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" SPRING COLLEGE ON GEOMAGNETISM AND AERONOMY "

(2 - 27 March 1987)

" Magnetospheric processes "

presented by :

J.G. ROEDERER
Geophysical Institute
University of Alaska
Fairbanks, AK 99775-0800
U.S.A.

Magnetospheric Processes

Juan G. Roederer
University of Alaska
Fairbanks AK 99775-0800 USA

Part I: Introduction

Why study the magnetosphere?

The magnetosphere is the outer envelope of the earth system or *geosphere*. We define the geosphere as that region of space in which the medium is physically tied to Planet Earth. From the center of the earth until a few hundreds of km above the surface, the link is established by gravitational forces; above the ionosphere until the boundary of the magnetosphere (magnetopause) the tying forces are mainly magnetic.

Quite generally, the gravitational and magnetic fields are the two basic large-scale "organizing agents" in the Universe. Gravitation dominates the behavior of all condensed matter; magnetic fields dominate the behavior of all collision-less plasmas; both, of course, cooperate in this dynamic control. Therefore, by studying the terrestrial magnetosphere, we can observe and measure *in situ* in our own "backyard" processes that occur elsewhere in the universe and are of fundamental astrophysical importance (e.g., solar flares, the formation of planets and of the solar system; acceleration processes; the envelopes of neutron stars and black holes; galactic jets and arcs, etc.)

The study of the magnetosphere is also of great practical importance. Circumterrestrial space has become an economic resource of importance to all countries, advanced and developing. Human activity and technological systems are expanding outwards. Satellite systems are now crucial for communications, navigation, remote sensing and weather prediction. Transatmospheric transportation and human colonies in space are not far in the future. But the medium in which these systems operate, or will operate, is hostile. Just as the solid and fluid earth poses hazards to humans through earthquakes and storms, the space around earth poses hazards from solar and magnetospheric activity (orbital degradation from atmospheric density increases; radiation damage from radiation belts and solar flare particles; induction effects on ground conductors from varying ionospheric electric currents, etc.) Furthermore, near-earth space has become prone to "pollution": industrial effluents, rocket exhaust, orbiting debris, electromagnetic waves from powerful transmitters and from the electric power grids, shock waves from powerful explosions, all affect the upper atmosphere and near-earth space.

In short, it is important to be able to predict "weather and climate in space", and to understand and be able to mitigate the effects of "space pollutants". This requires understanding how the magnetosphere works.

General characteristics of the magnetosphere

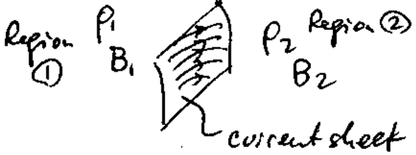
A magnetosphere arises from the interaction of a continuous stream of collisionless plasma (fully ionized, low-density, very hot gas) with a magnetized object. In this interaction a cavity is "carved out" in the plasma stream, inside of which the magnetic field of the magnetized body dominates. The cavity is comet-shaped, with a long tail. In principle the upstream boundary of the cavity is determined by the equilibrium between the magnetic pressure $B^2/8\pi\mu_0$ inside and the kinetic pressure ρv^2 (ρ: mass density) of the impinging plasma.

All magnetized planets (Mercury, Earth, Jupiter, Saturn, Uranus) have magnetospheres, of varying sizes. Jupiter's is the biggest (if it could be seen, it would appear to us of the same angular size as the sun or the moon!). The sun has a magnetosphere (called heliosphere) arising from its own motion through the interstellar plasma; some galaxies have magnetospheres. Basically, a magnetosphere can be "plasma-stream dominated" (i.e., mainly controlled by the plasma flow, like the Earth magnetosphere) or "rotation-dominated" (controlled by the (rotating) magnetized body, like Jupiter's or the heliosphere).

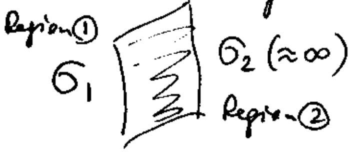
Although magnetospheres differ very much from each other in scale and appearance, they do have many characteristics in common. One striking characteristics, shared with other plasma systems in the Universe, is the appearance of discrete regions separated by sharp boundaries. Another characteristic is that in its temporal, dynamic behavior, a natural plasma system seems to proceed in discrete, sudden steps: whenever nature creates a plasma system, she arranges for catastrophic processes or instabilities in which magnetic energy is converted in a very short time into the kinetic energy of a very small fraction of plasma particles.

The following is a summary of some basic processes that can be found in all plasma systems such as magnetospheres:

Two types of boundaries between discrete regions:



<u>Case I</u>: Two regions of different ρ (density) and $\vec{B}$: role of <u>current</u> sheet separating them is crucial. Quite generally, electric currents in plasma systems tend to occur in two-dimensional sheets (that's why discrete regions appear!) Like the membrane of a biological cell, current sheets regulate the coupling and interactions between the regions which they separate.

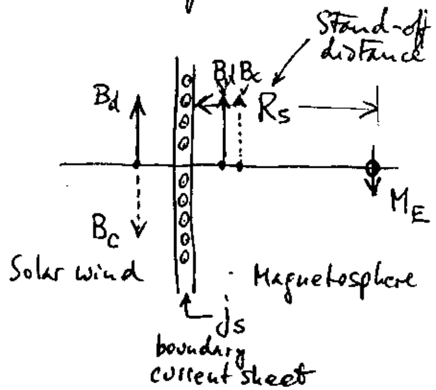


<u>Case II</u>: Two regions of different conductivity σ (e.g., ionosphere and magnetosphere). Separation is not so "sharp" as in Case I, but it plays an important role in acceleration and dissipation mechanisms, and in particle source and sink mechanisms.

Dynamo mechanisms (see later chapter)
Magnetic reconnection (conversion of magnetic energy into kinetic energy)
Energy and particle transfer processes
Parallel electric fields (acceleration along field lines)
Inductive acceleration ($\partial B/\partial t$, formation of ring current, see later chapter)
Bulk acceleration of plasma (shocks, jets or plasmoids)
Dissipation processes (joule heating, turbulence)
Generation of B-fields (Turbulent convection in MHD systems)

The main topology of the magnetospheric magnetic field is shown schematically on <u>Fig 2</u> of the accompanying article "Global Problems....". Note the four possible regions of field line topology. Note above all how the interplanetary magnetic field will control the field topology in the magnetosphere and many dynamic processes therein. <u>Fig 4</u> of the article "Global Problems...." shows the principal plasma regions in the magnetosphere.

The overall <u>size</u> of the magnetospheric cavity is controlled by the balance between the solar wind dynamic pressure $\rho_{sw}V_{sw}^2$ and the magnetic field pressure $\frac{B^2}{8\pi\mu_0}$ inside the boundary (the "Chapman-Ferraro problem"). Consider the figure. The boundary current j_s (<u>linear</u> density in Amp/m) must be such as to cancel the effect of the dipole field B_d right "outside", in the Solar Wind (<u>neglect</u> SW field).



thus $B_c = \frac{1}{2}\mu_0 j_s$ (field of an infinite planar sheet of current – see proper directions in figure). But this must be equal (and opposite) to $B_d = M_E/R_s^3$ (R_s is the "stand-off distance" of the magnetosphere, the geocentric distance to the subsolar point of the "nose" of the magnetosphere – usually measured in earth radii ($1\ R_e = 6371$ km)). B_c on the inside is upwards and adds to B_d, leading to a field immediately behind the boundary $B = 2M_E/R_s^3$. This is the resulting B-field that balances the SW pressure. It is a compressed field. Thus, the condition is

$$\rho_{sw} V_{sw}^2 = \frac{1}{8\pi\mu_0}\frac{4M_E^2}{R_s^6} \quad \text{and} \quad \boxed{R_s = \left[\frac{1}{2\pi\mu_0}\frac{M_E^2}{\rho_{sw}V_{sw}^2}\right]^{1/6}}$$

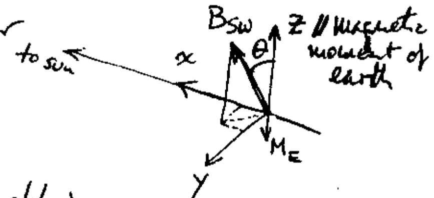
Note how R_s varies with $\rho_{sw}^{-1/6}$ (weak dependence) and $V_{sw}^{-1/3}$ (a bit stronger dependence). A convenient scale size is the quantity

$$l_0 = \left[\frac{M^2}{\mu_0 \rho V^2}\right]^{1/6}$$

allowing to scale magnetospheres of different planets of magnetic moment M, subjected to a flow of plasma ρ, V.

Concerning energy transfer to the magnetosphere a "reasonable" solar wind parameter (according to correlation studies between solar wind and energy dissipation in the magnetosphere) is the parameter

$$\boxed{\epsilon = l_0 V_{sw} B_{sw}^2 \sin^4\left(\frac{\theta}{2}\right)}$$



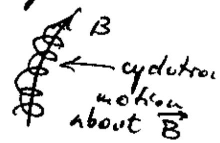
l_0 is the scale size given above. To obtain ϵ in Megawatt, set $l_0 = 2\times10^4$ and express V in km/sec and B in nanotesla). For $\epsilon \lessapprox 10^4$ MW, we have a quiet magnetosphere. For $\epsilon \sim 10^4$–10^5 MW, polar cap activity enhancement; 10^5–10^6: substorm; 10^6–10^7: storm with Dst ~100 nT; $\gtrsim 10^7$ MW: big storm, Dst > 200 nT. Note that for $\theta = 0$ (northward B_{sw}), $\epsilon \equiv 0$, for southward field ($\theta = 180$) ϵ is maximum.

Part II: Solar wind – magnetosphere – ionosphere dynamo

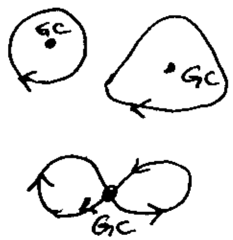
Review of particle drifts

(For details see Roederer, *Dynamics of Geomagnetically Trapped Radiation*, Springer-Verlag, New York, Heidelberg, Berlin, 1970)

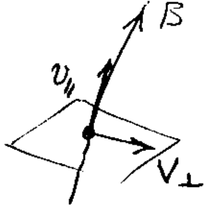
Charged particles in magnetic fields under certain conditions of energy and field configuration, travel in very complicated orbits. However, again under certain conditions, usually fulfilled in magnetospheric plasmas, it is possible to simplify the description of their motion by "averaging out" certain periodicities. One such periodicity, the first one to appear and the one of highest frequency, is a motion about the magnetic field, called cyclotron motion. Since the scale-size (gyroradius) of the cyclotron motion is usually very small compared to the scale-size of the regions through which the particle travels, it is often enough to know the point about which the particle gyrates, averaging out the detailed gyro-motion. That point is called guiding center, and it is introduced in the following way.



First we define the guiding center system as that frame of reference in which an observer sees the particle execute a closed periodic orbit. If such a frame of reference can be found, we say that the first adiabatic condition is fulfilled, and that frame is unique. The orbits in that Guiding Center System need not be circles (see figure) in the general case. The guiding center (GC) is the centroid of the closed orbit.



The description of particle motion is always referred to frames of references oriented in such a way as to have one axis // to $\vec{B}$. All vectors are decomposed into parallel and perpendicular components.



The particle velocity is

$$\vec{v} = \vec{v}_\parallel + \vec{v}_\perp$$

$\vec{v}_\perp$ is the rapidly gyrating velocity which we want to average out. For the GC velocity (which we will always designate with capital letters) we write

$$\vec{V}_{GC} = \vec{V}_\parallel + \vec{V}_\perp$$

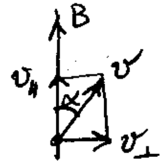
Obviously $\vec{V}_\parallel = \vec{v}_\parallel$. The perpendicular velocity $\vec{V}_\perp$ is called the drift velocity $\vec{V}_D$ of the particle. Obviously $\vec{V}_D = \langle \vec{v}_\perp \rangle$, with the average $\langle\,\rangle$ taken over the gyration period. V_D is an instantaneous drift velocity. It changes with time and as the particle spirals along a field line. Thus in general $\dot{\vec{V}}_D \neq 0$. $\dot{V}$ is the acceleration of the GS system, which therefore is not necessarily an inertial frame of reference.

The general equation of motion of a particle in a magnetic and electric field is given by:

$$\boxed{\vec{F} = m\vec{a} = q\vec{v}\times\vec{B} + q\vec{E}}$$ q: electric charge

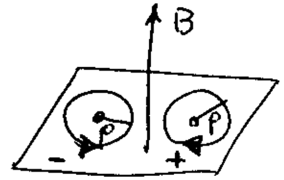
This is to be used to derive the expressions for drift velocities and other adiabatic quantities.

Case of a uniform B-field, no E



α: pitch angle of particle

Since in that case $F = qv_\perp B$ is always $\perp$ $\vec{B}$ and $\vec{v}$, the kinetic energy of the particle is constant. Since $F_\parallel \equiv 0$, $v_\parallel = $ c and therefore also $v_\perp = $ const. The GC will move along B, $\vec{V}_{GC} = \vec{v}_\parallel$, and in the GC system the motion will be circular, uniform. The drift velocity is zero. Note the senses of gyration. We have



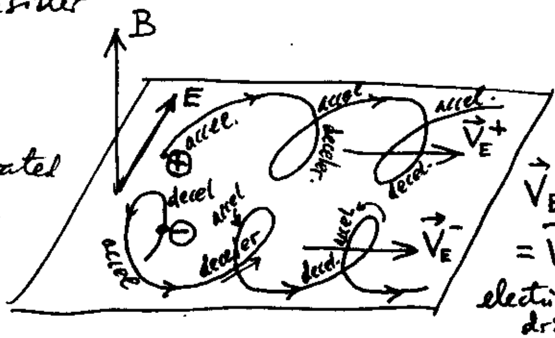
Gyroradius (or cyclotron, Larmor radius) $\boxed{\rho_c = \dfrac{mv_\perp}{qB} = \dfrac{p_\perp}{qB}}$ (relativistic expression)

Cyclotron period $\boxed{\tau_c = \dfrac{2\pi m}{qB}}$ (independent of particle energy only in nonrelativistic case. m is relativistic mass)

Cyclotron frequency (angular) $\boxed{\omega_c = \dfrac{qB}{m}}$

Case of uniform B, uniform E ($\perp$B)

Examine figure. Consider particles with $v_\parallel \equiv 0$. Note how during a cyclotron turn the particle gets accelerated and decelerated by the electric force $q\vec{E}$, and how drift arises.



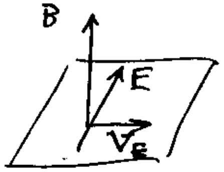
$\vec{V}_E^+ = \vec{V}_E^- = \vec{V}_E$ electric field drift

… V_E is as follows.

Consider the perpendicular equation ($\vec{F}_\parallel \equiv 0$):

$$\vec{F}_\perp = q\vec{E} + q\vec{v}_\perp \times \vec{B}$$

We need a frame of reference in which orbit is closed. Obviously, it will be one in which $\vec{E} \equiv 0$. So, all we have to do is find a $\vec{V}_E$ such that the electric field E^* in the frame that moves with V_E is zero.



Using the transformation rules (classical electromagnetism)

$$\text{new frame of ref.}\ \begin{cases} \vec{B}^* = \vec{B} \\ \vec{E}^* = \vec{E} + \vec{V}\times\vec{B} \end{cases} \ \text{old frame of reference} - V\text{: velocity of new frame w/respect to old}$$

we must have $E^* \equiv 0 = \vec{E} + \vec{V}_E \times \vec{B}$

i.e.

$$\boxed{\vec{V}_E = \frac{\vec{E}\times\vec{B}}{B^2}} \qquad \underset{\text{m/sec}}{|V_E|} = \frac{|E| \leftarrow \text{Volt/m}}{|B| \leftarrow \text{Tesla}}$$

will be the drift velocity of a particle in a $\vec{B}$ and $\vec{E}$ ($\perp B$) field (If E not $\perp B$, still the same!). Note that $\vec{V}_E$ is independent of charge, mass and energy of the particle! This means that all particles will drift with the same velocity, regardless of their class and energy. This is why V_E also represents the bulk velocity of a collisionless plasma, perpendicular to B. Note the "first degree" of confining property of a magnetic field: particles can freely flow along B, but they will drift to first order collectively in their perpendicular motion. Careful: there are other

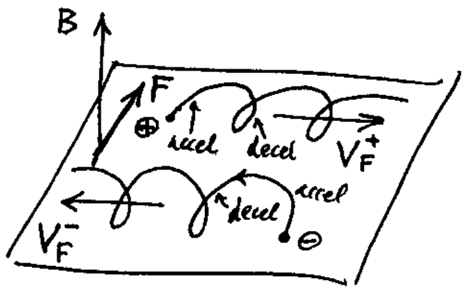
… that do depend on the particles, their charge, energy, etc. But in absence of all external forces, V_E is the first order drift for all particles.

Consider the converse: if I see a plasma, or a group of particles, move with bulk velocity $\vec{V}_\perp$ perpendicular to B, I can be sure that the electric field ($\perp$ B) in a frame of reference that moves with velocity $\vec{V}_\perp$, is zero (we can say nothing about $E_\parallel$)

[As a corollary: If I have a group of particles in which I can hold the ions in place, only the electrons will drift. This will now lead to an electric current perpendicular to B and E called Hall current. If we have n electrons per cc with charge $|e|$, the Hall current density will be $-ne\frac{E\times B}{B^2}$.]

Case of a force $\vec{F} \perp \vec{B}$ (no E)

Consider a non-electric force $\vec{F}$ and examine in the figure how the drift arises. The GC system is now found as that frame of reference in which an electric field E^* appears that exactly cancels the effect of the force F:



$qE^* = -F$; but $\vec{E}^* = \vec{E} + \vec{V}\times\vec{B}$ in which $\vec{E} \equiv 0$ by def.

Therefore, the force drift is

$$\boxed{\vec{V}_F = \frac{\vec{F}\times\vec{B}}{qB^2}}$$

It still does not depend on energy, but it does depend on electric charge. In response to an external force, the particles respond with a transverse drift, with positive and negative particles going into opposite directions. This will lead to an electric current of density

$$\boxed{\vec{j}_F = \sum_k n_k q_k \vec{V}_{Fk} = \sum n_k \frac{\vec{F}\times\vec{B}}{B^2} = \frac{n\vec{F}\times\vec{B}}{B^2} = \frac{\vec{f}\times\vec{B}}{B^2}}$$

n_K: number density, q_K charge of particles of class k.

j is the force-drift current, $\vec{f}$: force density.

Case of an accelerated Guiding Center system

Consider a guiding center system that is accelerated, i.e., the case of a time-dependent electric field $\partial\vec{E}/\partial t \neq 0$. In that case the electric drift (or bulk velocity) will be $\dot{\vec{V}}_E \neq 0$. The GC frame is non-inertial; like in all accelerated frames of reference, each body of mass m will experience in that frame of reference an inertial force $-m\dot{\vec{V}}_E$. This inertial force will give rise to yet another drift, also called polarization drift:

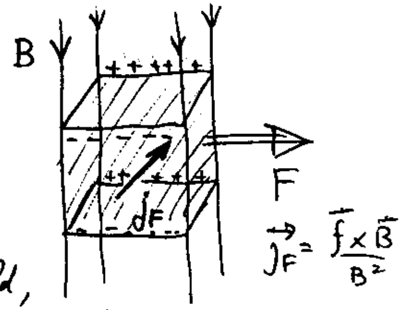
$$\boxed{\vec{V}_{in} = -\frac{m\dot{\vec{V}}_E\times\vec{B}}{qB^2}}$$

Since it is charge-dependent, a current will appear (polarization current):

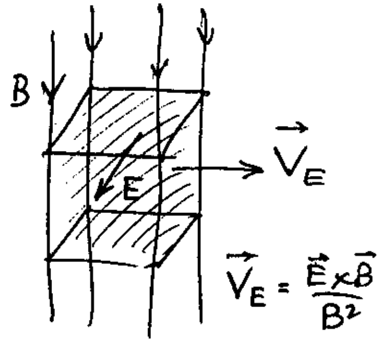
$$\boxed{\vec{j}_P = \sum_k n_k q_k \vec{V}_{ink} = -\frac{(\sum n_k m_k)\dot{\vec{V}}_E\times\vec{B}}{B^2} = -\rho\frac{\dot{\vec{V}}_E\times\vec{B}}{B^2}}$$

ρ is mass density.

Acceleration of plasma

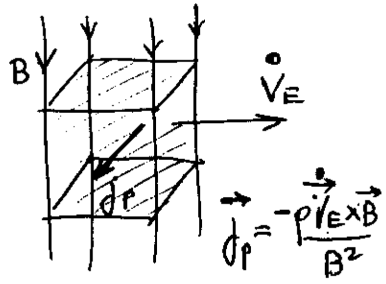


Consider an ultrasimplified case of a blob of plasma sitting in a uniform magnetic field as shown in the figure. Initially, there is no E-field, and we assume that there is no motion of particles parallel to B ($v_\parallel = 0$). There will be no drift; all particles are gyrating around their GC's which are at rest. I now apply an external force F (of force density f (force per unit volume)). Let's analyze what happens step by step (although in reality, all steps happen together!). First, the particles will start a force drift and a drift current $\vec{j}_F = \vec{f}\times\vec{B}/B^2$ as shown in the figure will arise. In other words, if I push or pull on a plasma, initially, the plasma does not react in the direction of the force: rather, I will squeeze out the ions and electrons sideways! The charges will accumulate on the sides as shown. This, however, will lead to an electric field, which in turn will lead to a drift $\vec{V}_E = \frac{\vec{E}\times\vec{B}}{B^2}$ (bulk motion) which does happen to be in the direction of the original external force $\vec{f}$. In other words now the plasma is beginning to react in bulk fashion as we should expec



he electric field, however, is time-dependent, and so is $\dot{V}_E$ (increasing). This will lead to a polarization current

$$\vec{j}_P = -\frac{\rho \dot{\vec{V}}_E \times \vec{B}}{B^2}$$



which is directed opposite to the original force drift current $\vec{j}_F$ (see figures). The electric field will settle in on a value such that both currents cancel each other (dynamic equilibrium). In that state (which arises very quickly) we have

$$\vec{j}_P + \vec{j}_F \equiv 0 = \frac{(\vec{f} - \rho\dot{\vec{V}})\times B}{B^2}, \text{ or } \boxed{\rho\dot{\vec{V}} = \vec{f}}$$

Therefore, in the end, the plasma blob follows Newton's equation! But in reality several concurrent steps are involved.

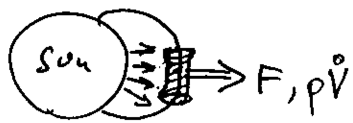
The behavior of a real plasma is, of course, much more complicated. First, there is a condition for all this to apply, which is that the Alfvén velocity $V_A = \frac{B}{\sqrt{\mu_0 \rho}} \ll c$. Regions where $\rho \to 0$ (i.e. on the boundaries) do not comply with this condition: the boundaries will "peel off" in the real case – only the core of the blob will behave as described above. Also, because of $v_\parallel \neq 0$ there will be an expansion of plasma along B.

In general, all the above is a result of one of the basic MHD equations:

$$\rho\dot{\vec{V}} = \vec{j}\times\vec{B} + \vec{f}$$

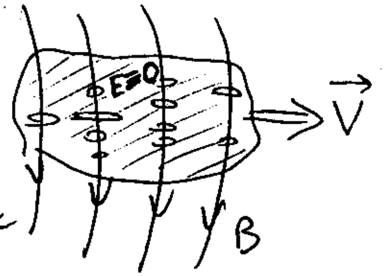
The initial step in our "Kindergarten" description would correspond to $\dot{V} \approx 0$ and $\vec{f} + \vec{j}\times\vec{B} \approx 0$. j would be mainly the drift current, and $j \times B$ would represent the initial reaction of the plasma to the external force. In the final state, when both currents j_P and j_F cancel each other we have $\vec{j} = 0$ and $\rho\dot{V} = f$!

In spite of our oversimplified model, it represents all basic features of a plasma that is being accelerated by an external force, for instance, plasma in the lower corona, on which an external force is acting caused by the dissipation (turbulent) of acoustic waves that originate in the photosphere. Thus, the solar wind is generated.



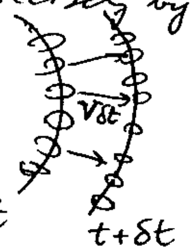
As the solar wind plasma flows out, it carries embedded in it the solar magnetic field. What does this mean? What is meant when one says that the solar magnetic field lines move with, or are "frozen into", the solar wind?

In a frame of reference that moves with the solar wind bulk velocity the electric field is zero (see p. 9). All particles are in beautiful circular orbits around the magnetic field, in that frame of reference.



Therefore, if I paint a field line with probe particles in that frame, these particles will remain on that field line in that frame. Therefore, the field line is moving with the plasma!

Great care is to be taken with the concept of moving field lines, though (see footnote on p. 5 of article "Global Problems...") Field lines are mathematical, not physical artifacts. They even are not relativistically invariant! There are infinite ways to define field line velocity (e.g., in a static dipole I can define a velocity field that has all field lines rotating around the dipole axis like crazy, and nothing changes in the field geometry!). The only reasonable thing is to define a phenomenological field line velocity, precisely by using probe particles of vanishing kinetic energy (so as to avoid other types of drift) and of $v_{\parallel} \equiv 0$. If I place these particles on a field line at time t, I can watch where they (their guiding centers) will be at time $t + \delta t$. If, and only if, they again are found all on one field line, I declare that field line to be the "same one" as the one on which the particles were at time t. This is what we have done with the solar wind field lines. Whenever the relation $\vec{E} + \vec{V} \times \vec{B} = 0$ holds, field lines move with the plasma ("frozen-in" condition).



Serious problems arise the moment I have a resistive medium on the portion of the field line. Those fond of persisting in using "spaghetti physics" have introduced the concept of field line "cutting", "slipping", etc. Bon appetit!

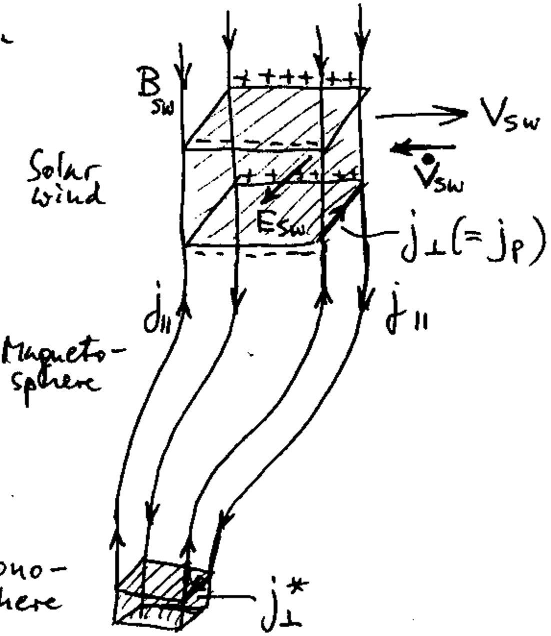
Once the solar wind plasma has been accelerated in the base of the corona, it enters a "cruise mode" of motion away from the sun, in which $f \simeq 0$, $j \simeq 0$ and $\vec{V} = \text{const.}$ It is important to realize that, as viewed from a system fixed to the Earth, the solar wind has an electric field

$$\boxed{\vec{E}_{SW} = -\vec{V}_{SW} \times \vec{B}_{SW}}$$

$\vec{B}_{SW}$ is the interplanetary magnetic field ("IMF"), $\vec{E}_{SW}$ is the interplanetary electric field. While $\vec{V}_{SW}$ is always directed radially out from the sun, $\vec{B}_{SW}$ can vary greatly and so will $\vec{E}_{SW}$.

Coupling of plasmas

Consider a blob of plasma in the solar wind coupled to the ionosphere on field lines of class II in Fig 2 of the paper. The moment we connect the polarization charges to the ionosphere along field lines (which we assume electric equipotentials – in principle, we cannot maintain a parallel electric field in a collisionless plasma), parallel electric currents will arise as shown in the figure, because the ionosphere can furnish nearly arbitrary numbers of cold electrons to cancel all positive charge accumulations. The result is a decrease of E_{SW}, hence a deceleration of the plasma blob $\dot{V}_{SW}$ as shown, the appearance of a polarization current because of $\rho \dot{V} = j_P \times B$, the appearance of a drag



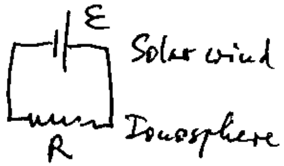
the appearance of a conduction (Pedersen) current $j_\perp^*$ in the ionosphere (closing the current circuit), the appearance of a force $j_\perp^* \times B^*$ on the patch of the ionosphere trying to accelerate it in the direction of solar wind motion, and above all, the appearance of an electric field E^* in the ionosphere, which is simply the solar wind electric field mapped down along field lines.

Let's examine some of the above statements in more detail.

First of all, if we want to maintain the SW plasma flowing with constant speed, we must apply an external force to cancel the drag force (this indeed happens in the so-called magnetosheath region near the magnetopause). Now we have

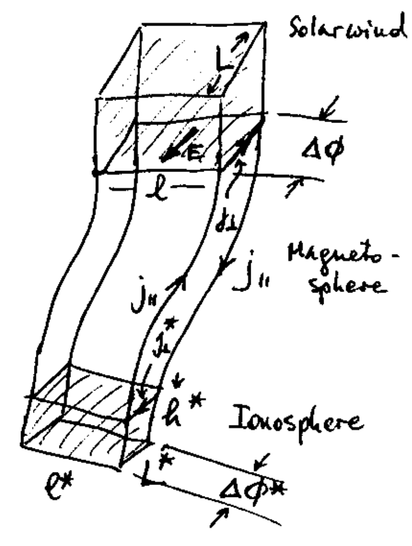
$$f_{ext} + \vec{j}_\perp \times \vec{B} = 0 = \rho \dot{\vec{V}} \qquad (\vec{V} = \text{const})$$

in the solar wind plasma blob. The external force required to maintain constant $\vec{V}_{sw}$ does <u>work</u> on the system. The power it delivers is dissipated in the ionosphere by the conduction current (and also used to accelerate that portion of the ionosphere). In short, we have a <u>dynamo</u> process! Note that the transverse current in the solar wind (figure on p 16) runs <u>against</u> the electric field (emf!!), the transverse current in the ionosphere runs in the same direction as E^*. We have a simple equivalent circuit as shown.



This constitutes the basic <u>coupling mechanism</u> between solar wind-magnetosphere-ionosphere.

It is important to consider a few quantitative relationships (see figure).



Since we consider the field lines as equipotentials, the electric potential difference $\Delta\phi$ between the front and back sides of both patches will be the same (starred quantities are those of the ionosphere)

$$\Delta\phi^* = \Delta\phi = E_{sw} L .$$

Thus the electric field in the ionosphere will be

$$E^* = E_{sw}\frac{L}{L^*} = V_{sw} B_{sw} \frac{L}{L^*}$$

The current in the ionosphere $j_\perp^*$ will be

$$j_\perp^* = \sigma E^* = \sigma V_{sw} B_{sw} \frac{L}{L^*}$$

σ is height-integrated Pedersen conductivity

On the other hand, for the closed circuit we must have for the <u>total</u> currents

$$j_{sw}\, l\, h = j_\perp^*\, l^* h^*$$

The <u>total</u> drag force on the SW plasma blob will thus be

$$F_D = f\, L\, l\, h = j_{sw} B_{sw}\, L\, l\, h$$ and replacing

$$\boxed{F_D = \sigma \frac{l^* h^*}{L^*} L^2 B^2 V}$$

$\sigma \frac{l^* h^*}{L^*}$ plays the role of an equivalent resistance. The proportionality with V shows that this force has the character of a <u>viscous</u> drag.