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SPRING COLLEGE IN MATERIALS SCIENCE

ON

"METALLIC MATERIALS"

(11 May - 19 June 1987)

ELECTRONIC STRUCTURE AND PHASE STABILITY
(Part II)

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These are preliminary lecture notes, intended only for distribution to participants.

II BAND STRUCTURE CALCULATIONS

II 1 Introduction.

{ Our aim is to discuss the main assumptions
with special attention concerning the
energy calculations and their limitations

- (i) general principles of a band str. calc.
[local / functional] approximation
- (ii) one electron states for periodic potential
- (iii) simple models to get general trends
(Chapter III)

II 2 DENSITY FUNCTIONAL FORMALISM.

2.1 Pb: find the solutions $\Psi_0(r, \dots, r_N)$, E_0
($n=0$ = GROUND STATE) for
 $\mathcal{H} = T + \mathcal{H} + V_{ext}$ (N el $\rightarrow \infty$!) ?
increase with

$$\begin{cases} T = -\frac{\hbar^2}{2m} \sum_i \nabla_i^2 & \text{KINETIC ENERGY} \\ U = \frac{1}{2} \sum_{i,j} \frac{e^2}{|r_i - r_j|} & \text{COULOMB ENERGY} \\ V_{ext} = \sum_i V_{ext}(r_i) & \text{EXT. POT. (NUCLEI)} \end{cases}$$

Two theorems

- (a) The ground state energy E_0 (or the thermodynamic potential) is a universal functional of the density $n(r) = N \int \delta \Psi_0(r, r_2, \dots, r_N) |^2 dr_2 \dots dr_N$
- (b) The ground state energy E_0 is the same as the corresponding energy for non interacting el. in the potential $V(r)$

(a) V_{ext} is a functional of $n(r)$ (obtained from)

$$\begin{aligned} \text{if: } V_{ext} \rightarrow \Psi_0, E_0 &\Rightarrow n(r) \Rightarrow E_0' < \langle \Psi | \mathcal{H} | \Psi \rangle = E_0 + \int n(r) V(r) dr \\ \text{and } V_{ext} \rightarrow \Psi_0, E_0 &\Rightarrow E_0 < \langle \Psi | \mathcal{H} | \Psi \rangle = E_0' + \int n(r) V(r) dr \\ &\downarrow \\ &V_{ext} = V_{ext} \end{aligned}$$

$$\begin{cases} E(n(r)) = \int V_{ext}(r) n(r) dr + F(n(r)) \\ F(n(r)) = \frac{1}{2} \int \frac{n(r) n(r')}{|r - r'|} + G(n(r)) \end{cases}$$

(b) introduce an auxiliary system of non interacting electrons with the same density $n(r)$ as the real density

$$\mathcal{H}_0 = T + V \quad V = \sum_i V(r_i) ?$$

$$n(r) = \sum_k f_k |\psi_k(r)|^2$$

$$\mathcal{H}_0 = \sum_i \mathcal{H}_0(i) \quad \begin{cases} \mathcal{H}_0 = -\frac{\hbar^2}{2m} \nabla^2 + V \\ \mathcal{H}_0 \psi_k = E_k \psi_k \end{cases}$$

$$\begin{cases} G(n(r)) = T_0(n) + E_{xc}(n(r)) \\ T_0(n) = \langle \Psi_0^0 | T | \Psi_0^0 \rangle \quad (\Psi_0^0 \text{ G.S for } \mathcal{H}_0) \end{cases}$$

$$\begin{cases} E(n(r)) = T_0(n(r)) + \int V_{ext}(r) n(r) dr + \frac{1}{2} \int \frac{n(r) n(r')}{|r - r'|} dr + E_{xc} \\ E(n(r)) = \langle \Psi_0^0 | \mathcal{H} | \Psi_0^0 \rangle = T_0(n(r)) + \int V(r) n(r) dr \end{cases}$$

$$\text{GROUND STATE ENERGY: } \frac{\delta E}{\delta n(r)} = 0 \quad (\text{minimization})$$

$$V(r) = V_{ext}(r) + V_{Coul}(r) + V_{xc}(r)$$

(c) Local density approximation

$$E_{xc}(n(r)) = \int n(r) \epsilon_{xc}(n(r)) dr$$

$\epsilon_{xc}(n)$ for an homogeneous el gas

(d) Extension to the magnetic systems
 $E_{xc}(n, m)$ $m = n_\uparrow - n_\downarrow$

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The effects of exchange and correlations being included in a potential $V_{xc}(r)$

$$V(r) = V_{ext} + \underbrace{e^2 \int \frac{n(r')}{|r-r'|} dr'}_{\text{Coulomb potential}} + V_{xc}(r)$$

The local density approximation

$$V_{xc}(r) \approx \frac{d}{dn} (n(r) \epsilon_{xc}(n)) \big|_{n=n(r)} = \mu_{xc}(n(r))$$

$\epsilon_{xc}(n)$ is the EXCHANGE CORRELATION ENERGY PER ELECTRON FOR AN HOMOGENEOUS GAS OF DENSITY n ... which is accurate by 1 unit!

$$\text{exchange part } V_x = \frac{d}{dn} (n(r) n^{1/3} \frac{3}{2} (\frac{2}{\pi})^{1/3}) = \mu_x$$

$$\approx \frac{2}{3} V_{Slater}$$

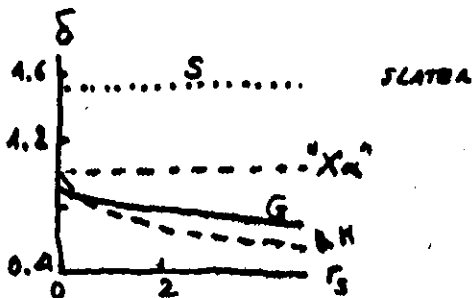
$$\mu_{xc}(n) = \beta(n) \mu_x(n)$$

$$\beta(r_s) = 1 + 0.545 r_s \log(1 + 11.4/r_s)$$

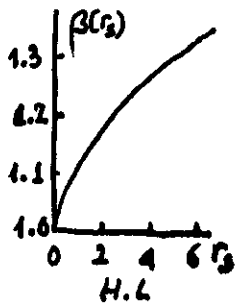
It can be extended to magnetic systems

$$V_{xc}^{\pm}(r_s, \zeta) = \mu_x(r_s) (\beta(r_s) \pm \frac{\zeta(r_s) \zeta}{1 \pm \zeta^2})$$

$$\delta(r_s) = 1 - 0.036 r_s - 1.36 r_s / (1 + 0.1 r_s) \quad \delta_0 = 0.993$$



$V_{Slater} = \langle V_x \rangle$ average of an exchange potential for a homogeneous gas $\approx n^{1/3}$
 (1) $\gamma_{xc} = \alpha V_{Slater}$ α fitted to some of exp.



$$\zeta = \frac{M}{N}$$

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3 Discussion

THE L.D.A. ALLOW TO DETERMINE GROUND STATE PROPERTIES

- $E_0, n(r), m(r), B, \chi \dots$
- THE AUXILIARY FUNCTION $\psi_h(u)$ HAVE NO ϕ MEANING... "IN PRINCIPLE" BUT...
- IT'S TEMPTING TO IDENTIFY $(E_h, \psi_h(u))$ TO THE ELEMENTARY EXCITATIONS (h, c) FOR THE REAL SYSTEM.
- IT WORKS WHEN THE EFFECTS OF CORRELATION ARE NOT TOO IMPORTANT

Ex 1 Cu ψ_h Ni

Ex 2 E_g for Si or Ge As (th. 25 eV E_g)

4. FORCES AND PRESSURE RELATIONS

homog. compression or: monob. $1/r$

$$p = - \frac{dE_0}{dV} \quad p(V_0) = 0 \quad V_0 = N \Omega_0$$

$$B = \text{bulk modulus} = K^{-1} = - \frac{dp}{d \log V} \big|_{V_0}$$

$$E_c = \text{cohesive energy per atom} = - \int p dV = -3 \int p K dV$$

(s = radius of the atomic sphere).

T_h (Andersson)

$$p = - \delta E_{bs} / \delta V$$

$$E_{bs} = \int_0^{6r} n(r) \epsilon dr$$

δ = variation with a frozen potential in the atomic sphere approximation (ASA) (neglecting the interstitial region)

$$\Omega_0 = \frac{4\pi}{3} s^3$$

$$3pV = - \frac{\delta E_{bs}}{\delta \log s} = \sum_i 3p_i V \quad (\text{spherical atoms as in the ASA})$$

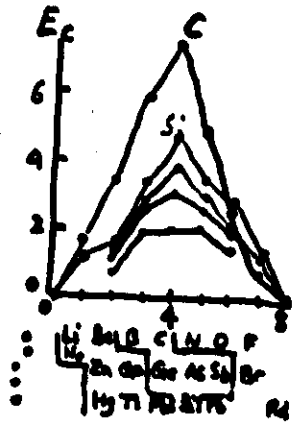
$$3p_i V = \int_0^{6r} n_i(r) \epsilon \phi_i^2(r, s) \{ (D_i(r) + \epsilon) (D_i(r) - \epsilon) + (\epsilon - \epsilon_{xc}) s^2 \}$$

$D_i(r) = \frac{s \phi_i'(r, s)}{\phi_i(r, s)}$ (At Hara, Moriyasu, Hodges)

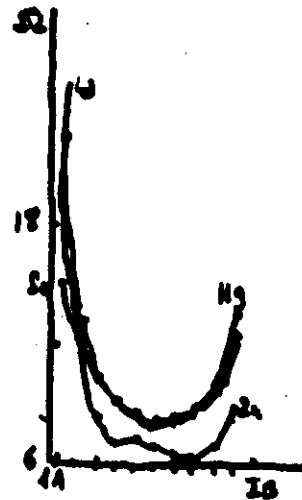
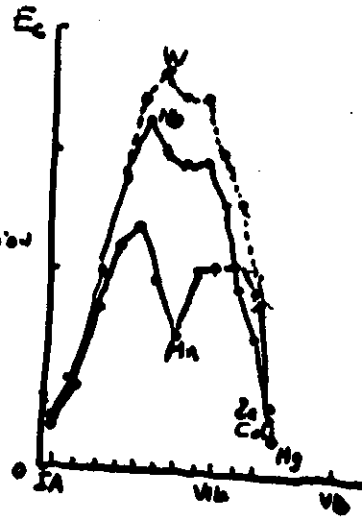
⇒ EQUATION OF STATE $p(V) = p(V)$ easy to calculate for

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COHESIVE ENERGIES (exp)
AND ATOMIC VOL

"sp"
ELEMENTS



TRANSITION
METALS



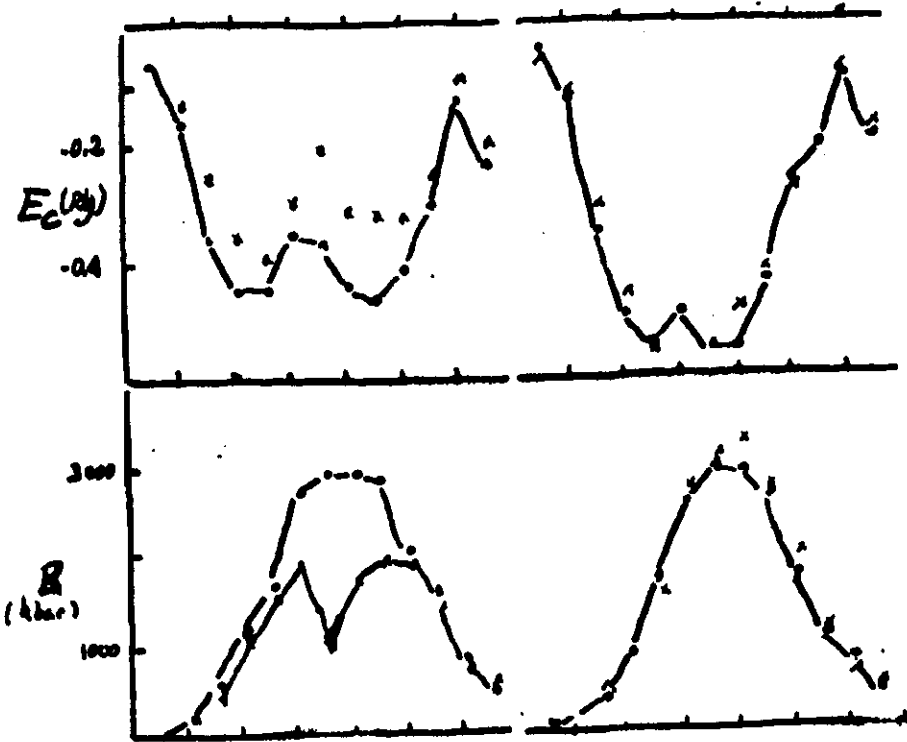
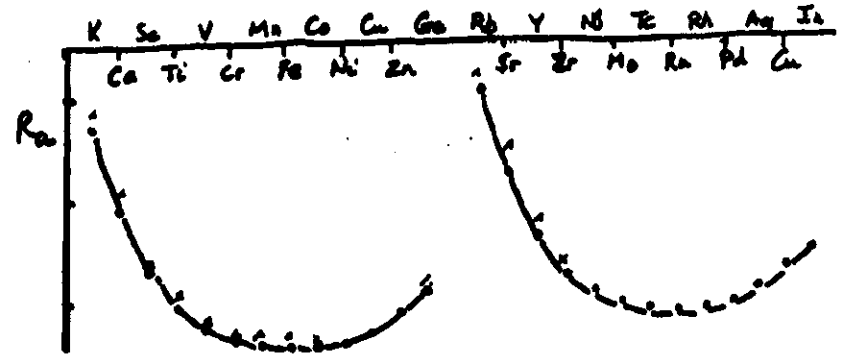
NOTE THAT : - THE COHESIVE ENERGY IS MAX WHEN
THE 'sp' OR d BANDS ARE HALF FILLED
- THE ATOMIC VOLUME IS MINIMUM WHEN
THESE BANDS ARE HALF FILLED (apart
from zero energy state)

$$E_c = E_{at} - E$$

Ω is the atomic volume

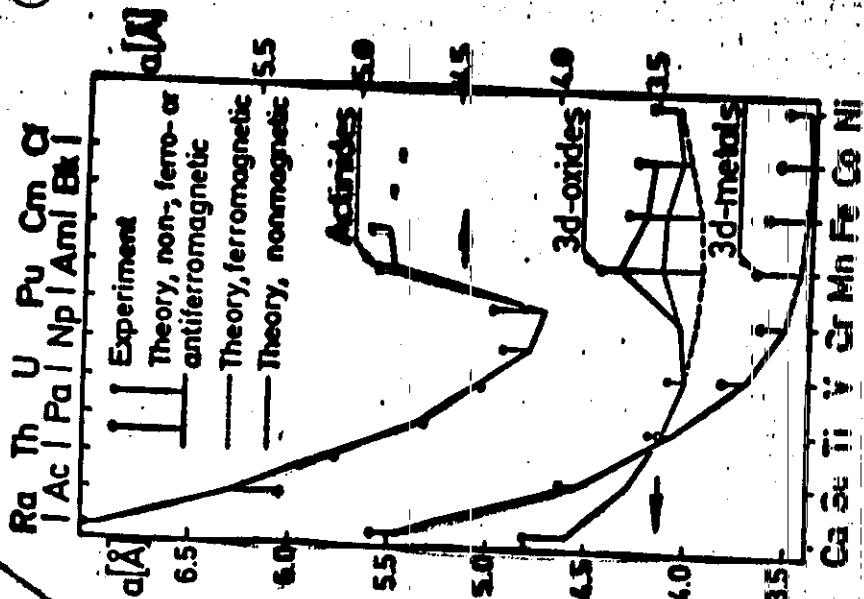
-6- CALCULATION
OF THE COHESIVE ENERGY, VIGNER SEITZ RADIUS AND
BULK MODULUS FOR THE 1st AND 2nd SERIES
NOTE THE DEVIATIONS DUE TO "STRONG" CORRELATIONS
AND MAGNETISM IN THE 7th SERIES

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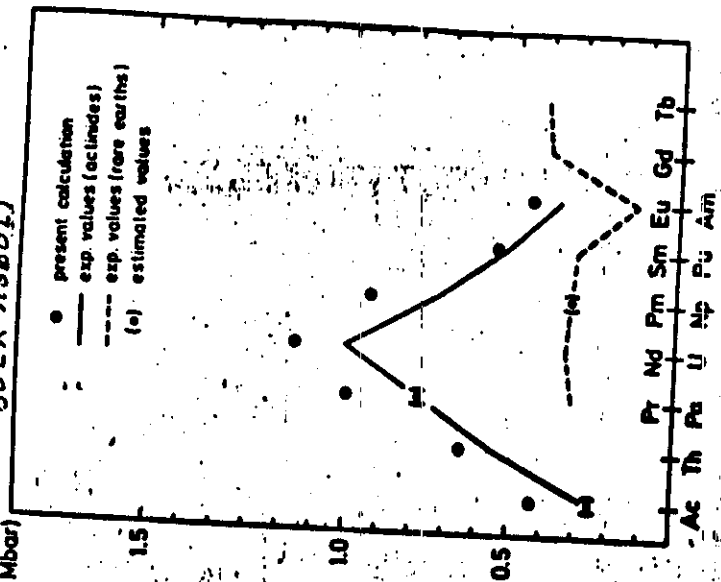
1st series
Cohesive energy, Vigner-Seitz radius R_v and bulk compressibility
of transition metals of 1st and 2nd series (calc. & exp)

(from OK Anderson)
(1985)



BULK MODULI

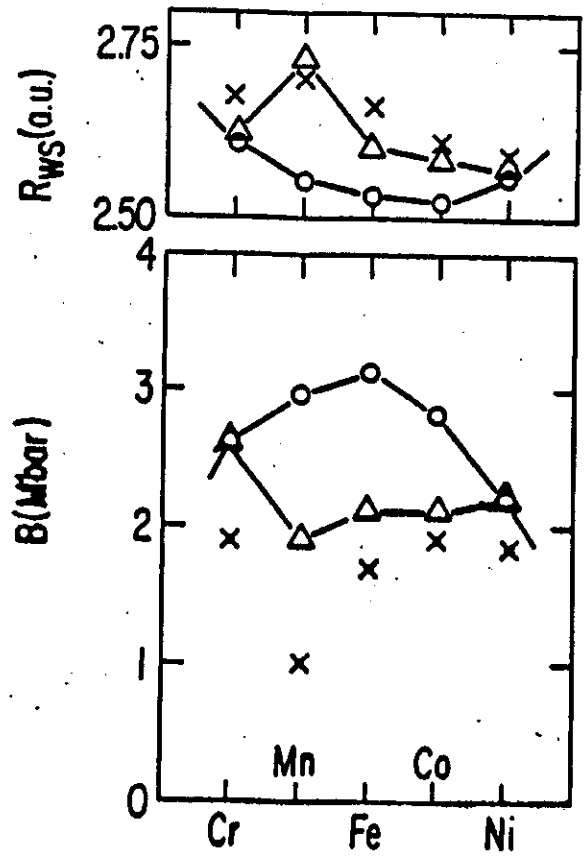
B (Mbar)



NOTE THE EFFECT OF MAGNETISM (↑ INCREASE a)

... LOCALIZATION, BUT, FOR LIGHTER & HEAVY "AN ACTINIDES?"

THE EFFECT OF MAGNETISM ON ATOMIC VOLUMES



(from Jeynes et al)

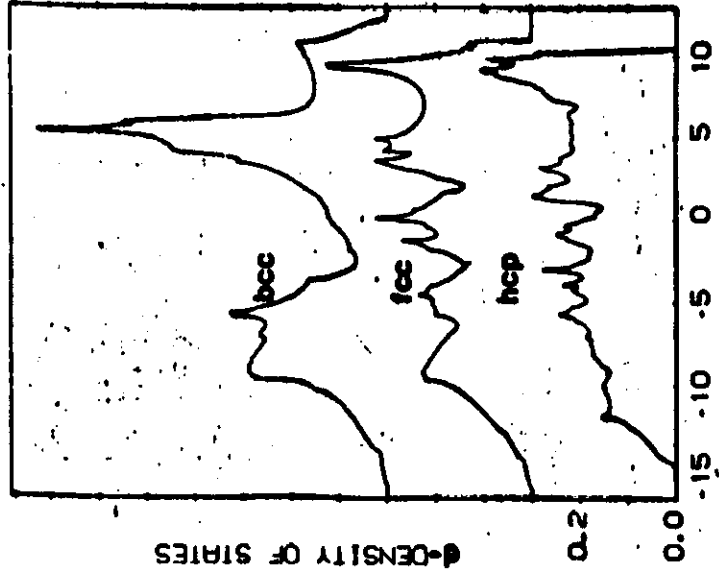
$\times$ exp
 $\circ$ magn calc
 $\circ$ paramag "

$$\Delta E_m = E(m) - E(n) = \Delta E_{el} - \frac{\hbar^2}{2} \frac{V}{m^2}$$

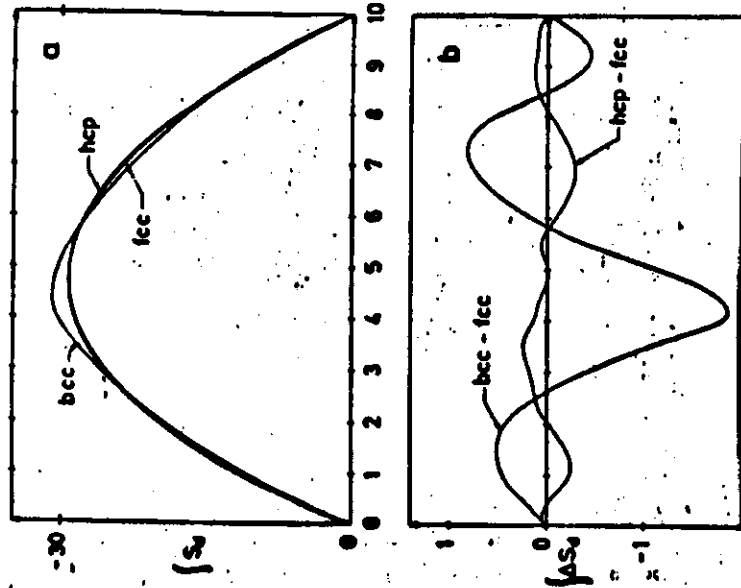
$$-\frac{\partial \Delta E_{magn}}{\partial \Omega} \Rightarrow P_m = -\frac{\partial \Delta E_{el}}{\partial \Omega}$$

$$P_m = \frac{\partial \Delta E_{el}(m)}{\partial \Omega} > 0$$

μ D.O.S.



STRUCTURAL STABILITY (2d)



STRUCTURAL STABILITY FROM CANONICAL μ BANDS OF STATES FOR TRY

STRUCTURAL STABILITY FROM CANONICAL μ BANDS

NOTE THAT THE bcc IS THE MOST STABLE STATE (FOR $N=9, 10$) IN CON-
TRAST WITH THE hcp

from O.K. Anderson

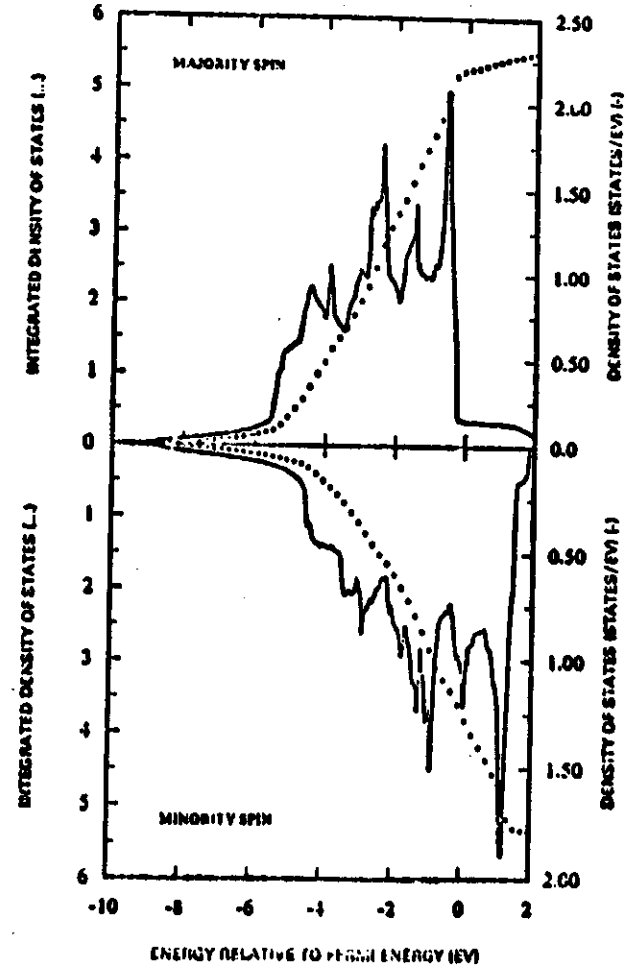
D.O.S. OF FERROMAGNETIC COBALT

THE μ BANDS ARE (ROUGHLY) RIGIDLY SHIFTED:

$$\begin{aligned} \Delta E(m) &= \epsilon_{\uparrow} - \epsilon_{\downarrow} \approx m \quad m \approx 0.5-0.7 \text{ eV} \\ m &= n_{\uparrow} - n_{\downarrow} \end{aligned}$$

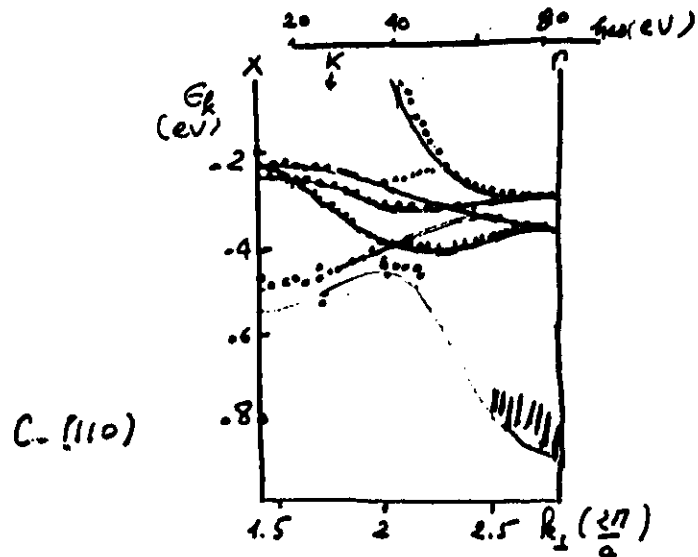
EFFECTS OF SPIN POLARIZATION

Cobalt



Cobalt is a strong ferromagnet
i.e. $n_{\uparrow} = 5$ (filled majority band)
 $m = n_{\uparrow} - n_{\downarrow} = 2.7$

EXCITATIONS AND ONE EL STATES



THE ONE ELECTRON STATES E_k AS CALCULATED BY "BURDICK ARE IN AGREEMENT WITH THE EXCITATIONS OBTAINED EXPERIMENTALLY BY ANGELAR RESOLVED PHOTOEMISSION (ARPES) (from Treg et al 1973)

III.50.

IN NICKEL THE EXCITATIONS OBTAINED BY ARPES ARE STILL RELATIVELY WELL DEFINED BUT THEIR SPECTRUM (i.e. E_k) IS NARROWER THAN THE L.D.A ONE ELECTRON SPECTRUM. CORRELATIONS INTRODUCE QUANTITATIVE BUT NO QUALITATIVE CHANGES IN THE SPECTRUM ON ONE EL. EXC.

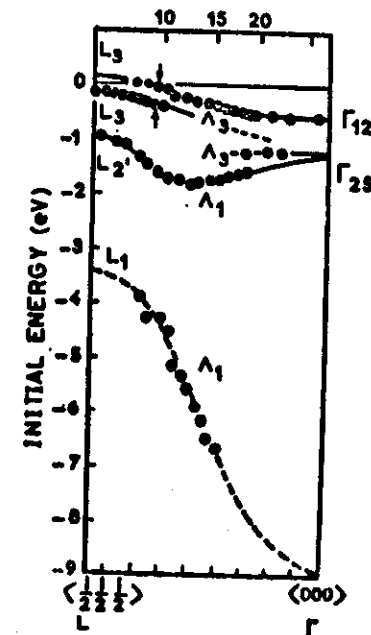
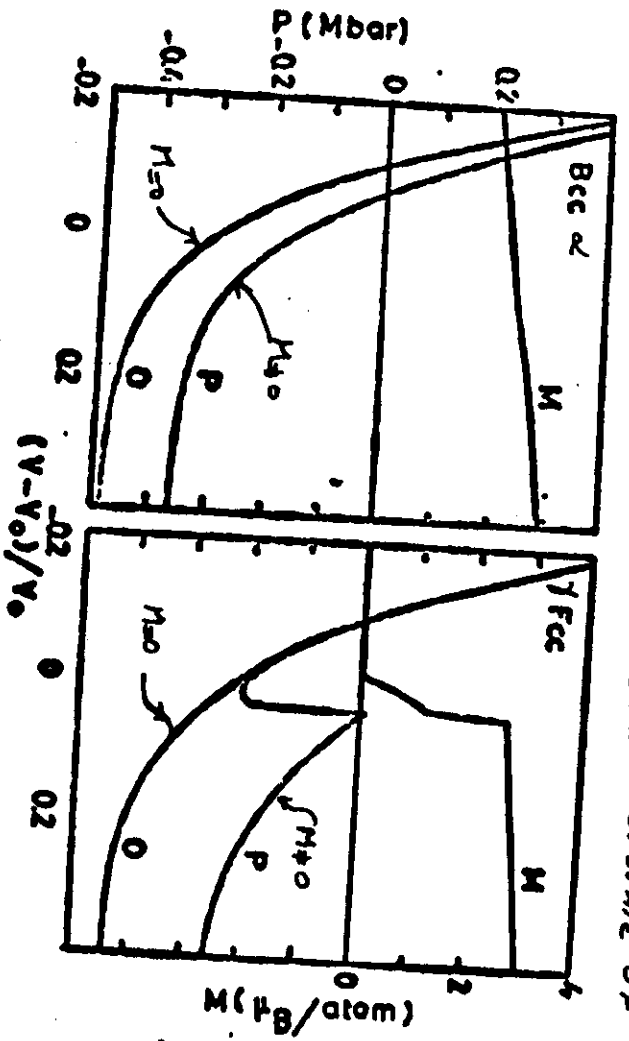


Fig. 2c : Angular photoemission of ferromagnetic nickel (from [3]). Two 'd' states observed for k near L (noted $\uparrow$) are identified to spin splitted states ($A_{1\uparrow}$, $A_{1\downarrow}$). The band width and the spin splitting are much smaller than the corresponding quantities deduced theoretically by band calculations. (See below)

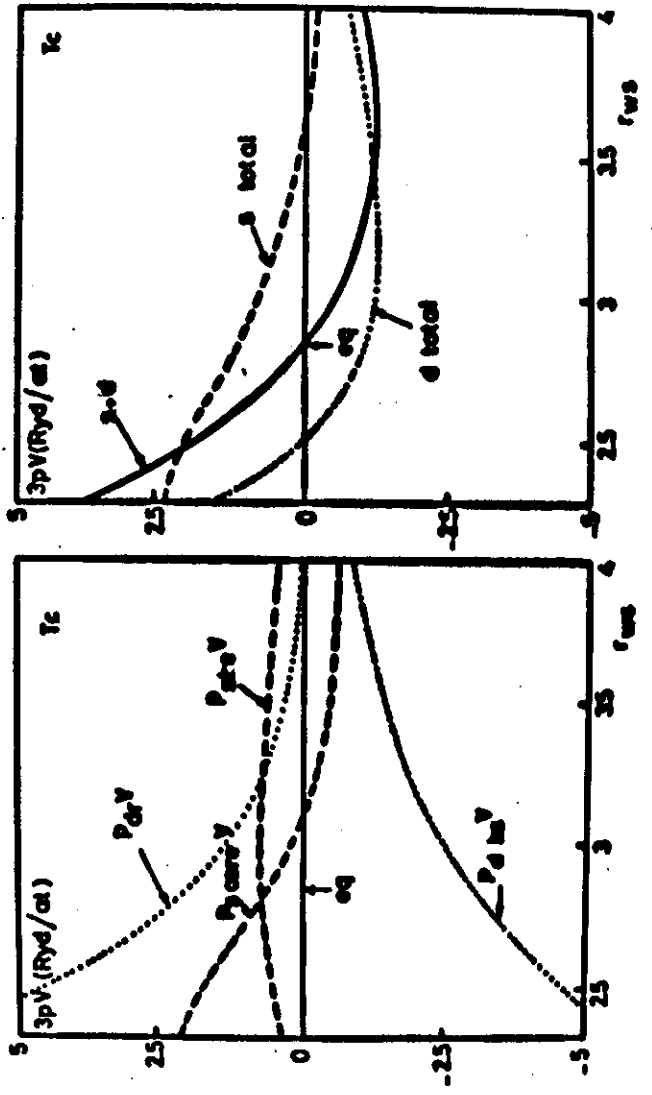
ROLE OF THE MAGNETISM (Fe₂SiO₄)
ON THE EQUATION OF STATE OF Fe



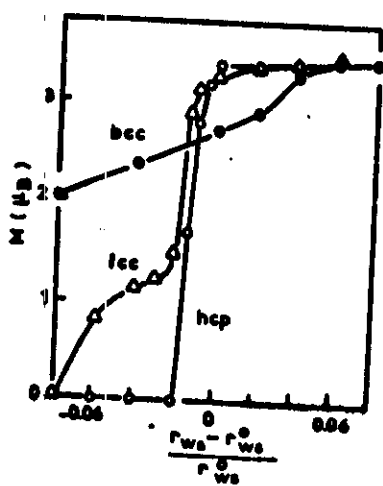
Use R_{ad} in
 M_{ad} calc. to
 establish M at
 $P=0$ (at $M=0$)
 $\beta = \alpha(1/M) \cdot M=0$
 and $M=2p_0 / \alpha t(Q)$
 $E(2) - E(4) = 2M_{ad}$

The solution "0" is obtained under the constraint $M=0$, is imposed

negative (unphysical) M for $P=0$ (unphysical) P
 is not a solution of the equation of state
 "0" is the solution of the equation of state
 for $M=0$, is imposed



THE CHANGE OF MANN MOMENTS WITH VOLUME (as calculated by the LDA) OR P.



4 APPROXIMATIONS FOR THE CALCULATION OF THE ONE ELECTRON STATES

① LDA $\left\{ \begin{aligned} \hat{h} &= -\frac{\hbar^2}{2m} \nabla^2 + V \\ V &= V_{ext} + V_{coul} + V_{xc} \end{aligned} \right.$

$\hat{h} \psi_a = E_a \psi_a$, $n(r) = \sum_a |\psi_a|^2$ and $V_{coul}(n)$, $V_{xc}(n)$

② THE DISTRIBUTION OF ATOMS ON A GIVEN LATTICE IS KNOWN (known). USUALLY SPECIAL ASSUMPTIONS ARE DONE FOR THE SELF CONSISTENT CALCULATION OF E_a ACCORDING TO THE CONSIDERED LATTICE AND AT. (use of the variat. meth)

WAVE FUNCTION L.C.A.O. CHOICE

$\psi_a(r) = \sum_{\lambda} A_{\lambda i} \phi_{\lambda}^i(r)$

$\phi_{\lambda}^i(r) = \phi^i(r - r_{\lambda})$

λ = SITE, i = index for the orbitals of the considered set $\{i\}$.

$\{i\}$ {SLATER FUNCTIONS, GAUSSIAN}

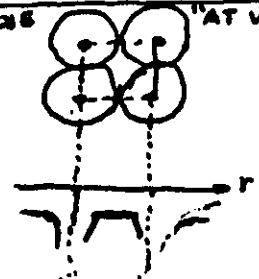
AS LOCALIZED AS POSS

2: they are not stable!

THE ENERGY LEVELS ARE GIVEN BY:

$\det | \hat{h}_{\lambda\lambda'} - E S_{\lambda\lambda'} | = 0$
A SIMPL. SEC. EQ.

POTENTIAL MUFFIN TIN APPROX. CHOICE "AT VOL"



V_0 / Spherical / for G at opt approx. if r is in the Int. reg.

VALID FOR COMPACT STRUCTURES
OPEN STRUCTURES?
SURFACES?
MOLECULES?

LOCALIZED ORBITALS CAN BE OBTAINED FROM LINEARIZED ATO.

EX: KKR APW for perfect periodic systems.

IF WE ARE INTERESTED IN A LIMITED RANGE OF BN LINEARIZED METHODS ARE USED A.S.A.

L. ATO
L. APW

CONSERVING PSEUDOPOTENTIALS

INTERCALATED GRAPHITE COMPOUNDS
- ANISOTROPY
- BONDING

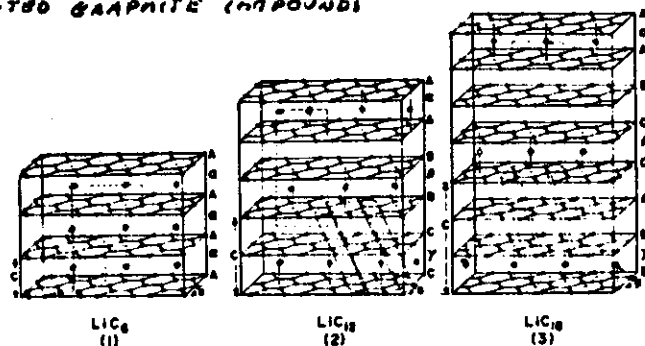
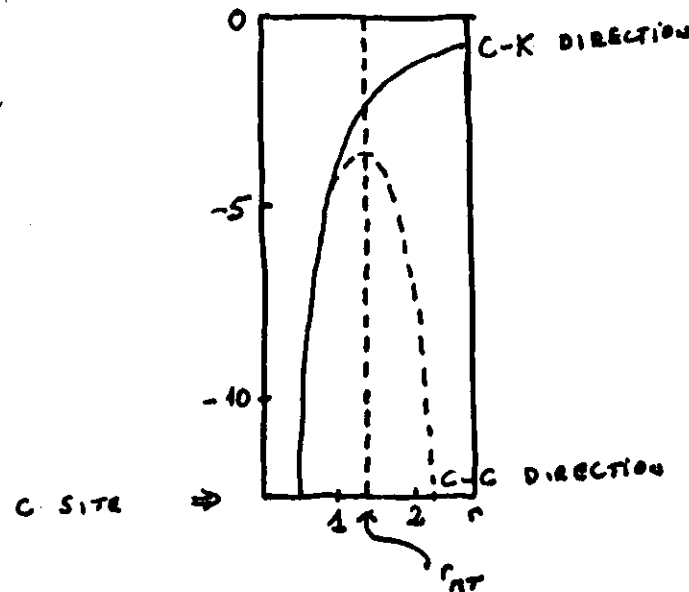
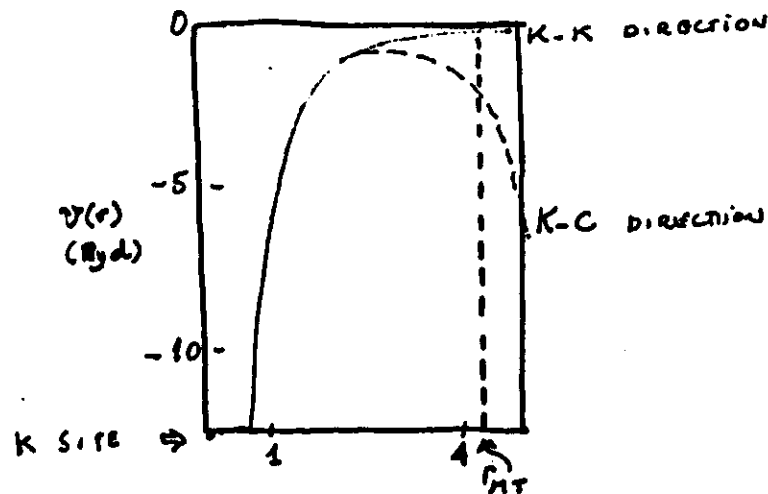


FIG. 1. Structures of Li-intercalated graphite compounds LiC_8 (stage 1), LiC_{12} (stage 2), and LiC_{16} (stage 3) assumed in present study. Carbon atoms are denoted by filled circles connected along nearest-neighbor bonds. Li atoms are denoted by unfilled circles. Primitive unit cells are indicated for each structure. Dashed rectangle in each structure denotes the plane used in contour plots shown in Figs. 2-7.

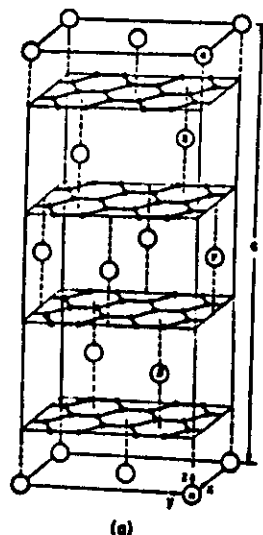
TABLE I. Lattice parameters for Li-intercalated graphite compounds.

LiC_8 (structure based on Ref. 9)	LiC_{12} and LiC_{16} (assumed structures continued)
Primitive lattice vectors: $\vec{T}_1 = a\sqrt{3}[(\sqrt{3}/2)\hat{x} + \frac{1}{2}\hat{y}]$, $\vec{T}_2 = a\sqrt{3}[-(\sqrt{3}/2)\hat{x} + \frac{1}{2}\hat{y}]$, $\vec{T}_3 = c\hat{z}$, $a = 2.485 \text{ \AA}$, $c_1 = 3.706 \text{ \AA}$	Atomic locations within unit cells: C atoms: $\pm \left[\frac{1-2p_n}{3} \left((\vec{T}_1 + \vec{T}_2) + p_n \vec{T}_3 \right) \right]$ $\pm \left[\frac{1-2p_n}{3} \left(\vec{T}_1 - \frac{2p_n}{3} \vec{T}_2 + p_n \vec{T}_3 \right) \right]$ $\pm \left[\frac{-2p_n}{3} \vec{T}_1 - \left[\frac{1+2p_n}{3} \right] \vec{T}_2 + p_n \vec{T}_3 \right]$ $\pm \left[-\left[\frac{1+2p_n}{3} \right] (\vec{T}_1 + \vec{T}_2) + p_n \vec{T}_3 \right]$ $\pm \left[-\left[\frac{1+2p_n}{3} \right] \vec{T}_1 - \frac{2p_n}{3} \vec{T}_2 + p_n \vec{T}_3 \right]$ $\pm \left[\frac{-2p_n}{3} \vec{T}_1 + \left[\frac{1-2p_n}{3} \right] \vec{T}_2 + p_n \vec{T}_3 \right]$ $p_n = \frac{c_1}{2c_n}$ where $n=2$ for LiC_{12} and 3 for LiC_{16}
Atomic locations within unit cell: C atoms: $\pm \left\{ \frac{1}{2}(\vec{T}_1 + \vec{T}_2) + \frac{1}{2}\vec{T}_3 \right\}$ $\pm \left\{ \frac{1}{2}\vec{T}_1 + \frac{1}{2}\vec{T}_2 \right\}$ $\pm \left\{ \frac{1}{2}\vec{T}_1 + \frac{1}{2}\vec{T}_3 \right\}$ Li atom: 0	Additional C atoms in LiC_{16} : $\pm \left\{ -\frac{1}{2}(\vec{T}_1 + \vec{T}_2) + \frac{1}{2}\vec{T}_3 \right\}$ $\pm \left\{ -\frac{1}{2}\vec{T}_1 + \frac{1}{2}\vec{T}_2 \right\}$ $\pm \left\{ \frac{1}{2}\vec{T}_1 + \frac{1}{2}\vec{T}_3 \right\}$ Li atom: 0
Symmetry directions: $\Gamma \leq k \leq A$ $\vec{k} = x\vec{Q}_1$ $0 \leq x \leq \frac{1}{2}$ $\Gamma \leq k \leq M$ $\vec{k} = x(\vec{Q}_1 - \vec{Q}_2)$ $0 \leq x \leq \frac{1}{2}$ $\Gamma \leq k \leq K$ $\vec{k} = x(\vec{Q}_1 + \vec{Q}_2)$ $0 \leq x \leq \frac{1}{2}$	Symmetry directions: $\Gamma \leq k \leq A$ $\vec{k} = x\vec{Q}_1$ $0 \leq x \leq \frac{1}{2}$ $\Gamma \leq k \leq M$ $\vec{k} = x(\vec{Q}_1 - \vec{Q}_2)$ $0 \leq x \leq \frac{1}{2}$ $\Gamma \leq k \leq K$ $\vec{k} = x(\vec{Q}_1 + \vec{Q}_2)$ $0 \leq x \leq \frac{1}{2}$
LiC_{12} and LiC_{16} (assumed structures) Primitive lattice vectors: $\vec{T}_1 = a\sqrt{3}[(\sqrt{3}/2)\hat{x} + \frac{1}{2}\hat{y}]$, $\vec{T}_2 = a\sqrt{3}[-(\sqrt{3}/2)\hat{x} + \frac{1}{2}\hat{y}]$, $\vec{T}_3 = (2a/\sqrt{3})\hat{y} + c_n\hat{z}$, $a = 2.485 \text{ \AA}$, $c_1 = 7.056 \text{ \AA}$ (LiC_{12}), $c_1 = 10.406 \text{ \AA}$ (LiC_{16})	

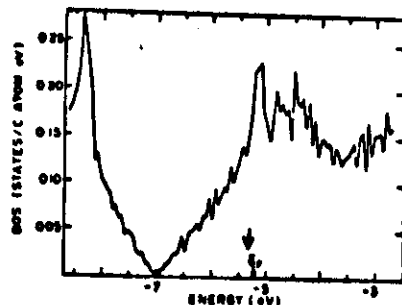
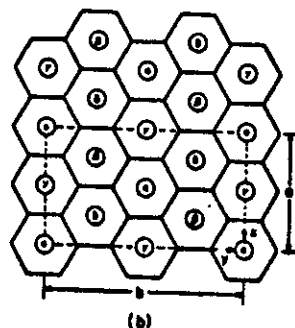
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ANISOTROPIC COMPOUNDS
AND MT APPROXIMATIONS



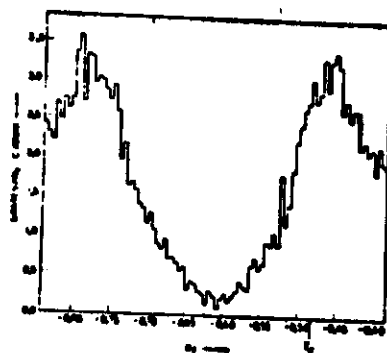
DONOR TO ACCEPTOR
RIGID BAND MODEL



• C ATOM
○ Li ATOM
PRIMITIVE TRANSLATIONS
 $T_1 = \frac{1}{2}a_1 + \frac{1}{2}a_2$
 $T_2 = \frac{1}{2}a_1 + \frac{1}{2}a_2$
 $T_3 = \frac{1}{2}a_1 + \frac{1}{2}a_2$



KC₈
de Vries et al



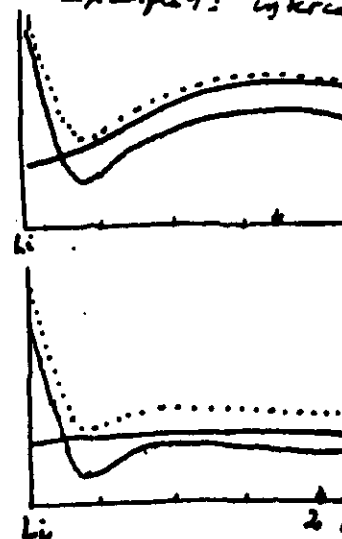
LiC₆
Holworth et al (1988)

AS FAR AS D.O.S ARE CONCERNED Li, K ACT AS
A DONOR i.e. THE Fermi LEVEL WHICH WAS IN THE
PSEUDO GAP OF GRAPHITE GOES UP TO ACCOMMODATE
THE SUPPLEMENTARY ELECTRONS OF Li.
APPLY THE RIGID BAND MODEL IS VALID!

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DEFINITION OF SINGLE TRANSFER IN A COMPOUND?

1. For the total charge of each unit cell... and its conductivity
Example 1: intercalate compounds LiC₆



— superimposed at
densities from ionic
Li⁺ 2s⁰ C 2s¹ 2p_x² 2p_y² 2p_z¹
LiC₆ — band str

--- neutral
Li 2s¹ C 2s¹ 2p_x² 2p_y² 2p_z¹
[IN THIS CASE THE Li WAVE FUNCTION
OVERLAPS STRONGLY WITH C SO THAT
THE NOTION OF CEE HAS NO MORE
CEE OR MEANING.]
center of C hexagon

Example 2 PAH c.f. later (chapter III)

Example 3 C₆₀, C₇₀, C₈₄...

Example 4 CT in T.M. compounds

2. IN THE ATOMIC CELL AND IN T.M. THE CHARGE TRANSFER
IS IN GENERAL SMALL EXCEPT IN SOME SPECIAL CASES
WITH STRONG ELECTRONEGATIVITY & WHERE IT CAN
ATTAIN 0.4, 0.5

Ex: YPd₃, vacancy in FeAl □ 0.5

3. EXAMPLE OF THE DISTRIBUTION OF CHARGES IN C₆₀Pd

	non self-consistent calc	self-consistent
Q(C ₆₀)	14.36 (MT 11.71)	14.05 (10.4)
Q(Pd)	9.93 (MT 18.17)	9.84 (5.1)
Q(average)	0.65	0.75

THE SELF-CONSISTENCY REDUCES THE CHARGE TRANSFER
COEFFICIENT QUASI NEUTRAL.

FOR PERFECT METALS, THE POTENTIAL ~~WELL~~)
IS PERIODIC AND THE ELECTRONIC STATES DEFINE
ENERGY BANDS. FOR THE STUDY OF ALLOYS, TWO
QUESTIONS APPEAR:

- 1- WHAT ARE THE NEW FEATURES INTRODUCED BY THE DISORDER IN AN ALLOY? ARE BANDS STILL NEAR NUCLEI?
- 2- ARE SUCH FEATURES DIFFERENT GOING FROM ONE HOST TO ANOTHER?
- CAN THE ENERGY BE CALCULATED IN AN UNIFIED WAY?

1 → FOR THE FIRST QUESTION, IT'S CONVENIENT TO
STUDY SUCCESSIVELY TWO LIMITS:

SEE
PR MARCH
LECTURES

STUDY SUCCESSFULLY TWO STATIONS
 ⇒ LOW CONCENTRATION LIMIT (OUT OF THE SCOPE OF THE
 DILUTE ALLOY STUDY OF THE SPATIAL PERT
 DILUTE ALLOYS: " " " ENERGY SPECTRA

* CONCENTRATED ALLOYS:
WILL SHOW THE GENERAL FEATURES ASSOCIATED WITH DISORDERED OR DISORGANIZED OF ENERGIES, EN

2 → FOR THE EXPLICIT CALCULATION OF ENERGIES, ON-
TROPIC... IT'S (UP IN ROW!) NECESSARY TO USE
APPROXIMATIONS WHICH ARE DIFFERENT FOR
EACH CLASS OF METALS, BE DEFINING ACCORDING
TO THEIR ELECTRONIC STRUCTURE... AND TO THEIR
PHYSICAL PROPERTIES. THESE APPROXIMATIONS WILL BE
USEFUL TO UNDERSTAND PHYSICALLY THE RESULTS WE OBTAIN
BY MORE PRECISE HARTREE-FOCK AND STRUCTURE CALC.

CLASSIFICATION OF METALS

• NORMAL METALS

those which are bonded by 'sp' electrons with 'd' or 'f' shells playing no significant role.

conduction electrons are nearly free.

{E practically given by F.E model perturbed only near the Brillouin zone.

$\psi_k(r)$ practically constant in most of the volume but oscillates in the core region to be orthogonalized to the inner states.

• TRANSITION METALS

ATOMS ARE CHARACTERIZED BY

fn states localized ($r \sim 0.5 \text{ \AA}$)

(n+1)s " EXTENDED AS VAL FUNCTION I OF NORM. METALS

MINERALS ARE CHARACTERIZED BY

NARROW Δ^2 BAND WIDTH (tight binding like)

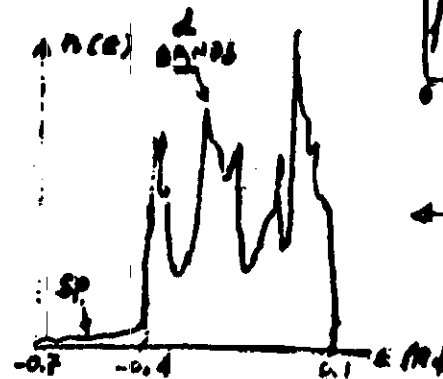
_____ bands

Li Be C
Na Mg Al Si P

Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br
Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sb	Te	I	
La	Hf	Ta	W	Re	Os	Pt	Au	Hg	Tl	Pb	Bi	Po		

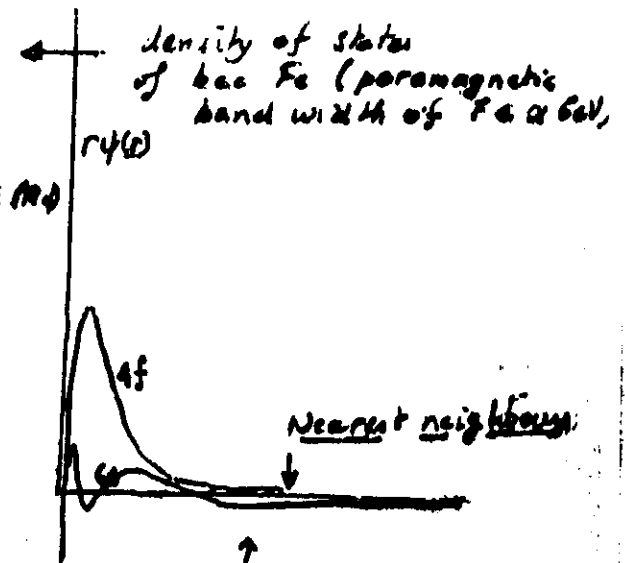
Ac Th Pa U Np Pu Am Cm

atomic function
of V



Large 3.s overlaps
 => wide 'sp' bands
 relatively small d. on
 => narrow 'd' ba

← density of states
of bare Fe (paramagnetic
band width of Fe $\approx 6\text{eV}$),



atomic states of Ga
small overlap between neighbouring
4f shells $\Rightarrow$ LOCALISED STATES

• RARE EARTH -24-

ATOMIC STATE CHARACTERIZED BY

4f electrons localized as compared with 5d.6s

METALS CHARACTERIZED BY

{f, localized states
and bonds similar to transition elements (5d.6s)

• ACTINIDES

INTERMEDIATE SITUATION BETWEEN TRANSITION METALS AND R.E.
ALONG THE ACTINIDE SERIES THE 5f SHELLS BECOME MORE CONTRACTED WHEN THE ACTINIDE IS HEAVIER

- AT THE BEGINNING OF THE SERIES, 5f HAVE EXTENSIONS (relative to the nearest neighbour distance b) COMPARABLE TO d FUNCTIONS OF TRANSITION METALS (5d.6s) HYBRIDIZED BANDS
- IN THE SECOND PART (of Cu) THEY ARE LOCALIZED AS 4f STATES OF RARE EARTHS

TWO EXTREME MODELS FOR THE BAND STRUCT

FREE ELECTRON

↓
NEARLY FREE ELECTRON 1st order perturbation theory
valid for normal metals
(small perturbation $V(r)$)

↓
$$v(k) = \frac{1}{\Omega} \int_{\Omega} V(r) e^{-i k \cdot r} d^3r$$

Ω atomic volume, & reciprocal lattice vectors
provided we replace potential V by pseudopotential (Hafner)
(cf. Hafner)

↑
TIGHT BINDING APPROX
(LCAO)

• the properties of transition metals are roughly determined by d electrons which are well represented by LCAO (qualitatively)

↑
ATOMIC LIMIT

r_0 = atomic radius for valence electrons
h = distance between two nearest neighbours

$r_0 > b$

$r_0 < b$

$r_0 \ll b$