

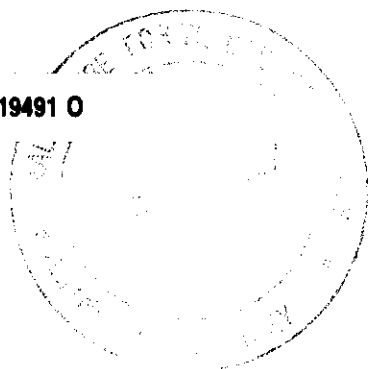
INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION



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WINTER COLLEGE ON

ATOMIC AND MOLECULAR PHYSICS

(9 March - 3 April 1987)

SYMMETRY PROPERTIES OF MOLECULAR ENERGY LEVELS
SPECTROSCOPY OF IONS AND FREE RADICALS

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SYMMETRY PROPERTIES OF MOLECULAR ENERGY LEVELS

PRIMARY CONCERN: INVARIANCE OF THE HAMILTONIAN
TWO APPROACHES, VIA TRANSFORMATION PROPERTIES

(1) OF THE HAMILTONIAN UNDER PERMUTATIONS OF IDENTICAL PARTICLES + INVERSION (PI-GROUPS), (2) OF THE NUCLEI AND THEIR DISPLACEMENTS (POINT GROUPS)

GROUP: SET OF ELEMENTS (OPERATIONS, OPERATORS, ...) WHICH SATISFY CERTAIN COMBINATION RULES

SYMMETRY OPERATIONS:

- PI GROUPS: PERMUTATIONS + INVERSION
- POINT GROUPS: BF INVERSION $\bar{E}$ (POINT), PROPER C_n AND IMPROPER S_n ROTATIONS (LINE; n MEANS ROTATION OVER $2\pi/n$) AND REFLECTIONS $\sigma_v, \sigma_h, \sigma_d$ (PLANE)
- σ_v PLANE CONTAINS C_n AXIS
- σ_h PERPENDICULAR TO C_n AXIS
- σ_d BISECTING PLANE

REPRESENTATION: A SET OF MATRICES REPRESENTING THE EFFECT OF THE GROUP OPERATIONS ON A SET OF VECTORS, FUNCTIONS, ... BASIS SQUARE $n \times n$ MATRICES (n = DIMENSION OF THE REPRESENTATION), THEIR PRODUCTS MUST FOLLOW COMBINATION RULES OF THE GROUP; THE SAME DIMENSION FOR ALL ELEMENTS OF THE GROUP.

IRREDUCIBLE REPRESENTATION: A SET OF SQUARE MATRICES OF SMALLEST POSSIBLE DIMENSION ($n = 1, 2, 3, \dots$) WHICH FORM A REPRESENTATION OF THE GROUP; THIS DIMENSION CANNOT BE REDUCED BY ANY TRANSFORMATION

GROUP NOMENCLATURE

C_n ROTATIONAL SYMMETRY FOR ROTATION OVER AN ANGLE $2\pi/n$

D_n AS ABOVE, BUT THERE ARE ALSO n C_2 AXES PERPENDICULAR TO C_n

SUFFIX v THERE IS A (NORMALLY n) σ_v PLANE
 h THERE IS A HORIZONTAL SYMMETRY PLANE

EX. $C_{2v}, C_{3v}, D_{2h}, \dots, C_{\infty v}, D_{\infty h}$

CUBIC GROUPS HAVE SPECIAL SYMBOLS:
 T_d = TETRAHEDRAL .. O_h = OCTAHEDRAL

NOMENCLATURE IR-REPRESENTATIONS *)

1-DIM.	A - SYM B - ANTISYM.	) FOR C_n	Σ
SUBSCRIPT	1 - SYM. 2 - ANTISYM.		
PRIME	' - SYM. " - ANTISYM.	) FOR σ_h OR S_n	

*) DETERMINED BY THE PROPERTY OF THE TRACE OF THE REPRESENTATION MATRICES

1, 2, 3 DIMENS. ... E
3 DIMENS. ... F

(SUBSCRIPTS & PRIMES
AS ABOVE;
 $E_1, E_2, E', \dots, F_1, F_2$)

→ FUNDAMENTAL RULE: DIM. OF A REPRESENTATION
EQUALS (ESS.) DEGENERACY OF THE Q.M. EIGEN
VALUE(S).

A, B - NON DEGENERATE
E - DOUBLY DEGENERATE
F - TRIPLY DEGENERATE

GROUPS: C_{2v}, C_{2h} ... ONLY 1-DIM. REPR.
 $C_{3v}, C_{3h}, \dots, C_{\infty v}$... 1 & 2 DIM. REPR.
 D_{2h}, D_{2d} ... ONLY 1-DIM. REPR.
 $D_{3h}, D_{3d}, \dots, D_{\infty h}$... 1 & 2 DIM. REPR.
 T_d, O_h ... 1, 2, 3 DIM. REPR.

EX. OF GROUPS & IR

$C_{2v} (H_2O)$	SYN. OPERATIONS SPECIES	$E, C_2, \sigma_v, \sigma_v'$ A_1, A_2, B_1, B_2 ($2 = A_1$)
$C_{3v} (NH_3)$	SYN. OPERATIONS SPECIES	$E, 2C_3, 3\sigma_v$ A_1, A_2, E ($2 = A_1$)
$C_{\infty v} (OH)$	SYN. OPERATIONS SPECIES	E, C_∞, σ_v $\Sigma^+, \Sigma^-, \Pi, \Delta, \dots$ ($2 = \Sigma^+$) $A_1, A_2, E_1, E_2, \dots$
D_{2h}	SYN. OPERATIONS SPECIES	$E, C_2, C_2', C_2'', \sigma, \sigma', \sigma'', i$ ($2 = E$) $A_g, B_g, B_g', B_g'', \dots$

*) VALUES OF THE PRESCRIBED TRACES (= CHARACTERS)
WHICH CHARACTERIZE IRRED. REPRESENTATIONS
CAN BE FOUND IN MANY BOOKS

$D_{3h} (BH_3)$ SYN. OPERATIONS SPECIES $E, 2C_3, 3C_2, \sigma_h, 2\sigma_v, 3\sigma_v'$
 $A_1', A_2', E', A_1'', A_2'', E''$ ($2 = A_1''$)

$D_{\infty h} (H_2)$ SYN. OPERATIONS SPECIES E, C_∞, σ_v, i ($2 = \Sigma^+$)
 $\Sigma_g^+, \Sigma_g^-, \Pi_g, \Delta_g, \dots$

$T_d (CH_4)$ SYN. OPERATIONS SPECIES $E, 8C_3, 3C_2, 6S_4, 6\sigma_d$
 A_1, A_2, E, F_1, F_2

CONVENTION: WHEN MOLECULAR CONFIGURATION
AND/OR MOLECULAR HAMILTONIAN HAS SYMMETRY
OF A CERTAIN GROUP IR. REPRESENTATIONS ($A_1, A_2,$
 $B_1, E, F_1, F_2, \dots$) ARE USED TO ORDER MOLECULAR
STATES.

GENERALLY TRANSITION IS ALLOWED WHEN

$$\langle f | A | i \rangle \neq 0$$

i.e. IS INVARIANT UNDER ALL SYMMETRY OPE
RATIONS OF THE MOLECULAR SYN. GROUP. i.e.
WHEN THE PRODUCT OF THE SYMMETRIES OF
 $|f\rangle$ AND $|i\rangle$ HAS THE SYMMETRY OF $A =$
 $= \mu_e(E_1), \mu_m(M_1), Q(E_2), \dots$

EXPERIMENTS ARE PERFORMED IN A LABORATORY (SPACE)
FIXED (SF) FRAME. FUNDAMENTAL SYMMETRY OPE
RATIONS IN THIS FRAME ARE:

SF INVERSION ($\hat{I}$) → PARITY $\hat{I}\psi = \pm\psi$

PERMUTATION ($\hat{P}$) OF IDENTICAL NUCLEI (= ELECTRONS)

$$\hat{P}\psi = \begin{cases} +\psi & \text{FOR BOSE-EINSTEIN} \\ -\psi & \text{FERMI-DIRAC STAT.} \end{cases}$$

DIATONICS

IF COUPLING BETWEEN ELECTRONIC (ψ_e), VIBRATIONAL (ψ_v), ROTATIONAL (ψ_r) AND NUCLEAR WAVEFUNCTIONS (MOTIONS) ABSENT OR WEAK THE MOL. WAVE-FUNCTION CAN BE WRITTEN AS:

$$\Psi = \psi_e \psi_v \psi_r \psi_i$$

$$\mathbb{I}\Psi = \begin{cases} +\Psi & \text{POSITIVE} \\ -\Psi & \text{NEGATIVE} \end{cases} \text{ STATES}$$

THIS APPLIES TO ALL DIATONIC MOLECULES;

$$\mathbb{P}\Psi = \begin{cases} +\Psi & \text{SYMMETRIC (S)} \\ -\Psi & \text{ANTISYMMETRIC (a)} \end{cases} \text{ STATES}$$

SELECTION RULES:

PARITY CHANGE E1 M1 E2 ...
YES NO NO ...

EXCHANGE (P) $s \leftrightarrow s, a \leftrightarrow a, s \not\leftrightarrow a$
(CONSERVATION OF THE NUCLEAR SPIN SYM.)

ELECTRONIC ORBITAL STATES

SYM. OPERATIONS: $C_e(z), \sigma_v, \hat{i}$ (HOMONUCLEAR)

$$C_e(z)$$

MATRIX REPRESENTATION

$$C_e(z) = \begin{pmatrix} \cos n\alpha & -\sin n\alpha \\ \sin n\alpha & \cos n\alpha \end{pmatrix} \quad \Sigma = n\alpha$$

NON-DEGENERATE REPR. (STATES)

(E) INVARIANT UNDER $C_e \rightarrow$ TRANSFORM WITH
 $n=0 \rightarrow \Sigma$ STATES

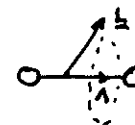
DOUBLY DEGENERATE REPR. (STATES): THE TWO

STATES TRANSFORM AS

$$\begin{aligned} 1. \alpha &\rightarrow \pi \\ 2. \alpha &\rightarrow \Delta \\ 3. \alpha &\rightarrow \Phi \text{ ETC.} \end{aligned}$$



i.e. $n = \Lambda = \pm 1, \pm 2, \dots$



ENERGY DEPENDS ONLY ON Λ^2 ,
HENCE THE DOUBLE DEGENERACY (Λ -DEGENERACY)
 C_e LABEL NOT SUFFICIENT TO SPECIFY THE STATE COMPLETELY

$$\sigma_v$$

$$\sigma_v^2 = 1 \rightarrow \sigma_v \psi_e = \pm \psi_e \rightarrow \Sigma^+, \Sigma^-$$

$\uparrow$ PARITY OF THE ORBITAL
REL. STATE

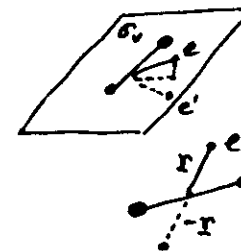
NOT FOR DEGENERATE STATES

$$\hat{i}$$

$$\hat{i}^2 = 1 \rightarrow \hat{i} \psi_e = \pm \psi_e$$

$+$ = GRADE
 $-$ = UNGRADE

FOR ALL STATES: $\Sigma_{g,u}^{\pm}, \Pi_{g,u}, \dots$



PARITY: CAN BE WORKED OUT ASS. ψ_i HAS A DEFINITE
(+) PARITY AND USING THE EQUIVALENCE

$$\mathbb{I} \equiv \sigma_v$$

$$\begin{aligned} \mathbb{I}\Psi &= (\mathbb{I}\psi_e)(\mathbb{I}\psi_v)(\mathbb{I}\psi_r) \\ &= (\sigma_v \psi_e)(+\psi_v)(-)^3 \psi_r \text{ ETC.} \end{aligned}$$

EXCHANGE: USE THE EQUIVALENCE

$$\mathbb{P} \equiv \mathbb{I} \cdot \hat{i} = \sigma_v \cdot \hat{i}$$

$$\Psi = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(+)(-)^3\left(\frac{1}{2}\right) = \begin{cases} + & \text{FOR BE} \\ - & \text{FOR FD} \end{cases}$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow \\ \psi_e & \psi_v & \psi_n & \psi_x \end{matrix} \begin{cases} + & \text{FOR ORTHO} \\ - & \text{FOR PARA} \end{cases}$$

ORTHO : $I_1 + I_2 = 1$
 PARA : $I_1 + I_2 = 0$

EX. (H_2) , Σ^+ , $I = 1/2$ THREE NUCL. SPIN STATES FOR ORTHO H_2 :
 $\uparrow\uparrow, \uparrow\downarrow, \downarrow\uparrow, \downarrow\downarrow$ SYM. (+)
 ONE STATE FOR PARA $\uparrow\downarrow - \downarrow\uparrow$ ANTISYM. (-)
 $(\frac{1}{2})(+)(+)(-)^3\left(\frac{1}{2}\right) = (-)$ ORTHO $J = 1, 3, 5, \dots$
 PARA $J = 0, 2, 4, \dots$

INTENSITY RATIO $R = \frac{q(\text{ORTHO})}{q(\text{PARA})} = \frac{3}{1}$

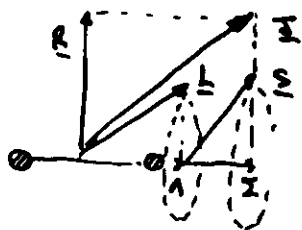
O_2 , Σ_g^- , $I = 0$ (BE)

$(\frac{1}{2})(-)(+)(-)^3\left(\frac{1}{2}\right) = (+) \rightarrow J = 1, 3, 5, \dots$
 $(\psi_x (+) \text{ FOR } I_2 = 0)$ LINES MISSING

ELECTRONIC SPIN

HUND'S COUPLING CASES

(a) $L \ll \Lambda, S \ll \Lambda$



CONSERVED:
 $J, \Lambda, \Sigma(S); \Omega = |\Lambda + \Sigma|$

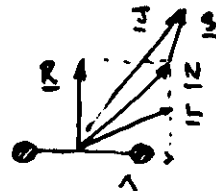
$$\begin{cases} L, S, \dots, -S \\ L, \dots, -L \end{cases}$$

STATES: $2S+1 \Lambda_\Omega$

EX. ${}^2\Pi_{1/2}, {}^2\Pi_{3/2}$

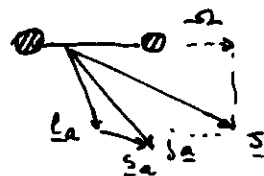
ANALOGOUS TO RUSSEL-SAUNDERS COUPLING

(b) S DECOUPLED FROM Λ



$N = R + \Lambda \dots J = N + S \rightarrow J = N+S, \dots, |N-S|$
 CONSERVED: $J, \Lambda; N$ ASSUMED
 (IF $N \ll S$ COUPLING WEAK)

(c) $L \ll S$ DECOUPLED FROM Λ



CONSERVED $J, \Omega \rightarrow 2S+1 \Omega^2$
 THESE ARE LIMITING CASES -
 REAL MOLECULES ARE ALL
 INTERMEDIATE

SPIN-ORBIT

$$\mathcal{H}_{so} = \sum_{i,j} \gamma_{ij} (\mathbf{r}_i \cdot \mathbf{r}_j) \mathbf{L}_i \cdot \mathbf{S}_j$$

$$\Rightarrow \Lambda \cdot S \Rightarrow \Lambda \Sigma$$

PRODUCES MULTIPLETS; EX. ${}^2\Pi_{1/2}, {}^2\Pi_{3/2}$
 INDICATED BY Ω

$\Lambda > 0$ NORMAL
 $\Lambda < 0$ INVERTED

> FINE STRUCTURE $\left(\begin{smallmatrix} \Pi_{1/2} \\ \Pi_{3/2} \end{smallmatrix} \right)$ LOWER
 SPECTRUM CONSISTS OF SEPERATE LADDERS OF
 LEVELS; TRANSITIONS BETWEEN MULTIPLETS (LADDERS)
 POSSIBLE IF $\Delta\Omega = \pm 1$ VALID

SPIN-ROTATION

$$\mathcal{H}_{sr} = \gamma N \cdot S$$

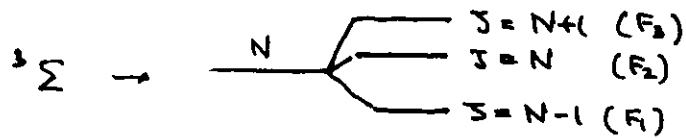
IMPORTANT IN ${}^2\Sigma, {}^3\Sigma, \dots$ STATES (PRESENT ALSO IN $\Pi, \dots$)

$$J = N + S \rightarrow N \cdot S = \frac{1}{2} (J^2 - N^2 - S^2)$$

$$\Rightarrow \frac{1}{2} [J(J+1) - N(N+1) - S(S+1)]$$

ALL LEVELS SPLIT INTO $2S+1$ SUBLEVELS

$${}^2\Sigma \rightarrow \begin{matrix} N & \swarrow & \searrow \\ & J = N + 1/2 & (F_2) \\ & J = N - 1/2 & (F_1) \end{matrix}$$



Σ - DOUBLING, TRIPLING, ...

$$O_2 \dots \gamma = -252.7 \text{ MHz}$$

SPIN-SPIN ($^3\Sigma, \dots$)

$$\mathcal{H}_{ss} = \sum_{ij} \vec{r}_{ij}^{-5} [(\vec{S}_i \cdot \vec{S}_j) r_{ij}^2 - 3(\vec{r}_{ij} \cdot \vec{S}_i)(\vec{r}_{ij} \cdot \vec{S}_j)]$$

$$\Rightarrow \lambda(3S^2 - S^2)$$

$$O_2 \dots \lambda = 39667 \text{ MHz}$$

Λ -DOUBLING ($\Pi, \Delta, \dots$)

$$\mathcal{H} = B\vec{R}^2 + A\vec{L} \cdot \vec{S} \dots \vec{R} = \vec{J} - \vec{L} - \vec{S}$$

SPLITTING OF ROTATIONAL LEVELS (INTO DOUBLETS OF OPPOSITE PARITIES); REMOVAL OF THE Λ -DEGENERACY OF ELECTRONIC STATES WITH $\Lambda \neq 0$
ORIGIN: INTERACTION OF ELECTRONIC MOTION AND ROTATION (BREAKDOWN BO-APPROX.)

• MIXING OF STATES ($\Pi, \Sigma, \Pi, \Delta, \dots$)

SPLITTINGS:

$^1\Pi$	$qJ(J+1)$
$^2\Pi$ case (b)	$qN(N+1)$
$^2\Pi_{1/2}$	(a) $a(J+1/2)$
$^2\Pi_{3/2}$	(a) $b(J-1/2)(J+1/2)(J+3/2)$
$^3\Pi$	(b) $qN(N+1)$

$\equiv X.OH$

8

VIBRATION NOT EXCITING

ROTATION A LITTLE

NO L, S



$$|JM\rangle = Y_{JM}(\theta)$$

$L, S \neq 0$



MOL. BEHAVES LIKE A SYMMETRIC TOP WITH $K = J_z$

$$|J, M\rangle = D_{M,0}^J(\omega, \beta, \gamma)$$

SELECTION RULES (FOR ψ_e, ψ_v, ψ_r)

→ HYPERFINE STRUCTURE

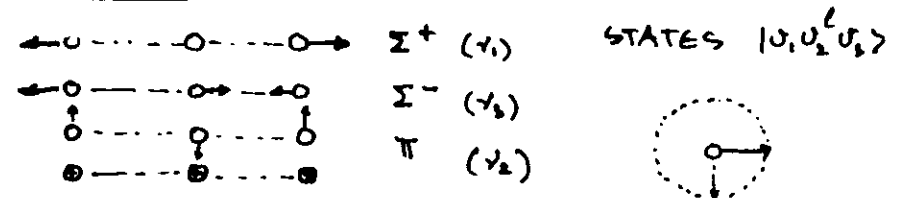
TRIATOMICS

(1) LINEAR ($HCO^+, N_2H^+, \dots$)

GROUPS: $C_{\infty v}, D_{\infty h}$

VIBR. ELECTRONIC SPECIES: $\Sigma^+, \Pi, \Delta, \dots$ (ev. g, u)

VIBRATIONS ($3N-5=4$)



COMBINATION OF TWO $\perp$ VIBRATION (LET SAY 90° OUT OF PHASE) CAN BE SEEN AS A VIBRATIONAL ANGULAR MOMENTUM; ITS MAGNITUDE

$$L \dots l = 0, 2, 4, \dots, 0 \text{ OR } 1$$

ENERGY DEPENDS ONLY ON $l^2 \rightarrow$ l -DEGENERACY

ELECTRONIC STATES

AS LONG AS THE MOLECULE REMAINS LINEAR

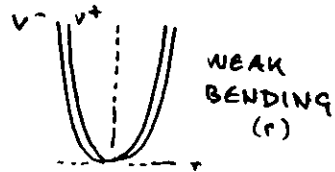
(ν_1, ν_3 VIBRATIONS) OR THE ELECTRONIC STATE IS NON-DEGENERATE SITUATION IS THE SAME AS IN DIATOMIC MOLECULES. QUITE DIFFERENT IN DEGENERATE ELECTRONIC STATES WHEN THE ν_2 (BENDING) VIBRATION IS EXCITED

JENNER-TELLER EFFECT

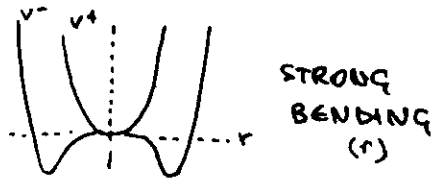
DURING THIS VIBRATION e.g. $D_{\infty h}$ (LIN.) SYMMETRY IS LOWERED TO C_{2v} IN WHICH THERE ARE NO DEGENERATE ELECTRONIC STATES. SO THERE WILL BE SPLITTING OF VIBR. LEVELS IN DEG. ELECTRONIC STATES INTO PAIRS OF SPECIES A_1, A_2, B_1, B_2 OF C_{2v}

REASON:

ELECTRONIC POT. (SOLUTION OF THE ELECTRONIC WAVE EQUATION) IS DIFFERENT IN THE LINEAR AND IN THE BENT CONFIGURATION



WEAK BENDING (r)

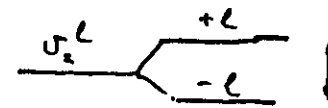


STRONG BENDING (r)

PRIMARILY DUE TO THE DIPOLE MOMENT (μ) INDUCED BY THE BENDING

GENERAL RULE: ESS. DEGENERACY IS TOTALLY OR PARTLY REMOVED WHEN THE SYMMETRY IS LOWERED; ONLY PERTURBATION OF SYMMETRY LOWER THAN THE "ORIGINAL" ONE CAN SPLIT DEGENERATE LEVELS.

L-DOUBLING



REMOVAL OF L-DEGENERACY BY THE VIBRATION - ROTATION (CORIOLIS-TYPE) INTERACTION

$$X_e(i) \approx k$$

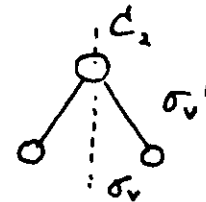
$$\Delta E = qJ(3H)$$

L-DEGENERACY CAN ALSO BE REMOVED BY L-TYPE RESONANCES (INTERACTION BETWEEN DEG. AND NON-DEG. VIBRATIONAL MODES DUCED BY α -ROTATION)

BENT CONFIGURATION ($NH_2, CH_2, \dots, H_2O$)

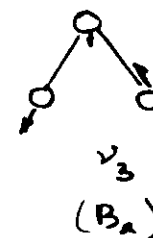
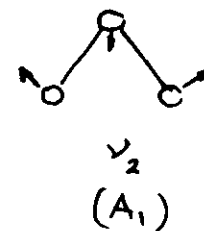
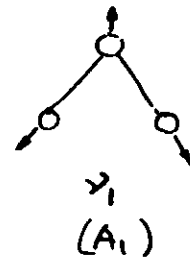
NOTE: MOLECULE CAN BE BENT IN ONE (e.g. GROUND) AND LINEAR IN ANOTHER (e.g. EXCITED) ELECTRONIC (OR EVEN VIBR.) STATE.

SYMMETRY: C_{2v} ; ν_2 E STATES: A_1, A_2, B_1, B_2 (WITH MULTIPLICITIES)



LONG PAIR

VIBRATIONS; LEVELS (ν_1, ν_2, ν_3)



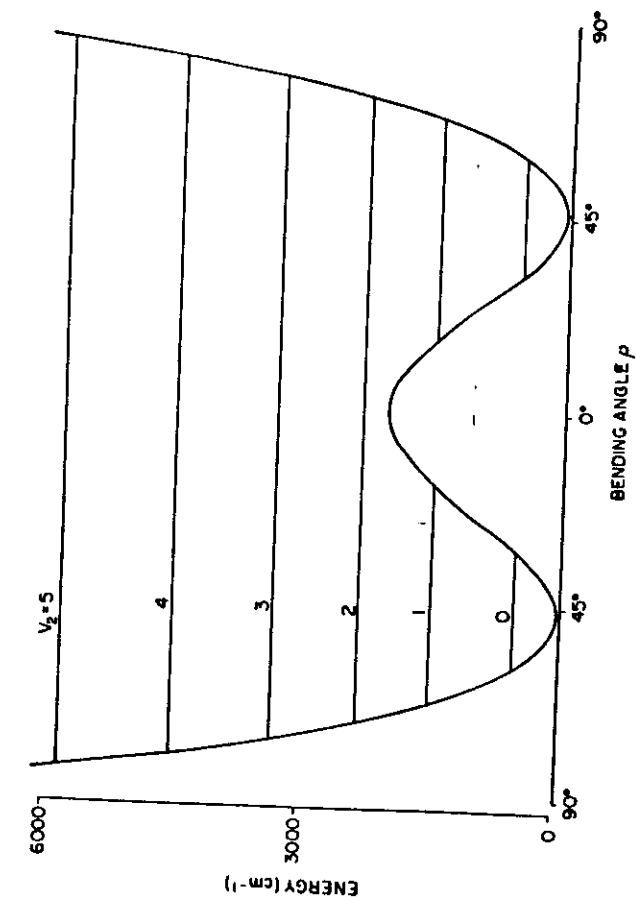


FIG. 3. The experimentally determined effective bending potential energy function for CH_2 in the $v_1 = v_3 = 0$ level of the X^3B_1 ground state. Used by permission [48].

3. NH_2

There: NH_2 and Ramsay [considerable Ramsay a state hav cm^{-1}) we observed

Dubois, 1 ($\text{C}_6\text{H}_5\text{SiH}_3$ in SiH_4 system [the group: trum con: The two: tion. Fr Renner e be ~8000 matrices) of the mo:

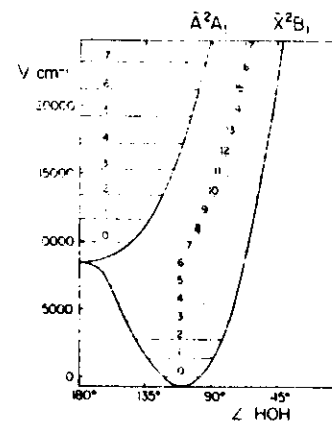


Figure 5. Bending potential curves for the X^3B_1 and A^2A_1 states of H_2O^+ (cf. the caption for figure 1).

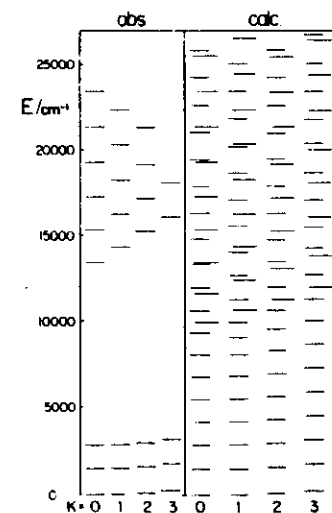


Figure 6. Observed and calculated vibronic level patterns in H_2O^+ (cf. the caption for figure 2); observed levels marked with broken lines are taken from the photoelectron data [9].

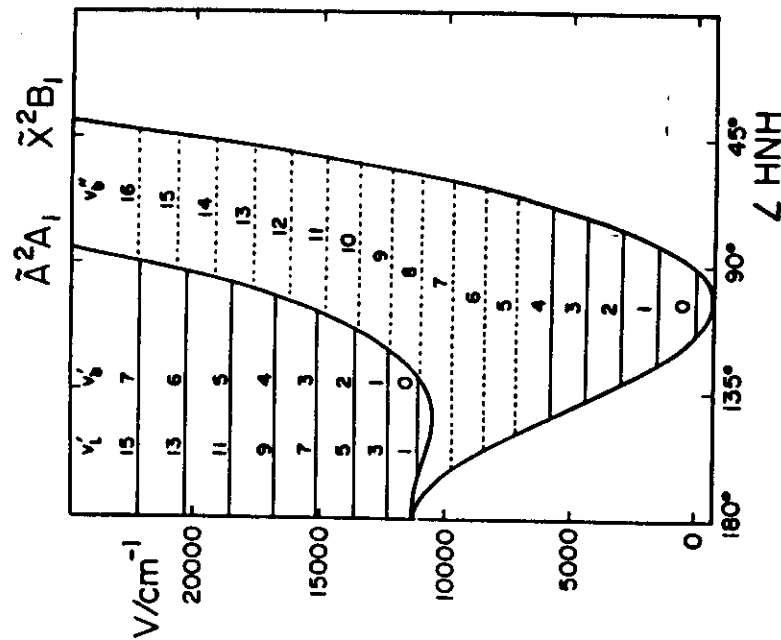


FIG. 4. Bending potential energy curves for the $\tilde{X}^2B_1$ and $\tilde{A}^2A_1$ states of NH_2 . The positions of the observed (—) and calculated (---) $K = 0$ bending levels are indicated. Vibrational numberings of the excited state levels are given for both linear (v_2') and bent (v_2'') notations (adapted from and used by permission [75]).

FIG. 5.
served levels
om

VIBRATIONAL & ELECTRONIC SPECTRA SIMPLE
BECAUSE ALL EL. & FUNDAMENTAL VIBRATIONAL
STATES NON-DEGENERATE

ROTATIONAL SPECTRA - COMPLICATED;
ASYMMETRIC TOP .. LEVELS $J_{K_1, K_2} = J_c$

$$J = K_1 - K_2$$

SYMMETRY OF THE ROTATIONAL LEVELS
DETERMINED BY THE IR OF THE V-GROUP
(E, C_2^x, C_2^y, C_2^z)

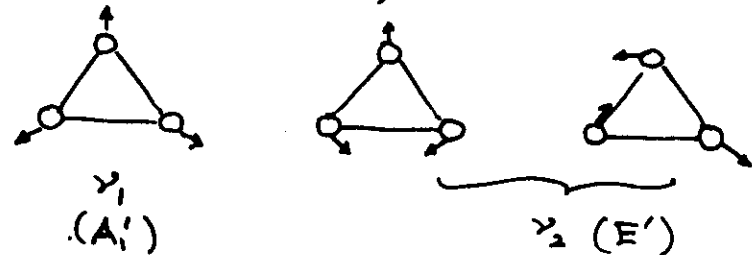
FOR SYMMETRIC BENT ($H_2O, CH_2, \dots$)
THERE ARE TWO ($I_a = 1/2$) OR MORE ($I_a = 1, \dots$;
EX. D_2O) MOLECULAR SPECIES (ORTHO,
PARA, META, ...)

BENT SYMMETRICAL ($H_3^+, H_3, \dots$)

SYMMETRY D_{3h} ; V & E SPECIES: A_1'', A_2'', E''

→ THERE CAN BE BOTH NON-DEG. AND DEGE-
NERATE VIBR. & ELECTRONIC STATES

VIBRATIONS ($3N - 6 = 3$)



VIBR. LEVELS ($0, 1, 2, \dots$) $l = 0, 2, 4, \dots, 0$ OR 1
 l - DEGENERATE

ℓ -DOUBLING: $\rightarrow$ SPLITTING OF ℓ -DEG. LEVELS

ROTATIONAL: CORIOLIS COUPLING BETWEEN A NON-DEGENERATE AND A DEGENERATE MODE DUE TO ROTATION ABOUT THE X, Y AXES; ALSO $\nu_6, \nu_7 \in R_2$

VIBRATIONAL: ANHARMONIC COUPLING BETWEEN A NON-DEG. AND A DEGENERATE MODE

ROTATION: SYMMETRIC TOP

$$E = B J(J+1) - (B-A)K^2 \rightarrow K\text{-DEGENERATE}$$

K-TYPE DOUBLING: SPLITTING OF ROTATIONAL LEVELS WITH $K = 3n; n=1, 2, \dots$

DUE TO THE TERM $\mathcal{H}_K = h_3 (J_+^6 + J_-^6)$ ($C_{3v}, \dots$)

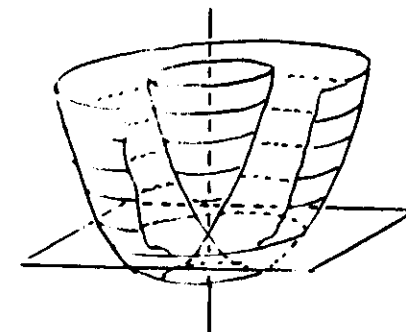
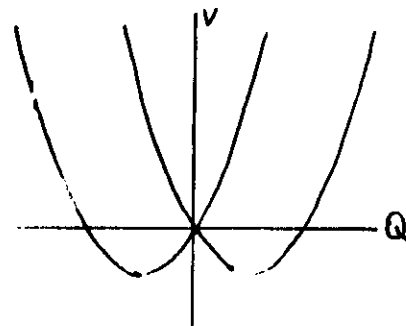
WHICH GIVES MATRIX ELEMENTS OFF-DIAGONAL IN K

JAHN-TELLER EFFECT

THEOREM: FOR A NON-LINEAR MOL. IN A DEGENERATE ELECTRONIC STATE, THERE EXISTS A DISTORTION OF THE NUCLEI ALONG AT LEAST ONE NON-TOTALLY SYMMETRIC NORMAL COORDINATE THAT RESULTS IN A SPLITTING OF THE POTENTIAL ENERGY FUNCTION SO THAT THE POTENTIAL MINIMUM IS NO LONGER AT THE SYMMETRICAL POSITION

$$\mathcal{H}'_{JT} = \dots k_i^{+-} Q_i^+ + k_i^{-+} Q_i^- \quad \text{LINEAR JT-} \\ + q_i^{++} Q_i^2 + q_i^{--} Q_i^2 \quad \text{QUADR. JT-}$$

k_i, q_i : CONSTANTS DEPENDING ON THE ELECTRONIC STRUCTURE; $Q_i^2 = Q_i^+ Q_i^- + Q_i^- Q_i^+$



WITH ONLY LINEAR INTERACTION

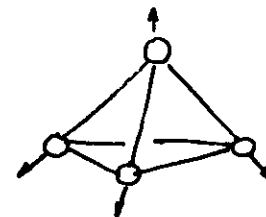
WHEN QUADRATIC TERMS ARE INCLUDED THERE ARE THREE (FOR $C_{3v}, D_{3h}, \dots$) BOWLS SEPARATED BY SADDLE POINTS, ALL OF WHICH ARE JOINED TOGETHER IN THE CENTER BY A CONICAL PEAK.

WHEN THE INTERACTION IS WEAK MOLECULE CAN TUNNEL BETWEEN THE BOWLS, FOR STRONG INTERACTION MOLECULE BECOMES NON-SYMMETRIC VERY COMPLICATED SPLITTING & PERTURBATION IS

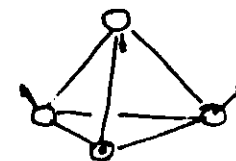
FOUR-ATOMICS ($H_2O^+, CF_3, \dots, NH_3$)

ONLY C_{3v} SPECIES A_1, A_2, E

VIBRATIONS ($3N-6 = 6$) $2A_1 (\nu_1, \nu_2) + 2E (\nu_3, \nu_4)$



$\nu_1(A_1)$



$\nu_2(A_1)$

VIB. LEVELS ($0, \nu_2, \nu_3^l, \nu_4^l$)

18
ESSENTIALLY SAME INTERACTIONS (BUT THERE IS ONLY ONE DEGENERATE VIBRATION) AND STRUCTURE OF ENERGY LEVELS AS IN X_3 + INVERSION POT. SEE HO_3^+

FIVE-ATOMICS ($NH_4, \dots, CH_4$)

ONLY T_d SYMMETRY (C_{3v} AS ABOVE)

VIB. & ELECT. SPECIES ... A_1, A_2, E, F_1, F_2
(1, 2 SYM/ANTISYM. σ_d, S_4)

VIBRATIONS $\nu_1(A_1), \nu_2(E), \nu_3(F_2), \nu_4(F_2)$
($3N-6=9$)
VIB. LEVELS ($0, U_2^2, U_3^2, m_3, U_4^2, m_4$)

ROTATION SPHERICAL TOP

$$E_r = BJ(J+1)$$

DEGENERACY $(2J+1)^2$

PERTURBATIONS:

$$T^4 = (70)^{1/2} T_0^4 + 5T_{+4}^4 + 5T_{-4}^4$$

$$T^6 = (14)^{1/2} T_0^6 - 7T_{+4}^6 - 7T_{-4}^6$$

$$T = C [(T^4 \cos \phi + T^6 \sin \phi) \cos \Theta + T^8 \sin \Theta]$$

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 T_d SYMMETRY

$A_1, A_2 \dots 1x$

$E \dots 2x$

$(T_1, T_2) F_1, F_2 \dots 3x$

$$J=54 \dots 2J+1 = 109$$

FINE STRUCTURE

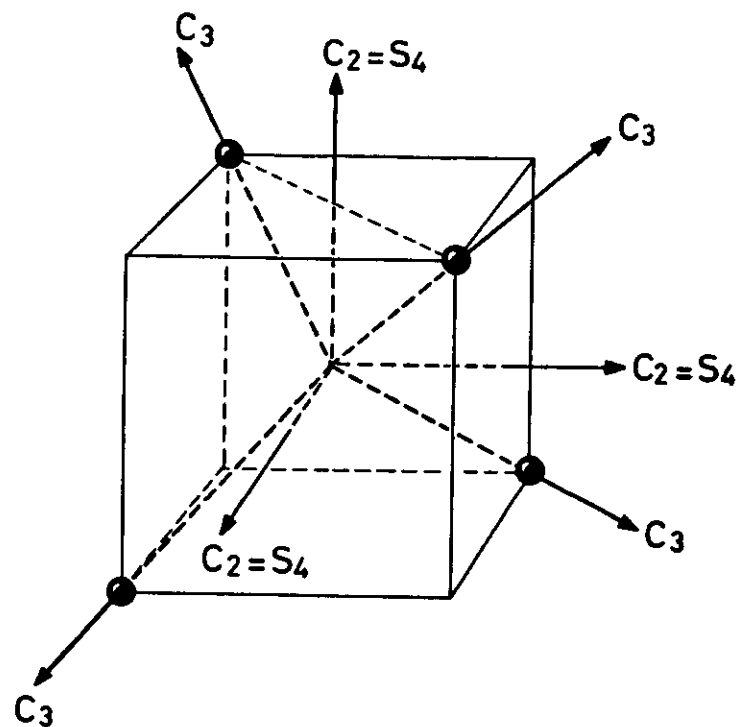
$$5A_1 + 5A_2 + 9E + 13F_1 + 14F_2 \uparrow$$

AT LEAST 46 LEVELS EXPECTED

→ ONLY 15-20 LINES OBSERVED
(FOR CF_4)

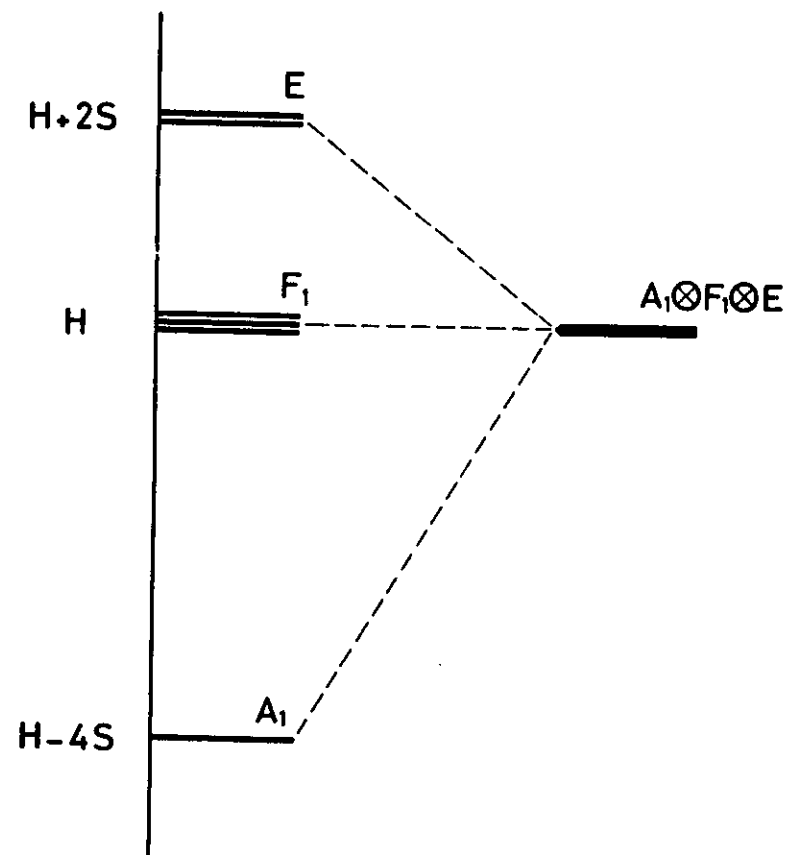
$J=54 \dots K=47$	$S' = 120 \text{ MHz}$
48	2.6 MHz
:	
52	9.4 Hz
53	9.7 mHz
54	3.5 mHz

SPLITTING OF THE CLUSTERS:
= SUPERFINE STRUCTURE



T_d	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$	
A_1	1	1	1	1	1	
A_2	1	1	1	-1	-1	
E	2	-1	2	0	0	
F_1	3	0	-1	1	-1	(R_x, R_y, R_z)
F_2	3	0	-1	-1	1	(x, y, z)

splitting of a $A_1 \otimes F_1 \otimes E$
 C_4 cluster



PRODUCTION METHODS

DISCHARGE - HALLOW CATHOD
- POSITIVE COLUMN

REACTION - F ATOMS FROM F_2 , CF_4 + He DISCH.
- O FROM O_2 DISCHARGE
- H FROM H_2 OR H_2O
DISCHARGE (USUALLY MICROWAVE)

LASER PHOTOLYSIS

- EXCIMER LASER (REMPI)

SURFACE SPUTTERING

- ACCELERATED ATOMIC IONS

ION BEAMS

EXTRACTION FROM AFTERGLOW (PENNING)

CONTROLLED ELECTRON IMPACT OF
MOLECULAR (SUPERSONIC) BEAMS

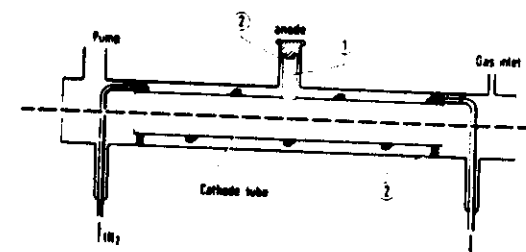
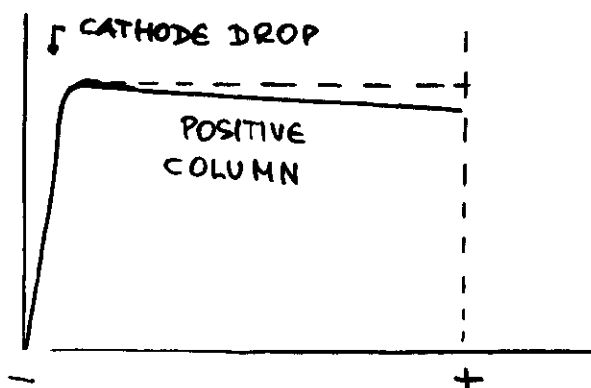


Fig. 1 Glass absorption cell with cold hollow cathode.

1. Glass insert 2. Teflon seals

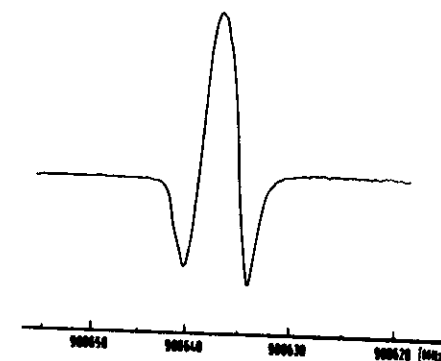


Fig. 2 The $J' = 11 + J = 10$ rotational transition of HCO^+ ($v = 0$) observed using frequency modulation. Integration time 1 s, cell pressure 4 Pa, discharge current 300 mA.

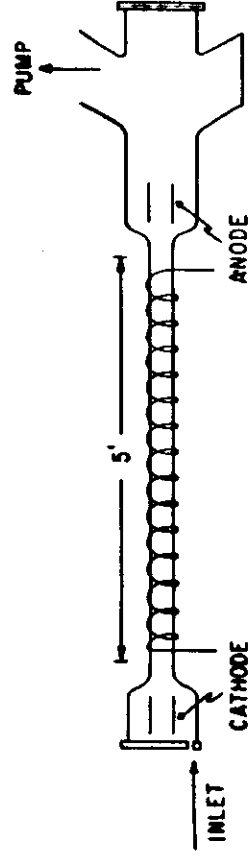
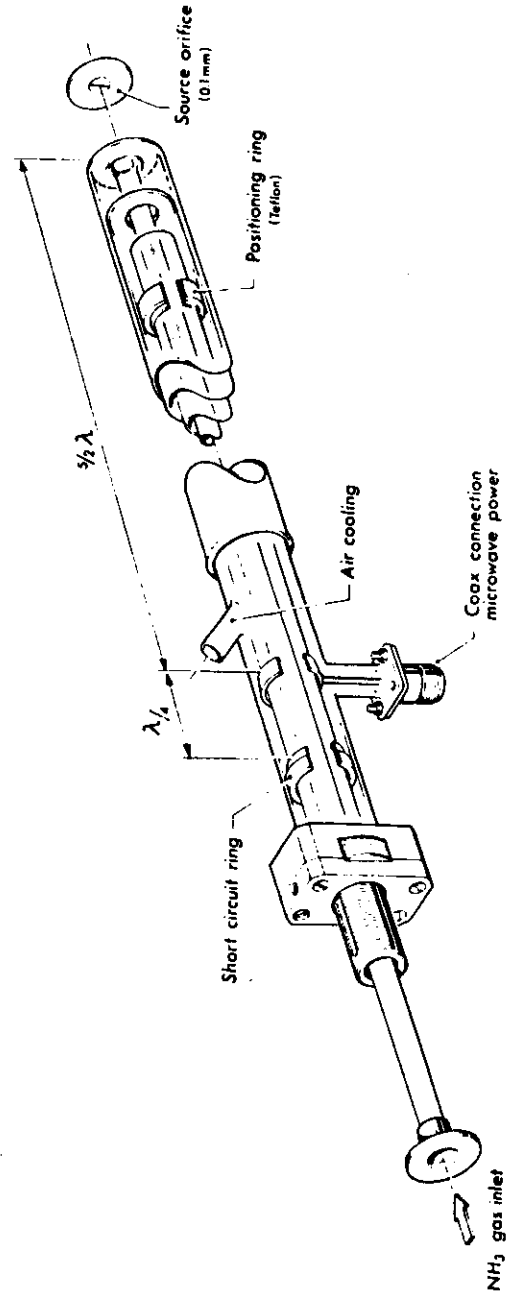
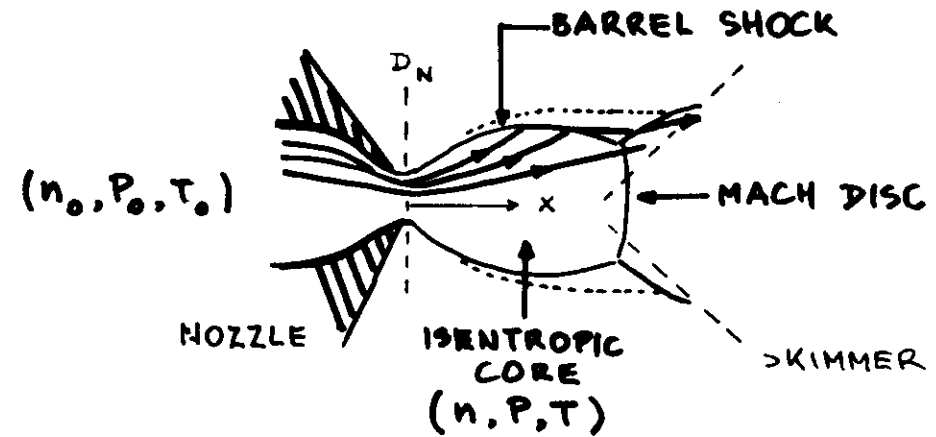
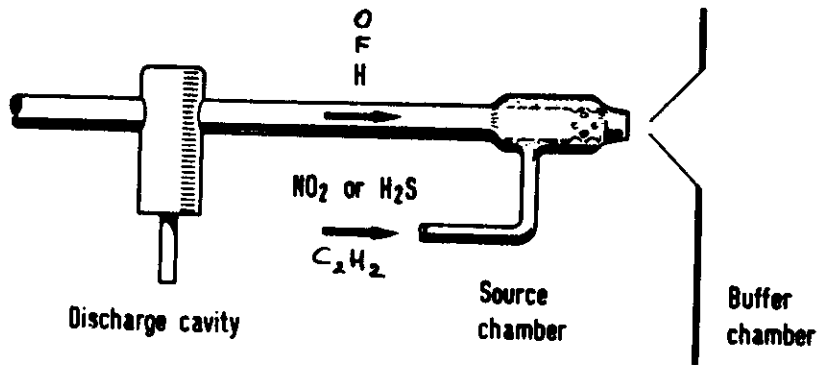


FIG. 1. Diagram of apparatus for the production of molecular ions.



SUPERSONIC NOZZLE BEAMS



ISENTROPIC ($ds=0$) EXPANSION FROM HYDRODYNAMIC TO FREE MOLECULAR FLOW ($n(x) \propto x^{-2}$). EXPANSION IS SO FAST THAT THE GAS CANNOT EXCHANGE ENERGY WITH WALLS ETC., AND ANISOTROPIC ($T_{||}, T_{\perp}$)

RELEVANT QUANTITIES:

SPECIFIC HEAT RATIO $\gamma = C_p/C_v$
(5/3 MONATOMIC, 7/5 DIATOMIC, $\rightarrow$ 1 POLYATOMIC)

LOCAL SPEED OF SOUND $a = (\gamma kT/m)^{1/2}$

MOST PROBABLE VELOCITY $\alpha = (2kT/m)^{1/2}$

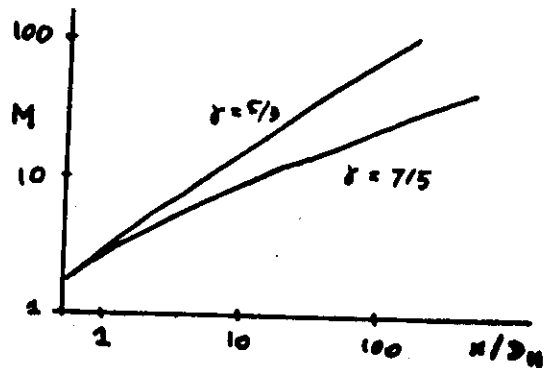
STREAM VELOCITY u

SPEED RATIO $S = u/\alpha$ (u/α_s)

MACH NUMBER $M = U/a = (2/\gamma)^{1/2} S$

$$M = A \left(\frac{x}{D_N} \right)^{\gamma-1} - \frac{1}{2} \left(\frac{\gamma+1}{\gamma-1} \right) / A \left(\frac{x}{D_N} \right)^{\gamma-1}$$

$$(A = A(\gamma) \dots 3.26 - 6.44)$$



TERMINAL MACH NUMBER

$$M_T \approx 1.17 K_{N_N}^{-0.4} = 133 (P_0 D_N)^{0.4} \quad (Ar)$$

$$(K_{N_N} = \lambda_0 / D_N; P_0 \text{ IN ATM., } D_N \text{ IN CM})$$

POSITION MACH DISK $x_M \approx 0.67 (P_0 / P_b)^{1/2} D_N$

(P_b - BACKGROUND PRESSURE, BEHIND THE SKINER)

TERMINAL STREAM VELOCITY

$$U_M = \left(\frac{2}{\gamma-1} \right)^{1/2} a_0 = \left(\frac{\gamma}{\gamma-1} \right) a_0$$

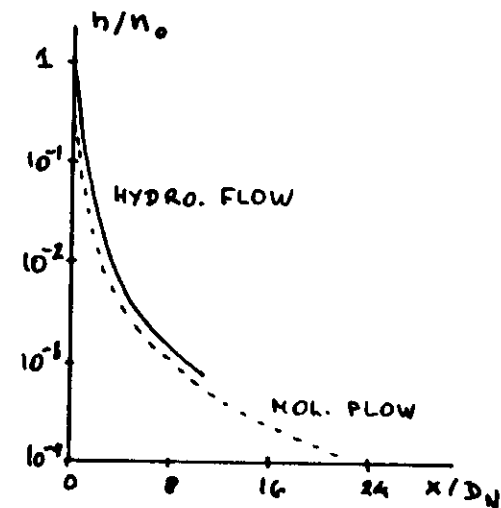
$\rightarrow \sqrt{3}$ FOR MONATOMIC GAS

HIGH INTENSITY

(PRECOLLIMATION, DIRECTED FLOW, LOWERING OF a AND HENCE INCREASE OF M VIA COOLING)

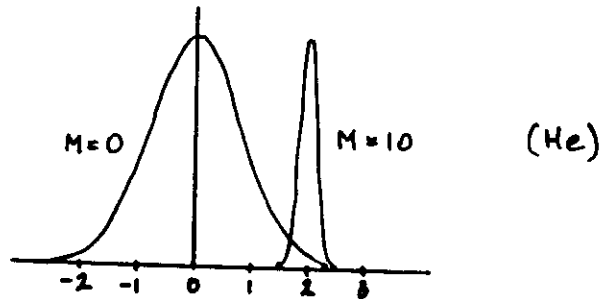
$$\frac{I(0)_N}{I(0)_{OR}} \approx \left(\frac{\gamma}{2} M_s^2 + \frac{3}{2} \right) \quad (M_s > 4)$$

$$\frac{n}{n_0} = \left[1 + \frac{\gamma-1}{2} M_s^2 \right]^{-1/(\gamma-1)} \quad (\text{FREE JET})$$



NARROW VELOCITY DISTRIBUTION

$$n(u) = u^2 \exp \left[-5^2 \left(\frac{u}{u_0} - 1 \right)^2 \right]$$



$$\frac{\Delta u}{u} \propto \frac{\alpha_3}{u_0} = \left(\frac{2}{\gamma} \right)^{1/2} M_0^{-1}$$

$$\frac{\Delta u}{u_0} \propto \frac{\alpha_3}{u_0} \propto \left(\frac{2}{\gamma} \right)^{1/2} M_0^{-1} \quad (\text{ANG. SPREAD})$$

$$\chi_{\frac{u}{u_0}} = M_0 \left[\frac{\gamma}{3 + \frac{\gamma}{2} (\gamma - 1) M_0^2} \right] = M_0 \left(\frac{\gamma}{3} \frac{T_0}{T_0} \right)^{1/2}$$

$$\rightarrow \left(\frac{2}{3} \frac{\gamma}{\gamma - 1} \right)^{1/2} \quad \text{FOR } M_0 \rightarrow \infty$$

= 1.53 DIATOMIC

1.29 MONATOMIC GAS

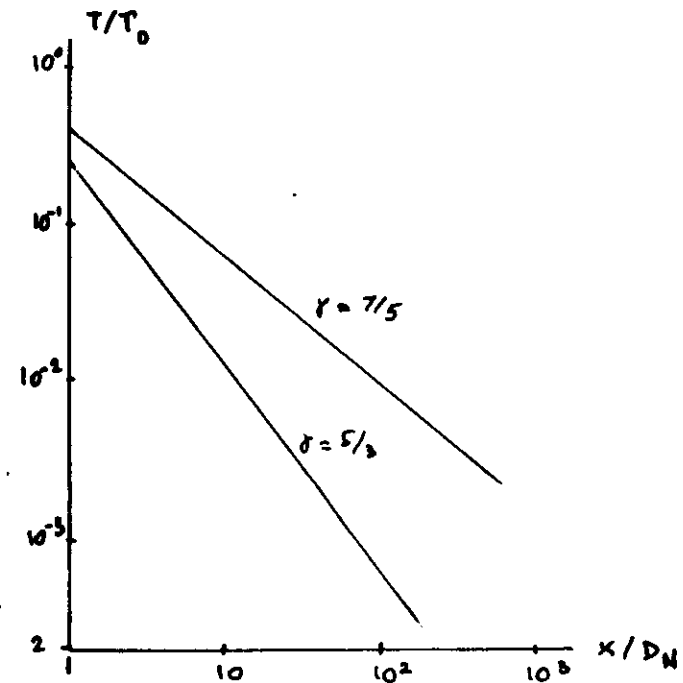
LOW TEMPERATURE

ISENTROPIC REGIME $\left(\frac{\partial T}{\partial v} \right)_s < 0$

CONSERVATION OF ENTHALPY

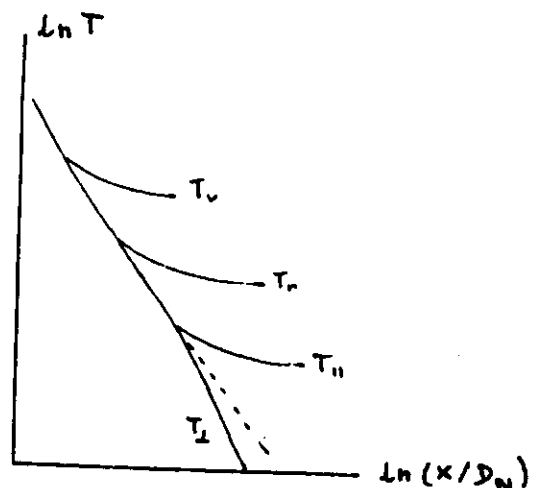
$$h_0 = \frac{1}{2} m \bar{u}^2 + \frac{3}{2} k T_{\parallel} + \frac{3}{2} k T_{\perp} + \int_0^{T_{\parallel}} C_p dT + \int_0^{T_{\perp}} C_v dT$$

$$\frac{T}{T_0} = \left[1 + \left(\frac{\gamma - 1}{2} \right) M^2 \right]^{-1}$$



$$T(x) \propto x^{-2(\gamma-1)}$$

FULL EXPANSION



$$T_{II} = \frac{1}{2} \frac{m}{k} \left(\frac{U}{S} \right)^2$$

$$\rightarrow \frac{\gamma}{\gamma-1} \frac{T_0}{q^2} \quad (q \gg 1)$$

$$\propto T_0^{1.3} (P_0 D_N)^{-1.0}$$

He, Ne, Ar

$$\propto T_0^{0.8} (P_0 D_N)^{0.6}$$

SF₆, ...

CONDENSATION PHENOMENA

- FORMATION OF CLUSTERS
((H₂)_n, Ne Ar, ...)

SEEDED BEAMS

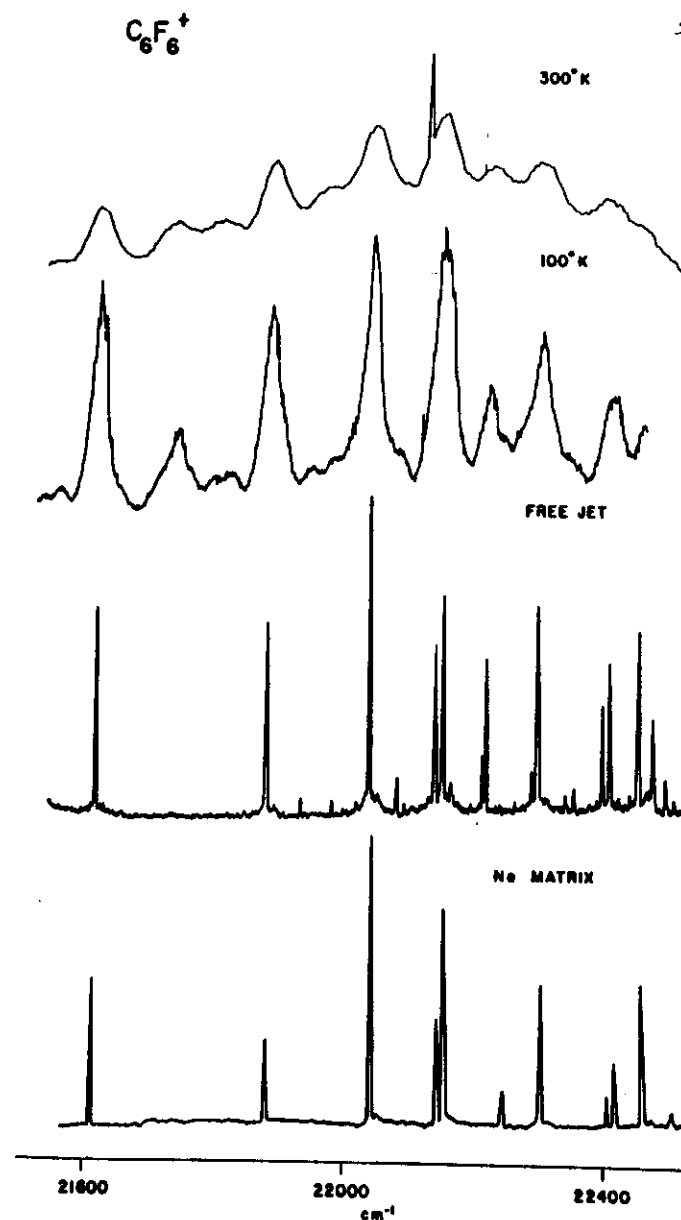


Figure 2.

Laser excitation spectra of C₆F₆⁺ taken under a variety of conditions. Preceding upward from the bottom (i) C₆F₆⁺ in a Ne matrix at 4°K, (ii) C₆F₆⁺ produced via 2-photon ionization by an ArF laser in a free jet expansion, (iii) C₆F₆⁺ produced by Penning ionization with the flow system at liquid N₂ temperature, (iv) same as (iii) except flow system at room temperature. The Ne matrix spectrum has been shifted in absolute frequency so that its origin coincides with those of the gas phase spectra.

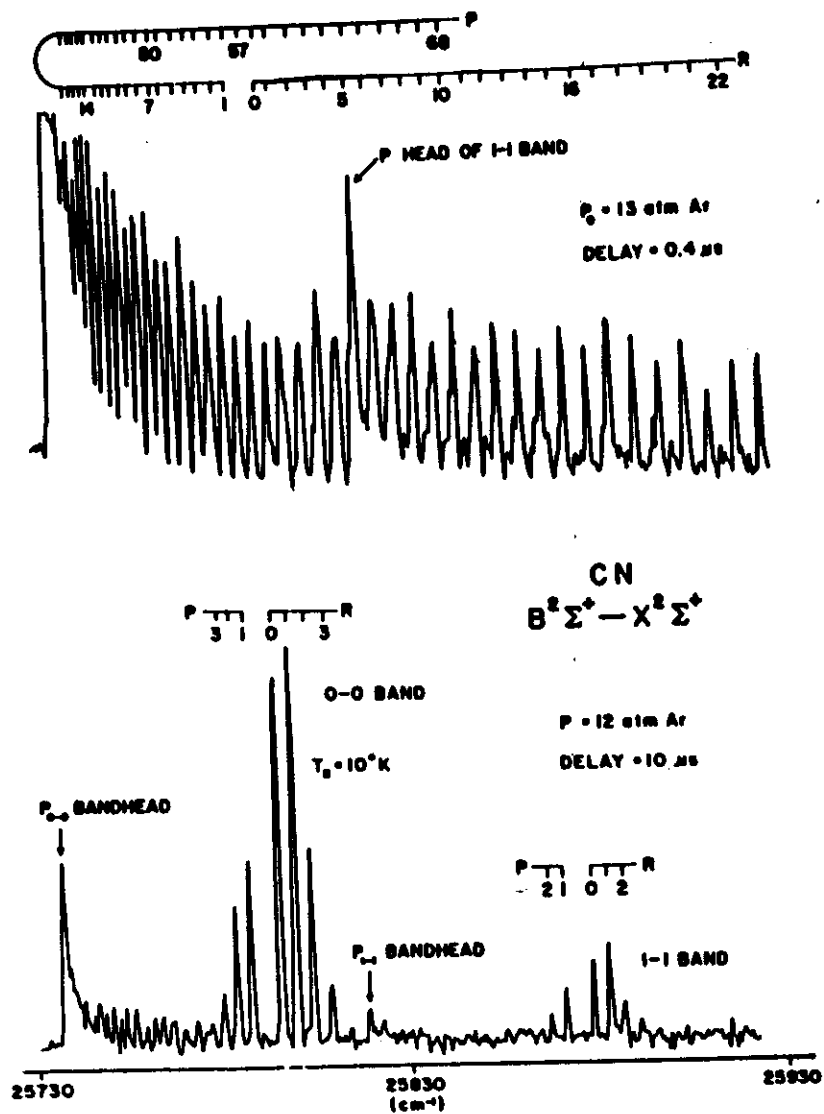


Fig. 6. Laser-induced fluorescence spectra of the CN radical produced by the ArF laser photolysis of BrCN in air.

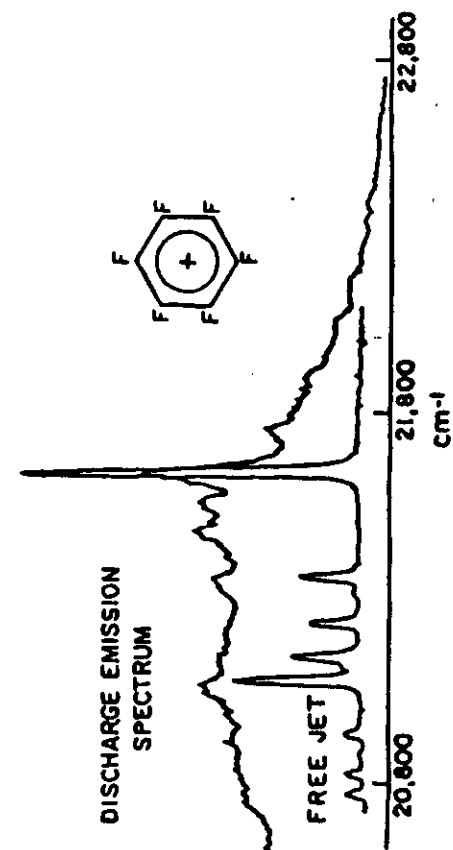


Fig. 3. Wavelength resolved emission spectra of $C_6F_6^+$. The top spectrum was obtained by electron impact ionization and excitation of C_6F_6 in a cell at roughly 100 millitorr pressure. The lowest trace was obtained from $C_6F_6^+$ in a supersonic jet, laser excited.

equilibrium configuration, from D_{6h} symmetry by the Jahn-Teller effect. A detailed discussion of these studies. Fig. 4 shows in some detail

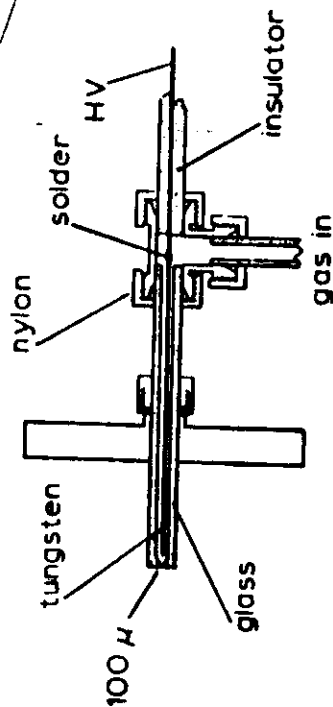


Fig. 1. Schematic of corona discharge excited nozzle.

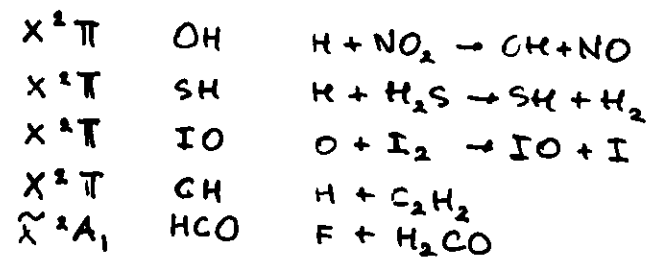
DETECTION SPECTROSCOPIES

- EMISSION (VISIBLE, UV)
 - HALLOW CATHODE DISCHARGE
 - ELECTRON IMPACT FLUORESCENCE
 - AFTERGLOW PENNING IONIZATION
 - CHEMILUMINESCENCE
 - ELECTRON IMPACT IONIZATION OF ION. BEAM
 - (- LASER INDUCED FLUORESCENCE)
- PHOTODETACHMENT (NEG. IONS)
- PHOTOELECTRON
- MATRIX ISOLATION
- MICROWAVE + BEAM MASER + ESR
- FAR INFRARED (ESP. LIGHT SPECIES)
- RADIO ASTRONOMY
- LASER MAGNETIC/ELECTRIC RESONANCE
- IR LASER - COLOUR CENTER
 - DIODE
 - DIFFERENCE FREQUENCY
- LASER DOPPLER (ION BEAMS)
- RF-SPECTROSCOPY OF TRAPPED IONS
- LIF

MODULATION TECHNIQUES:

ZEEMAN (PARAMAGNETIC SPECIES ONLY)
 FREQUENCY
 CONCENTRATION
 VELOCITY (IONS ONLY)

REACTIVE PRODUCTION (EXAMPLES)



DISCHARGE

