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SPRING COLLEGE IN MATERIALS SCIENCE

ON

"METALLIC MATERIALS"

(11 May - 19 June 1987)

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ELECTRONIC STRUCTURE AND PHASE STABILITY  
(Part I)

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These are preliminary lecture notes, intended only for distribution to participants.

ELECTRONIC STRUCTURE  
AND  
PHASE STABILITY  
METALLIC SYSTEMS

I INTRODUCTION

- 1a) "build" new phases from at, clust, md-  
 - det (descr) their struct  
 - predict their occurrence:  
 their properties

«design»



- b) Use of {  
 • electronic structure  
 • statistical physics

c) Examples:

- (L) HIGH TEMPERATURE ALLOYS  
 superalloys (heterogeneous)  $\delta, \gamma'$   
 (U) NEW SUPERCONDUCTORS  $La_{2-x}M_xCuO_4$   
 «ceramics»  $Y_{1-x}Ca_xCuO_4$   
 (11c) AMORPHOUS  $NiZr, FeP, \dots$

APL6

2 ORDER-DISORDER

2-3 TOPOLOGICAL DISORDER

- ORIGIN
- (A) Crystalline: 3x periodic  
 (B) quasicrystalline  
 "LRO" → lattice  
 $n_{at}(r)$   
 «ground state» → H.T. order → «ensemble average»
- TOPOLOGICAL DISORDER
- (C) AMORPHOUS  
 (D) LIQUID  
 "S.R.O."  
 $g_{at}(r)$

2-2 ALLOY PHASES AND CHEMICAL DISORDER

- START FROM SIMPLE LATTICES
- BUILD ORDERED STRUCTURES  $A_cB_{1-c}$  (O.S.)  
 FROM SIMPLE TO MORE AND MORE COMPLEX  
 BY ADDING PERIODIC ID DEFECTS

Examples

- 1a)  $CuAu$  ( $L1_0$ )  $CoFe$  ( $B2$ ) simple O.S.  
 $Cu_3Au$  ( $L1_2$ ) fcc bcc
- b) substoichiometric alloys  
 "simple ordering on a sublattice"  
 ex: carbides, nitrides.
- c) long periods  $CuAu, CuPd, TiAl_3$   
 TOWARD AN  $\infty$  OF STRUCTURES.

2a) polytypes

- Co-V alloys
  - Mg alloys,  $Zr(FeCr)_2$
- ABC (a)      ABA (b)

3 ORDER - DISORDER amorphous alloys ③

2-3 Low dimensionality. ( $\rightarrow$  2D, 1D)

- 2D: Intercalation chemistry  
 Examples • TM dichalcogenides  
 • graphite + metal (alkaline RE...)  
 • molecules

1D  
 organic compounds  
 (transport magnetism)

• D  
 small particles  
 (catalysis magnetism)

2-4. ARTIFICIAL STRUCTURES (AS)

- 1D periodic modulations ABAB...  
 - composition superlattices  
 - coherent and incoherent interfaces  
 - periodic and aperiodic  
 order in A.S vs order in quasicrystals?

x:  $GeSi$ ,  $AlAs$  bp:  $RuIr$ ,  $NiCr$ ...

2-5 ORDER } VS SEGREGATION  
 MIXING }

- Ex 1  $CuNi$  (mixing-seg)  $\leftarrow$  diff<sup>n</sup> Li  
 Ex 2  $CuPt_{1-x}$  order-seg vs concentration

2-6 PHASE STABILITY ( $\phi$  st)

• Compare the (free) energies of the various  $\phi$ ?  
 to det de  $\phi$  st

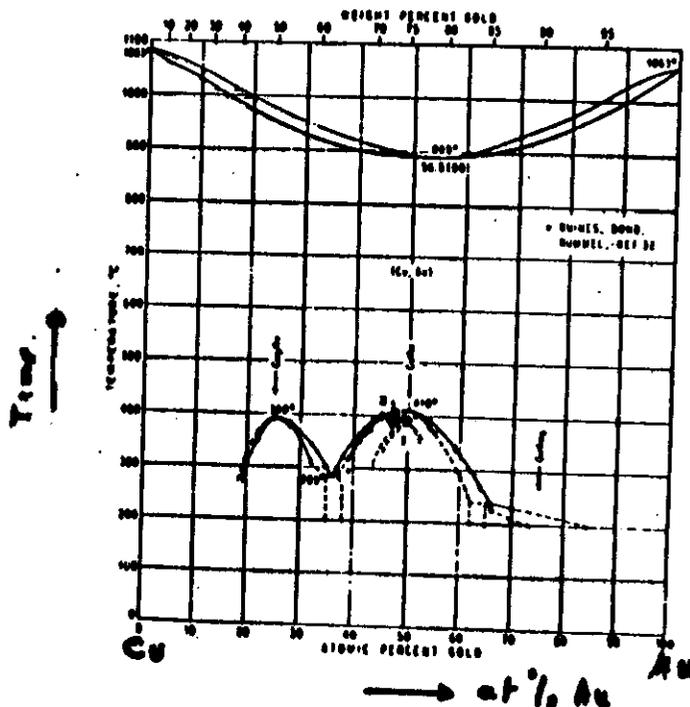
$\rightarrow$  various orders of the comp. lattice  $>$  st st  
 $\rightarrow$  compare for  $\phi$  lattices, diff

$\Delta H$  (or  $\Delta E$ )  $\Delta E = E - Q E_0 - \phi E_1$

mixing energy  
 formation "

IN THE FOLLOWING TRANSPARENTS, WE PRESENT QUALITATIVELY SOME REPRESENTATIVE EXAMPLES OF THE ORDERED STRUCTURES: WE WILL STUDY LATER THEIR EXAMPLES FROM AN EL STR POINT OF VIEW

ONE OF THE SIMPLEST PHASE DIAGRAM WITH O.S.



d'après Hensen (1988)

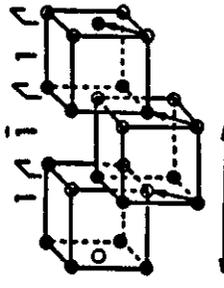
• THREE ORDERED PHASES

- $CuAu_2$   $L1_0$   
 fcc latt.  $Cu_3Au$   $L1_2$  + "LONG PERIODS"  $CuAu_3$   
 $CuAu$

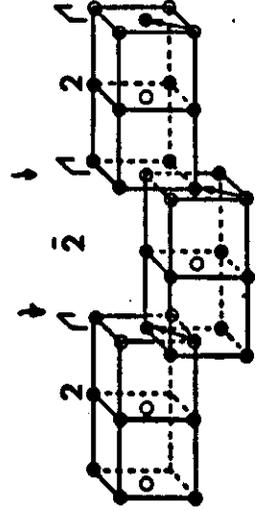




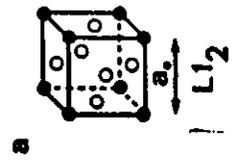
APB



b DO<sub>22</sub> M=1



c DO<sub>23</sub> M=2



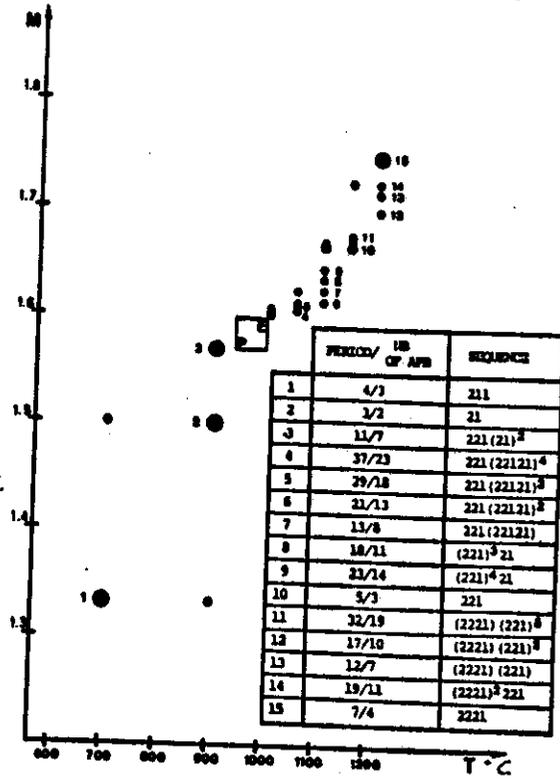
● Ti  
○ Al



Succession of  
PURE (001) PLANES  
 $2a, 2a, \dots, (2n+1)a$   
AND OF MIXED  
PLANES  
 $2a, 0, a, \dots, na$   
WITH PURE  
PLANE

CONSTRUCTION OF L.P. FOR AB<sub>3</sub> ALLOYS  
THE MIXED PLANES ONLY ARE REPRESENTED.

②



TiAl<sub>3</sub>

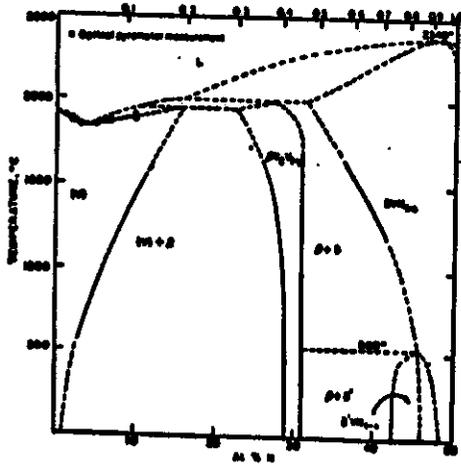
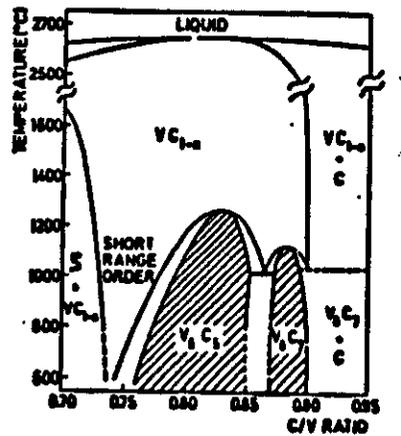
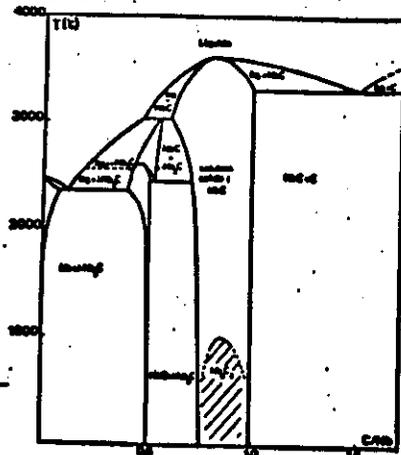
Fig 19

Loiseau et al  
1995

PERIODS OF THE OBSERVED L.P. FOR VA  
RIOUS T (observation by HREM)

③

PHASE DIAGRAMS OF SUBSTOICHIOMETRIC CARBIDES



9

ORDER IN SUBSTOICHIOMETRIC CARBIDES AND NITRIDES (STRUCTURE "NaCl" B1)

10

- ONE OF THE SUBLATTICES IS ONLY OCCUPIED BY TM ATOMS
- ONE OF THE FCC SUBLATTICES IS OCCUPIED BOTH BY C (or N) AND BY VACANCIES. THE VACANCIES ARE ORDERED ON THIS FCC SUBLATTICE ACCORDING TO THE CLASSICAL O.S. WE DISCU. PREVIOUS

[O.S. for vacancies on  $L_0$ ]  
[metallic f.c.c. sublattice]

- carbides
- $L_1$  "L1"  $Ti_2C, Zr_2C, Y_2C, Sc_2C$
  - $A_2$  "A2"  $Gd_2C, Dy_2C, Ho_2C, Er_2C$
  - $A_1$  "A1"  $Nb_6C_5, V_6C_5, Ta_6C_5$

- nitrides
- $D_{02}$  "D02"  $Nb_4N_3$
  - $A_1$  "A1"  $Ti_2N, Mo_2N$

(from de Novy et al)

- Note:  $L_1$  IS THE STRUCTURE OBTAINED ON AN FCC LATTICE BY OCCUPYING THE SUCCESSIVE (111) PLANES BY A AND B ATOMS  
 $A_2$  IS THE STRUCTURE OBTAINED FROM  $L_1$  BY INTRODUCING THE PREVIOUS ABB ON ALL PLANES  $z = n/2$ .
- Note also that in METALLIC SYSTEMS, THERE IS ONLY ONE " $L_1$ " STRUCTURE EXPERIMENTALLY OBSERVED ("G.P.M") WHEREAS MANY SUBSTOICHIOMETRIC CARBIDES PRESENT THIS STRUCTURE AS A STABLE O.S. WHY? SEE CHAPTER II.



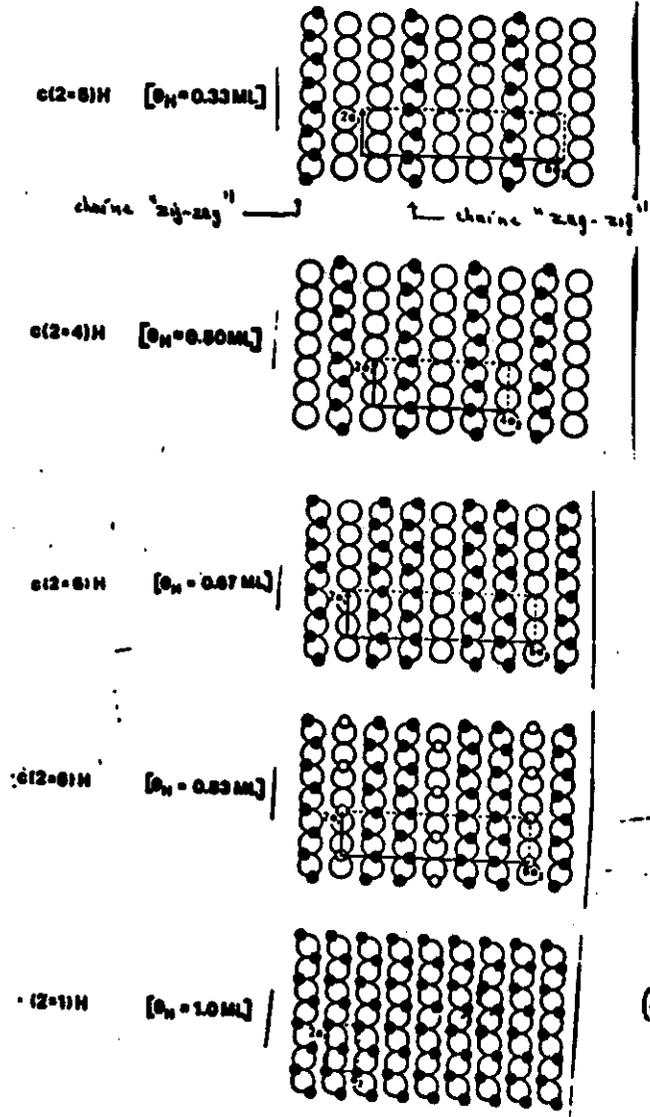
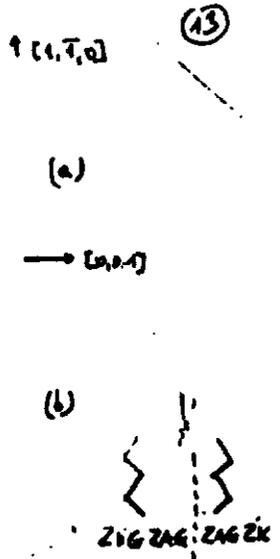


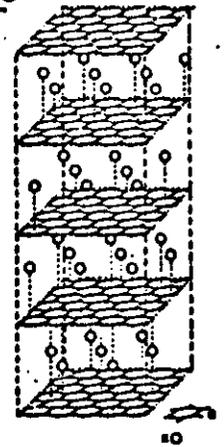
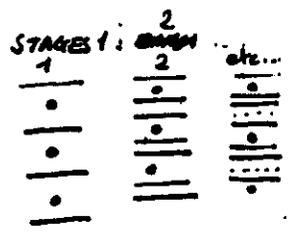
Fig. 21 (d'après [90c])

Arrangements des adatoms d'hydrogène sur la surface (110) de nickel pour les structures ordonnées observées ( $0 < \theta < 1$ ). Les grands cercles correspondent aux atomes de Ni, tandis que les petits cercles (noirs et blancs) correspondent aux atomes H. Notez qu'il y a trois types d'arrangements atomiques différents correspondant à la structure  $c(2 \times 6)$ .



INTERCALATED

GRAPHITES



Model for  $C_xK$  according to Rüdorff and Schulze (1954) showing the stacking of graphite layers (networks of small solid balls) and of potassium layers (networks of large hollow balls). The graphite and intercalate layers are arranged in an AA'AyAA' stacking sequence, where A refers to the graphite layers and the greek letters to the intercalate layers.

Fig. 10b (d'après [10])

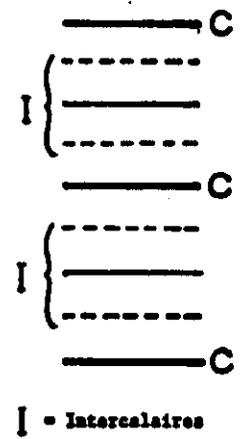
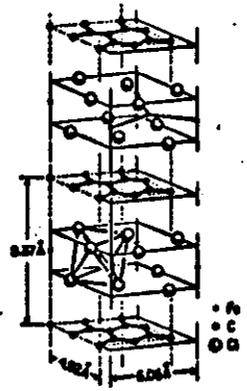


Fig. 10c



Schematic diagram of stage 1 graphite- $FeCl_3$ , showing intercalate layers with the bulk  $FeCl_3$  structure sandwiched between graphite planes. The intercalate layers are incommensurate with the graphite and the pertinent lattice constants are indicated. The octahedral coordination of the ferric ion is shown in the lower sandwich and the two inequivalent ferric ions are indicated on the upper sandwich.

Fig. 10d (d'après [10])



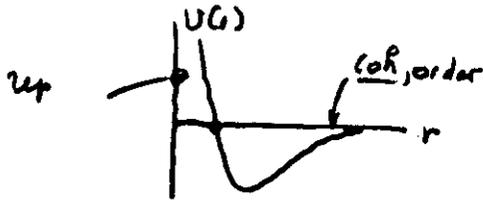
(17)

3 interatomic potentials, pair and multiple.  
order interactions, and  $\phi$  stability.  
alloy  $A_c B_{1-c}$

1. We assume  $\sum_{i,j} v^{ij}(r)$



$$E_{conf} = \frac{1}{2} \sum_{i,j} v^{ij}(r)$$



2. On a lattice  $\rightarrow$  Ising model

• a configuration  $\equiv \{P_i^i\}$

$$P_i^i = \begin{cases} 1 & \text{if site } i \text{ occ by } i \\ 0 & \text{otherwise} \end{cases}$$

$$E_{conf} = \frac{1}{2} \sum_{i,j} \sum_{\lambda,\mu} v_{\lambda\mu}^{ij} P_i^i P_j^j$$

• BINARY ALLOY is A, B

$$P_i^B = 1 - P_i^A \quad P_i^A = P_i$$

$$E_{conf} = \frac{1}{2} \sum_{i \neq j} v_{\lambda\mu} P_i P_j = \bar{\mu} \sum_i P_i$$

$$v_{\lambda\mu} = v_{\lambda\lambda}^{AA} + v_{\lambda\lambda}^{BB} - 2v_{\lambda\lambda}^{AB}$$

• ISING NOTATION

$$P_i = \frac{1 + \sigma_i}{2} \Rightarrow E_{conf} = \frac{1}{2} \sum_{i \neq j} v_{\lambda\mu} \sigma_i \sigma_j - J_{ij} \sigma_i \sigma_j$$

(18)

IN SUCH A CASE WE ARE INTERESTED BY:

- THE DETERMINATION OF THE ATOMIC ORDER (possible intermetallic compounds / the most stable at eq wt% C? order or segregation?..)

- THE SHORT RANGE ORDER (SRO) IS DETERMINED BY:

$$\langle P_i^i P_j^j \rangle = \begin{cases} S_{ij} c_i & \text{if } i=j \\ c_i c_j - c_i c_j \lambda^{ij} (1-\lambda) \end{cases}$$

(probability to have i at  $\frac{1}{2}$  and j at  $\frac{1}{2}$ )

WHERE  $\langle P_i^i \rangle = c_i$  IS THE CONCENTRATION OF ATOM

FOR A BINARY ALLOY  $A_c B_{1-c}$ :

$$\langle P_i^A P_j^A \rangle = c_A^2 + c_A c_B S(\lambda) \quad S^A(\lambda) = S(\lambda) \\ \langle P_i^A P_j^B \rangle = c_A c_B - c_A c_B S(\lambda) \quad \text{SRO PARAMETERS}$$

$S(\lambda)$  decreases with  $\lambda$  (oscillating) and tends to zero when  $\lambda \rightarrow \infty$

•  $S_1 > 0$  (1st ord)  $\rightarrow$  CLUSTERING (more A at than the average when one of its neighbors is occ by A)  
 $S_1 < 0$   $\rightarrow$  ORDERING (less...; more... B).

4 GENERAL TRENDS FOR SEGREGATION AND ORDER

GENERAL: EMPIRICAL "RULES":

1. DISORDERED SYSTEMS AND COMPOUND



$\Delta E_1 \approx A(\Delta r)^2$   
 $(r = \text{at volume})$   
 $\Delta E_2 \approx -\frac{N}{R} \Delta \phi^2$   
 $\Delta E = -P \Delta \phi^2 + Q \Delta n_{ws}^{2/3}$  (MIEDEMA)

2 ORDERED STRUCTURE ON A LATTICE

FOR A GIVEN CONCENTRATION C  
TWO PARAMETERS (AT LEAST):

- AVERAGE NUMBER OF ELECTRONS PER ATOM ( $e/a$ ) HUME ROTHERY.

$\bar{N} = c_A N_A + c_B N_B$

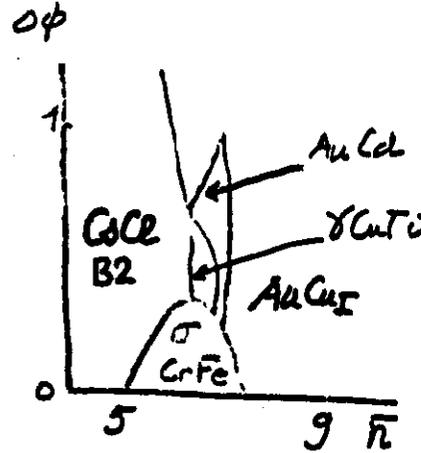
- DIFFERENCE OF ELECTRON NEGATIVITIES

$\Delta \phi$

- DIFFERENCE OF VALENCE ( $\Delta N$ )

↓

"STRUCTURAL MAPS"



3 EXAMPLE: PARAMETERS CHARACTERIZING AN ALLOY OF TRANSITION METALS.

FOR ORDERING ON A GIVEN LATTICE (fcc for example):

- (i) CONCENTRATION C DETERMINES A SET OF POSSIBLE STRUCTURES
- (ii) THE ( $e/a$ ) FILLING OF THE BANDS

$\bar{N} = c_A N_A + c_B N_B$

- (iii)  $\Delta N = N_A - N_B$  CHARACTERIZES THE FLUCTUATION OF POTENTIAL

- (iv)  $\Delta S = \pm 1, \pm 2, 0$  CORRESPONDS TO THE SERIES ( $S = 1, 2, 3$ ).

$\Delta N \approx \Delta E_d \sim \Delta \phi$   
 $\Delta E_d = E_A - E_B = \text{difference between the A and B energy levels in the metal}$   
 $\Delta V_d = W_A - W_B = f(\Delta Z, \Delta S)$

(21)

NOTE THAT A GOOD SEPARATION IS OBTAINED BY 2D MAPS FOR ALL CONCENTRATIONS!

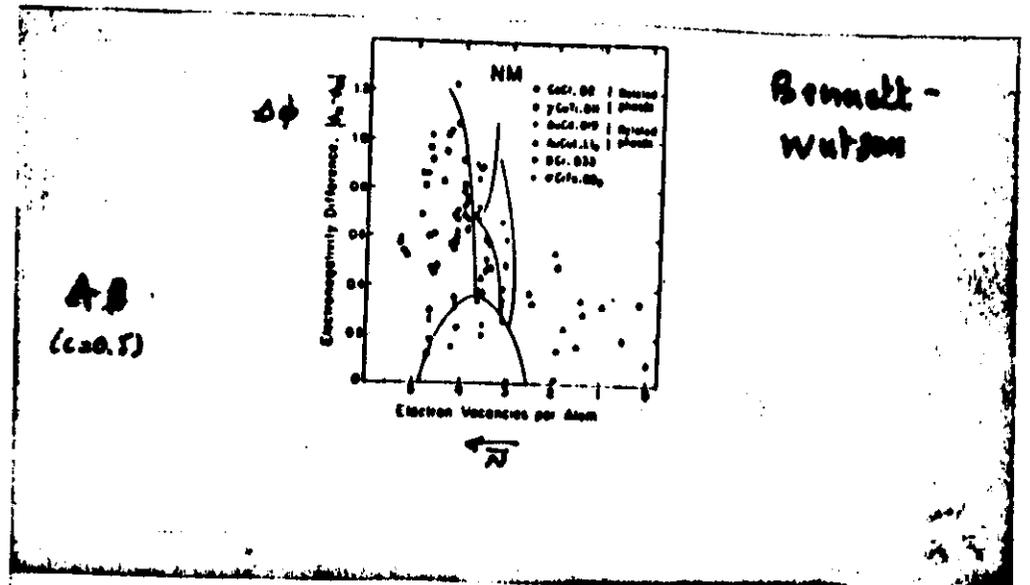
4. STRUCTURAL MAPS AND ELECTRONEGATIVITY SCALE FOR p.d. COMPOUNDS (Pettifor)

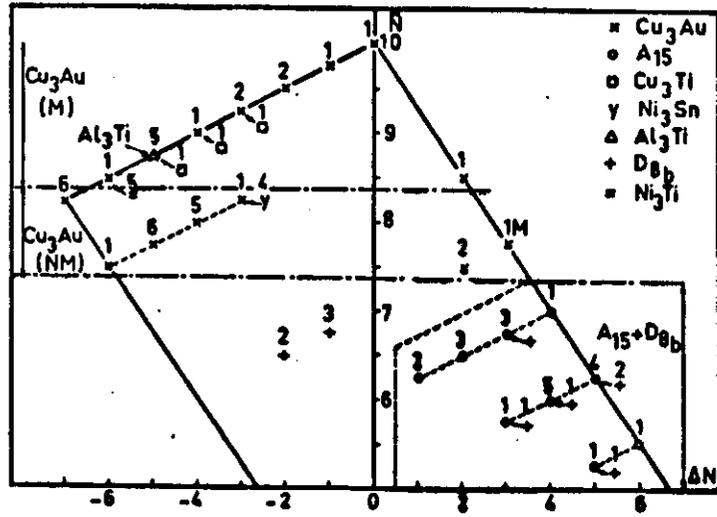
5 GLASS FORMING ABILITY

$\Delta H_m$  /  $r/R$  (Giesse)

and more complicated representation...

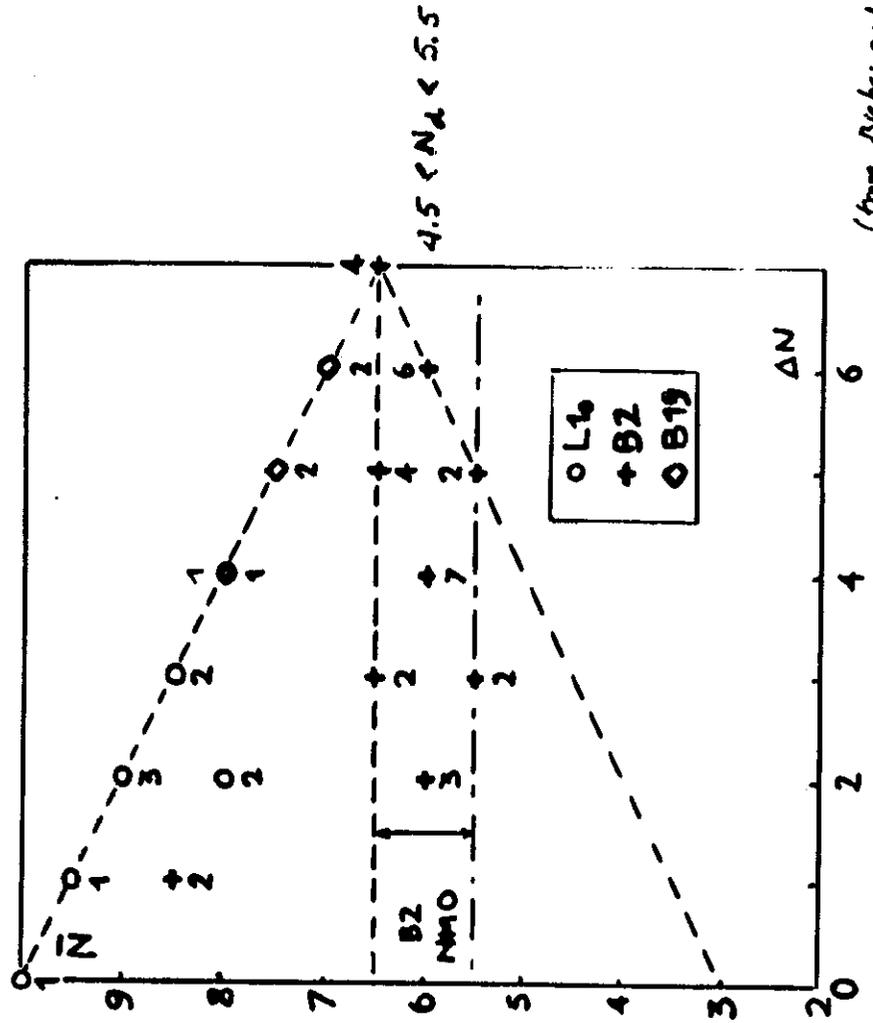
(22)





(from Breder and Gauthier)

Fig. IV.7.3. :  
 Cartes structurales phénoménologiques ( $N$ ,  $\Delta N$ ) pour les alliages de métaux de transition  $A_3B$  ;  $\text{Cu}_3\text{Au}$  (M) et  $\text{Cu}_3\text{Au}$  (NM) correspondent respectivement aux composés ferromagnétiques et paramagnétiques. Les nombres représentent le nombre de systèmes pour lesquels chaque SO a été observé.



CHEMICAL SCALE FOR THE STUDY OF THE STABILITY OF AB COMPOUNDS

(2)

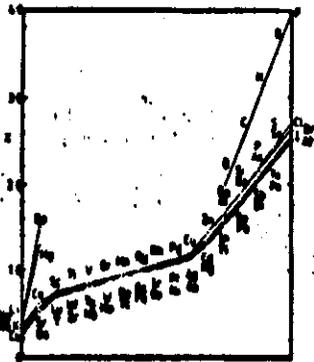


Fig. 1. - The chemical scale.

THE COARDI  
POLYDROMIC  
TOTAL MAP

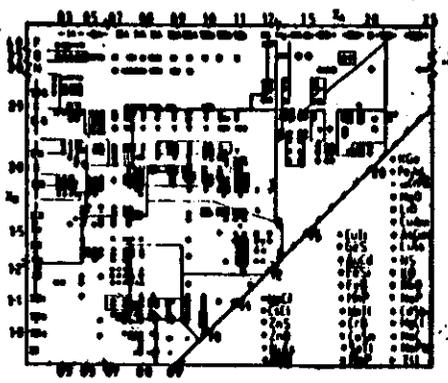


Fig. 2. - The  $(N_A, N_B)$  structure map for 374 AB compounds.

structural behavior under pressure of the sp bonded III-V compounds AlP, AlAs, GaP, and GaAs has been investigated from first principles by Frey and Gahn (16). Here we wish to study the pd bonded AB compounds which are located near the middle of Fig. 2. In particular, we wish to understand (13)

STRUCTURAL MAP (After) FOR sp-d COMPOUNDS

(26)

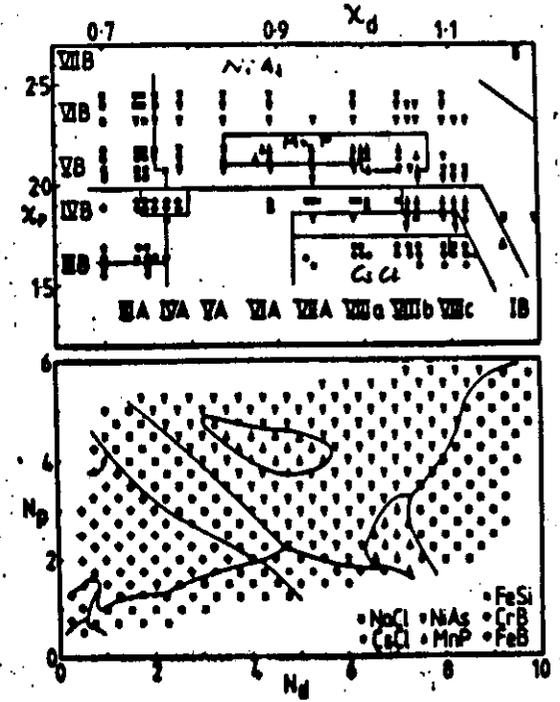


Fig. 3 -  
Upper panel: The structure map  $(N_A, N_B)$  for 140 pd bonded AB compounds, where  $N_A$  and  $N_B$  are the values of the chemical scale for the A and B elements respectively. The sp elements are not included in the present figure.

Lower panel: The theoretical structure map  $(N_d, N_p)$  where  $N_d$  and  $N_p$  are the number of d and p valence electrons respectively on the CdI lattice.

(From Ashcroft)

I: INTRODUCTION

- Topological and chemical orders.
- eg. 2D and metastable structures: order of compounds and artificial materials. General trends (structural maps).
- Phase stability and pair potentials? If such potential, 3 the results of Statistical  $\phi$  can be used (grand state,  $\phi$  diag. ...)

II ELECTRONIC STRUCTURE: GENERAL PRINCIPLES

- The Density Functional formalism and the L.D.A.
- Discussion. Validity of the results for G.S. and excitations (??) and some examples.
- Force theorem and the equation of state (T00K)
- The current approximations for the calculation of the one electron state - Brief comments on the deficiency of charge transfer.

III ELECTRONIC STRUCTURE OF ORDERED AND DISORDERED SYSTEMS: GENERAL TRENDS

- Introduction: the tight binding method
- Local electronic structure and recursion (moments) method.
- General features of the electronic structure of (concentrated) disordered alloys and ordered compounds. A simple method for the calculation of completely disordered alloys: the CPA.
- Discussion of the roles of the (1) topological disorder (2) chemical order of compounds (3) disorder in crystalline systems (4) topological disorder of O or D. alloys (5) ODD in interstitial compounds and  $\text{La}_2\text{CuO}_4$  systems ... ON THE D.O.S

IV PHASE STABILITY, PAIR POTENTIALS AND MULTIATOM INTERACTIONS

- Introduction: the various approaches for the expansion of the total energy in terms of multiatom interactions (MAI)
- The relevant energies for the study of  $\phi$  stability. Order of magnitude
- Some examples:
  - formation energy of x and a alloys
  - structural stability of pure metals
  - relative stability of  $\phi$  OS
- Qualitative discussion of the corresponding energies in terms of the D.O.S
  - cohesive energy (Friedel)
  - mixing energy for disordered alloys
  - structural stability
  - ordering energies
- Interatomic potentials and multiatom interactions
  - the various approaches
  - the general relation between perturbation expansion and MAI (cluster expansions)
  - the Generalized Perturbation method, principles and qualitative discussion of the results. (X. GONZALEZ)
  - the Linearized Green Function method (LPG) (DIXON)
  - the generalized pseudopotential formalism (GPF)

V PHASE STABILITY AND  $\phi$  DIAGRAMS USING CLUSTER EXPANSIONS.

- Introduction: the simultaneous use of electronic structure and of stat  $\phi$  results. In T00K and T $\infty$ K
- GROUND STATE PHASE STABILITY
  - Using models and O.S. stability: some examples (fcc lattice, mostly with APB equilibrium, long periods, polytypes...)
  - Structural maps (G.S.M) for simple structures using the CPA. (Homogeneous states)
  - Ground state phase stability in binary alloys (heterogeneous states)
- PHASE DIAGRAMS CALCULATIONS
  - Determination of the configurational entropy (CVM, Monte Carlo) Simf.
  - Prototype phase diagrams.
  - The liquid state of TM alloys from their electronic structures
  - a simple 1st order  $\phi$  diag. calculation
  - The role of vibrational and magnetic entropies.

VI CONCLUSION(S)

