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" TRANSIONOSPHERIC RADIOWAVE PROPAGATION "

presented by :

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Transionospheric Radiowave Propagation

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ABSTRACT

In this paper the fundamentals of transionospheric radiowave propagation are reviewed. The approximations which apply to the Appleton-Hartree derivation for refractive index are given and their range of applicability is described, along with some examples of actual satellite systems which make corrections for the effects of the ionosphere in navigation and ranging systems. Some problems are given which embody much of the information described in this lecture; the solutions to these problems are also given.

INTRODUCTION

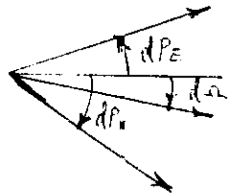
Propagation of radio waves through the earth's ionosphere has been important since the early days of rocket observations of the earth's upper atmosphere. With the launch of the first artificial earth satellites, many of the effects which the ionosphere has on radio waves which pass through it became apparent, even at frequencies much higher than those normally considered for high frequency ground to ground propagation. Today, almost thirty years after the launch of the first Sputnik, we still have satellite systems which have limitations to their accuracy due to the effects of the ionosphere. In the following sections the major effects are described which the ionosphere can have on radio waves which must pass through it.

FARADAY POLARIZATION ROTATION

The electric vector of a plane polarized wave can be represented by a vector having constant direction and sinusoidally varying amplitude:

$$\vec{E} = R_E \, e^{j(kz - \omega t)}$$

or by two vectors having constant amplitude and rotating continuously in opposite directions. If this plane polarized wave satisfies several approximations as it travels through the ionosphere it will undergo rotation of the resultant plane of polarization because the two counter rotating vectors will rotate at different velocities due to the bi-refringent property of the ionosphere.



$$dP_E + d\Omega = dP_o - d\Omega$$

$$d\Omega = \frac{dP_o - dP_E}{2}$$

The approximations which must be satisfied for simple Faraday rotation of linearly polarized waves to occur in the ionosphere are illustrated by considering the Appleton-Hartree expression for the phase refractive index of the ionosphere:

$$\mu^2 = 1 - \frac{X}{1 - jZ - \frac{Y_T^2}{2(1-X-jZ)} \pm \sqrt{\frac{Y_T^4}{4(1-X-jZ)^2} + Y_L^2}}$$

where $Z = \frac{\nu}{w}$, $X = \left(\frac{w_N}{w}\right)^2$, $Y = \frac{w_H}{w}$, $Y_T = Y\sin\theta$, $Y_L = Y\cos\theta$

$$w_N^2 = \frac{e^2 N}{\epsilon_0 m}$$

θ is the angle between the ray path and the earth's magnetic field. Typical values in the ionosphere are

$$w_H = 1.5\times10^6\ Hz,\ \nu < 10^4\ Hz,\ N \approx 10^6\ el/cm^3$$

For a satellite telemetry transmitter at 136 MHz, (until recently a commonly used satellite telemetry band),

$$w = 1.36\times10^8\ Hz,\ Z \ll 1,\ X \ll 1$$

The Appleton-Hartree expression for phase refractive index then becomes:

$$\mu^2 = 1 - \frac{2X}{2 \pm \sqrt{Y_T^4 + 4Y_L^2}}$$

where

$$Y_T = Y\sin\theta = \frac{1.5}{136}\sin\theta$$
$$Y_L = Y\cos\theta = \frac{1.5}{136}\cos\theta$$

For quasi-longitudinal propagation (Q.L.) $4Y_L^2 \gg Y_T^4$

$$\frac{4Y_L^2}{Y_T^4} \approx 100 = \frac{4(.011\cos\theta)^2}{(.011\sin\theta)^4}$$

This condition is satisfied for all values of θ) 87°. In other words, the propagation angle can be within $\gtrsim 3^\circ$ of transverse with respect to the magnetic field and the propagation of a linearly polarized radio wave through the ionosphere at 136 MHz will still be Q.L. (quasi-longitudinal).

If this condition is met, as it is for most viewing directions from any station on the earth, monitoring the polarization of linearly polarized radio waves from an artificial earth satellite, the refractive index reduces to:

$$\mu^2 = 1 - \frac{X}{1 \pm Y_L}$$

From the binomial expansion

$$\mu \approx 1 - \frac{X}{2(1 \pm Y_L)} \approx 1 - \frac{X}{2} \pm \frac{XY_L}{2}$$

where the - sign represents the refractive index of the ordinary ray and the + sign represents the refractive index of the extraordinay ray.

$$d\Omega = \frac{\pi}{\lambda}(\mu_o - \mu_E)\,dl\ \text{(radians)} = \frac{\pi}{\lambda}(XY_L)\,dl = \frac{Ne^3\mu_0 H\cos\theta\,dl}{2w^2 c\epsilon_0 m^2}$$

$$= \frac{K}{f^2}\int_{PATH} NB\cos\theta\,dl,\ \text{where } K = 2.36\times10^{-5},\ (B \text{ is in gammas})$$

It is justified to remove $B\cos\theta$ from under the integral and replace it with an average value $\overline{B\cos\theta}$, since this quantity changes very little over the height interval, where N changes by several orders of magnitude.

The quantity $B\cos\theta$ is a function only of the longitudinal component of the earth's magnetic field strength and the zenith angle of the satellite. The earth's magnetic field intensity, B, changes by less than approximately 1% even during a large magnetic storm and the variation of this quantity with height is generally less than 1% per 10 kilometers of height change of the ionosphere. Thus, any effect of removing it from under the integral sign is a second order one, in most cases. Then we have:

$$d\Omega = \frac{K}{f^2}\,\overline{B\cos\theta}\int_{PATH} N\,dl$$

In order to refer all measurements to an equivalent vertical electron column we define

$$\int_{VERTICAL} N\,dl = \int_{PATH} N\sec i\,dl \quad \text{or} \quad d\Omega = \frac{K}{f^2}\,\overline{B\cos\theta\sec i}\int_{\substack{EQUIVALENT\\VERTICAL}} N\,dh$$

$$\text{or} \int N\,dh = \frac{d\Omega\, f^2}{KM},\ \text{where } M = \overline{B\cos\theta\sec i}$$

This last form is the standard relationship as used by many workers to make measurements of the equivalent vertical electron content of the earth's ionosphere.

THE DIFFERENTIAL DOPPLER EFFECT IN THE EARTH'S IONOSPHERE

The Doppler shift of a radio wave traversing the ionosphere can be expressed as:

$$\Delta f = \frac{f_1}{c} v_r$$

where $v_r = \frac{\lambda}{2\pi}\frac{dP}{dt}$; that is, v_r is the linear rate of change of the phase path $P = \frac{2\pi}{\lambda}\int \mu\, dl$

$$\mu \simeq 1 - \frac{X}{2} \pm \frac{XY_L}{2}, \text{ but } \frac{XY_L}{2} << \frac{X}{2}, \text{ since } Y_L << 1.$$

$$\therefore v = \frac{f_1}{c}\frac{d}{dt}\int\left(1 - \frac{X}{2}\right) dl, \quad P = \Delta\phi_P = \frac{1}{2\lambda}\int X\, dl \text{ (cycles)}$$

$$\Delta f = \frac{f_1}{c}\frac{d}{dt}\left[\int dl - \int \frac{X}{2}\, dl\right]$$

$\uparrow$ GEOMETRIC TERM

$$\frac{1}{2}\int X\, dl = \frac{K}{2f^2}\int N\, dl$$

Thus, the Doppler shift, at velocities much lower than relativistic ones, consists of the usual geometric term, plus a term proportional to the time rate of change of the electron content, $\int N\, dl$ along the path.

Now, if two, coherently derived frequencies are used in a satellite system, the "free space" Doppler cancels out; that is assume two frequencies

f_2 and f_1, such that $f_2 = m * f_1$

$$\text{Defining } f_B = \frac{\Delta f_2}{m} - \Delta f_1 = \frac{d}{dt}\left[\frac{f_2}{mc}\int dl - \frac{f_2}{mc}\int \frac{X_2}{c}\, dl - \right.$$

$$\left.\frac{f_1}{c}\int dl - \frac{f_1}{2c}\int X_1\, dl\right] = \frac{f_1}{2c}\frac{d}{dt}\left[\int X_2\, dl - X_1\, dl\right] =$$

$$\frac{K}{2cf_1}\frac{d}{dt}\left[N\, dl - \frac{N}{m^2}\, dl\right]. \quad f_B = \frac{K}{2cf_1}\frac{d}{dt}\left[\left(1 - \frac{1}{m^2}\right) N\, dl\right] =$$

$$\frac{40.3}{cf_1}\frac{(m^2-1)}{m^2}\frac{d}{dt}\int N\, dl.$$

From the last equation we obtain the time derivative of $\int N\, dl$

We can actually measure the differential phase of the two carriers as a function of time; therefore, we can measure relative changes in $\int N\, dl$ very accurately.

Since there are generally many cycles of differential phase shift at two, widely spaced frequencies, the absolute value of $\int N\, dl$ cannot be determined unambiguously from the differential Doppler technique, but the time rate of change of $\int N\, dl$ can be measured precisely.

The differential Doppler method has been a standard method of making measurements of electron density height profiles using rockets launched into the earth's ionosphere since the late 1940s. As the rocket ascended into the ionosphere the differential phase would yield a continuously increasing phase advance, thus the total density up to the rocket height was continuously measured.

The differential Doppler technique is the standard method of correcting for the first order effects of rate of change of electron content of the ionosphere on satellite navigation systems. The TRANSIT system uses several satellites in approximate 1,000 km height polar orbits, transmitting coherently derived signals near 150 and 400 MHz. The position of a user on the earth's surface is determined by the Doppler shift received from a station over the approximate ten to twenty minute duration of a single satellite pass. The contribution to range rate error due to the ionosphere can be the greatest error source of the TRANSIT system if not corrected for using the dual frequency range rate information.

THE GROUP DELAY EFFECT IN THE EARTH'S IONOSPHERE

The transit time of a radio wave through the ionosphere at frequencies of approximately 100 MHz and higher can be expressed as:

$$t = \frac{1}{c}\int \mu_g\, dl, \text{ where } \mu_g \simeq 1 + \frac{X}{2}. \quad (\mu_g * \mu_\phi = c^2)$$

$$t = \frac{1}{c}\int\left(1 + \frac{X}{2}\right) dl, \quad \Delta t = \frac{1}{2c}\int N\, dl = \frac{40.3}{cf^2}\int N\, dl = \frac{40.3}{cf^2}\int N_{sec\,i}\, dl$$

$$t = \frac{40.3}{cf^2}\int N \sec i \, dl$$

The differential group delay can be expressed in the following manner:

$$\delta(\Delta t) = \frac{K}{c}\left[\frac{1}{f_1^2} - \frac{1}{f_2^2}\right]\int N dl = \Delta t_1 \frac{(f_2^2 - f_1^2)}{f_2^2}$$

$$\text{or } \Delta t_1 = \frac{f_2^2}{(f_2^2 - f_1^2)}\,\delta(\Delta t)$$

That is, by measuring the differential group delay at two frequencies, f_1 and f_2, the absolute delay at f_1 can be determined.

The NAVSTAR-Global Positioning System (GPS) uses high orbit satellites transmitting on dual, coherently derived, frequencies of approximately 1.2 and 1.6 GHz. The GPS satellites, however, also transmit modulated signals such that, in addition to allowing a differential Doppler measurement to be made to correct for the effects of the ionospheric range rate errors, a differential group delay measurement can be made. The combination of the differential group delay measurement, which gives absolute TEC, and the differential carrier phase, which gives relative changes in TEC to high accuracy, is a powerful combination of methods to study the dynamics of the ionosphere.

Some typical numbers for ionospheric group delay and phase advance, for an $\int N dl = 3.5 \times 10^{17}$ el/m^2 are:

Frequency	group delay (seconds)	phase advance (cycles)
200 MegaHertz	1,175 nanoseconds (352.5 meters)	235
1000 MegaHertz	47 nanoseconds (14.1 meters)	47

$$\Delta t_g = \frac{40.3}{cf^2}\int N dl \quad \text{(seconds)}$$

$$\Delta\phi_p = \frac{1}{2\lambda}\int X dl \text{ (cycles)} = \frac{K}{\lambda f^2}\int N dl \text{ ; since } f\lambda = c$$

$$\Delta\phi_p = \frac{40.3}{cf}\int N dl \quad \text{(cycles)}$$

$$\Delta \tau_g \propto \frac{1}{f^2}, \text{ but } \Delta\phi_p \propto \frac{1}{f}$$

$\Delta\phi_p$ is therefore much more sensitive to changes in $\int N dl$ than is Δt_g.

CONCLUSIONS

The effects of Faraday rotation, differential Doppler and group delay on radio waves passing through the ionosphere have been described. These effects, plus scintillation of radio waves, are the major ones on modern satellite ranging systems.

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ILLUSTRATIVE PROBLEMS

1. A radio astronomy antenna is located at a site near the earth's magnetic equator, where the earth's magnetic field is 0.352 Gauss, and operates at a frequency as low as 20 MHz. An important study of the sense of polarization of emission from portions of the galaxy is to be measured using this linearly polarized array. How close to a direction transverse to the earth's magnetic field can the radio astronomy antenna be used and still be assured of receiving quasi-longitudinal propagation at the lowest operational frequency of the antenna?

Suggest methods of reducing, or even completely eliminating, the effects of the earth's ionosphere on the measurement of the sense of the polarization of different regions of the galaxy with this array.

2. The group delay of a modulated signal at 1 GHz from a geostationary satellite was measured at 0322 U.T. to be 100 nanoseconds. At the same time the relative carrier phase advance was 12 radians, with a linear rate of increase of 3.0 radians every 27 seconds. The group delay measurement equipment failed at 0325 U. T. The carrier phase advance data continued increasing at the same rate until 0340 when it suddenly changed to 1.7 radians every 51 seconds, continuing at this rate of increase until 0359 U. T.

A lower orbit satellite at a height of approximately, 20,000 km passed directly in front of the geostationary satellite at 0355 U. T. While there were no group delay measurements available from this second satellite, it carried a precise clock from which users needed to calibrate the ionospheric time delay at 1.575 GHz. Calculate the ionospheric time delay for users of the precise clock at 0355 U. T.

3. If you were to design a satellite-borne clock for diseminating time to users all over the earth to an accuracy of 1 nanosecond, what means would you use to insure that the effects of the ionosphere were minimized? Discuss the relative cost advantages versus accuracy of each method you propose. How accurately would you have to know the orbit of the satellite(s)?

4. A radar designed to detect and track missiles as they rise over the horizon operates on a frequency of 300 MHz. If the mean height of the ionospheric time delay occurs near 350 km, and a measurement of electron content up to the height of 1,000 km vertically, is to be used to correct for the slant time delay at 1 degree above the horizon, calculate the correction factor which must be multiplied to the vertical measurement to yield the near-horizon value. The missile is also assumed to be at a height o 1000 km.

Discuss the major objection to the use of this procedure.