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" SPRING COLLEGE ON GEOMAGNETISM AND AERONOMY "

(2 - 27 March 1987)

" Magnetospheric processes" - II part

presented by :

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These are preliminary lecture notes, intended for distribution to participants only.

Magnetospheric Processes
(Continuation)

Juan G. Roederer

Part III: Magnetospheric Convection

General electric field configuration

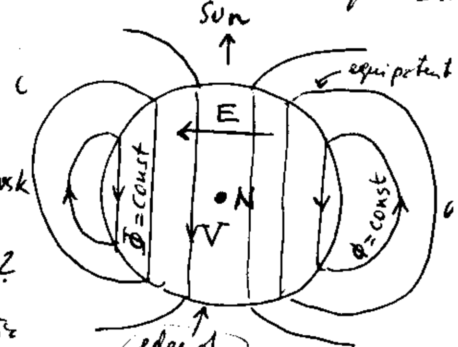
In the previous section we have seen that the solar wind electric field (as seen in a frame of reference fixed to the Earth) is "projected" along the interconnected field lines (called open field lines) onto the polar ionosphere. In other words, the electric field in the polar cap is impressed from outside, by the solar wind dynamo process.

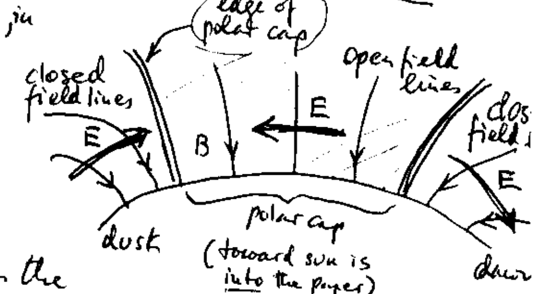
One can use a magnetospheric field model on which one superposes an external (solar wind) magnetic field to determine numerically general properties of the configuration of the polar cap and the electric potentials in it. Many of the numerical findings have been confirmed experimentally, at least qualitatively.

Some results are as follows:

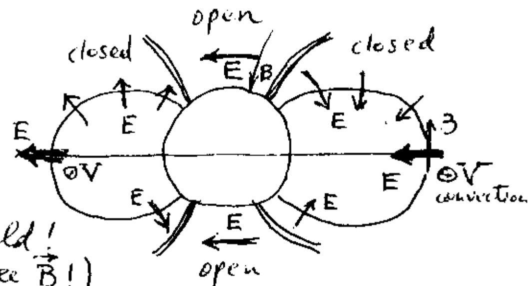
1. In general, the polar cap electric field is directed from dawn to dusk (see dipole case on figures shown at end of Part II). This causes a general convection of the ionosphere in the polar cap from the sunward side to the night edge (antisunward convection given by $V = -E^* \times B^*$; E^*, B^* fields in the ionosphere. The force on the ionospheric plasma is the drag force resulting from the solar wind-magnetosphere-ionosphere dynamo).
2. The size of the polar cap (region of open or interconnected field lines) is controlled by B_z (component of interplanetary B parallel to the magnetic moment of the earth). Size is greatest for southward B_z ($B_z < 0$), tends to zero for completely northward directed B ($B_z > 0$). Note, however, that _in reality_ the polar cap never disappears. For a completely northward-directed B_{sw} the field topology is not yet fully known!
3. The dawn-dusk component of B_{sw} shifts the whole polar cap in the dawn-dusk direction (north and south polar caps in opposite ways).
4. The potential difference across the polar cap is usually of several tens of KV, up to 70 or 80 KV during times of intense, due southward interplanetary field.

Assuming that $\vec{E}$ is constant inside the polar cap (models show that for a _due_ southward interplanetary field), how is the rest of the electric field in the ionosphere determined? One can state a simple electrostatic problem in the ionosphere, in which the boundary of the polar cap represents the boundary of a domain: _given_ the E (Φ) inside, what will the E (Φ) be outside that matches the inside on the boundary? The result is a field of the type shown in the top figure (looking _down_ on the polar cap). The field that arises in the closed field line region is shown in the bottom figure (vertical slice along the dawn-dusk meridian). Note that the boundary of closed/open field lines is a source/sink of $\vec{E}$ ($\nabla \cdot \vec{E} \neq 0$).



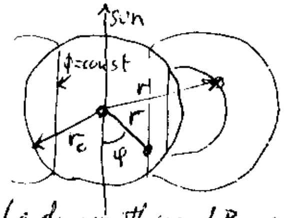


It is important to realize that projecting the E-field configuration to the equatorial surface along the closed field lines, one again obtains a dawn-dusk electric field! But now the convection is sunward (see $\vec{B}$!)



Assuming a potential in the polar cap of the form

$$\Phi(r,\varphi) = \Phi_0 \frac{r}{r_0} \sin\varphi$$



we obtain a uniform electric field (a due southward B_{sw} produces this). The potential difference across the polar cap will be $2\Phi_0$. Φ = const are the $r\sin\varphi$ = const. lines, which represent the equipotentials and convection lines. The solution of the electrostatic problem for the potential outside the polar cap gives

$$(\text{for } r > r_0) \quad \Phi(r,\varphi) = \Phi_0 \left(\frac{r_0}{r}\right)^k \sin\varphi$$

with k arbitrary, but related to the ionospheric conductivity in the real case.
If one projects these equipotentials along field lines to the equatorial surface, one obtains, for a dipole field

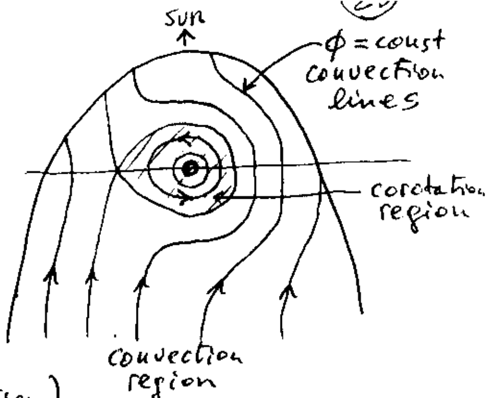
$$\Phi(r,\varphi) = \Phi_0 \left(\frac{r}{r_c}\right)^{k/2} \sin\varphi$$

Now r, φ are coordinates on the equatorial surface. Note that for $k = 2$ this leads to a uniform dawn-dusk electric field in this surface!
But this cannot be the only electric field in the closed field line region. We must add the corotational field (see previous section) that is imposed by the corotating ionosphere, of potential $\Phi_c = -\frac{\omega M_E}{r}$

The resulting potential on the equatorial surface is of the type

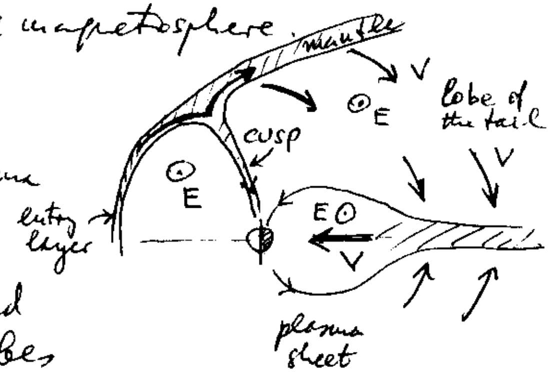
$$\boxed{\Phi(r,\varphi) = C_2 r \sin\varphi - \frac{C_1}{r}}$$

(A general discussion is given in Roederer, Dynamics of Geomagnetically Trapped Radiation).

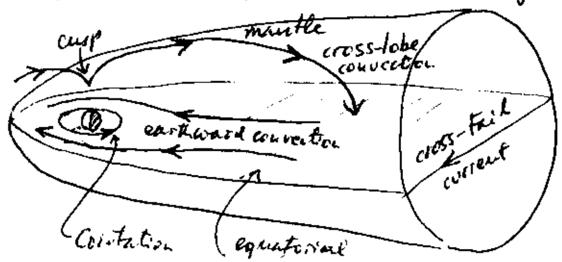


In the above equation we have assumed $k = 2$. Note the separation into a corotation region (home of the plasmasphere!) and a convection region responsible for transport of plasma from the tail toward the sunward side of the magnetosphere.

A general dawn-dusk electric field has a global effect on the flow of plasma from the dayside entry layer through the cusp into the mantle, and from there across the lobes into the plasmasheet, and from there toward the inner magnetosphere.



In summary, the externally controlled electric field is a fundamental organizing and modulating agent in the magnetosphere, regulating to a large extent the dynamics of the entire region.

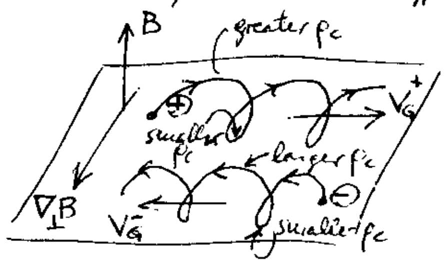


Ring current injection

In Part II we discussed "first order" particle drifts which were independent of particle energy. They thus prevail in low energy particle populations such as those that make up a plasma.

We now must introduce an energy-dependent drift which will play an important role for particles that are being accelerated during the process of general convection. We still will limit our discussion to particles that only move perpendicularly to B, i.e., to the case $v_\parallel \equiv 0$.

Consider a magnetic field that increases in magnitude in the direction shown by the transverse gradient $\vec\nabla_\perp B$ (gradient vector of magnitude of B).



Injecting a positive particle, it will gyrate with a curvature radius ρ_c that will be larger on the upper loop than on the lower loop. A drift arises, called gradient drift, of expression (see Roederer, Dynamics...)

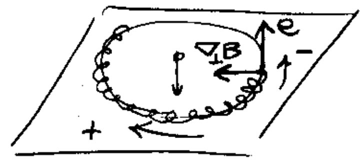
$$\vec V_G = \frac{1}{2}\frac{m v_\perp^2}{qB^3}\,\vec B \times \vec\nabla_\perp B$$

Introducing $\vec e = \frac{\vec B}{B}$, unit vector $\parallel \vec B$, we write for the non-relativistic case ($T_\perp = \frac{1}{2} m v_\perp^2$ is transverse kinetic energy):

$$\boxed{\vec V_G = \frac{T_\perp}{qB^2}\,\vec e \times \vec\nabla_\perp B}$$

Note that for $T_\perp \to 0$, $V_G \to 0$ and all other drifts will prevail.

In a dipole field, on the equatorial surface, this results in a drift around the earth, ions westward, electrons eastward. A current is established (ring current!), of density



$$\vec j_G = \sum n_k q_k \vec V_{Gk} = \sum n_k T_{\perp k}\,\frac{\vec e \times \vec\nabla_\perp B}{B^2} = \mathcal{E}_\perp\,\frac{\vec e \times \vec\nabla_\perp B}{B^2}$$

$\mathcal{E}_\perp$ is the perpendicular kinetic energy density. This is the current density for equatorial ($v_\parallel \equiv 0$) particles only. In the next section we will determine the general expression for $v_\parallel \neq 0$).

First we need to introduce the concept of the first adiabatic invariant, a quantity that is conserved under certain conditions and which relates to the particle's cyclotron motion. The condition is that B change very little during one cyclotron turn, or

$$\frac{\tau_c \dot B}{B} \ll 1 \quad \text{and} \quad \rho_c \frac{\nabla_\perp B}{B} \ll 1$$

It can be shown that, then,

$$\boxed{M = \frac{p_\perp^2}{2m_0 B} = \text{const.}}$$

m_0: rest mass. M is called the magnetic moment of the particle. It indeed is the equivalent magnetic moment of the particle in its cyclotron turn. $p_\perp$ is the transverse momentum, it really must be computed in the guiding center system.

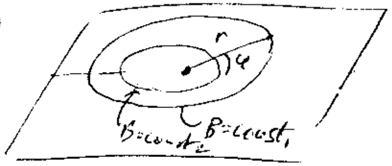
In the non-relativistic approximation

$$\boxed{M = \frac{T_\perp}{B}}$$

This expression and the conservation of M can be used at once to determine drift trajectories of equatorial particles.

Pure B-field, no E:

In absence of other forces, $v_\perp = \text{const}$ and $T_\perp = \text{const}$. Conservation of M thus leads to $B = \text{const}$ along the particle's drift trajectory.



If we have a magnetic field $B = B(r, \varphi)$ (not necessarily a dipole) we can determine drift trajectories at once: they are the $B = \text{const}$ lines on the equatorial surface!

B-field and E-field

Suppose we have a B field which on the equatorial surface is $B = B(r,\varphi)$ and an electric field derived from a potential $\Phi = \Phi(r, \varphi)$. Now the transverse kinetic energy $T_\perp$ is no longer conserved, but the total energy is:

$$W = T_\perp + q\Phi = \text{const} = T_{\perp 0} + q\Phi_0 \text{ (initial values)}$$

But $T_\perp = MB$ with $M = \frac{T_{\perp 0}}{B_0}$ (initial values)

Thus

$$\boxed{MB(r,\varphi) + q\Phi(r,\varphi) = \text{const}}$$

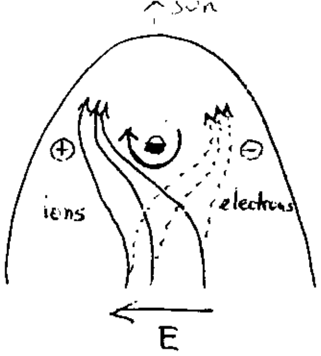
gives the equation of the drift trajectory (just solve for $r = r(\varphi)$ for the particular values of M and W.)

Note that for high energy, M will be very large and the first term will always prevail over the second one: $B(r,\varphi) = \text{const}$ gives the drift trajectories of high energy particles. If on the other hand $M \to 0$, the drift trajectories are the equipotentials $\Phi(r,\varphi) = \text{const}$. Note that positive particles will have different trajectories than negative particles in general. Only in the extremes will they coincide (but the sense of drift will be opposite for the large M case).

Note also that the energy $T_\perp$ of the particles increases or decreases at the expense of the electrostatic potential energy $q\Phi$. But note that one can also interpret the change in $T_\perp$ as due to the change in the local $\underline{B}$ (because $T_\perp = MB$!). Both are of course equivalent, but the first "looks" like an electrostatic acceleration, the second as a cyclotron acceleration!

Let us consider now an electric potential like the one given on p. 24, in presence of a dipole, or dipole-like field, and discuss typical drift trajectories of equatorial ($v_\parallel = 0$) particles of low energy starting in the tail of the magnetosphere. Both protons and electrons will first drift along $\Phi = \text{const}$ lines (electric drift) toward the earth. But as they enter a more intense B-field, their kinetic energy $T_\perp$ will increase proportionately, and the gradient drift will become noticeable, taking protons toward the dusk side and electrons toward the dawn side. This of course will be reflected in the actual drift trajectories computed using the constancy of M and W.

Typical trajectories in the equatorial plane look like shown in the figure. Note that there is a separation of charges caused by the gradient drift which takes over once the particles have been convected by the E-drift to the higher B regions and thus, been energized.

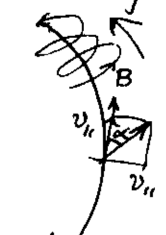


The process shown in the figure represents the basic mechanism of <u>substorm acceleration of particles</u>. In that case, the dawn-dusk electric field is greatly enhanced (probably highly localized around the midnight meridian) during a limited time interval (say 1/2 hr or less). This enhancement causes a sudden earthward injection of particles and the appearance of a <u>partial</u> ring current on the night side (see top figure). When the E-field recovers to its normal value, the protons (and some electrons) have a chance of remaining in a <u>closed</u> drift orbit around the earth. This represents the full ring current, the <u>basis</u> for the Dst change in the magnetic field. The ring current decays mainly through charge exchange of the ions with ambient neutral atoms.

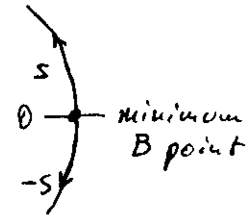
The above process is also the first stage in the formation of the Van Allen radiation belts.

Part IV : Trapped Particles

So far we have only dealt with particles on the equatorial surface with 90° pitch angles, i.e., $v_\parallel = 0$. We must now turn to the real case of a particle injected into a magnetic field with an arbitrary pitch angle α. This particle (it's <u>guiding center</u>) will move along the magnetic field line (let us neglect for the moment all drifts $V_\perp$).



We can use the conservation of the magnetic moment M to find out about the type of motion of the particle <u>along</u> the B line. Let us consider a B-field constant in time and no other forces. In that case the velocity of the particle and its total kinetic energy T will remain constant ($\vec{f} = q\vec{v} \times \vec{B}$ cannot change $|v|$, only its direction). Let us also introduce the arc length s along the field line as our spatial variable, measured from the point where B is minimum along that line (the equatorial point). Now $B = B(s)$.



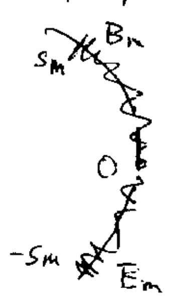
Since $M = \dfrac{T_\perp}{B} = \text{const}$ and B increases away from 0, $T_\perp$ will increase too, but of course must do so at the expense of $T_\parallel = \frac{1}{2} m v_\parallel^2$, because $T = T_\perp + T_\parallel = \text{const}$. We can write

$$T_\parallel(s) = T - M B(s)$$

where $T = T_{\text{initial}}$

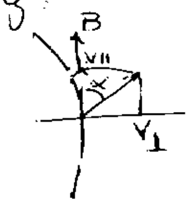
$M = \dfrac{T_{\perp\,\text{initial}}}{B_{\text{initial}}}$

As the particle spirals up the field line, a point will be reached where $T_\parallel = 0$. This is the mirror point, where all the energy is in transverse form. The particle will reverse its motion, pass through the original point and go to the other mirror point.



Only closed field lines have two mirror points and thus can trap particles. A particle on an open field line may come in (say from the solar wind), mirror near the earth, but on its outward motion it'll go out into the solar wind again.

Let us express everything in terms of pitch angle, which is an easily measurable quantity:



$$T_\parallel = \tfrac{1}{2} m v^2 \cos^2\alpha = T\cos^2\alpha$$

$$T_\perp = \tfrac{1}{2} m v^2 \sin^2\alpha = T\sin^2\alpha$$

$$M = \frac{T\sin^2\alpha_0}{B_0} \quad \text{(zero indicates initial point)}$$

Replacing in the expression for $T_\parallel$ on the previous page.

$$T\cos^2\alpha(s) = T - T\,\frac{\sin^2\alpha_0}{B_0}\,B(s)$$

or

$$\boxed{\frac{\sin^2\alpha(s)}{B(s)} = \frac{\sin^2\alpha_0}{B_0}}$$

Note how the pitch angle varies along a field line, and how it reaches 90° at the point s_m given by

$$\boxed{B(s_m) = B_m = \frac{B_0}{\sin^2\alpha_0}} \qquad B_m \text{ is the mirror field.}$$

From this expression we conclude that

$$\boxed{M = \frac{T}{B_m}}$$

Since in this case (no $\partial B/\partial t$, no other forces) $T = \text{const}$, we also will have $B_m = \text{const}$. The mirror field is an adiabatic invariant in this case.

Note that in a trapping geometry we have another periodicity: the bounce motion up and down the field line. The bounce period will be

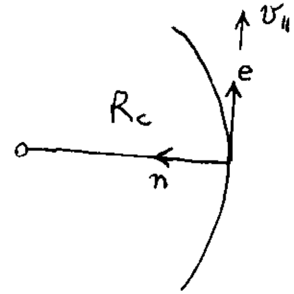
$$\tau_b = \oint \frac{ds}{v_\parallel} = \frac{2}{v}\int_{-s_m}^{s_m} \frac{ds}{\cos\alpha(s)} = \frac{2}{v}\int_{-s_m}^{s_m} \frac{ds}{\sqrt{1 - \frac{B(s)}{B_m}}}$$

between mirror points

All the above was done neglecting drifts. In the real case, the particle will, in addition to its bounce motion, drift away from the original field line.

The bounce motion leads to a new drift that we must now examine.

As a particle's guiding center moves along a curved field line, it has a centripetal acceleration given by



$$\vec{a}_c = \frac{v_\parallel^2}{R_c}\,\vec{n}$$

with R_c = radius of curvature of the field line

This means that the guiding center system is an accelerated frame of reference. In it, an inertial force $-\frac{m v_\parallel^2}{R_c}\vec{n}$ will appear acting on the particle which in turn leads to a drift (see p. 11)

$$\vec{V}_c = \frac{-m v_\parallel^2}{R_c}\,\frac{\vec{n}\times\vec{B}}{qB^3}$$

Field-geometrical calculations show that there is a relation between $\frac{1}{R_c}\vec{n}$ and the transverse gradient of B:

$$\frac{\vec{n}}{R_c} = \frac{\vec{\nabla}_\perp B}{B}$$

Thus, the curvature drift velocity becomes

$$\boxed{\vec{V}_c = \frac{m v_\parallel^2}{qB^2}\,\vec{e}\times\vec{\nabla}_\perp B}$$

and the total gradient-curvature drift (see pp 26, 27)

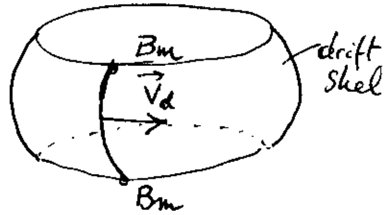
$$\boxed{\vec{V}_{GC} = \frac{m}{2qB^2}\left(v_\perp^2 + 2v_\parallel^2\right)\vec{e}\times\vec{\nabla}_\perp B}$$

with an associated current

$$\boxed{\vec{J}_{GC} = \left(\mathcal{E}_\perp + 2\mathcal{E}_\parallel\right)\frac{\vec{e}\times\vec{\nabla}_\perp B}{B^2}}$$ $\mathcal{E}$: energy densities

For particles trapped in the closed field line region this represents the ring current density.

A particle bouncing up and down a closed field line and mirroring at B_m generates a drift shell in its drift motion. This now leads to the third periodicity in adiabatic motion, the drift period.



There is a second adiabatic invariant that is a constant of motion under certain conditions which pertains to the bounce motion. It can be shown (see Roederer, Dynamics ....) that

$$\boxed{J = \oint p_\parallel\, ds}$$ is conserved

The integral is taken during a full bounce period along the field line in question, between one mirror point to the other and back. For a B-field constant in time and in absence of other forces

$$\oint p_\parallel\, ds = 2mv\int_m^{m'}\cos\alpha(s)\, ds = 2p\int_m^{m'}\sqrt{1-\frac{B(s)}{B_m}}\, ds$$

The integral

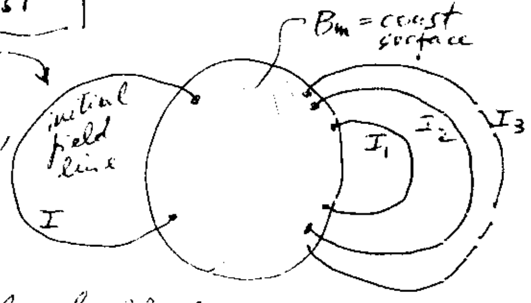
$$\boxed{I = \int_m^{m'}\sqrt{1-\frac{B(s)}{B_m}}\, ds}$$

is a purely field-geometric quantity and is constant, too, under these conditions.

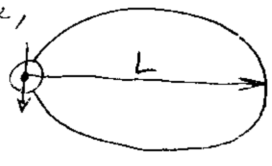
In summary, we have two adiabatic constants that for a $\partial B/\partial t = 0$, $E = 0$ conditions can be used to determine drift shells without having to integrate the drift velocity $\vec{V}_D$. These quantities are field-geometric expressions:

$$\boxed{\begin{aligned} B_m &= \text{const} \\ I &= \text{const} \end{aligned}}$$

Take the initial field line. At any other time or longitude, the particle will be occupying a field line with its mirror points on the $B_m = \text{const}$ surface. Of all the possible field lines, it will be that one for which the value I computed between mirror points is equal to the I-value on initial field line between mirror points. A computer program can be written that searches for the correct value I among approximations I_1, I_2, etc.



I is not a very intuitive quantity. McIlwain introduced a function of I, B_m which is more "visible", at least in a dipole field. It is a relation $L(I, B_m)$ whose value remains constant along a dipole field line, and which represents the radial distance (in earth radii) to the equatorial point.



This function has been tabulated (what is

tabulated is the relation $\frac{L^3 B_m}{M_E} = f\left(\frac{I^3 B_m}{M_E}\right)$).

For a non-dipole field $L = L(I, B_m)$ is no longer exactly constant along a field line (but it always is an adiabatic invariant), and it will no longer be exactly the distance to the equatorial point. However, McIlwain has shown that for an internal field with multipole terms, the effect of the latter on L is extremely small (mainly due to the fact that their greatest effect on $\vec{B}$ is near the earth, i.e., near the mirror points, i.e. near the field line region where the integrand $\sqrt{1 - \frac{B(s)}{B_m}}$ in I becomes very small!).

L and B_m are the most common invariant parameters to map trapped particle fluxes.