENSITY.—From eq (1), (22), and (33c) one may write the following three forms of computational equations for mass density ρ:

$$\rho = \frac{P \cdot M_0}{R^* \cdot T_M} = \frac{P \cdot M}{R^* \cdot T} = \frac{\Sigma (n_i \cdot M_i)}{N_A}. \quad (42)$$

The altitude-dependent variations of this quantity are shown in figure 7 along with those of pressure.

1.3.5 MOLE VOLUME.—Mole volume v_m of air is defined as the volume of one mole of air, where one mole of air is the amount consisting of a number of neutral particles equal to N_A. In SI units, the quantity v_m should specify the number of cubic meters containing one kilomole of air. Since M has the dimensions of kg/kmol, and ρ has the dimensions of kg/m^3, the ratio M/ρ, with the units m^3/kmol, provides the definition of mole volume. In terms of eq (1), (22), and (33c), this ratio may be equated to the following series of expressions:

$$v_m = \frac{R^* \cdot M \cdot T_M}{M_0 \cdot P} = \frac{R^* \cdot T}{P} = \frac{N_A}{\Sigma n_i} \quad (43)$$

This quantity, while not tabulated in this Standard, is shown graphically in figure 8.

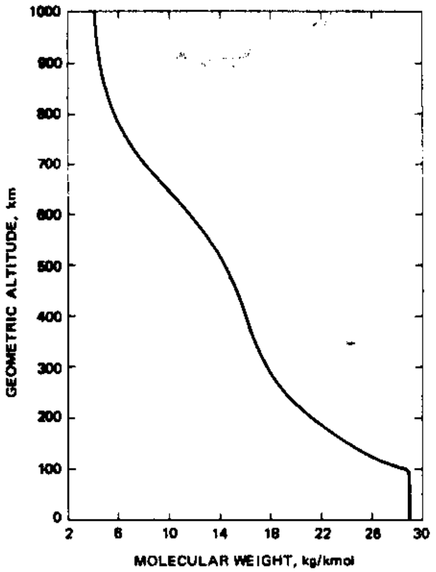

Figure 6. Mean molecular weight as a function of geometric altitude

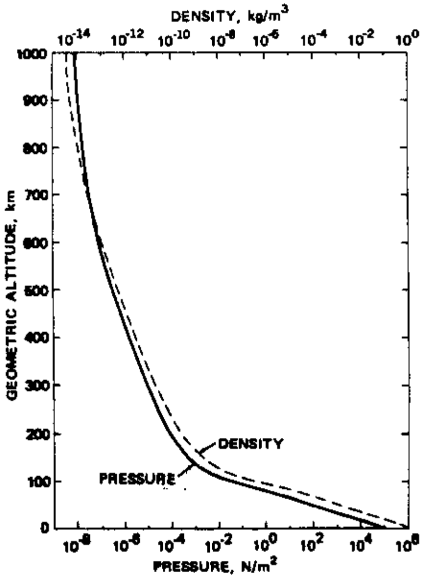

Figure 7. Total pressure and mass density as a function of geometric altitude

1.3.6 SCALE HEIGHT

Pressure Scale Height.—The quantity H_P $R^* \cdot T$ $(g \cdot M)$, which has dimensions of length, is a quantity commonly associated with the concept of scale height, and is the defining form of pressure scale height H_P used in this model, such that with eq (20) and (22) we may write

$$H_P = \frac{R^* \cdot T_M}{g \cdot M_0} = \frac{R^* \cdot T}{g \cdot M} = \frac{R^* \cdot T \cdot \Sigma n_i}{g \cdot \Sigma(n_i \cdot N_i)} \cdot \quad (44)$$

The reciprocal of this quantity, which appears on the right-hand side of eq (5) is seen to equal the slope of the function $\ln P$ versus Z at height Z in the regions where hydrostatic equilibrium or diffusive equilibrium holds. In the present model, this condition is true for heights below 80 km (complete mixing) and essentially true above approximately 120 km, where diffusive equilibrium is nearly satisfied and where each individual species is governed by eq (4).

In the region 80 to 120 km, where the transition from a completely mixed atmosphere to one in diffusive equilibrium takes place, the situation is complicated by the competition between three processes: molecular diffusion, eddy diffusion, and dissociation of molecular oxygen. These processes result in a vertical transport, such that eq (4) and (5) are no longer exactly true in this 40-km layer. Since molecular nitrogen is the dominant species in this altitude range, however and since this species has a zero transport velocity in this model, the pressure scale height is still a good indicator of the rate of change of the pressure in this height region.

It should be noted that eq (4) and (5) also become invalid at very high altitudes (the exospheric region) due to the infrequent collisions between neutral particles. Thus, in this region, the significance of H_P as a measure of $d\ln P$ dZ again loses validity.

In eq (44) both g and T_M or all three of g, T, and M are functions of Z, such that H_P is the local value of geometric pressure scale height.

This quantity, which is the particular scale height tabulated in this Standard, and which is plotted in figure 9, is frequently but incorrectly associated with the altitude increment over which the pressure decreases by exactly a factor of 1/e. The conditions necessary for the pressure to decrease by exactly that factor over an altitude increase of a single pressure scale height, would be for the variables T, g, and M all to remain constant over that altitude interval. Since g may never be constant over any altitude interval, this particular concept of pressure decrease can rarely if ever apply exactly to H_P.

Density Scale Height.—Because of the relationship between H_P and the slope of $\ln P$ versus Z, it is convenient to apply the name geometric density scale height H_ρ to the negative reciprocal of the slope of $\ln\rho$ versus Z. Using the equation of state to relate $d\ln\rho$ *to* $d\ln P$, one may define

$$H_\rho = \frac{H_P}{1 + H_P \cdot \left(\frac{d\ln T_M}{dZ}\right)} = \frac{H_P}{1 + H_P \cdot \left(\frac{d\ln T}{dZ} - \frac{d\ln M}{dZ}\right)} \quad (45)$$

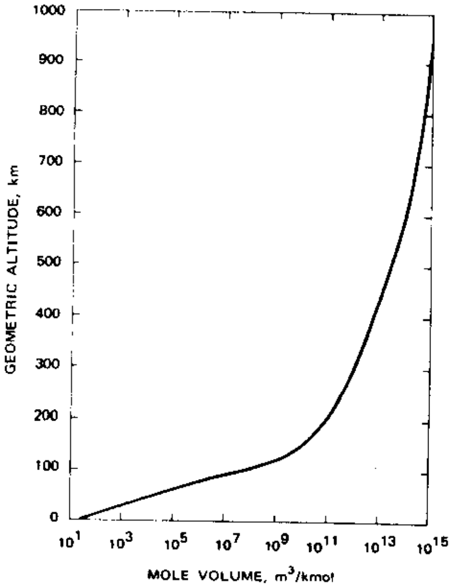

Figure 8. Mole volume as a function of geometric altitude

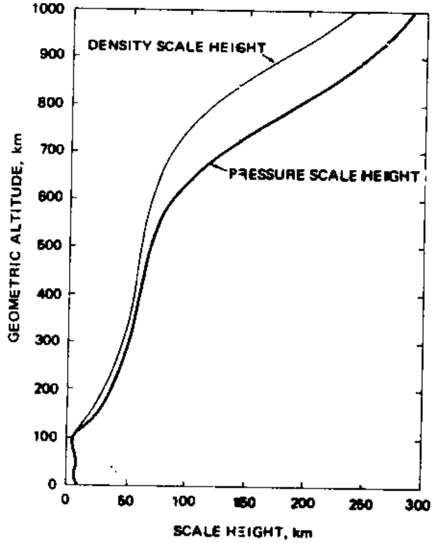

Figure 9. Pressure scale height and density scale height as a function of geometric altitude

The relationships implied between H_ρ and $d\ln\rho/dZ$ are subject to the same limitations as those between H_P and $d\ln P/dZ$ expressed above, i.e., H_ρ is only an approximation to $(d\ln\rho/dZ)^{-1}$ between 80 and 120 km, and in the exosphere, where the approximation becomes less valid with increasing altitude.

Within these limitations it is apparent that in layers where T_M does not change with changing altitude i.e., where $(d\ln T_M/dZ) = 0$, H_ρ is equal to H_P. Within such layers, the slope of $\ln\rho$ versus Z, at any particular altitude Z, is identical to the slope of $\ln P$ versus Z.

While density scale height is not tabulated in this Standard, values of this quantity are shown graphically with H_P in figure 9.

1.3.7 MEAN AIR-PARTICLE SPEED.—The mean air-particle speed V is the arithmetic average of the speeds of all air particles in the volume element being considered. All particles are considered to be neutral. For a valid average to occur, there must, of course, be a sufficient number of particles involved to represent mean conditions. Pressure and temperature gradients within the volume must also be negligible. The analytical expression for V is closely related to that for the speed of sound, and is proportional to the ratio T/M. Thus, in terms of eq (20) and (22), we write

$$V = \left[\frac{8 \cdot R^* \cdot T_M}{\pi \cdot M_0}\right]^{1/2} = \left[\frac{8 \cdot R^* \cdot T}{\pi \cdot M}\right]^{1/2} = \left[\frac{8 \cdot R^* \cdot T \cdot \Sigma n_i}{\pi \cdot \Sigma(n_i \cdot M_i)}\right]^{1/2}. \quad (46)$$

The variation of particle speed with geometric altitude is shown in figure 10.

1.3.8 MEAN FREE PATH.—The mean free path L is the mean value of the distances traveled by each of the neutral particles, in a selected volume, between successive collisions with other particles in that volume. As in the case of V, a meaningful average requires that the selected volume be big enough to contain a large number of particles. The computational form for L is

$$L = \frac{2^{1/2} \cdot R^* \cdot M \cdot T_M}{2\pi \cdot N_A \cdot \sigma^2 \cdot M_0 \cdot P} = \frac{2^{1/2} \cdot R^* \cdot T}{2\pi \cdot N_A \cdot \sigma^2 \cdot P} = \frac{2^{1/2}}{2\pi \cdot \sigma^2 \cdot \Sigma n_i} \quad (47)$$

where σ is the effective collision diameter of the mean air molecules. The adopted value of σ, i.e., 3.65×10^{-10} m is suitable for that part of the atmosphere below about 86 km, which is dominated by N_2 and O_2. Above this height, the value of σ, which depends upon composition in a complicated manner, begins to change significantly so that tabulations with four significant figures are no longer valid. At great altitudes, this expression for L is valid only under assumptions that hold M, T_M, P, and σ constant throughout the volume used. Figure 11 depicts the mean free path in terms of altitude.

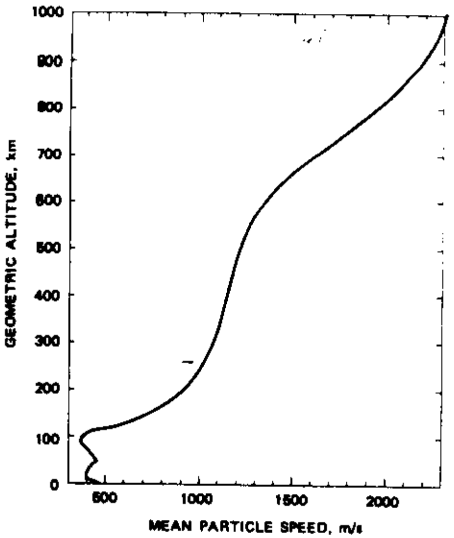

Figure 10. Mean air-particle speed as a function of geometric altitude

1.3.9 MEAN COLLISION FREQUENCY.—The mean collision frequency ν is the average speed of the air particles within a selected volume divided by the mean free path L of the particles within that volume. That is,

$$\nu = \frac{V}{L} \quad (48)$$

and in computational form:

$$\nu = 4N_A \cdot \sigma^2 \cdot \left[\frac{\pi \cdot M_0 \cdot P^2}{R^* \cdot M^2 \cdot T_M}\right]^{1/2} = 4N_A \cdot \sigma^2 \cdot \left[\frac{\pi \cdot P^2}{R^* \cdot M \cdot T}\right]^{1/2} = 4N_A \cdot \sigma^2 \cdot \left[\frac{\pi \cdot P^2 \cdot \Sigma n_i}{R^* \cdot T \cdot \Sigma(n_i \cdot M_i)}\right]^{1/2}. \quad (49)$$

Note that σ is again involved in this quantity, and hence ν has limitations similar to those of mean free path. The foregoing expressions are taken to apply to neutral particles only, since no considerations involving charged particles are introduced for purposes of developing the tables and graphs of this standard.

Figure 12 graphically displays the variation of collision frequency with altitude. See section 1.3.7 for a discussion of the assumptions under which eq (49) is valid at great altitudes.

1.3.10 Speed of Sound.—The expression adopted for the speed of sound C_s is

$$C_s = \left(\frac{\gamma \cdot R^* \cdot T_M}{M_0}\right)^{1/2} \qquad (50)$$

where γ is the ratio of specific heat of air at constant pressure to that at constant volume; and is taken to be 1.40 exact (dimensionless), as defined in table 2. Eq (50) for speed of sound applies only when the sound wave is a small perturbation on the ambient condition. Calculated values for C_s have been found to vary slightly from experimentally determined values.

The limitations of the concept of speed of sound due to extreme attenuation are also of concern. The attenuation which exists at sea level for high frequencies applies to successively lower frequencies as atmospheric pressure decreases, or as the mean free path increases. For this reason, the concept of speed of sound (except for frequencies approaching zero) progressively loses its range of applicability at high altitudes. Hence, the main tables listing the values for speed of sound terminate at 86 km. Figure 13 shows the variation with altitude of the computed speed of sound.

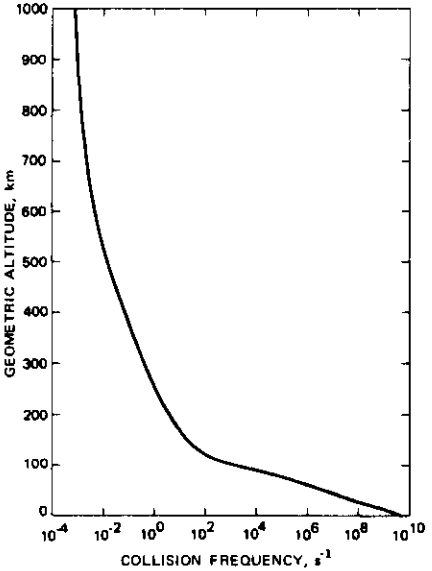

Figure 12. Collision frequency as a function of geometric altitude

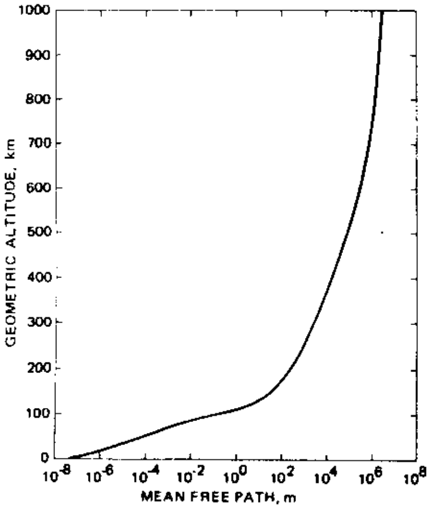

Figure 11. Mean free path as a function of geometric altitude

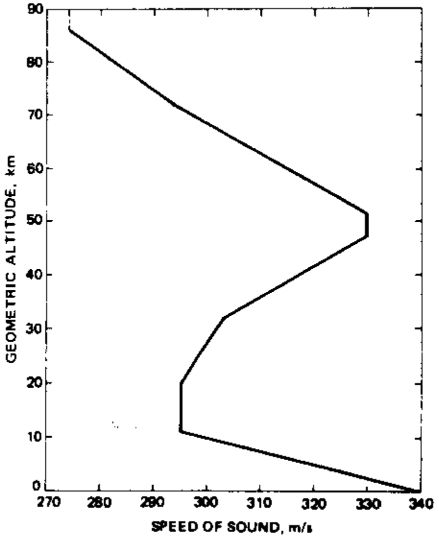

Figure 13. Speed of sound as a function of geometric altitude

1.3.11 Dynamic Viscosity.—The coefficient of dynamic viscosity μ (N · s m^{-2}) is defined as a coefficient of internal friction developed where gas regions move adjacent to each other at different velocities. The following expression, basically from kinetic theory, but with constants derived from experiment, is used for computation of the tables:

$$\mu = \frac{\beta \cdot T^{3/2}}{T + S}. \qquad (51)$$

In this equation β is a constant equal to 1.458×10^{-6} kg/(s · m · K$^{1/2}$) and S is Sutherland's constant, equal to 110.4 K, both defined in table 2B. Because of the empirical nature of this equation, no attempt has been made to transform it into one involving T_M.

Eq (51) fails for conditions of very high and very low temperatures, and under conditions occurring at great altitudes. Consequently, tabular entries for coefficient of dynamic viscosity are terminated at 86 km. For these reasons caution is necessary in making measurements involving probes and other objects which are small with respect to the mean free path of molecules particularly in the region of 32 to 86 km.

The variation of dynamic viscosity with altitude is shown in figure 14.

1.3.12 Kinematic Viscosity.—Kinematic viscosity η is defined as the ratio of the dynamic viscosity of a gas to the density of that gas; that is,

$$\eta = \frac{\mu}{\rho}. \qquad (52)$$

Limitations of this equation are comparable to those associated with dynamic viscosity, and consequently tabular entries of kinematic viscosity are also terminated at the 86-km level. See figure 15 for a graphical representation of the variation of kinematic viscosity with altitude.

1.3.13 Coefficient of Thermal Conductivity.—The empirical expression adopted for purposes of developing tabular values of the coefficient of thermal conductivity k_t for heights up to the 86-km level is as follows:

$$k_t = \frac{2.64638 \times 10^{-3} \cdot T^{3/2}}{T + 245.4 \times 10^{-(12/T)}} \qquad (53)$$

This expression differs from that used in the U. S. Standard Atmosphere, 1962 in that the numerical constant has been adjusted to accommodate a conversion of the related energy unit from the temperature-dependent kilogram calorie to the invariant joule. Thus, the values of k_t in units of J/(m · s · K) or W/(m · K) are greater than the values of k_t in units of kcal/(m · s · K) by a factor of exactly 4.18680×10^3, when the kilocalorie is assumed to be the one for 15°C. Kinetic-theory de-

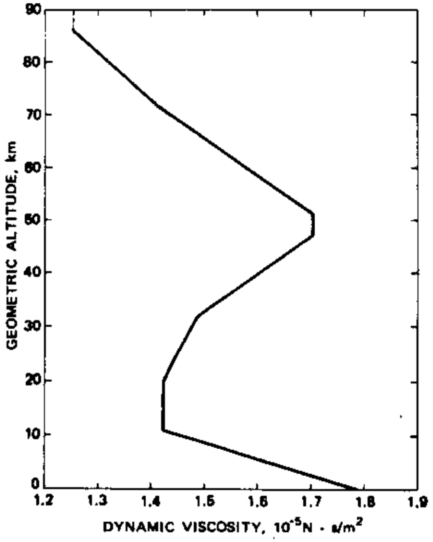

Figure 14. Dynamic viscosity as a function of geometric altitude

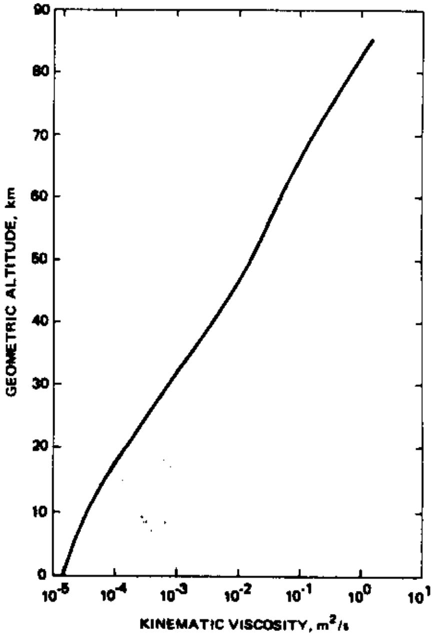

Figure 15. Kinematic viscosity as a function of geometric altitude

terminations of thermal conductivity of some monatomic gases agree well with observation. For these gases thermal conductivity is directly proportional to the dynamic viscosity. Modification of the simple theory has accounted in part for differences introduced by polyatomic molecules and by mixtures of gases. Tabular entry of values for coefficient of thermal conductivity is terminated at 86 km. The variation with height of this quantity is shown in figure 16.

1.4 SELECTED TABULAR VALUES OF ATMOSPHERIC PROPERTIES AND CONVERSION FACTORS FOR METRIC TO ENGLISH UNITS

1.4.1 SEA-LEVEL VALUES.—The sea-level values of fifteen of the atmospheric properties discussed in this Standard are listed in table 10. The sea-level values for g, P, and T are defined quantities; the remainder are quantities calculated from the preceding equations.

1.4.2 CONVERSION OF METRIC TO ENGLISH UNITS.—For those who have a need to work in the English System of units, the conversion factors listed in table 11 are applicable to the atmospheric parameters tabulated or shown graphically in this Standard. For other transformations, see Mechtly (1973).

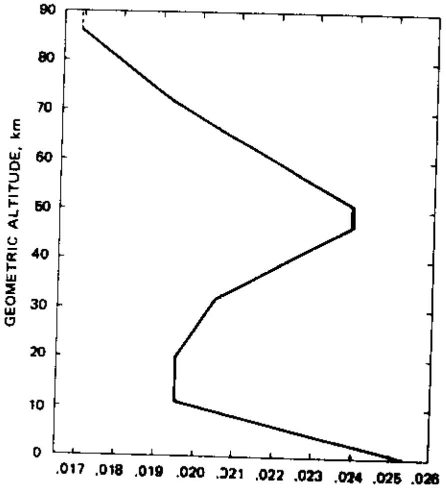

Figure 16. Coefficient of thermal conductivity as a function of geometric altitude

TABLE 10.—Seal-level values of atmospheric properties

Symbol	Sea-level value
$C_{s,0}$	3.40294×10^{2} m/s
g_0	9.80665 m/s²
$H_{P,0}$	8.4345×10^{3} m
$k_{t,0}$	2.5326×10^{-2} J/(s·m·K) or W/(m·K)
L_0	6.6328×10^{-8} m
$V_{m,0}$	2.3643×10^{1} m³/kmol
M_0	2.89644×10^{1} kg/kmol
N_0	2.5470×10^{25} m⁻³
P_0	1.01325×10^{5} N/m²
T_0	2.8815×10^{2} K
V_0	4.5894×10^{2} m/s
η_0	1.4607×10^{-5} m²/s
μ_0	1.7894×10^{-5} kg/(m·s)
ν_0	6.9193×10^{9} s⁻¹
ρ_0	1.2250 kg/m³

TABLE 11.—Metric to English conversion factors for properties of The U. S. Standard Atmosphere, 1976

Symbol	To convert from metric units	to English units	divide by
C_s	m/s	ft/s	$3.048^{*} \times 10^{-1}$
g	m/s²	ft/s²	$3.048^{*} \times 10^{-1}$
H_P	m	ft	$3.048^{*} \times 10^{-1}$
k_t	W/(m·K)	BTU/(ft·s·°R)	$6.226477504 \times 10^{-3}$
L	m	ft	$3.048^{*} \times 10^{-1}$
v_m	m³/kmol	ft³/lbmol	$6.242796057 \times 10^{-2}$
M	kg/kmol	lb/lbmol	1.000^{*}
N	m⁻³	ft⁻³	$3.531466672 \times 10^{1}$
P	mb	in Hg (32°F)	3.386389×10^{1}
T or T_M	K	°R	$5/9^{*}$
V	m/s	ft/s	$3.048^{*} \times 10^{-1}$
η	m²/s	ft²/s	$9.290304 \times 10^{-2*}$
μ	N·s/m²	lb/(ft·s)	1.488163944
ν	s⁻¹	s⁻¹	1.000^{*}
ρ	kg/m³	lb/ft³	1.6018463×10^{1}

*exact definition