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WINTER COLLEGE ON
ATOMIC AND MOLECULAR PHYSICS

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PLASMA DIAGNOSTICS

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PLASMA DIAGNOSTICS

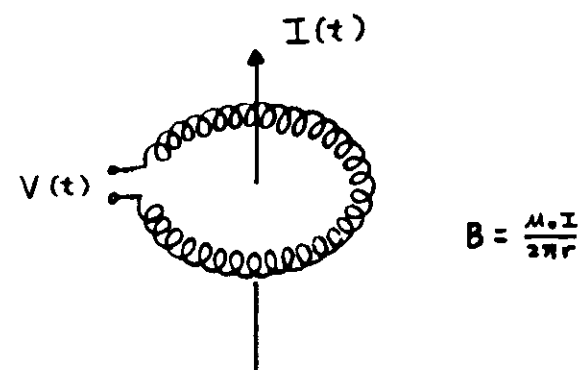
C.S.Wong

1. Rogowski Coil
2. Magnetic Probe
3. High Voltage Probe
resistive divider
capacitive divider
4. Electric Probe
single and double probes
computer controlled system
pulsed system
triple probe

Emphases given to : basic principles
application examples

DDRII CDC
01/25/87 14.12.11.

PRINCIPLE OF ROGOWSKI COIL :



Voltage induced :

$$V(t) = \left(-\frac{\mu_0 N A}{2\pi r} \right) \frac{dI}{dt}$$

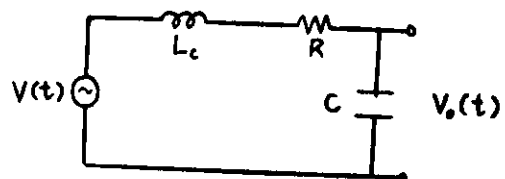
where N is no. of turns;

A is the minor cross-sectional
area; and

r is the major radius of the torus.

2 MODES OF OPERATION :

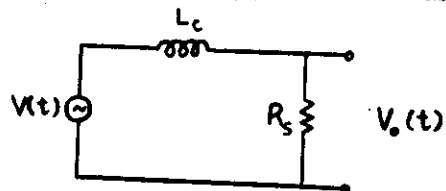
(1) With RC-integrator



$$V_o(t) = \left(-\frac{\mu_p NA}{2\pi r RC} \right) I(t)$$

with $RC \gg \omega^{-1}$; and
 $R \gg \omega L_c$

(2) Self-integrating (current transformer)



$$V_o(t) = \left(-\frac{R_s}{N} \right) I(t)$$

with $R_s \ll \omega L_c$

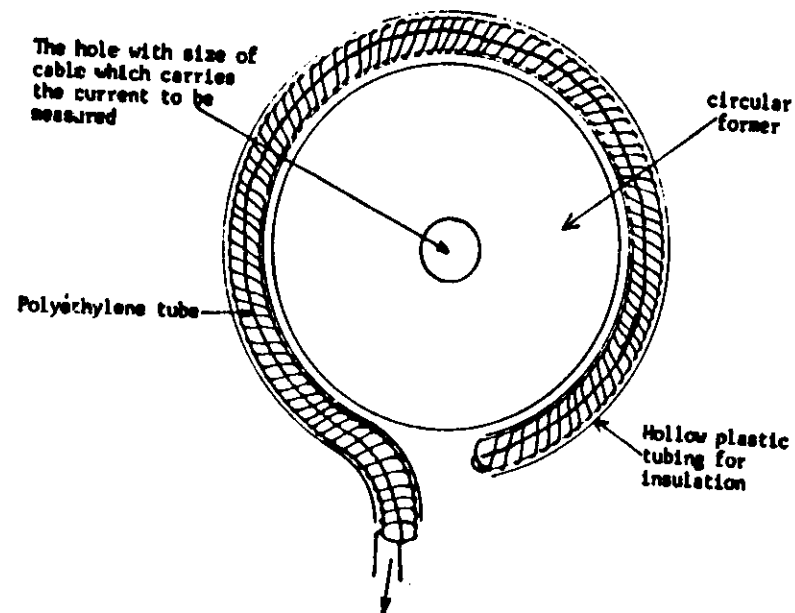
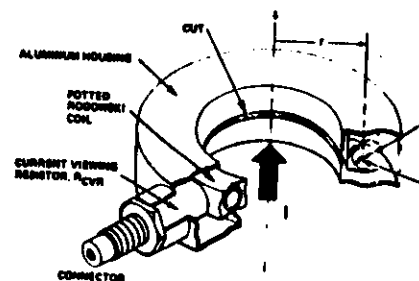


Fig. 3.1 The construction of a Rogowski coil. (C.Y. Ong, M.Sc. Thesis)



RESPONSE OF ROGOWSKI COIL

For self-integrating coil :

$$2\tau < t_r < L/R_s$$

τ is transit time of signal along coil.

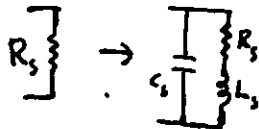
For a coil with electrostatic shielding,

$$\tau = \sqrt{LC} \cdot 2\pi$$

where L , C are inductance and capacitance per unit length of the coil.

Factors affecting response :

1. $R_s > \sqrt{L/C}$.
- $V_o(t)$ becomes oscillatory.
2. Imperfect R_s .
- $V_o(t)$ becomes oscillatory.
3. Skin effect (high frequency I).
 L increases $\Rightarrow \tau$ increases.



For RC-integrator mode :

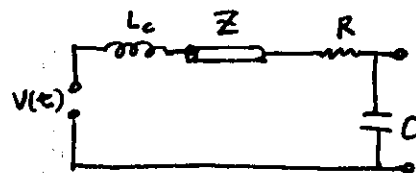
$$L_c/R < t_r < RC$$

If long connecting cable with impedance

Z is used, then $R = Z$ (usually 50Ω);

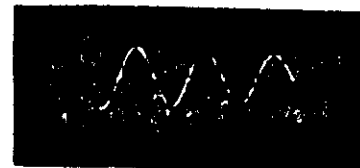
and $t_r > L_c/50$.

$$L_c \sim \mu H \Rightarrow t_r \sim \mu s$$



CALIBRATION OF ROGOWSKI COIL :

- best done in situ using an ideal LCR discharge



For a LCR discharge,

$$I(t) = I_0 \exp(-\alpha t) \sin \omega t$$

$$\text{where } I_0 = V_0 \sqrt{C/L}$$

$$\alpha = R/(2L)$$

$$\omega = 1/\sqrt{LC}$$

From current waveform obtained,

$$\ln(V_2/V_1)$$

$$\alpha = \frac{\ln(V_2/V_1)}{T}$$

$$V_1 \cdot I_1 = I_0 \exp(-\alpha T/4)$$

Coil constant :

$$K_1 = \frac{2\pi C V_0}{T V_1} \left(\frac{V_2}{V_1} \right)^{1/4}$$

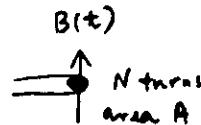
C , V_0 are known; and V_1 , V_2 , T are measured from waveform

THE MAGNETIC PROBE

1. Principle - same as Rogowski coil

Voltage induced :

$$V(t) = NA \left(\frac{dB}{dt} \right)$$



2. Small in size

- measure localised field
hence field distribution

3. L_c small

$R_s \ll \omega L_c$ only true at large ω

=> Self integrating mode not possible
With RC integrator,

$$V_o(t) = \left(\frac{NA}{RC} \right) B(t)$$

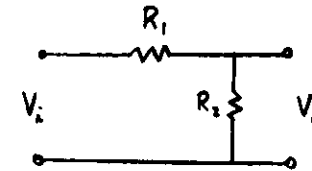
4. Response : $L_c/Z < t_r < RC$

5. Electrostatic shielding

- against capacitive pick-up
of high frequency plasma potential

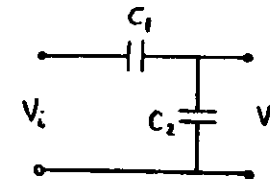
HIGH VOLTAGE PROBE

1. Resistive divider



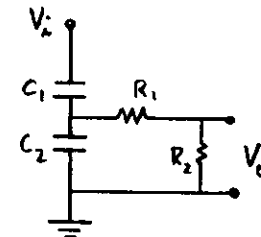
$$\frac{V_o}{V_i} = \frac{R_2}{R_1 + R_2}$$

2. Capacitive divider



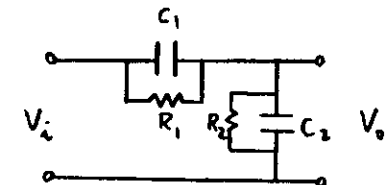
$$\frac{V_o}{V_i} = \frac{C_1}{C_1 + C_2}$$

3. Hybrid : capacitive-resistive



$$\frac{V_o}{V_i} = \left(\frac{C_1}{C_1 + C_2} \right) \left(\frac{R_2}{R_1 + R_2} \right)$$

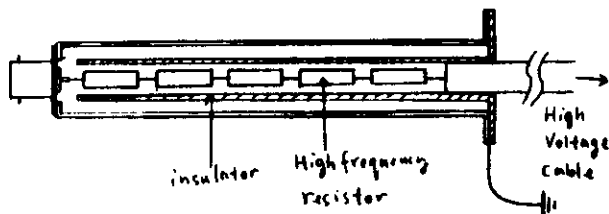
4. Hybrid : capacitive//resistive



$$\frac{V_o}{V_i} = \frac{C_1}{C_2} = \frac{R_2}{R_1}$$

THE RESISTIVE PROBE

An example :



Probe response affected by :

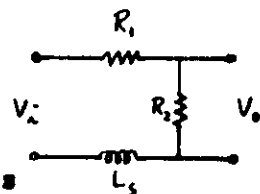
(i) Stray inductance.

$$t_r > L_s / R_2$$

$$R_2 = 50 \Omega$$

$$\text{For } L_s \sim \mu\text{H} \Rightarrow t_r > 100 \text{ ns}$$

$$\text{For } L_s \sim 10 \text{ nH} \Rightarrow t_r > \text{ns}$$

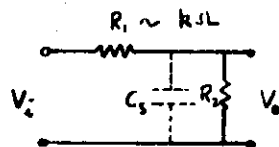


(ii) Stray capacitance.

$$t_r > R_1 C_s$$

$$\text{For } C_s \sim \text{pF} \Rightarrow t_r > \text{ns}$$

$$\text{For } C_s \sim \text{nF} \Rightarrow t_r > \mu\text{s}$$



(iii) Transit time.

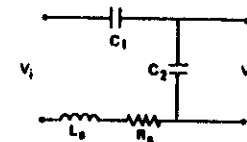
$$t_r > \sqrt{LC} \cdot l$$

Shielding may increase L and C

THE CAPACITIVE PROBE

Factors affecting response :

1. Stray inductance.



$$\text{Natural frequency : } \omega_0 = 1/\sqrt{L_s C}$$

$$C = C_1 C_2 / (C_1 + C_2)$$

$$\text{Upper frequency limit : } \omega < \omega_0$$

2. Terminating impedance.

$$\text{Lower frequency limit : } \omega > 1/ZC_2$$

Connect direct to scope output.

$$Z = 1 \text{ M}\Omega$$

$$\text{Take } C_s = 100 \text{ pF}$$

$$\Rightarrow \omega > 10 \text{ kHz}$$

When long cable connection required,

need to couple to a resistive stage

--> capacitive-resistive probe

3. Transit time.

$$t_r > \sqrt{LC} \cdot l$$

EFFECT OF STRAY INDUCTANCE

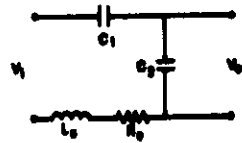


FIG. 1. The equivalent circuit of an imperfect capacitive divider with stray inductance and stray resistance.

Circuit equation:

$$L_s \frac{d^2 Q}{dt^2} + R_s \frac{dQ}{dt} + \frac{Q}{C} = V_1$$

where $C = C_1 C_2 / (C_1 + C_2)$

Take $V_1 = V_m \sin(\omega t)$

Let $\tau = t / \sqrt{L_s C}$; $q = Q / Q_0$; $\delta_s = R_s / \sqrt{L_s / C}$;
 $\gamma = \omega / \omega_0$; $\omega_0 = 1 / \sqrt{L_s C}$; $Q_0 = C V_m$

Then $\frac{d^2 q}{d\tau^2} + \delta_s \left(\frac{dq}{d\tau} \right) + q = \sin(\gamma \tau)$

Solution: $q(\tau) = \left(\frac{1}{1 - \gamma^2} \right) [\sin(\gamma \tau) - \sin \tau] \quad (\delta_s \approx 0)$

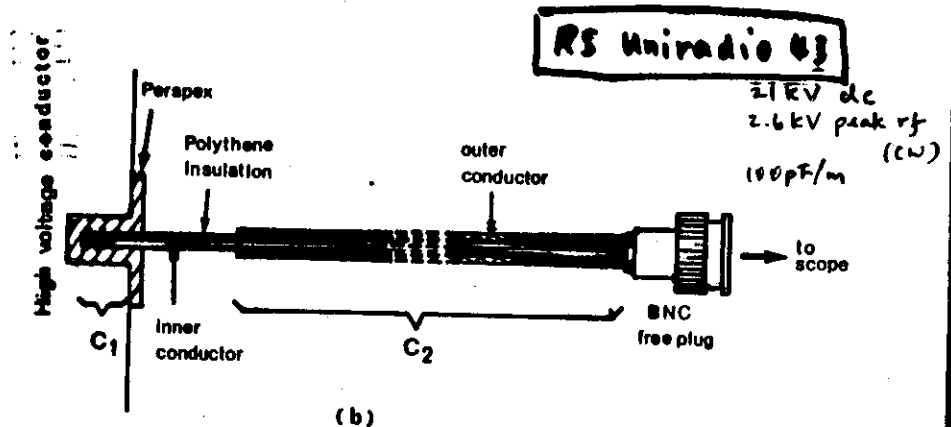
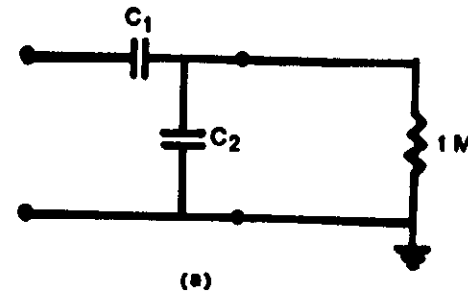
Output voltage: $v_o(\tau) = \beta q(\tau)$

where $\beta = C_1 / (C_1 + C_2)$

Rev. 3.1. Instrum. 56, 761 (1985) Fig. 1(a)

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CAPACITIVE DIVIDER FROM COAXIAL CABLE



RSE/NISDERG/MAY 88/FIG. 1/F: 55 1.

CAPACITIVE-RESISTIVE PROBE

2 possible modes :

(i) $(R_1 + R_2)C_2 > \tau$

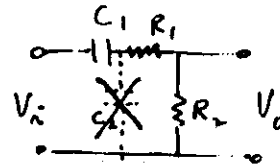
where τ is characteristic time of interest \Rightarrow voltage divider mode

$V_o \approx V_i$

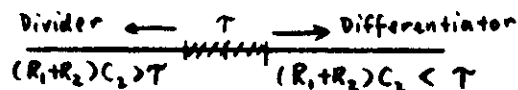
(ii) $(R_1 + R_2)C_2 < \tau$

 \Rightarrow differentiator mode

$V_o \approx R_2 C_1 (dV_i / dt)$



For $(R_1 + R_2)C_2 \approx \tau$

 \Rightarrow mixture of 2 modes

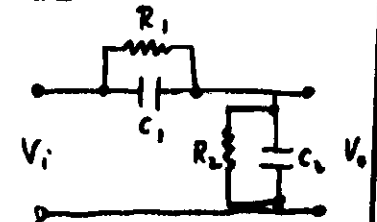
Requirement of long connecting cable,

$\Rightarrow R_2 = 50 \Omega$

Then with $R_1 = 5000 \Omega$ and $C_2 = 100 \text{ pF}$,the probe acts as voltage divider only for voltage pulses with $\tau < 500 \text{ ns}$

CAPACITIVE//RESISTIVE PROBE

1. Required to fix $\frac{R_2}{R_1} = \frac{C_1}{C_2}$

Take $R_2 = 50 \Omega$ (cable impedance),

$C_1 = 10 \text{ pF}$,

$C_2 = 100 \text{ pF}$.

$\Rightarrow R_1 = 500 \Omega$

2. For $\omega > 1/R_1 C_1$,

 \Rightarrow capacitive

3. For $\omega < 1/R_1 C_1$,

 \Rightarrow resistive

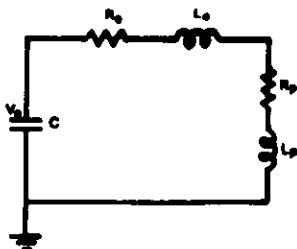
4. For $\omega \approx 1/R_1 C_1$,

Response of the probe may be nonlinear(?)

5. Upper frequency limit

Set by either transit time, or stray inductance
(Refer to capacitive probe)

PLASMA AS ACTIVE CIRCUIT ELEMENT



L_0 : circuit inductance

R_0 : circuit resistance

$L_p(t)$: plasma inductance

$R_p(t)$: plasma resistance

Circuit equation :

$$V_0 - \frac{\int I dt}{C} = \frac{d}{dt} [(L_0 + L_p)I] + I(R_0 + R_p)$$

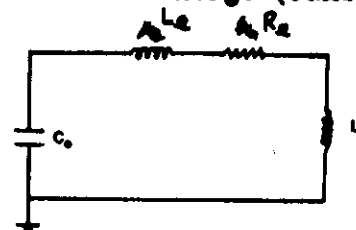
R_0 is usually negligible

Voltage across plasma :

$$V_p(t) = \frac{d}{dt} (L_p I) + I R_p$$

DEDUCTION FROM $I(t)$

1. Ideal LCR discharge (calibration)



Measure T and α from $I(t)$

$$T = 2\pi \sqrt{LC}, L = L_0 + L_L$$

C and L_L are known

$\Rightarrow L_0$, the circuit inductance

$\alpha = R_0/(2L)$, the damping factor

$\Rightarrow R_0$, the circuit resistance

2. Plasma discharge

Voltage across plasma :

$$V_p(t) = V_0 - \frac{\int I dt}{C} - L_0 \frac{dI}{dt}$$

-- dependent on $\int I dt$ and dI/dt

From $I(t)$ measured, obtain $\int I dt$ and dI/dt

$\Rightarrow V_p(t)$, the plasma voltage waveform

DEDUCTION FROM $I(t)$ AND $V_p(t)$

With $I(t)$ measured; and $V_p(t)$ measured
or deduced from $I(t)$

1. Assume plasma to be INDUCTIVE

Plasma inductance :

$$L_p(t) = \frac{\int V_p dt}{I}$$

L_p is related to geometry of plasma.
e.g. the linear Z-pinch

$$L_p(t) = \frac{\mu_0 l}{2\pi} \ln\left(\frac{r_0}{r}\right)$$

for coaxial geometry

$$\Rightarrow r(t) = r_0 \exp\left(-\frac{2\pi L_p}{\mu_0 l}\right)$$

-- the plasma trajectory

2. If plasma is RESISTIVE

i.e. $R_p \gg \omega L_p$, or L_p is fixed

Bulk plasma resistance :

$$R_p(t) = [V_p(t) - L_p(dI/dt)]/I(t)$$

Plasma conductivity $\sigma = l/(R_p A)$

$$\sigma \propto T_e^{3/2}$$

$\Rightarrow T_e$, the electron temperature

e.g. Coaxial flashlamp

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ENERGY CONSIDERATION

Rate of electrical energy input :

$$\begin{aligned} W_e &= I(t)V_p(t) \\ &= I^2 \frac{dL_p}{dt} + L_p I \left(-\frac{dI}{dt}\right) + I^2 R_p \end{aligned}$$

Magnetic energy : $E_m = L_p I^2/2$

Rate of change : $W_m = \frac{I^2}{2} \left(-\frac{dL_p}{dt}\right) + L_p I \left(-\frac{dI}{dt}\right)$

Hence rate of energy input to the plasma :

$$\begin{aligned} W_p &= \frac{I^2}{2} \left(-\frac{dL_p}{dt}\right) + I^2 R_p \\ &\text{(dynamic) (Joule)} \end{aligned}$$

Dynamic - compression or expansion(configuration)

Joule heating - due to resistivity

At any instant t , from $I(t)$ and $V_p(t)$.

E_e , E_m and E_p can be calculated

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CONCEPT OF PLASMA POTENTIAL

In an ideal plasma,
charge neutrality \Rightarrow zero electrostatic field

But, the system of particles possesses
potential energy

Plasma at potential ϕ_p with respect to ground

2 consequences :

- (i) If an external charge is placed into the plasma, the plasma particles re-distribute to shield off the effect of the alien particle to within a distance λ_D from the particle.

(Debye shielding)

$$\lambda_D = 6.9 \sqrt{\left[\frac{T}{(1+Z)n_e} \right]}$$

- (ii) Imagine the alien particle suddenly disappears, then the system of particles try to restore equilibrium.
 \Rightarrow oscillation about equilibrium with frequency

$$\omega_p = \sqrt{\left(\frac{n_e e^2}{\epsilon_0 m_e} \right)}$$

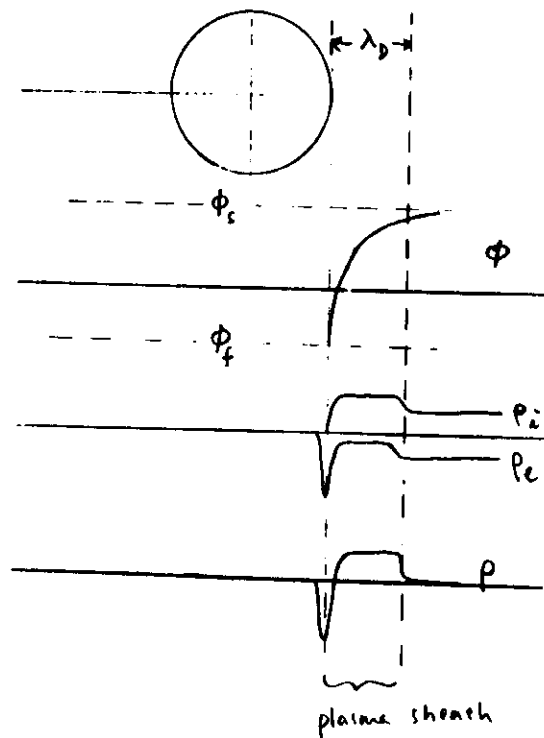
ASSUMPTIONS OF PROBE THEORY

1. Plasma is infinite, homogeneous and neutral
2. Particles have Maxwellian velocity distn.
3. No collision between particles within the vicinity of the probe
4. Particles hitting the probe are absorbed (no deflection, no emission of particle)
5. The bulk of the plasma is not disturbed by the presence of the probe
6. No strong magnetic field in the plasma

FLOATING CONDUCTOR IN PLASMA

At equilibrium,

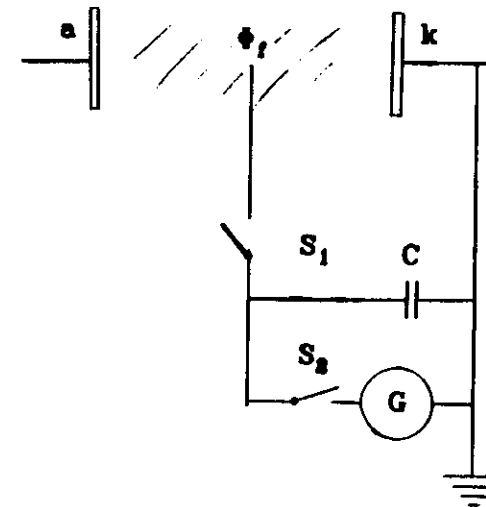
- the conductor becomes negatively charged due to electron attachment.
- at floating potential ϕ_f (negative)
- shielded by positive plasma sheath from the bulk of the plasma.
(no. ions > no. electrons in the sheath)
- thickness of the sheath = λ_D



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MEASUREMENT OF FLOATING POTENTIAL

For d.c. plasma (e.g. glow discharge)



When S_1 is closed,

capacitor C charged to potential ϕ_f

Then open S_1 , and close S_2 ;

C discharged through galvanometer G

Deflection = $k(C\phi_f)$

k can be obtained by calibration
using a known voltage source

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BIASED CONDUCTOR (PROBE) IN PLASMA

1. $\phi_p < \phi_f$ (ion saturation region)
 $I_p = I_{ri}$, constant
2. $\phi_p \rightarrow \phi_f$
no. of e reaching probe increases
 I_p decreases ($I_p = I_{ri} - i_e$)
3. $\phi_p = \phi_f$
 $i_e = I_{ri} \Rightarrow I_p = 0$
4. $\phi_f < \phi_p < \phi_e$ (e retardation region)
more e are reaching the probe
but they are retarded by the field
5. $\phi_p > \phi_e$ (e acceleration region)
 $I_p = I_{re}$
6. $\phi_p \gg \phi_e$ (secondary emission region)
 e accelerated to high energy
secondary particle emission from probe surface
 \rightarrow probe starts to glow
 I_p becomes very large

THE SINGLE PROBE

(The e retardation region)

For this region, electron current given by:

$$i_e = I_{re} \exp\left[-\frac{e(\phi_p - \phi_f)}{kT_e}\right]$$

Probe current $I_p = i_e - I_{ri}$

From probe characteristic: (ϕ_p, I_p) ,

$\rightarrow (\phi_p, i_e)$

Plot $\ln i_e$ vs ϕ_p

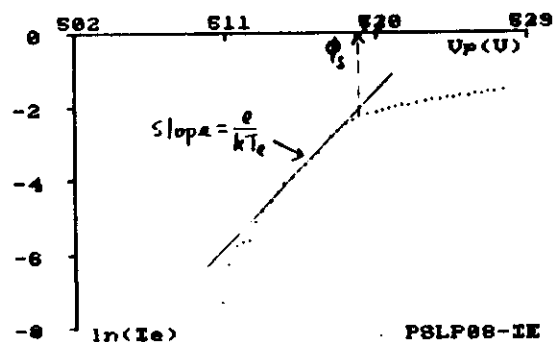
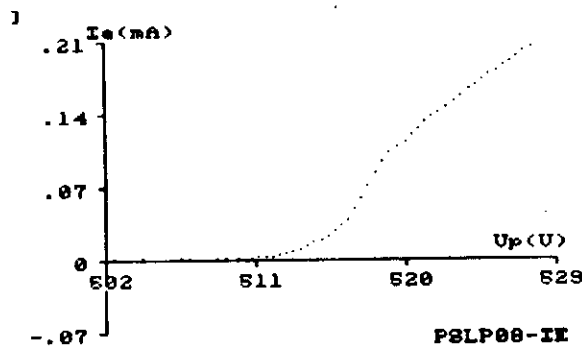
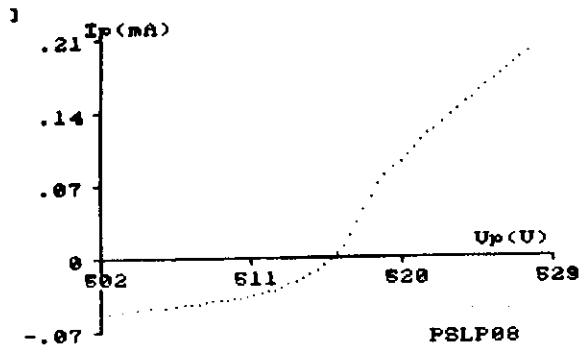
$$\text{Gradient} = e/kT_e$$

$\Rightarrow T_e$, the electron temperature

At $\phi_p = \phi_e$ obtain i_{er}

$$i_{er} = A n_e e \sqrt{\left(-\frac{kT_e}{2\pi m_e}\right)}$$

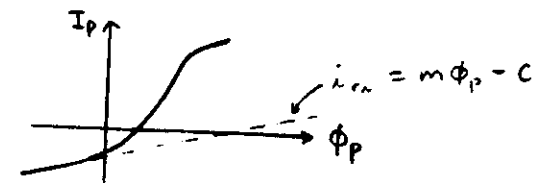
$\Rightarrow n_e$, the electron density



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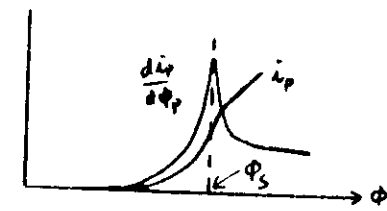
POSSIBLE COMPLICATIONS

1. i_{r1} is not constant.



Then $i_{r1} = m\phi_p - c$
which is a linear function of ϕ_p

2. The point $\phi_p = \phi_s$ not clearly observed
-- determined from change of gradient



3. Absence of exponential retardation region
==> plasma not Maxwellian

It is required to determine the EEDF before
application of classical probe theory

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EXPERIMENTAL DIFFICULTIES

1. Unsteady plasma condition.

Impossible to complete a set of probe characteristic by the point-by-point method

-- use pulsed probe

2. Probe contamination

(gas molecules attachment; metallic coating)

-- can be cleaned by sputtering

3. For plasma of temperature > 10 eV,

ion bombardment of probe may occur

--> secondary particles emission

--> glow or arc between probe and plasma

4. Plasma perturbation

condition of plasma obtained may differ from the actual condition

5. Electrons not completely absorbed

-- e.g. reflection

6. Photoemission - due to uv

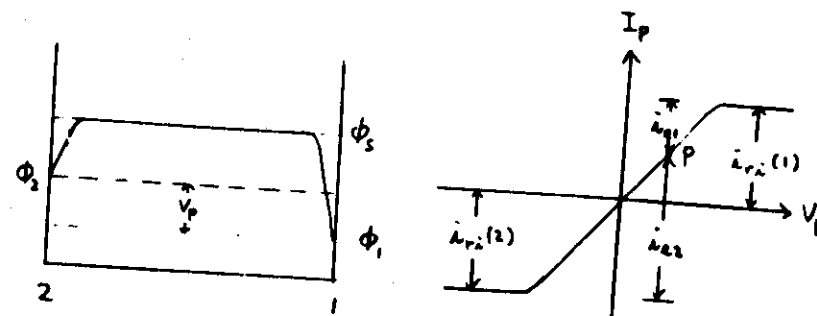
THE DOUBLE PROBE (1)

Ideally, assume

(i) 2 exactly identical probes

(ii) same space potential

Then,



At P :

$$i_{e1} = i_{r1} \exp \left[- \frac{e(\phi_2 - \phi_1)}{kT_e} \right], \quad i_{e1} = i_{r1} - I_p$$

$$i_{e2} = i_{r2} \exp \left[- \frac{e(\phi_2 - \phi_2)}{kT_e} \right], \quad i_{e2} = i_{r2} + I_p$$

$$i_{e1} + i_{e2} = i_{r1}(1) + i_{r1}(2) = 2i_{r1}$$

$$1 + \frac{i_{e1}}{i_{e2}} = \frac{2i_{r1}}{i_{e2}}$$

$$\exp \left[- \frac{e(\phi_2 - \phi_1)}{kT_e} \right] = - \frac{2i_{r1}}{I_p + i_{r1}} - 1 \quad (V_p = \phi_2 - \phi_1)$$

THE DOUBLE PROBE (2)

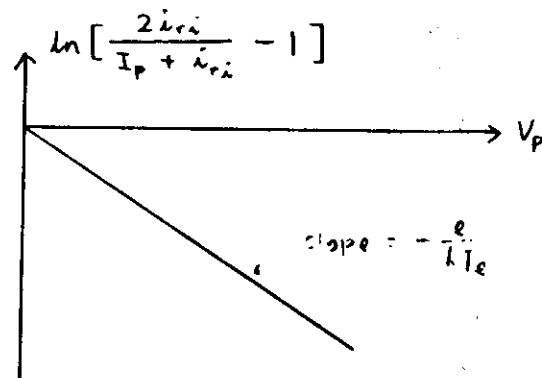
Experimentally,

from the double probe characteristic,

-- measure i_{ri}

-- obtain a set of points (V_p , I_p)

-- plot $\ln\left[-\frac{2i_{ri}}{I_p + i_{ri}} - 1\right]$ vs V_p



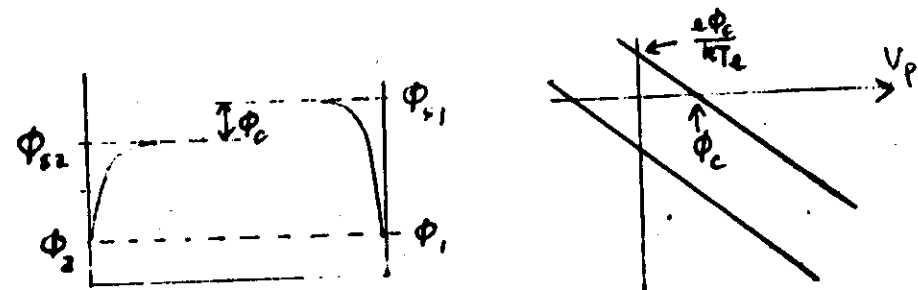
Slope = $-e/kT_e$

$\Rightarrow T_e$, the electron temperature

COMPLICATIONS IN DOUBLE PROBE CHARACTERISTICS (1)

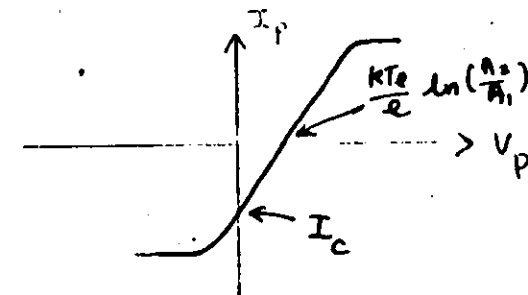
1. Non-uniform space potential

$$\Rightarrow \exp\left[-\frac{e(V_p - \phi_c)}{kT_e}\right] = \frac{2i_{ri}}{I_p + i_{ri}} - 1$$



2. Non-identical probe areas : $A_1 \neq A_2$

$$\Rightarrow \left(\frac{A_2}{A_1}\right) \exp\left(-\frac{eV_p}{kT_e}\right) = \frac{2i_{ri}}{I_p + i_{ri}} - 1$$



$$\text{At } V_p = 0, I_p = \left(\frac{A_1 - A_2}{A_1 + A_2}\right) i_{ri}$$

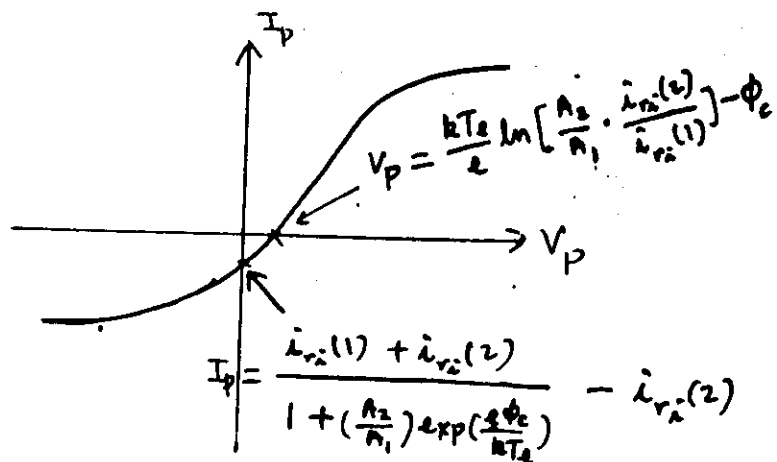
$$\text{At } I_p = 0, V_p = \frac{kT_e}{e} \ln\left(\frac{A_2}{A_1}\right)$$

• $i_{r11} \neq i_{r12}$

$$\Rightarrow \exp\left(-\frac{eV_p}{kT_e}\right) = \frac{i_{r11} + i_{r12}}{I_p + i_{r12}} - 1$$

In general, the experimental double probe characteristic can be given by :

$$\left(\frac{A_2}{A_1}\right) \exp\left[-\frac{e(V_p - \phi_c)}{kT_e}\right] = \frac{i_{r11} + i_{r12}}{I_p + i_{r12}} - 1$$



Unfortunately, T_e can still be determined from:

Plot $\ln\left[\frac{i_{r11} + i_{r12}}{I_p + i_{r12}} - 1\right]$ vs V_p

slope = $-\frac{e}{kT_e} \Rightarrow T_e$

ALTERNATIVE METHOD OF DOUBLE PROBE CHARACTERISTIC INTERPRETATION

$$\frac{i_{e2}}{i_{e1}} = \frac{i_{r12} + I_p}{i_{r11} - I_p} = \frac{A_2}{A_1} \exp\left[-\frac{e(V_p - \phi_c)}{kT_e}\right]$$

Upon differentiation with respect to V_p ,

$$\Rightarrow \frac{dI_p}{dV_p} = \frac{e}{kT_e} \left[\frac{(I_p + i_{r12})(i_{r11} - I_p)}{i_{r11} + i_{r12}} \right]$$

At $I_p = 0$,

$$\left(\frac{dI_p}{dV_p}\right)_{I_p=0} = \frac{e}{kT_e} \left[\frac{i_{r12}i_{r11}}{i_{r11} + i_{r12}} \right]$$

From the probe characteristic,

measure i_{r11} , i_{r12} and $\left(\frac{dI_p}{dV_p}\right)_{I_p=0} \Rightarrow T_e$

Assume $T_1 \ll T_e$ and $n_e \approx n_1$

then $i_{r1} \approx 2An_e e / (kT_e/m_1)$; where $i_{r1} = \frac{1}{2}(i_{r11} + i_{r12})$

$$\Rightarrow n_e$$

(a) Single

PROBE TECHNIQUES - IMPLEMENTATION

1. Manual, point-by-point
 - slow, tedious, inaccurate
2. microcomputer-controlled data acquisition
 - require fast ADC and DAC
 - preferably 16-bits for better resolution
3. Pulsed system
 - for dc plasma, use ms sweep
 - => probe characteristic captured in ms
 - can be extended to obtain (dI_p/dV_p) directly by using a simple RC differentiator
 - can also be coupled to a computer via a digitiser or an oscilloscope with digitiser storage
 - sweeping rate limited to

$$\frac{dV}{dt} \leq \frac{I_{ri}}{C_s}$$

where C_s is the stray capacitance between the probe and the reference electrode (cathode or anode)

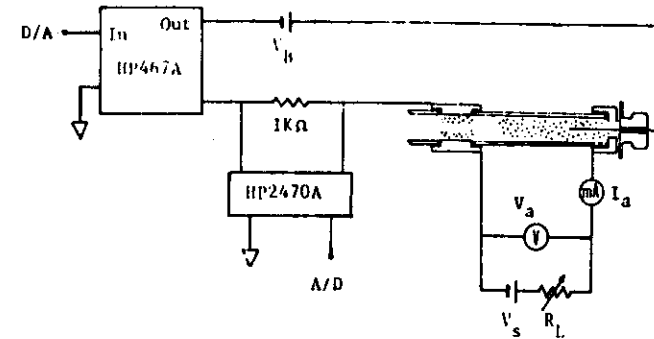


Fig. 2. Experimental set-up of the microcomputer-based Langmuir probe measurement of the glow discharge.

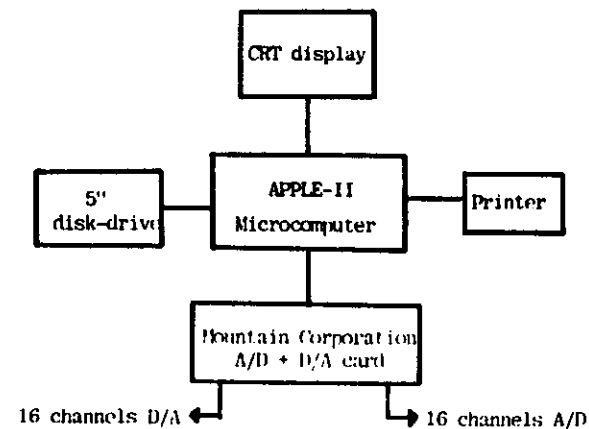


Fig. 3. Block diagram of the data acquisition system.

Microcomputer controlled double probe

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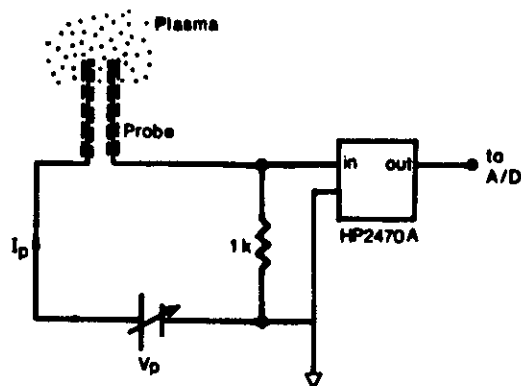


Fig. 2: Experimental set-up of the double Langmuir probe measurement of a plasma.

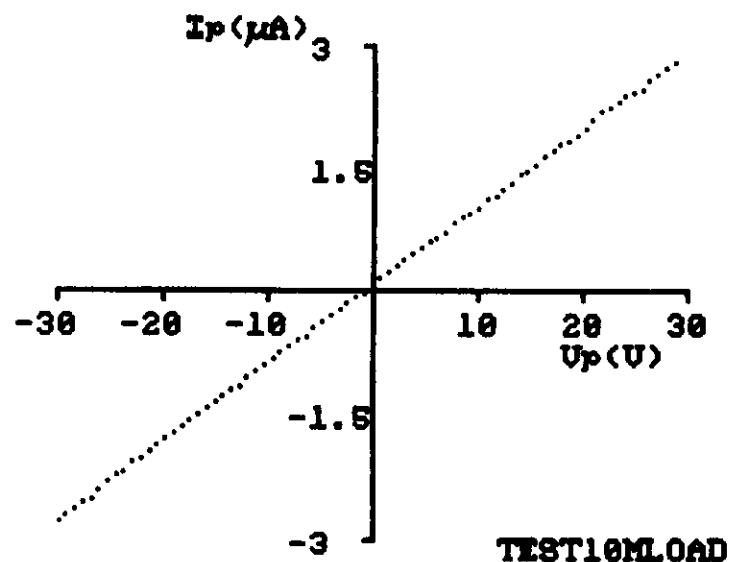
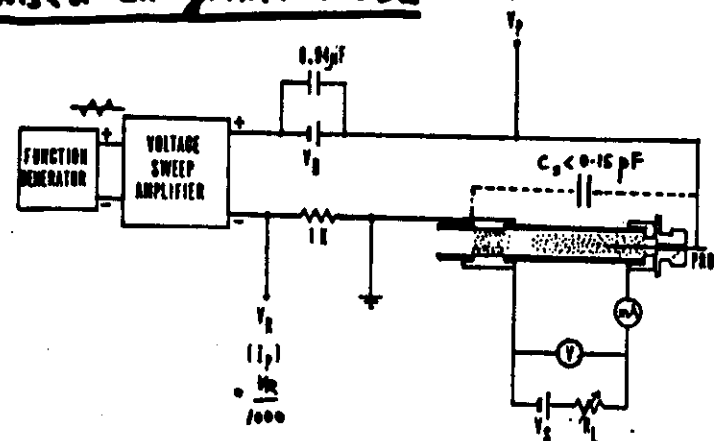


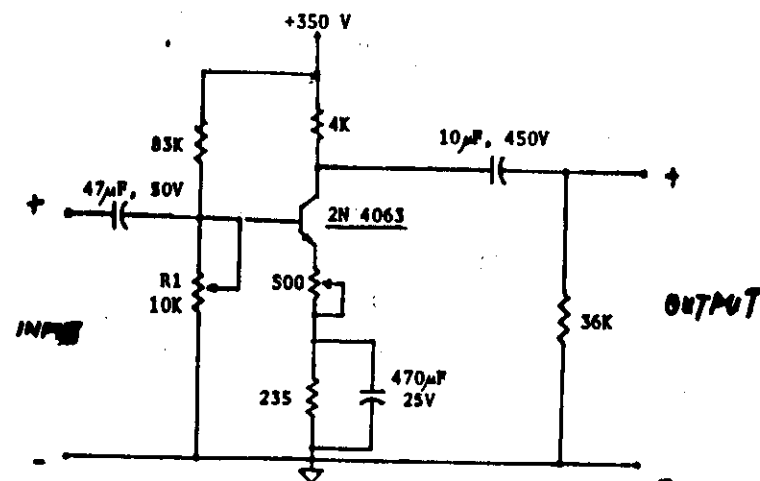
Fig. 3: Ohm's Law simulation test of the microcomputer-controlled Langmuir probe.

Pulsed Langmuir Probe

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Glow Discharge and Pulsed Langmuir Probe System



Amplifier to supply voltage sweep to probe

(W.D. Friedman, Rev. Sci. Instrum., 42, 963 (1971))

Acknowledgement

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Simple method of pulsed plasma discharge current analysis

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A simple method is proposed to analyze the current waveform of a pulsed plasma discharge from which the voltage waveform and hence the temporal evolution of the plasma inductance can be deduced. As an example, the application of the method to a typical plasma focus discharge is illustrated.

The measurements of the discharge current and the anisotropic voltage induced across the plasma are essential in the experimental studies of pulsed plasma discharges. The discharge current can be measured conveniently by using an Rogowski coil operated as a current transformer.¹ Such a measurement is considered as the most basic diagnostic in the investigation of a pulsed plasma device. On the other hand, the measurement of the transient voltage across the plasma is often found to be more difficult than the current measurement. Recently, several designs of high-performance capacitive voltage dividers for measuring this transient voltage have been reported.^{2,3} In these designs, the voltage probes are made an integral part of the plasma device. This may be difficult to implement in some situations, particularly in some existing experiments in small laboratories, in order to obtain sufficient information to enable the study of the discharge dynamics even without direct measurement of the transient voltage. We propose here a simple method to analyze the experimental discharge current waveform from which the voltage waveform can be deduced. Even in the case where the voltage waveform is being measured experimentally, this method of current analysis can be useful as a check to check the result of the voltage measurement.

Pulsed plasma devices are frequently powered by capacitor discharge, and a typical equivalent circuit of the discharge circuit is as shown in Fig. 1. In this circuit, C is the capacitance of the storage capacitor; L_s and R_s are, respectively, the inductance and resistance of the circuit external to the plasma. The effect of R_s is usually negligible as compared to the effect of L_s . The plasma is represented by a time-varying resistance $R_p(t)$ and a time-varying inductance $L_p(t)$ in series.

When the capacitor is charged to a voltage V_0 and then

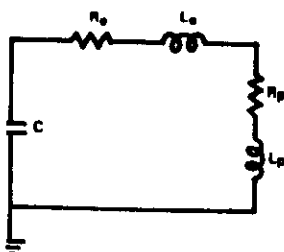


Fig. 1. Equivalent circuit of a pulsed plasma discharge.

discharged through the circuit, the instantaneous current $I(t)$ that flows through the circuit is described by the circuit equation as follows

$$V_0 - \frac{dI(t)}{dt} = \frac{d}{dt} [L(t)I(t)] + I(t)R_p(t), \quad (1)$$

where $L(t) = L_s + L_p(t)$ is the total inductance of the circuit. Rearranging Eq. (1) gives the voltage across the plasma at any instant as

$$V_p(t) = V_0 - \frac{dI(t)}{dt} - L_s \frac{dI}{dt}. \quad (2)$$

This equation can be normalized by introducing the following dimensionless parameters:

$$\tau = t/t_c, \quad i(\tau) = I(t)/I_m, \quad v_p(\tau) = V_p(t)/V_0$$

where $t_c = \sqrt{LC}$ is the capacitor characteristic time and $I_m = V_0/\sqrt{C/L_s}$ is the short-circuited peak current. The normalized form of Eq. (2) is

$$v_p(\tau) = 1 - \tau \frac{di}{d\tau} - \frac{di}{d\tau}. \quad (3)$$

Hence the instantaneous transient voltage induced across the plasma of a pulsed plasma discharge can be deduced if the explicit form of its current waveform is known. In practice, the form of $i(\tau)$ in Eq. (3) can be obtained by digitizing and then curve-fitting the measured current waveform. This will be illustrated in the following by analyzing the current waveform of a typical plasma focus discharge using the method outlined above.

A typical set of current and voltage waveforms obtained for the University of Malaya Dense Plasma Focus (UMDPF)⁴ is shown in Fig. 2. The current waveform has

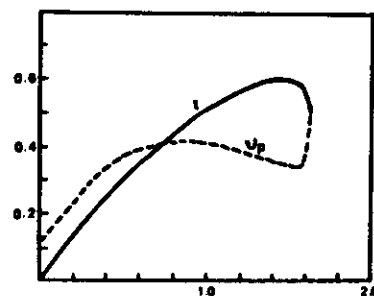


Fig. 2. Experimental current and voltage waveforms (in normalized forms) of a typical plasma focus discharge.

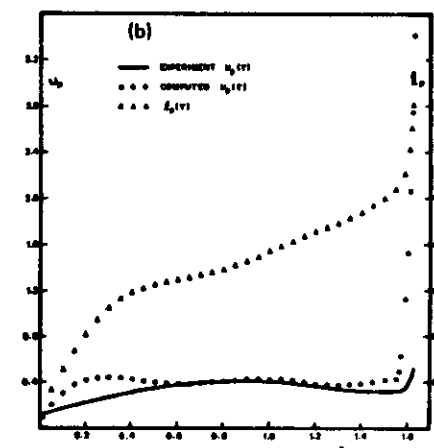
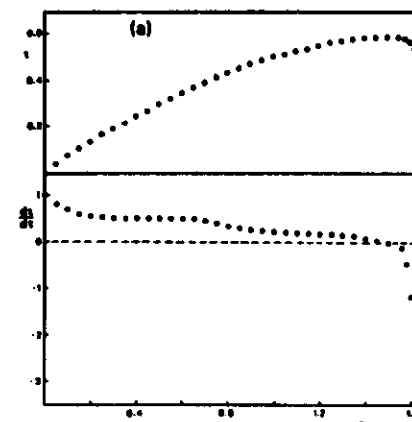


Fig. 3. Results obtained from the analysis of the plasma focus discharge current waveform in Fig. 2. (a) The curve-fitted $i(\tau)$ and the corresponding $di/d\tau$. (b) Comparison of the experimental and the computed voltage waveforms and the temporal evolution of the focus tube inductance by assuming an inductive model for the focus discharge.

been carefully traced onto a graph paper and digitized to produce a set of points (τ, i) to be curve-fitted into polynomial of $i(\tau)$. We shall confine ourselves to the interval between $\tau = 0$, to $\tau = 1.628$, where the current has reached the minimum of the dip.

Using the least-square curve-fitting technique, we obtain the following two polynomials of $i(\tau)$ for two time regimes:

Regime I. $\tau = 0-1.54$ (positive gradient),

$$\begin{aligned} i(\tau) = & -2.8169 \times 10^{-3} + 0.9303\tau - 1.6059\tau^2 \\ & - 2.7603\tau^3 + 1.5817\tau^4 - 0.6485\tau^5 \\ & + C.4259\tau^6 + 0.4689\tau^7 - 0.0966\tau^8 \\ & - C.2278\tau^9 + 0.0832\tau^{10}, \end{aligned} \quad (4)$$

Regime II. $\tau = 1.54-1.628$ (current-dip, negative gradient)

$$\begin{aligned} i(\tau) = & -161.88 + 144.628\tau + 60.389\tau^2 - 24.935\tau^3 \\ & - 11.016\tau^4 - 41.092\tau^5 + 15.713\tau^6 + 4.649\tau^7 \\ & + 4.363\tau^8 - 2.825\tau^9. \end{aligned} \quad (5)$$

It should be noted from these two polynomials that (i) at $\tau = 0$, $i \approx 0$ which is expected experimentally; (ii) it can be deduced from Eq. (4) that $di/d\tau = 0.9303$ at $\tau = 0$ which is close to unity; (iii) the value of i at $\tau = 1.54$ is calculated to be 0.5935 from Eq. (4) and 0.6013 from Eq. (5) which are in good agreement; and (iv) similarly, the value of $di/d\tau$ at $\tau = 1.54$ is calculated to be -4.3×10^{-3} from Eq. (4) and 0.064 from Eq. (5) which can both be approximated to zero. These considerations, together with point by point comparison of the computed and experimental values of i at various τ indicates that the two polynomials (4) and (5) are accurate representation of the current waveform in the two respective time regimes.

The results obtained from the analysis of the current waveform in Fig. 2 are presented in Fig. 3. The curve-fitted current waveform, together with the corresponding $di/d\tau$ are shown in Fig. 3(a); whereas the voltage waveforms, both computed and experimental, are shown in Fig. 3(b). The computed voltage waveform shows three distinctive regions: (i) the initial slow rise from $\tau = 0$ to $\tau = 0.25$, (ii) the roughly flat plateau region from $\tau = 0.25$ to $\tau = 1.54$, and (iii) the sharp spike which occurs from $\tau = 1.54$ to $\tau = 1.628$. In the measured voltage waveform, however, the voltage is seen to rise more slowly than the computed one, especially in the region of the sharp voltage spike. At the tip of the voltage spike, the measured value is only 0.53 whereas the computed value is as high as 3.4. This is believed to be due to the inadequate response of the probe used.⁵ The features of the voltage waveform computed from the current analysis in fact resemble those observed by others,^{6,7} although their observed voltage spikes are still low compared to our computation.

With the measured current waveform (curve-fitted into polynomials) and the voltage waveform computed as described above, the temporal evolution of the focus tube inductance can be deduced if an inductive discharge model is assumed. In terms of $v_p(\tau)$ and $i(\tau)$, the normalized form of the tube inductance is given by

$$L_p(\tau) = \frac{\int v_p(\tau) di}{i(\tau)}. \quad (6)$$

This has been computed for the example considered here and the result obtained is shown in Fig. 3(b). The sharp rise in L_p corresponding to the current dip is a feature commonly observed for a typical plasma focus discharge. The rise in the focus tube inductance at the start of the discharge is unexpected from a consideration of the focus dynamics. This is probably due to the fact that at the start of the discharge, the tube resistance is not negligible and hence the inductive model is not valid. This tube resistance, however, will soon drop to a negligibly low level as the plasma is heated up. This is most likely to occur during the lift-off phase of the focus discharge dynamics.

We thank T. Y. Tou for providing the set of current and voltage waveforms of the UMDPF1 for the above analysis.

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Effect of stray inductance on capacitive-voltage divider

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The effect of stray circuit inductance on the response of a capacitive-voltage divider to fast-rising voltage pulses is investigated. Results obtained from the analysis of the capacitive divider's equivalent circuit indicate an upper-frequency limit of only 1% of the natural frequency of the circuit.

The capacitive-voltage divider is now being widely used in many laboratories for measuring high-transient voltage pulses such as those encountered in pulsed plasma experiments.¹⁻³ It is a common practice to design the capacitive-voltage divider as an integral part of the device studied so as to minimize the stray inductance. However, it has been pointed out by Fawkes and Rowe⁴ that even a stray inductance of a few hundredths of a microhenry was sufficient to cause nonlinearity in the high-frequency response of this type of voltage divider. Hence the effect of the stray inductance must be taken into account in any realistic design consideration of the capacitive divider. This is particularly important in the measurements of the extremely sharp voltage spikes of possibly less than 10-ns rise times in the plasma focus⁵ and in the vacuum spark.⁶

Ideally, the capacitive-voltage divider consists of only two components: capacitors C_1 and C_2 . The attenuation factor of the divider is given by

$$V_o/V_i = C_1/(C_1 + C_2), \quad (1)$$

where $C_1 < C_2$. For such an ideal situation, the rise time of the voltage divider is only limited by the transit time of the voltage signal between the output and input terminals. The presence of leakage resistances across the capacitors only affects the low-frequency response of the capacitive divider. Since C_1 and C_2 are usually in the picofarad range, the presence of a stray resistance of milliohm magnitude in series with the capacitors will have practically no effect on the

divider's response. This is due to the fact that the RC time constant of such a circuit is only of the order of 10^{-15} s, which is much shorter than the time scale of any possible transient phenomenon that may be studied in the laboratory.

On the other hand, the presence of a stray inductance in the divider's circuit may result in the output voltage waveform being superimposed with high-frequency ringing due to the natural LC oscillation; and the response of the divider thus becomes nonuniform.

The equivalent circuit of an imperfect capacitive-voltage divider consisting of a stray inductance L , and a stray resistance R , is represented as shown in Fig. 1. Its circuit equation can be written as

$$L \frac{d^2Q}{dt^2} + R \frac{dQ}{dt} + \frac{Q}{C} = V_i, \quad (2)$$

where Q is the instantaneous charge flow in the circuit and C is the effective capacitance given by

$$C = C_1 C_2 / (C_1 + C_2).$$

For simplicity, a sinusoidal input voltage [$V_i = V_m \sin(\omega t)$] where V_m is the peak voltage and ω is the frequency] has been chosen so that a simple analytical solution can be obtained for Eq. (2). For a more general analysis of the circuit, the method of Laplace transform may be used.

Equation (2) can be normalized using the following dimensionless parameters:

$$\tau = t/\sqrt{LC}; \quad q = Q/Q_0; \quad \delta = R/\sqrt{L/C};$$

$$\gamma = \omega/\omega_0, \quad \omega_0 = 1/\sqrt{LC}, \quad Q_0 = C_1V_m.$$

The normalized circuit equation thus obtained is

$$\frac{d^2q}{dt^2} + \delta \frac{dq}{dt} + q = \sin(\gamma t). \quad (3)$$

γ is the characteristic parameter of the circuit while δ , is the damping factor. The solution of Eq. (3) above gives $q(t)$, the normalized instantaneous charge that is flowing in the circuit. The output voltage across C_2 can then be computed as

$$v_o(t) = \beta q(t), \quad (4)$$

where $\beta = C_1/(C_1 + C_2)$ is the ideal attenuation factor of the capacitive divider. It must be pointed out that in this case L_s represents the total inductance of the circuit considered. If part of this inductance is across the output terminals in series with C_2 , then the output voltage becomes

$$v_o(t) = \beta q(t) + k(d^2q/dt^2), \quad (4a)$$

where k is a number less than unity. The response characteristics of the divider in this configuration is, however, identical to that represented by Fig. 1.

Equation (3) above has three possible solutions depending on the value of δ . The three regimes of δ , are

- (a) $\delta = 2$ (critically damped);
- (b) $\delta < 2$ (underdamped); and
- (c) $\delta > 2$ (overdamped).

In the case of the capacitive-voltage divider considered here, $C \sim \text{pF}$; $R_s \sim \text{m}\Omega$; and for $L_s \sim \text{nH}$, $\delta \sim 10^{-3}$ which is much less than 2. Thus the situation here can be approximated to the case of zero damping. Taking $\delta = 0$, the solution of Eq. (3) can be written as

$$q(t) = A [\sin(\gamma t) - \gamma \sin(t)], \quad (5)$$

where $A = 1/(1 - \gamma^2)$. It can be seen from Eq. (5) that for small γ , we have for $0 < t < \pi/\gamma$,

$$q(t) \approx \sin(\gamma t).$$

This corresponds to the situation of negligible stray inductance ($\omega \ll \omega_0$) where the response of the divider can be considered to be instantaneous.

On the other hand, for large γ , the second term of Eq. (5) becomes important and for sufficiently large γ , it may even dominate over the first term. Under such a condition, the LCR circuit will oscillate at its natural frequency ω_0 after being perturbed by the high-frequency input voltage. The divider, therefore, is unable to follow the temporal development of the input voltage.

From the above discussion, it is clear that the capacitive-voltage divider will be able to give perfect response only if there is no stray inductance present in its circuit. Since there will inevitably be some amount of stray inductance in the circuit of a practical capacitive divider, it is important to

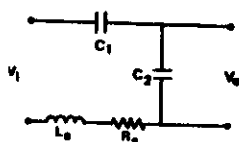


FIG. 1. The equivalent circuit of an imperfect capacitive-voltage divider with stray inductance and stray resistance.

determine the upper-frequency limit of a particular divider setup. This can be done by an investigation of the divider's response for various values of γ .

The output waveform of the capacitive divider has been computed from Eqs. (4) and (5) for various values of γ and α shown in Figs. 2(a)-(d). It can be seen from Fig. 2(a) that the output waveform for $\gamma = 0.3$ has been severely distorted. For smaller γ , the output waveform becomes more sinusoidal but it is superimposed with a high-frequency ringing as can be seen from Fig. 2(b). For $\gamma = 0.01$, the output wave-

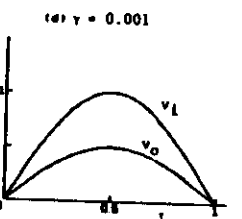
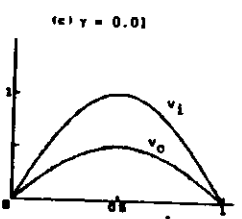
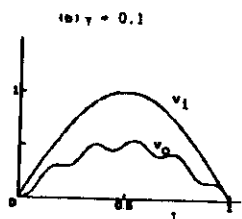
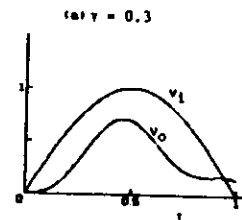


FIG. 2. The input (v_1) and output (v_o) voltage waveforms of an undamped capacitive voltage divider for various values of γ for $\beta = 0.5$. (a) $\gamma = 0.3$; (b) $\gamma = 0.1$; (c) $\gamma = 0.01$; and (d) $\gamma = 0.001$.

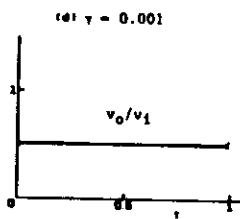
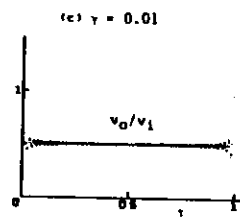
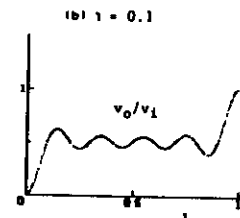
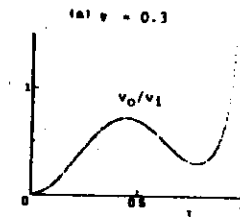


FIG. 3. Plots of v_o/v_1 corresponding to the results shown in Fig. 2.

form can be considered to be a reasonably accurate reproduction of the input waveform, despite the fact that small-amplitude but high-frequency oscillation is still present [Fig. 2(c)].

The above description of the response of the imperfect capacitive-voltage divider can be further clarified by looking at the time evolution of the ratio v_o/v_1 as shown in Fig. 3(a)-(d). Ideally, one expects v_o/v_1 to be constant and equal to β . This is almost true in the case of $\gamma = 0.001$ [Fig. 3(d)]. In the case of $\gamma = 0.01$, the nonuniform response of the divider gives rise to oscillation of the value of v_o/v_1 about the level of 0.5 and this oscillation is most severe near $t = 0$ and $t = 1$ where $v_1 \rightarrow 0$. For the main portion of the plot of v_o/v_1 against t , the value of v_o/v_1 oscillates about 0.5 with an amplitude of approximately 0.01.

From the above discussion, it seems reasonable to fix the upper limit of γ at 0.01 for a practical capacitive divider setup. Hence the condition for optimum response of the capacitive divider can be expressed as

$$\omega < 0.01 \omega_0$$

For a divider with $C \sim \text{pF}$ and $L_s \sim \text{nH}$, we have

$$\omega < 10^9 \text{ rad/s.}$$

Equivalently, the fastest input pulse rise time that the capacitive divider can respond to with sufficient accuracy is

$$t_r > 10 \text{ ns.}$$

The condition for optimum response derived above provides a useful guideline for the design of a capacitive-voltage divider for the measurement of fast voltage pulses. It is clear that in order to relax the constraint on the minimum stray inductance that may be introduced, it is necessary to make C , the overall capacitance, small. In practice, the overall capacitance of the divider is limited to a value governed by the stray capacitance of the setup, which is usually of the order of pF. On the other hand, in order to increase the attenuation factor of the divider, it is required to keep C_2 large compared to C_1 . This is usually limited by geometrical constraint. In many applications of the capacitive-voltage divider, water, which has a dielectric constant of ≈ 80 , is used as dielectric for the capacitor C_2 . This may, however, make the capacitor more leaky than those using material such as Mylar as the dielectric, thus affecting the low-frequency limit of the divider.

The author is grateful to Professor S. Lee and Dr. S. P. Chia for their enthusiasm and useful discussions. This work is supported by an F-grant of the University of Malaya (F31/79).

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Simple nanosecond capacitive voltage divider

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The construction of a simple and inexpensive capacitive voltage probe by using coaxial cable is described. The probe has been tested to a frequency response of better than 30 MHz for sinusoidal voltage input and a rise time of < 20 ns for single-pulse input. The probe is ideal for applications in small experiments involving low-energy and high-voltage capacitor discharge of nanosecond rise time.

The capacitive voltage divider is now being widely used in many laboratories to monitor the transient voltage pulses encountered in various plasma and laser experiments. Several successful designs of the capacitive voltage probe have been reported.¹⁻⁴ This note describes an extremely simple version of the capacitive voltage divider which can be made

out of coaxial cable. Such a voltage probe can be incorporated into small plasma and laser devices or some fast pulsed electronic circuits to provide an inexpensive method of measuring nanosecond voltage pulses in these devices with sufficient accuracy.

Ideally, a capacitive voltage divider consists of two ca-

pacitors connected in series as shown in Fig. 1 (a). In the present design, these capacitors are made from a single coaxial cable as shown in Fig. 1 (b). The capacitor C_1 is formed by embedding a short length (l_1) of the inner conductor of the cable, together with the insulating polythene tube, into the body of the high-voltage component of the discharge system; while the rest of the length of the coaxial cable (l_2) acts as the capacitor C_2 . The attenuation ratio (output:input) of this divider is

$$\beta = C_1/(C_1 + C_2).$$

In this case, C_2 is connected directly across the input of the oscilloscope which has a typical input resistance of 1 M Ω . If required, the attenuation can be increased tenfold by connecting a 10 M Ω resistor in series to the oscilloscope's input.

The response characteristics of the above capacitive voltage divider is determined primarily by C_2 . At the low-frequency end, the response of the probe is limited by the decay time constant of C_2 , $R_i C_2$, where R_i is the input resistance of the oscilloscope used; while the high-frequency end is limited by the transit time for signal to travel along the length of C_2 . For a single-voltage pulse of rise time t_r , the probe will be able to respond accurately if

$$t_r > (l_2/v),$$

where v is the velocity of propagation of electrical signal along the coaxial cable, which is approximately 2×10^8 m/s.

The probe constructed for illustration here uses a RS Uniradio 43 coaxial cable. This cable is rated at 21 kV dc, 2.6 kV peak r.f.c.w., and has a capacitance of 100 pF/m in its manufactured form. With l_2 fixed at 1 m, $C_2 = 100$ pF, and the ultimate limit of the probe's response time is

$$t_r > 5 \text{ ns.}$$

The attenuation ratio of the probe can be calculated if C_1 is

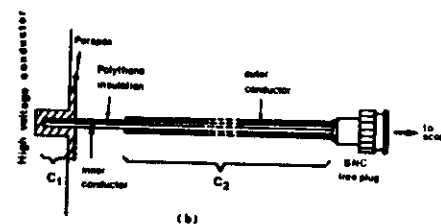
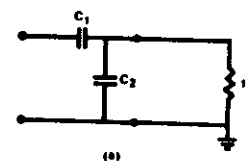


FIG. 1. (a) The equivalent circuit of the capacitive voltage divider connected directly to the oscilloscope. (b) Schematic of the capacitive voltage divider which is made from a single coaxial cable.

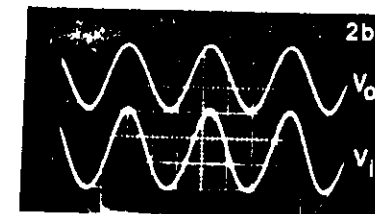
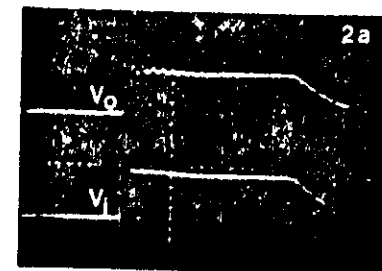


FIG. 2. (a) The response of the capacitive voltage divider to a 20-ns rise-time step voltage function. Upper trace is the output waveform V_0 , while the lower trace is the input waveform V_1 . (Vertical scale: 0.2 V/cm for V_0 , 20 V/cm for V_1 . Horizontal scale: 100 ns/cm.) (b) The response of the capacitive voltage divider to a 30-MHz sinusoidal waveform. Upper trace is the output waveform while lower trace is the input waveform. (Vertical scale: 0.5 V/cm for V_0 , 50 V/cm for V_1 . Horizontal scale: 10 ns/cm.)

known. In practice, this ratio is obtained by calibrating the probe with a voltage of known amplitude.

Figure 2 (a) shows the response of the present capacitive voltage probe to a square pulse of 20-ns rise time. It can be seen that the probe is able to respond accurately to the voltage pulse. The ringing at the plateau region of the voltage pulse is probably due to the presence of a stray inductance which is contributed by the cable itself. Similarly, Fig. 2 (b) shows the accurate response of the probe to a 30-MHz sinusoidal waveform with peak-to-peak amplitude of 130 V. The attenuation ratio of the probe calculated from these measurements is 1/110.

The pulse hold-off voltage of the probe is determined mainly by the insulation of C_1 . In this design, the insulation of C_1 is enhanced by adding a small Perspex cap to cover the part of the coaxial cable which is embedded in the high-voltage conductor as shown in Fig. 1 (b). In this manner the probe's hold-off voltage can be considerably higher than the rated values. However, since it is desirable to keep the length of C_2 short in order to minimize the stray inductance as well as the signal transit time, the measuring oscilloscope must be stationed close to the high-voltage point of the device being studied. It is therefore recommended that the probe be used for experiments involving low-energy capacitor discharge (<20 kV). Despite this limitation, the probe can be considered a useful tool for a number of plasma and laser experiments as well as some pulsed control electronic circuits

where nanosecond voltage pulses of kilovolts amplitudes need to be measured. The simplicity of the probe's construction ensures its quick implementation with minimum expenses.

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J. Fiz. Mat. 6, 115 (1985)
Research Note

MICROCOMPUTER-BASED LANGMUIR PROBE SYSTEM

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Abstract

A simple microcomputer-based data acquisition system is set up to obtain the I-V characteristic of the Langmuir probe. The system is ideal for extensive probe data collection in the study of steady-state plasmas.

The Langmuir probe is a simple but effective tool for the diagnostic of fairly large volume, relatively cold and low density plasmas. From the I-V characteristic of the probe, plasma parameters such as the electron temperature, electron density, space potential and electron energy distribution can be deduced. For a time-varying plasma, the pulsed Langmuir probe¹⁻³ must be used. In the case of a steady-state plasma, it is usually sufficient to use the point-by-point technique, although the pulsed method can also be employed. However, the process of obtaining the Langmuir probe characteristic by the point-by-point method and its analysis is tedious and time-consuming if it is done manually. This is particularly true in cases where the Langmuir probe is employed as a basic diagnostic and extensive data are required to be collected concurrently with other diagnostics of the plasma.

In this note, a simple microcomputer-based data acquisition system for the Langmuir probe is described. The system employs an Apple-II microcomputer to control the variation of the probe's bias potential as well as to monitor the probe current automatically.

The data acquisition system consists of a 64k RAM Apple-II microcomputer which is supported by a high-resolution CRT display unit, two mini floppy disk-drives and a dot matrix printer with high-resolution graphic capability (Fig. 1). Data acquisition is done via a multi-channel A/D + D/A interface card (Mountain Computer Inc.) which can be plugged directly into the expansion slot of the Apple-II. This interface card provides 16 channels each of analog-to-digital and digital-to-analog converters. The range of its analog input/output level is -5V to +5V, with a digital equivalence of 0 to 255.

The experimental set-up of the microcomputer-based double Langmuir probe measurement of a plasma is as shown in Fig. 2. A 1 k Ω resistor is connected in place of the usual microammeter for measuring the probe current, I_p . The potential drop across the 1 k Ω resistor is expected to be of the order of mV only and hence it must be amplified before it is fed into an A/D channel of the computer. This is done by using a HP2470A data amplifier.

The probe potential V_p is supplied by using the HP467A power amplifier acting as a variable power supply. The output of the power amplifier can be varied within the range of -30V to +30V by applying a voltage of -3V to +3V to its input. Thus when used in conjunction with a digital-to-analog converter, the HP467A power amplifier functions as a low voltage digitally controlled power supply. This arrangement is good enough for the present purpose. If higher voltages are required, the digitally controlled power supply described by Fanelli and Morangelli⁴ may be employed.

It is the present intention to apply the above microcomputer-controlled Langmuir probe system to study the plasma of a dc glow discharge. However, in order to test the reliability of the system, a simulation experiment is first performed by replacing the plasma with a 10 M Ω resistor

($\pm 10\%$). For such a simulation system, the current flow in the circuit varies linearly with voltage (Ohm's Law). This has been obtained as shown in Fig. 3. The slope of the straight line obtained is $10.5 \text{ MS}\Omega$ which is consistent with the resistance of the load used.

As an example to illustrate the operation of the microcomputer-controlled Langmuir probe system, the double probe characteristic of a hot-cathode dc glow discharge is obtained as shown in Fig. 4. The feature of this probe characteristic is typical to that obtained with a contaminated and non-symmetrical double probe. It is well known⁶ that surface contamination of the Langmuir probe has the effect of smoothing out the "knee" of the characteristic; whereas the intercept of the curve on the V_p -axis at point C is caused by the difference in the plasma potentials at the locations of the two probes. The electron temperature of the plasma can be determined from the slope of the characteristic at point C where $I_p=0$. This has been obtained to be $T_e = (2.0 \pm 0.2) \text{ eV}$ in the above case, which is typical for the glow discharge plasma considered here.

We have illustrated above the operation of a microcomputer-based Langmuir probe system for the measurement of the electron temperature of a glow discharge plasma. As can be seen from Fig. 4, more points at small voltage increments of 0.4 V have been obtained for a single characteristic, which is not possible if the same procedures are performed manually. Thus the utilization of microcomputer in Langmuir probe measurement has greatly enhanced its accuracy and effectiveness as a basic diagnostic in the study of steady-state plasmas.

Acknowledgement

We are grateful to Professor Lee Sing for encouragement and support. This work is supported by a F-grant of the University of Malaya (F 31/79).

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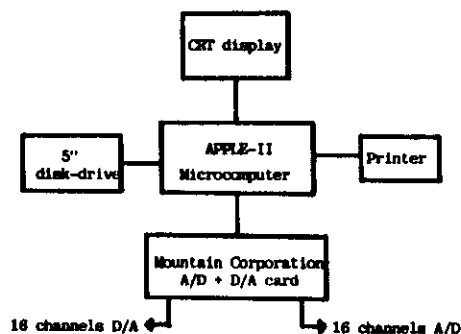


Fig. 1: Schematic of the microcomputer-based data acquisition system.

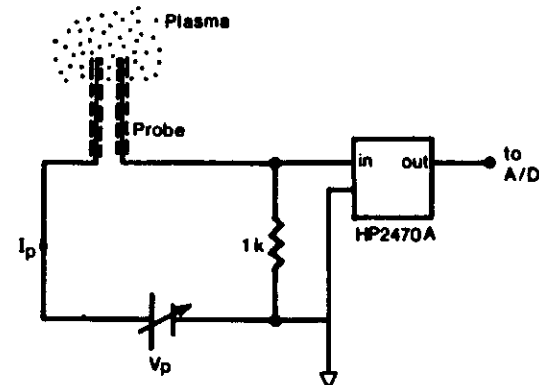


Fig. 2: Experimental set-up of the double Langmuir probe measurement of a plasma.

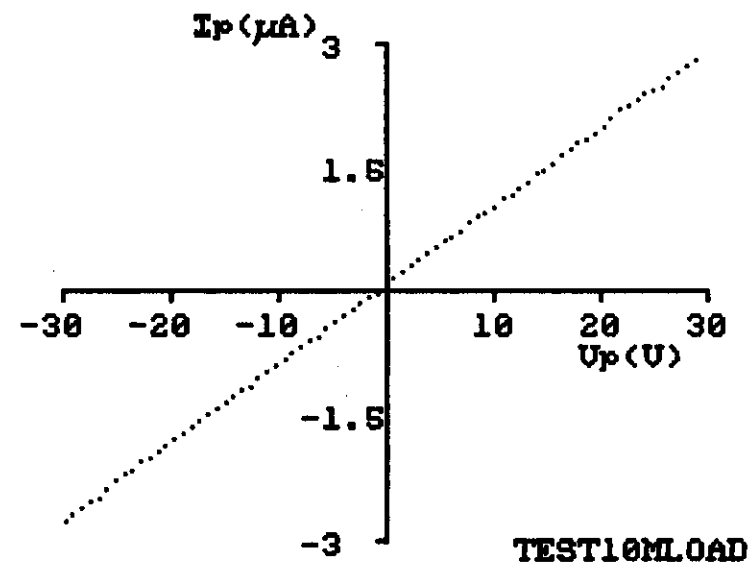


Fig. 3: Ohm's Law simulation test of the microcomputer-controlled Langmuir probe.

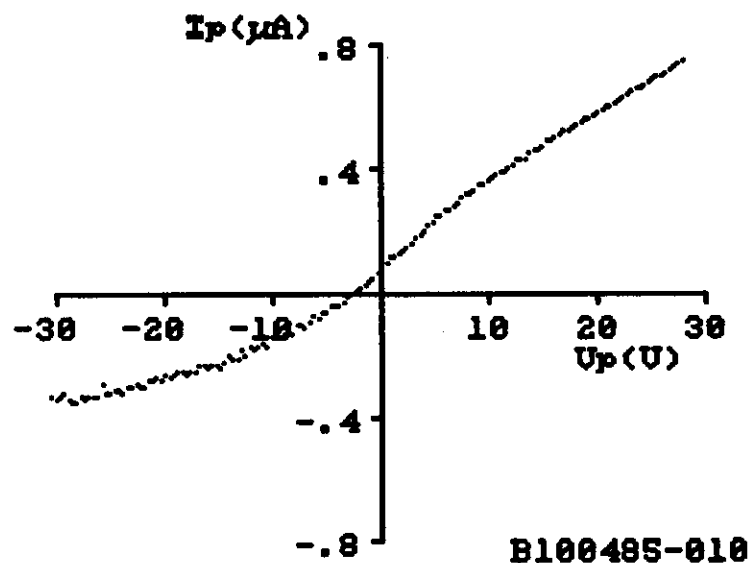


Fig. 4: A double probe characteristic of the hot-cathode dc glow discharge plasma obtained by the computer.

SECOND TROPICAL COLLEGE ON APPLIED PHYSICS

EXPERIMENT

GLOW DISCHARGE MEASUREMENT USING A MICROCOMPUTER-CONTROLLED LANGMUIR PROBE

C.S.Wong

1. Aim of the experiment.

In the glow discharge experiment¹ of the First Tropical College on Applied Physics, the measurement of the Langmuir probe characteristic had been obtained by the point-by-point manual method. In the present experiment, this will be obtained by a microcomputer-controlled data acquisition system.

2. The glow discharge system.

The glow discharge tube used in this experiment is of the cold cathode type consisting of ring electrodes as shown in Fig. 1. The tube is pumped to a base pressure of $\sim 10^{-4}$ mbar before helium gas is filled in. With this discharge tube, steady discharge can be maintained at an interelectrode voltage drop of 400 to 800 volts in helium at filling pressure in the range of 2 to 20 mbar. In this experiment, the discharge is maintained at 10 mbar helium. For a discharge current of $I_d = 50$ mA, the voltage drop across the electrodes is 675 V.

The glow discharge circuitry used in this experiment is shown in Fig. 2. The discharge current can be controlled by

adjusting either the limiting resistor R_L or the power supply output V_g . The voltage drop across the electrodes and the discharge current are monitored by analog voltmeter and ammeter.

3. The Langmuir probe.

The Langmuir probe is made from molybdenum wire of 1 mm diameter. The Mo wire is inserted into a small thin-wall glass tube of inner bore diameter slightly more than the wire until the tip of the wire become flush with one end of the glass tube. The Mo wire together with the glass tube is then heated in gas flame until the Mo wire becomes red-hot and the glass tube becomes soften and begins to shrink. This procedure will ensure that there is no space left between the Mo wire and the inner wall of the glass tube. The tip of the wire-glass assembly now forms a plane probe geometry. This probe is supported by inserting it into another glass tube of bigger bore diameter and is introduced into the plasma through a perspex holder from the entrance on the anode (Fig. 1).

4. The microcomputer-controlled data acquisition system.

The data acquisition system consists of a 64k RAM Apple-II microcomputer which is supported by a high-resolution CRT display unit, two mini floppy disk-drives and a dot matrix printer with high-resolution graphic capability (Fig. 3). Data acquisition is done via a multi-channel A/D \rightarrow D/A interface card (Mountain Computer Inc.) which can be plugged directly into the expansion slot of the Apple-II. This interface card provides 16 channels each of analog-to-digital and digital-to-analog converters. The

range of its analog input/output levels is $\pm 5V$ to $\pm 5V$, with a digital equivalence of 0 to 255. This gives a resolution of not better than 32 mV for the system.

The experimental set-up of the microcomputer-based Langmuir probe measurement of a plasma is as shown in Fig. 2. A 1 k Ω resistor is connected in place of the usual micro-ammeter for measuring the probe current, I_p . The potential drop across the 1 k Ω resistor is expected to be of the order of mV only, and hence it must be amplified before it is fed into an A/D channel of the computer. This is done by using a HP2470A data amplifier.

The probe potential V_p is supplied by using a battery pack connected in series with a HP467A power amplifier acting as a variable power supply. The output of the battery pack can be selected between 0 to 900 V in step of 22.5 V. The output of the power amplifier can be varied within the range of ± 30 V to ± 30 V by applying a voltage of ± 3 V to ± 3 V to its input. Thus when used in conjunction with a digital-to-analog converter, the HP467A power amplifier functions as a low voltage digitally controlled power supply. This arrangement is good enough for the present purpose.

5. Experimental procedures.

5.1. Getting the glow.

Before the start of the experiment, the glow discharge tube should have been pumped to a pressure of $\sim 10^{-4}$ mbar. The pump is then cut-off and helium gas is let in to a pressure of 25 mbar (end of scale of the pressure gauge). With only the rotary pump, the gas is evacuated for about 5 minutes to a pressure of \sim

10^{-2} mbar. Helium gas is filled in again and the procedure may be repeated two to three times so that a reasonably pure volume of helium is obtained. The pressure of the chamber is finally maintained at 10 mbar of helium.

The power supply V_g is now turn on and its output is increased slowly. Initially, before the gas breaks down, the voltmeter V_a indicates the output voltage of power supply V_g . However, at $V_g \sim 1.2$ kV, gas breakdown will occur and current starts to flow through the discharge tube as indicated by the reading on the milliammeter. V_a now shows the voltage drop across the electrodes and it is less than V_g . A glow should now be visible. Note the shape and the colour of the glow and also identify the positive column. Is the probe immersed in the positive column of the discharge? Now adjust R_L or V_g so as to obtain the appropriate current and note down the values of the following quantities :

$V_g =$ volts;
 $V_a =$ volts;
 $I_a =$ mA; and
 $P =$ mbar.

5.2. Determining the suitable starting voltage for the data acquisition.

Depending on its location along the axis of the discharge tube, the floating potential of the probe (ϕ_f) may vary between the range of $(V_a - 300)$ to V_a while the electron retardation region of the probe characteristic lies between ϕ_f and the space potential ϕ_s at the location of the probe. At a probe potential

of $V_p > \phi_s$, electrons will be attracted towards the probe surface, bombarding it to produce a faint glow at the tip of the probe. This glow is used as an indicator to choose the suitable starting probe potential V_p . It is learnt from experience that the suitable starting potential can be fixed at 2 step below the potential above which a glow begins to be visible at the tip of the probe.

5.3. Sweeping the probe potential - determination of the probe characteristic.

We are now ready to obtain a set of the probe characteristic for the helium discharge. The sweeping of the probe potential and the measurement of the corresponding probe current are controlled by using a BASIC programme (FILENAME : DATA ACQUISITION(S)). The sequence of operations of this programme is as follows :

(i) At the start of a RUN, the programme will request the user to input various parameters such as V_a , I_a , V_g , P etc and a user created data serial number.

(ii) The programme then proceeds to perform the data acquisition. At each step, the programme sends an analog output, via a D/A channel, to the input terminal of the HP467A power amplifier which then produces an amplified output voltage V_0 in series to V_p . Thus the effective probe potential is $V_p = V_0 + V_g$. If the amplification factor selected is $\times 10$, then the minimum voltage step-size is 0.39 V. The choice of the most suitable sweep voltage range and the step-size will be obtained after some trial-and-error processes so as the best result is

achieved.

(iii) After the completion of the data acquisition, the programme will request for the input of I_a , V_a and P again. This allows the user to check if these parameters have changed during the data acquisition.

(iv) Upon receiving the requested parameters, the computer begins to print out a listing of the probe data collected ($I_p - V_p$).

(v) Finally, the programme ask the user to decide if the present set of data collected should be stored into disk memory for further analysis. This allows the user to discard data which are not correct. If a positive reply is given, the programme will store the data under the filename of the data serial number input at the start of the RUN. The programme then resets to the start and it is now ready for the next sequence of data acquisition.

The above sequence of events is illustrated in the flow chart in Fig. 4.

5.4. Plotting the probe characteristic.

Using the data listing obtained in step (iv) of the operation of the data acquisition programme above, the probe characteristic obtained for the helium glow discharge can now be plotted. You may also use the programme PLOT I-V(S) to plot the characteristic. An example of the I-V characteristic plotted by this programme will be shown in Appendix B.

6. Analysis.

From the probe characteristic obtained above, deduce the electron temperature T_e , the electron density n_e and the space potential ϕ_s of the helium positive column. (Refer to Ref.1 for the interpretation of the probe characteristic).

Reference.

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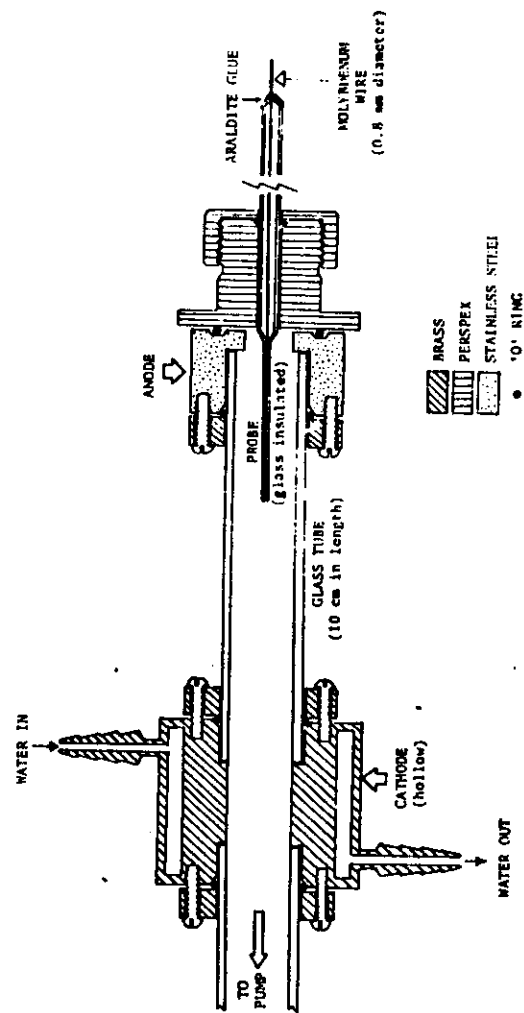


Fig. 1. Schematic of the glow discharge tube.

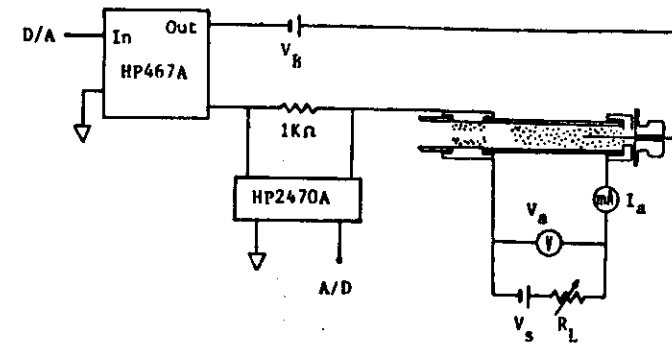


Fig. 2. Experimental set-up of the microcomputer-based Langmuir probe measurement of the glow discharge.

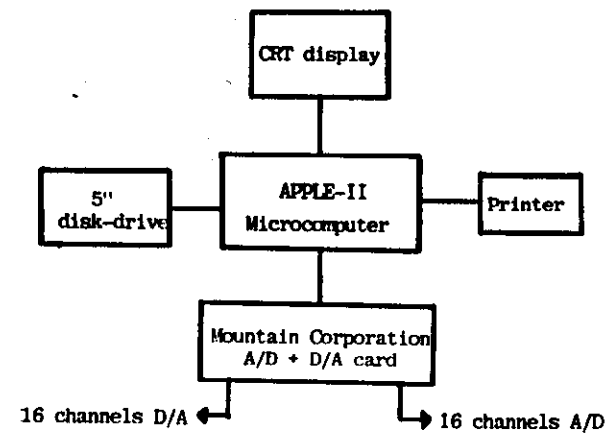


Fig. 3. Block diagram of the data acquisition system.

Appendix A : Programme listing of DATA ACQUISITION(S).

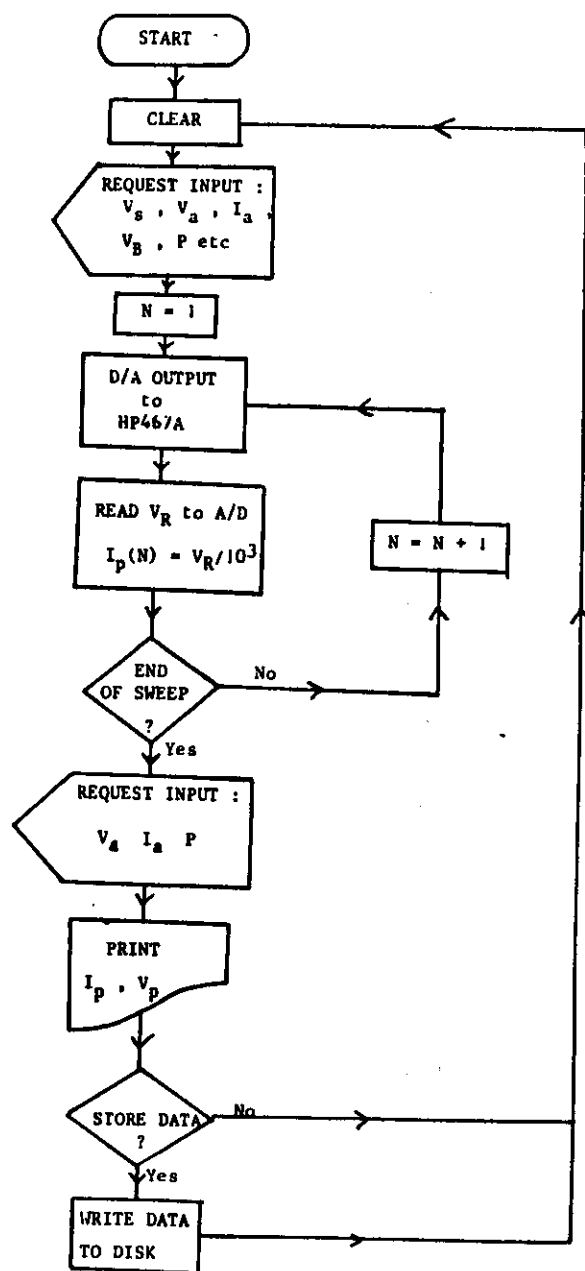


Fig. 4. Flow chart showing the sequence of data acquisition.

```

1 TEXT : HOME
10 REM
20 REM *** NAME: DATA ACQUISITION(S)
30 REM
35 REM *** DATA ACQUISITION OF THE SINGLE LANGMUIR PROBE
36 REM *** GLOW DISCHARGE
38 REM
40 REM ***FOR Vp, USE D/A OUTPUT AMPLIFIED BY HP467A POWER AMP
45 REM
50 M1 = 500: REM Ip IS AVERAGED OVER M1 DATA
60 DIM IP(256),VF(256)
70 REM
80 REM A/D+D/A CARD IN SLOT 5
90 REM
100 A = 49370: REM D/A CH 10 TO OUTPUT Vp
140 S3 = 49360: REM CHANNEL NUMBER FOR Ip IS 0
260 REM
374 PRINT "*****"
375 PRINT "* SINGLE LANGMUIR PROBE MEASUREMENT OF *"
377 PRINT "* GLOW DISCHARGE *"
378 PRINT "*****"
380 PRINT : PRINT
390 INPUT "DATA SERIAL NUMBER(Bddmmyy-aaa) : "IF$: PRINT
392 INPUT "DISCHARGE VOLTAGE(V) = "IVZ: PRINT
410 INPUT "TYPE OF GAS USED IS "IN$: PRINT
415 INPUT "AMPLIFICATION FACTOR OF POWER AMP = "IPF: PRINT
425 INPUT "RESISTANCE FOR MONITORING Ip(ohms) = "IRI: PRINT
426 INPUT "BASE PROBE PONTETIAL(V) = "IVB: PRINT
427 INPUT "AMPLIFICATION FACTOR OF OP-AMP = "IAF: PRINT
430 INPUT "INITIAL VESSEL PRESSURE(mbar) = "IPI: PRINT
435 INPUT "DISCHARGE CURRENT(mA) = "IMI: PRINT
440 INPUT "MEASURED ANODE POTENTIAL(IN V) = "IMV: PRINT
450 REM
770 PRINT : PRINT "Data acquisition in progress....."
803 PRINT : PRINT
805 REM
806 REM *** DATA ACQUISITION ***
807 REM
808 II = 0
810 FOR I = 50 TO 140 STEP 2
811 II = II + 1
812 POKE A,I
815 FOR PAUSE = 1 TO 1000: NEXT PAUSE
820 SY = 0
960 FOR J3 = 1 TO M1
970 Y = PEEK (S3):Y = PEEK (S3)
980 SY = Y + SY
990 NEXT J3
1260 REM
1270 REM *** CALCULATIONS
1280 REM
1310 V3 = ((SY / M1) * 10 / 255 - 5)
1320 VP(II) = (I * 10 / 255 - 5) * PF + VB
1330 IP(II) = V3 / (AF * RI)
1425 NEXT I

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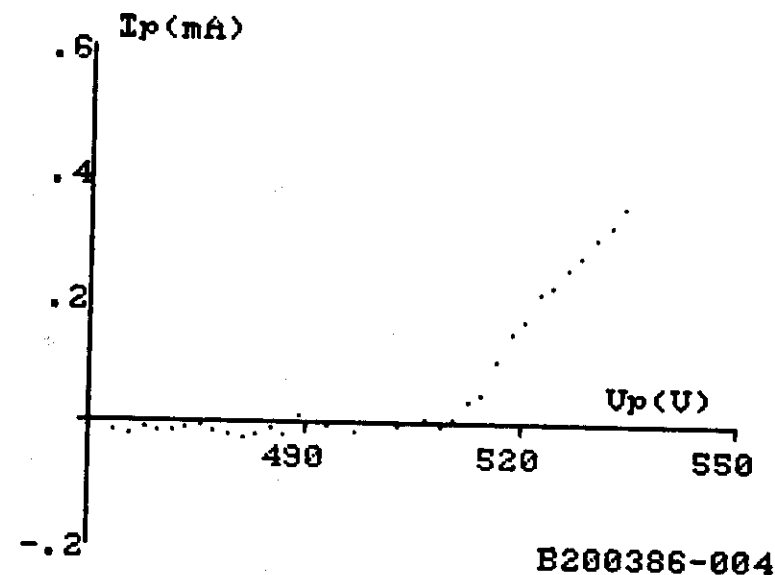


```

1435 HONE
1470 FLASH : PRINT "DATA ACQUISITION COMPLETED": NORMAL
1475 PRINT : PRINT
1500 INPUT "FINAL VESSEL PRESSURE(mbar) = " : PF: PRINT
1502 INPUT "FINAL DISCHARGE CURRENT(mA) = " : FI: PRINT
1504 INPUT "FINAL ANODE POTENTIAL(V) = " : FV: PRINT
1510 P = (FI + FF) / 2
1512 MI = (MI + FI) / 2
1514 MV = (MV + FV) / 2
1520 REM
1530 REM *** PRINT DATA
1540 REM
1550 PR# 1
1564 PRINT "*****"
1565 PRINT "*"
1570 PRINT "* SINGLE LANGMUIR PROBE MEASUREMENT OF UMGD-B *"
1575 PRINT "*"
1580 PRINT "SER.NO.: " : IF#1 " "
1590 PRINT "*"
1592 PRINT "*****"
1595 PRINT : PRINT
1596 PRINT "Discharge voltage = " : IV: " V"
1597 PRINT "Discharge current = " : MI: " mA"
1598 PRINT "Anode potential = " : MV: " V"
1601 PRINT "Type of gas used is " : INS
1602 PRINT "Pressure = " : PI: " mbar"
1605 PRINT : PRINT
1610 PRINT "Up(V)"; TAB( 20); "Ip(uA)"
1620 PRINT "-----"; TAB( 20); "-----"
1630 FOR I = 1 TO II
1640 PRINT VP(I); TAB( 20); IP(I) * 1E4
1650 NEXT I
1660 PRINT : PRINT
1670 PR# 0
1680 PRINT
1682 INPUT "DO YOU WISH TO STORE THE DATA?(Y/N) " : JC#
1685 PRINT
1690 IF JC# = "Y" THEN GOTO 1850
1692 GOTO 2010
1695 REM
1697 REM *** TO STORE DATA
1698 REM
1699 DS = CHR$(4): REM CHR$(4)=CTRL-D
1700 PRINT DS: "OPEN " : IF#
1701 PRINT DS: "WRITE " : IF#
1702 PRINT II
1703 PRINT P
1704 PRINT MV
1705 PRINT MI
1706 PRINT NS
1707 FOR I = 1 TO II
1708 PRINT IP(I): PRINT VP(I)
1709 NEXT I
1710 PRINT DS: "CLOSE"
1711 RUN

```

Appendix B : An example of the I-V characteristic of the Langmuir probe plotted by the computer.



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